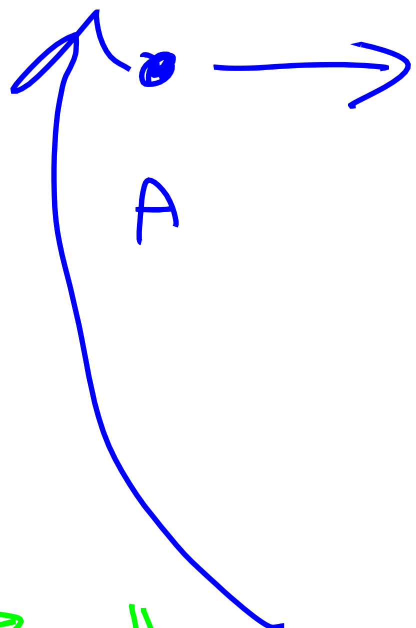
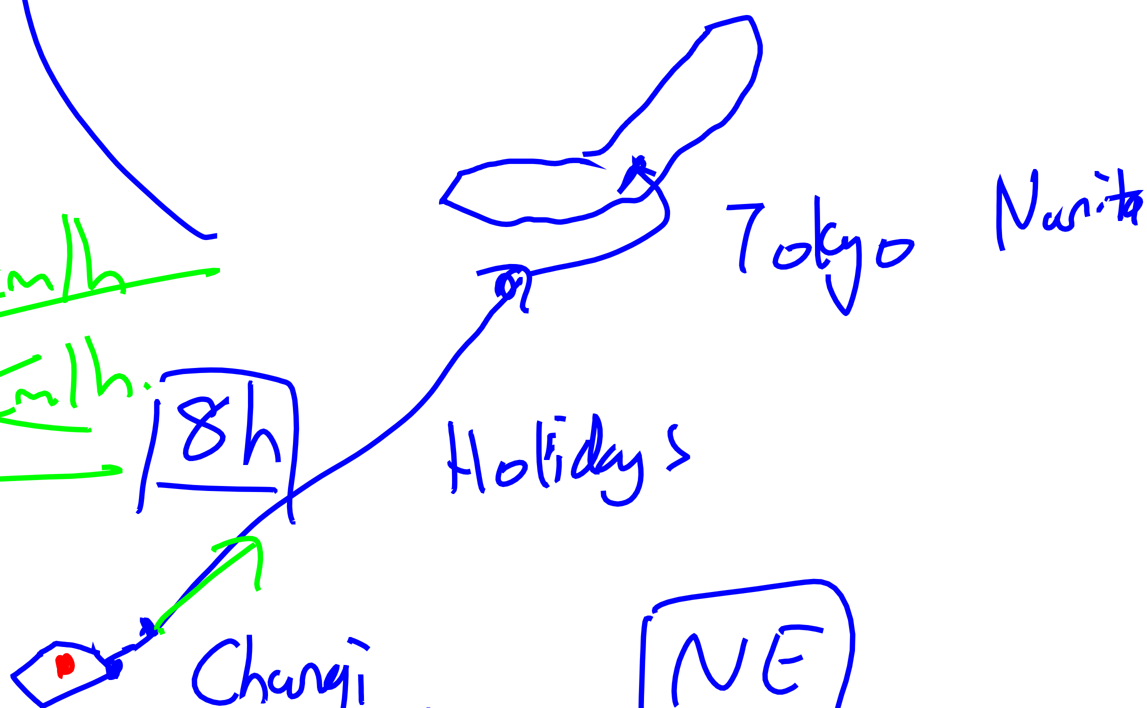
Kinematics



800km/h

~~800km/h.~~

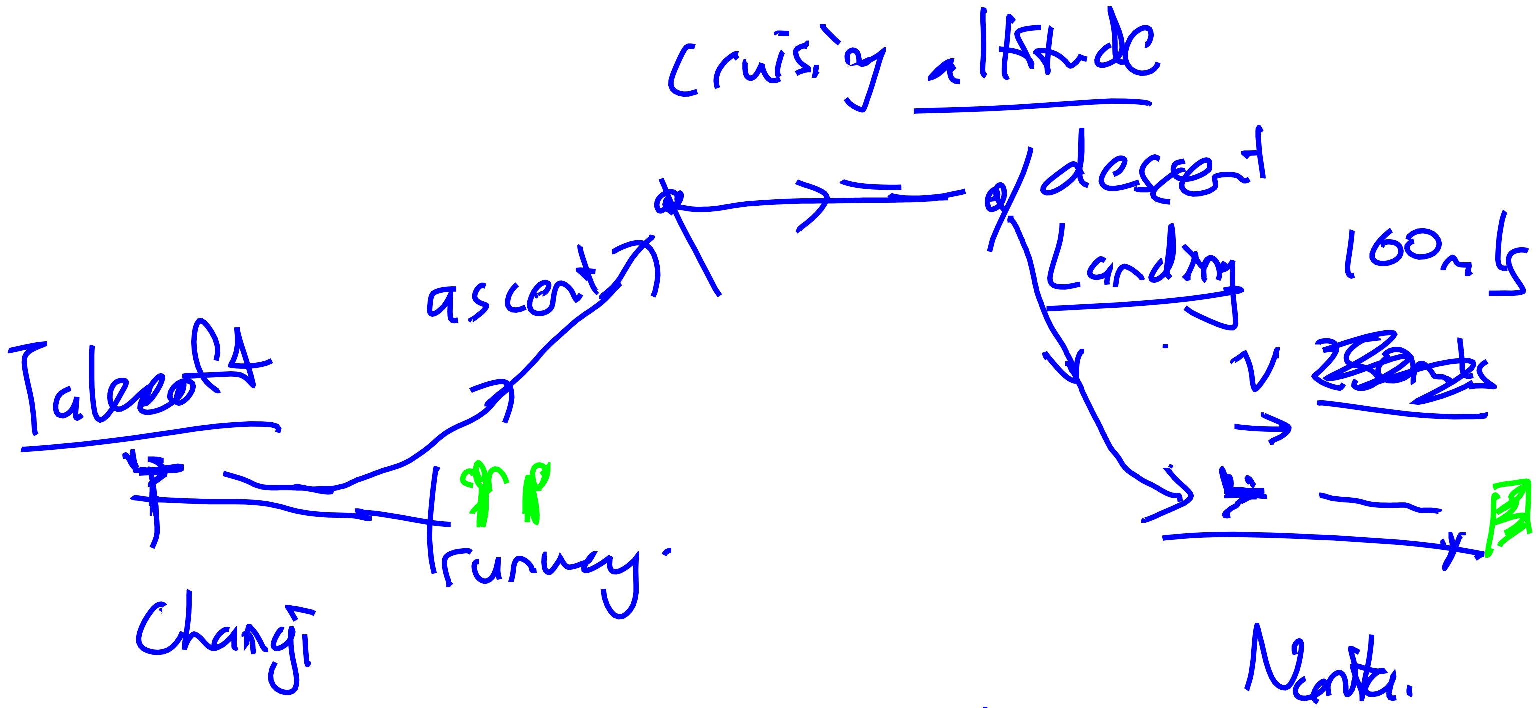
$\boxed{8h}$



$\boxed{NE}$

quantities

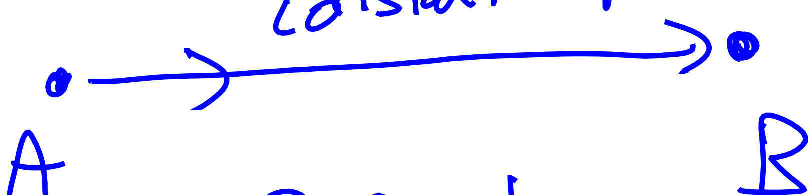
Vectors = Quantities w/ magnitude & direction.



How long does your runway need to be?

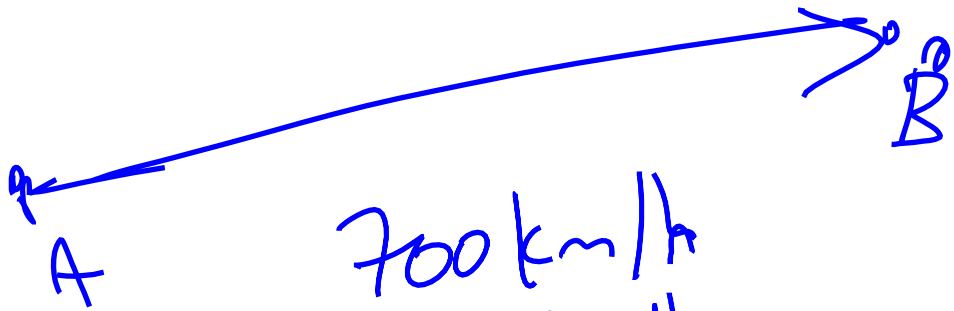
Cruising

roughly constant speed. $\underline{\underline{v}}$



3500 km

3500 km



700 km/h

– 800 km/h

displacement.

$$S = v\,t$$

velocity.

$$v = \frac{\text{distance}}{\text{time}}$$

$$\text{time} = \frac{\text{distance}}{v}$$

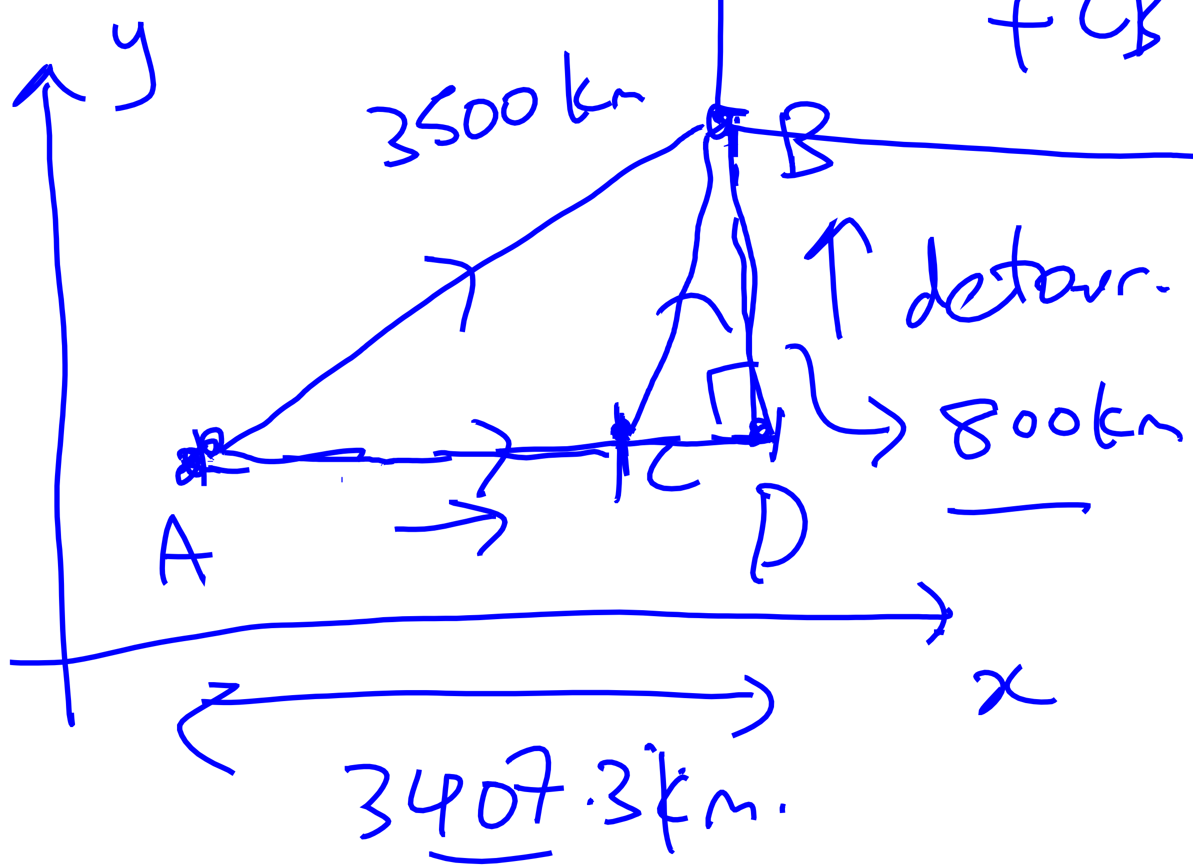
$$= \frac{3500\,(\text{km})}{700\,(\text{km/h})} = \underline{5\,h}$$

cruising alt

detour.

Direction becomes impt.

$$\vec{AB} = \vec{AC} + \vec{CB}$$



$$AD = \sqrt{3300^2 - 800^2}$$

$$= \underline{3407.3\,km}$$

5sf.

$$AC = 3000\,km.$$

$$BC = \sqrt{407.3^2 + 800^2}$$

$$BC\,(km) = \left(\sqrt{805{,}929}\right) km.$$

$$= 897.7\,km.$$

$A\overset{C}{\cancel{B}} + BC$

$= 3000\text{ km} + 897.7\text{ km}$

$= 3897.7\text{ km}$

extra time taken

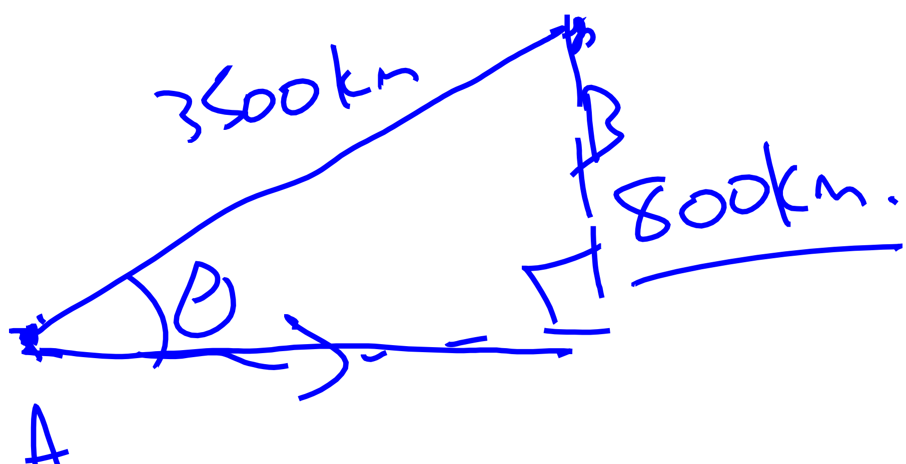
$$= \frac{3897.7 - 3500}{700\text{ km/h}}$$

$= 0.568\text{h}$

$= \underline{34.1\text{ min}}$

Detour example

↳ took ~ 1/2 hr longer to get from A to B.



what is displacement from A to B?

method 1 of rcp vectors.

$\boxed{3500\ km.}$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$= \frac{800}{3500}$$

$$\theta = \arcsin\left(\frac{800}{3500}\right)$$

$$\sin^{-1}\left(\frac{800}{3500}\right)$$

$$\theta = 0.2306 \text{ rad}$$

w.r.t

+ve x dir

θ is also 13.24°

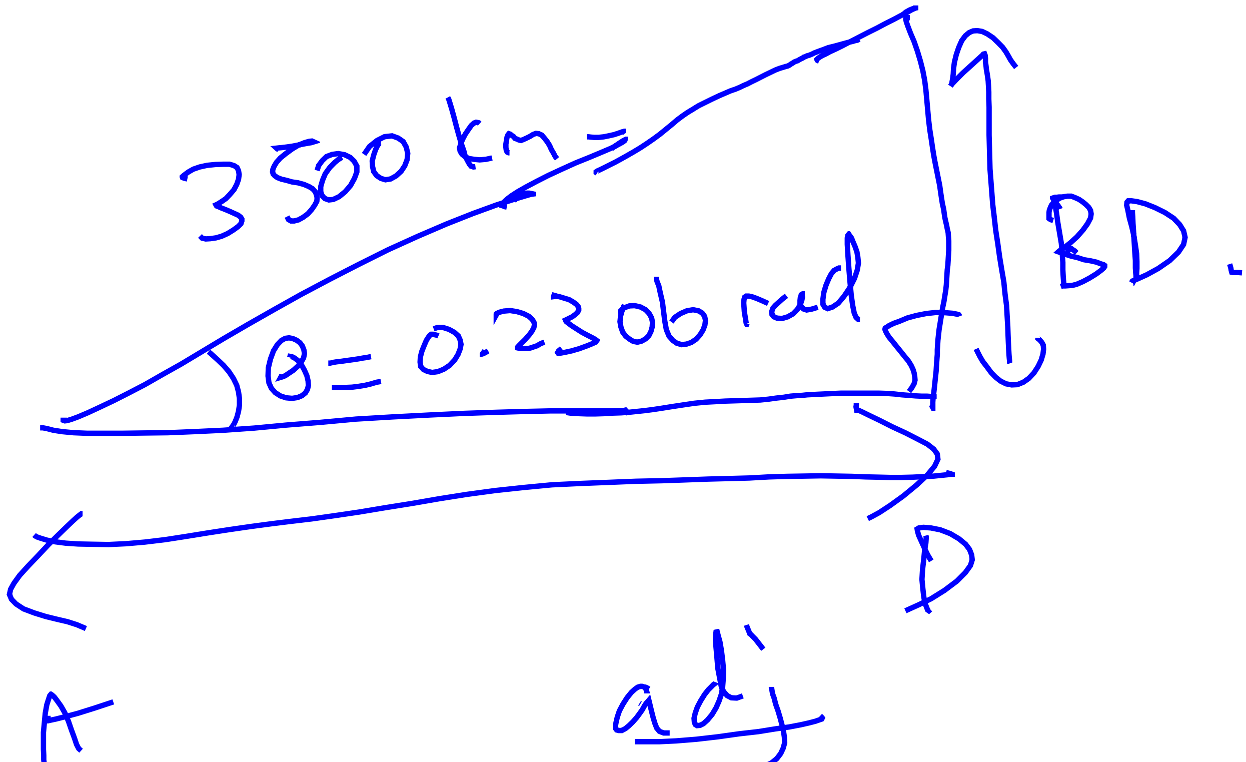
method 2: use "coordinates" in 2D

$$\vec{S} = \begin{bmatrix} S_x \\ S_y \end{bmatrix}$$

where

$S_x = 3407.3\ \text{km}$

$S_y = 800\ \text{km}.$



$$\cos\theta = \frac{adj}{hyp.}$$

$$AD = 3500 \times \cos\theta$$

$$= 3500 \times \cos(0.2306\text{ rad})$$

$$= \underline{3407.3}\text{ km.}$$

$$BD = 3500 \times \sin(0.2306 \text{ rad})$$

$$= 800 \text{ km}$$

Kinematics 2 vector addition.

$$\vec{S} = \vec{AB} = \begin{bmatrix} 2407.3 \\ 800 \end{bmatrix} \text{km.}$$

show

$$\vec{AB} = \vec{AC} + \vec{CB}$$

$$\vec{AC} = \begin{bmatrix} 3000 \\ 0 \end{bmatrix} \text{km.}$$

$$\vec{CB} = \begin{bmatrix} 407.3 \\ 800.0 \end{bmatrix} \text{km}$$

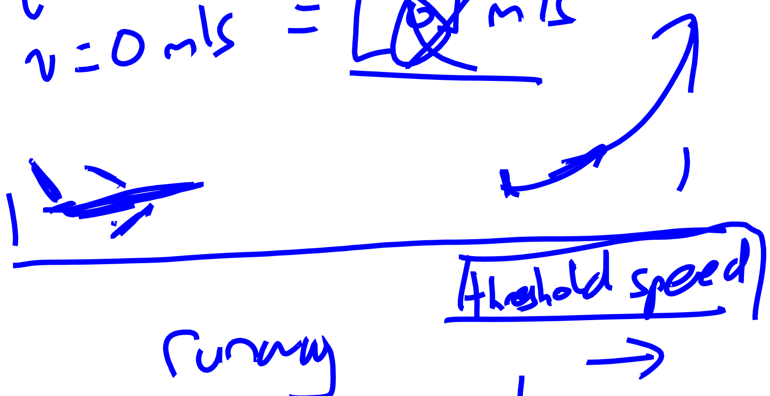
$$\vec{AB} = \begin{bmatrix} 3000 \\ 0 \end{bmatrix} \text{km} + \begin{bmatrix} 407.3 \\ 800 \end{bmatrix}$$

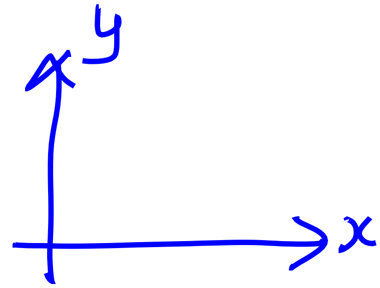
$$= \begin{bmatrix} 3407.3 \\ 800 \end{bmatrix} \text{km.}$$

Takeoff / Landing

$t = 0\,s$

$v = 0\,m/s = \begin{bmatrix}0\\0\end{bmatrix} m/s$





$\rightarrow$

v_x

$t = 0\,s$

$v_x = 0\,m/s.$

?

$\boxed{100\,m/s}$

(1) we need to hit 100m/s.

(2) we need to accelerate the plane...

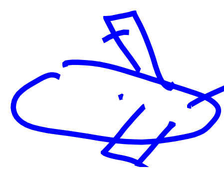
how much acceleration the plane can take $\rightarrow 0.7\,m/s^2$ ($0.7\,ms^{-2}$)

$\boxed{a_x}$

$(\vec{a})$ acceleration is rate of chg of $(\vec{v})$ velocity w.r.t time.

acceleration limits

- Human body
- plane structure



fuselage.

main limitation

pilots → up to $\underline{9g}$ of acceleration.

$9 \times \cancel{9.81}$ 10

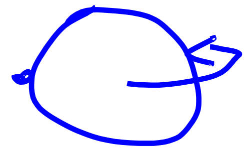
$= 90 \, m/s^2$

g = gravitational acceleration of earth.

$\underline{10.0 \, m/s^2}$

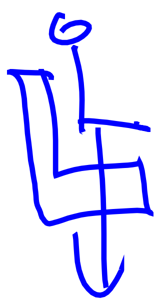
9.80

$- 9.81 \, m/s^2$



gravity at equator is weaker than polar region.
↳ bulge effect

passenger → 0.2-0.5g max



→ 0.5g   10kg.

→ 30kg   5kg.

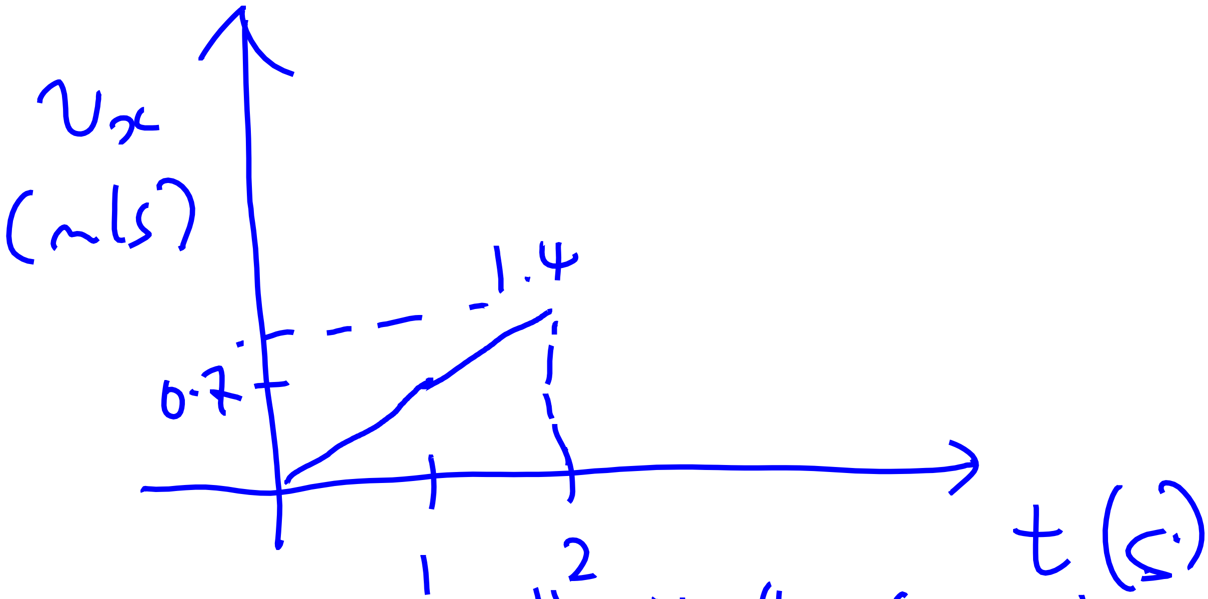
60kg → 0.08g.   0.08g.

for passenger comfort → 0.7 m/s² acc max

1.4 m/s²

for acceleration problem. → assume const acceleration

use v_x t graph.

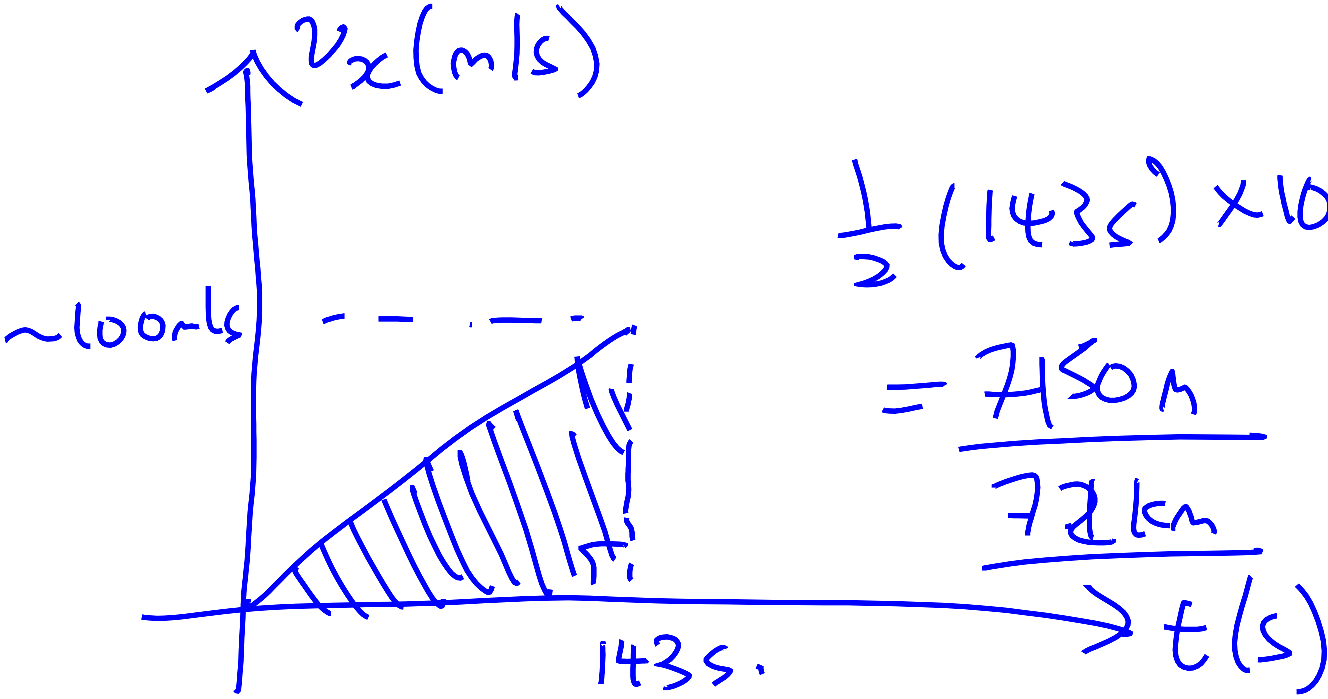


"Delta." → Greek letter used to denote chg in physics.

$$a_x = \frac{\Delta v_x}{\Delta t} =$$

$$0.7\,ms^{-2} = \frac{100\,m/s - 0\,m/s}{t - 0\,s}.$$

$$t = \frac{100}{0.7} = \underline{143\,s}$$

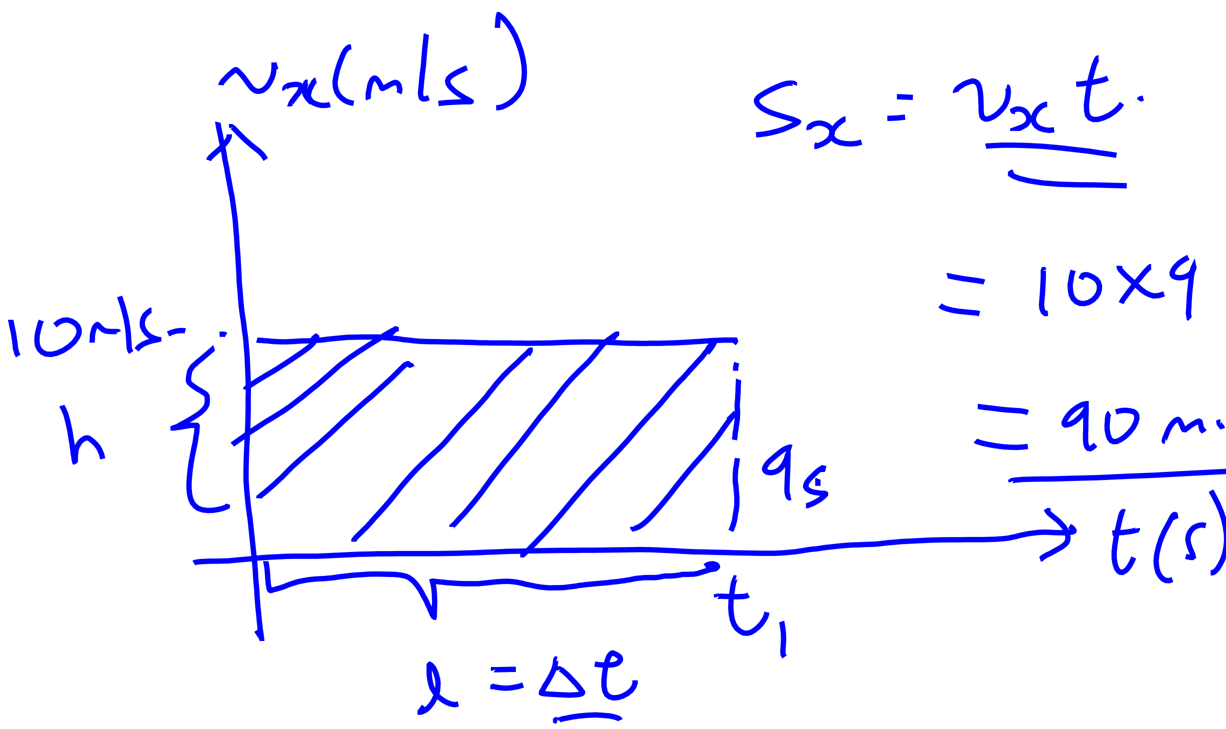


$\frac{1}{2}(143\,s) \times 100\,m/s$

$= 7150\,m$

$7.2\,km$

How to calc distance.



$s_x = v_x t$

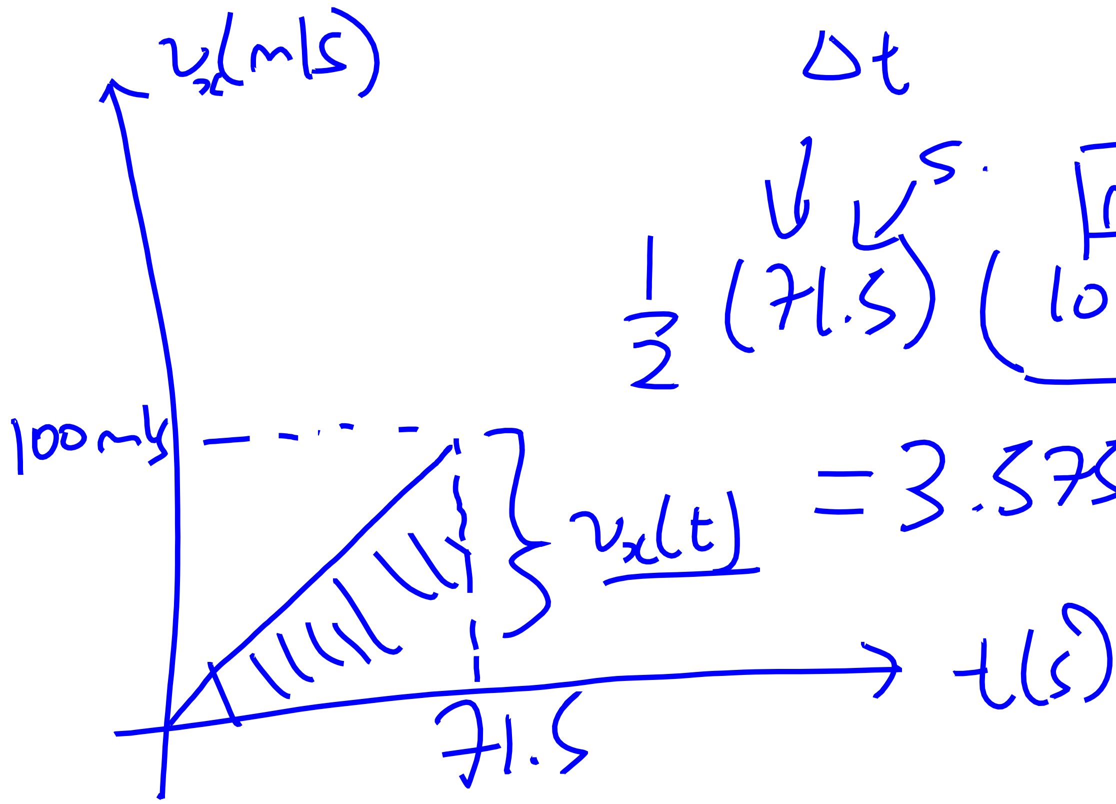
$= 10 \times 9$

$= 90\,m.$

in $\underline{\text{velocity}}$ - time graph.
v_x

↳ area under graph = $\underline{\text{displacement}}$.
s_x

lets try with $\underline{\underline{1.4\ m/s^2}}$.



Δt

s. $\boxed{m/s}$.

$\frac{1}{2}(71.5)(100)$

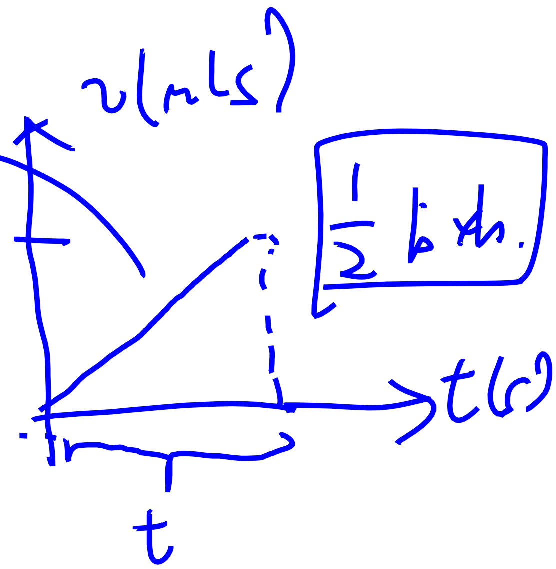
$= 3.575\ \underline{\underline{km}}$.

runways are limited by space constraints.

For $\boxed{\text{const acceleration}}$.

$$\boxed{S_x = \frac{1}{2} a_x t^2}$$

$v_x = at$

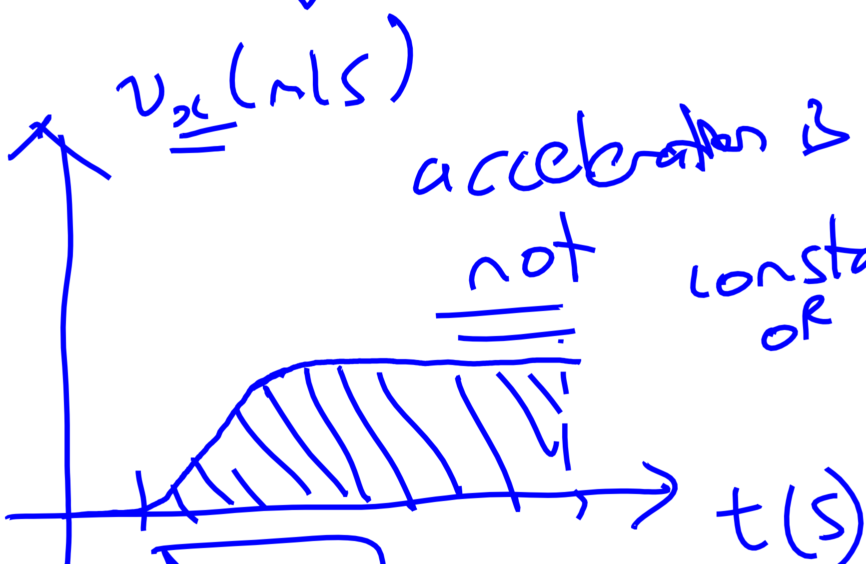


$\boxed{\frac{1}{2} b h}$.

$$v_x(t) = at.$$

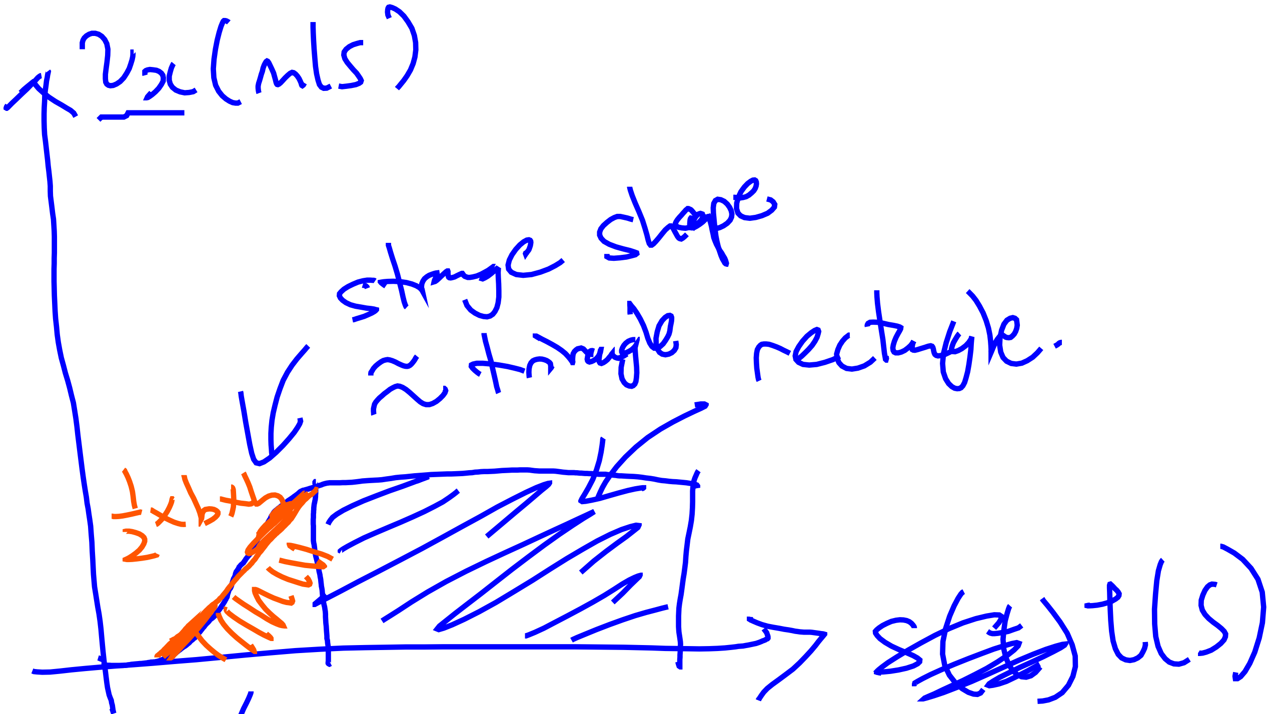
(velocity time graph).

The graph is most impt.

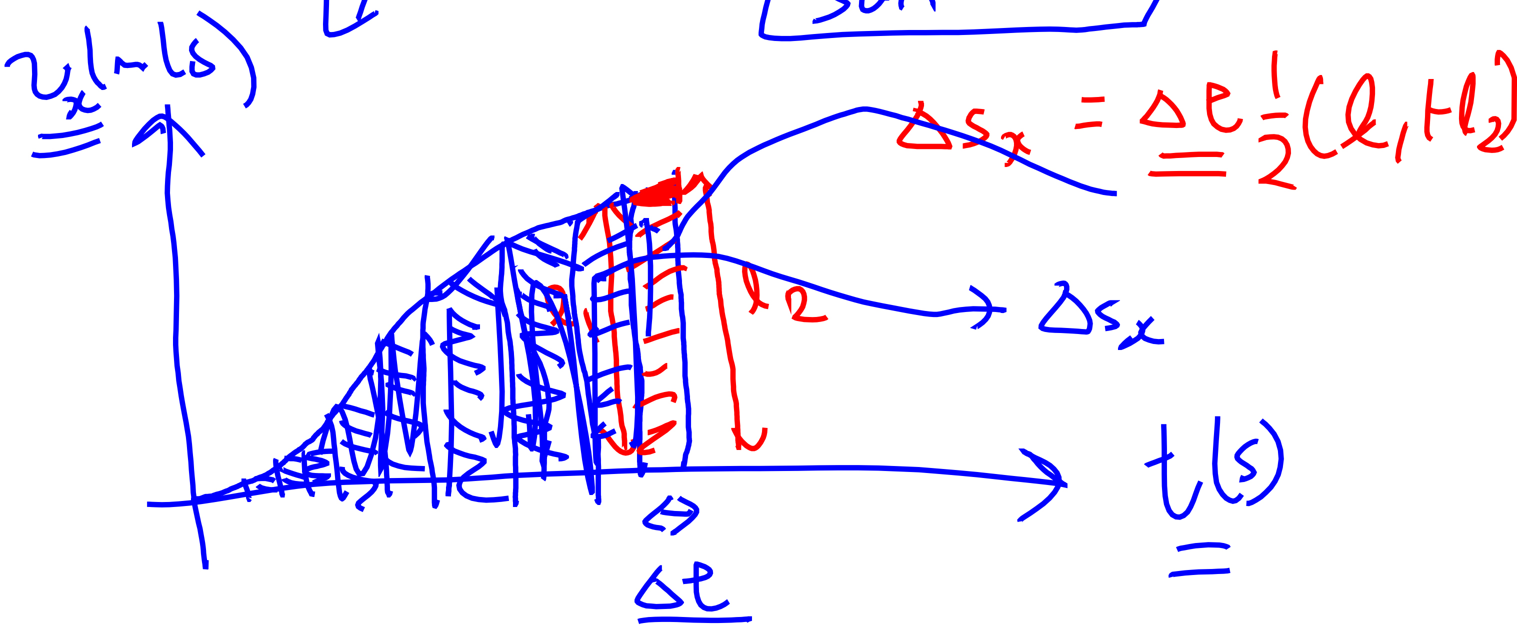


acceleration is **not** constant most of the time.

$\boxed{S_x}$



$$\text{Area} = \boxed{\sum_{\text{sum}} \Delta s_x}$$



If time interval is short,
(Δt)

then there is less curvature

$\boxed{\text{taking limits. } \Delta t \to 0}$ ✓

$\boxed{\begin{array}{ll} \sum v_x \, \Delta t & \\ \text{sum} & \lim \Delta t \to 0 \end{array}}$

integration

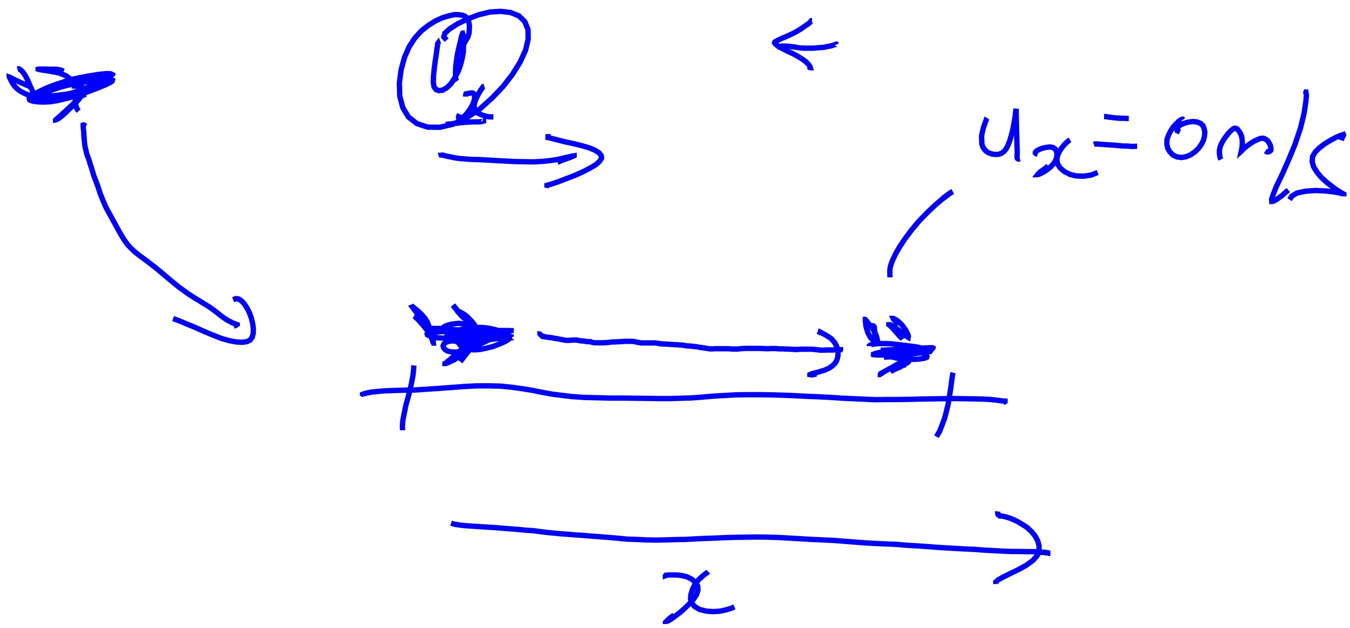
area under graph $= \displaystyle\int_{t=0}^{t=t_1} v_x \, dt.$

distorted "s" = Sum

Kinematics 3

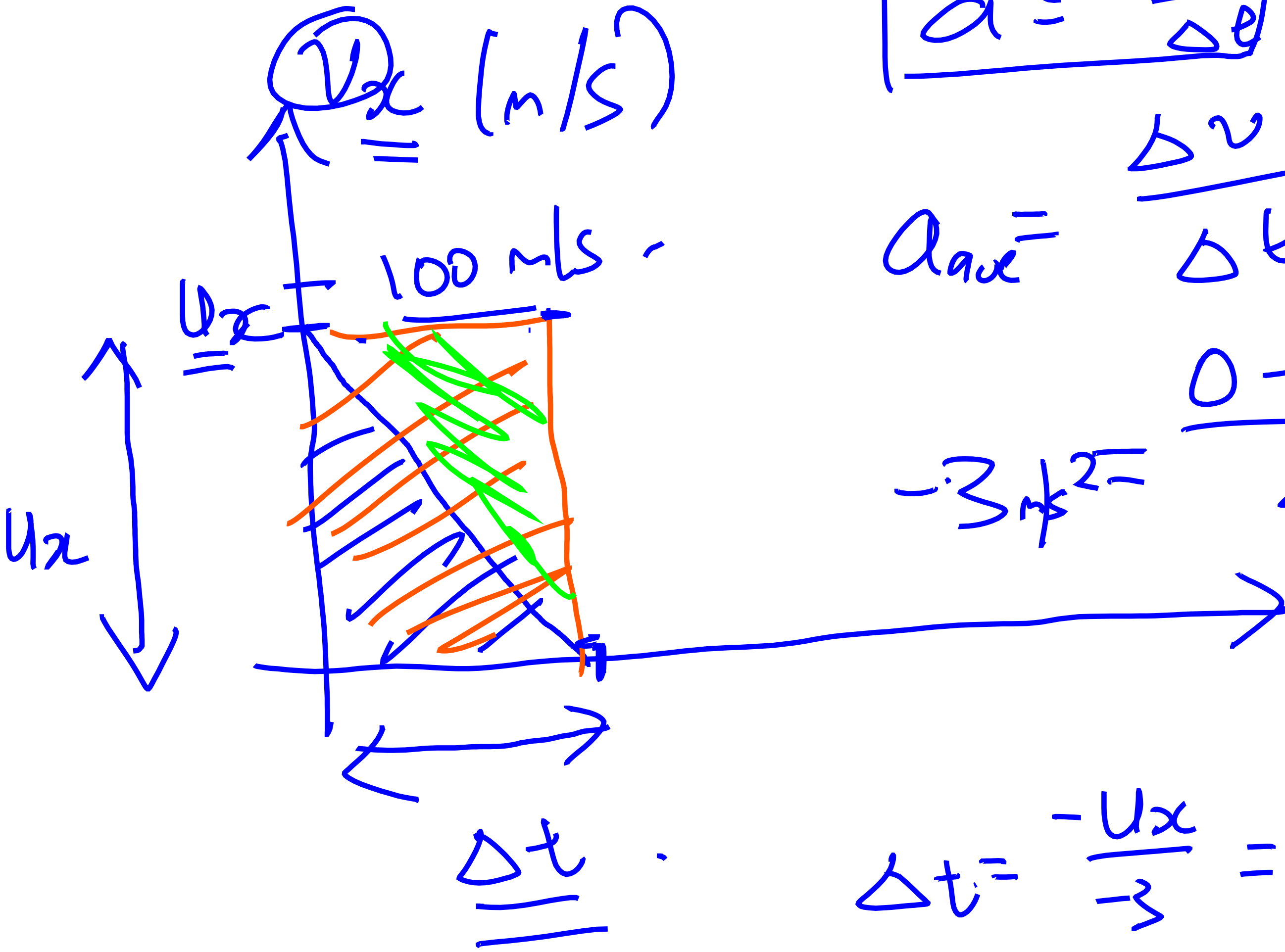
landing case

-ve acceleration.



tolerable deceleration:

$a_x = -3\,m/s^2$



$$\boxed{a = \frac{-u_x}{\Delta t}} \qquad u_x = -a\Delta t$$

$$a_{ax} = \frac{\Delta v}{\Delta t} = \frac{0 - u_x}{\Delta t}$$

$$-3\,m/s^2 =$$

$$\Delta t = \frac{-u_x}{-3} = \frac{100}{3} \simeq 33.34\,s \simeq \underline{34\,s}$$

$$\vec{v} = \begin{bmatrix} \underline{u} \\ v \\ \underline{w} \end{bmatrix} \begin{array}{l} v_x \\ v_y \\ v_z \end{array}$$

3D

$$\Delta s = \left(100 \, \text{m/s} \times 34 \, \text{s}\right) \frac{1}{2}$$

$$= (3.4 \, \text{km}) \frac{1}{2}$$

$$= \underline{1.7 \, \text{km}}$$

$$\Delta s = u_x \Delta t - \frac{1}{2} \underline{u_x} \, \underline{\Delta t}.$$

in terms of acceleration.

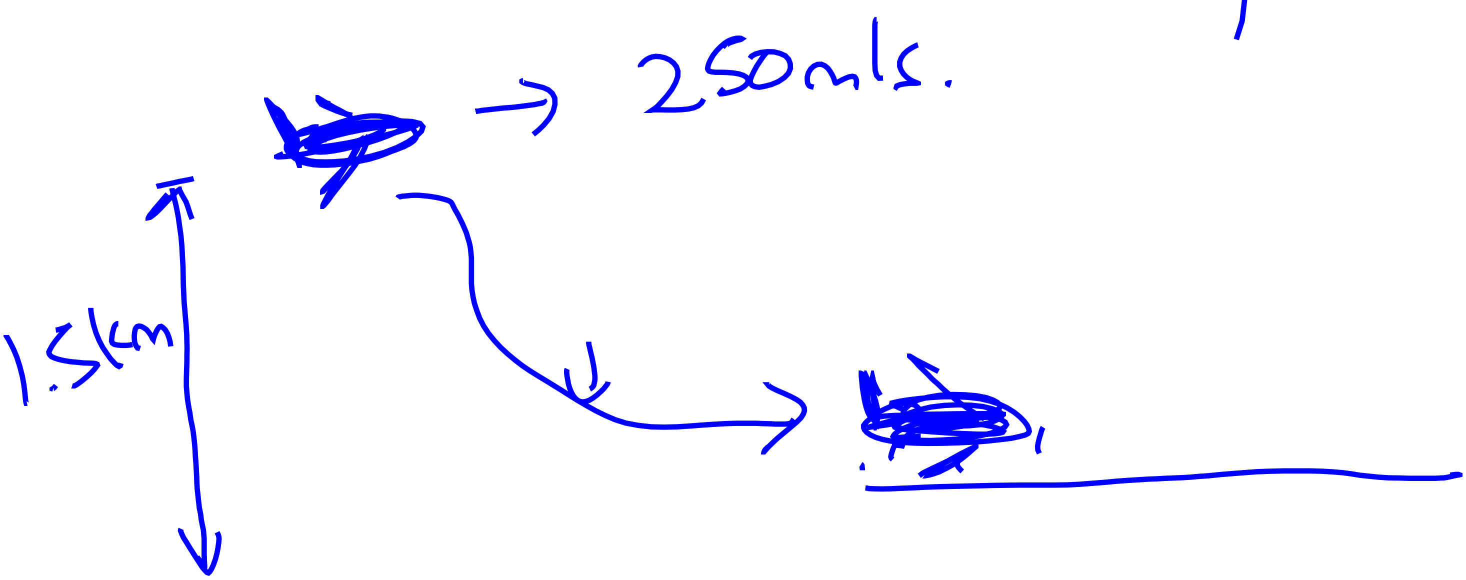
$$u_x = -a_x \underline{\Delta t}$$

$$\Delta s = u_x \Delta t - \frac{1}{2} (-a_x) \Delta t \Delta t$$

$$\boxed{\Delta s = u_x \Delta t + \frac{1}{2} a_x \Delta t^2}$$

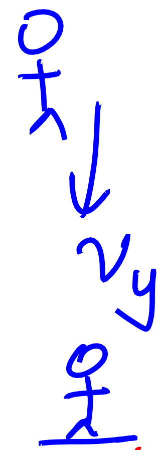
$$\boxed{s = ut + \frac{1}{2}at^2.}$$

Ascent	Descent



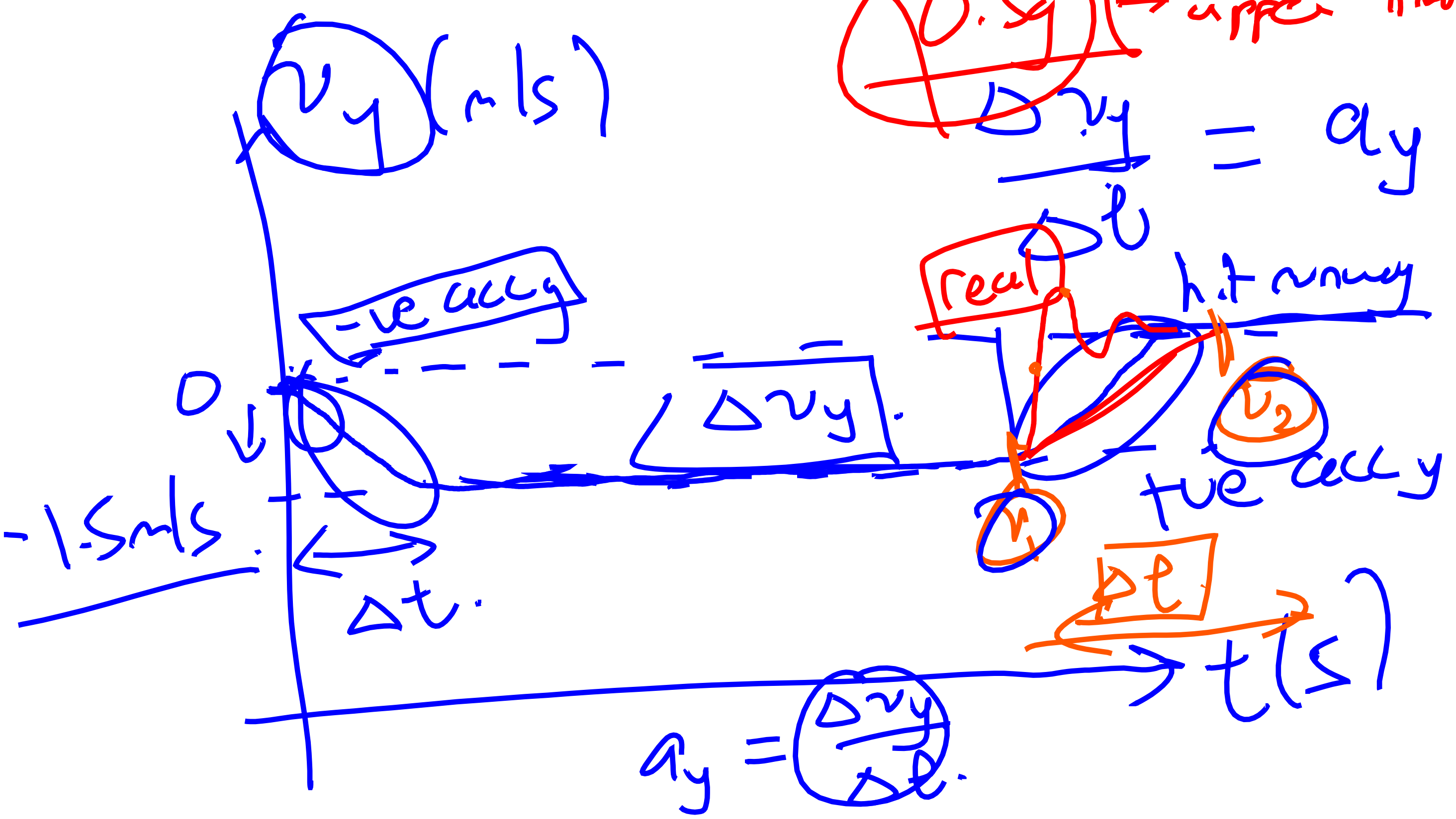
+v

$\boxed{\downarrow g}$



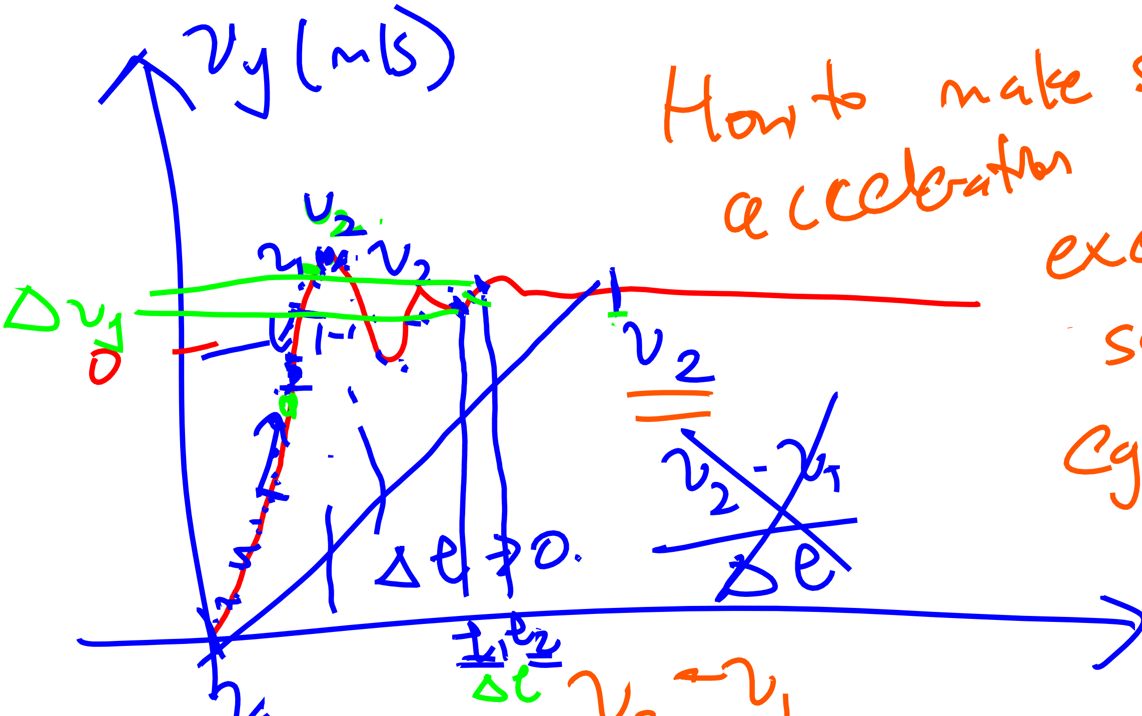
~~0.3g~~ → upper limit

$$\frac{\Delta v_y}{t} = a_y.$$



$$a_y = \frac{\Delta v_y}{\Delta t}.$$

$$= \frac{v_{y2} - v_{y1}}{t_2 - t_1}.$$



How to make sure acceleration won't exceed some value?

e.g. $\boxed{0.3g}$.

$$a_y = \frac{v_2 - v_1}{\Delta t}.$$

$$t_2 = \Delta t + t_1$$

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{v_{y2} - v_{y1}}{t_2 - t_1}$$

$\to \boxed{\Delta t \to 0}$

$\boxed{\text{Instantaneous acceleration}}$

$\boxed{\text{differentiation}}$

$$a_y = \lim_{\Delta t \to 0} \frac{\Delta v_y}{\Delta t} = \frac{dv_y}{dt}$$

Eg. of a derivative

$v \cancel{y} = a t^{3}$.

$v \cancel{y} = a t^{n}$ where n is some integer. (+ve integer).

how do find gradient at any point in time?

$$\lim_{\Delta t \to 0} \frac{\Delta \cancel{y} v}{\Delta t} = \frac{d \cancel{y} v}{dt} = \frac{a(t+\Delta t)^n - a t^n}{(t+\Delta t) - t}$$

$$= \lim_{\Delta t \to 0} \frac{a(t+\Delta t)^n - a t^n}{\Delta t}.$$

$$\lim_{\Delta t \to 0} \frac{a(t+\Delta t)^n - a t^n}{\Delta t}$$

$\boxed{\text{Binomial expansion.}}$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

$$= \binom{n}{0} x^n + \binom{n}{1} x^{n-1} y + \underline{\binom{n}{2}} x^{n-2} y^2 + \cdots$$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

$y \Rightarrow \Delta t$

$x \Rightarrow t.$

factorial:

$3! = 3 \times 2 \times 1$

$5! = 5 \times 4 \times 3 \times 2 \times 1$

$1! = 1$.

$$\boxed{0! = 1}$$

$$\binom{n}{0} = \frac{n!}{\cancel{0!}\,(n-0)!} = 1$$

$$\binom{n}{1} = \frac{n!}{1!\,(n-1)!} = n$$

$$\binom{n}{2} = \frac{n!}{2!\,(n-2)!}$$

$$= \frac{n(n-1)}{2}$$

$$(t + \underline{\underline{\Delta t}})^n \qquad \lim_{\Delta t \to 0}$$

v. small

$$= t^n + n t^{n-1} \Delta t + \frac{n(n-1)}{2} t^{n-2} \Delta t^2 + \cdots$$

$$a\left[\frac{(t+\Delta t)^n - t^n}{\Delta t}\right]$$

$$= a\left[\frac{\cancel{t^n} + nt^{n-1}\Delta t + \frac{n(n-1)}{2}t^{n-2}\Delta t^2 + \dots - \cancel{t^n}}{\Delta t}\right]$$

$$= a\left[nt^{n-1} + \cancel{\frac{n(n-1)}{2}t^{n-2}\Delta t}^{\,0} + \dots \cancel{(\Delta t)^{?}}^{\,0}\right]$$

$$= \boxed{a\,n\,t^{n-1}}$$

$$\frac{d\,at^n}{dt} = a n t^{n-1}$$