Let's attempt stable marriage.

$$[MAN, WOMAN]$$
$$Reflexive[T] == \{r : T \leftrightarrow T \mid \mathrm{id}\ T \subseteq r\}$$
$$Transitive[T] == \{r : T \leftrightarrow T \mid r \fatsemi r \subseteq r\}$$
$$Preorder[T] == Reflexive[T] \cap Transitive[T]$$
$$Antisymmetric[T] == \{r : T \leftrightarrow T \mid r \cap r^{\sim} \subseteq \mathrm{id}\ T\}$$
$$PartialOrder[T] == Preorder[T] \cap Antisymmetric[T]$$
$$TotalOrder[T] == \{r : PartialOrder[T] \mid r \cup r^{\sim} = T \times T\}$$

StableMarriage

$$male\_pref : MAN \pfun TotalOrder[WOMAN]$$
$$female\_pref : WOMAN \pfun TotalOrder[MAN]$$

$$MAN \in \mathbb{F}\ MAN$$
$$WOMAN \in \mathbb{F}\ WOMAN$$
$$\#\ MAN = \#\ WOMAN$$

Initially, no preferences are expressed

StableMarriageInit

$$StableMarriage'$$

$$male\_pref' = \varnothing$$
$$female\_pref' = \varnothing$$

EnterWomanPref

$$\Delta StableMarriage$$
$$name? : WOMAN$$
$$inpref? : TotalOrder[MAN]$$

$$name? \notin dom\ female\_pref$$
$$female\_pref' = female\_pref \cup \{name? \mapsto inpref?\}$$
$$male\_pref' = male\_pref$$

EnterManPref

$$\Delta StableMarriage$$
$$name? : MAN$$
$$inpref? : TotalOrder[WOMAN]$$

$$name? \notin dom\ male\_pref$$
$$male\_pref' = male\_pref \cup \{name? \mapsto inpref?\}$$
$$female\_pref' = female\_pref$$

Marry

$\Xi StableMarriage$

$wife! : MAN \rightarrowtail\!\!\!\!\!\!\rightarrow WOMAN$

$\mathrm{dom}\, male\_pref = MAN$

$\mathrm{dom}\, female\_pref = WOMAN$

$\forall\, m, n : MAN \mid m \neq n \bullet (wife!m \mapsto wife!n \in male\_pref\ m \lor n \mapsto m \in female\_pref\ (wife!n))$