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Part I

Problem statement

Pipe networks can be analogous to Electrical Circuits. Therefore, it can be tempting to use electrical flow solvers for pipe networks.

1 Unit Comparison

Voltage is energy per unit charge unit wise. $\frac{J}{C}$ Joules per coulomb.

Whereas Pressure can be thought of as Energy per unit volume. $\frac{J}{m^3}$ Joules per meter cubed. That would be the unit of pascals.

$$(workDone)\ Joules = (pressure)\ Pa * (volume)\ m^3$$
$$(pressure)\ Pa = \frac{(workDone)\ Joules}{(volume)\ m^3}$$

Compare this to voltage:

$$(voltage)\ Volts = \frac{(workDone)\ Joules}{(Charge)\ Coulomb}$$

Likewise for volumetric flowrate, this is

$$(flowrate)\ m^3/s = \frac{(vol)\ m^3}{(time)\ s}$$

And for current,

$$(current)\ C/s = \frac{(Charge)\ Coulomb}{(time)\ s}$$

2 nonlinearities in flow resistance

However, the components exhibiting flow resistance often do not obey Ohm's law.

$$electrical\ resistance\ (\Omega) = \frac{V\ (Volts)}{I\ (Ampere)}$$
$$flow\ resistance = \frac{\Delta P\ (Pa)}{\dot{V}\ (m^3)}$$

For ohm's law, the ratio V/I reduces to a constant, but flow resistance often reduces to some expression.

If we were to use Fanning friction factor for pipe (cite perry's handbook)

$$f = \frac{\Delta P}{(\frac{4L}{D})\ \frac{1}{2}\rho u^2}$$

$$\Delta P = f(\frac{4L}{D})\ \frac{1}{2}\rho u^2$$

In Perry's chemical engineering handbook, the formula used for fanning's friction factor by Churchill is:

$$f = 2\left[\left(\frac{8}{Re}\right)^{12} + \left(\frac{1}{A+B}\right)^{3/2}\right]^{1/12}$$

Where:

$$A = \left[2.457 \ln \frac{1}{\left(\frac{1}{(7/Re)^{0.9}} + 0.27\frac{\varepsilon}{D}\right)}\right] \quad ; \; B = \left(\frac{37530}{Re}\right)^{16}$$

$$Re = \frac{ux}{\nu} = \frac{\dot{V}x}{A_{XS}\nu}$$

Where A_{XS} represents cross sectional area. We can see that this is strongly non linear with respect to volumetric flowrate. abcde

Part II

Newton Raphson method for Nonlinear Equations

3 one equation example of Newton Raphson

We want to solve if we have some equation y=f(x)

$$f(x) = 0$$

We are going to make use of derivatives to help us get there:

We do a first order Taylor series approximation of derivatives

$$\frac{df(x)}{dx} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

we want to make it such that:

$$f(x + \Delta x) \approx 0$$

So we have this:

$$f(x) >> f(x + \Delta x)$$

$$\frac{df(x)}{dx} = \frac{-f(x)}{\Delta x}$$

$$\frac{df(x)}{dx} = \frac{-f(x)}{\Delta x}$$

Solve for Δx:

$$\Delta x = \frac{-f(x)}{\frac{df(x)}{dx}}$$

Once we solve for Δx, we can update:

$$f(x_{i+1}) = f(x + \Delta x)$$

and repeat the method after we solve for $f(x_{i+1})$

4 Newton Raphson for multiple equations

$$f_1(x_1, x_2, x_3) = 0$$
$$f_2(x_1, x_2, x_3) = 0$$
$$f_3(x_1, x_2, x_3) = 0$$

We need derivatives as well if we want to solve above system of eqns:

$$df_1 = f_1(x_1 + \Delta x_1, x_2 + \Delta x_2, x_3 + \Delta x_3) - f_1(x_1, x_2, x_3)$$

Let's expand the LHS:

$$df_1 = \frac{\partial f_1}{\partial x_1}\Delta x_1 + \frac{\partial f_1}{\partial x_2}\Delta x_2 + \frac{\partial f_1}{\partial x_3}\Delta x_3$$

$$\frac{\partial f_1}{\partial x_1}\Delta x_1 + \frac{\partial f_1}{\partial x_2}\Delta x_2 + \frac{\partial f_1}{\partial x_3}\Delta x_3 = f_1(x_1 + \Delta x_1, x_2 + \Delta x_2, x_3 + \Delta x_3) - f_1(x_1, x_2, x_3)$$

We need to do this for f_2 and f_3 as well:

$$\frac{\partial f_2}{\partial x_1}\Delta x_1 + \frac{\partial f_2}{\partial x_2}\Delta x_2 + \frac{\partial f_2}{\partial x_3}\Delta x_3 = f_2(x_1 + \Delta x_1, x_2 + \Delta x_2, x_3 + \Delta x_3) - f_2(x_1, x_2, x_3)$$

$$\frac{\partial f_3}{\partial x_1}\Delta x_1+\frac{\partial f_3}{\partial x_2}\Delta x_2+\frac{\partial f_3}{\partial x_3}\Delta x_3 = f_3(x_1+\Delta x_1, x_2+\Delta x_2, x_3+\Delta x_3)-f_3(x_1, x_2, x_3)$$

Let's write this in matrix form

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \end{bmatrix} = \begin{bmatrix} f_1(x_1+\Delta x_1, x_2+\Delta x_2, x_3+\Delta x_3) \\ f_2(x_1+\Delta x_1, x_2+\Delta x_2, x_3+\Delta x_3) \\ f_3(x_1+\Delta x_1, x_2+\Delta x_2, x_3+\Delta x_3) \end{bmatrix} - \begin{bmatrix} f_1(x_1, x_2, x_3) \\ f_2(x_1, x_2, x_3) \\ f_3(x_1, x_2, x_3) \end{bmatrix}$$

Again, we need to approx:

$$\begin{bmatrix} f_1(x_1+\Delta x_1, x_2+\Delta x_2, x_3+\Delta x_3) \\ f_2(x_1+\Delta x_1, x_2+\Delta x_2, x_3+\Delta x_3) \\ f_3(x_1+\Delta x_1, x_2+\Delta x_2, x_3+\Delta x_3) \end{bmatrix} \approx \vec{0}$$

And we just solve:

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \end{bmatrix} = - \begin{bmatrix} f_1(x_1, x_2, x_3) \\ f_2(x_1, x_2, x_3) \\ f_3(x_1, x_2, x_3) \end{bmatrix}$$

We shall say that the Jacobian matrix is:

$$\vec{J} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{bmatrix}$$

We need to find the inverse in order to solve the above equation

$$\vec{J}^{-1}$$

So that we can find:

$$\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \end{bmatrix} = -\vec{J}^{-1} \begin{bmatrix} f_1(x_1, x_2, x_3) \\ f_2(x_1, x_2, x_3) \\ f_3(x_1, x_2, x_3) \end{bmatrix}$$

Once we find $\vec{\Delta x}$, we can update the values:

$$\vec{\Delta x} = \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \end{bmatrix}$$

This is the part where we find the next iteration

$$\overrightarrow{x_{i+1}} = \overrightarrow{x_i} + \overrightarrow{\Delta x}$$

$$\overrightarrow{x_i} = \begin{bmatrix} x_{1(i)} \\ x_{2(i)} \\ x_{3(i)} \end{bmatrix}$$

$$\overrightarrow{x_{i+1}} = \begin{bmatrix} x_{1(i+1)} \\ x_{2(i+1)} \\ x_{3(i+1)} \end{bmatrix}$$

Keep substituting until the solution is found within tolerance.

Part III

Matrices

Part IV

Matrices

$$\begin{bmatrix} 1 & 2 & 3 \\ a & b & c \end{bmatrix}$$