Homework 6 Due: Friday, November 20, 2015 @ 5:00:00 PM in E218

For each problem:

i) Find the exact solution for the deflection, $w(x)$.

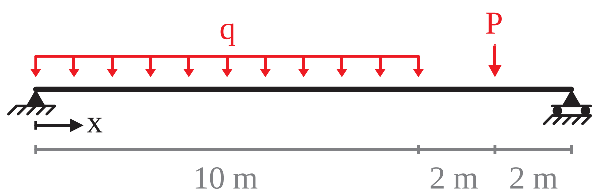
ii) Define and apply Boundary Conditions to solve for d_1 and d_0:

$$\bar{w}(x) = d_3 x^3 + d_2 x^2 + d_1 x + d_0$$

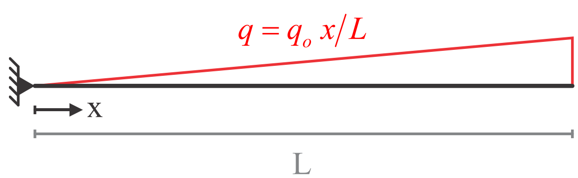
iii) Solve for the remaining unknown coefficients using:
   a. The Variational Principle
   b. The Method of Weighted Residuals

iv) Plot your three solutions on one graph and discuss how they compare.

1. Beam: q = 2 kN/m and P = 10 kN. E = 200,000 MPa, I = 1 x 10^9 mm^4. Note that the exact solution will be piecewise. The approximate solution, while not piecewise, will require piecewise integration. **25 pts.**



2. Truss: A = 100 mm^2, E = 200,000 MPa, L = 10 m, q_o = 2 kN/m. **15 pts.**



Note: The axial deformation, w, of a truss element loaded by a distributed axial load, q, is governed by the differential equation:

$$q = -EA\frac{d^2 w}{dx^2}$$

The total potential energy for a truss element can be expressed as:

$$\Pi = \tfrac{1}{2} EA \int_L \left(\frac{dw}{dx}\right)^2 dx - \int_L (qw)\, dx$$

Homework 7 Due: Monday, December 7, 2015 @ 5:00:00 PM in E218

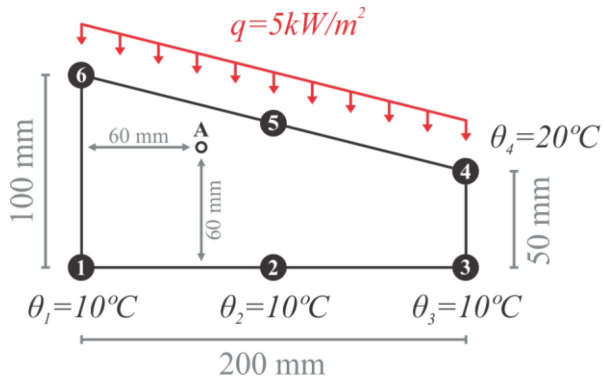
Solve **Question 1** by hand:

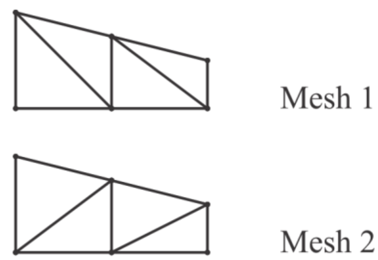
- i) Redraw the diagram and label your elements & degrees of freedom.
- ii) Generate the force vector as well as the element <u>and</u> global stiffness matrices.
- iii) Solve for the nodal temperature values and the temperature at point A.

Code in the heat triangle element into MATLAB and perform an analysis for **Questions 1 and 2**:

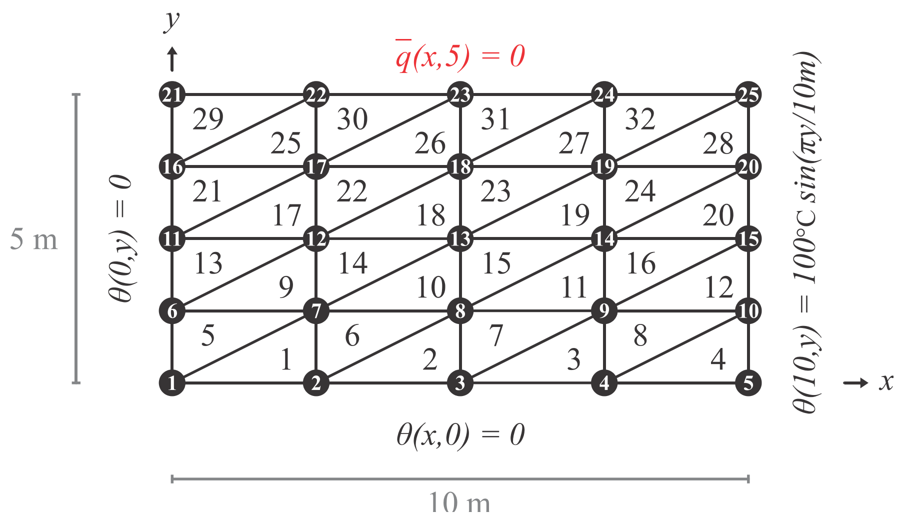
- iv) Provide labeled images of your input geometry and a plot of your temperature distribution.
- v) Compare (tabulate and discuss) your hand calculations with your MATLAB results.
- vi) Provide a short description of your code changes and attach a printout <u>only</u> of the parts of the code that you changed.

1. 2D heat: Perform all analyses for both meshes. t = 10 mm. $k_x = k_y = 50$ W/m°C. Q = 0. **25 pts**





2. 2D heat: Instead of hand calculations, compare your answer to the exact solution. Q = 0. **15 pts**

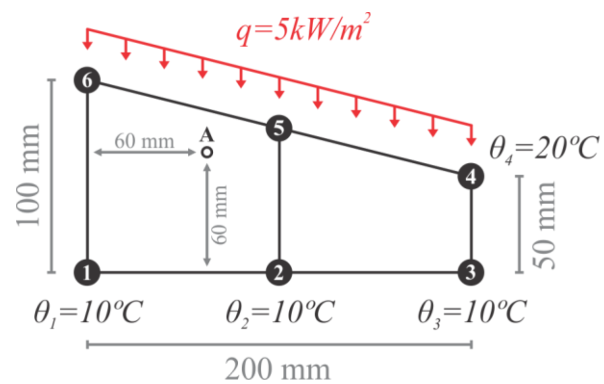


Exact solution: $\theta(x,y) = \dfrac{100°C}{\sinh(\pi)} \sin\left(\dfrac{\pi y}{10m}\right) \sinh\left(\dfrac{\pi x}{10m}\right)$

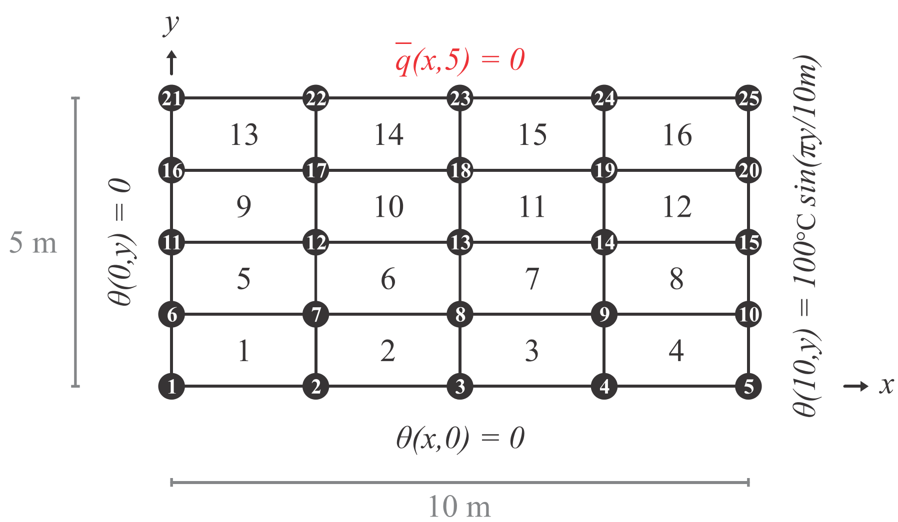
Homework 8 Due: Monday, December 7, 2015 @ 5:00:00 PM in E218

Solve **Question 1** by hand:
i) Redraw the diagram and label your elements & degrees of freedom.
ii) Generate the force vector as well as the element and global stiffness matrices.
iii) Solve for the nodal temperature values and the temperature at point A.
Code in the heat quad element into MATLAB and perform an analysis for **Questions 1 and 2**:
iv) Provide labeled images of your input geometry and a plot of your temperature distribution.
v) Compare (tabulate and discuss) your hand calculations with your MATLAB results as well as your calculations from Homework 7.
vi) Provide a short description of your cod changes and attach a printout only of the parts of the code that you changed.

1. 2D heat: t = 10 mm. $k_x = k_y = 50$ W/m℃. Q = 0. **25 pts**



2. 2D heat: Instead of hand calculations, compare your answer to the exact solution. Q = 0. **15 pts**



Exact solution: $$\theta(x,y)=\frac{100°C}{\sinh(\pi)}\sin\left(\frac{\pi y}{10m}\right)\sinh\left(\frac{\pi x}{10m}\right)$$

Homework 9 Due: Friday, December 11, 2015 @ 5:00:00 PM in E218

For both **Question 1 and 2**:

- i) Redraw the diagram and label your elements & degrees of freedom.
- ii) State whether the problem is Plane Stress or Plane Strain (and why).
- iii) Calculate the applied force vector.

Solve **Question 1** by hand:

- iv) Generate all element stiffness matrices as well as $[K_{UU}]$ and $[K_{US}]$ without generating $[K_G]$.
- v) Solve for the nodal displacements.
- vi) Find the displacements, $\{\varepsilon\}$, $\{\sigma\}$, σ_1, σ_2, and θ_p at point A.

Code the elastic triangle into MATLAB and perform an analysis for **Questions 1 and 2**:

- vii) Provide images of the input and deformed geometry as well as plots of σ_x, σ_y, τ_{xy}, σ_1 & σ_2.
- viii) Provide a description of your code changes and only the modified parts of your code.

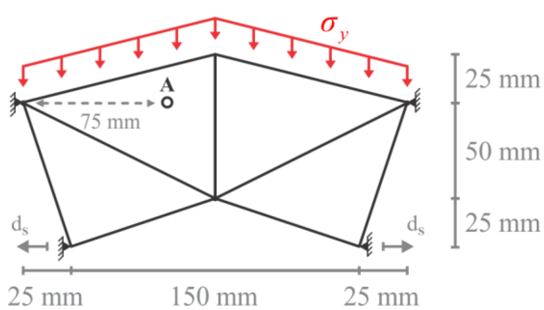
Perform an analysis with SAP2000 for **Questions 1 and 2**:

- ix) Provide images of the input and deformed geometry as well as plots of σ_x, σ_y, τ_{xy}, σ_1 & σ_2.

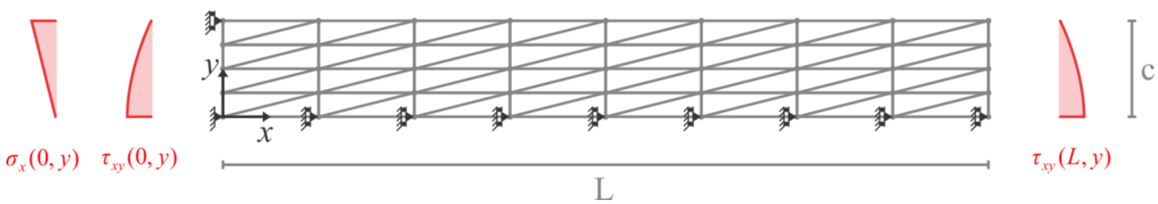
Finally, for both **Questions 1 and 2**:

- x) Tabulate the displacements from your MATLAB, SAP2000, and hand calculations.
- xi) Compare and discuss your results based on the tabulated displacements and the stress plots.

1. 2D Elasticity: t = 10 m, E = 200 000 MPa, $\upsilon = 0.3$, $d_s = 0.1$ mm, and $\sigma_y = 50$ MPa down. **35 pts**



2. 2D Elasticity: L = 16 m, c = 2 m, t = 10 mm, E = 10 000 MPa, $\upsilon = 0.3$, and P = -10 kN. **25 pts**



The non-zero applied surface tractions are defined in global coordinates as:

$$\sigma_x(0,y) = -\frac{3PL}{2tc^3}y \qquad \tau_{xy}(0,y) = \tau_{xy}(L,y) = \frac{3P}{4tc^3}\left(c^2 - y^2\right)$$

Use the exact solution, instead of hand calculations, for your comparison:

$$\begin{Bmatrix} u_x \\ u_y \end{Bmatrix} = \frac{P}{4Etc^3}\begin{Bmatrix} y(3x^2 - 6Lx + 2.3c^2 - 2.3y^2) \\ (L-x)^3 - L^3 + x\left(5.5c^2 + 3L^2\right) + 0.9(L-x)y^2 \end{Bmatrix}$$

$$\sigma_x = -3P(L-x)y/2tc^3$$
$$\sigma_y = 0$$
$$\tau_{xy} = 3P(c^2 - y^2)/4tc^3$$

Homework 10 Due: Friday, December 18, 2015 @ 5:00:00 PM in E218

For both **Question 1 and 2**:
- i) Redraw the diagram and label your elements & degrees of freedom.

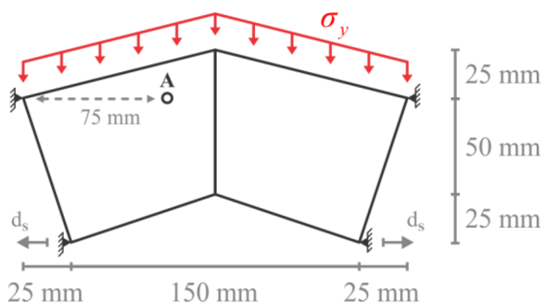
Solve **Question 1** by hand:
- ii) Generate all required element matrices. For your first integration point, find [D], [J_e], [$J_{e,1}$], |$J_{e,1}$|, [$B_{e,1}$], and [$K_{e,1}$]. Then find [$K_{e,2}$], [$K_{e,3}$], and [$K_{e,4}$] as well as the final [K_e].
- iii) Find [K_{UU}] and [K_{US}] without solving for [K_G]. Then, solve for the nodal displacements.
- iv) Find the displacements, {ε}, {σ}, σ_1, σ_2, and θ_p at point A.

Code the elastic quad into MATLAB and perform an analysis for **Questions 1 and 2**:
- v) Provide images of the input and deformed geometry as well as plots of σ_x, σ_y, τ_{xy}, σ_1 & σ_2.
- vi) Provide a description of your code changes and only the modified parts of your code.

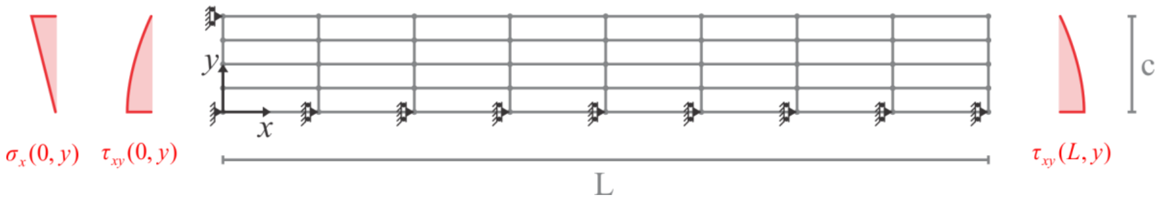
Perform an analysis with SAP2000 for **Questions 1 and 2**:
- vii) Provide images of the input and deformed geometry as well as plots of σ_x, σ_y, τ_{xy}, σ_1 & σ_2.

Finally, for both **Questions 1 and 2** (incorporating results from HW9):
- viii) Tabulate the displacements from your MATLAB, SAP2000, and hand calculations.
- ix) Compare and discuss your results based on the tabulated displacements and the stress plots.

1. 2D Elasticity: t = 10 m, E = 200 000 MPa, υ = 0.3, d_s = 0.1 mm, and σ_y = 50 MPa down. **35 pts**



2. 2D Elasticity: L = 16 m, c = 2 m, t = 10 mm, E = 10 000 MPa, υ = 0.3, and P = -10 kN. **20 pts**



The non-zero applied surface tractions are defined in global coordinates as:

$$\sigma_x(0,y) = -\frac{3PL}{2tc^3}y \qquad \tau_{xy}(0,y) = \tau_{xy}(L,y) = \frac{3P}{4tc^3}\left(c^2 - y^2\right)$$

Use the exact solution, instead of hand calculations, for your comparison:

$$\begin{Bmatrix} u_x \\ u_y \end{Bmatrix} = \frac{P}{4Etc^3}\begin{Bmatrix} y(3x^2 - 6Lx + 2.3c^2 - 2.3y^2) \\ (L-x)^3 - L^3 + x\left(5.5c^2 + 3L^2\right) + 0.9(L-x)y^2 \end{Bmatrix}$$

$$\sigma_x = -3P(L-x)y/2tc^3$$
$$\sigma_y = 0$$
$$\tau_{xy} = 3P(c^2 - y^2)/4tc^3$$