$$ds * dA * \frac{\text{rate of energy absorbed}}{\text{volume}} = (I|_s - I|_{s+ds})dA$$

$$\int_{I_\lambda e^{\tau_\lambda}|_{\tau_\lambda=0}}^{I_\lambda e^{\tau_\lambda}|_{\tau_\lambda=\tau_\lambda}} (dI_\lambda e^{\tau_\lambda}) = \int_{\tau_\lambda=0}^{\tau_\lambda=\tau_\lambda} S(\tau_\lambda, \hat{s}) e^{\tau_\lambda} d\tau_\lambda$$

$$\frac{\partial I_1}{\partial \tau_\lambda} = \frac{\partial}{\partial \tau_\lambda}(2\pi \int_{-1}^{1} \mu I_\lambda(\tau_\lambda) d\mu)$$

$$\frac{\partial I_1}{\partial \tau_\lambda} = 2\pi (\int_{-1}^{1} \mu \frac{\partial}{\partial \tau_\lambda} I_\lambda(\tau_\lambda) d\mu)$$

Part I
OpenFOAM Radiation Models

1 Spherical Harmonics Basic Concepts

1.1 Expanding out the Intensity Field

$$I(r, \hat{s}) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} I_l^m(r) Y_l^m(\hat{s})$$

Let's try writing out from l=0 to l=3

1.1.1 l=0 (1 term)

$$I(r, \hat{s})(l = 0) = \sum_{m=-0}^{0} I_0^m(r) Y_0^m(\hat{s}) = I_0^0(r) Y_0^0(\hat{s})$$

1.1.2 l=1 (3 terms)

$$I(r, \hat{s})(l = 1) = \sum_{m=-1}^{1} I_1^m(r) Y_1^m(\hat{s})$$

$$= I_1^{-1}(r) Y_1^{-1}(\hat{s}) + I_1^0(r) Y_1^0(\hat{s}) + I_1^1(r) Y_1^1(\hat{s})$$

1.1.3 l=2 (5 terms)

$$I(r,\hat{s})(l=2) = \sum_{m=-2}^{2} I_2^m(r)Y_2^m(\hat{s})$$

$$= I_2^{-2}(r)Y_2^{-2}(\hat{s}) + I_2^{-1}(r)Y_2^{-1}(\hat{s}) + I_2^0(r)Y_2^0(\hat{s}) + I_2^1(r)Y_2^1(\hat{s}) + I_2^2(r)Y_2^2(\hat{s})$$

1.1.4 l=3 (7 terms)

$$I(r,\hat{s})(l=3) = \sum_{m=-3}^{3} I_3^m(r)Y_3^m(\hat{s})$$

$$= I_3^{-2}(r)Y_3^{-2}(\hat{s}) + I_3^{-1}(r)Y_3^{-1}(\hat{s}) + I_3^0(r)Y_3^0(\hat{s}) + I_3^1(r)Y_3^1(\hat{s}) + I_3^2(r)Y_3^2(\hat{s})$$

$$+I_3^{-3}(r)Y_3^{-3}(\hat{s}) + I_3^{-3}(r)Y_3^{-3}(\hat{s})$$

1.2 Legendre's ODE

$$\frac{1}{\Theta(\theta)}\frac{\partial^2\Theta(\theta)}{\partial\theta^2} = \frac{1}{\Theta(\theta)}(\sin^2\theta\frac{\partial^2\Theta(x)}{\partial x^2} + \sin\theta\frac{\partial\Theta(x)}{\partial x}\frac{\partial\sin\theta}{\partial x})$$

$$\frac{1}{\Theta(\theta)}\frac{\partial^2\Theta(\theta)}{\partial\theta^2} = \frac{1}{\Theta(\theta)}(\sin^2\theta\frac{\partial^2\Theta(x)}{\partial x^2} + \sin\theta\frac{\partial\Theta(x)}{\partial x}\frac{\text{-cos}\theta}{\sin\theta})$$

$$\frac{1}{\Theta(\theta)}\frac{\partial^2\Theta(\theta)}{\partial\theta^2} = \frac{1}{\Theta(\theta)}(\sin^2\theta\frac{\partial^2\Theta(x)}{\partial x^2} + -\cos\theta\frac{\partial\Theta(x)}{\partial x})$$

In terms of x

$$\frac{1}{\Theta(\theta)}\frac{\partial^2\Theta(\theta)}{\partial\theta^2} = \frac{1}{\Theta(\theta)}((1-x^2)\frac{\partial^2\Theta(x)}{\partial x^2} + -x\frac{\partial\Theta(x)}{\partial x})$$

And the other term:

$$K_1 + \frac{m^2}{1-x^2} = \frac{1-x^2}{\Theta(\theta)}\frac{\partial^2\Theta(x)}{\partial x^2} - \frac{x}{\Theta(\theta)}\frac{\partial\Theta(x)}{\partial x} - \frac{x}{\Theta(\theta)}\frac{\partial\Theta(x)}{\partial x}$$

$$K_1 + \frac{m^2}{1-x^2} = \frac{1-x^2}{\Theta(\theta)}\frac{\partial^2\Theta(x)}{\partial x^2} - \frac{2x}{\Theta(\theta)}\frac{\partial\Theta(x)}{\partial x}$$

$$\Theta(\theta)(K_1 + \frac{m^2}{1-x^2}) = (1-x^2)\frac{\partial^2\Theta(x)}{\partial x^2} + -x\frac{\partial\Theta(x)}{\partial x}$$

2 P1 Model

2.1 How to use:

2.1.1 Introduction

The P1 model is part of spherical harmonics model and it does well in optically thick medium [**?**].

Part II
Bibliography