

1 Introduction

The classical approach to pattern formation modelling, in particular the phenomena of the symmetry breaking and *de novo* pattern formation, is based on the search for system components whose interactions correspond to Turing-type models, *i.e.* they contain two diffusing substances with sufficiently different diffusion coefficients and an appropriate scheme of interactions, e.g., an activator and an inhibitor from the Gierer-Meinhardt model.

Often, the identification of such molecular substances does not succeed, as was the case in research on Hydra patterning, where for two decades an inhibitor for the Wnt signalling pathway fulfilling the assumptions of the Gierer-Meinhardt model has been sought to explain spontaneous symmetry breaking and pattern formation within this signalling pathway. This motivated the group of Anna Marciniak-Czochra to look for alternative mechanisms for pattern formation in Hydra, in parallel to mathematical modelling and analysis of the Wnt signalling pathway.

One mechanism we have proposed is based on mechano-chemical interactions in the tissue, [1]. Another concept proposes to focus on taking into account the coupling of intercellular communication via diffusing substances coupled to non-linear interactions within or on the cell surface [CITATION2]. The latter, leads to coupling of reaction-diffusion equations describing cell-to-cell communication with space-dependent ordinary equations describing cell-localised processes.

Such models may exhibit a range of unexpected phenomena such as emergence of patterns with jump discontinuity [CITATION3] and DDI-induced finite- or infinite-time mass concentration [CITATION4]. So far reaction-diffusion-ODE systems and their ability for pattern formation have been systematically studied only in case of coupling a scalar reaction-diffusion equation to a scalar or a system of ODEs. Comprehensive analytical results on the stability of such systems can be found in [CITATION5].

The aim of this project is to investigate the role of a second diffusive component. To streamline the analysis, we focus on a system coupling a scalar ODE with two reaction-diffusion equations.