$A == [ns : \mathbb{F}\,\mathbb{N}_1]$
$AInit == [A' \mid ns' = \varnothing]$
$New == [\Delta A;\ n? : \mathbb{N}_1 \mid ns' = ns \cup \{n?\}]$
$MSF == [\Xi A;\ m! : \mathbb{N}_1 \mid ns \neq \varnothing;\ m! = max\ ns]$

AM2SF

ΞA
$m1!, m2! : \mathbb{N}_1$

$\# ns > 1$
$m1! = max\ ns$
$m2! = max\,(ns \setminus \{m1!\})$

Store the two max seen so far as they are observed. Must be $c = 0 \land d = 0$ to ensure two MSF are unique.

$C5 == [c, d : \mathbb{N} \mid (c = 0 \land d = 0) \lor c < d]$
$C5Init == [C5' \mid c' = 0 \land d' = 0]$
$C5MSF == [\Xi C5;\ m! : \mathbb{N} \mid m! = c]$

LI5

$A;\ C5$

$c = 0 \Rightarrow ns = \varnothing$
$(c > 0 \land d = 0) \Rightarrow ns = \{c\}$
$d > 0 \Rightarrow (\{c, d\} \subseteq ns \land c = max\ ns \land d = max(ns \setminus \{c\}))$

C5New

ΔC
$n? : \mathbb{N}_1$

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if $n? > c$ **then** $(c' = n? \land d' = c)$
else (**if** $(n? > d \land n? < c)$ **then** $c' = c \land d' = n?$
else $c' = c \land d' = d$)

C5MSF2

$\Xi C5$
$ma!, mb! : \mathbb{N}$

$ma! = c$
$mb! = d$