CM3109: Combinatorial Optimisation Report

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1 Task 1: Setting SA parameters

1.1 Definition:

A ranking R_2 is in the neighbourhood N of R_1 if and only if it can be obtained by swapping two adjacent participants in R_1 with the other participants remaining in identical positions. The first and last participants in R are classed as adjacent participants, therefore if the last element of R_1 is involved in the swap, it is swapped with the first element, creating a wrap-around effect. Each ranking with n participants will have exactly n neighbours.

1.2 Example:

Let $R_1 = [\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}]$

The neighbourhood $N(R_1)$ have the following 4 neighbours:

$[\mathrm{B}, \mathrm{A}, \mathrm{C}, \mathrm{D}]$ - swapped A and B

$[\mathrm{A}, \mathrm{C}, \mathrm{B}, \mathrm{D}]$ - swapped B and C

$[\mathrm{A}, \mathrm{B}, \mathrm{D}, \mathrm{C}]$ - swapped C and D

$[D, B, C, A]$ – swapped D and A (demonstrating the wrap-around effect)

The neighbouring ranking R_2 could be any ranking in this neighbourhood.

1.3 Justification:

In a ranking, only the two adjacent elements that are swapped need to be re-evaluated instead of reevaluating every edge in the ranking, this makes it easier to compute their *Kemeny Score*.

Whilst any initial solution will need to have a *Kemeny Score* calculated for each participant in the ranking, any neighbour succeeding it can be calculated more efficiently. When two elements are swapped, their relative ordering with respect to other elements in the list changes. However, the relative ordering of all other pairs of elements remains unchanged. Therefore, only the scores of the swapped elements need to be updated. The scores of the other elements remain valid from the previous calculation. Therefore, in the main simulated annealing loop, the ΔC can be obtained by calculating neighbour score by evaluating only the *Kemeny Scores* of the two swapped elements and comparing its sum to the current ranking.

1.3.1 Example Cost Calculation

Consider $R_1 = [A, B, C, D]$ and its neighbour ranking $R_2 = [A, B, D, C]$.

Consider the following weighted tournament matrix T:

	A	***B***	***C***	***D***
A	0	5	14	0
B	0	0	0	0
C	0	2	0	9
D	3	0	0	0

Calculating the *Kemeny Scores* of R_1 would return $c(R_1, T) = [0,0,2+9,3] = 14$

This is the initial solution. As R_2 is a neighbour of R_1 with the edges C and D swapped, the *Kemeny Scores* at edges A and B will remain the same, as their relative ordering is kept the same between the rankings therefore, we can preserve these values ($[0,0]$).

To calculate the *Kemeny Scores* of R_2 we can either:

Calculate the *Kemeny Score* at each edge of R_2

$$c(R_2, T) = [0,0,2,3] = 5$$

Calculate the new *Kemeny Scores* of the swapped edges C and D, and add the preserved values.

$$[0,0] + c\left(R_{2_{C,D}}, T\right) = [0,0] + [2,3] = 5$$

Observe that the *Kemeny Score* returned through both calculations is the same, however the first method requires the computation of the *Kemeny Score* of 4 edges, whereas the second method only requires the computation of 2 edges, therefore is more computationally efficient.

2 Task 3: Selecting best parameters and discussion

2.1 Results

TI: 1

TL: 20

a: 0.9999

num_non_improve/stop_criterion: 20000

2.2 Screenshots

1. Alain Prost	1. Alain Prost	1. Niki Lauda	1. Niki Lauda	1. Alain Prost
2. Niki Lauda	2. Niki Lauda	2. Alain Prost	2. Alain Prost	2. Niki Lauda
3. Elio de Angelis	3. Elio de Angelis	3. Rene Arnoux	3. Rene Arnoux	3. Rene Arnoux
4. Rene Arnoux	4. Rene Arnoux	4. Elio de Angelis	4. Elio de Angelis	4. Elio de Angelis
5. Corrado Fabi	5. Corrado Fabi	5. Corrado Fabi	5. Corrado Fabi	5. Corrado Fabi
6. Michele Alboreto	6. Michele Alboreto	6. Michele Alboreto	6. Derek Warwick	6. Derek Warwick
7. Derek Warwick	7. Derek Warwick	7. Derek Warwick	7. Michele Alboreto	7. Michele Alboreto
8. Nelson Piquet	8. Nelson Piquet	8. Nelson Piquet	8. Nelson Piquet	8. Nelson Piquet
9. Patrick Tambay	9. Patrick Tambay	9. Patrick Tambay	9. Patrick Tambay	9. Patrick Tambay
10. Andrea de Cesaris	10. Andrea de Cesaris	10. Andrea de Cesaris	10. Andrea de Cesaris	10. Andrea de Cesaris
11. Mauro Baldi	11. Mauro Baldi	11. Mauro Baldi	11. Mauro Baldi	11. Mauro Baldi
12. Thierry Boutsen	12. Teo Fabi	12. Teo Fabi	12. Teo Fabi	12. Thierry Boutsen
13. Teo Fabi	13. Thierry Boutsen	13. Thierry Boutsen	13. Thierry Boutsen	13. Teo Fabi
14. Riccardo Patrese	14. Riccardo Patrese	14. Riccardo Patrese	14. Riccardo Patrese	14. Riccardo Patrese
15. Jo Gartner	15. Jo Gartner	15. Jo Gartner	15. Jo Gartner	15. Nigel Mansell
16. Gerhard Berger	16. Gerhard Berger	16. Gerhard Berger	16. Gerhard Berger	16. Stefan Johansson
17. Nigel Mansell	17. Nigel Mansell	17. Nigel Mansell	17. Nigel Mansell	17. Gerhard Berger
18. Keke Rosberg	18. Keke Rosberg	18. Keke Rosberg	18. Keke Rosberg	18. Jo Gartner
19. Ayrton Senna	19. Ayrton Senna	19. Ayrton Senna	19. Ayrton Senna	19. Keke Rosberg
20. Eddie Cheever	20. Eddie Cheever	20. Eddie Cheever	20. Eddie Cheever	20. Ayrton Senna
21. Marc Surer	21. Marc Surer	21. Marc Surer	21. Marc Surer	21. Eddie Cheever
22. Jonathan Palmer	22. Jonathan Palmer	22. Jonathan Palmer	22. Jonathan Palmer	22. Marc Surer
23. Martin Brundle	23. Martin Brundle	23. Martin Brundle	23. Martin Brundle	23. Jonathan Palmer
24. Huub Rothengatter	24. Huub Rothengatter	24. Huub Rothengatter	24. Huub Rothengatter	24. Martin Brundle
25. Jacques Laffite	25. Jacques Laffite	25. Jacques Laffite	25. Jacques Laffite	25. Huub Rothengatter
26. Stefan Bellof	26. Stefan Bellof	26. Stefan Bellof	26. Stefan Bellof	26. Jacques Laffite
27. Francois Hesnault	27. Francois Hesnault	27. Francois Hesnault	27. Francois Hesnault	27. Piercarlo Ghinzani
28. Stefan Johansson	28. Stefan Johansson	28. Stefan Johansson	28. Stefan Johansson	28. Stefan Bellof
29. Piercarlo Ghinzani	29. Piercarlo Ghinzani	29. Piercarlo Ghinzani	29. Piercarlo Ghinzani	29. Manfred Winkelhock
30. Manfred Winkelhock	30. Manfred Winkelhock	30. Manfred Winkelhock	30. Manfred Winkelhock	30. Philippe Streiff
31. Philippe Streiff	31. Johnny Cecotto	31. Johnny Cecotto	31. Johnny Cecotto	31. Francois Hesnault
32. Johnny Cecotto	32. Philippe Streiff	32. Philippe Streiff	32. Philippe Streiff	32. Johnny Cecotto
33. Philippe Alliot	33. Philippe Alliot	33. Philippe Alliot	33. Philippe Alliot	33. Philippe Alliot
34. Mike Thackwell	34. Pierluigi Martini	34. Pierluigi Martini	34. Mike Thackwell	34. Pierluigi Martini
35. Pierluigi Martini	35. Mike Thackwell	35. Mike Thackwell	35. Pierluigi Martini	35. Mike Thackwell
Kemeny Score: 62	Kemeny Score: 62	Kemeny Score: 62	Kemeny Score: 62	Kemeny Score: 63
Runtime: 66 ms	Runtime: 63 ms	Runtime: 73 ms	Runtime: 61 ms	Runtime: 76 ms

2.3 Analysis

2.3.1 Method

I created *test_params()*, which accepts a set of parameter ranges and tests each parameter combination 20 times to find their average *Kemeny Score* using following ranges:

```
parameter_ranges.TI = new double[] {1, 2, 3, 4, 5};
parameter_ranges.TL = new int[] {10, 20, 30, 40, 50, 60, 70, 80, 90, 100};
parameter_ranges.a = new double[] {0.8, 0.85, 0.9, 0.95, 0.99, 0.995, 0.999, 0.9995, 0.9999};
parameter_ranges.stop_criterion = new int[] {100, 500, 1000, 2000, 4000, 6000, 8000, 10000, 12000, 14000, 16000, 18000, 20000};
```

2.3.2 TI

The optimal TI was 1 (fig.1). TI and a do not have much influence on the *Kemeny Score* as long as a is close to 1 (fig.3). Regarding *TI* and *Stop Criterion* combinations, at specific points $TI = 1\, Stop\ Criterion = 20000$, and $TI = 4\, Stop\ Criterion = 16000$ the *Kemeny Score* is significantly better than any other combination (fig.4). TI and TL combinations where both values are greater tend to have higher quality rankings (fig.5), however it is likely that $TI = 1$s slightly worse interactions with TL are superseded by its combination with $Stop\ Criterion = 20000$.

2.3.3 TL

The optimal TL was 20 (fig.1) presumably due to is its interaction at $Stop\ Criterion = 20000$ in *Figure 7*, reaching the lowest *Kemeny Score* which appears to supersede TLs combination with TI where higher TLs towards 100 appear to be optimal (fig.5). When combining TL with a, TLs value doesn't matter that much, only that a is kept close to 1 (fig.6).

2.3.4 a

The optimal a was 0.9999 (fig.1). In general, it appears as if the higher the a the lower the *Kemeny Score*. When interacting with TI, TL, and *Stop Criterion* the only factor that seems to improve the final *Kemeny Score* is how close a is 1, closer leading to a higher score (fig.2, 3, 6).

2.3.5 Stop Criterion

The optimal *Stop Criterion* was 20000 (fig.1). Various *Stop Criterions* can produce wildly different *Kemeny Scores* when interacting with TI and TL, seemingly with no pattern. For example, when $TL = 20$ and $Stop\ Criterion = 20000$ the *Kemeny Score* improves drastically, likely the factor contributing the most towards my optimal values (fig.7). Slightly less optimal TLs and *Stop Criterions* are scattered where TL is around 60-80 at various *Stop Criterions* from 5000-20000 (fig.7). Similarly, in *Figure 4*, at $TI = 1\, Stop\ Criterion = 20000$, and $TI = 4\, Stop\ Criterion = 16000$ the *Kemeny Score* improves dramatically.

2.3.6 Conclusion

The $TI = 1$ and $TL = 20$ interaction with $Stop\ Criterion = 20000$ combined with the good performance of a values closer to 1 appear to be the largest influences on my best combination.

2.4 Local Optima Discussion

My solution never achieved a sub-62 score with any parameter combination, meaning the global optima likely has a *Kemeny Score* of 62. The rankings achieving 62 vary each time indicating there are multiple global optima, necessarily implying the presence of multiple local optima.

There are likely not many local optima as *Kemeny Rankings* aren't sensitive, swapping two drivers won't yield a wildly different score meaning small adjustments won't create many local improvements that don't move towards the global optimum. A common local optimum found are rankings with a *Kemeny Score* of 63.

2.5 Figures

```
Best Score: 62.25
Best Parameters: TI=1.0 TL=20 a=0.9999 stop_criterion=20000
```

Figure 1: test_params() output.

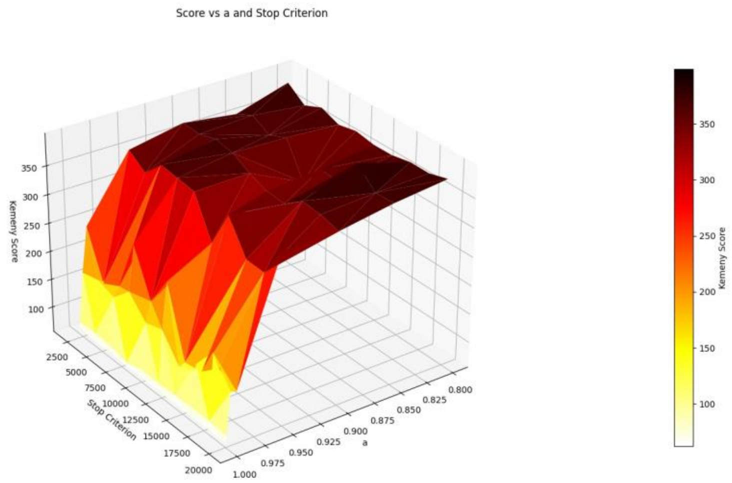

Figure 2: Score vs a and Stop Criterion 3d heatmap.

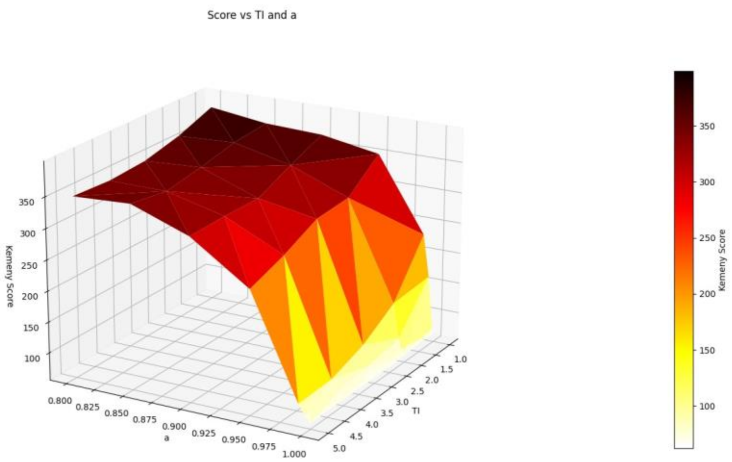

Figure 3: Score vs TI and a 3d heatmap.

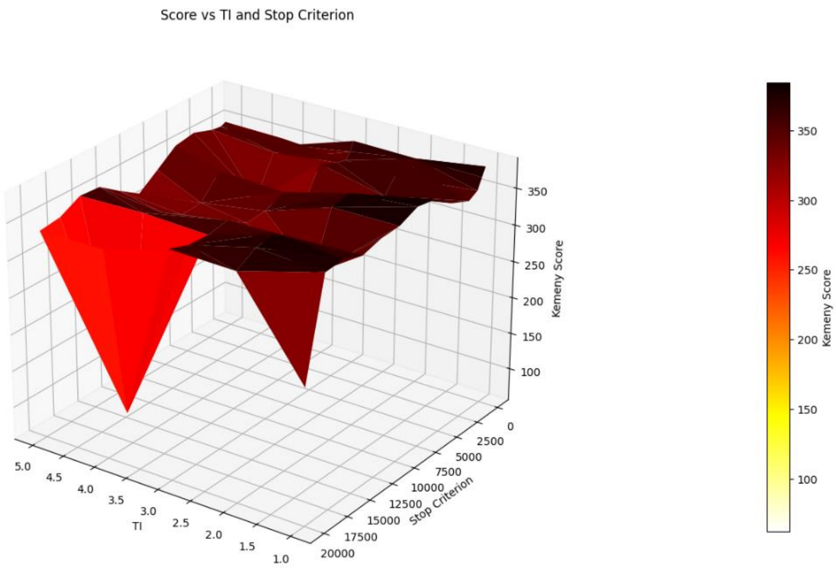

Figure 4: Score vs TI and Stop Criterion 3d heatmap.

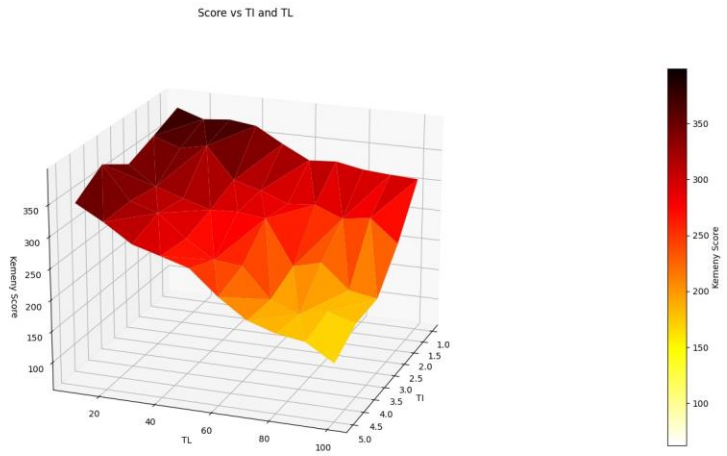

Figure 5: Score vs TI and TL 3d heatmap.

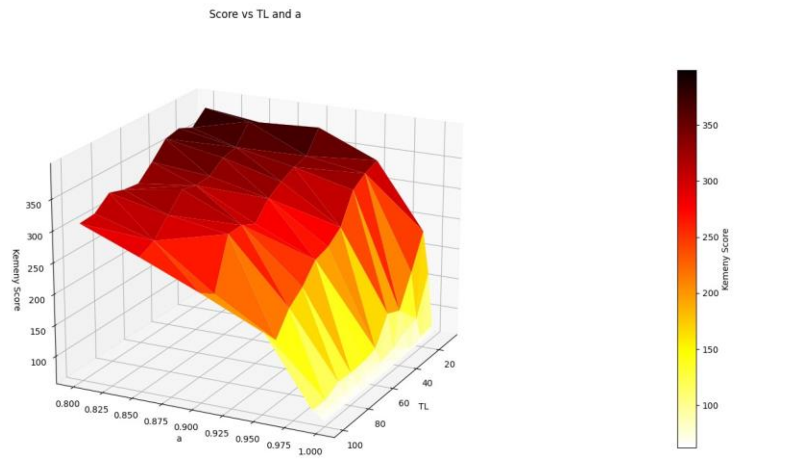

Figure 6: Score vs TL and a 3d heatmap.

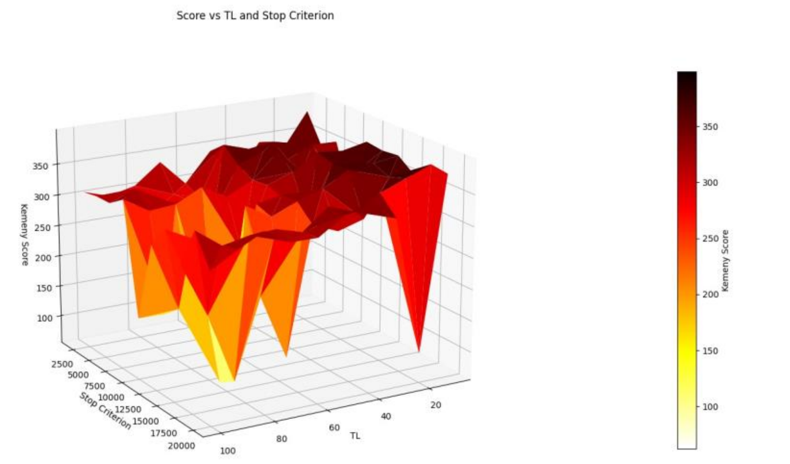

Figure 7: Score vs TL and Stop Criterion 3d heatmap.