

$$\begin{aligned}
A &== [ns : \mathbb{F} \mathbb{N}_1] \\
AInit &== [A' \mid ns' = \emptyset] \\
New &== [\Delta A; n? : \mathbb{N}_1 \mid ns' = ns \cup \{n?\}] \\
MSF &== [\Xi A; m! : \mathbb{N}_1 \mid ns \neq \emptyset; m! = \max ns]
\end{aligned}$$

$AM2SF$
ΞA $m1!, m2! : \mathbb{N}_1$
$\# ns > 1$ $m1! = \max ns$ $m2! = \max (ns \setminus \{m1!\})$

Store the two max seen so far as they are observed. Must be $c = 0 \wedge d = 0$ to ensure two MSF are unique.

$$\begin{aligned}
C5 &== [c, d : \mathbb{N} \mid (c = 0 \wedge d = 0) \vee c < d] \\
C5Init &== [C5' \mid c' = 0 \wedge d' = 0] \\
C5MSF &== [\Xi C5; m! : \mathbb{N} \mid m! = c]
\end{aligned}$$

$LI5$
$A; C5$
$c = 0 \Rightarrow ns = \emptyset$ $(c > 0 \wedge d = 0) \Rightarrow ns = \{c\}$ $d > 0 \Rightarrow (\{c, d\} \subseteq ns \wedge c = \max ns \wedge d = \max (ns \setminus \{c\}))$

$C5New$
ΔC $n? : \mathbb{N}_1$
if $n? > c1$ then $c' = n? \wedge d' = c$ else (if $(n? > d \wedge n? < c)$ then $c' = c \wedge d' = n?$ else $c' = c \wedge d' = d$)

$C5MSF2$
$\Xi C5$ $ma!, mb! : \mathbb{N}$
$ma! = c$ $mb! = d$