46755 – Renewables in Electricity Markets

Lecture 8: Optimal offering strategy for renewable energy resources in electricity markets

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March 31, 2025

Learning objectives of this lecture

After this lecture, you are expected be able to:

- Explain the differences between **price-take**r and **price-maker** market participants.
- Formulate and solve a decision-making problem under uncertainty using stochastic programming to derive **optimal offers** for a price-taking wind farm.
- Conduct ex-post out-of-sample and cross-validation analyses to assess the **quality of offering decisions**.

Offering strategy problem

In previous lectures, we discussed the sequential clearing of various electricity markets, including:

Reserve market → **Day-ahead market** → **Intra-day market** → **Balancing market**

- To be cleared by the system operator, e.g., Energinet
- To be cleared by the market operator, e.g., Nord Pool

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Before clearing the aforementioned markets, each market participant on the supply side, such as a wind farm W1, a conventional generator G1, or a power producer who owns both, needs to determine its **optimal participation strategy** in various markets.

To do so, we formulate and solve an "offering strategy problem"!

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Before clearing the aforementioned markets, each market participant on the supply side, such as a wind farm W1, a conventional generator G1, or a power producer who owns both, needs to determine its **optimal participation strategy** in various markets.

To do so, we formulate and solve an "offering strategy problem"!

- This problem determines in which market to participate, how much quantity to offer, and at what price!
- The objective is to maximize the (expected) profit!
- Similarly, demand-side participants should formulate and solve their own "bidding strategy problem"!

Price-taker vs. price-maker

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Price-taker vs. price-maker

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The price-taking participant does ***not*** consider how its offering decisions impact the market-clearing outcomes in its offering strategy problem.

- In other words, it does ***not*** model the market-clearing price as a **function** of its offering price or offering quantity.

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A potential market offering strategy approach:

The price-taking participant **forecasts** the clearing prices of various markets for the next day and optimizes the offering quantity to be sold (or bought) during different time periods. Recall, its goal is to maximize the expected profit.

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A market in which all players are *price-takers* is called a "perfectly competitive" market.

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→ In electricity markets with a limited number of players, the existence of perfect competition is merely an **assumption**.

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With the definition of a price-taker participant in mind:

Who then is a price-maker (also known as a strategic) participant?

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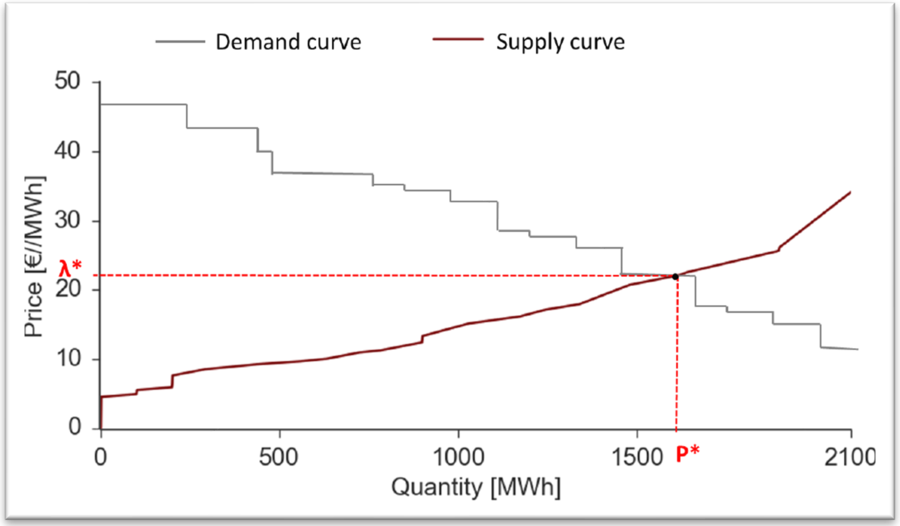
If market outcomes have been influenced to its benefit, this implies that the price-making participant has exercised "market power".

Illustrative example



Let's assume the marginal producer (the most expensive generator dispatched) offers truthfully at its true production cost. As a result of the market-clearing outcomes,

- the total demand supplied (P*) is approximately 1600 MW,
- the market-clearing price (λ*) is around €22/MWh.

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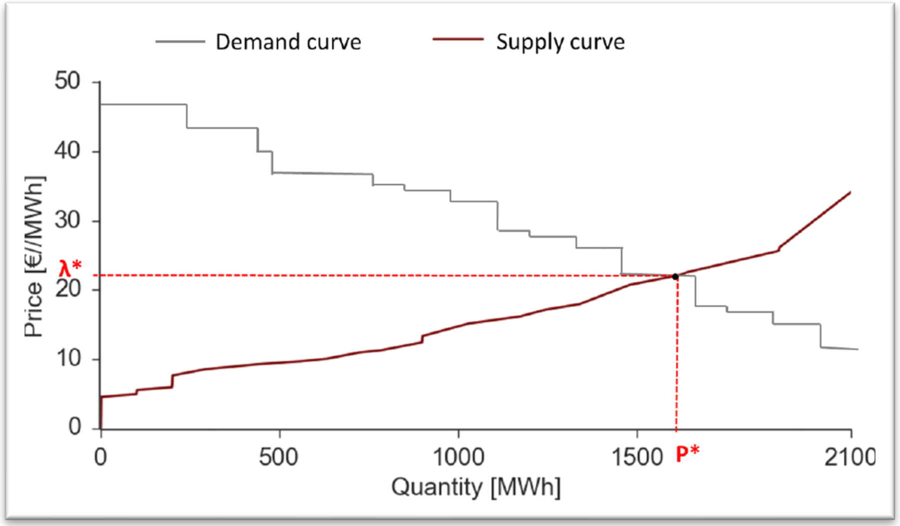

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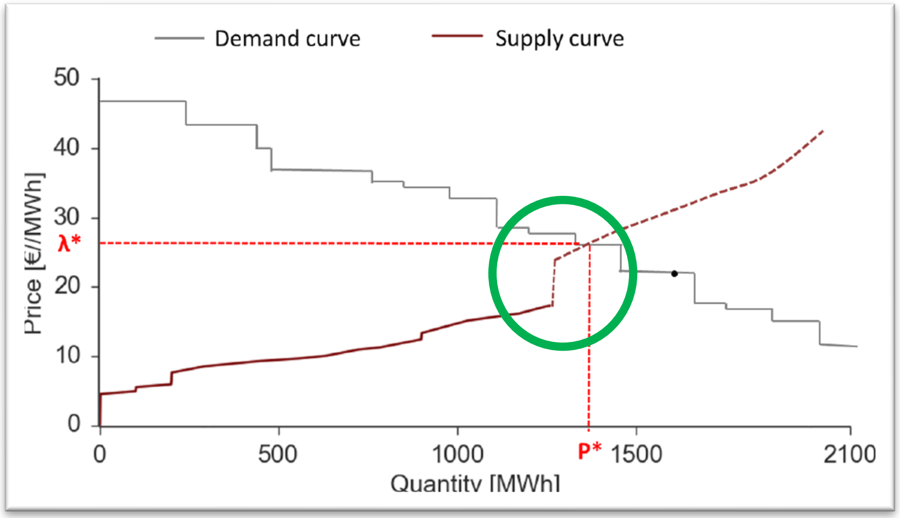

What is the interpretation of this case?

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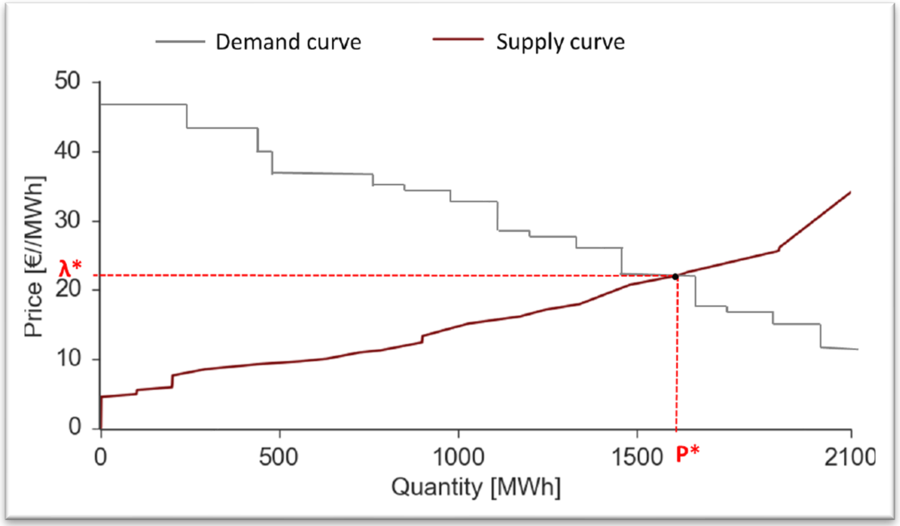

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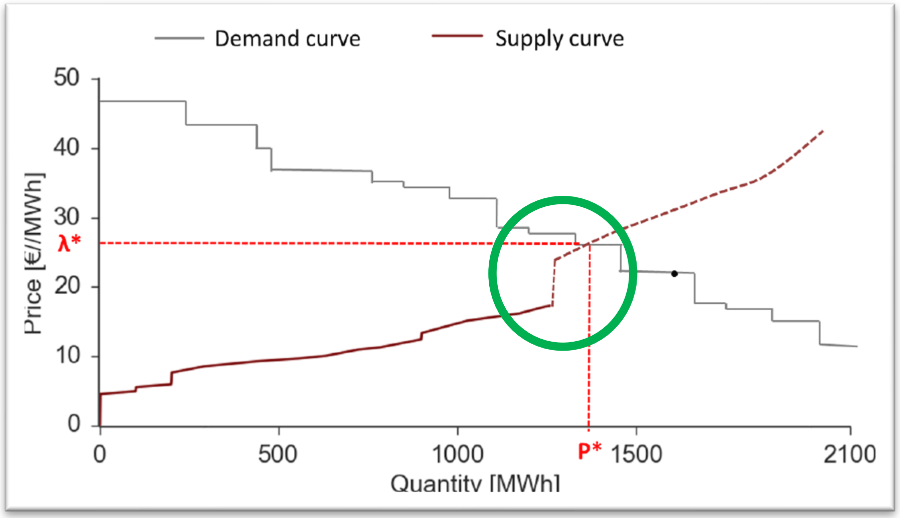

The marginal producer offers at a comparatively higher price (the so-called strategic offering), which leads to:

- The total demand supplied (P*) decreases to around 1350 MW,
- the market-clearing price (λ*) increases to €28/MWh.

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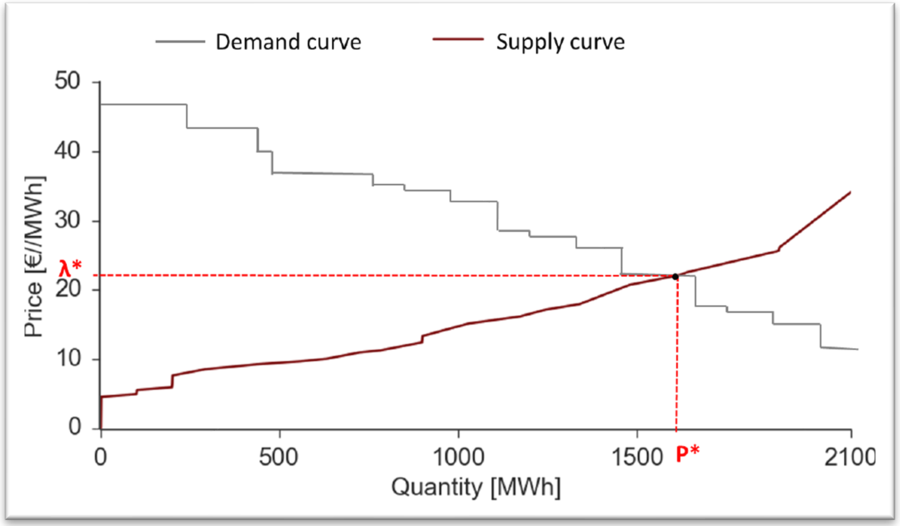

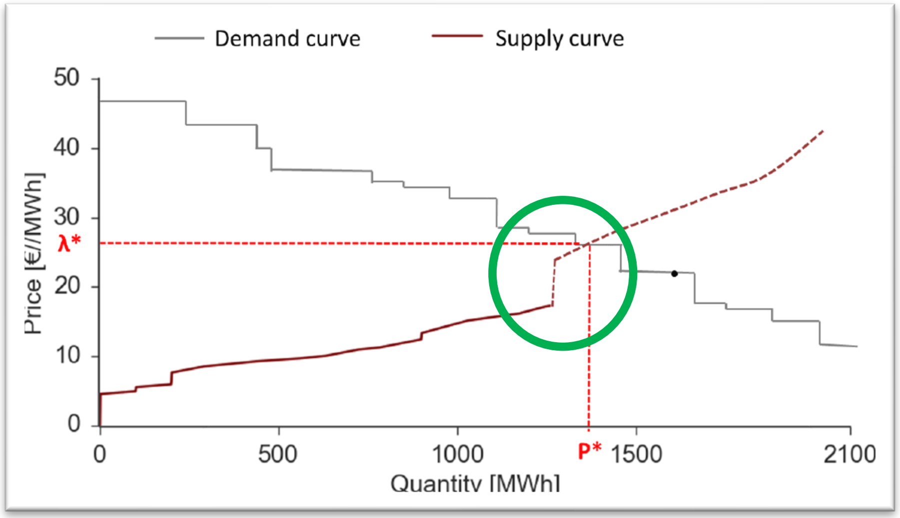

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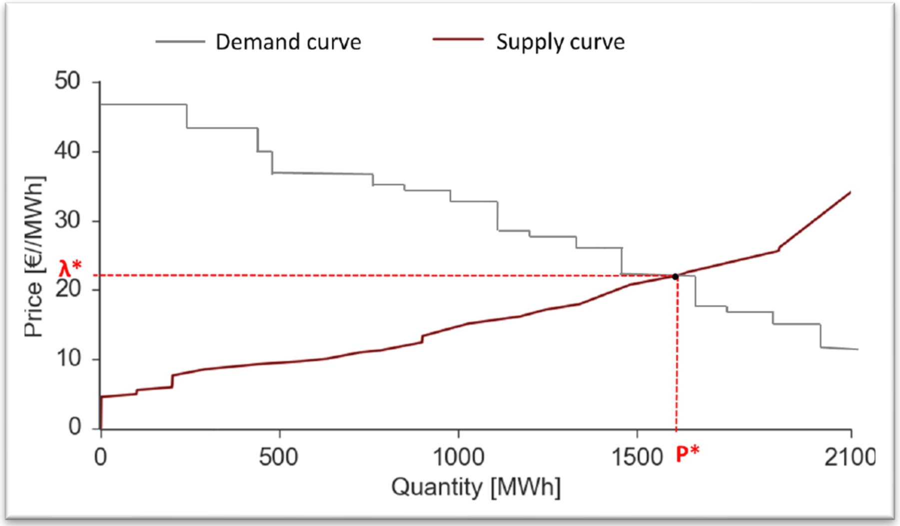

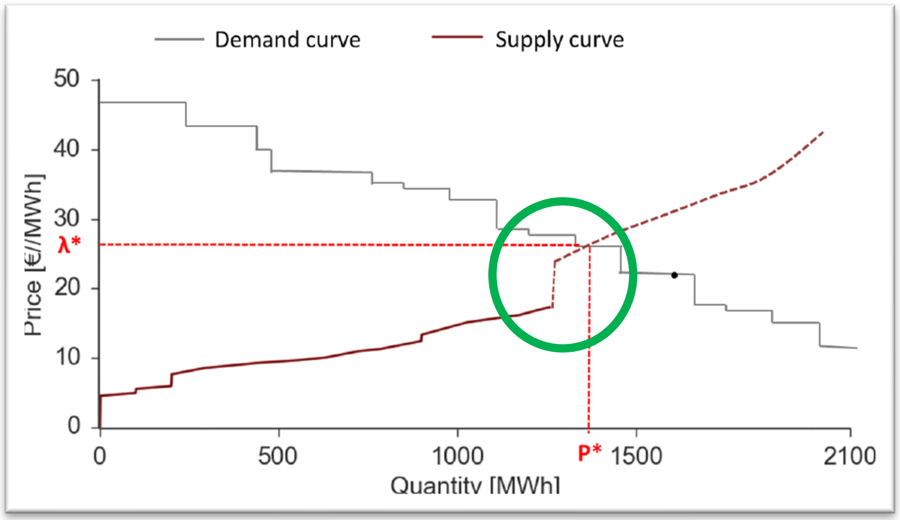

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Perhaps, depending on the quantity produced by the marginal producer and the market-clearing price!

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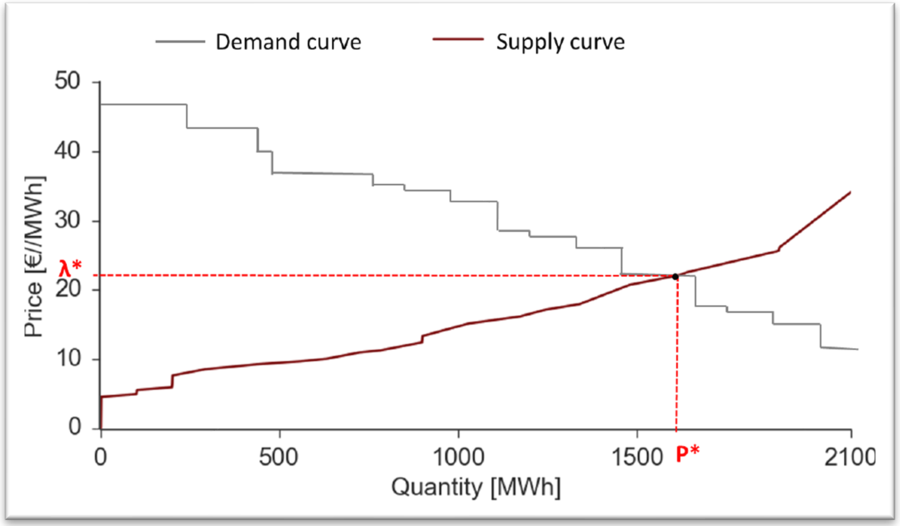

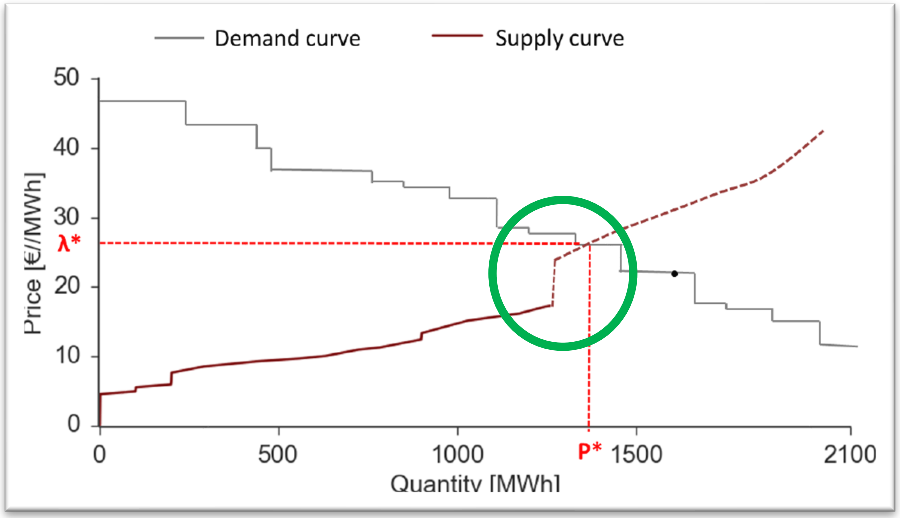

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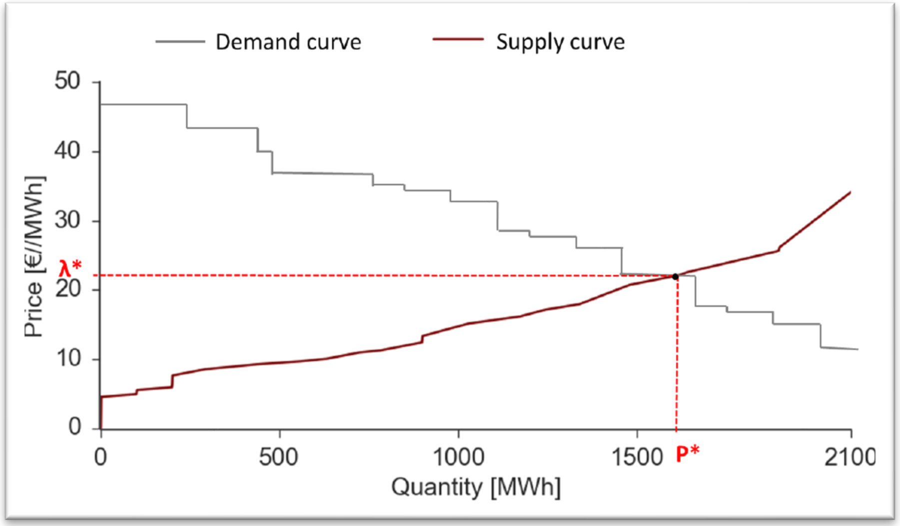

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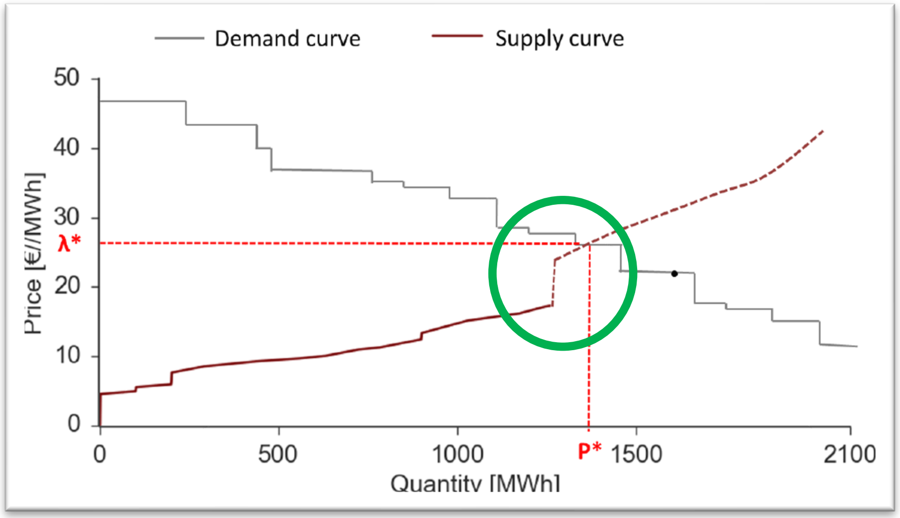

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Definitely! They are now paid at a higher price.

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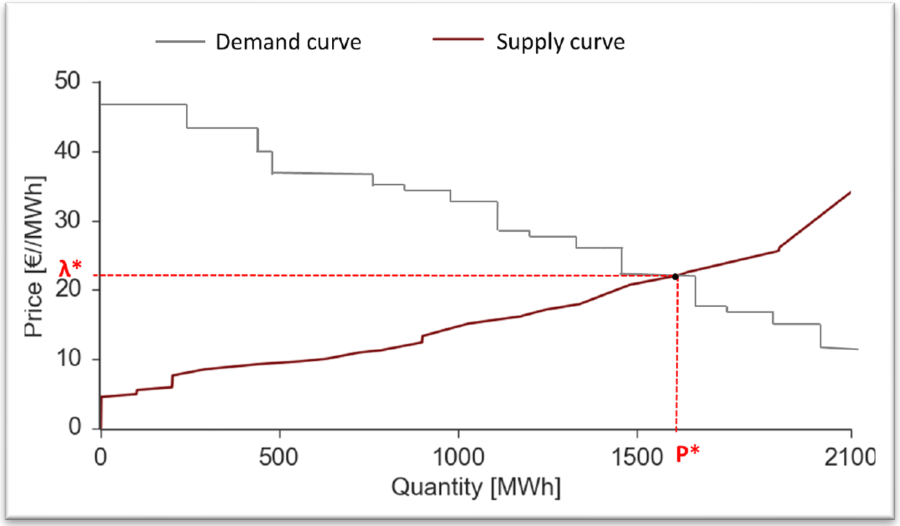

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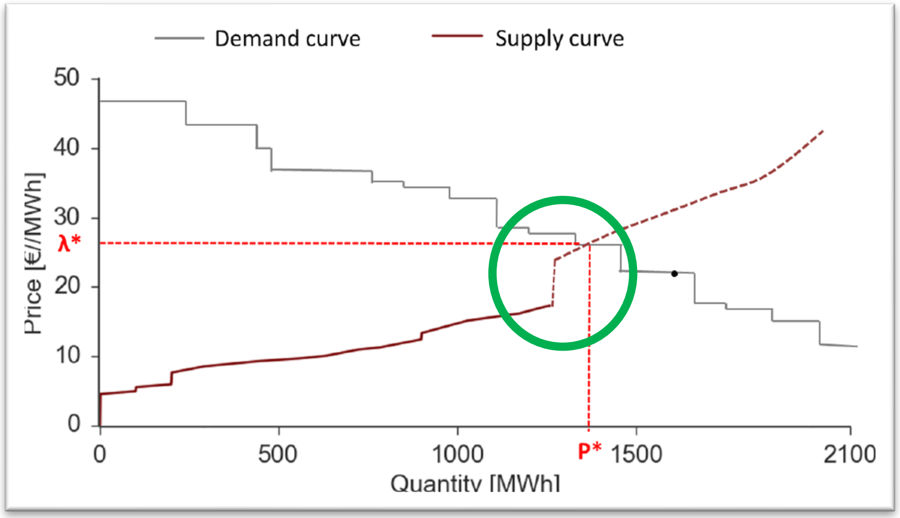

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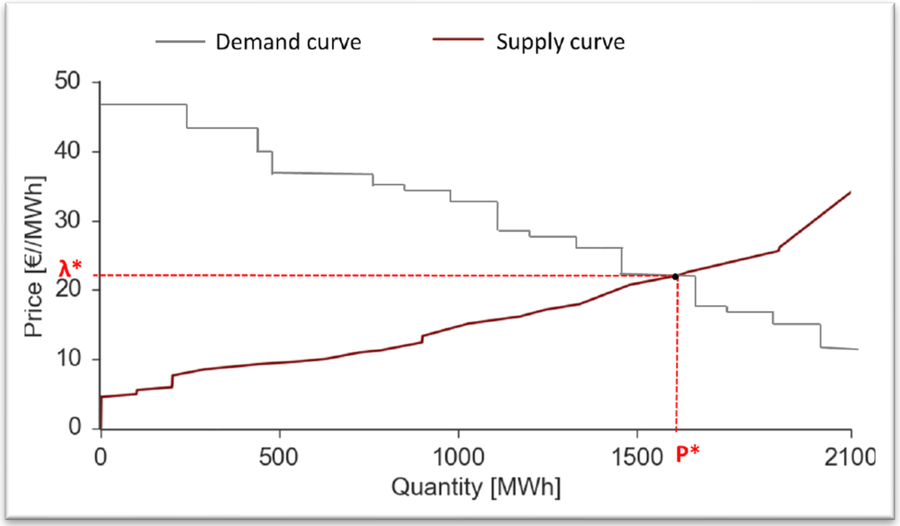

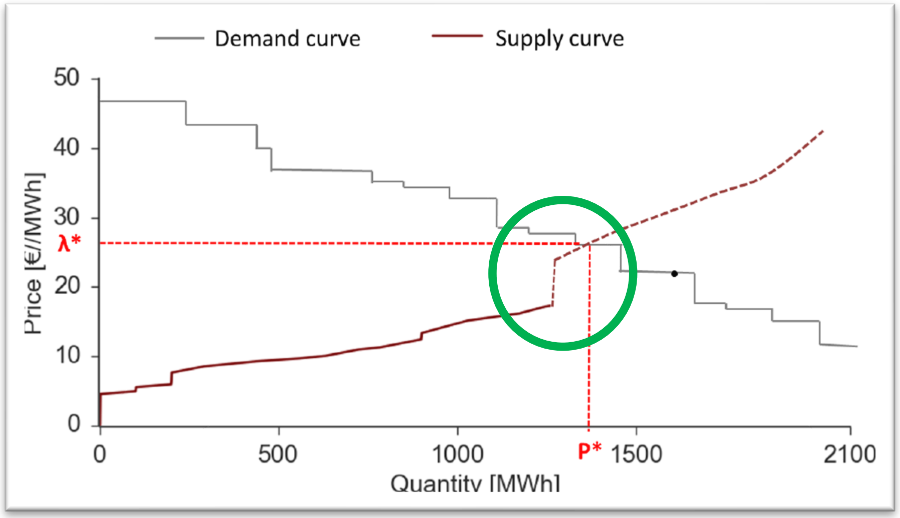

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Yes, strategic behavior reduces social welfare (the area between the supply and demand curves).

How to model imperfect competition?

How to model <u>imperfect</u> competition?

Common models:

1. Cournot competition model
2. Bertrand competition model
3. Conjectural variations model
4. Supply function model
5. etc

Cournot competition

- Each producer assumes that it can alter market-clearing outcomes through its production level [1]. In other words, producers compete based on quantities.

[1] H. R. Varian. Microeconomic Analysis. Norton & Company, New York, 1992.

Cournot competition

- Each producer assumes that it can alter market-clearing outcomes through its production level [1]. In other words, producers compete based on quantities.
- **Assumption**: The market price is considered an affine function of the total production.

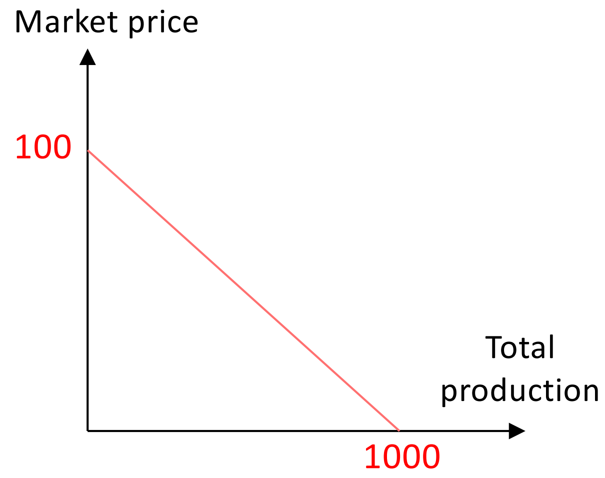
Example with two price-maker (strategic) producers:

Market price = 100 – 0.1(production 1 + production 2)*

Each producer maximizes its own revenue, i.e.,

*Revenue of producer 1 = production 1 * market price.*

*Revenue of producer 2 = production 2 * market price.*



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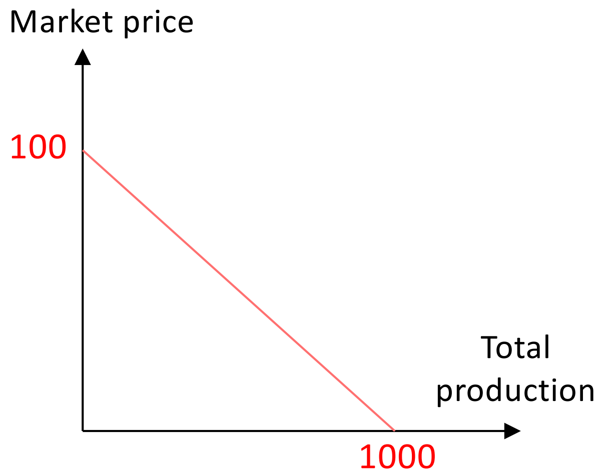

Note that the market price depends on the production strategies of both producers.

In an **oligopoly**, there are multiple strategic participants, each maximizing its own benefit. **Strategic interaction** occurs among these participants.

The above example is a **duopoly**, where there are two strategic producers.

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Other models

Bertrand competition model

- ✓ Similar to Cournot, the market price is considered an affine function of the total production.
- ✓ However, unlike Cournot, each producer assumes that it can alter market-clearing outcomes through its offer price. In other words, producers compete on pricing.

Conjectural variations model

- ✓ An upgraded version of the Cournot model
- ✓ The production strategy of each producer impacts not only the market price but also the production strategy of rivals, which are modeled by given reaction parameters.
- ✓ These reactions parameters reflect the competitiveness level of the underlying market, ranging from perfect competition to a monopoly (or a cartel).

Supply function model

- ✓ Each producer submits its supply function offer to the market, which includes both a price and a production quantity offer.
- ✓ This model provides a more accurate description of the functioning of real-world electricity markets when compared with other imperfect competition models, such as Cournot, Bertrand, or conjectural variations.

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REMIT

ACER (the EU Agency) and the national (energy) regulatory authorities protect energy markets from abuse, building trust that energy markets work well for businesses and citizens. It is important that wholesale energy markets function well and that prices are determined in a fair way.

ACER European Union Agency for the Cooperation of Energy Regulators

The Regulation on Wholesale Energy Market Integrity and Transparency (REMIT) came into force in 2011 to support open and fair competition in the European wholesale energy markets.

Link: https://www.acer.europa.eu/remit/about-remit

An example

ACER
European Union Agency for the Cooperation of Energy Regulators

THE AGENCY ELECTRICITY GAS GREEN DEAL REMIT DOCUMENTS NEWS & ENGAGEMENT DATA EXTRANET

Home / News

6.3.2025

REMIT breach: Energi Danmark fined for manipulating the Nordic wholesale electricity market

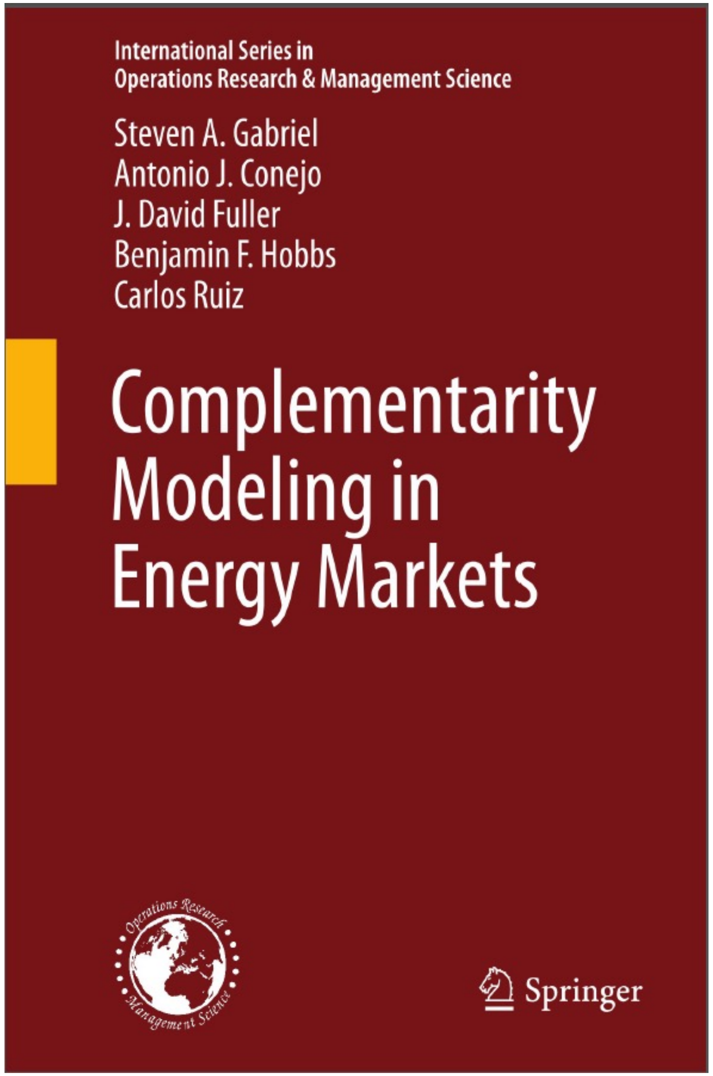
Share on:

“According to energy regulator (DUR) and the Danish state prosecutor, the misconduct, which took place on 3 January 2020, included five cases of electricity market manipulation and one attempt to do so in violation of Article 5 of REMIT. ***Through its behaviour, called 'cross-zonal capacity hoarding', the company acquired all, or a significant share of, the capacity available on an electricity transmission connection between two bidding areas by trading with itself.*** *In this way, Energi Danmark prevented other market participants from using the capacity, thereby creating or increasing a price difference between the two bidding areas.”*

Source: ACER [link]

Further reading

Interested in learning about strategic offering and imperfect market equilibrium?



Further reading

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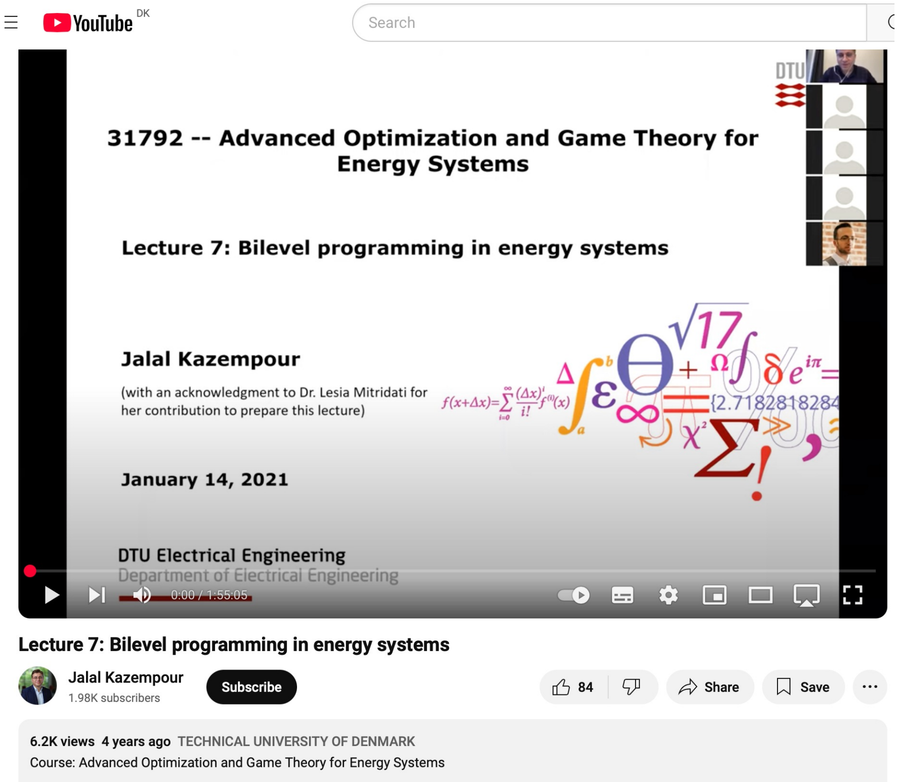
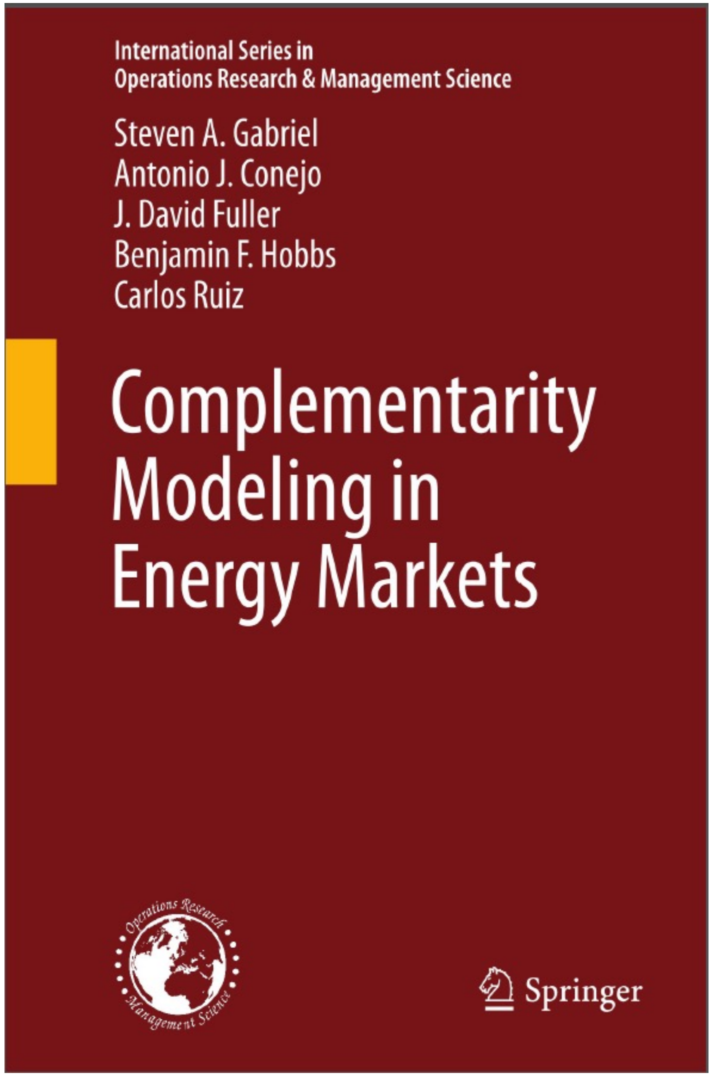
A video recording on strategic offering: [link]





Note

We will focus solely on investigating price-taking offering strategy for the remainder of this course.

Price-taking offering strategy

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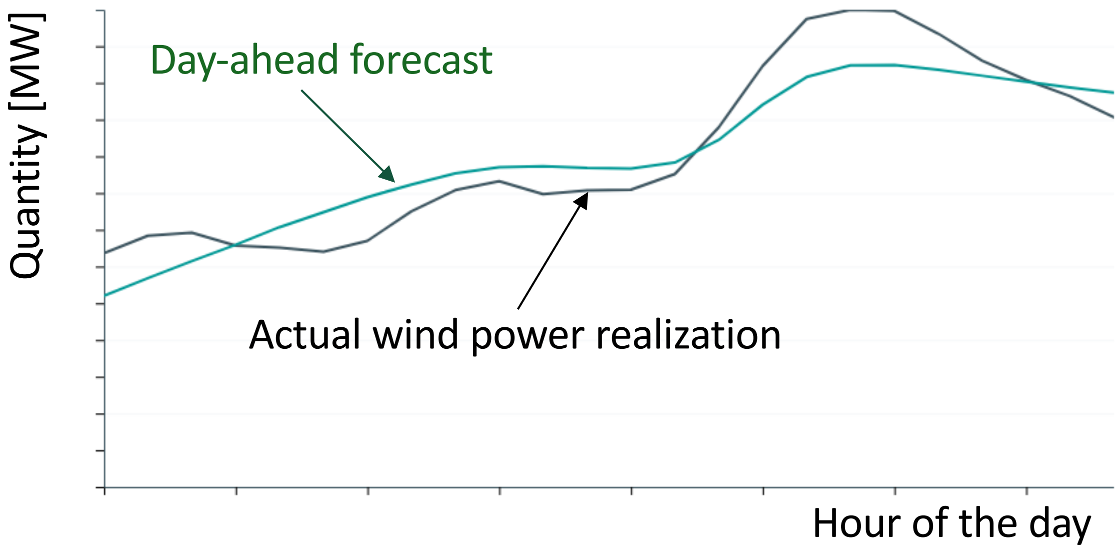

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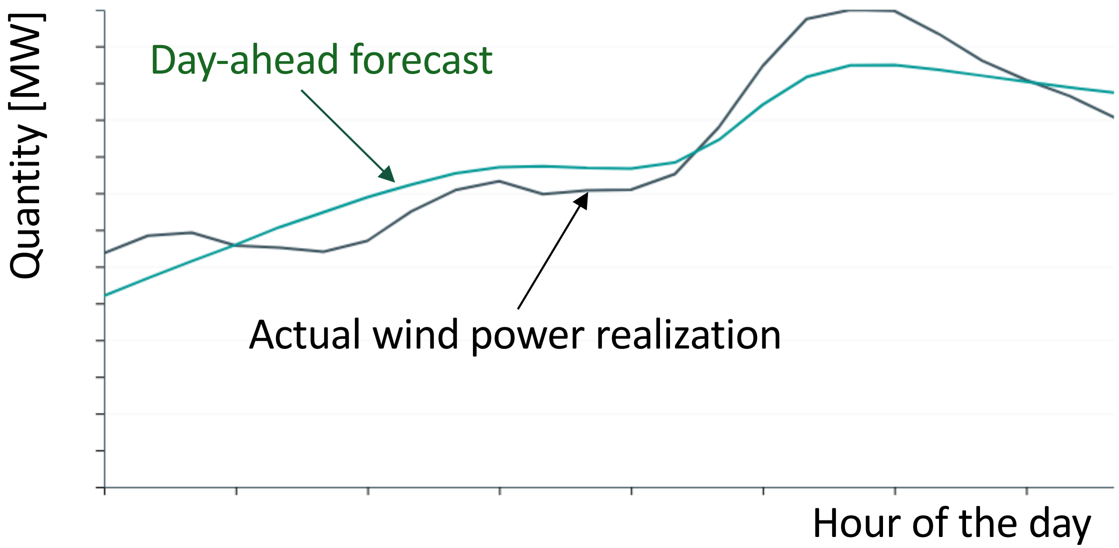

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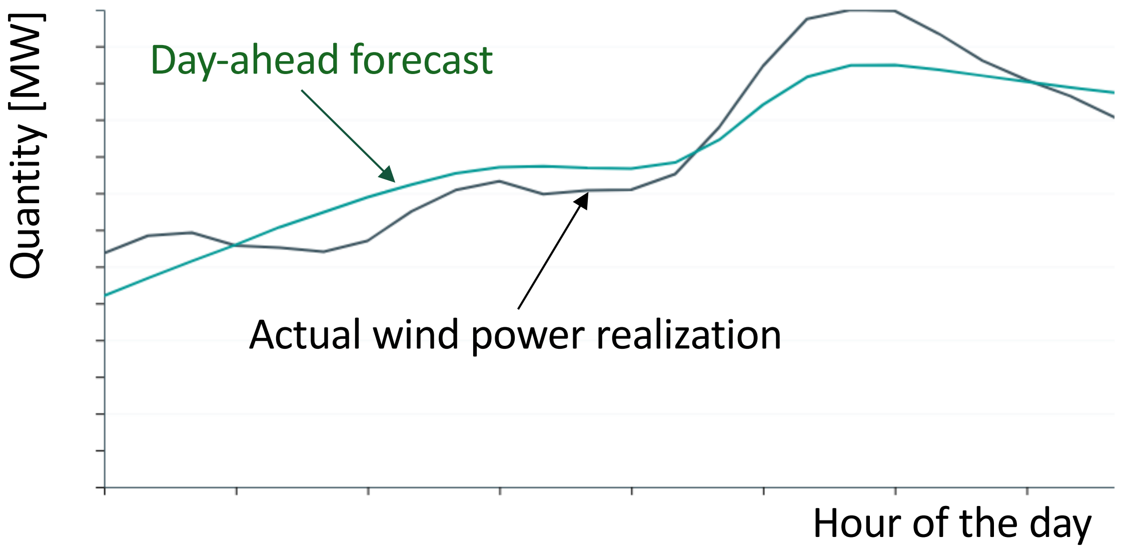

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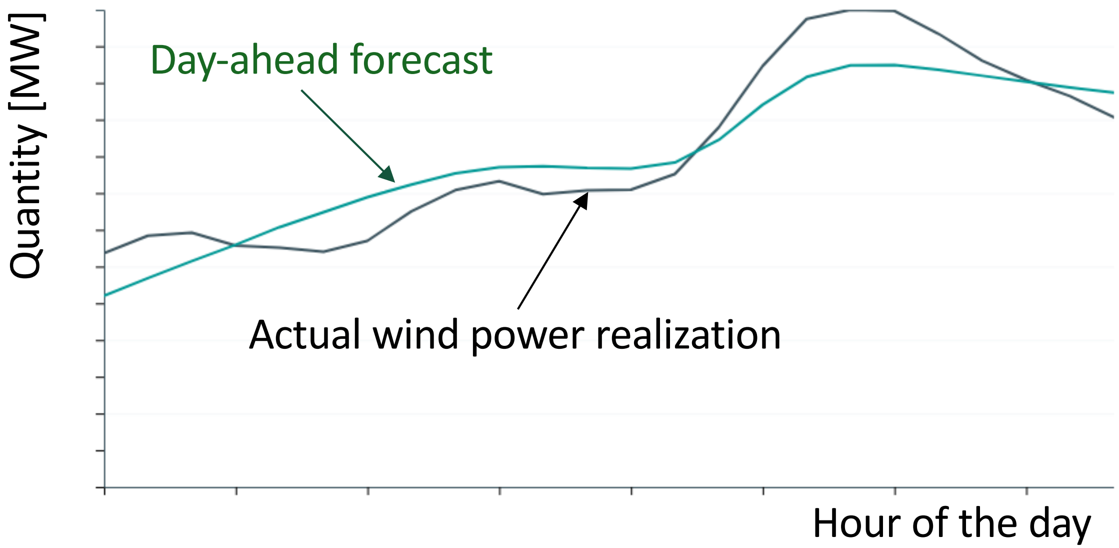

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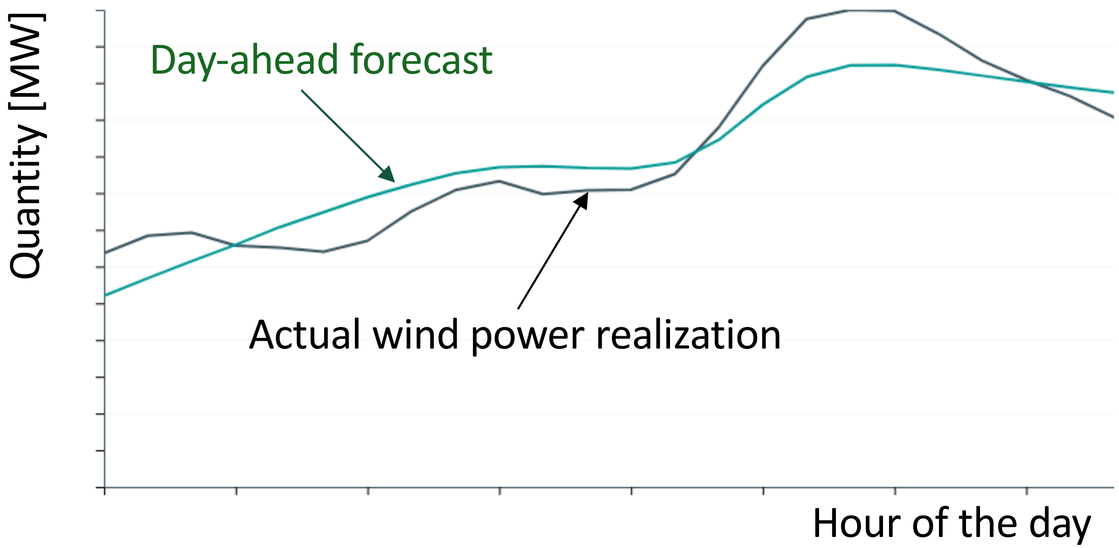

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Discussion: Should the wind farm offer its day-ahead forecast as the offering quantity in the day-ahead market? Not necessarily. Why?

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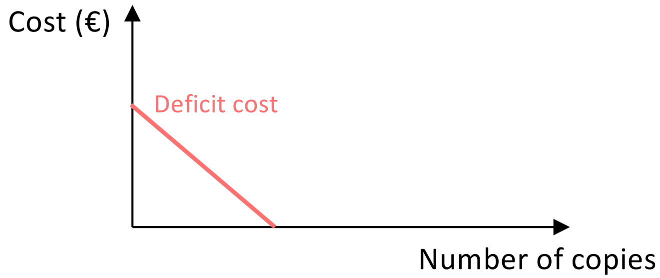

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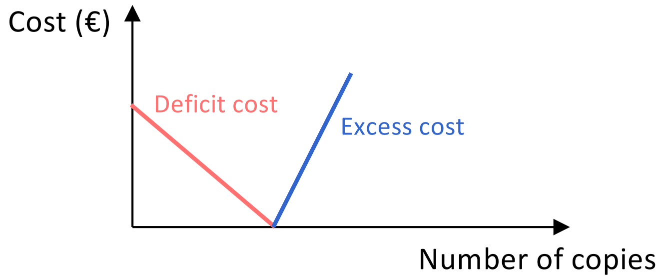

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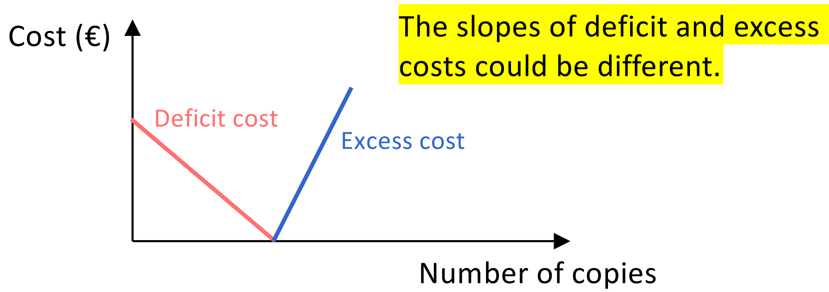

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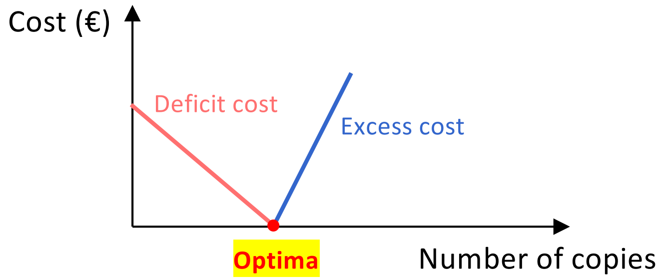


- Too many papers: Unsold copies will become worthless at the end of the day.
- Not enough papers: There is a loss of opportunity to sell more papers.

Wind power trading as a “newsvendor” problem

Newsvendor problem:

A newspaper vendor must decide each morning how many copies of the day's paper to purchase, given the uncertainty in demand.

A wind farm must decide each morning how much wind power to sell, given the uncertainty in supply.

- Too many papers: Unsold copies will become worthless at the end of the day.
- Not enough papers: There is a loss of opportunity to sell more papers.

Wind power trading as a “newsvendor” problem

Newsvendor problem:

A newspaper vendor must decide each morning how many copies of the day's paper to purchase, given the uncertainty in demand.

A wind farm must decide each morning how much wind power to sell,
given the uncertainty in supply.

- Too many papers: Unsold copies will become worthless at the end of the day.
- Not enough papers: There is a loss of opportunity to sell more papers.

Can we apply this to the wind power trading problem?

Wind power trading as a "newsvendor" problem

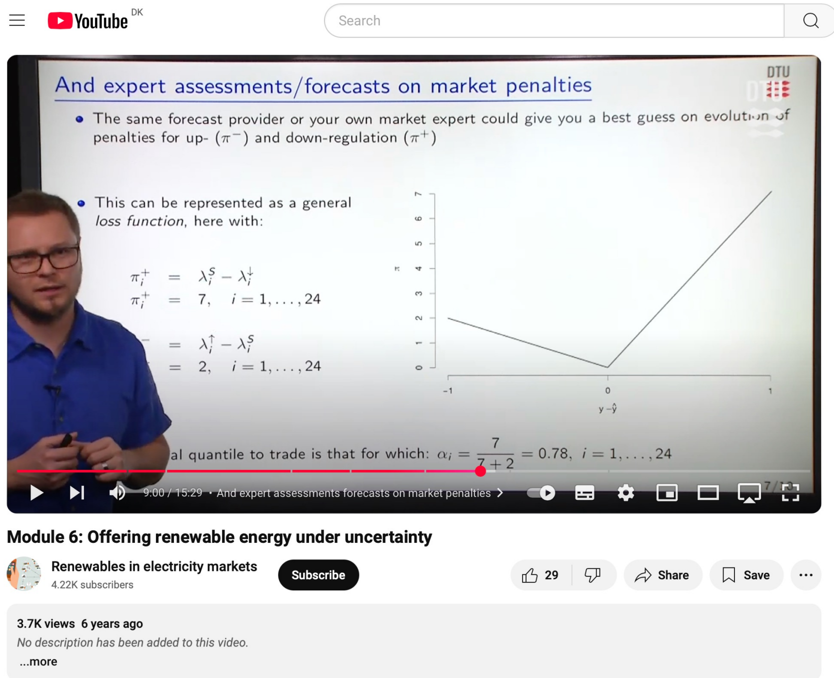
- Too many papers: Unsold
- Not enough papers: There

Recall from previous lectures that under a **two-price** balancing market, there is no profit opportunity for a wind farm that creates imbalances with respect to its day-ahead quantity offer.

- Wind power trading under a **two-price** scheme resembles a newsvendor problem, where the wind farm must make an informed offering decision in the "day-ahead" stage to minimize costs in the balancing stage.
- This is **not** be the case under a **one-price** scheme, as the wind farm can potentially earn a profit in the balancing market.

Wind power trading as a “newsvendor” problem

Under certain assumptions, wind power trading under uncertainty, given a **two-price** balancing scheme, can be analytically solved as a newsvendor problem.



For your study:

Video recording of Pierre Pinson: [link]

P. Pinson, C. Chevallier and G. N. Kariniotakis, "Trading Wind Generation From Short-Term Probabilistic Forecasts of Wind Power," *IEEE Transactions on Power Systems*, vol. 22, no. 3, pp. 1148-1156, Aug. 2007

Price-taking offering strategy

For simplicity, let's consider only the day-ahead and balancing markets, with both markets using the uniform pricing scheme.

Price-taking offering strategy

For simplicity, let's consider only the day-ahead and balancing markets, with both markets using the uniform pricing scheme.

Input data:

Offering strategy optimization problem

(To be solved by a <u>price-taking</u> wind farm)

Objective: Expected profit maximization

Results:

Price-taking offering strategy

For simplicity, let's consider only the day-ahead and balancing markets, with both markets using the uniform pricing scheme.

Input data:

- Day-ahead wind power forecast per each time period (MW)
- Day-ahead market-clearing price forecast per each time period (€/MWh)
- Balancing market-clearing price forecast per each time period (€/MWh)

Offering strategy optimization problem

(To be solved by a <u>price-taking</u> wind farm)

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Offering strategy optimization problem

(To be solved by a price-taking wind farm)

Objective: Expected profit maximization

Results:

Production quantity offer (MW) in the day-ahead market (note that the offer price is assumed to be zero).

Price-taking offering strategy

For simplicity, let's consider only the day-ahead and balancing markets, with both markets using the uniform pricing scheme.

Input data:

- Day-ahead wind power forecast per each time period (MW)
- Day-ahead market-clearing price forecast per each time period (€/MWh)
- Balancing market-clearing price forecast per each time period (€/MWh)

Offering strategy optimization problem

(To be solved by a price-taking wind farm)

Objective: Expected profit maximization

Results:

Production quantity offer (MW) in the day-ahead market (note that the offer price is assumed to be zero).

Assumption (to be relaxed later):

All forecasts used as input data are single-point values (deterministic forecast).

Offering problem of a price-taking wind farm

$$\underset{p_t^{\mathrm{DA}},\ I_t^{\mathrm{B}},\ \Delta_t}{\text{Maximize}} \quad \sum_{t=1}^{24} \left(\lambda_t^{\mathrm{DA}}\ p_t^{\mathrm{DA}} + I_t^{\mathrm{B}}\right)$$

subject to:

$$0 \leq p_t^{\mathrm{DA}} \leq P^{\mathrm{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

$$I_t^{\mathrm{B}} = \begin{cases} \lambda_t^{\downarrow}\ \Delta_t & \text{if } \Delta_t \geq 0 \\ \lambda_t^{\uparrow}\ \Delta_t & \text{if } \Delta_t < 0 \end{cases} \quad \forall t = \{1, 2, ..., 24\}$$

$$\Delta_t = p_t^{\mathrm{real}} - p_t^{\mathrm{DA}} \quad \forall t = \{1, 2, ..., 24\}$$

Offering problem of a price-taking wind farm

Forecasts (input data) for each time period *t*

$$\underset{p_t^{\mathrm{DA}},\ I_t^{\mathrm{B}},\ \Delta_t}{\text{Maximize}} \quad \sum_{t=1}^{24} \left(\boxed{\lambda_t^{\mathrm{DA}}}\, p_t^{\mathrm{DA}} + I_t^{\mathrm{B}} \right)$$

Day-ahead market price forecast

subject to:

$$0 \leq p_t^{\mathrm{DA}} \leq P^{\mathrm{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

$$I_t^{\mathrm{B}} = \begin{cases} \lambda_t^{\downarrow}\ \Delta_t & \text{if } \Delta_t \geq 0 \\ \lambda_t^{\uparrow}\ \Delta_t & \text{if } \Delta_t < 0 \end{cases} \quad \forall t = \{1, 2, ..., 24\}$$

$$\Delta_t = \boxed{p_t^{\mathrm{real}}} - p_t^{\mathrm{DA}} \quad \forall t = \{1, 2, ..., 24\}$$

Wind power forecast

Offering problem of a price-taking wind farm

$$\underset{p_t^{\mathrm{DA}},\ I_t^{\mathrm{B}},\ \Delta_t}{\text{Maximize}} \quad \sum_{t=1}^{24} \left(\lambda_t^{\mathrm{DA}} \boxed{p_t^{\mathrm{DA}}} + I_t^{\mathrm{B}}\right)$$

Main result: How much production quantity (MW) should the wind farm offer in time *t* in the day-ahead market?

subject to:

$$0 \leq p_t^{\mathrm{DA}} \leq P^{\mathrm{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

$$I_t^{\mathrm{B}} = \begin{cases} \lambda_t^{\downarrow}\ \Delta_t & \text{if } \Delta_t \geq 0 \\ \lambda_t^{\uparrow}\ \Delta_t & \text{if } \Delta_t < 0 \end{cases} \quad \forall t = \{1, 2, ..., 24\}$$

$$\Delta_t = p_t^{\mathrm{real}} - p_t^{\mathrm{DA}} \quad \forall t = \{1, 2, ..., 24\}$$

Offering problem of a price-taking wind farm

$$\underset{p_t^{\mathrm{DA}},\ I_t^{\mathrm{B}},\ \Delta_t}{\text{Maximize}} \quad \sum_{t=1}^{24} \overbrace{(\underbrace{\lambda_t^{\mathrm{DA}}\ p_t^{\mathrm{DA}}}_{\substack{\text{Income (price*quantity) in} \\ \text{time } t \text{ in the day-ahead market}}} + I_t^{\mathrm{B}})}^{\substack{\text{Profit of the wind farm in time } t \text{ from} \\ \text{day-ahead and balancing markets}}}$$

Payoff (could be positive, zero, or negative) in time t in the balancing market

subject to:

$$0 \leq p_t^{\mathrm{DA}} \leq P^{\mathrm{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

$$I_t^{\mathrm{B}} = \begin{cases} \lambda_t^{\downarrow}\ \Delta_t & \text{if } \Delta_t \geq 0 \\ \lambda_t^{\uparrow}\ \Delta_t & \text{if } \Delta_t < 0 \end{cases} \quad \forall t = \{1, 2, ..., 24\}$$

$$\Delta_t = p_t^{\mathrm{real}} - p_t^{\mathrm{DA}} \quad \forall t = \{1, 2, ..., 24\}$$

Offering problem of a price-taking wind farm

$$\underset{p_t^{\mathrm{DA}},\ I_t^{\mathrm{B}},\ \Delta_t}{\text{Maximize}} \quad \sum_{t=1}^{24} \left(\lambda_t^{\mathrm{DA}}\ p_t^{\mathrm{DA}} + I_t^{\mathrm{B}}\right)$$

subject to:

Nominal (installed) capacity of the farm

$$0 \leq p_t^{\mathrm{DA}} \leq \boxed{P^{\mathrm{nom}}} \quad \forall t = \{1, 2, ..., 24\}$$

→ Lower and upper bounds on the production quantity (MW) to be offered in time *t* in the day-ahead market.

$$I_t^{\mathrm{B}} = \begin{cases} \lambda_t^{\downarrow}\ \Delta_t & \text{if } \Delta_t \geq 0 \\ \lambda_t^{\uparrow}\ \Delta_t & \text{if } \Delta_t < 0 \end{cases} \quad \forall t = \{1, 2, ..., 24\}$$

$$\Delta_t = p_t^{\mathrm{real}} - p_t^{\mathrm{DA}} \quad \forall t = \{1, 2, ..., 24\}$$

Offering problem of a price-taking wind farm

$$\underset{p_t^{\text{DA}},\ I_t^{\text{B}},\ \Delta_t}{\text{Maximize}} \quad \sum_{t=1}^{24} \left(\lambda_t^{\text{DA}}\ p_t^{\text{DA}} + I_t^{\text{B}}\right)$$

subject to:

$$0 \leq p_t^{\text{DA}} \leq P^{\text{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

$$I_t^{\text{B}} = \begin{cases} \lambda_t^{\downarrow}\ \boxed{\Delta_t} & \text{if } \Delta_t \geq 0 \\ \lambda_t^{\uparrow}\ \Delta_t & \text{if } \Delta_t < 0 \end{cases} \quad \forall t = \{1, 2, ..., 24\}$$

$$\boxed{\Delta_t = p_t^{\text{real}} - p_t^{\text{DA}}} \quad \forall t = \{1, 2, ..., 24\}$$

Imbalance in real-time (MW) = Wind power forecast – Day-ahead schedule

Offering problem of a price-taking wind farm

$$\underset{p_t^{\mathrm{DA}},\ I_t^{\mathrm{B}},\ \Delta_t}{\text{Maximize}} \quad \sum_{t=1}^{24} \left(\lambda_t^{\mathrm{DA}}\ p_t^{\mathrm{DA}} + I_t^{\mathrm{B}}\right)$$

subject to:

$$0 \leq p_t^{\mathrm{DA}} \leq P^{\mathrm{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

BBalancing price (if the wind farm has excess wind power in the balancing stage).

$$I_t^{\mathrm{B}} = \begin{cases} \boxed{\lambda_t^{\downarrow}} \Delta_t & \text{if } \Delta_t \geq 0 \\ \boxed{\lambda_t^{\uparrow}} \Delta_t & \text{if } \Delta_t < 0 \end{cases} \quad \forall t = \{1, 2, ..., 24\}$$

Balancing price (if the wind farm has a wind power deficit in the balancing stage).

$$\Delta_t = p_t^{\mathrm{real}} - p_t^{\mathrm{DA}} \quad \forall t = \{1, 2, ..., 24\}$$

Offering problem of a price-taking wind farm

$$\underset{p_t^{\mathrm{DA}},\ I_t^{\mathrm{B}},\ \Delta_t}{\text{Maximize}} \quad \sum_{t=1}^{24} \left(\lambda_t^{\mathrm{DA}}\ p_t^{\mathrm{DA}} + I_t^{\mathrm{B}}\right)$$

subject to:

$$0 \leq p_t^{\mathrm{DA}} \leq P^{\mathrm{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

$$I_t^{\mathrm{B}} = \begin{cases} \boxed{\begin{matrix} \lambda_t^{\downarrow}\ \Delta_t \\ \lambda_t^{\uparrow}\ \Delta_t \end{matrix}} & \begin{matrix} \text{if } \Delta_t \geq 0 \\ \text{if } \Delta_t < 0 \end{matrix} \end{cases} \quad \forall t = \{1, 2, ..., 24\}$$

$$\Delta_t = p_t^{\mathrm{real}} - p_t^{\mathrm{DA}} \quad \forall t = \{1, 2, ..., 24\}$$

Question:

Do we assume a **two-price** or a **one-price** scheme in the balancing stage?

Offering problem of a price-taking wind farm

$$\underset{p_t^{\mathrm{DA}},\ I_t^{\mathrm{B}},\ \Delta_t}{\text{Maximize}} \quad \sum_{t=1}^{24} \left(\lambda_t^{\mathrm{DA}}\ p_t^{\mathrm{DA}} + I_t^{\mathrm{B}}\right)$$

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$$\Delta_t = p_t^{\mathrm{real}} - p_t^{\mathrm{DA}} \quad \forall t = \{1, 2, ..., 24\}$$

Question:

Do we assume a **two-price** or a **one-price** scheme in the balancing stage?

Answer:

A two-price scheme, since depending on the form of imbalance the wind farm creates (excess or deficit), it may face different balancing prices.

Offering problem of a price-taking wind farm

$$\underset{p_t^{\mathrm{DA}},\ I_t^{\mathrm{B}},\ \Delta_t}{\text{Maximize}} \quad \sum_{t=1}^{24} (\lambda_t^{\mathrm{DA}}\ p_t^{\mathrm{DA}} + I_t^{\mathrm{B}})$$

subject to:

$$0 \leq p_t^{\mathrm{DA}} \leq P^{\mathrm{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

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Question:

Do we assume a **two-price** or a **one-price** scheme in the balancing stage?

Answer:

A two-price scheme, since depending on the form of imbalance the wind farm creates (excess or deficit), it may face different balancing prices.

This problem will be extended in Assignment 2, where, depending on whether the imbalance is "desired" (i.e., helping the system), the balancing price will be equal to the day-ahead price.

Offering problem of a price-taking wind farm

$$\underset{p_t^{\mathrm{DA}},\ I_t^{\mathrm{B}},\ \Delta_t}{\text{Maximize}} \quad \sum_{t=1}^{24} \left(\lambda_t^{\mathrm{DA}}\ p_t^{\mathrm{DA}} + I_t^{\mathrm{B}}\right)$$

subject to:

$$0 \leq p_t^{\mathrm{DA}} \leq P^{\mathrm{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

Income for the wind farm in the balancing stage due to power excess, which is calculated as the balancing price (in the down-regulation status) multiplied by the imbalance.

$$I_t^{\mathrm{B}} = \begin{cases} \boxed{\lambda_t^{\downarrow}\ \Delta_t} & \text{if } \Delta_t \geq 0 \\ \boxed{\lambda_t^{\uparrow}\ \Delta_t} & \text{if } \Delta_t < 0 \end{cases} \quad \forall t = \{1, 2, ..., 24\}$$

Cost for the wind farm in the balancing stage due to the power deficit, which is equal to the balancing price (in the up-regulation status) times the imbalance.

$$\Delta_t = p_t^{\mathrm{real}} - p_t^{\mathrm{DA}} \quad \forall t = \{1, 2, ..., 24\}$$

Offering problem of a price-taking wind farm

$$\underset{p_t^{\mathrm{DA}},\ I_t^{\mathrm{B}},\ \Delta_t}{\text{Maximize}} \quad \sum_{t=1}^{24} \left(\lambda_t^{\mathrm{DA}}\ p_t^{\mathrm{DA}} + I_t^{\mathrm{B}}\right)$$

subject to:

$$0 \leq p_t^{\mathrm{DA}} \leq P^{\mathrm{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

$$I_t^{\mathrm{B}} = \begin{cases} \boxed{\lambda_t^{\downarrow}}\ \Delta_t & \text{if } \Delta_t \geq 0 \\ \lambda_t^{\uparrow}\ \Delta_t & \text{if } \Delta_t < 0 \end{cases} \quad \forall t = \{1, 2, ..., 24\}$$

$$\Delta_t = p_t^{\mathrm{real}} - p_t^{\mathrm{DA}} \quad \forall t = \{1, 2, ..., 24\}$$

Question:

Under a **two-price** balancing scheme, what is the balancing price (in comparison to the day-ahead price)?

Offering problem of a price-taking wind farm

$$\underset{p_t^{\text{DA}},\ I_t^{\text{B}},\ \Delta_t}{\text{Maximize}} \quad \sum_{t=1}^{24} \left(\lambda_t^{\text{DA}}\ p_t^{\text{DA}} + I_t^{\text{B}}\right)$$

subject to:

$$0 \leq p_t^{\text{DA}} \leq P^{\text{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

$$I_t^{\text{B}} = \begin{cases} \boxed{\lambda_t^{\downarrow}}\ \Delta_t & \text{if } \Delta_t \geq 0 \\ \lambda_t^{\uparrow}\ \Delta_t & \text{if } \Delta_t < 0 \end{cases} \quad \forall t = \{1, 2, ..., 24\}$$

$$\Delta_t = p_t^{\text{real}} - p_t^{\text{DA}} \quad \forall t = \{1, 2, ..., 24\}$$

Question:

Under a **two-price** balancing scheme, what is the balancing price (in comparison to the day-ahead price)?

Answer:

- **Equal to the day-ahead price** if the imbalance is desired (helps the system). This happens when the system has a deficit and requires up-regulation, meaning wind power excess contributes positively.
- **Less than the day-ahead price** if the imbalance is undesired (does not help the system). This occurs when the system already has an excess supply and requires down-regulation services.

Offering problem of a price-taking wind farm

$$\underset{p_t^{\mathrm{DA}},\ I_t^{\mathrm{B}},\ \Delta_t}{\text{Maximize}} \quad \sum_{t=1}^{24} \left(\lambda_t^{\mathrm{DA}}\ p_t^{\mathrm{DA}} + I_t^{\mathrm{B}}\right)$$

subject to:

$$0 \leq p_t^{\mathrm{DA}} \leq P^{\mathrm{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

$$I_t^{\mathrm{B}} = \begin{cases} \boxed{\lambda_t^{\downarrow}}\ \Delta_t & \text{if } \Delta_t \geq 0 \\ \lambda_t^{\uparrow}\ \Delta_t & \text{if } \Delta_t < 0 \end{cases} \quad \forall t = \{1, 2, ..., 24\}$$

$$\Delta_t = p_t^{\mathrm{real}} - p_t^{\mathrm{DA}} \quad \forall t = \{1, 2, ..., 24\}$$

Question:

And what can we say about this price under a **one-price** scheme?

Offering problem of a price-taking wind farm

$$\underset{p_t^{\mathrm{DA}},\ I_t^{\mathrm{B}},\ \Delta_t}{\text{Maximize}} \quad \sum_{t=1}^{24} \left(\lambda_t^{\mathrm{DA}}\ p_t^{\mathrm{DA}} + I_t^{\mathrm{B}}\right)$$

subject to:

$$0 \leq p_t^{\mathrm{DA}} \leq P^{\mathrm{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

$$I_t^{\mathrm{B}} = \begin{cases} \boxed{\lambda_t^{\downarrow}}\ \Delta_t & \text{if } \Delta_t \geq 0 \\ \lambda_t^{\uparrow}\ \Delta_t & \text{if } \Delta_t < 0 \end{cases} \quad \forall t = \{1, 2, ..., 24\}$$

$$\Delta_t = p_t^{\mathrm{real}} - p_t^{\mathrm{DA}} \quad \forall t = \{1, 2, ..., 24\}$$

Question:

And what can we say about this price under a **one-price** scheme?

Answer:

The balancing price depends on the system's needs. If the system experiences a power supply deficit, the balancing price will be higher than the day-ahead price. Conversely, if there is an excess in power supply, the balancing price will be lower than the day-ahead price.

Offering problem of a price-taking wind farm

$$\underset{p_t^{\mathrm{DA}},\ I_t^{\mathrm{B}},\ \Delta_t}{\text{Maximize}} \quad \sum_{t=1}^{24} \left(\lambda_t^{\mathrm{DA}}\ p_t^{\mathrm{DA}} + I_t^{\mathrm{B}}\right)$$

What is the balancing price in this case?

subject to:

$$0 \leq p_t^{\mathrm{DA}} \leq P^{\mathrm{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

$$I_t^{\mathrm{B}} = \begin{cases} \lambda_t^{\downarrow}\ \Delta_t & \text{if } \Delta_t \geq 0 \\ \boxed{\lambda_t^{\uparrow}}\ \Delta_t & \text{if } \Delta_t < 0 \end{cases} \quad \forall t = \{1, 2, ..., 24\}$$

$$\Delta_t = p_t^{\mathrm{real}} - p_t^{\mathrm{DA}} \quad \forall t = \{1, 2, ..., 24\}$$

Offering problem of a price-taking wind farm

$$\underset{p_t^{\mathrm{DA}},\ I_t^{\mathrm{B}},\ \Delta_t}{\text{Maximize}} \quad \sum_{t=1}^{24} (\lambda_t^{\mathrm{DA}}\ p_t^{\mathrm{DA}} + I_t^{\mathrm{B}})$$

subject to:

$$0 \le p_t^{\mathrm{DA}} \le P^{\mathrm{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

$$I_t^{\mathrm{B}} = \begin{cases} \lambda_t^{\downarrow}\ \Delta_t & \text{if } \Delta_t \ge 0 \\ \boxed{\lambda_t^{\uparrow}}\ \Delta_t & \text{if } \Delta_t < 0 \end{cases} \quad \forall t = \{1, 2, ..., 24\}$$

$$\Delta_t = p_t^{\mathrm{real}} - p_t^{\mathrm{DA}} \quad \forall t = \{1, 2, ..., 24\}$$

Under a **two-price** balancing scheme:

- The balancing price will be equal to the day-ahead price if the imbalance is desired (i.e., it helps the system). In other words, if the system has an excess supply and requires down-regulation, the wind power deficit contributes positively.
- The balancing price will be higher than the day-ahead price if the imbalance is undesired (i.e., it does not help the system). In other words, if the system also has a supply deficit and requires up-regulation, the wind power shortfall adds to the problem.

Under a **one-price** balancing scheme:

- The balancing price can be lower, equal to, or higher than the day-ahead price, depending on the system's needs.

Offering problem of a price-taking wind farm

$$\underset{p_t^{\mathrm{DA}},\ I_t^{\mathrm{B}},\ \Delta_t}{\text{Maximize}} \quad \sum_{t=1}^{24} \left(\lambda_t^{\mathrm{DA}}\ p_t^{\mathrm{DA}} + I_t^{\mathrm{B}}\right)$$

An assumption to remove the need for balancing price forecast:
For simplicity, let's assume that in the power deficit condition, the balancing price is 20% higher than the day-ahead price, whereas in the power excess condition, it is 10% lower than the day-ahead price.

subject to:

$$0 \leq p_t^{\mathrm{DA}} \leq P^{\mathrm{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

$$\boxed{I_t^{\mathrm{B}} = \begin{cases} \lambda_t^{\downarrow}\ \Delta_t & \text{if } \Delta_t \geq 0 \\ \lambda_t^{\uparrow}\ \Delta_t & \text{if } \Delta_t < 0 \end{cases} \quad \forall t = \{1, 2, ..., 24\}}$$

$$\Delta_t = p_t^{\mathrm{real}} - p_t^{\mathrm{DA}} \quad \forall t = \{1, 2, ..., 24\}$$

Offering problem of a price-taking wind farm

$$\underset{p_t^{\mathrm{DA}},\ I_t^{\mathrm{B}},\ \Delta_t}{\text{Maximize}} \quad \sum_{t=1}^{24} (\lambda_t^{\mathrm{DA}}\ p_t^{\mathrm{DA}} + I_t^{\mathrm{B}})$$

subject to:

$$0 \leq p_t^{\mathrm{DA}} \leq P^{\mathrm{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

These factors (0.9 and 1.2) discourage participation in the balancing market, incentivizing the wind farm to offer its most accurate forecast as the production quantity in the day-ahead market.

$$\boxed{I_t^{\mathrm{B}} = \begin{cases} 0.9\ \lambda_t^{\mathrm{DA}}\ \Delta_t & \text{if } \Delta_t \geq 0 \\ 1.2\ \lambda_t^{\mathrm{DA}}\ \Delta_t & \text{if } \Delta_t < 0 \end{cases} \quad \forall t = \{1, 2, ..., 24\}}$$

$$\Delta_t = p_t^{\mathrm{real}} - p_t^{\mathrm{DA}} \quad \forall t = \{1, 2, ..., 24\}$$

Offering problem of a price-taking wind farm

$$\underset{p_t^{\mathrm{DA}},\ I_t^{\mathrm{B}},\ \Delta_t}{\text{Maximize}} \quad \sum_{t=1}^{24} (\lambda_t^{\mathrm{DA}}\ p_t^{\mathrm{DA}} + I_t^{\mathrm{B}})$$

subject to:

$$0 \leq p_t^{\mathrm{DA}} \leq P^{\mathrm{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

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$$\Delta_t = p_t^{\mathrm{real}} - p_t^{\mathrm{DA}} \quad \forall t = \{1, 2, ..., 24\}$$

Question:

Is this also the case under the "one-price" scheme (where both factors are identical)? In other words, does the wind farm have an incentive to submit its most accurate forecast in the day-ahead stage?

Offering problem of a price-taking wind farm

$$\underset{p_t^{\mathrm{DA}},\ I_t^{\mathrm{B}},\ \Delta_t}{\text{Maximize}} \quad \sum_{t=1}^{24} (\lambda_t^{\mathrm{DA}}\ p_t^{\mathrm{DA}} + I_t^{\mathrm{B}})$$

subject to:

$$0 \leq p_t^{\mathrm{DA}} \leq P^{\mathrm{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

These factors (0.9 and 1.2) discourage participation in the balancing market, incentivizing the wind farm to offer its most accurate forecast as the production quantity in the day-ahead market.

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$$\Delta_t = p_t^{\mathrm{real}} - p_t^{\mathrm{DA}} \quad \forall t = \{1, 2, ..., 24\}$$

Question:

Is this also the case under the "one-price" scheme (where both factors are identical)? In other words, does the wind farm have an incentive to submit its most accurate forecast in the day-ahead stage?

Answer: No!

Offering problem of a price-taking wind farm

$$\underset{p_t^{\mathrm{DA}},\ I_t^{\mathrm{B}},\ \Delta_t}{\text{Maximize}} \quad \sum_{t=1}^{24} \left(\lambda_t^{\mathrm{DA}}\ p_t^{\mathrm{DA}} + I_t^{\mathrm{B}}\right)$$

subject to:

$$0 \leq p_t^{\mathrm{DA}} \leq P^{\mathrm{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

These factors (0.9 and 1.2) discourage participation in the balancing market, incentivizing the wind farm to offer its most accurate forecast as the production quantity in the day-ahead market.

$$I_t^{\mathrm{B}} = \begin{cases} 0.9\ \lambda_t^{\mathrm{DA}}\ \Delta_t & \text{if } \Delta_t \geq 0 \\ 1.2\ \lambda_t^{\mathrm{DA}}\ \Delta_t & \text{if } \Delta_t < 0 \end{cases} \quad \forall t = \{1, 2, ..., 24\}$$

$$\Delta_t = p_t^{\mathrm{real}} - p_t^{\mathrm{DA}} \quad \forall t = \{1, 2, ..., 24\}$$

Recall: The wind farm may not have a perfect forecast of its production in the day-ahead stage. Therefore, it cannot avoid participating in the balancing market. We will revisit this topic in a few slides!

Offering problem of a price-taking wind farm

Note: This constraint is non-linear and non-convex due to the variable-dependent condition. In other words, there is a variable after the "if" statement. We generally prefer to work with convex optimization, as it guarantees a global (rather than local) optimal solution.

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Let’s convexify (in this case, linearize) this constraint in the next slide!

Linear offering problem of a price-taking wind farm

$$\underset{p_t^{\mathrm{DA}},\ \Delta_t,\ \Delta_t^{\uparrow},\ \Delta_t^{\downarrow}}{\text{Maximize}} \quad \sum_{t=1}^{24} \left(\lambda_t^{\mathrm{DA}}\ p_t^{\mathrm{DA}} + \boxed{0.9\ \lambda_t^{\mathrm{DA}}\ \Delta_t^{\uparrow} - 1.2\ \lambda_t^{\mathrm{DA}}\ \Delta_t^{\downarrow}} \right)$$

subject to:

$$0 \leq p_t^{\mathrm{DA}} \leq P^{\mathrm{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

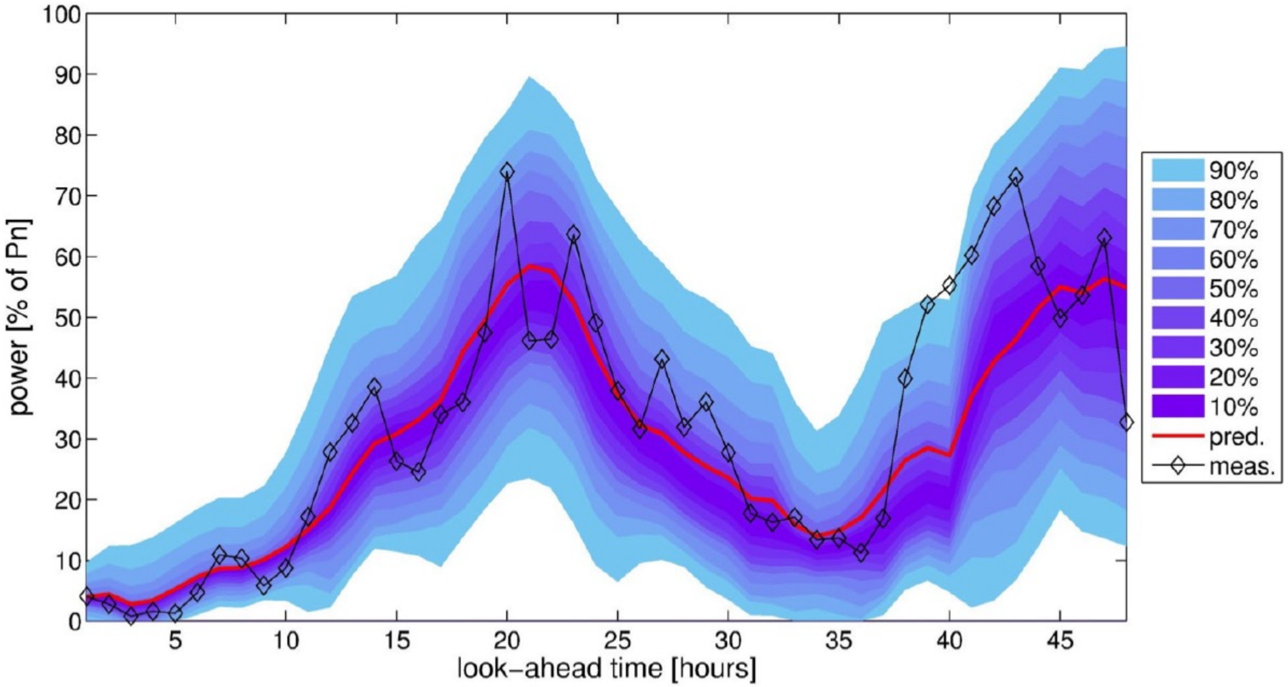
$$\Delta_t = p_t^{\mathrm{real}} - p_t^{\mathrm{DA}} \quad \forall t = \{1, 2, ..., 24\}$$

$$\boxed{\begin{aligned} &\Delta_t = \Delta_t^{\uparrow} - \Delta_t^{\downarrow} \quad \forall t = \{1, 2, ..., 24\} \\ &\Delta_t^{\uparrow} \geq 0 \quad \forall t = \{1, 2, ..., 24\} \\ &\Delta_t^{\downarrow} \geq 0 \quad \forall t = \{1, 2, ..., 24\} \end{aligned}}$$

We introduce two additional non-negative auxiliary variables: one for power excess and another for power deficit.

Modeling uncertainty

We almost never have a "perfect" deterministic wind power forecast (especially in the day-ahead stage), but we can obtain a probabilistic forecast!



from pierrepinson.com

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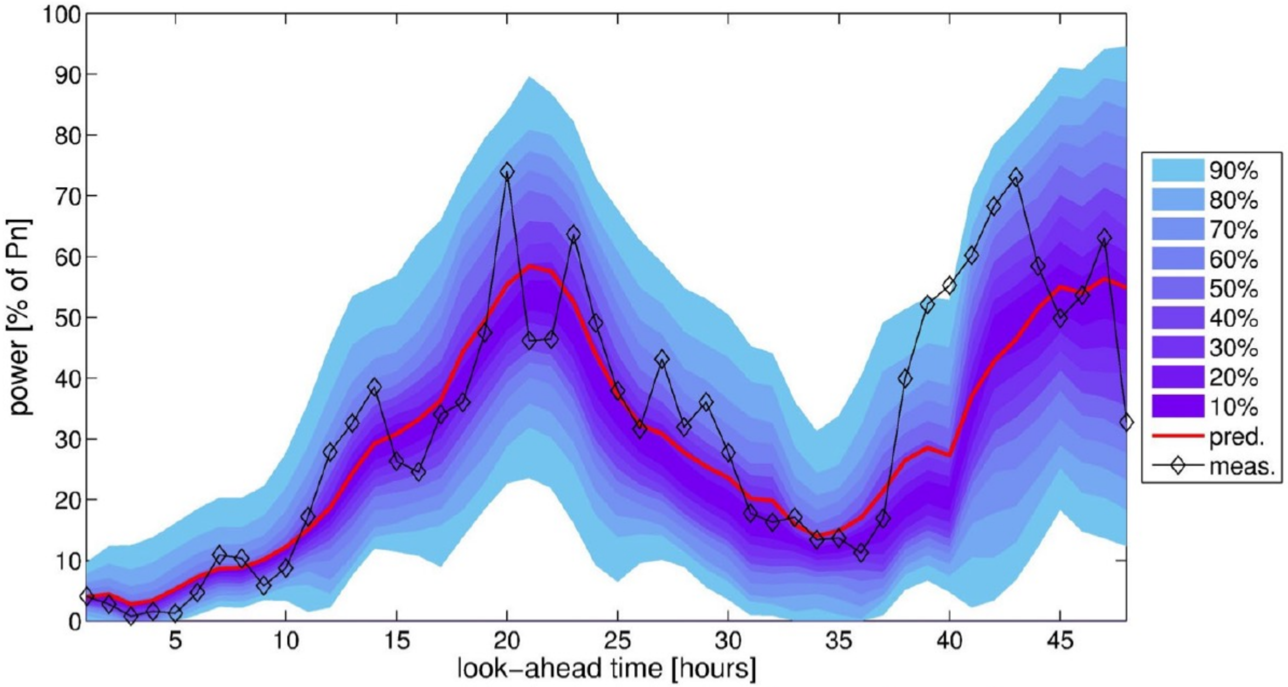

from pierrepinson.com

- Similarly, the day-ahead market price is also uncertain.
- The balancing market price is uncertain as well, but for simplicity, we have already linked it to the day-ahead price!

Modeling uncertainty

Let's model uncertainty using a finite number of scenarios!

- Recall we have two sources of uncertainty: Wind power forecasts and day-ahead market prices in every time t.

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For example, assume there are ***two*** scenarios with the same probability (weight). In other words, the probability of each scenario is 0.5.

- Note that the sum of the probabilities of all scenarios should equal 1.

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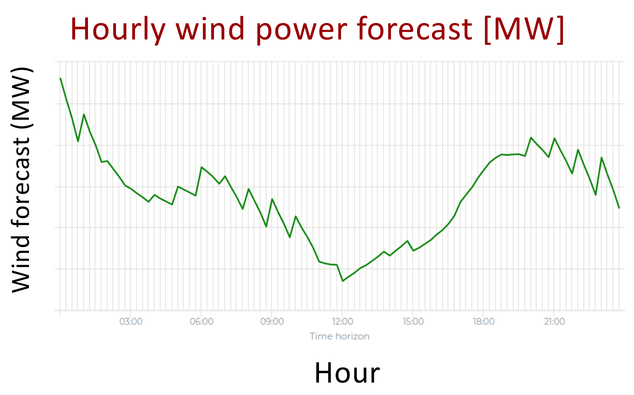
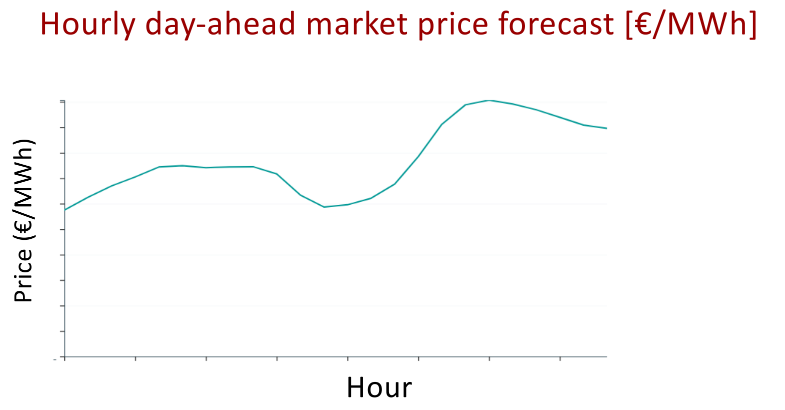
Scenario 1:





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Hourly wind power forecast [MW]

Hourly day-ahead market price forecast [€/MWh]

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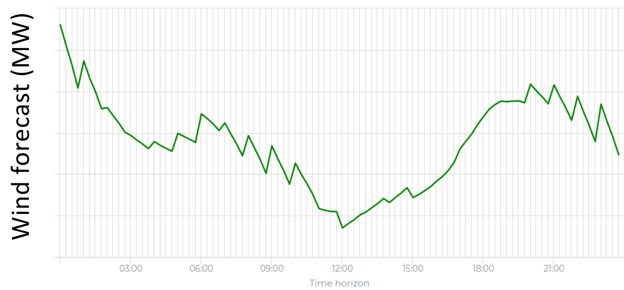
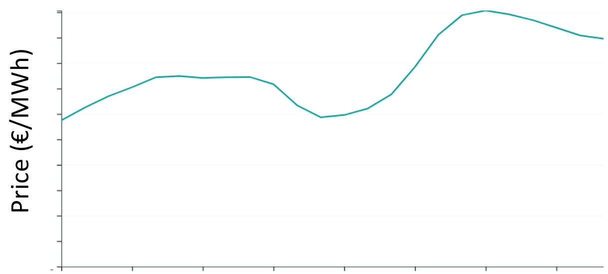
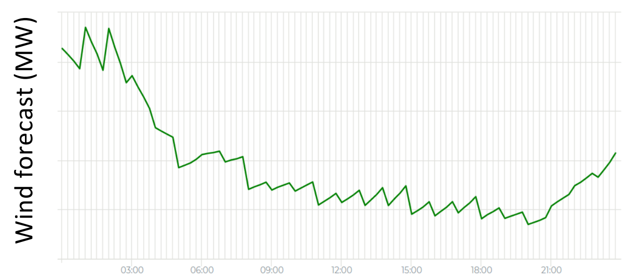
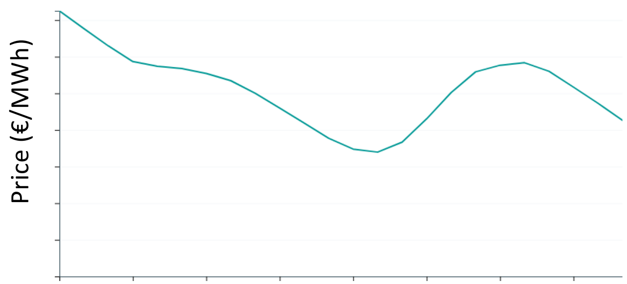




Scenario 2:





Hour

Hour

Stochastic offering strategy problem

Imagine that in the day-ahead stage, based on the available forecast, we have 4 potential scenarios, each with identical probabilities:

Stochastic offering strategy problem

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Scenarios are generated based on the available information in the day-ahead stage, but they will only be realized in the balancing stage.

$\xrightarrow{0.25}$ Scenario 1

$\xrightarrow{0.25}$ Scenario 2

$\xrightarrow{0.25}$ Scenario 3

$\xrightarrow{0.25}$ Scenario 4

Stochastic offering strategy problem

Imagine that in the day-ahead stage, based on the available forecast, we have 4 potential scenarios, each with identical probabilities:

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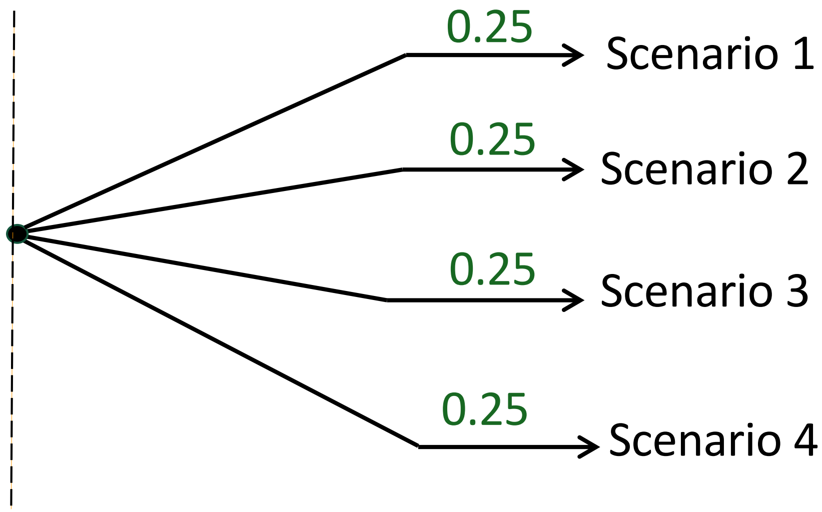
A decision must be made in the day-ahead stage, accounting for the potential scenarios in expectation.



Stochastic offering strategy problem

Imagine that in the day-ahead stage, based on the available forecast, we have 4 potential scenarios, each with identical probabilities:

Look-ahead strategy

Scenarios are generated based on the available information in the day-ahead stage, but they will only be realized in the balancing stage.

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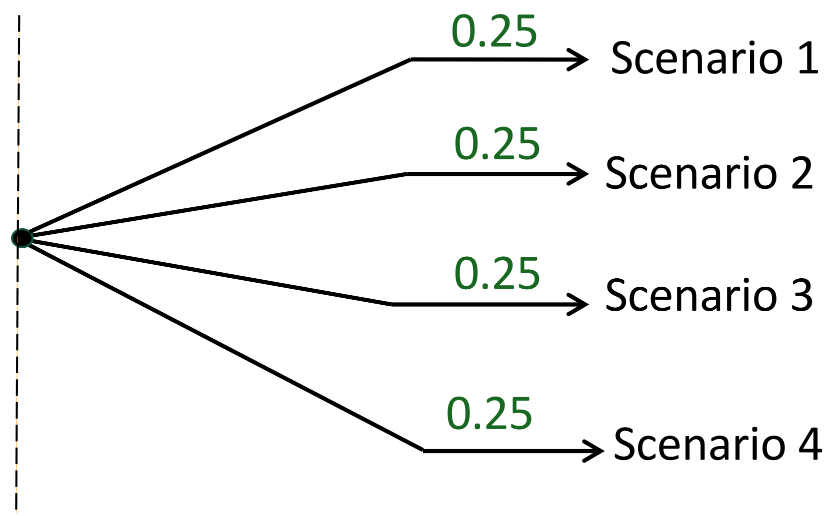
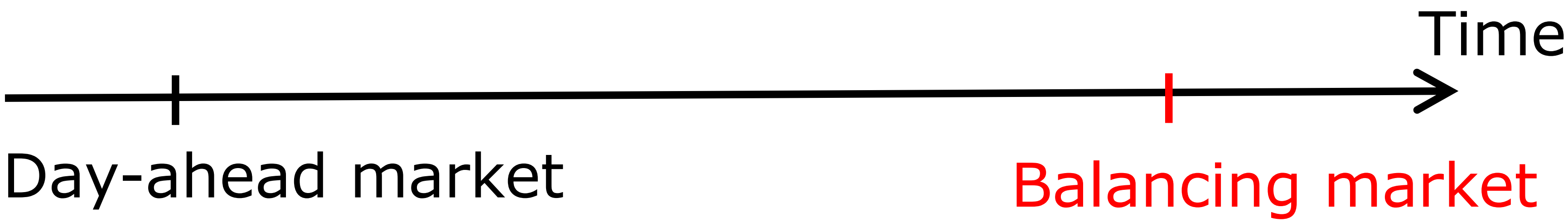
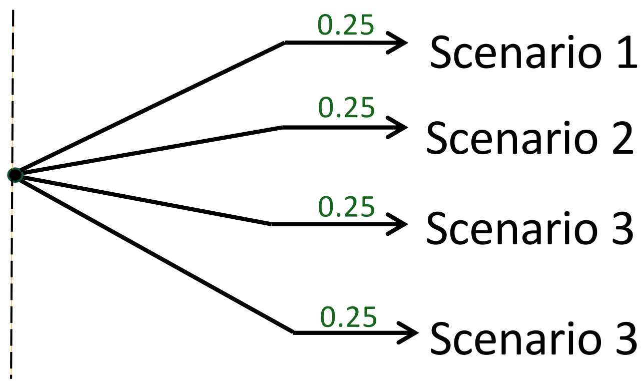


Stochastic offering strategy problem



Offering strategy based on stochastic forecasts, e.g.,



The true realization of wind power production

Stochastic offering strategy problem

$$\underset{p_t^{\mathrm{DA}},\ \Delta_{t\omega},\ \Delta_{t\omega}^{\uparrow},\ \Delta_{t\omega}^{\downarrow}}{\text{Maximize}} \quad \sum_{t=1}^{24} \sum_{\omega=1}^{|\omega|} \pi_{\omega} \left(\lambda_{t\omega}^{\mathrm{DA}}\ p_t^{\mathrm{DA}} + 0.9\ \lambda_{t\omega}^{\mathrm{DA}}\ \Delta_{t\omega}^{\uparrow} - 1.2\ \lambda_{t\omega}^{\mathrm{DA}}\ \Delta_{t\omega}^{\downarrow} \right)$$

subject to:

$$0 \le p_t^{\mathrm{DA}} \le P^{\mathrm{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

$$\Delta_{t\omega} = p_{t\omega}^{\mathrm{real}} - p_t^{\mathrm{DA}} \quad \forall t = \{1, 2, ..., 24\},\ \forall \omega = \{1, 2, ..., |\omega|\}$$

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subject to:

Set of time period (hours)

$$0 \le p_t^{\mathrm{DA}} \le P^{\mathrm{nom}} \quad \boxed{\forall t = \{1, 2, ..., 24\}}$$

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Set of scenarios

$$\Delta_{t\omega} = p_{t\omega}^{\mathrm{real}} - p_t^{\mathrm{DA}} \quad \forall t = \{1, 2, ..., 24\},\ \boxed{\forall \omega = \{1, 2, ..., |\omega|\}}$$

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Number of scenarios (cardinality)

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Stochastic offering strategy problem

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Probability of scenario *w* (input data)

subject to:

$$0 \le p_t^{\mathrm{DA}} \le P^{\mathrm{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

Set of scenarios

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Uncertain parameter: Forecasted day-ahead market price (€/MWh) in time *t* under scenario *w* (input data).

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Set of scenarios

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Stochastic offering strategy problem

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Uncertain parameter: Wind power forecast (MW) in time *t* under scenario *w* (input data)

$$\Delta_{t\omega}^{\downarrow} \ge 0 \quad \forall t = \{1, 2, ..., 24\},\ \forall \omega = \{1, 2, ..., |\omega|\}$$

Stochastic offering strategy problem

$$\underset{p_t^{\mathrm{DA}},\ \Delta_{t\omega},\ \Delta_{t\omega}^{\uparrow},\ \Delta_{t\omega}^{\downarrow}}{\boxed{\text{Maximize}}} \quad \sum_{t=1}^{24} \sum_{\omega=1}^{|\omega|} \pi_\omega \left(\lambda_{t\omega}^{\mathrm{DA}}\ p_t^{\mathrm{DA}} + 0.9\ \lambda_{t\omega}^{\mathrm{DA}}\ \Delta_{t\omega}^{\uparrow} - 1.2\ \lambda_{t\omega}^{\mathrm{DA}}\ \Delta_{t\omega}^{\downarrow} \right)$$

subject to:

All variables are indexed by scenario w, except for P^{DA}. Why?

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Stochastic offering strategy problem

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Answer: Recall that in the day-ahead stage, we don't know which scenario will be realized. The wind farm should submit a single value to the day-ahead market (P^{DA}) as its production offer in each hour. Therefore, it cannot be scenario-indexed. However, considering the scenarios allows it to make a more informed (i.e., uncertainty-aware) offering decision!

Stochastic offering strategy problem

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Profit **in expectation** (weighted by probabilities)

subject to:

$$0 \leq p_t^{\mathrm{DA}} \leq P^{\mathrm{nom}} \quad \forall t = \{1, 2, ..., 24\}$$

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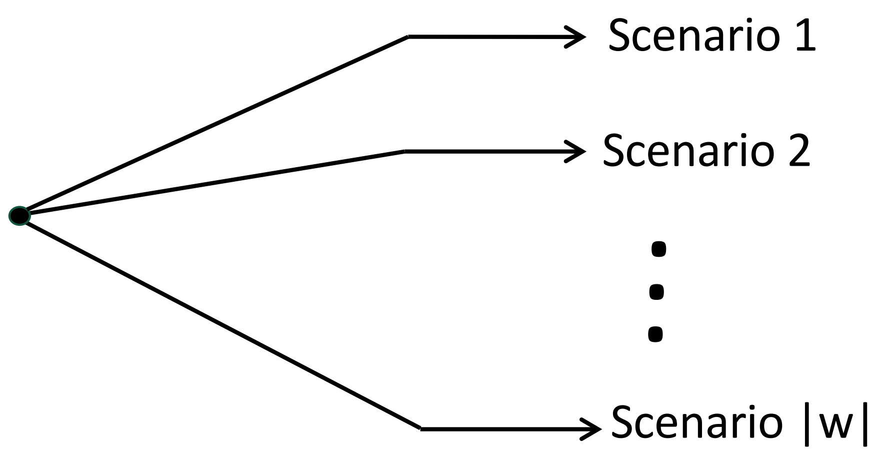
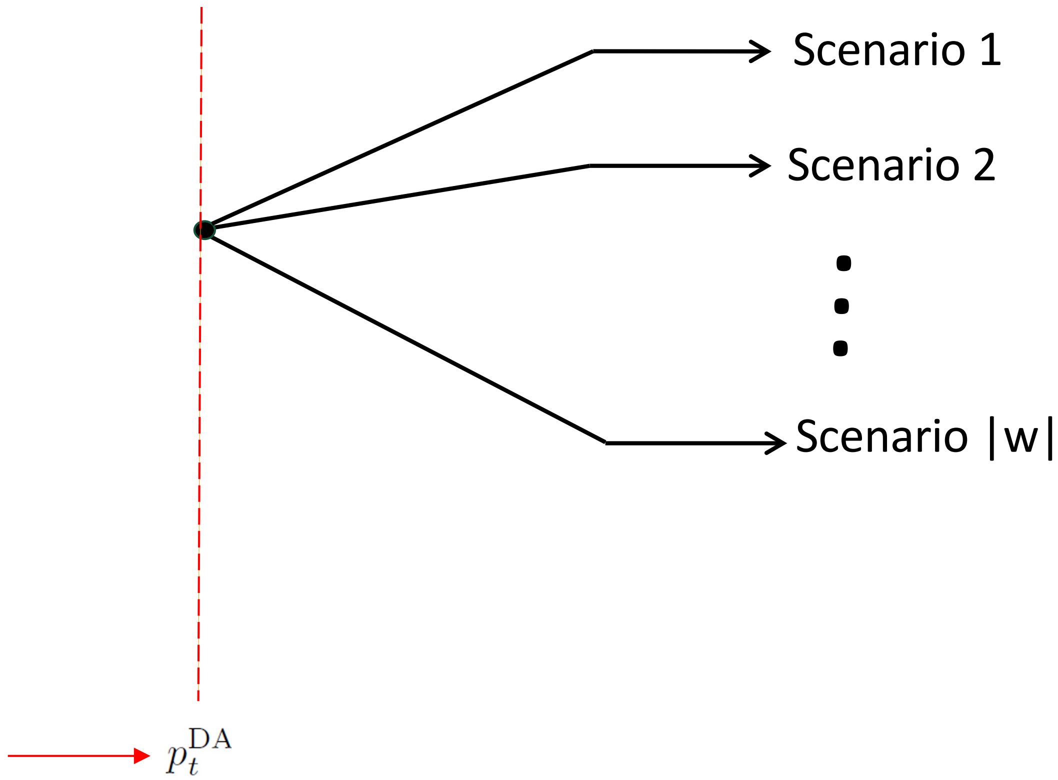
Stochastic offering strategy problem



Stochastic offering strategy problem

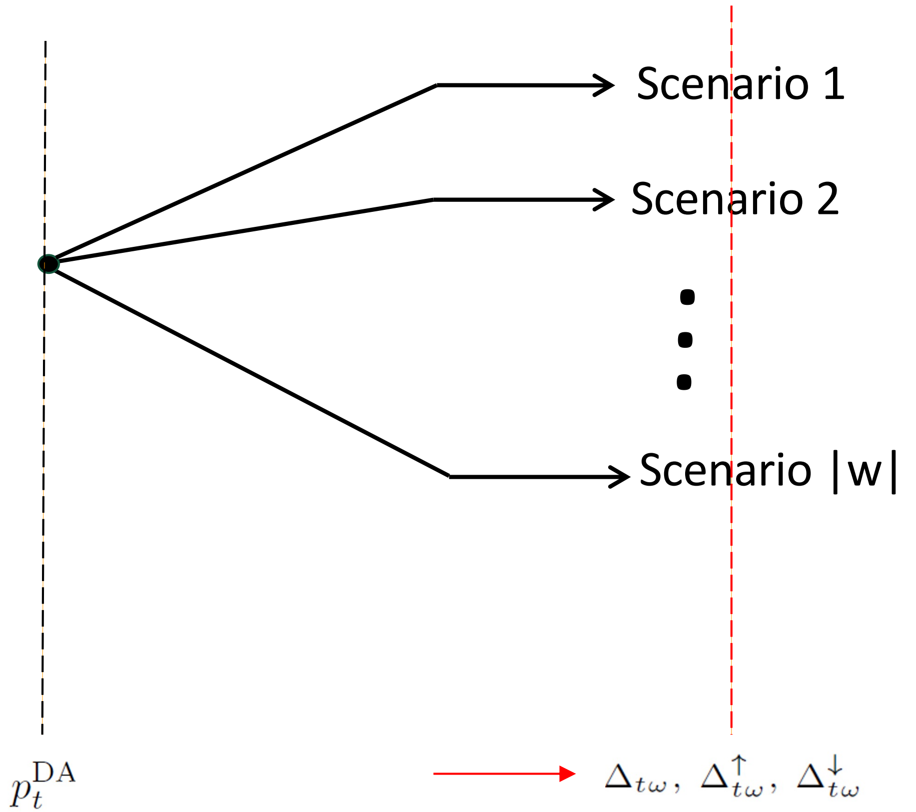
First-stage (here-an-now)
decisions (not indexed by scenario)



Stochastic offering strategy problem

First-stage (here-an-now)
decisions (not indexed by scenario)

Second-stage (wait-and-see)
decisions (indexed by scenario)



Ex-post out-of-sample analysis

Assume we have N scenarios, for example, 2000 scenarios.

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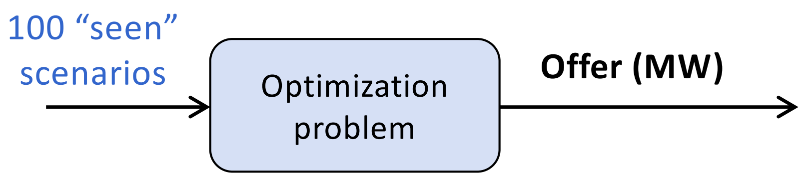

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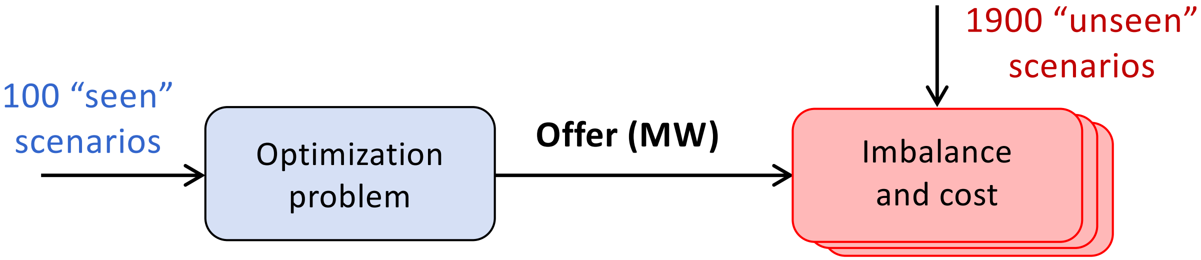
- **Step 1 (in-sample analysis):** Let's select a random subset of these scenarios, for example, 100 scenarios (each with a probability of 0.01), to solve the stochastic optimization problem and determine the optimal quantity offer (MW) of the wind farm in the day-ahead market.

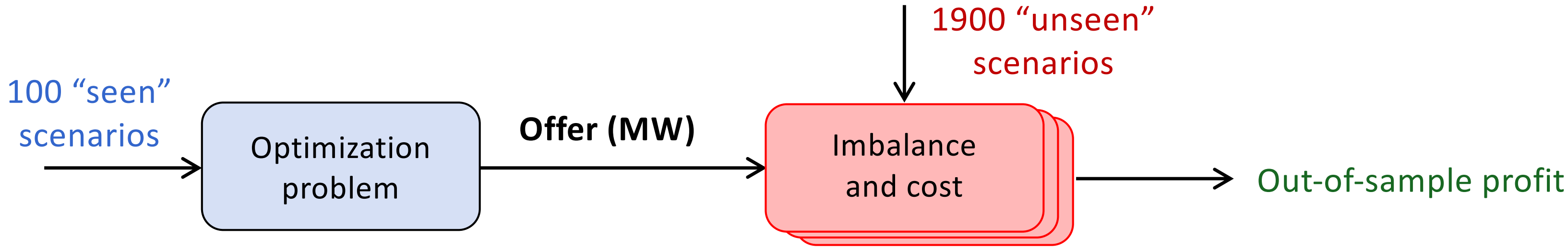


- **Step 2 (Out-of-sample analysis):** This is an ex-post analysis to assess the **quality** of the offering decision made. Recall there are 1900 "unseen" scenarios. For a given offering decision, and for each of the unseen scenarios, we calculate the imbalance and the corresponding cost incurred.

Ex-post out-of-sample analysis

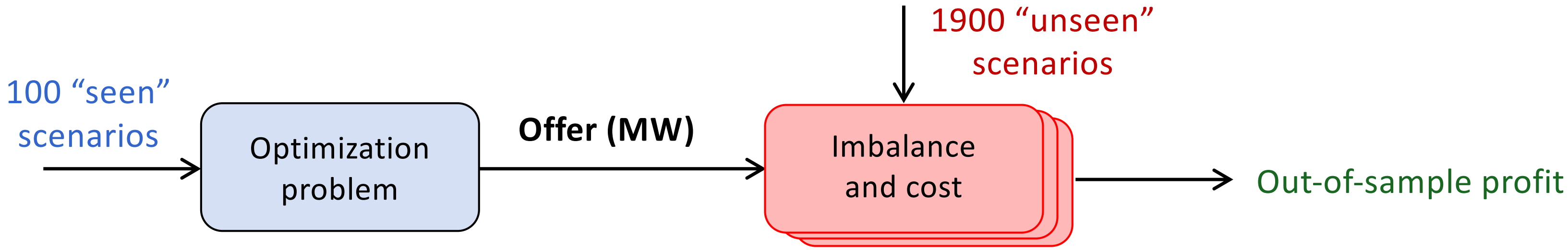
Assume we have *N* scenarios, for example, 2000 scenarios.

- **Step 1 (in-sample analysis):** Let's select a random subset of these scenarios, for example, 100 scenarios (each with a probability of 0.01), to solve the stochastic optimization problem and determine the optimal quantity offer (MW) of the wind farm in the day-ahead market.



- **Step 2 (Out-of-sample analysis):** This is an ex-post analysis to assess the **quality** of the offering decision made. Recall there are 1900 "unseen" scenarios. For a given offering decision, and for each of the unseen scenarios, we calculate the imbalance and the corresponding cost incurred.
- **Step 3 (Out-of-sample profit):** We can now calculate the "out-of-sample" cost as the day-ahead profit in Step 1 plus the <u>average</u> payoff in Step 2.

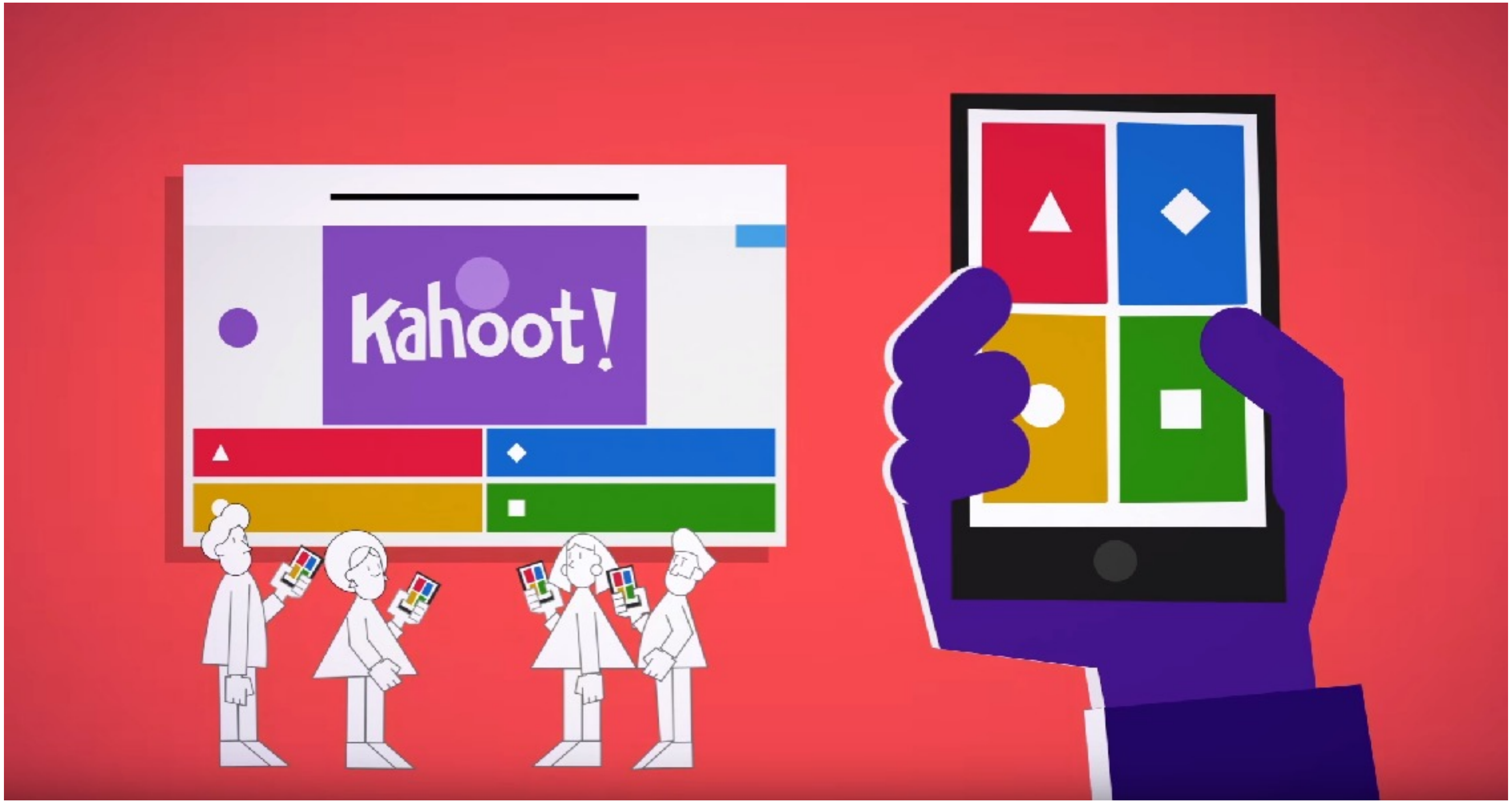
K-fold cross validation



For cross-validation (here, 20-fold cross validation):

Let’s divide the original 2000 scenarios into 20 folds. We run the out-of-sample analysis 20 times, each time with one of the folds for in-sample analysis (Step 1) and the other 19 folds for out-of-sample analysis (Step 2). This way, we eliminate any **bias** from the selection of in-sample and out-of-sample scenarios.

- The ex-post profit is the average profit obtained over the 20 out-of-sample analyses.


Kahoot!

Thanks for your attention!

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