CEE 361

Tehila Stone | Theo Dimitrasopoulos

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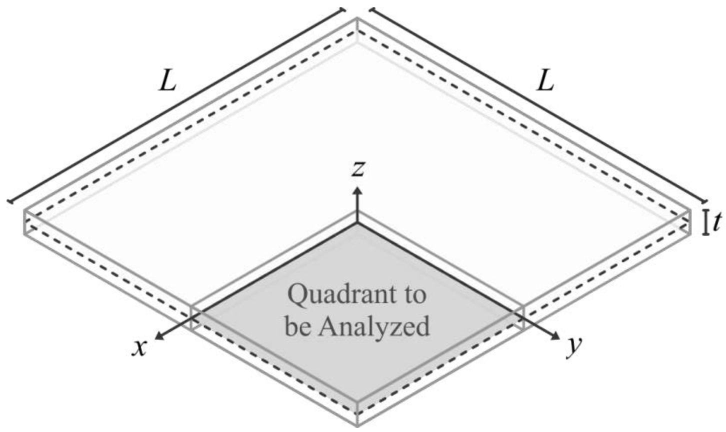
You are required to implement the plate quad element into your MATLAB program and analyze one quadrant of the plate shown below. L = 10 m, t = 100 mm, E = 30 x 10^6 MPa, and υ = 0.3.



Your analysis should include permutations of all of the following conditions:

i) **Supports** (in both cases, all perimeter displacements are restrained)

a. Edges are fully clamped (all perimeter rotations are restrained)

b. Edges are simply supported (all perimeter rotations are released)

ii) **Loading**

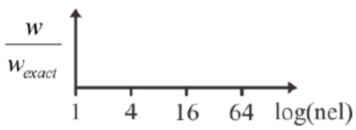
a. Distributed load of q = 10 Pa downward applied to the entire plate

b. Concentrated load of P = 1 kN downward applied to the plate center

iii) Mesh Refinement: quadrant is modelled with 1, 4, 16, and 64 elements

iv) Gaussian Quadrature: shear stiffness found using 1x1 and 2x2 integration

As part of your analysis, you need to include graphs and tabulations of the normalized center deflection of the plate against the logarithm of the number of elements (nel) used. The exact solutions for the center deflection are provided below:



Exact deflection at plate center, w_{exact}, for $\upsilon = 0.3$			
Simply Supported		**Clamped**	
$0.0444\frac{qL^4}{Et^3}$	$0.1267\frac{PL^2}{Et^3}$	$0.0138\frac{qL^4}{Et^3}$	$0.0611\frac{PL^2}{Et^3}$
Distributed	Concentrated	Distributed	Concentrated

Your deliverable is a report which should include, but does not need to be limited to: the above convergence studies, code changes, code description, deformed geometries, displacement tabulations, stress or stress resultant analyses (in plot and/or tabulated form), and discussions. While an exhaustive set of results is not expected (irrelevant/redundant content will likely cost you points), you should include a sufficient cross-section of results to demonstrate the element performance.

One report and one set of code are expected per group. Discussion or sharing of work between groups is not permitted. Groups are to contain no more than two students unless permission is received from the instructors. A hard copy of the report is to be submitted in E218 and a single .zip file containing all MATLAB scripts is to be sent to fabieng@princeton.edu by the deadline.

0. Explanations on notation

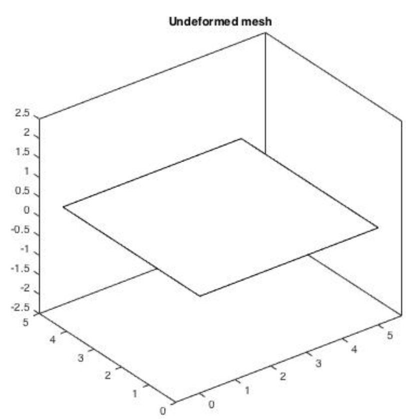
Chapter 0 is to serve as an acquaintance with the notation and methods used to present and visualize the data acquired from our permutations:

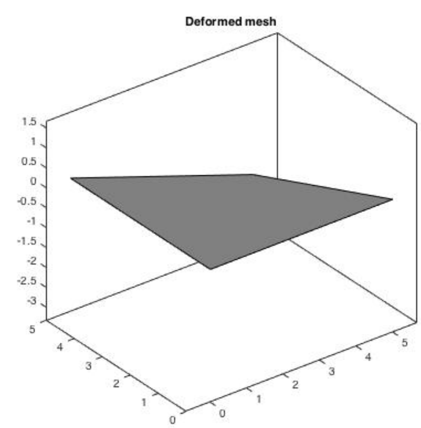
- Regarding number notation, all powers of ten were presented as e##, which is equivalent to $10^{\#\#}$
- All permutations used 2x2 quadrature and 1x1 quadrature is explained in chapter V, describing some of the strange behaviors seen in the graphics in chapters I-IV and what caused them.
- All quantities presented in the report use m, kN, rads and any combination of the three as their units, including graphics.
- The element is modeled as a two-dimensional plate element, lacking a third dimension in the analysis.
- Moments and Shears are always taken at the plate center, or point 9 in this case (a diagram of our numbering is provided in the appendix)

I. Clamped Condition for distributed load of 10 Pa downward applied to the entire plate.

The method followed implemented a set of loops that applied a load that was derived by multiplying the entire distributed load with the area under analysis (25 m^2) and divided by the number of total points on the mesh. The points along the bottom and right side of the element were clamped, meaning all displacements and rotations were constrained.

For a one-element division the following were obtained:





Nodal Displacements:

node Displacement(x,y,θ; m,m,rad respectively)

3 -2.3188e-05 -1.473434e-05 -1.46669e-05

Moment:

0

0

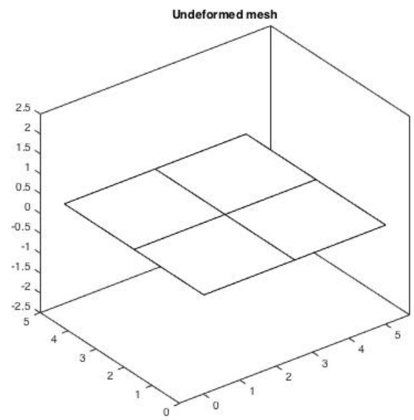
0

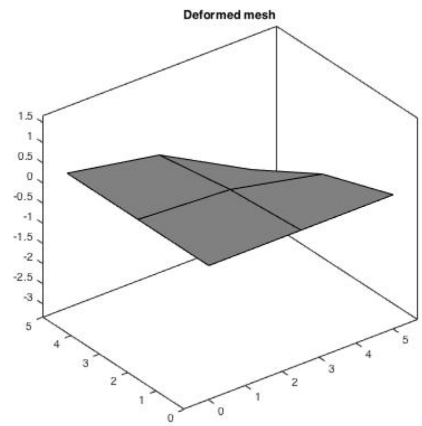
Shear:

0

0

For a four-element division the following were obtained:





Nodal Displacements:

node	Displacement(x,y,θ; m,m,rad respectively)		
3	-4.39344e-5	-1.365293e-05	-6.74751e-06
6	-1.66587e-05	-5.978767e-06	-1.05225e-05
7	-1.12138e-05	-7.095827e-06	-2.53943e-06
9	-3.60221e-06	-2.303440e-06	-2.29872e-06

Moment:

0

0

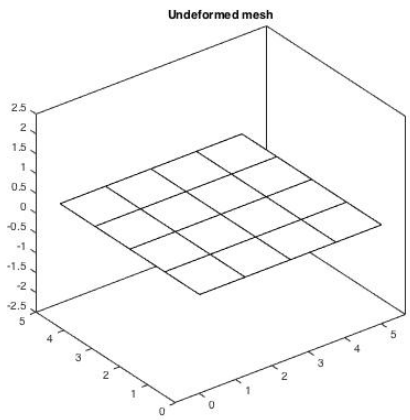
0

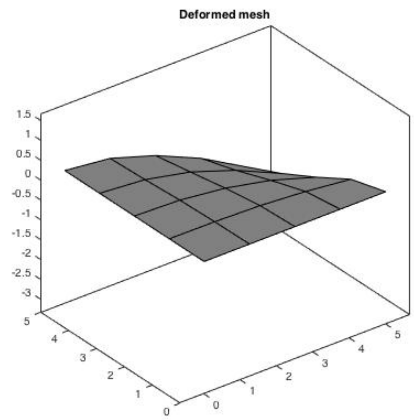
Shear:

0

0

For a sixteen-element division the following were obtained:





Nodal Displacements:

node	Displacement(x,y,θ; m,m,rad respectively)		
3	-8.58949e-05	-1.253481e-05	-6.36802e-06
6	-4.01585e-05	-7.245269e-06	-8.5763e-06
7	-2.58045e-05	-7.679855e-06	-2.74592e-06
9	-9.15406e-06	-3.284892e-06	-2.93058e-06
12	-1.33077e-05	-2.225503e-06	-7.52575e-06
13	-6.47667e-05	-8.905038e-06	-7.00889e-06
14	-5.20325e-05	-8.936392e-06	-3.69497e-06
15	-6.84932e-06	-4.341658e-06	4.23232e-08
17	0	0	9.0071e-07
18	-5.54785e-07	-3.564168e-07	-1.24976e-06
19	-2.25484e-06	-7.227601e-07	-5.33293e-07

20	-6.59012e-06	-2.035748e-06	-5.07637e-06
21	-2.15400e-05	-4.558579e-06	-5.31466e-06
22	-3.80671e-05	-8.021299e-06	-5.17376e-06
23	-1.76219e-05	-4.953394e-06	-2.43078e-06
24	-4.90075e-06	-3.109405e-06	-1.28336e-06
25	-1.9897e-06	-1.260726e-06	-5.66066e-07

Moment:

0

0

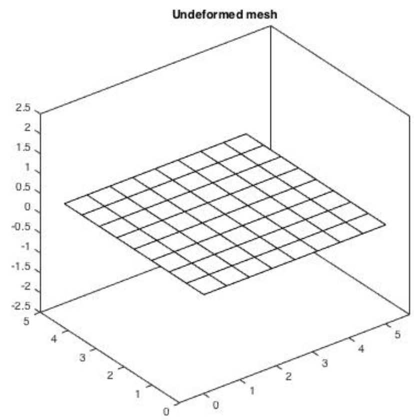
0

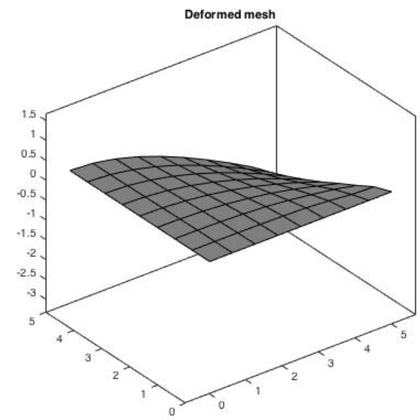
Shear:

0

0

For a sixty-four-element division the following were obtained:





Nodal Displacements:

node	Displacement(x,y,θ; m,m,rad respectively)		
3	-0.000192445	-1.255895e-05	-4.89994e-06
6	-0.000103917	-8.418367e-06	-7.38556e-06
7	-6.02998e-05	-8.691931e-06	-1.84522e-06
9	-2.63095e-05	-4.150039e-06	-2.72188e-06
12	-4.77662e-05	-5.281564e-06	-6.31832e-06
13	-0.000153426	-1.088017e-05	-6.1216e-06
14	-0.000120902	-1.102095e-05	-3.5689e-06
15	-1.68541e-05	-5.333781e-06	-3.07688e-07
17	0	0	2.2673e-07
18	-1.60505e-06	-6.408612e-07	-3.71936e-07

19	-8.16852e-06	-1.582929e-06	-2.18082e-06
20	-2.3964e-05	-3.014220e-06	-3.63187e-06
21	-5.93335e-05	-6.334050e-06	-4.73863e-06
22	-9.28505e-05	-8.966046e-06	-4.18263e-06
23	-4.48419e-05	-6.577044e-06	-2.44037e-06
24	-1.2458e-05	-3.811614e-06	-9.06989e-07
25	-6.51235e-06	-2.087984e-06	-9.30809e-07
29	0	0	-7.59255e-06
30	-1.87526e-05	7.185802e-08	-1.16347e-05
31	-7.51774e-05	-5.275905e-06	-1.08157e-05
32	-0.000129672	-8.696994e-06	-8.92263e-06
33	-0.000173897	-1.083061e-05	-6.84666e-06
34	-0.000155438	-1.087999e-05	-5.53702e-06
35	-8.87833e-05	-9.342287e-06	-3.50046e-06
36	-3.59565e-05	-6.753899e-06	-1.28112e-06
37	-4.24025e-06	-2.679955e-06	2.67956e-07
41	0	0	2.08081e-07
42	-7.16936e-08	-5.100677e-08	-2.50587e-07
43	-3.21771e-07	-1.027878e-07	-5.79002e-10
44	-1.05206e-06	-3.683305e-07	-8.71439e-07
45	-2.17765e-06	-3.405416e-07	-1.1734e-06
46	-3.99570e-06	-8.230162e-07	-2.74715e-06
47	-8.95192e-06	-2.316764e-06	-5.44506e-06

48	-1.57499e-05	-1.987779e-06	-2.63712e-06
49	-3.40569e-05	-3.392093e-06	-9.40743e-06
50	-5.76436e-05	-5.861142e-06	-5.78544e-06
51	-7.99573e-05	-6.749327e-06	-8.37746e-06
52	-0.000102059	-8.831908e-06	-5.65107e-06
53	-0.000121604	-9.270622e-06	-6.75376e-06
54	-0.000139483	-1.100999e-05	-4.58948e-06
55	-0.000107358	-9.350733e-06	-5.00768e-06
56	-7.85306e-05	-8.923985e-06	-3.00837e-06
57	-5.30375e-05	-7.261370e-06	-2.75162e-06
58	-3.17303e-05	-6.234059e-06	-1.40357e-06
59	-1.51271e-05	-4.314689e-06	-7.83552e-07
60	-4.14034e-06	-2.634008e-06	-3.328e-07
61	-3.24440e-06	-2.046433e-06	-2.36185e-07
62	-2.42950e-06	-1.551962e-06	-2.8413e-07
63	-1.63045e-06	-1.022661e-06	-2.23544e-07
64	-8.90139e-07	-5.754886e-07	-2.51174e-07
65	-3.11436e-07	-1.897077e-07	-3.39932e-07
66	-4.16160e-06	-9.677626e-07	-1.54312e-06
67	-1.49513e-05	-2.702964e-06	-4.64031e-06
68	-4.09488e-05	-4.715068e-06	-6.90108e-06
69	-7.65316e-05	-7.344239e-06	-6.1509e-06
70	-6.69808e-05	-7.436614e-06	-4.31919e-06

71	-2.64968e-05	-5.067118e-06	-1.91695e-06
72	-9.50094e-06	-2.915193e-06	-9.61813e-07
73	-3.78658e-06	-1.248085e-06	-7.88366e-07
74	-8.95764e-06	-2.047395e-06	-1.72079e-06
75	-1.69796e-05	-3.026537e-06	-3.17936e-06
76	-2.76163e-05	-3.734853e-06	-3.61843e-06
77	-4.11163e-05	-5.217995e-06	-4.95485e-06
78	-5.45505e-05	-6.589389e-06	-3.5722e-06
79	-3.57957e-05	-5.321953e-06	-3.2851e-06
80	-2.07230e-05	-4.220387e-06	-1.75149e-06
81	-1.48102e-05	-3.153916e-06	-2.00138e-06

Moment:

0

0

0

Shear:

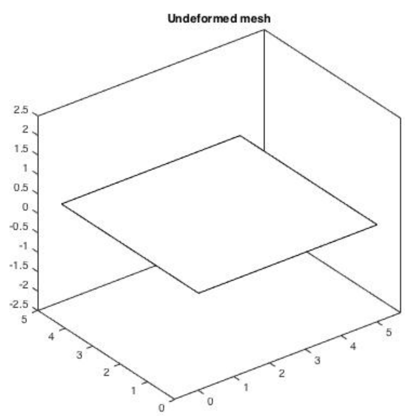
0

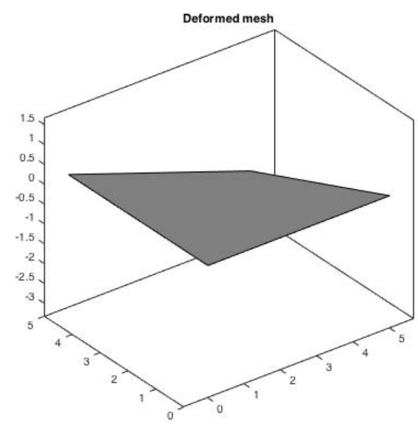
0

II. Clamped Condition for a concentrated load of 1kN downward applied to plate center

The method followed implemented a single applied load of 1kN downward at point 9 of our mesh (plate center or top left corner of the bottom right plate element under consideration). The points along the bottom and right side of the element were clamped, meaning all displacements and rotations were constrained.

For a one-element division the following were obtained:





Nodal Displacements:

node	Displacement(x,y,θ; m,m,rad respectively)		
3	-0.000371008	-2.357494e-04	-0.00023467

Moment:

0

0

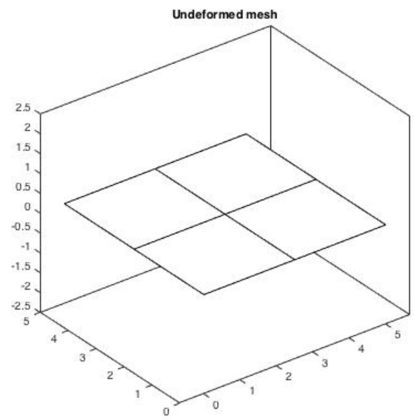
0

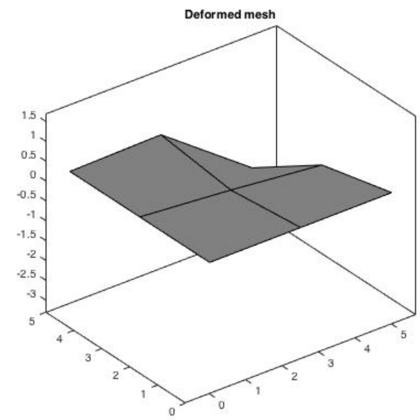
Shear:

0

0

For a four-element division the following were obtained:





Nodal Displacements:

node	Displacement(x,y,θ; m,m,rad respectively)		
3	-0.000658963	-4.419187e-04	-0.000190744
6	-0.000177958	-1.021598e-05	-0.00011305
7	1.754950e-05	1.213625e-05	0.000111129
9	-7.979330e-05	-5.146773e-05	-5.02272e-05

Moment:

0

0

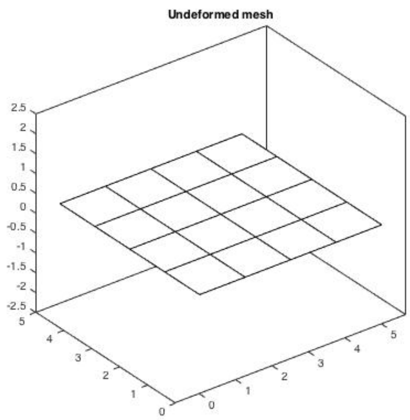
0

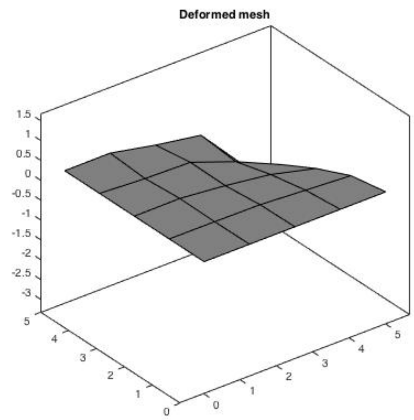
Shear:

0

0

For a sixteen-element division the following were obtained:





Nodal Displacements:

Node	Displacement(x,y,θ; m,m,rad respectively)		
3	-0.00131106	-6.509762e-04	-0.000189404
6	-0.000437553	-1.308059e-04	-0.000111916
7	-0.000184001	-1.239123e-04	-0.00010965
9	-8.59442e-05	-6.466750e-05	-3.57364e-05
12	-0.000128992	3.011384e-07	-6.66093e-05
13	-0.000810387	-2.120479e-05	-0.000125014
14	-0.000334631	3.010319e-05	0.000121662
15	3.06182e-06	3.767991e-06	7.71686e-05
17	0	0	1.60298e-05
18	-4.41712e-06	-3.647318e-06	-1.83213e-05
19	-1.35155e-05	-1.455952e-06	6.65457e-06

20	-7.16677e-05	-3.609281e-05	-6.05475e-05
21	-0.000211361	-1.364606e-05	-4.50032e-05
22	-0.000407089	-2.329287e-04	-7.84217e-05
23	-7.45689e-05	2.053435e-05	4.22734e-05
24	-5.0957e-05	-3.406569e-05	-4.40124e-05
25	5.98668e-06	5.215490e-06	8.37281e-06

Moment:

0

0

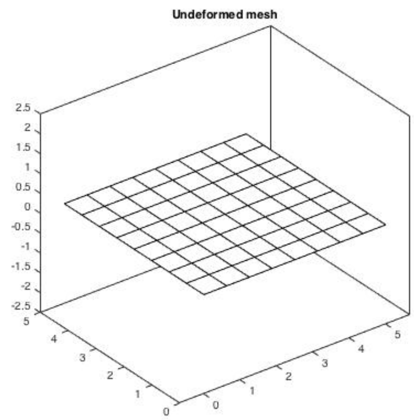
0

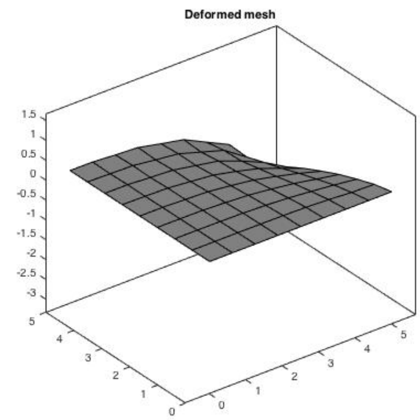
Shear:

0

0

For a sixty-four-element division the following were obtained:





Nodal Displacements:

Node	Displacement(x,y,θ; m,m,rad respectively)		
3	-0.00276497	-8.650769e-04	-0.000182314
6	-0.00104307	-1.607475e-04	-8.95294e-05
7	-0.000337436	-1.475383e-04	-8.54121e-05
9	-0.000186893	-7.342358e-05	-2.6759e-05
12	-0.00042475	-6.751816e-05	-5.63305e-05
13	-0.00181054	-3.338587e-04	-0.000117108
14	-0.00100053	-3.272049e-04	-0.000111012
15	-7.58242e-05	-4.674519e-05	-5.88898e-05
17	0	0	-7.51542e-07
18	-8.19455e-06	-8.260578e-06	-3.89717e-06
19	-5.86501e-05	-2.646540e-05	-1.84524e-05

20	-0.000208386	-4.990300e-05	-3.2701e-05
21	-0.000530773	-1.297137e-04	-4.82452e-05
22	-0.000838832	-2.491159e-04	-5.40661e-05
23	-0.000297119	-1.244960e-04	-4.81338e-05
24	-6.85862e-05	-4.389853e-05	-3.27628e-05
25	-3.74844e-05	-2.527359e-05	-1.26457e-05
29	0	0	-6.91872e-05
30	-0.00016671	1.084845e-05	-0.000102434
31	-0.000704782	-7.273834e-06	-0.000123009
32	-0.00140171	-1.506159e-05	-0.000139197
33	-0.00222862	-1.527169e-05	-0.00015002
34	-0.00146408	3.682958e-05	0.000100081
35	-0.000532216	2.716388e-05	9.51032e-05
36	-0.000131857	1.423069e-05	7.94355e-05
37	-6.44507e-06	-4.544981e-07	3.9855e-05
41	0	0	4.22669e-06
42	3.48281e-08	-8.311697e-07	-3.78822e-06
43	-4.36499e-07	1.289837e-06	3.20833e-06
44	-6.52429e-06	-5.854039e-06	-7.39492e-06
45	-1.35565e-05	2.061153e-06	-5.48703e-06
46	-3.24692e-05	-1.473423e-05	-2.36354e-05
47	-7.70847e-05	-1.293594e-05	-4.57545e-05
48	-0.000139892	-2.742497e-05	-2.27022e-05

49	-0.000304234	-9.765990e-06	-8.62873e-05
50	-0.000532642	-1.005050e-04	-5.66686e-05
51	-0.000765577	-1.699729e-05	-9.28856e-05
52	-0.001032	-2.170224e-04	-7.46095e-05
53	-0.00126244	-1.637591e-05	-7.42834e-05
54	-0.00150834	-4.381429e-04	-7.94287e-05
55	-0.00086769	2.8584130e-05	3.35263e-05
56	-0.000575811	-2.1056530e-04	-7.33342e-05
57	-0.000282518	2.1567450e-05	5.54695e-05
58	-0.000171292	-8.915773e-05	-5.82095e-05
59	-4.86375e-05	8.4243990e-06	4.42432e-05
60	-2.56131e-05	-1.984812e-05	-2.90291e-05
61	-4.82888e-06	4.247927e-07	1.64944e-05
62	-1.5715e-05	-1.318669e-05	-1.03249e-05
63	-2.97446e-06	9.257488e-07	2.76152e-06
64	-5.10905e-06	-5.512063e-06	-1.96592e-06
65	2.16325e-08	1.623245e-06	-1.18652e-07
66	-2.11005e-05	1.395844e-06	-6.32699e-06
67	-0.000116948	-9.232186e-06	-3.46629e-05
68	-0.000354401	-1.412736e-05	-6.17502e-05
69	-0.000676957	-1.069605e-05	-4.58857e-05
70	-0.000474216	1.509533e-05	1.10642e-05
71	-0.000123725	1.178252e-05	2.96457e-05

72	-3.54115e-05	3.517255e-06	1.30582e-05
73	-1.39872e-05	1.882098e-06	-1.25858e-06
74	-5.62831e-05	-3.054741e-05	-1.42798e-05
75	-0.000113547	-3.983233e-06	-1.81787e-05
76	-0.000223249	-6.734510e-05	-3.08464e-05
77	-0.000317877	-7.391090e-06	-3.08681e-05
78	-0.000432213	-1.419281e-04	-4.01819e-05
79	-0.000222349	6.280832e-06	2.92894e-06
80	-0.000126993	-6.419420e-05	-2.93881e-05
81	-7.77515e-05	2.026184e-06	-6.55825e-07

Moment:

0

0

0

Shear:

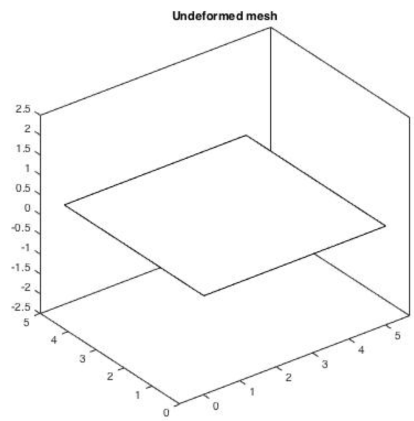
0

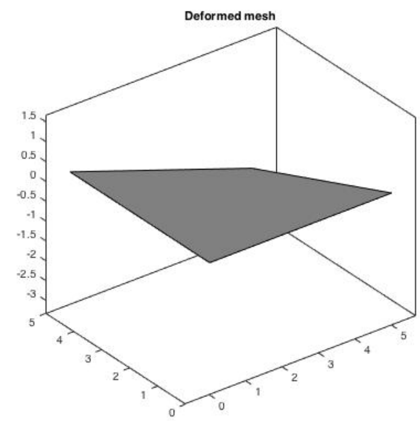
0

III. Simply supported Condition for distributed load of 10 Pa downward applied to the entire plate.

The method followed implemented a set of loops that applied a load that was derived by multiplying the entire distributed load with the area under analysis (25 m^2) and divided by the number of total points on the mesh. The points along the bottom and right side of the element were simply supported, meaning all displacements were constrained and the rotation was free.

For a one-element division the following were obtained:





Nodal Displacements:

Node	Displacement(x,y,θ; m,m,rad respectively)		
1	0	0	-8.72642e-05
2	0	0	4.52624e-05
3	-0.000101925	-6.461804e-05	-4.52624e-05
4	0	0	2.26461e-05

Moment:

-0.0373

-0.1243

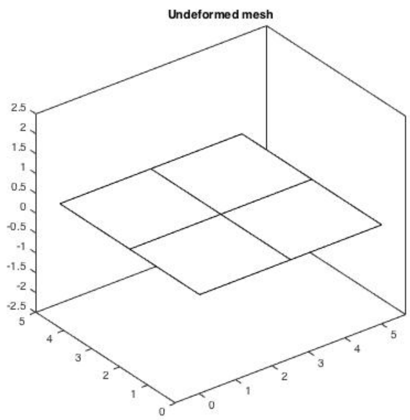
0.0839

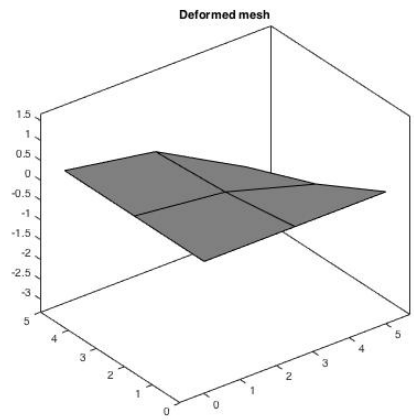
Shear:

-100.6897

-52.2261

For a four-element division the following were obtained:





Nodal Displacements:

Node	Displacement(x,y,θ; m,m,rad respectively)		
1	0	0	5.06891e-06
2	0	0	-9.56736e-06
3	-7.27591e-05	-2.396893e-05	-2.34009e-06
4	0	0	5.79805e-06
5	0	0	-9.00469e-06
6	-4.1516e-05	-1.126177e-05	-1.11086e-05
7	-1.74768e-05	-1.108189e-05	-5.74917e-06
8	0	0	5.49338e-07
9	-1.18754e-05	-7.531819e-06	-4.15507e-06

Moment:

0.0079

0.0263

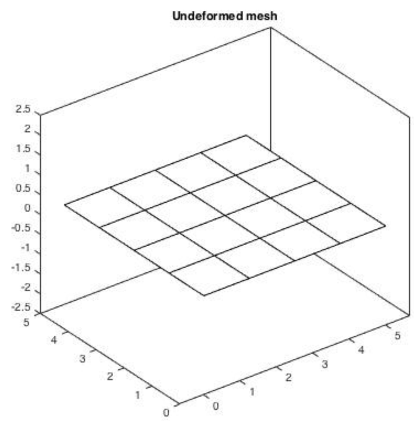
-0.0049

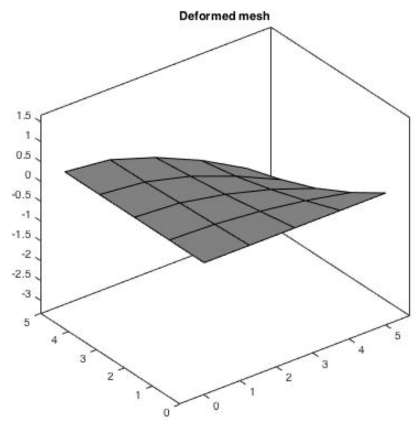
Shear:

-5.8488

-11.0393

For a sixteen-element division the following were obtained:





Nodal Displacements:

Node	Displacement(x,y,θ; m,m,rad respectively)		
1	0	0	-4.30764e-06
2	0	0	-1.88855e-05
3	-0.000132698	-1.905879e-05	-9.98893e-06
4	0	0	-3.22374e-06
5	0	0	-8.07762e-06
6	-8.18004e-05	-1.227328e-05	-1.45545e-05
7	-4.10923e-05	-1.255902e-05	-5.8147e-06
8	0	0	-4.52541e-06
9	-2.38904e-05	-7.304303e-06	-7.92412e-06
10	0	0	3.04887e-06

11	0	0	-4.37239e-06
12	-4.44931e-05	-6.575943e-06	-7.66315e-06
13	-0.000109827	-1.307292e-05	-3.00829e-06
14	-8.17742e-05	-1.322841e-05	-1.54002e-07
15	-1.06303e-05	-6.748303e-06	3.92985e-06
16	0	0	4.7197e-06
17	0	0	5.94297e-06
18	-3.3697e-06	-2.137047e-06	-6.82384e-06
19	-1.22384e-05	-3.488251e-06	1.96005e-06
20	-2.59289e-05	-5.194713e-06	-1.37149e-05
21	-4.89292e-05	-8.567005e-06	-2.29478e-06
22	-6.80645e-05	-1.340266e-05	-1.00379e-05
23	-3.31299e-05	-8.749873e-06	2.26408e-06
24	-9.67329e-06	-6.125585e-06	-6.03209e-06
25	-6.17971e-06	-3.921259e-06	3.62072e-06

Moment:

0.0156

0.0519

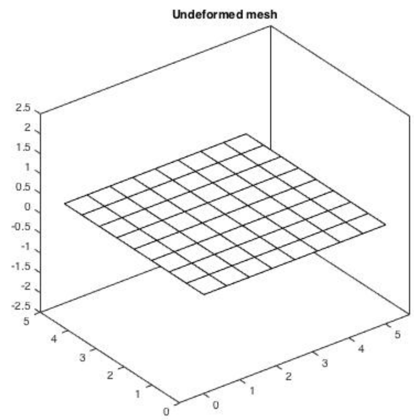
0.0041

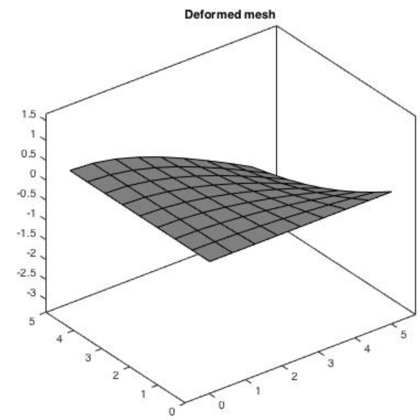
Shear:

-4.9704

-21.7911

For a sixty-four-element division the following were obtained:





Nodal Displacements:

Node	Displacement(x,y,θ; m,m,rad respectively)		
1	0	0	0.000120007
2	0	0	0.000104782
3	-0.000264692	-1.650893e-05	0.000114389
4	0	0	0.000120407
5	0	0	0.000115759
6	-0.000166962	-1.136843e-05	0.000109713
7	-8.56586e-05	-1.217737e-05	0.000118265
8	0	0	0.000119985
9	-5.08126e-05	-7.264126e-06	0.000116519
10	0	0	0.000118873

11	0	0	0.000111143
12	-9.17966e-05	-6.692921e-06	0.000106698
13	-0.000223274	-1.452708e-05	0.000112614
14	-0.000169326	-1.483794e-05	0.000115998
15	-2.37836e-05	-7.597014e-06	0.000120239
16	0	0	0.000120133
17	0	0	0.000119937
18	-7.46793e-06	-2.267536e-06	0.000118849
19	-2.68252e-05	-3.872134e-06	0.000115927
20	-5.50245e-05	-5.193389e-06	0.000111744
21	-0.000102703	-9.468658e-06	0.000113295
22	-0.000140167	-1.256906e-05	0.000114938
23	-7.06161e-05	-9.946197e-06	0.000117266
24	-2.03644e-05	-6.139002e-06	0.000119122
25	-1.43062e-05	-4.337102e-06	0.000118974
26	0	0	-0.000120366
27	0	0	-0.000122667
28	0	0	-0.000126602
29	0	0	-0.000131977
30	-4.7468e-05	-3.628425e-06	-0.000134406
31	-0.00013172	-8.836299e-06	-0.00013173
32	-0.000197289	-1.235408e-05	-0.00012881
33	-0.000245169	-1.443728e-05	-0.000126503

34	-0.000215698	-1.455392e-05	-0.000125308
35	-0.000125394	-1.300647e-05	-0.00012328
36	-5.11053e-05	-9.720711e-06	-0.000121109
37	-5.88914e-06	-3.746066e-06	-0.00011977
38	0	0	-0.000120147
39	0	0	-0.000119864
40	0	0	-0.000119674
41	0	0	-0.000119561
42	-9.90363e-07	-6.277133e-07	0.000119292
43	-3.76995e-06	-1.134959e-06	-0.000120816
44	-8.06409e-06	-1.588859e-06	0.000117105
45	-1.36005e-05	-1.922588e-06	-0.00012393
46	-2.02917e-05	-2.321315e-06	0.000113283
47	-2.80427e-05	-2.594094e-06	-0.000128462
48	-3.69387e-05	-3.047419e-06	0.000108095
49	-7.22295e-05	-5.712262e-06	-0.000130859
50	-0.00010459	-8.363700e-06	0.000110069
51	-0.000133338	-9.949264e-06	-0.000128617
52	-0.000158564	-1.219629e-05	0.000112497
53	-0.000180181	-1.279356e-05	-0.000126478
54	-0.000199076	-1.478307e-05	0.000114508
55	-0.000155368	-1.293369e-05	-0.000124586
56	-0.000115158	-1.256528e-05	0.00011653

57	-7.87106e-05	-1.054824e-05	-0.00012241
58	-4.75357e-05	-9.223032e-06	0.000118588
59	-2.27402e-05	-6.500029e-06	-0.000120642
60	-6.25354e-06	-3.953482e-06	0.000119742
61	-5.33341e-06	-3.389310e-06	-0.000120312
62	-4.60949e-06	-2.916370e-06	0.000119583
63	-3.72917e-06	-2.368555e-06	-0.000120262
64	-2.88436e-06	-1.826058e-06	0.000119414
65	-1.9444e-06	-1.234527e-06	-0.000120274
66	-1.58808e-05	-3.064646e-06	-0.00012244
67	-3.98832e-05	-4.404158e-06	-0.000126087
68	-8.00484e-05	-7.198068e-06	-0.000127337
69	-0.000122613	-1.060064e-05	-0.000125792
70	-0.000103323	-1.078366e-05	-0.000124021
71	-4.24853e-05	-7.882169e-06	-0.000121776
72	-1.73339e-05	-5.151057e-06	-0.000120774
73	-1.09473e-05	-3.285775e-06	-0.000120791
74	-2.33654e-05	-4.589397e-06	0.00011743
75	-3.92472e-05	-5.482343e-06	-0.000123536
76	-5.82941e-05	-6.596570e-06	0.000114146
77	-7.50195e-05	-8.078441e-06	-0.000125067
78	-9.01147e-05	-1.000731e-05	0.000115374
79	-6.11856e-05	-8.338320e-06	-0.000122974

80	-3.67477e-05	-7.159712e-06	0.000117868
81	-3.02499e-05	-5.768413e-06	-0.000122109

Moment:

-0.0864

-0.2879

-0.1154

Shear:

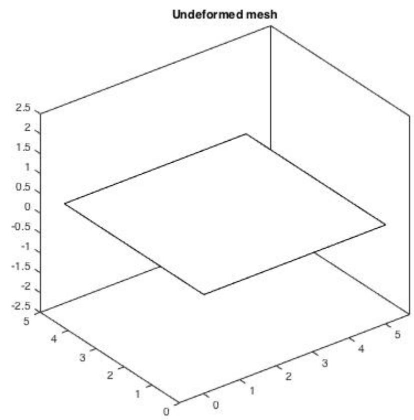
-138.4698

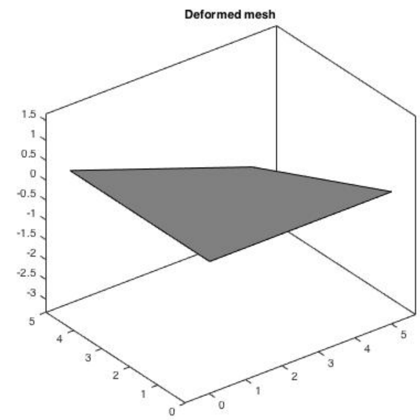
-120.9024

IV. Simply supported condition a concentrated load of 1kN downward applied to plate center.

The method followed implemented a single applied load of 1kN downward at point 9 of our mesh (plate center or top left corner of the bottom right plate element under consideration). The points along the bottom and right side of the element were simply supported, meaning all displacements were constrained and the rotation was free.

For a one-element division the following were obtained:





Nodal Displacements:

Node	Displacement(x,y,θ; m,m,rad respectively)		
1	0	0	-0.00139623
2	0	0	0.000724199
3	-0.0016308	-1.033889e-03	-0.000724199
4	0	0	0.000362338

Moment:

-0.5969

-1.9896

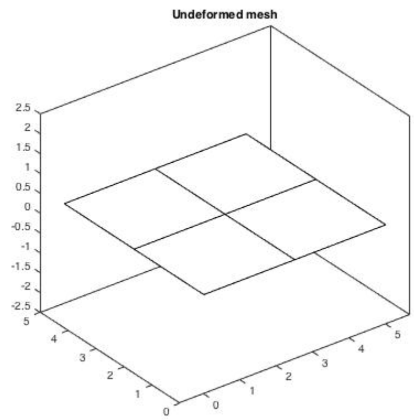
1.3425

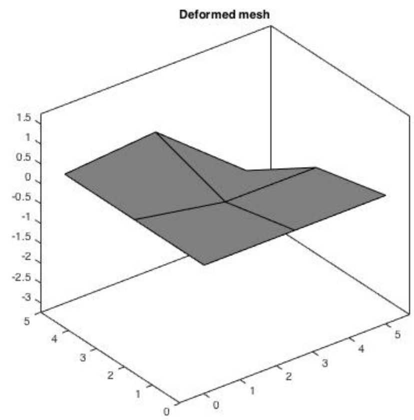
Shear:

-1.6110 e+03

-0.8356 e+03

For a four-element division the following were obtained:





Nodal Displacements:

Node	Displacement(x,y,θ; m,m,rad respectively)		
1	0	0	-0.00046992
2	0	0	-0.000398618
3	-0.00121781	-9.429267e-04	-9.49259e-05
4	0	0	0.00019502
5	0	0	0.000243056
6	-0.000323461	1.945726e-04	9.65035e-05
7	0.000133964	8.545261e-05	6.1336e-05
8	0	0	0.0003729
9	-0.000314589	-1.999109e-04	-0.000345337

Moment:

0.3285

1.0951

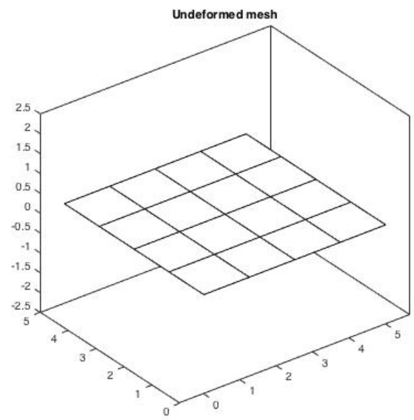
0.4518

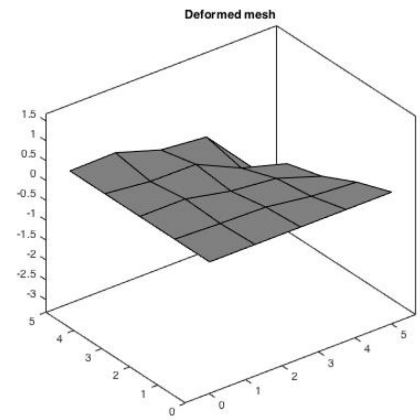
Shear:

-542.2171

-459.9459

For a sixteen-element division the following were obtained:





Nodal Displacements:

Node	Displacement(x,y,θ; m,m,rad respectively)		
1	0	0	-0.000247708
2	0	0	-0.00019151
3	-0.0018917	-1.068114e-03	4.19715e-05
4	0	0	0.000352112
5	0	0	-0.000160094
6	-0.000846185	-3.665292e-04	-0.000139246
7	-0.000397432	-2.816625e-04	0.00014891
8	0	0	-0.000121632
9	-0.000272936	-1.748150e-04	-0.000103246
10	0	0	0.000152091

11	0	0	2.283e-05
12	-0.000363992	7.245898e-05	-5.70864e-05
13	-0.00113639	2.373692e-04	-0.000194801
14	-0.000526151	2.008660e-04	-0.000124493
15	2.16846e-05	1.537492e-05	-0.000185345
16	0	0	-2.7722e-05
17	0	0	0.000230009
18	-4.97717e-05	-3.241541e-05	-0.000165785
19	-8.10873e-05	1.292812e-05	9.0758e-05
20	-0.000268867	-1.323237e-04	-0.00017539
21	-0.000409543	8.885171e-05	-2.30738e-05
22	-0.000763206	-4.726887e-04	-5.09908e-05
23	-0.000159096	8.889146e-05	2.52595e-05
24	-0.000147862	-9.540429e-05	-3.19625e-05
25	-2.25807e-07	1.333607e-06	8.82899e-05

Moment:

0.1578

0.5261

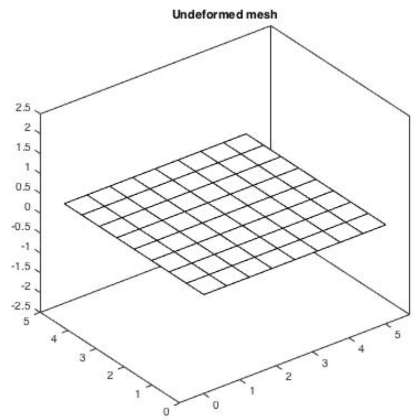
0.2382

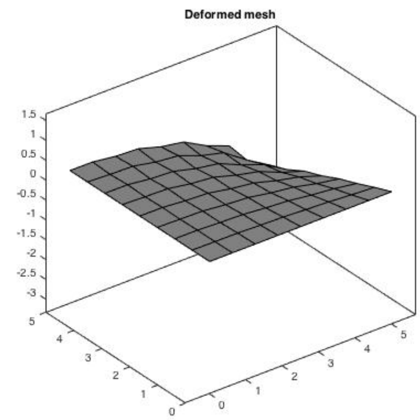
Shear:

-285.8181

-220.9740

For a sixty-four-element division the following were obtained:





Nodal Displacements:

Node	Displacement(x,y,θ; m,m,rad respectively)		
1	0	0	0.000632112
2	0	0	0.000683829
3	-0.003394	-1.247495e-03	0.00089351
4	0	0	0.00121379
5	0	0	0.000710572
6	-0.00151778	-3.670280e-04	0.000733672
7	-0.000569514	-2.828272e-04	0.00102244
8	0	0	0.000749363
9	-0.000389263	-1.658089e-04	0.000767044
10	0	0	0.00069743
11	0	0	0.000699795

12	-0.000743042	-1.696420e-04	0.000695703
13	-0.00236132	-6.360677e-04	0.000802678
14	-0.00141127	-5.603033e-04	0.000980918
15	-0.000164607	-1.010785e-04	0.00107295
16	0	0	0.000915952
17	0	0	0.000660146
18	-5.817e-05	-3.165947e-05	0.000714538
19	-0.000205961	-7.957696e-05	0.000723959
20	-0.000443727	-1.301774e-04	0.00071282
21	-0.000869907	-2.783295e-04	0.000756986
22	-0.00122732	-4.570948e-04	0.000847571
23	-0.000522116	-2.479739e-04	0.000846829
24	-0.000153603	-9.520956e-05	0.000863162
25	-0.000110835	-6.402917e-05	0.000767003
26	0	0	-0.000677743
27	0	0	-0.000743632
28	0	0	-0.000803392
29	0	0	-0.000873786
30	-0.000357886	3.473135e-05	-0.000914774
31	-0.00108759	1.088176e-04	-0.000947953
32	-0.00185937	1.925403e-04	-0.00101862
33	-0.00270999	2.799261e-04	-0.0011364
34	-0.00186553	2.750237e-04	-0.000988274

35	-0.000774011	1.545560e-04	-0.00100632
36	-0.000225734	6.372370e-05	-0.00103671
37	-7.35406e-06	-7.988801e-07	-0.00111555
38	0	0	-0.001027
39	0	0	-0.000816062
40	0	0	-0.000692816
41	0	0	-0.000636037
42	-8.89067e-06	-6.640961e-06	0.000676125
43	-1.9931e-05	2.802085e-07	-0.000714153
44	-6.12175e-05	-2.689563e-05	0.000708943
45	-9.06402e-05	8.591781e-06	-0.000772176
46	-0.000158425	-5.190604e-05	0.000707089
47	-0.00020915	2.010814e-05	-0.00083651
48	-0.000294322	-7.455785e-05	0.000691283
49	-0.000563709	5.501010e-05	-0.000887138
50	-0.000885673	-2.354224e-04	0.000716872
51	-0.00112672	1.172450e-04	-0.000928091
52	-0.00147481	-4.340248e-04	0.000775856
53	-0.00165725	1.869577e-04	-0.000993715
54	-0.00200486	-7.237678e-04	0.000886111
55	-0.00118571	2.017905e-04	-0.000934013
56	-0.000889758	-3.876921e-04	0.000890458
57	-0.000446827	1.054913e-04	-0.000911622

58	-0.000324961	-1.815619e-04	0.000911613
59	-8.83961e-05	3.013718e-05	-0.000933773
60	-6.31011e-05	-4.394037e-05	0.000963816
61	-6.66442e-06	-2.486442e-07	-0.000888026
62	-4.39684e-05	-3.167677e-05	0.000811121
63	-9.22789e-06	-2.457277e-06	-0.000765782
64	-2.65365e-05	-1.958207e-05	0.000719601
65	-6.61723e-06	-2.249586e-06	-0.000699097
66	-9.54667e-05	8.857599e-06	-0.000755443
67	-0.000286013	2.948621e-05	-0.000812951
68	-0.000618927	6.498836e-05	-0.000857767
69	-0.00098555	1.217534e-04	-0.000898772
70	-0.000712199	1.288470e-04	-0.000890315
71	-0.000219547	5.491500e-05	-0.000866178
72	-7.48843e-05	1.434162e-05	-0.000812036
73	-5.40789e-05	3.837351e-06	-0.000745403
74	-0.000176637	-8.270674e-05	0.00073765
75	-0.000259116	3.135037e-05	-0.000791457
76	-0.000464006	-1.621825e-04	0.000733966
77	-0.000542777	6.964302e-05	-0.00084215
78	-0.000723859	-2.861421e-04	0.000795781
79	-0.000384521	6.970271e-05	-0.000832368
80	-0.000271214	-1.403176e-04	0.000797531

81	-0.000171763	2.679939e-05	-0.00079245

Moment:

-0.5636

-1.8786

-0.6078

Shear:

-729.3627

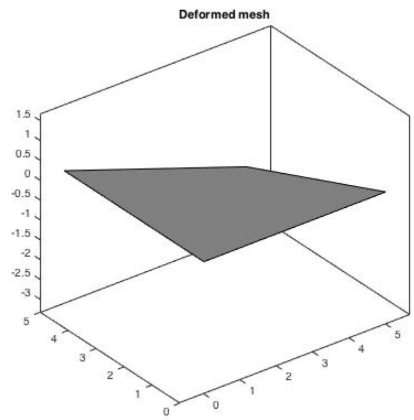
-789.0358

V. Shear-locking: what exactly is it and how we prevent it from happening.

Looking at some of the graphs above, such as the 4, 16 and 64-element division for the simply supported condition with a 1kN concentrated load applied at the center of the plate, we notice that some of the subdivisions look distorted and the element as a whole discontinuous and "wrinkled". When 2x2 shear quadrature is being implemented for the shear, the shear stiffness of the structure is predicted to be more than tensile stress and the bending stress much lower than the tensile stress. Our selection of a quad element as the base for the analysis was not a good one in this case, as the quad is not a very "smart" shape, and is therefore unable to acquire the correct shape for the shear stress within the plate. A simple solution to this would be to add a point in the middle of each subdivision and connect it to the four nodes of the quad subdivision, thus turning each quad to 4 triangular elements, which will not be susceptible to shear locking.

To prevent shear locking from happening, we can under-integrate for the shear stiffness matrix and move from a 2x2 Gaussian integration to a 1x1 Gaussian integration, in which case the shear stiffness contribution is not over-exaggerated and the end-result looks "smoother" across the plate element. We are switching from an nglx=ngly=2 to a nglx=ngly=1 and remove the extraneous r and s values to acquire the new values for 1x1 quadrature. The following graphics include only the deformed shapes of all the scenarios above and the accompanying values underneath them.

The adjusted one-element cases are as follows:



Clamped condition | Concentrated Load

Nodal Displacements:

Node Displacement(x,y,θ; m,m,rad respectively)

3 -0.00058521 -5.885470e-04 -0.000581872

Moment:

0

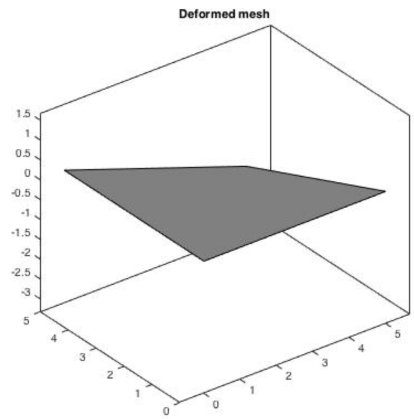
0

0

Shear:

0

0



Clamped Condition | Distributed Load

Nodal Displacements:

Node Displacement(x,y,θ; m,m,rad respectively)

3 -3.65756e-05 -3.678418e-05 -3.6367e-05

Moment:

0

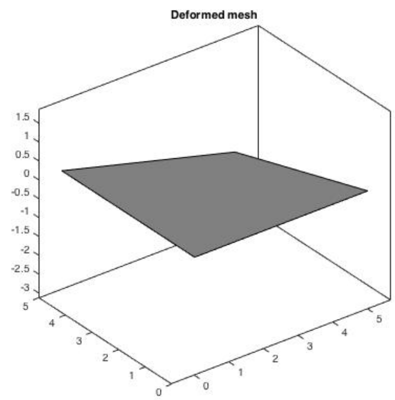
0

0

Shear:

0

0



Simply Supported Condition | Concentrated Load

Nodal Displacements:

Node	Displacement(x,y,θ; m,m,rad respectively)		
1	0	0	-0.00340205
2	0	0	0.00173002
3	-0.00257235	-2.572350e-03	-0.00173002
4	0	0	0.000829697

Moment:

-1.4258

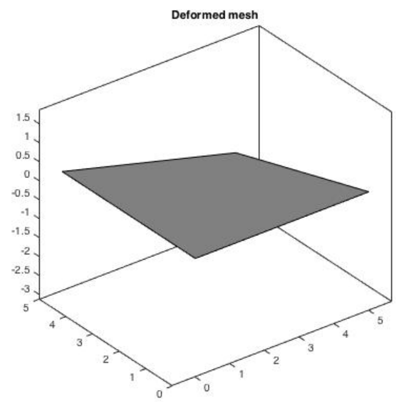
-4.7528

3.2712

Shear:

-3.9255e+03

-1.9962e+03



Simply Supported Condition | Distributed Load

Nodal Displacements:

Node Displacement(x,y,θ; m,m,rad respectively)

1	0	0	-0.000212628
2	0	0	0.000108126
3	-0.000160772	-1.607719e-04	-0.000108126
4	0	0	5.1856e-05

Moment:

-0.0891

-0.2970

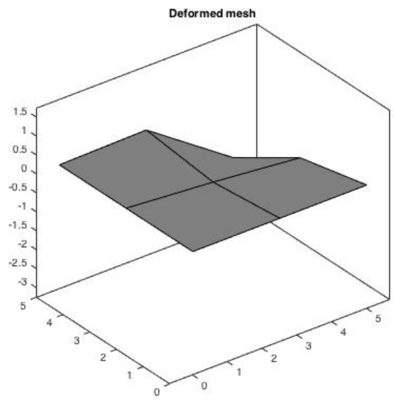
0.2044

Shear:

-245.3407

-124.7614

The adjusted 4-element cases are as follows:



Clamped Condition | Concentrated Load

Nodal Displacements:

Node	Displacement(x,y,θ; m,m,rad respectively)		
3	-0.0010411	-1.106763e-03	-0.000468712
6	-0.000281955	-2.140917e-05	-0.000283327
7	2.69896e-05	3.317781e-05	0.000271559
9	-0.000125877	-1.312747e-04	-0.000123691

Moment:

0

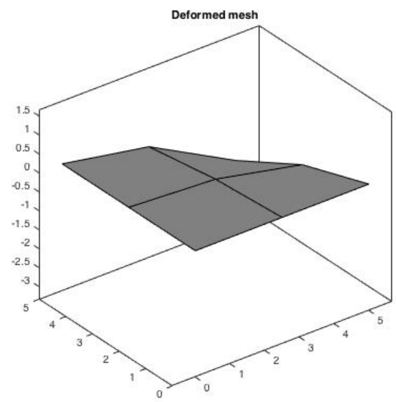
0

0

Shear:

0

0



Clamped Condition | Distributed Load

Nodal Displacements:

Node	Displacement(x,y,θ; m,m,rad respectively)		
3	-6.9412e-05	-3.406575e-05	-1.67839e-05
6	-2.63653e-05	-1.490699e-05	-2.61264e-05
7	-1.7729e-05	-1.764678e-05	-6.34833e-06
9	-5.68298e-06	-5.804499e-06	-5.77552e-06

Moment:

0

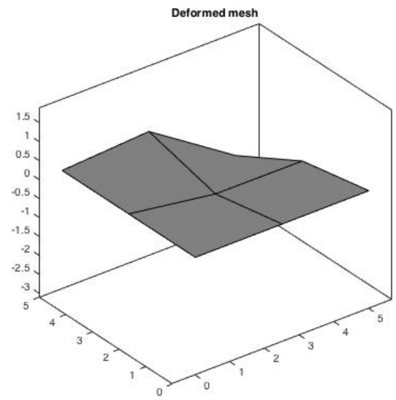
0

0

Shear:

0

0



Simply Supported Condition | Concentrated Load

Nodal Displacements:

Node Displacement(x,y,θ; m,m,rad respectively)

1	0	0	-0.000607703
2	0	0	-0.000430834
3	-0.00192065	-2.348151e-03	0.000326136
4	0	0	0.00104503
5	0	0	4.47977e-05
6	-0.000510346	4.854131e-04	-0.000320753
7	0.000210646	2.138559e-04	-0.000409839
8	0	0	0.000366638
9	-0.00049555	-4.984365e-04	-0.000298417

Moment:

0.3551

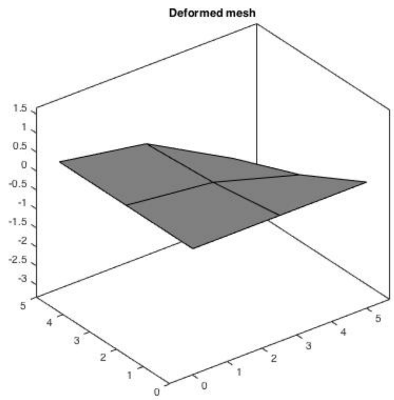
1.1836

0.5843

Shear:

-701.1986

-497.1181



Simply Supported Condition | Distributed Load

Nodal Displacements:

Node	Displacement(x,y,θ; m,m,rad respectively)		
1	0	0	-3.52473e-07
2	0	0	-3.67229e-05
3	-0.00011479	-5.965250e-05	-1.8758e-05
4	0	0	1.4792e-06
5	0	0	-9.46947e-06
6	-6.55068e-05	-2.803448e-05	-1.46847e-05
7	-2.75722e-05	-2.758493e-05	-1.36296e-06
8	0	0	1.42915e-05
9	-1.87334e-05	-1.875252e-05	-2.32821e-05

Moment:

0.0303

0.1009

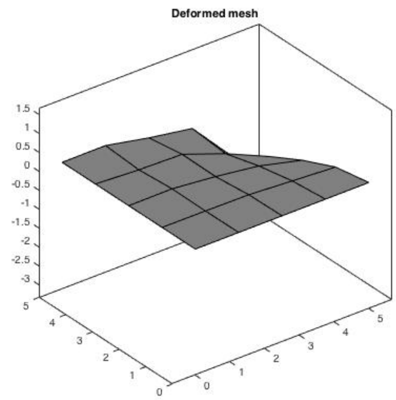
0.0003

Shear:

-0.4067

-42.3727

The adjusted 16-element cases are as follows:



Clamped Condition | Concentrated Load

Nodal Displacements:

Node Displacement(x,y,θ; m,m,rad respectively)

3	-0.00207161	-1.644155e-03	-0.000451146
6	-0.000691663	-3.411961e-04	-0.000270172
7	-0.000288963	-3.184538e-04	-0.000255702
9	-0.000135346	-1.694364e-04	-8.31627e-05
12	-0.00020509	9.160840e-06	-0.000169407
13	-0.0012803	-3.329063e-05	-0.000321799
14	-0.000528215	8.941540e-05	0.000283485
15	4.69829e-06	1.552484e-05	0.00018084
17	0	0	4.07175e-05
18	-7.02719e-06	-1.201621e-05	-4.4783e-05
19	-2.13718e-05	1.609097e-06	1.46305e-05
20	-0.000113263	-9.760886e-05	-0.0001486
21	-0.000334472	-2.245596e-05	-0.000118962

22	-0.000642809	-5.946837e-04	-0.000185425
23	-0.000117739	6.001575e-05	9.67311e-05
24	-7.93745e-05	-8.980158e-05	-0.000103438
25	9.36929e-06	1.775525e-05	1.7453e-05

Moment:

0

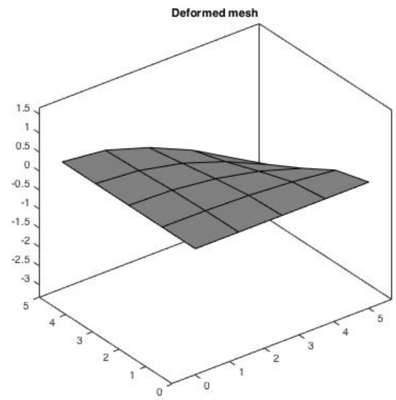
0

0

Shear:

0

0



Clamped Condition | Distributed Load

Nodal Displacements:

Node Displacement(x,y,θ; m,m,rad respectively)

3	-0.000135588	-3.129617e-05	-1.57801e-05
6	-6.33895e-05	-1.808028e-05	-2.12821e-05
7	-4.07095e-05	-1.913461e-05	-6.76561e-06
9	-1.44455e-05	-8.199204e-06	-7.24977e-06
12	-2.10255e-05	-5.520433e-06	-1.87306e-05
13	-0.000102234	-2.212045e-05	-1.74649e-05
14	-8.21135e	-05-2.219885e-05	-9.26914e-06
15	-1.08019e-05	-1.079921e-05	7.39687e-08
17	0	0	2.22965e-06
18	-8.7039e-07	-8.985988e-07	-3.08457e-06
19	-3.55655e-06	-1.796834e-06	-1.35319e-06
20	-1.04061e-05	-5.114118e-06	-1.2623e-05
21	-3.40047e-05	-1.130598e-05	-1.32807e-05
22	-6.00782e-05	-2.003149e-05	-1.28583e-05
23	-2.78079e-05	-1.231755e-05	-6.0695e-06

24	-7.7323e-06	-7.746233e-06	-3.17835e-06
25	-3.13532e-06	-3.132229e-06	-1.4426e-06

Moment:

0

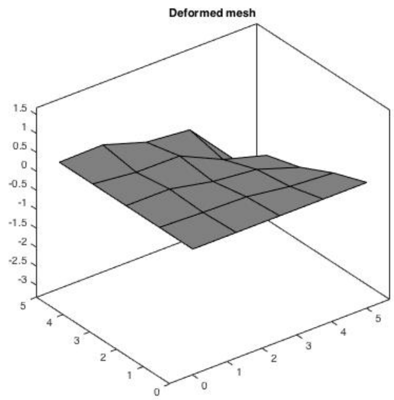
0

0

Shear:

0

0



Simply Supported Condition | Concentrated Load

Nodal Displacements:

Node	Displacement(x,y,θ; m,m,rad respectively)		
1	0	0	0.000482544
2	0	0	0.00061921
3	-0.00298277	-2.669817e-03	0.00121009
4	0	0	0.00197027
5	0	0	0.000695364
6	-0.00133384	-9.198901e-04	0.000751637
7	-0.000625578	-7.062831e-04	0.00147354
8	0	0	0.000794054
9	-0.00042936	-4.395688e-04	0.000840043
10	0	0	-0.000715782
11	0	0	-0.00103737
12	-0.000573975	1.846782e-04	-0.00123765
13	-0.00179228	6.013077e-04	-0.00158484
14	-0.000829554	5.072290e-04	-0.00141433

15	3.37759e-05	4.348479e-05	-0.00156213
16	0	0	-0.00116454
17	0	0	-0.000526207
18	-7.79596e-05	-8.312354e-05	0.000682453
19	-0.000128265	3.502035e-05	-0.000868675
20	-0.000423531	-3.324049e-04	0.000658677
21	-0.000646032	2.268096e-04	-0.0011545
22	-0.00120262	-1.183129e-03	0.000973203
23	-0.000250983	2.265697e-04	-0.00103703
24	-0.000231985	-2.419288e-04	0.00101969
25	-1.09154e-06	7.755279e-06	-0.000876633

Moment:

-0.5103

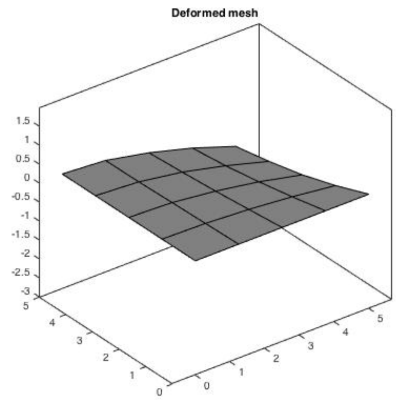
-1.7011

-0.4640

Shear:

-556.7838

-714.4755



Simply Supported Condition | Distributed Load

Nodal Displacements:

Node Displacement(x,y,θ; m,m,rad respectively)

1	0	0	0.000309161
2	0	0	0.000272919
3	-0.00020937	-4.743291e-05	0.000295041
4	0	0	0.000311872
5	0	0	0.000299789
6	-0.000129064	-3.054071e-05	0.000283681
7	-6.4832e-05	-3.122721e-05	0.000305419
8	0	0	0.000308619
9	-3.76933e-05	-1.815865e-05	0.000300168
10	0	0	-0.000312299
11	0	0	-0.000330784
12	-7.02046e-05	-1.637568e-05	-0.000338938
13	-0.000173286	-3.254416e-05	-0.000327367
14	-0.000129019	-3.293572e-05	-0.000320281

15	-1.67604e-05	-1.681508e-05	-0.000310093
16	0	0	-0.000308153
17	0	0	-0.00030512
18	-5.31397e-06	-5.318858e-06	0.000302924
19	-1.93085e-05	-8.692654e-06	-0.00031502
20	-4.09084e-05	-1.292912e-05	0.000285751
21	-7.71983e-05	-2.133015e-05	-0.000325604
22	-0.000107388	-3.334850e-05	0.000294904
23	-5.22675e-05	-2.180193e-05	-0.000314256
24	-1.52681e-05	-1.522535e-05	0.000304884
25	-9.74909e-06	-9.770041e-06	-0.000310883

Moment:

-0.2249

-0.7498

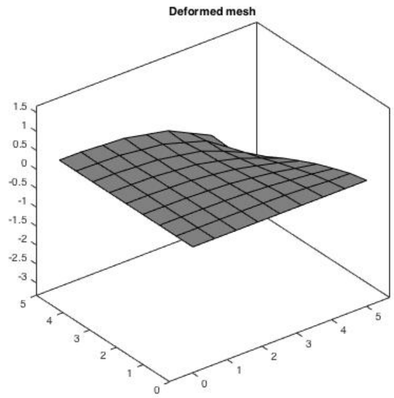
-0.2973

Shear:

-356.7252

-314.9079

The adjusted 64-element cases are as follows:



Clamped Condition | Concentrated load

Nodal Displacements:

Node Displacement(x,y,θ; m,m,rad respectively)

3	-0.00437853	-2.235432e-03	-0.000388991
6	-0.00165451	-4.496880e-04	-0.000201388
7	-0.000531158	-3.980657e-04	-0.000157669
9	-0.000296235	-2.081630e-04	-5.25982e-05
12	-0.000678465	-1.968670e-04	-0.000129623
13	-0.00286592	-8.966442e-04	-0.000255474
14	-0.00158274	-8.640379e-04	-0.000212531
15	-0.000115641	-1.337759e-04	-0.000109584
17	0	0	-3.61589e-06
18	-1.31069e-05	-2.923090e-05	-8.30855e-06
19	-9.32589e-05	-8.204353e-05	-4.24036e-05
20	-0.000331896	-1.473841e-04	-7.62938e-05
21	-0.000842125	-3.604627e-04	-0.000102926

22	-0.00132854	-6.665812e-04	-0.000102401
23	-0.00046956	-3.391517e-04	-9.18731e-05
24	-0.000106898	-1.246622e-04	-6.26952e-05
25	-5.90326e-05	-7.678110e-05	-2.30672e-05
29	0	0	-0.000176768
30	-0.000270647	3.857172e-05	-0.000263059
31	-0.00112118	1.996279e-05	-0.000321571
32	-0.00222124	1.887711e-05	-0.000372592
33	-0.0035262	3.401468e-05	-0.000417139
34	-0.00231615	1.515159e-04	0.000184357
35	-0.000841925	1.049508e-04	0.00017739
36	-0.000207679	5.809869e-05	0.000151615
37	-9.17156e-06	1.006122e-05	7.81864e-05
41	0	0	1.20727e-05
42	-9.285e-08	-4.483664e-06	-1.01657e-05
43	-5.81467e-07	7.931281e-06	7.90327e-06
44	-1.04474e-05	-2.151993e-05	-1.78179e-05
45	-2.16213e-05	1.393222e-05	-1.50191e-05
46	-5.17607e-05	-4.787809e-05	-5.80863e-05
47	-0.000123719	-2.056732e-05	-0.000116977
48	-0.000223866	-8.275590e-05	-5.43602e-05
49	-0.000486347	-6.202206e-07	-0.000223106
50	-0.000846774	-2.861088e-04	-0.000128571

51 -0.00121563	-3.196472e-07	-0.000250372
52 -0.00163525	-5.908222e-04	-0.000161645
53 -0.00199974	1.363158e-05	-0.000218957
54 -0.00238829	-1.152172e-03	-0.000156634
55 -0.00137376	1.173481e-04	4.38261e-05
56 -0.000910619	-5.626918e-04	-0.00014376
57 -0.000447238	8.196130e-05	0.000102815
58 -0.000268687	-2.443170e-04	-0.000113082
59 -7.67001e-05	3.616443e-05	8.60398e-05
60 -3.83758e-05	-5.748452e-05	-5.68629e-05
61 -8.48616e-06	9.924445e-06	3.07231e-05
62 -2.42649e-05	-4.123489e-05	-1.9072e-05
63 -5.03822e-06	9.694022e-06	2.61161e-06
64 -8.08145e-06	-1.993077e-05	-2.51253e-06
65 1.55685e-07	8.489393e-06	-8.91258e-08
66 -3.37085e-05	1.562911e-05	-1.88149e-05
67 -0.000186767	-4.420144e-06	-9.15935e-05
68 -0.000564338	-5.952923e-06	-0.000165115
69 -0.001074	1.398035e-05	-0.000136808
70 -0.000751792	7.320676e-05	-2.36089e-06
71 -0.000196396	5.057424e-05	5.01876e-05
72 -5.69548e-05	2.355290e-05	2.02714e-05
73 -2.24798e-05	1.606288e-05	-8.23066e-06

74 -8.92479e-05	-9.273427e-05	-2.85267e-05
75 -0.000181086	1.057865e-05	-5.42279e-05
76 -0.000354646	-1.941163e-04	-6.72532e-05
77 -0.000505454	1.147633e-05	-9.27618e-05
78 -0.000684845	-3.884402e-04	-7.88444e-05
79 -0.000353318	4.222687e-05	-1.18208e-05
80 -0.0002004	-1.811441e-04	-5.65075e-05
81 -0.000124131	2.395508e-05	-1.33989e-05

Moment:

m =

0

0

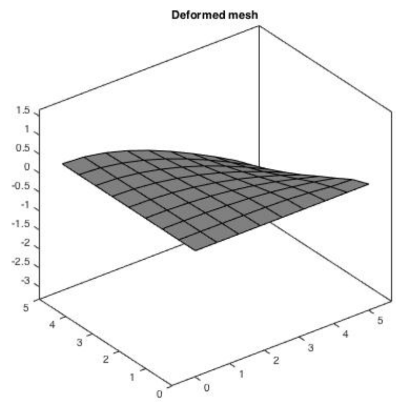
0

Shear:

s =

0

0



Clamped Condition | Distributed Load

Nodal Displacements:

Node Displacement(x,y,θ; m,m,rad respectively)

3 -0.000304063-3.145348e-05 -1.21833e-05

6 -0.000164276-2.105199e-05 -1.83106e-05

7 -9.52212e-05-2.179206e-05 -4.43907e-06

9 -4.15838e-05-1.050119e-05 -6.7021e-06

12 -7.56048e-05-1.316666e-05 -1.56969e-05

13 -0.000242441-2.722969e-05 -1.51759e-05

14 -0.000190984-2.764691e-05 -8.80161e-06

15 -2.65916e-05-1.335445e-05 -6.38431e-07

17 0 0 5.57456e-07

18 -2.53674e-06-1.651597e-06 -9.08644e-07

19 -1.29199e-05-4.075237e-06 -5.42694e-06

20 -3.79201e-05-7.672556e-06 -9.04933e-06

21 -9.37946e-05-1.599092e-05 -1.17287e-05

22 -0.000146708-2.255331e-05 -1.03361e-05

23 -7.08407e-05-1.653904e-05 -5.97062e-06

24 -1.96772e-05-9.560967e-06 -2.16886e-06

25 -1.02898e-05-5.275507e-06 -2.25384e-06

29 0 0 -1.90769e-05

30 -2.98011e-058.631662e-08 -2.90245e-05

31 -0.000118914-1.312675e-05 -2.6944e-05

32 -0.000204942-2.159665e-05 -2.22361e-05

33 -0.000274742-2.687021e-05 -1.70645e-05

34 -0.000245536-2.694118e-05 -1.38236e-05

35 -0.000140244-2.309583e-05 -8.84075e-06

36 -5.67853e-05-1.671307e-05 -3.32688e-06

37 -6.68259e-06-6.644788e-06 6.13797e-07

41 0 0 4.93019e-07

42 -1.12386e-07-1.418424e-07 -5.90308e-07

43 -5.06622e-07-2.278814e-07 -3.76747e-08

44 -1.66509e-06-9.762657e-07 -2.14082e-06

45 -3.44246e-06-7.752476e-07 -2.95081e-06

46 -6.32647e-06-2.173594e-06 -6.85851e-06

47 -1.42115e-05-5.697324e-06 -1.35804e-05

48 -2.49506e-05-5.010528e-06 -6.593e-06

49 -5.39393e-05-8.393111e-06 -2.34626e-05

50 -9.11664e-05-1.474602e-05 -1.44052e-05

51 -0.000126413-1.667883e-05 -2.09336e-05

52 -0.000161294-2.218759e-05 -1.4049e-05

53 -0.000192147-2.292935e-05 -1.68804e-05

54 -0.000220379-2.762838e-05 -1.14029e-05

55 -0.000169604-2.311021e-05 -1.25183e-05

56 -0.000124051-2.243933e-05 -7.40404e-06

57 -8.37873e-05-1.795850e-05 -6.95245e-06

58 -5.01084e-05-1.565087e-05 -3.38172e-06

59 -2.38941e-05-1.068959e-05 -2.0379e-06

60 -6.5364e-06-6.579595e-06 -7.68701e-07

61 -5.12877e-06-5.077647e-06 -6.45152e-07

62 -3.83668e-06-3.895964e-06 -6.67511e-07

63 -2.57694e-06-2.521371e-06 -6.00164e-07

64 -1.40482e-06-1.462891e-06 -6.0146e-07

65 -4.90323e-07-4.509853e-07 -8.57924e-07

66 -6.57932e-06-2.323927e-06 -3.86932e-06

67 -2.36725e-05-6.594790e-06 -1.15753e-05

68 -6.4781e-05-1.159200e-05 -1.71983e-05

69 -0.000120955-1.810568e-05 -1.53482e-05

70 -0.000105834-1.833035e-05 -1.0845e-05

71 -4.18672e-05-1.251022e-05 -4.87568e-06

72 -1.50171e-05-7.199496e-06 -2.45893e-06

73 -5.98529e-06-3.050139e-06 -2.00237e-06

74 -1.416e-05-5.219870e-06 -4.25359e-06

75 -2.6856e-05-7.410861e-06 -7.95131e-06

76 -4.3669e-05-9.505942e-06 -9.00129e-06

77 -6.50065e-05-1.281233e-05 -1.24158e-05

78 -8.62046e-05-1.663628e-05 -8.84544e-06

79 -5.65738e-05-1.311566e-05 -8.24315e-06

80 -3.27426e-05-1.064260e-05 -4.29309e-06

81 -2.34136e-05-7.745032e-06 -5.0642e-06

Moment:

0

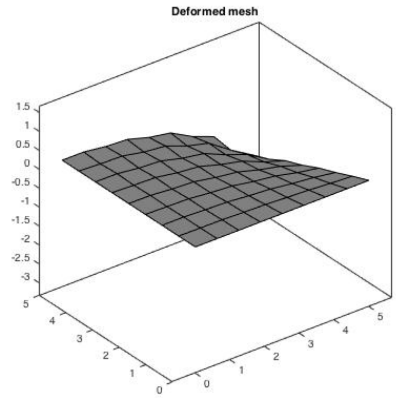
0

0

Shear:

0

0



Simply Supported Condition | Concentrated Load

Nodal Displacements:

Node Displacement(x,y,θ; m,m,rad respectively)

1 0 0 0.000341245

2 0 0 0.000457983

3 -0.00535221-3.150193e-03 0.00101889

4 0 0 0.00177395

5 0 0 0.00051937

6 -0.00239248-9.432349e-04 0.000591188

7 -0.000894773-7.226541e-04 0.00132854

8 0 0 0.000626819

9 -0.000612206-4.283374e-04 0.000670879

10 0 0 0.000489295

11 0 0 0.000493819

12 -0.00117125-4.384683e-04 0.00048988

13 -0.00372233-1.622210e-03 0.00077407

14 -0.00222278-1.420613e-03 0.00123245

15 -0.000256214-2.665854e-04 0.00144557

16 0 0 0.0010365

17 0 0 0.000409269

18 -9.09489e-05-8.648309e-05 0.000533782

19 -0.000323915-2.072937e-04 0.000555614

20 -0.000699035-3.352841e-04 0.000529037

21 -0.00137045-7.125673e-04 0.000646795

22 -0.00193351-1.162140e-03 0.000883936

23 -0.000821061-6.352202e-04 0.00088036

24 -0.000240108-2.514297e-04 0.000917206

25 -0.000173329-1.724282e-04 0.000669804

26 0 0 -0.000445128

27 0 0 -0.000602125

28 0 0 -0.000750492

29 0 0 -0.000928085

30 -0.0005643759.484317e-05 -0.00103254

31 -0.00171512.945603e-04 -0.00111965

32 -0.002932255.145786e-04 -0.00130429

33 -0.004274177.396768e-04 -0.00161079

34 -0.002941447.182805e-04 -0.0012533

35 -0.001218854.064053e-04 -0.00129143

36 -0.0003543161.759155e-04 -0.00135934

37 -1.11269e-059.811772e-06 -0.00154355

38 0 0 -0.00131192

39 0 0 -0.00079056

40 0 0 -0.000488802

41 0 0 -0.000350766

42 -1.37567e-05-1.926240e-05 0.000441419

43 -3.17891e-054.734772e-06 -0.000531521

44 -9.60873e-05-7.146367e-05 0.000516658

45 -0.0001432452.626163e-05 -0.000673335

46 -0.00024942-1.343915e-04 0.000511748

47 -0.0003299755.595595e-05 -0.000834376

48 -0.000463837-1.925217e-04 0.000474641

49 -0.0008890861.506024e-04 -0.000963877

50 -0.00139579-6.053320e-04 0.000543337

51 -0.001776973.164092e-04 -0.00107314

52 -0.00232428-1.108428e-03 0.000699485

53 -0.002613554.966788e-04 -0.00124813

54 -0.00315988-1.834486e-03 0.000987267

55 -0.001869085.276632e-04 -0.00110531

56 -0.00140058-9.859701e-04 0.000996324

57 -0.0007035572.809893e-04 -0.00104688

58 -0.000509717-4.683334e-04 0.00104548

59 -0.000138948.980927e-05 -0.00109667

60 -9.76793e-05-1.190680e-04 0.00116638

61 -1.14201e-051.027677e-05 -0.000974234

62 -6.81911e-05-8.898646e-05 0.000781252

63 -1.53997e-053.184790e-06 -0.000666521

64 -4.10986e-05-5.621603e-05 0.000550762

65 -1.09271e-05-2.627330e-07 -0.000498799

66 -0.000151223.085754e-05 -0.000634127

67 -0.0004514498.355123e-05 -0.000777194

68 -0.0009763481.779752e-04 -0.000892663

69 -0.001554293.258747e-04 -0.00100449

70 -0.001122713.414715e-04 -0.000989041

71 -0.0003461841.535214e-04 -0.000926527

72 -0.0001190825.042689e-05 -0.000785716

73 -8.62096e-052.050824e-05 -0.000613029

74 -0.000277266-2.179994e-04 0.000593247

75 -0.0004093089.116753e-05 -0.000726917

76 -0.000730534-4.177711e-04 0.000584339

77 -0.0008563491.908375e-04 -0.00085787

78 -0.00113965-7.315710e-04 0.000748206

79 -0.0006066491.910741e-04 -0.000838385

80 -0.000425705-3.652619e-04 0.000751368

81 -0.0002717598.165231e-05 -0.000733403

Moment:

-0.3775

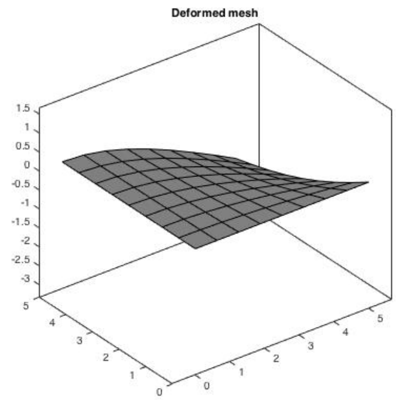
-1.2582

-0.3281

Shear:

-393.7457

-528.4432



Simply Supported Condition | Distributed Load

Nodal Displacements:

Node Displacement(x,y,θ; m,m,rad respectively)

1 0 0 9.18412e-05

2 0 0 5.39798e-05

3 -0.000417651-4.109599e-05 7.78724e-05

4 0 0 9.28462e-05

5 0 0 8.12704e-05

6 -0.00026343-2.829449e-05 6.62372e-05

7 -0.000135147-3.027574e-05 8.75105e-05

8 0 0 9.17833e-05

9 -8.01676e-05-1.805977e-05 8.31638e-05

10 0 0 8.90139e-05

11 0 0 6.97949e-05

12 -0.000144837-1.665828e-05 5.87398e-05

13 -0.000352282-3.615717e-05 7.34542e-05

14 -0.000267167-3.690597e-05 8.18729e-05

15 -3.75179e-05-1.887852e-05 9.2422e-05

16 0 0 9.2149e-05

17 0 0 9.16698e-05

18 -1.17807e-05-5.637470e-06 8.89589e-05

19 -4.23209e-05-9.628170e-06 8.16897e-05

20 -8.68122e-05-1.291755e-05 7.12855e-05

21 -0.000162037-2.355000e-05 7.51425e-05

22 -0.00022115-3.126018e-05 7.92316e-05

23 -0.000111415-2.472547e-05 8.50233e-05

24 -3.21342e-05-1.525293e-05 8.96396e-05

25 -2.25709e-05-1.077934e-05 8.92727e-05

26 0 0 -9.2732e-05

27 0 0 -9.84577e-05

28 0 0 -0.000108259

29 0 0 -0.00012165

30 -7.4897e-05-9.035337e-06 -0.00012765

31 -0.000207827-2.199562e-05 -0.000120999

32 -0.000311282-3.074914e-05 -0.000113741

33 -0.000386836-3.593227e-05 -0.000108016

34 -0.000340339-3.622532e-05 -0.000105059

35 -0.000197843-3.237619e-05 -9.99956e-05

36 -8.06212e-05-2.420390e-05 -9.45798e-05

37 -9.27818e-06-9.351959e-06 -9.12348e-05

38 0 0 -9.21967e-05

39 0 0 -9.14946e-05

40 0 0 -9.10312e-05

41 0 0 -9.07607e-05

42 -1.56149e-06-1.561067e-06 9.0084e-05

43 -5.94754e-06-2.827943e-06 -9.38722e-05

44 -1.27218e-05-3.954999e-06 8.46332e-05

45 -2.1457e-05-4.789194e-06 -0.000101611

46 -3.20136e-05-5.778219e-06 7.51081e-05

47 -4.42432e-05-6.460208e-06 -0.000112876

48 -5.82792e-05-7.585771e-06 6.21803e-05

49 -0.00011396-1.421915e-05 -0.000118862

50 -0.000165016-2.081529e-05 6.70985e-05

51 -0.000210374-2.476499e-05 -0.00011328

52 -0.000250176-3.035272e-05 7.31448e-05

53 -0.000284287-3.184419e-05 -0.000107957

54 -0.000314104-3.679311e-05 7.81566e-05

55 -0.000245137-3.221415e-05 -0.000103243

56 -0.000181693-3.126830e-05 8.31917e-05

57 -0.000124183-2.628273e-05 -9.78268e-05

58 -7.50005e-05-2.294229e-05 8.83158e-05

59 -3.58753e-05-1.620277e-05 -9.34227e-05

60 -9.87285e-06-9.807652e-06 9.11887e-05

61 -8.41449e-06-8.461167e-06 -9.26027e-05

62 -7.27294e-06-7.238992e-06 9.07992e-05

63 -5.8837e-06-5.908434e-06 -9.24782e-05

64 -4.54965e-06-4.535475e-06 9.03833e-05

65 -3.06739e-06-3.077633e-06 -9.25068e-05

66 -2.50545e-05-7.628443e-06 -9.78977e-05

67 -6.29233e-05-1.096193e-05 -0.000106977

68 -0.000126294-1.792585e-05 -0.000110079

69 -0.000193452-2.640115e-05 -0.000106239

70 -0.000163017-2.684267e-05 -0.000101836

71 -6.70301e-05-1.962584e-05 -9.62472e-05

72 -2.73482e-05-1.283891e-05 -9.37623e-05

73 -1.72712e-05-8.186911e-06 -9.38088e-05

74 -3.68626e-05-1.141856e-05 8.544e-05

75 -6.19198e-05-1.365675e-05 -0.000100631

76 -9.19704e-05-1.641508e-05 7.72578e-05

77 -0.000118359-2.010839e-05 -0.00010444

78 -0.000142177-2.490095e-05 8.03137e-05

79 -9.65336e-05-2.077376e-05 -9.92311e-05

80 -5.79796e-05-1.780963e-05 8.65278e-05

81 -4.77253e-05-1.436133e-05 -9.70759e-05

Moment:

-0.0445

-0.1483

-0.0883

Shear:

-105.9709

-62.2846

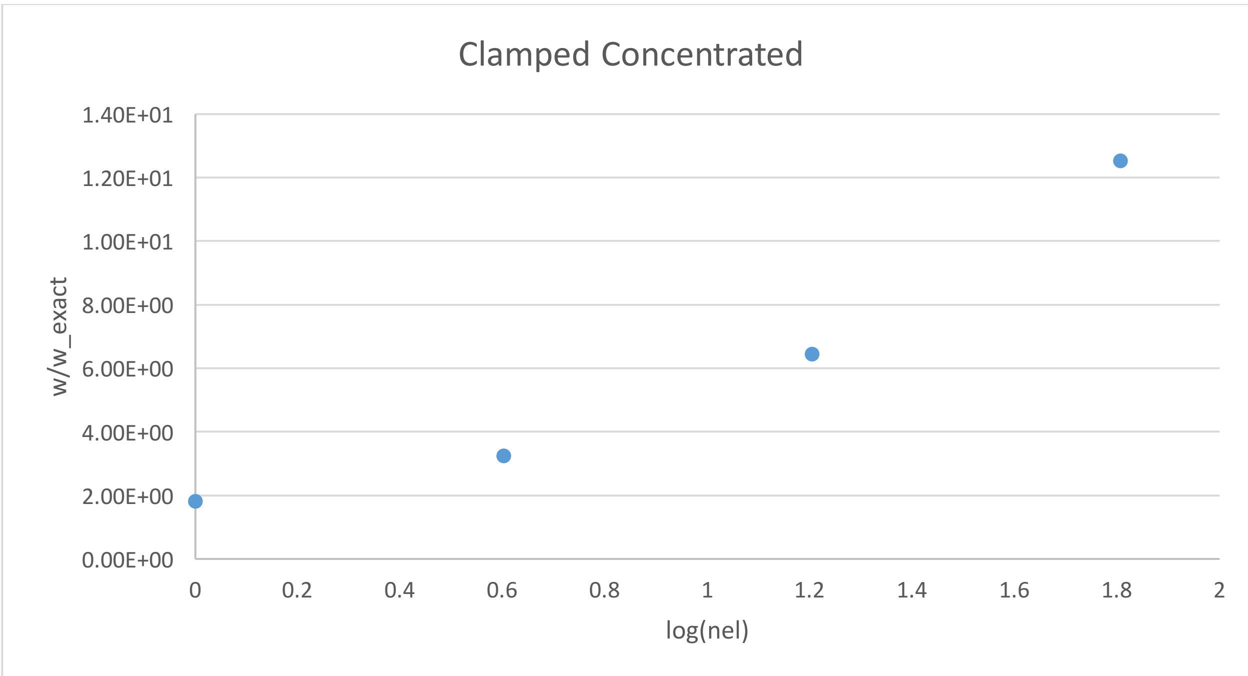
Comments: As it can be seen in most of the graphs above the plates have become smoother since the integration was changed to a 1x1 basis. The values, if analyzed also decrease in size (pay close attention to the values of inner elements, that created "kinks" in the plates), which is the reason why the plates look smoother in the first place.

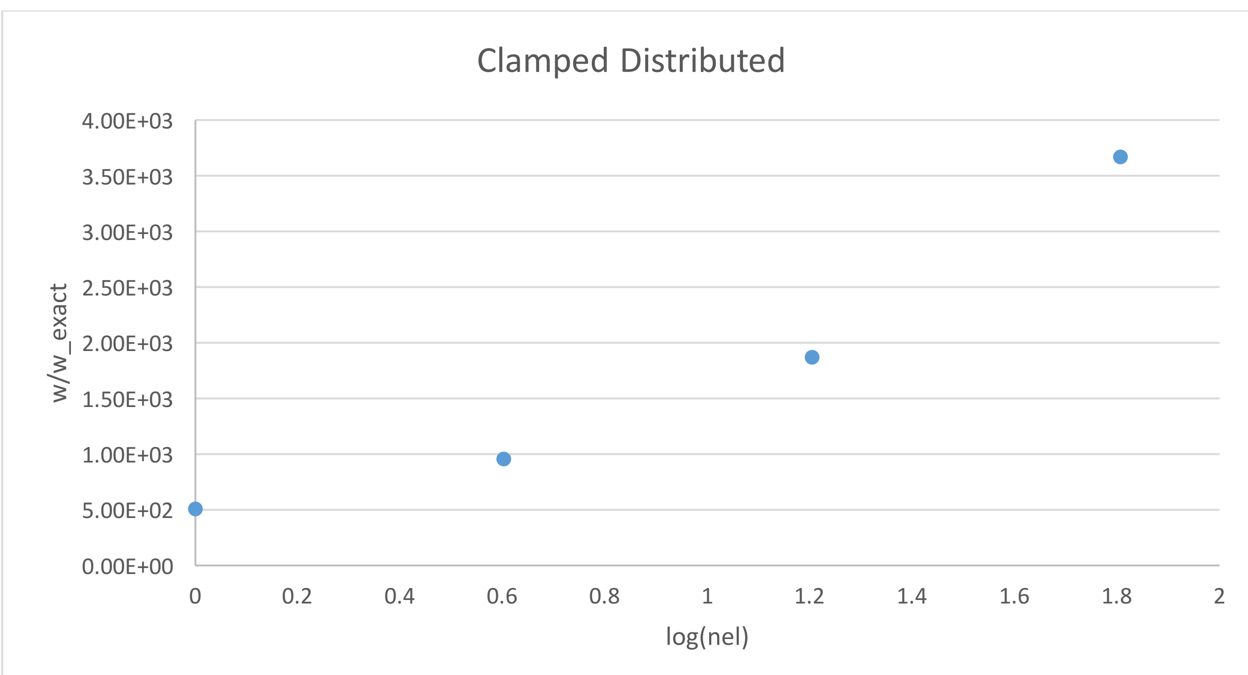
VI. Convergence studies on plate

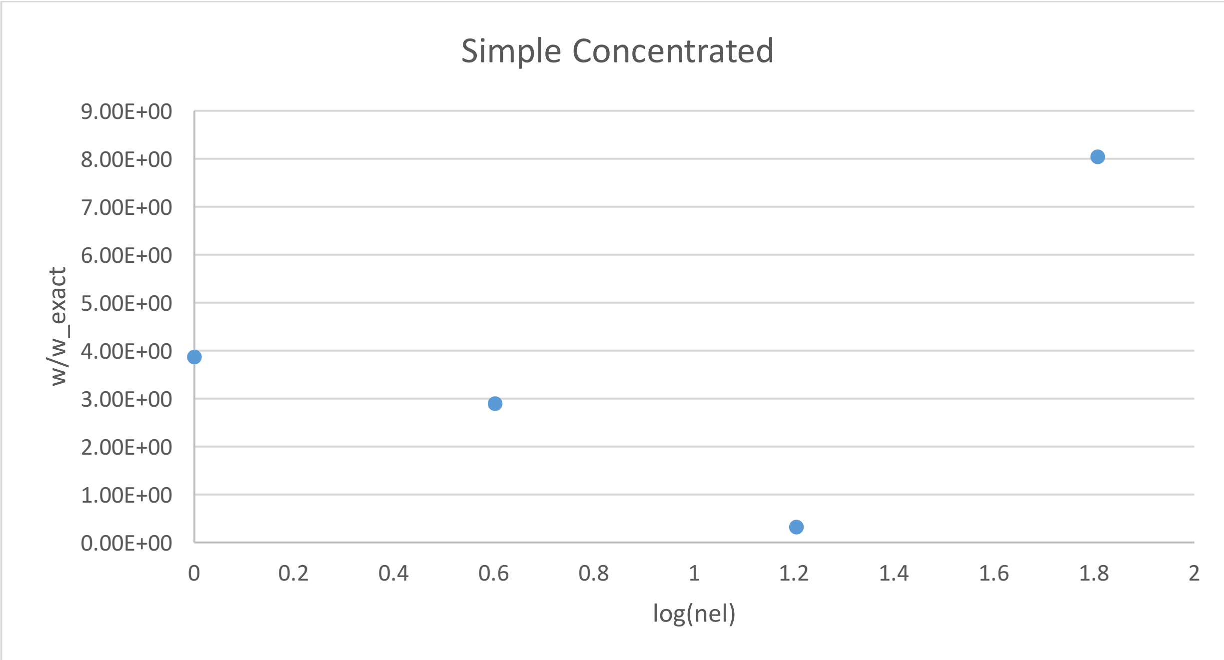
For the analysis as a whole, it seems that when the number of elements increases, the difference between exact and MATLAB-simulated values also increases. Concentrated load tended to produce more accurate values in MATLAB than distributed, which makes sense considering that we approximated the distributed load with point loads as opposed to in the case of the concentrated loading, where the exact value and point of application was put into the code.

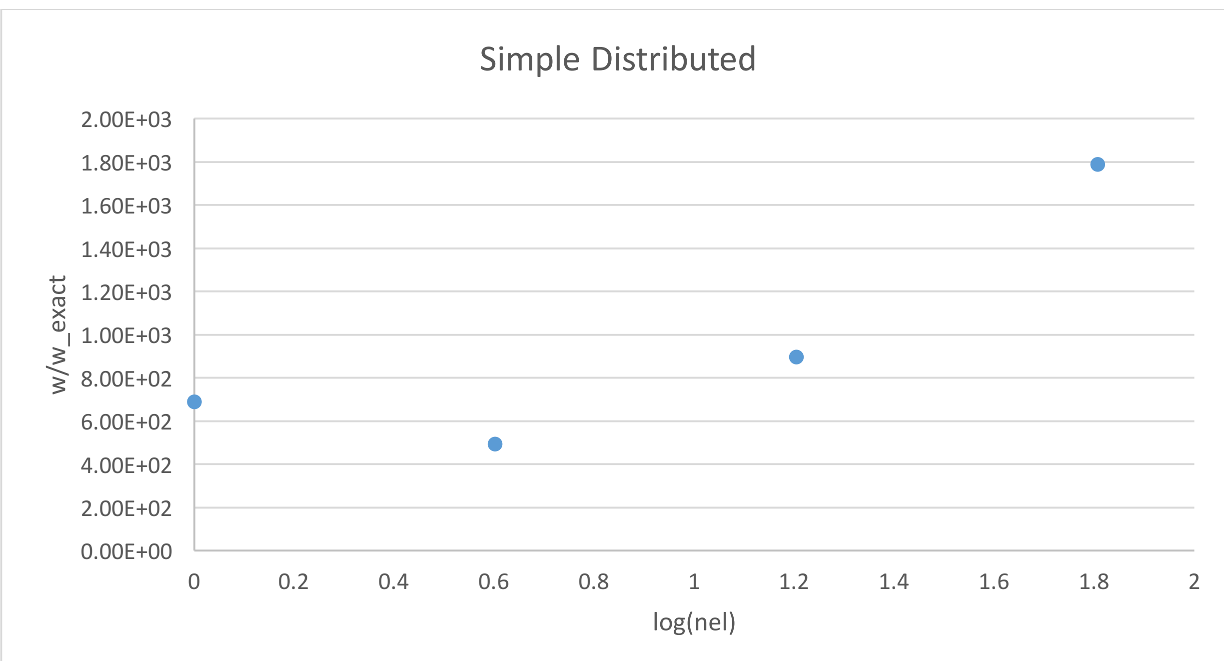
			Exact	MATLAB	Ratio	Log NEL	Average Ratio
1	Clamped	Dist	-0.000000046	-2.32E-05	5.04E+02	0	3.00E+02
		Con	-0.000203667	-0.000371008	1.82E+00	0	
	Simple	Dist	-0.000000148	-0.000101925	6.89E+02	0	
		Con	-0.000422333	-0.00163	3.86E+00	0	
4	Clamped	Dist	-0.000000046	-4.39E-05	9.55E+02	0.602059991	3.63E+02
		Con	-0.000203667	-0.000658963	3.24E+00	0.602059991	
	Simple	Dist	-0.000000148	-7.28E-05	4.92E+02	0.602059991	
		Con	-0.000422333	-0.00121781	2.88E+00	0.602059991	
16	Clamped	Dist	-0.000000046	-8.59E-05	1.87E+03	1.204119983	6.93E+02
		Con	-0.000203667	-0.00131106	6.44E+00	1.204119983	
	Simple	Dist	-0.000000148	-0.000132698	8.97E+02	1.204119983	
		Con	-0.000422333	-0.000132698	3.14E-01	1.204119983	
64	Clamped	Dist	-0.000000046	-0.000168829	3.67E+03	1.806179974	1.37E+03
		Con	-0.000203667	-0.00255106	1.25E+01		
	Simple	Dist	-0.000000148	-0.000264692	1.79E+03		
		Con	-0.000422333	-0.003394	8.04E+00		

The following graphs show the various permutations and each ratio vs. log(nel) in the permutation represented as a point.






Simple Concentrated
w/w_exact
9.00E+00
8.00E+00
7.00E+00
6.00E+00
5.00E+00
4.00E+00
3.00E+00
2.00E+00
1.00E+00
0.00E+00
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
log(nel)


Simple Distributed
w/w_exact
2.00E+03
1.80E+03
1.60E+03
1.40E+03
1.20E+03
1.00E+03
8.00E+02
6.00E+02
4.00E+02
2.00E+02
0.00E+00
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
log(nel)

VII. Code changes

Across the analysis, we created some separate files that where a single one before, such as **shape_quad.m**, which was replaced by **shape_quad_bending.m** and **shape_quad_shear.m** to calculate the bending and shear stiffness element contributions separately and then add them together in **Ke_Plate.m.**

The following are the snippets of code that were altered to fit the context and requirements of the current analysis (for the case of new functions, the entire code is included, and for the case of preexisting programs, only the snippets that were modified are highlighted in yellow)

```
Shape2_quad.m: for shape and derivative calculations (please refer to
attached file in accompanying .zip folder)

function [N, Nr, Ns] = shape2_quad(r,s)

    %o = [1; 1; 1; 1];2
    N = zeros(1,4);
    Nr = zeros(1,4);
    Ns = zeros(1,4);

    N1 = 1/4*(1-r(1))*(1-s(1));
    N2 = 1/4*(1+r(2))*(1-s(2));
    N3 = 1/4*(1+r(3))*(1+s(3));
    N4 = 1/4*(1-r(4))*(1+s(4));

    N(1,1) = N1;
    N(1,2) = N2;
    N(1,3) = N3;
    N(1,4) = N4;

    Nr(1,1) = 1/4*(-1+s(1));
    Nr(1,2) = 1/4*(1-s(2));
    Nr(1,3) = 1/4*(1+s(3));
    Nr(1,4) = 1/4*(-1-s(4));
```

```
    Ns(1,1) = 1/4*(-1+r(1));
    Ns(1,2) = 1/4*(-1-r(2));
    Ns(1,3) = 1/4*(1+r(3));
    Ns(1,4) = 1/4*(1-r(4));
```

Shape_quad_bending.m: to assemble the B matrix for Bending contributions.

```
function [Bblarge] = shape_quad_bending(r, s, xn, ien, nen)

    [N, Nr, Ns] = shape2_quad(r,s);

    Bb = zeros(3,3,nen);

    for i=1:nen
    Bb(1,2,i) = Nr(1,i);
    Bb(2,3,i) = Ns(1,i);
    Bb(3,2,i) = Ns(1,i);
    Bb(3,3,i) = Nr(1,i);
    %disp(Bb);
    end


    Bblarge=horzcat(Bb(:,:,1), Bb(:,:,2), Bb(:,:,3), Bb(:,:,4));

    disp(Bblarge);
end
```

Shape_quad_shear.m: to assemble the B matrix for Shear contributions.

```
function [Bslarge] = shape_quad_shear(r, s, xn, ien, nen)
x = zeros(1,nen);
y = zeros(1,nen);

    for i= 1:nen
        x(1,i) = xn(1, ien(i));
        y(1,i) = xn(2, ien(i));
    end

    [N, Nr, Ns] = shape2_quad(r,s);

    Bs = zeros(2,2,nen);
```

```
  for i=1:nen

      Bs(1,1,i)=Nr(1,i);
      Bs(1,2,i)=-N(1,i);
      Bs(2,1,i)=Ns(1,i);
      Bs(2,3,i)=-N(1,i);
   end


   Bslarge=horzcat(Bs(:,:,1), Bs(:,:,2), Bs(:,:,3), Bs(:,:,4));

   disp(Bslarge);
end
```

Jacobian_2d.m: to assemble the Jacobian matrix containing individual Jacobian values for the corner points of each quad.

```
function [J] = jacobian_2d(r, s, x, y, nen)
    J = zeros(2);
    J(1,1) = 1/4*((1-s(1))*(x(2)-x(1)) + (1+s(1))*(x(3)-x(4)));
    J(1,2) = 1/4*((1-s(2))*(y(2)-y(1)) + (1+s(2))*(y(3)-y(4)));
    J(2,1) = 1/4*((1-r(3))*(x(4)-x(1)) + (1+r(3))*(x(3)-x(2)));
    J(2,2) = 1/4*((1-r(4))*(y(4)-y(1)) + (1+r(4))*(y(3)-y(2)));

end
```

Ke_plate.m: to compute the final element stiffness matrix using the contributions and calculated values generated by the functions above.

```
function [ke]=Ke_plate(r,s,t,v,E,l,xn,ien,nen,ndf,nsd,point,weight,nglx,ngly)

k=5/6;

x = zeros(1,nen);
y = zeros(1,nen);
    for i= 1:nen
        x(1,i) = xn(1, ien(i));
        y(1,i) = xn(2, ien(i));
    end

        Db=zeros(3,3);
        Db=(3.2967e7)*[1 v 0; v 1 0; 0 0 0.5*(1-v)];
        Ds=zeros(2,2);
        Ds=(1.15385e7)*[1 0; 0 1];
        [J] = jacobian_2d(r,s, x, y, nen);
```

```
        detJ = det(J);
        ke  = zeros(12,12);
        keb = zeros(12,12);
        kes = zeros(12,12);

                for j=1:l

                    [Bblarge] = shape_quad_bending(r, s, xn, ien, nen);
                    [Bslarge] = shape_quad_shear(r, s, xn, ien, nen);
                    disp(Bblarge);
                    disp(Bslarge);

                    kjb=0.6220*Bblarge.'*Db*Bblarge;
                    kjs=0.6220*Bslarge.'*Ds*Bslarge;
                    keb = keb+t^3/12*kjb;
                    kes = kes+t*k*kjs;

                end
       ke = keb+kes;
       disp(keb);
       disp(kes);
       disp(ke);
end
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% final_project.m - 1/12/16                                %
% author: Tehila Stone | Theo Dimitrasopoulos               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear; % removes all variables from the workspace.

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                           DATA                           %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%
% Material %
%%%%%%%%%%%%

L = 10;     % meters
t = 0.1;    % meters
E = 30.0e6; % MPa
v = 0.3;

%%%%%%%%%%%%
%   Other  %
%%%%%%%%%%%%

icase = 0;

%%%%%%%%
% Mesh %
%%%%%%%%
nsd=2;       % number of space dimensions
ndf=3;       % number of freedom per node
nen=4;       % number of element nodes

nel=64;      % number of elements. Changes between 1,4,16 and 64
nnp=81;      % number of nodal points. Changes between 4,9,25 and 81
nglx = 2;
ngly = 2;
nglz = 0;
```

```
%%%%%%%%%%%%%%%%%%%%%
% Nodal coordinates %
%%%%%%%%%%%%%%%%%%%%%
% xn(i,N):= coordinate i for node N
% N=1,...,nnp
% i=1,...,nsd
xn=zeros(nsd,nnp);

% 1 or more elements
xn(1,2) = 5;
xn(1,3) = 5;
xn(2,3) = 5;
xn(2,4) = 5;

% 4 or more elements
xn(1,5) = 2.5;
xn(1,6) = 5;
xn(2,6) = 2.5;
xn(1,7) = 2.5;
xn(2,7) = 5;
xn(2,8) = 2.5;
xn(1,9) = 2.5;
xn(2,9) = 2.5;


% 16 or more elements
xn(1,10)=1.25;
xn(1,11)=3.75;
xn(1,12)=5;
xn(2,12)=1.25;
xn(1,13)=5;
xn(2,13)=3.75;
xn(1,14)=3.75;
xn(2,14)=5;
xn(1,15)=1.25;
xn(2,15)=5;
xn(2,16)=3.75;
xn(2,17)=1.25;
xn(1,18)=1.25;
xn(2,18)=1.25;
xn(1,19)=2.5;
xn(2,19)=1.25;
xn(1,20)=3.75;
xn(2,20)=1.25;
xn(1,21)=3.75;
xn(2,21)=2.5;
xn(1,22)=3.75;
```

```
xn(2,22)=3.75;
xn(1,23)=2.5;
xn(2,23)=3.75;
xn(1,24)=1.25;
xn(2,24)=3.75;
xn(1,25)=1.25;
xn(2,25)=2.5;

%64 or more elements
xn(1,26)=0.625;
xn(1,27)=1.875;
xn(1,28)=3.125;
xn(1,29)=4.375;
xn(1,30)=5;
xn(2,30)=0.625;
xn(1,31)=5;
xn(2,31)=1.875;
xn(1,32)=5;
xn(2,32)=3.125;
xn(1,33)=5;
xn(2,33)=4.375;
xn(1,34)=4.375;
xn(2,34)=5;
xn(1,35)=3.125;
xn(2,35)=5;
xn(1,36)=1.875;
xn(2,36)=5;
xn(1,37)=0.625;
xn(2,37)=5;
xn(2,38)=4.375;
xn(2,39)=3.125;
xn(2,40)=1.875;
xn(2,41)=0.625;
xn(1,42)=0.625;
xn(2,42)=0.625;
xn(1,43)=1.25;
xn(2,43)=0.625;
xn(1,44)=1.875;
xn(2,44)=0.625;
xn(1,45)=2.5;
xn(2,45)=0.625;
xn(1,46)=3.125;
xn(2,46)=0.625;
xn(1,47)=3.75;
xn(2,47)=0.625;
xn(1,48)=4.375;
xn(2,48)=0.625;
xn(1,49)=4.375;
```

```
xn(2,49)=1.25;
xn(1,50)=4.375;
xn(2,50)=1.875;
xn(1,51)=4.375;
xn(2,51)=2.5;
xn(1,52)=4.375;
xn(2,52)=3.125;
xn(1,53)=4.375;
xn(2,53)=3.75;
xn(1,54)=4.375;
xn(2,54)=4.375;
xn(1,55)=3.75;
xn(2,55)=4.375;
xn(1,56)=3.125;
xn(2,56)=4.375;
xn(1,57)=2.5;
xn(2,57)=4.375;
xn(1,58)=1.875;
xn(2,58)=4.375;
xn(1,59)=1.25;
xn(2,59)=4.375;
xn(1,60)=0.625;
xn(2,60)=4.375;
xn(1,61)=0.625;
xn(2,61)=3.75;
xn(1,62)=0.625;
xn(2,62)=3.125;
xn(1,63)=0.625;
xn(2,63)=2.5;
xn(1,64)=0.625;
xn(2,64)=1.875;
xn(1,65)=0.625;
xn(2,65)=1.25;
xn(1,66)=1.875;
xn(2,66)=1.25;
xn(1,67)=3.125;
xn(2,67)=1.25;
xn(1,68)=3.75;
xn(2,68)=1.875;
xn(1,69)=3.75;
xn(2,69)=3.125;
xn(1,70)=3.125;
xn(2,70)=3.75;
xn(1,71)=1.875;
xn(2,71)=3.75;
xn(1,72)=1.25;
xn(2,72)=3.125;
xn(1,73)=1.25;
```

```
xn(2,73)=1.875;
xn(1,74)=1.875;
xn(2,74)=1.875;
xn(1,75)=2.5;
xn(2,75)=1.875;
xn(1,76)=3.125;
xn(2,76)=1.875;
xn(1,77)=3.125;
xn(2,77)=2.5;
xn(1,78)=3.125;
xn(2,78)=3.125;
xn(1,79)=2.5;
xn(2,79)=3.125;
xn(1,80)=1.875;
xn(2,80)=3.125;
xn(1,81)=1.875;
xn(2,81)=2.5;

%%%%%%%%%%%%%%%%
% Connectivity %
%%%%%%%%%%%%%%%%
% ien(a,e)=N
% N: global node number - N=1,...,nnp
% e: element number - e=1,...,nel
% a: local node number - a=1,...,nen
% each set will be activated separately

ien=zeros(nen,nel);

% 1 element
% ien(1,1) = 1;
% ien(2,1) = 2;
% ien(3,1) = 3;
% ien(4,1) = 4;

% 4 elements
% ien(1,1)=1;     ien(1,2)=5;     ien(1,3)=9;     ien(1,4)=8;
% ien(2,1)=5;     ien(2,2)=2;     ien(2,3)=6;     ien(2,4)=9;
% ien(3,1)=9;     ien(3,2)=6;     ien(3,3)=3;     ien(3,4)=7;
% ien(4,1)=8;     ien(4,2)=9;     ien(4,3)=7;     ien(4,4)=4;

% 16 elements
% ien(1,1)=1;     ien(1,2)=10;    ien(1,3)=5;     ien(1,4)=11;
% ien(2,1)=10;    ien(2,2)=5;     ien(2,3)=11;    ien(2,4)=2;
% ien(3,1)=18;    ien(3,2)=19;    ien(3,3)=20;    ien(3,4)=12;
% ien(4,1)=17;    ien(4,2)=18;    ien(4,3)=19;    ien(4,4)=20;
%
```

```
% ien(1,5)=20;    ien(1,6)=21;    ien(1,7)=22;    ien(1,8)=23;
% ien(2,5)=12;    ien(2,6)=6;     ien(2,7)=13;    ien(2,8)=22;
% ien(3,5)=6;     ien(3,6)=13;    ien(3,7)=3;     ien(3,8)=14;
% ien(4,5)=21;    ien(4,6)=22;    ien(4,7)=14;    ien(4,8)=7;
%
% ien(1,9)=24;    ien(1,10)=16;   ien(1,11)=8;    ien(1,12)=17;
% ien(2,9)=23;    ien(2,10)=24;   ien(2,11)=25;   ien(2,12)=18;
% ien(3,9)=7;     ien(3,10)=15;   ien(3,11)=24;   ien(3,12)=25;
% ien(4,9)=15;    ien(4,10)=4;    ien(4,11)=16;   ien(4,12)=8;
%
% ien(1,13)=18;   ien(1,14)=19;   ien(1,15)=9;    ien(1,16)=25;
% ien(2,13)=19;   ien(2,14)=20;   ien(2,15)=21;   ien(2,16)=9;
% ien(3,13)=9;    ien(3,14)=21;   ien(3,15)=22;   ien(3,16)=23;
% ien(4,13)=25;   ien(4,14)=9;    ien(4,15)=23;   ien(4,16)=24;

% 64 elements
ien(1,1)=1;     ien(1,2)=26;    ien(1,3)=10;    ien(1,4)=27;
ien(2,1)=26;    ien(2,2)=10;    ien(2,3)=27;    ien(2,4)=5;
ien(3,1)=42;    ien(3,2)=43;    ien(3,3)=44;    ien(3,4)=45;
ien(4,1)=41;    ien(4,2)=42;    ien(4,3)=43;    ien(4,4)=44;

ien(1,5)=5;    ien(1,6)=28;    ien(1,7)=11;     ien(1,8)=29;
ien(2,5)=28;    ien(2,6)=11;    ien(2,7)=29;    ien(2,8)=2;
ien(3,5)=46;    ien(3,6)=47;    ien(3,7)=48;    ien(3,8)=30;
ien(4,5)=45;    ien(4,6)=46;    ien(4,7)=47;    ien(4,8)=48;

ien(1,9)=48;    ien(1,10)=49;    ien(1,11)=50;    ien(1,12)=51;
ien(2,9)=30;    ien(2,10)=12;    ien(2,11)=31;    ien(2,12)=6;
ien(3,9)=12;    ien(3,10)=31;    ien(3,11)=6;     ien(3,12)=32;
ien(4,9)=49;    ien(4,10)=50;    ien(4,11)=51;    ien(4,12)=52;

ien(1,13)=52;    ien(1,14)=53;    ien(1,15)=54;    ien(1,16)=55;
ien(2,13)=32;    ien(2,14)=13;    ien(2,15)=33;    ien(2,16)=54;
ien(3,13)=13;    ien(3,14)=33;    ien(3,15)=3;     ien(3,16)=34;
ien(4,13)=53;    ien(4,14)=54;    ien(4,15)=34;    ien(4,16)=14;

ien(1,17)=56;    ien(1,18)=57;    ien(1,19)=58;    ien(1,20)=59;
ien(2,17)=55;    ien(2,18)=56;    ien(2,19)=57;    ien(2,20)=58;
ien(3,17)=14;    ien(3,18)=35;    ien(3,19)=7;     ien(3,20)=36;
ien(4,17)=35;    ien(4,18)=7;     ien(4,19)=36;     ien(4,20)=15;

ien(1,21)=60;    ien(1,22)=38;    ien(1,23)=16;    ien(1,24)=39;
ien(2,21)=59;    ien(2,22)=60;    ien(2,23)=61;    ien(2,24)=62;
ien(3,21)=15;    ien(3,22)=37;    ien(3,23)=60;    ien(3,24)=61;
ien(4,21)=37;    ien(4,22)=4;     ien(4,23)=38;    ien(4,24)=16;

ien(1,25)=8;     ien(1,26)=40;   ien(1,27)=17;    ien(1,28)=41;
```

```
ien(2,25)=63;     ien(2,26)=64;    ien(2,27)=65;     ien(2,28)=42;
ien(3,25)=62;     ien(3,26)=63;    ien(3,27)=64;     ien(3,28)=65;
ien(4,25)=39;     ien(4,26)=8;     ien(4,27)=40;     ien(4,28)=17;

ien(1,29)=42;     ien(1,30)=43;     ien(1,31)=44;     ien(1,32)=45;
ien(2,29)=43;     ien(2,30)=44;     ien(2,31)=45;     ien(2,32)=46;
ien(3,29)=18;     ien(3,30)=66;     ien(3,31)=19;     ien(3,32)=67;
ien(4,29)=65;     ien(4,30)=18;     ien(4,31)=66;     ien(4,32)=19;

ien(1,33)=46;     ien(1,34)=47;     ien(1,35)=20;     ien(1,36)=68;
ien(2,33)=47;     ien(2,34)=48;     ien(2,35)=49;     ien(2,36)=50;
ien(3,33)=20;     ien(3,34)=49;     ien(3,35)=50;     ien(3,36)=51;
ien(4,33)=67;     ien(4,34)=20;     ien(4,35)=68;     ien(4,36)=21;

ien(1,37)=21;     ien(1,38)=69;     ien(1,39)=22;     ien(1,40)=70;
ien(2,37)=51;     ien(2,38)=52;     ien(2,39)=53;     ien(2,40)=22;
ien(3,37)=52;     ien(3,38)=53;     ien(3,39)=54;     ien(3,40)=55;
ien(4,37)=69;     ien(4,38)=22;     ien(4,39)=55;     ien(4,40)=56;

ien(1,41)=23;     ien(1,42)=71;     ien(1,43)=24;     ien(1,44)=61;
ien(2,41)=70;     ien(2,42)=23;     ien(2,43)=71;     ien(2,44)=24;
ien(3,41)=56;     ien(3,42)=57;     ien(3,43)=58;     ien(3,44)=59;
ien(4,41)=57;     ien(4,42)=58;     ien(4,43)=59;     ien(4,44)=60;

ien(1,45)=62;     ien(1,46)=63;     ien(1,47)=64;     ien(1,48)=65;
ien(2,45)=72;     ien(2,46)=25;     ien(2,47)=73;     ien(2,48)=18;
ien(3,45)=24;     ien(3,46)=72;     ien(3,47)=25;     ien(3,48)=73;
ien(4,45)=61;     ien(4,46)=62;     ien(4,47)=63;     ien(4,48)=64;

ien(1,49)=18;     ien(1,50)=66;     ien(1,51)=19;     ien(1,52)=67;
ien(2,49)=66;     ien(2,50)=19;     ien(2,51)=67;     ien(2,52)=20;
ien(3,49)=74;     ien(3,50)=75;     ien(3,51)=76;     ien(3,52)=68;
ien(4,49)=73;     ien(4,50)=74;     ien(4,51)=75;     ien(4,52)=76;

ien(1,53)=76;     ien(1,54)=77;     ien(1,55)=78;     ien(1,56)=79;
ien(2,53)=68;     ien(2,54)=21;     ien(2,55)=69;     ien(2,56)=78;
ien(3,53)=21;     ien(3,54)=69;     ien(3,55)=22;     ien(3,56)=70;
ien(4,53)=77;     ien(4,54)=78;     ien(4,55)=70;     ien(4,56)=23;

ien(1,57)=80;     ien(1,58)=72;     ien(1,59)=25;     ien(1,60)=73;
ien(2,57)=79;     ien(2,58)=80;     ien(2,59)=81;     ien(2,60)=74;
ien(3,57)=23;     ien(3,58)=71;     ien(3,59)=80;     ien(3,60)=81;
ien(4,57)=71;     ien(4,58)=24;     ien(4,59)=72;     ien(4,60)=25;

ien(1,61)=74;     ien(1,62)=75;     ien(1,63)=9;     ien(1,64)=81;
ien(2,61)=75;     ien(2,62)=76;     ien(2,63)=77;     ien(2,64)=9;
```

```
ien(3,61)=9;    ien(3,62)=77;    ien(3,63)=78;    ien(3,64)=79;
ien(4,61)=81;    ien(4,62)=9;    ien(4,63)=79;    ien(4,64)=80;

%%%%%%%%%%%%%%%%%%%%%%%
% Boundary conditions %
%%%%%%%%%%%%%%%%%%%%%%%
% prescribed displacement (essential boundary condition)
%
% idb(i,N)=1 if the degree of freedom i of the node N is prescribed
%          =0 otherwise
%
% 1) initialize idb to 0

idb=zeros(ndf,nnp);

% 2) enter the flag for prescribed displacement boundary conditions

% 1 or more elements
idb(1,1) = 1; idb(2,1) = 1; idb(3,1) = 1;
idb(1,2) = 1; idb(2,2) = 1; idb(3,2) = 1;
idb(1,4) = 1; idb(2,4) = 1; idb(3,4) = 1;

% 4 or more elements
idb(1,5) = 1; idb(2,5) = 1; idb(3,5) = 1;
idb(1,8) = 1; idb(2,8) = 1; idb(3,8) = 1;

% 16 or more elements
 idb(1,10) = 1;  idb(2,10) = 1;  idb(3,10) = 1;
 idb(1,11) = 1;  idb(2,11) = 1;  idb(3,11) = 1;
 idb(1,16) = 1;  idb(2,16) = 1;  idb(3,16) = 1;
 idb(1,17) = 1;  idb(2,17) = 1;  idb(4,17) = 1;

% 64 or more elements
 idb(1,38) = 1;  idb(2,38) = 1;  idb(3,38) = 1;
 idb(1,39) = 1;  idb(2,39) = 1;  idb(3,39) = 1;
 idb(1,40) = 1;  idb(2,40) = 1;  idb(3,40) = 1;
 idb(1,41) = 1;  idb(2,41) = 1;  idb(4,41) = 1;
 idb(1,26) = 1;  idb(2,26) = 1;  idb(3,26) = 1;
 idb(1,27) = 1;  idb(2,27) = 1;  idb(3,27) = 1;
 idb(1,28) = 1;  idb(2,28) = 1;  idb(3,28) = 1;
 idb(1,29) = 1;  idb(2,29) = 1;  idb(4,29) = 1;
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% prescribed nodal displacement boundary conditions %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% g(i,N): prescribed displacement for the dof i of node N
% initialize g
g=zeros(ndf,nnp);
% enter the values


%%%%%%%%%%%%%%%%%%%%%%%%%%%
% prescribed nodal forces %
%%%%%%%%%%%%%%%%%%%%%%%%%%%
% f(i,N): prescribed force for the dof i of node N
% initialize f
f=zeros(ndf,nnp);
% enter the values
%only one condition should be active at a time.

% concentrated load - for all meshes
%f(2,3)=-1;  %kN

% distributed load - 1 element
% for i = 1:4
%      f(2,i)=-0.0625;
% end

% distributed load - 4 elements
% for i = 1:9
%     f(2,i)=-0.02778;
% end

% distributed load - 16 elements
% for i = 1:25
%     f(2,i) = -0.01;
% end

% distributed load - 64 elements
for i = 1:81
   f(2,i) = -0.003086;
end
```

```
%-----------------------------------------------------------------

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% number the equations; build the id table %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
[id,neq]=number_eq(idb,nnp,ndf)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%      Gaussian Integration Parameters     %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
[point,weight]=gauss(nglx,ngly,nglz,nsd)

        r = zeros(4,1);
        s = zeros(4,1);
        r(1) = point(1,1);
        r(2) = point(2,1);
        r(3) = point(2,2);
        r(4) = point(1,2);
        s(1) = point(1,1);
        s(2) = point(1,2);
        s(3) = point(2,1);
        s(4) = point(2,2);


%the following content was only altered in terms of the names of the
stiffness element matrix (ke instead of Ke for example)

for e=1:nel
  [ke(:,:,e)] =
Ke_plate(r,s,t,v,E,l,xn,ien(:,e),nen,ndf,nsd,point,weight,nglx,ngly);
end;

disp(ke);


% Contribution of the prescribed displacements to the elemental force vector
%f=f-Ke*Ue;
```

(this section was omitted as no changes were made in the code.)

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                         Assembly operation                           %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


       (this section was omitted as no changes were made in the code.)


%%%%%%%%%%%%%%%%%%%%
%  Post-processing %
%%%%%%%%%%%%%%%%%%%%


       (this section was omitted as no changes were made in the code.)


%%%%%%%%%%%%%%%%%%%%
% Moment and Shear %
%%%%%%%%%%%%%%%%%%%%

[m] = zeros(3,1);
[s] = zeros(2,1);
kappa = zeros(3,1);
gamma = zeros(2,1);
Db=zeros(3,3);
Db=(3.2967e7)*[1 v 0; v 1 0; 0 0 0.5*(1-v)];
Ds=zeros(2,2);
Ds=(1.15385e7)*[1 0; 0 1];


% Calculate curvature kappa. Extracting directly from the displacement matrix
[kappa] = [Ucomp(2);
           Ucomp(6);
           Ucomp(3) + Ucomp(5)];

% Calculate shear gamma. Extracting directly from the displacement matrix
[gamma] = [-sqrt(Ucomp(2)^2+Ucomp(3)^2) + Ucomp(1);
           -sqrt(Ucomp(5)^2+Ucomp(6)^2) + Ucomp(4)];



% Calculate moment vector
[m] = (-t^3/12)*Db*kappa;
disp('Moment');
[m]
```

```
% Calculate shear vector
[s] = t*Ds*gamma;
disp('Shear');
[s]


%%%%%%%%%%%%%%%%%%%%
% plot the results %
%%%%%%%%%%%%%%%%%%%%

plot_results_shell(icase,xn,Ucomp,ien,nel,nen,nsd,ndf,nnp);
```

A. Appendix

This is the system that we used to number our plate section: