Chapter 1: Linear Systems and Matrices
Section: Inverse of a Matrix
Lecture #4

Lebanese University

Prof Ali WEHBE

Course Plan

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Definition

A system of linear equation is said to be **Homogeneous** if the constant terms of each equation is equal to **zero**, i.e. it is of teh form;

$$(S): \begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = 0, \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = 0, \\ \qquad\qquad\qquad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = 0, \end{cases}$$

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Theorem

Homogeneous system are always **consistent**. Moreover, if the number of equations are less than the number of unknowns, then the Homogeneous System has an infinite number of solutions.

Example (Supplementary 1)

Discuss according the value of a for which the consistence of the following system:

$$(S)\begin{cases} x & +2y & +z & = & 2, \\ 2x & -2y & +3z & = & 1, \\ x & +2y & -az & = & a. \end{cases}$$

Solution:

Example (Supplementary 2)

Discuss according the value of $\boldsymbol{a}$ for which the consistence of the following system:

$$(S)\begin{cases} x & +y & +7z & = & -7, \\ 2x & +3y & +17z & = & 11, \\ x & +2y & +(a^2+1)z & = & 6a. \end{cases}$$

Solution:

Example (Supplementary 3)

Discuss according the values of a and b for which the consistence of the following system:

$$(S)\begin{cases} ax & & +bz & = & 2, \\ ax & +ay & +4z & = & 4, \\ & ay & +2z & = & b. \end{cases}$$

Solution:

Example (Supplementary 4)

Solve the following system of non-linear equations for the unknowns angles α, β and γ, where $0 \leq \alpha \leq \frac{\pi}{2}$, $0 \leq \beta < 2\pi$ and $0 \leq \gamma \leq \pi$.

$$(S)\begin{cases} 2\sin\alpha & +\cos\beta & -\tan\gamma & = & 1, \\ -4\sin\alpha & +\cos\beta & +\tan\gamma & = & 0, \\ -2\sin\alpha & +3\cos\beta & +2\tan\gamma & = & 4. \end{cases}$$

Solution:

Example (Supplementary 5)

Find the values of a, b and c such that the graph of the polynomial $p(x) = ax^2 + bx + c$ passes through the points $(1, 2)$, $(-1, 6)$ and $(2, 3)$.

Solution:

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Let A be $n \times n$ Matrix.
The matrix A is said to be **nonsingular** or **Invertible** if there exists a matrix B such that:

$$AB = BA = I_n$$

The matrix B is called the inverse of A and we denote it by A^{-1}.

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a) The matrices $\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$ and $\begin{bmatrix} -\frac{1}{10} & \frac{2}{5} \\ \frac{3}{10} & -\frac{1}{5} \end{bmatrix}$ are the inverses of each other.

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Example

The matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ is a singular one. Why?

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$$AB = \begin{bmatrix} 1 & 4 & 3 \\ -1 & -2 & 0 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{6} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

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Thus, $B = A^{-1}$.

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b) $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix}$.

Solution:

Example

$[A|I_3]$

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$$[A|I_3] \Longrightarrow \begin{array}{lccc} \textbf{Step1}: & R_2 - R_1 & \longrightarrow & R_2. \\ \textbf{Step2}: & R_3 + 6R_1 & \longrightarrow & R_3. \\ \textbf{Step3}: & R_3 + 4R_2 & \longrightarrow & R_3. \\ \textbf{Step4}: & -R_3 & \longrightarrow & R_3. \\ \textbf{Step5}: & R_2 + R_3 & \longrightarrow & R_2. \\ \textbf{Step6}: & R_1 + R_2 & \longrightarrow & R_1. \end{array}$$

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$$A^{-1} = \begin{bmatrix} -2 & -3 & -1 \\ -3 & -3 & -1 \\ -2 & -4 & -1 \end{bmatrix}.$$

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Compute A^{-2} in two different ways for $A = \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}$.

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Compute A^{-2} in two different ways for $A = \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}$.

Solution: $A^{-2} = \begin{bmatrix} \frac{9}{2} & -\frac{5}{4} \\ -\frac{5}{2} & \frac{3}{4} \end{bmatrix}$.

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Find the inverse of AB with

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 3 \end{bmatrix}.$$

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Find the inverse of AB with

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<u>Solution:</u> $A^{-1} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ -1 & 1 & 0 \\ \frac{2}{3} & 0 & -\frac{1}{3} \end{bmatrix}$.

Example

Hence $(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 8 & -5 & -2 \\ -8 & 4 & 3 \\ 5 & -2 & -\frac{7}{3} \end{bmatrix}$.

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Theorem:

If C is an invertible matrix, then the following statements hold:

a) If$AC = BC \Longrightarrow A = B$.
b) If $CA = CB \Longrightarrow A = B$.

Example

Solve the following matrix equation $\boldsymbol{AX + 2B = B^T}$ where

$$\boldsymbol{A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}} \quad \text{and} \quad \boldsymbol{B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 3 \end{bmatrix}}$$

Solution

$$\boldsymbol{X = A^{-1}(B^T - 2B)}$$

so

$$\boldsymbol{X = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 4 \\ 3 & 3 & 3 \end{bmatrix} - 2 \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 3 \end{bmatrix} \right) = \cdots}$$

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The system $AX = b$ of n linear equations in n unknowns has a unique solution if and only if A is **Invertible**. In this case, the solution X is given by $X = A^{-1}b$.

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Example (Solve the system using the inverse of Matrices)

a)

$$(S)\begin{cases} x_1 & +4x_2 & +3x_3 & = & 12, \\ -x_1 & -2x_2 & & = & -12, \\ 2x_1 & +2x_2 & +3x_3 & = & 8. \end{cases}$$

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In the previous example, we have find that the inverse of A is given by

$$A^{-1} = \begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{6} \end{bmatrix}.$$

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Hence, $x_1 = 4$, $x_2 = 4$ and $x_3 = -\frac{8}{3}$ is the solution of system (S).

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Hence, $x_1 = 4$, $x_2 = 4$ and $x_3 = -\frac{8}{3}$ is the solution of system (S).

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Solution: Homework.

c)

$$(S)\begin{cases} 2x & +3y & +z & = & 4, \\ 3x & +3y & +z & = & 8, \\ 2x & +4y & +z & = & 5. \end{cases}$$

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Hence, $\boldsymbol{x_1 = 4}$, $\boldsymbol{x_2 = 4}$ and $\boldsymbol{x_3 = -\frac{8}{3}}$ is the solution of system $\boldsymbol{(S)}$.

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