APPLICATIONS OF GROVER'S SEARCH ALGORITHM

In part 2, I've considered the application of Grover's Search Algorithm to solve the Travelling Salesman Problem. The **Travelling Salesman Problem** (**TSP**) is the **challenge** of finding the shortest yet most efficient route for a person to take given a list of specific destinations. It is a well-known algorithmic **problem** in the fields of computer science and operations research.

So suppose we're given n cities and ${}^{n}C_{2}$ distances between them, we want 2 to find a tour of the cities of length at most D. The above problem can be formulated as follows:

Given a boolean formula of totally n variables in conjunctive normal form with at most 3 variables in each clause, ex. $(x1 \lor x3 \lor x7) \land (x1 \lor \bar{x}5 \lor x9) \land (x2 \lor \bar{x}3 \lor x11) \ldots$, we want to know if there exist a set of $\{xi, i = 1 \ldots n\}$ such that the whole formula is satisfied.

Using the 3SAT problem as an example, classically we have to search exhaustively and try every set of values, therefore the algorithm takes 2n time. Here we will show that the problem can be done using Grover's search algorithm in 2n/2 time. A better way to start is to find a maximal set of disjoint clauses in the formula. We can rename the variables and write the formula as $(x1 \vee x2 \vee x3) \wedge (x4 \vee x5 \vee x6) \wedge \ldots \wedge (xm-2 \vee xm-1 \vee xm) \wedge \ldots (xi \vee xn-1 \vee xn)$, where $1 < m < n$, and $1 < i < m$. In this form, we have a disjoint clauses set (from x1 to xm), and the rest of the formula are 2SAT. Since we know the polynomial time algorithm for 2SAT problems, we can easily solve the leftover part. Notice that we need to try only 7 values for each clause,and there are at most n clauses. The time is $7n/3 \approx 2.93n$. If we apply Grover's search algorithm, we 3 can do with time in O(2.93n/2). This is almost close to the best classical algorithm O(2.43n). This demonstrates that applying Grover's algorithm to a relatively simple classical algorithm can gain substantial speed up (however, not exponential speed up). It is possible that the Grover's algorithm and be used on the best classical algorithm can gain speed up that can not be done classically.

By combining Grover's Search with Quantum Phase Estimation, we can approximately count the number of targets in the set, i.e. the value of M.

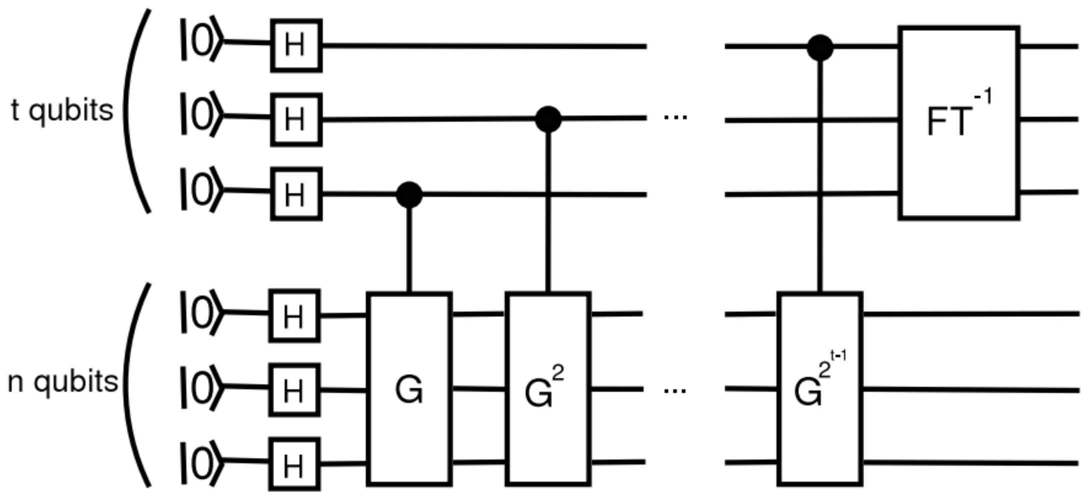

Figure 1: Circuit for quantum counting.

Due to time constraints, I haven't been able to work on the proposed problem. However, I'll continue working on the same post Hackathon!

Thanks!