CS2100 Tutorial 6

Boolean Algebra, Logic Gates & Simplification

take k-map sheet in front
will start at :05 as usual

Slides by Theodore, adapted from Prof. Aaron’s slides

Recap

- K-maps
- minterm and Maxterm
- Boolean Algebra laws

(Announcement: A1, A2, midterms)

Overview

Q1) Consensus theorem

Q2) Simplification

Q3) Intro to K-maps

Q4) More K-maps

Q1. Consensus theorem

$$x \cdot y + x' \cdot z + y \cdot z = x \cdot y + x' \cdot z$$

Note:
The dot is important!

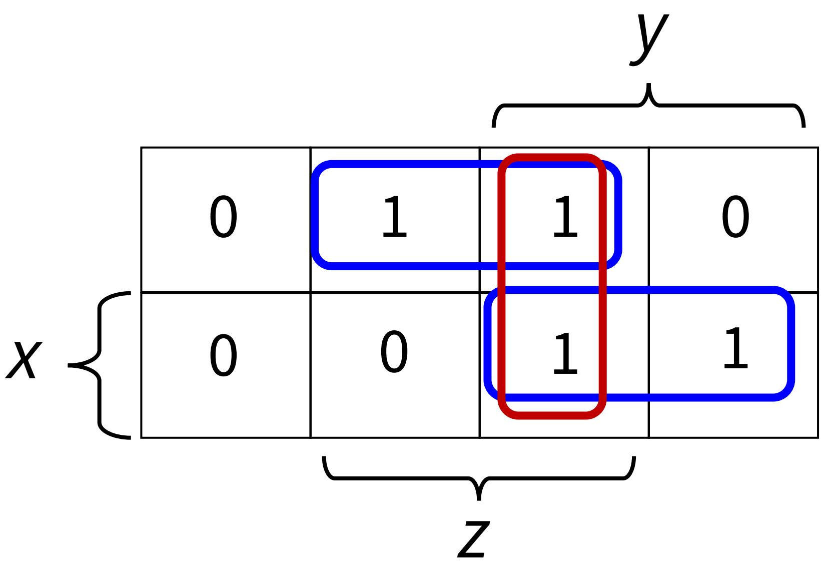
Q1. Consensus theorem

$$x \cdot y + x' \cdot z + y \cdot z = x \cdot y + x' \cdot z$$

$$\begin{array}{c|c|c|c|c|}
\multicolumn{1}{c}{} & \multicolumn{1}{c}{} & \multicolumn{1}{c}{} & \multicolumn{2}{c}{\overbrace{\hphantom{00000}}^{y}} \\ \cline{2-5}
 & 0 & 1 & 1 & 0 \\ \cline{2-5}
x \left\{ \right. & 0 & 0 & 1 & 1 \\ \cline{2-5}
\multicolumn{1}{c}{} & \multicolumn{1}{c}{} & \multicolumn{2}{c}{\underbrace{\hphantom{00000}}_{z}} & \multicolumn{1}{c}{}
\end{array}$$

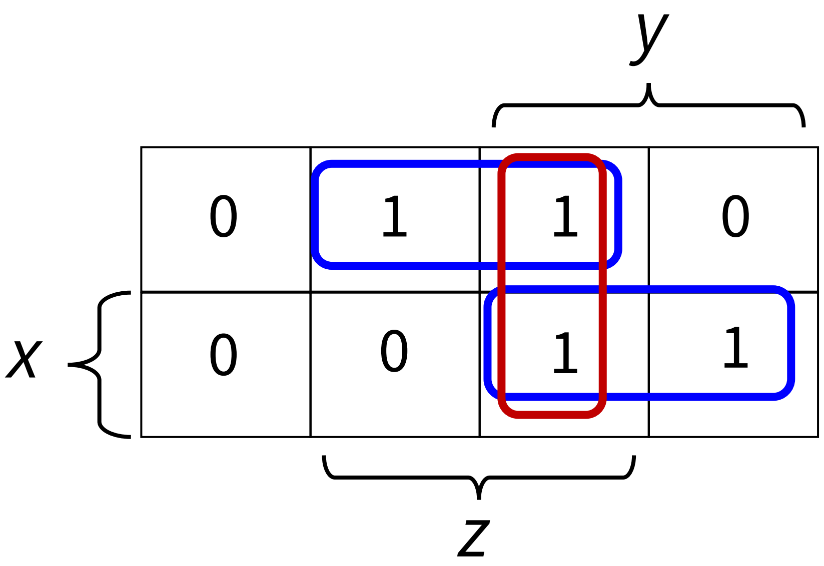
Q1. Consensus theorem

$$x\cdot y + x'\cdot z + y\cdot z = x\cdot y + x'\cdot z$$



Q1. Consensus theorem

$$x \cdot y + x' \cdot z + y \cdot z = x \cdot y + x' \cdot z$$



$$\begin{aligned}
& x \cdot y + x' \cdot z + y \cdot z \\
&\quad = x \cdot y + x' \cdot z + 1 \cdot y \cdot z && \text{[identity law]} \\
&\quad = x \cdot y + x' \cdot z + (x+x') \cdot y \cdot z && \text{[complement law]} \\
&\quad = x \cdot y + x' \cdot z + x \cdot y \cdot z + x' \cdot y \cdot z && \text{[distributive law]} \\
&\quad = x \cdot y + x \cdot y \cdot z + x' \cdot z + x' \cdot y \cdot z && \text{[commutative law]} \\
&\quad = (x \cdot y + x \cdot y \cdot z) + (x' \cdot z + x' \cdot y \cdot z) && \text{[associative law]} \\
&\quad = x \cdot y + x' \cdot z && \text{[absorption theorem 1]}
\end{aligned}$$

Q2a. Simplifying to SOP

$F(x,y,z) = (x+y \cdot z') \cdot (y'+y) + x' \cdot (y \cdot z'+y)$

$= (x+y \cdot z') \cdot 1 + x' \cdot (y \cdot z'+y)$ (by the complement law)

$= (x+y \cdot z') + x' \cdot (y \cdot z'+y)$ (by the identity law)

$= x + y \cdot z' + x' \cdot y$ (by absorption theorem 1)

$= x + x' \cdot y + y \cdot z'$ (by the commutative law)

$= x + y + y \cdot z'$ (by absorption theorem 2)

$= x + y$ (by absorption theorem 1)

Q2b. Simplifying to SOP

$G(p,q,r,s) = \prod M(5, 9, 13)$

$5 = (0101)_2$; $9 = (1001)_2$; $13 = (1101)_2$

Q2b. Simplifying to SOP

$G(p,q,r,s) = \prod M(5, 9, 13)$

$5 = (0101)_2; 9 = (1001)_2; 13 = (1101)_2$

$= (p+q'+r+s') \cdot (p'+q+r+s') \cdot (p'+q'+r+s')$ (by definition of maxterms)

Q2b. Simplifying to SOP

$5 = (0101)_2$; $9 = (1001)_2$; $13 = (1101)_2$

$$G(p,q,r,s) = \prod M(5, 9, 13)$$

$$= \boxed{(p+q'+r+s')} \cdot (p'+q+r+s') \cdot \boxed{(p'+q'+r+s')}$$ (by definition of maxterms)

$$= ((p \cdot p') + (q'+r+s')) \cdot (p'+q+r+s')$$ (by the distributive law)

$$(p+(q'+r+s')) \cdot (p'+(q'+r+s'))$$
$$= (p \cdot p') + (q'+r+s')$$

Distributive law:

$$A + (B \cdot C) = (A+B) \cdot (A+C)$$

or $(B \cdot C) + A = (B+A) \cdot (C+A)$

Q2b. Simplifying to SOP

Absorption theorem 2:

$A \cdot (A' + B) = A \cdot B$

$\boxed{q' \cdot (p'+q) = q' \cdot (q+p') = q' \cdot p' = p' \cdot q'}$

$G(p,q,r,s) = \prod M(5, 9, 13)$

$= (p+q'+r+s') \cdot (p'+q+r+s') \cdot (p'+q'+r+s')$ (by definition of maxterms)

$= ((p \cdot p') + (q'+r+s')) \cdot (p'+q+r+s')$ (by the distributive law)

$= (0 + (q'+r+s')) \cdot (p'+q+r+s')$ (by the complement law)

$= (q'+r+s') \cdot (p'+q+r+s')$ (by the identity law)

$= (q' \cdot (p'+q)) + (r+s')$ (by the distributive law)

$= p' \cdot q' + r + s'$ (by absorption 2)

Q3. K-maps

			B		
	m0	m1	m5	m4	
	m2	m3	m7	m6	C
A	m10	m11	m15	m14	
	m8	m9	m13	m12	
		D			

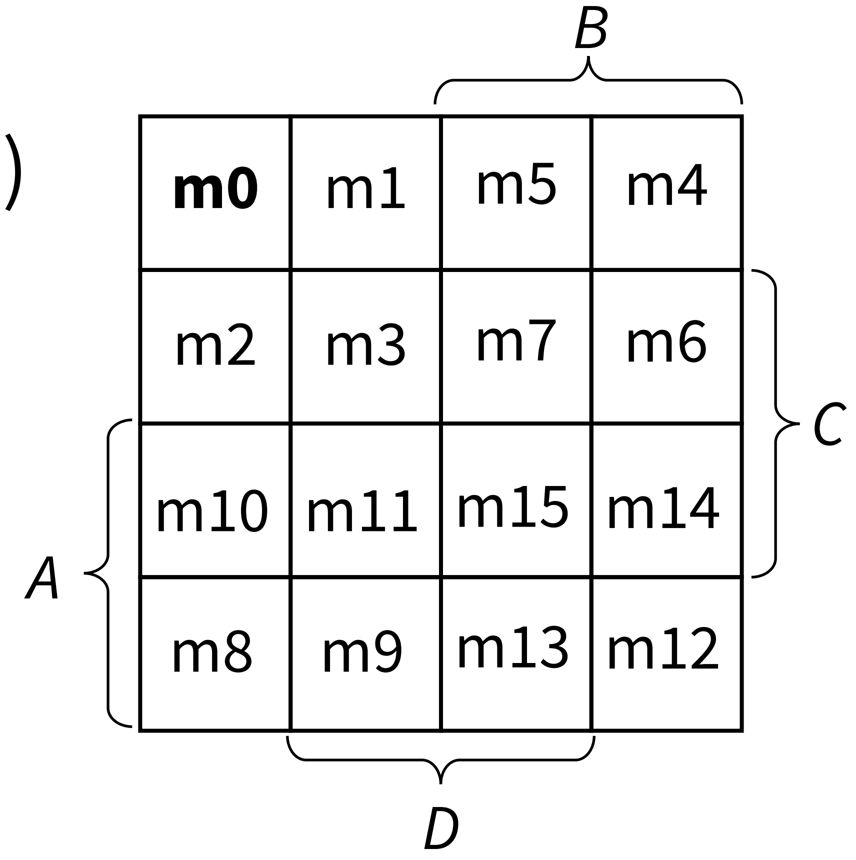
m1: 0001 = A’·B’·C’·D
m2: 0010 = A’·B’·C·D’
m3: 0011 = A’·B’·C·D

Q3. K-maps

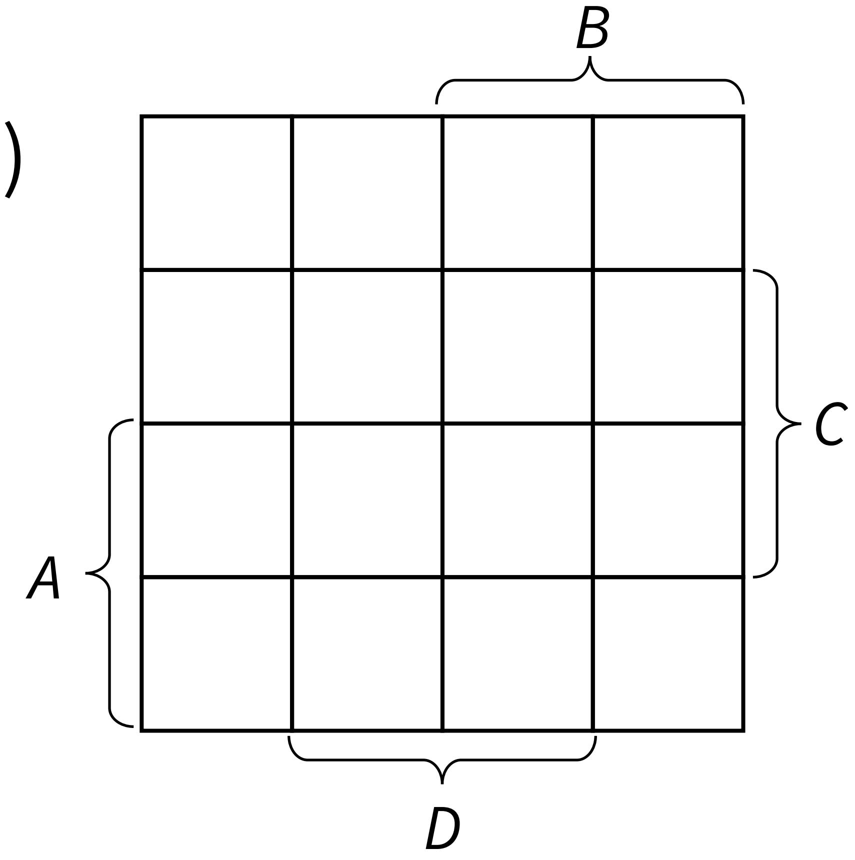
(b) $T(A,B,C,D) = \prod M(3,7,8,10,12,13) \cdot X(6,11,14,15).$

$T(A,B,C,D) = \Sigma m(0,1,2,4,5,9) + X(6,11,14,15).$

(a)



(c)



Q3. K-maps

(b) $T(A,B,C,D) = \prod M(3,7,8,10,12,13) \cdot X(6,11,14,15).$

$T(A,B,C,D) = \Sigma m(0,1,2,4,5,9) + X(6,11,14,15).$

(a)

m0	m1	m5	m4
m2	m3	m7	m6
m10	m11	m15	m14
m8	m9	m13	m12

(c)

1	1	1	1
1	0	0	X
0	X	X	X
0	1	0	0

Q3. K-maps

(b) $T(A,B,C,D) = \prod M(3,7,8,10,12,13) \cdot X(6,11,14,15)$.

$T(A,B,C,D) = \Sigma m(0,1,2,4,5,9) + X(6,11,14,15)$.

(a)

			B		
	m0	m1	m5	m4	
	m2	m3	m7	m6	C
A	m10	m11	m15	m14	
	m8	m9	m13	m12	
		D			

(c)

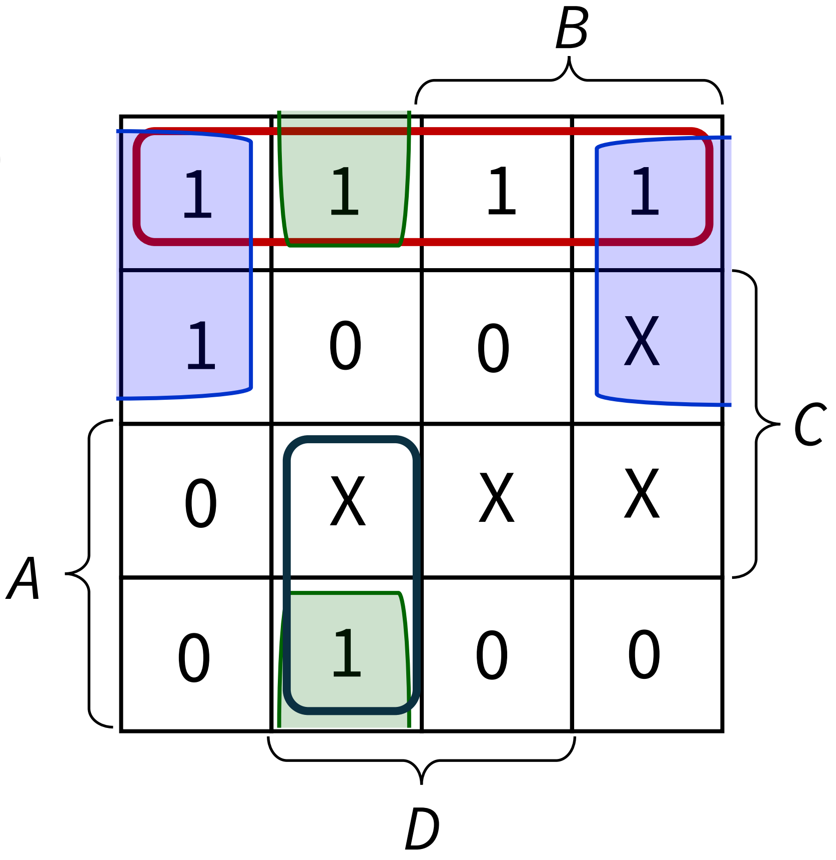
			B		
	1	1	1	1	
	1	0	0	X	C
A	0	X	X	X	
	0	1	0	0	
		D			

Q3. K-maps

(d) How many PIs?

(e) How many EPIs?

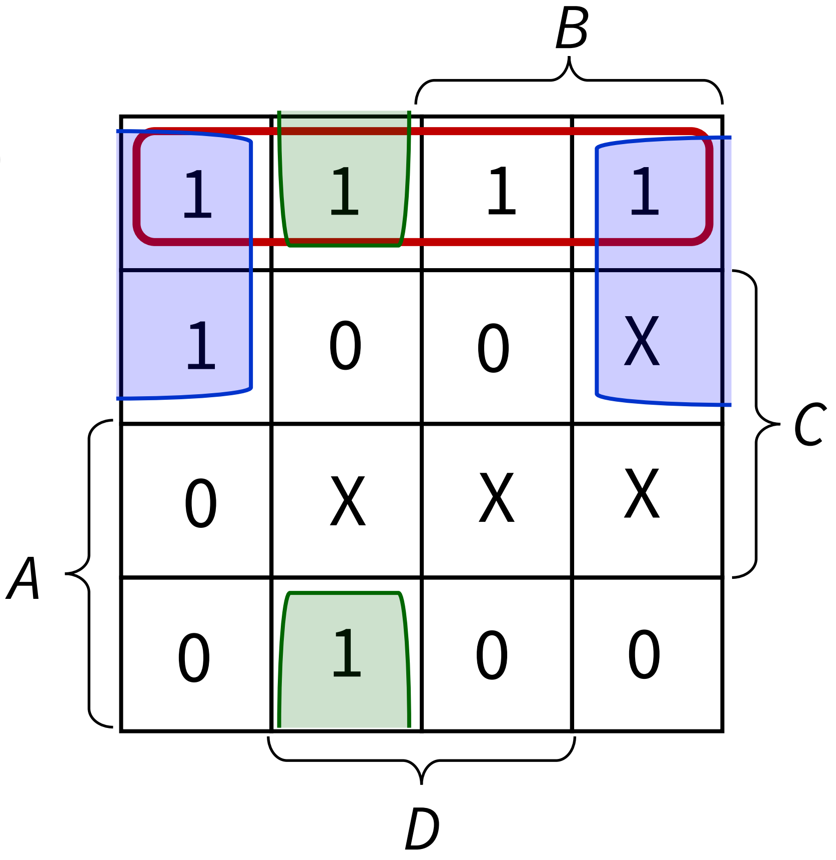
(c)



Q3. K-maps

(f) Simplified SOP expression:

(c)



Q3. K-maps

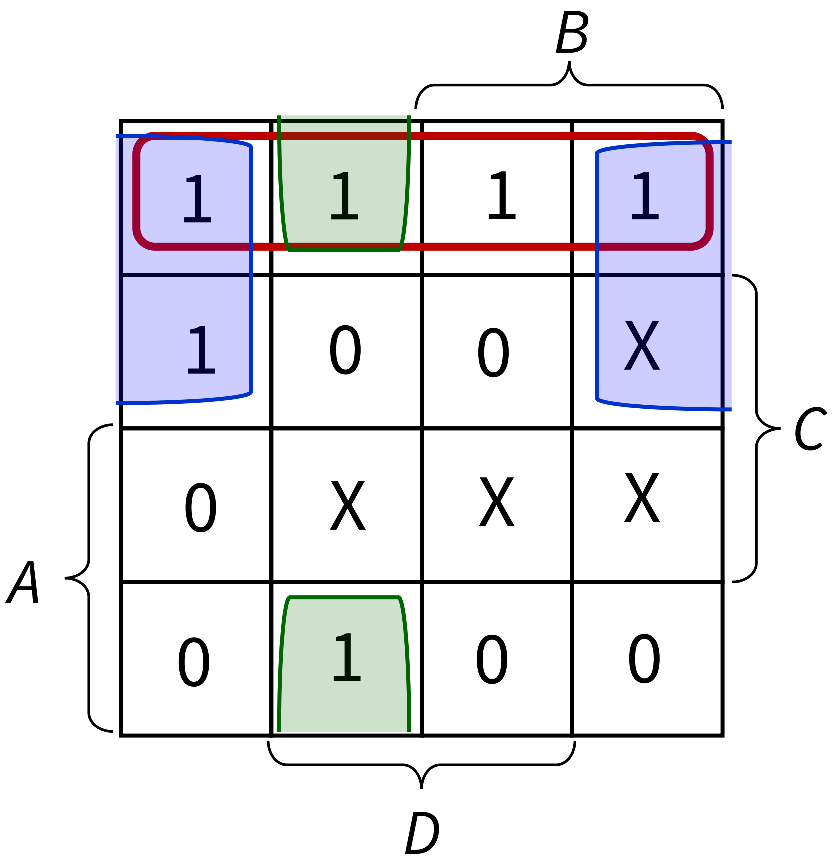
(f) Simplified SOP expression:

$A' \cdot D' + A' \cdot C' + B' \cdot C' \cdot D$

or

$A' \cdot D' + A' \cdot C' + A \cdot B' \cdot D$

(c)



Q3. K-maps

(g) K-map for T’:

(c) K-map for T:

1	1	1	1
1	0	0	X
0	X	X	X
0	1	0	0

Q3. K-maps

(g) K-map for T’:

<table>
<tr><td></td><td></td><td></td><td colspan="2">B</td><td></td></tr>
<tr><td></td><td>0</td><td>0</td><td>0</td><td>0</td><td></td></tr>
<tr><td></td><td>0</td><td>1</td><td>1</td><td>X</td><td rowspan="2">C</td></tr>
<tr><td rowspan="2">A</td><td>1</td><td>X</td><td>X</td><td>X</td></tr>
<tr><td>1</td><td>0</td><td>1</td><td>1</td><td></td></tr>
<tr><td></td><td></td><td colspan="2">D</td><td></td><td></td></tr>
</table>

(c) K-map for T:

<table>
<tr><td></td><td></td><td></td><td colspan="2">B</td><td></td></tr>
<tr><td></td><td>1</td><td>1</td><td>1</td><td>1</td><td></td></tr>
<tr><td></td><td>1</td><td>0</td><td>0</td><td>X</td><td rowspan="2">C</td></tr>
<tr><td rowspan="2">A</td><td>0</td><td>X</td><td>X</td><td>X</td></tr>
<tr><td>0</td><td>1</td><td>0</td><td>0</td><td></td></tr>
<tr><td></td><td></td><td colspan="2">D</td><td></td><td></td></tr>
</table>

Q3. K-maps

(g) K-map for T’:

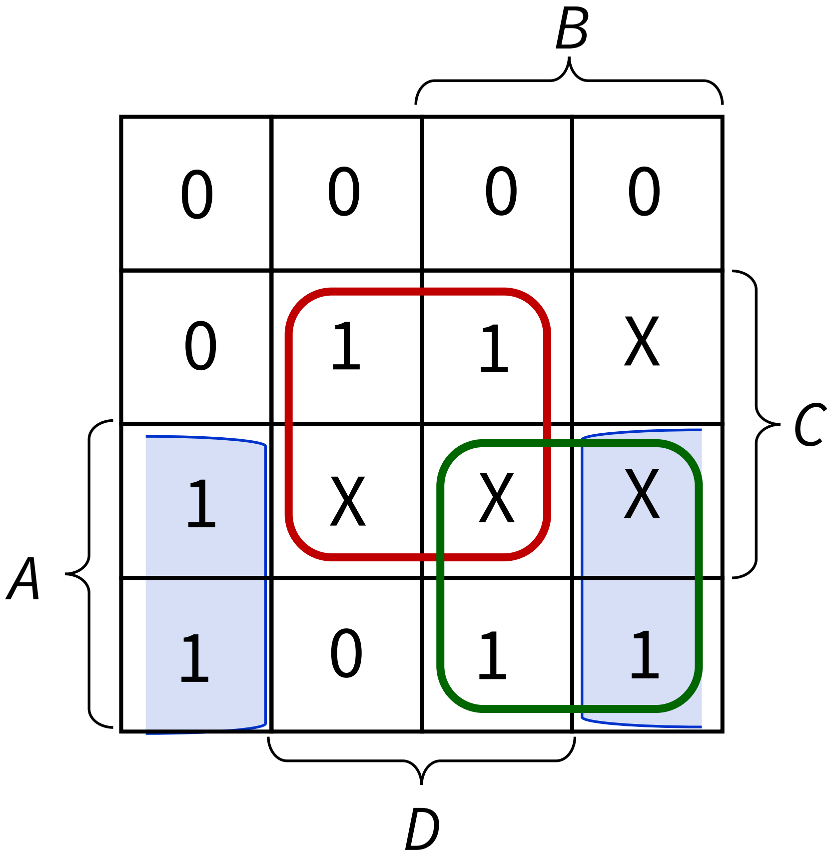
			B		
	0	0	0	0	
	0	1	1	X	*C*
A	1	X	X	X	
	1	0	1	1	
		D			

(c) K-map for T:

			B		
	1	1	1	1	
	1	0	0	X	*C*
A	0	X	X	X	
	0	1	0	0	
		D			

Q3. K-maps

(g) K-map for T’:



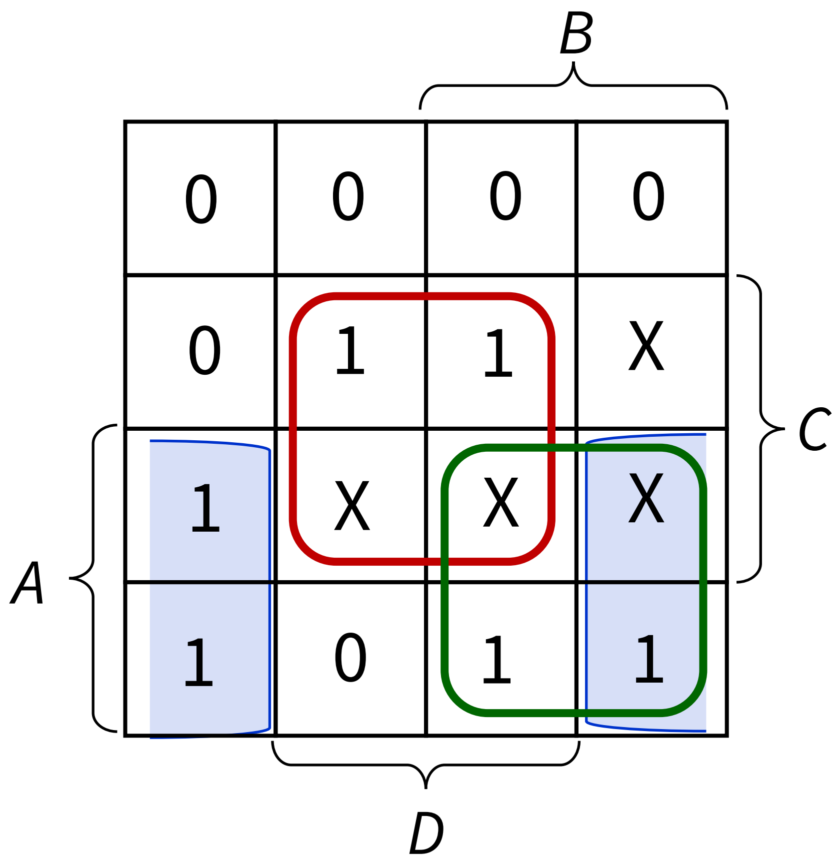
(g) Simplified POS expression:

T’ =

T =

Q3. K-maps

(g) K-map for T':



(g) Simplified POS expression:

$T' = A \cdot D' + C \cdot D + A \cdot B$

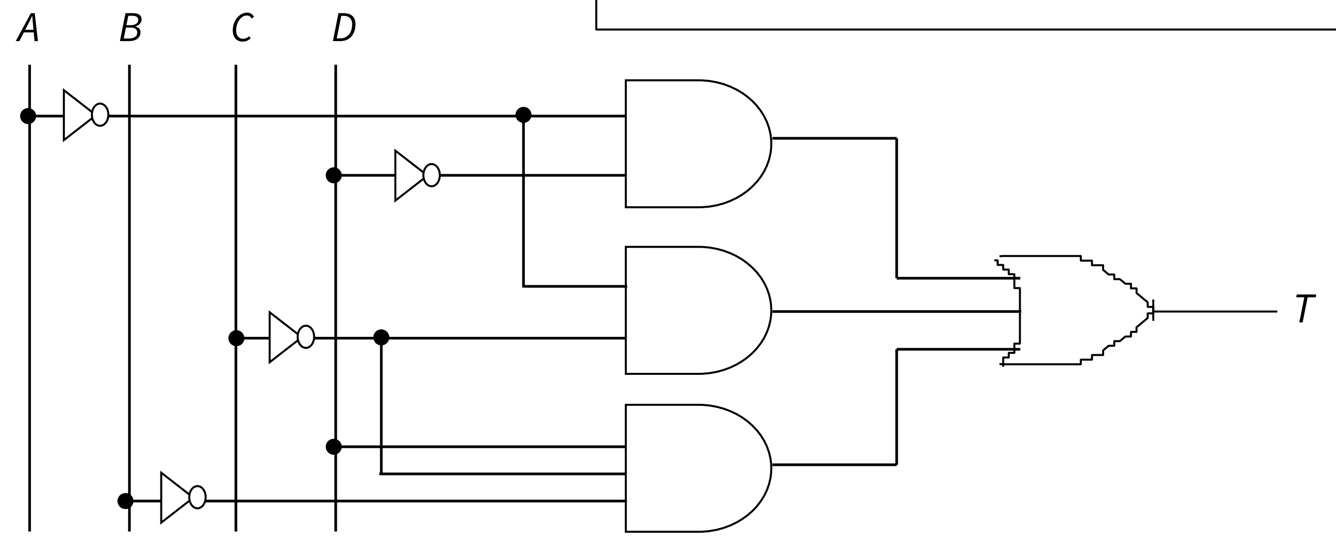
$T = (A \cdot D' + C \cdot D + A \cdot B)'$

$= (A' + D) \cdot (C' + D') \cdot (A' + B')$

Q3. K-maps

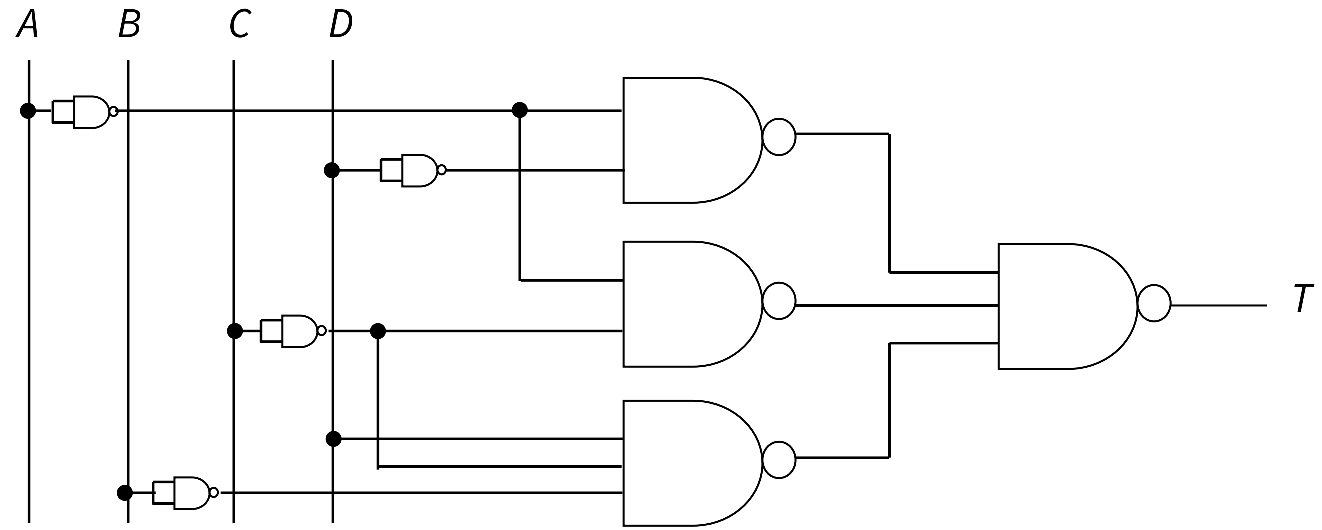
$T = A' \cdot D' + A' \cdot C' + B' \cdot C' \cdot D$

2-level AND-OR circuit:



Note: draw nicely :)
- Rulers for lines
- Thick dots for wire junctions
- Unfilled circle on NAND gates

2-level NAND circuit:



Q4. Circuits with don't-cares

KL, MN are 2-bit unsigned integers.

$X(K,L,M,N)$
$= 1$ if $KL = MN$
or 0 otherwise

$Y(K,L,M,N)$
$= 1$ if $KL \leq MN$
or 0 otherwise

$Z(K,L,M,N)$
$= 1$ if $KLM < LMN$
or 0 otherwise

Assume input 0000 will not occur.

K	L	M	N	X	Y	Z
0	0	0	0			
0	0	0	1			
0	0	1	0			
0	0	1	1			
0	1	0	0			
0	1	0	1			
0	1	1	0			
0	1	1	1			

K	L	M	N	X	Y	Z
1	0	0	0			
1	0	0	1			
1	0	1	0			
1	0	1	1			
1	1	0	0			
1	1	0	1			
1	1	1	0			
1	1	1	1			

Q4. Circuits with don't-cares

$X(K,L,M,N)$
$= 1$ if $KL = MN$
or 0 otherwise

$Y(K,L,M,N)$
$= 1$ if $KL \leq MN$
or 0 otherwise

$Z(K,L,M,N)$
$= 1$ if $KLM < LMN$
or 0 otherwise

Assume input 0000 will not occur.

K	L	M	N	X	Y	Z
0	0	0	0	d	d	d
0	0	0	1	0	1	1
0	0	1	0	0	1	1
0	0	1	1	0	1	1
0	1	0	0	0	0	1
0	1	0	1	1	1	1
0	1	1	0	0	1	1
0	1	1	1	0	1	1

K	L	M	N	X	Y	Z
1	0	0	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	1	1	0
1	0	1	1	0	1	0
1	1	0	0	0	0	0
1	1	0	1	0	0	0
1	1	1	0	0	0	0
1	1	1	1	1	1	0

Q4. Circuits with don't-cares

K	L	M	N	X	Y	Z
0	0	0	0	d	d	d
0	0	0	1	0	1	1
0	0	1	0	0	1	1
0	0	1	1	0	1	1
0	1	0	0	0	0	1
0	1	0	1	1	1	1
0	1	1	0	0	1	1
0	1	1	1	0	1	1

K	L	M	N	X	Y	Z
1	0	0	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	1	1	0
1	0	1	1	0	1	0
1	1	0	0	0	0	0
1	1	0	1	0	0	0
1	1	1	0	0	0	0
1	1	1	1	1	1	0

X

		N	M, N	M
	d	0	0	0
L	0	1	0	0
K, L	0	0	1	0
K	0	0	0	1

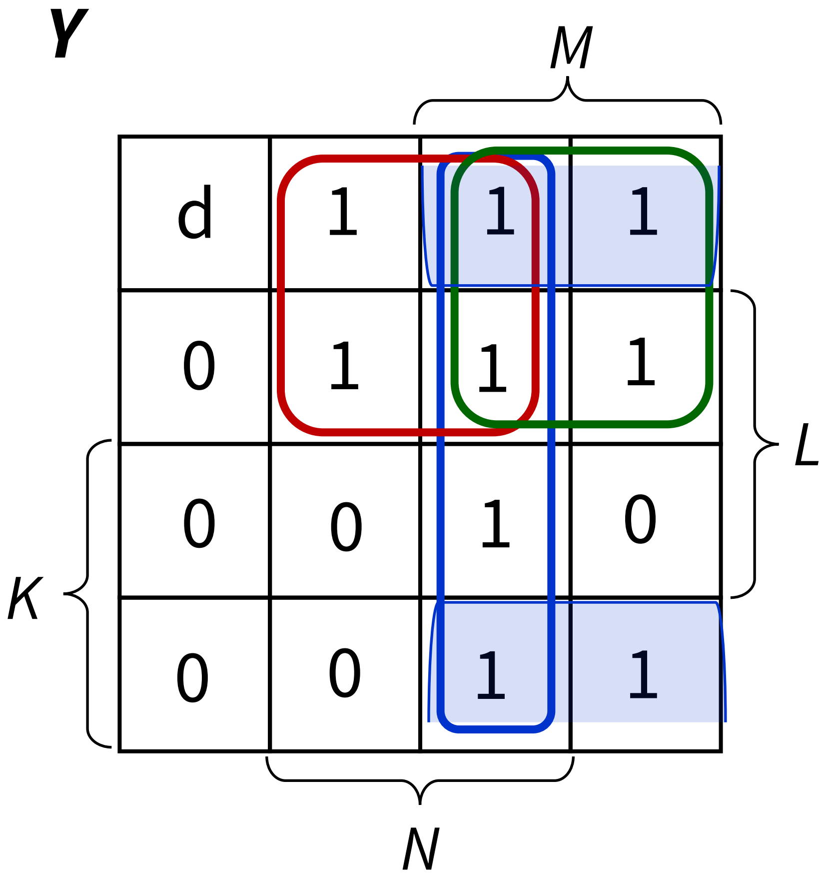
$X = K'\cdot L\cdot M'\cdot N + K\cdot L'\cdot M\cdot N' + K\cdot L\cdot M\cdot N$

$KLMN = 0000 \rightarrow X = 0$

Q4. Circuits with don't-cares

K	*L*	*M*	*N*	*X*	*Y*	*Z*
0	0	0	0	d	d	d
0	0	0	1	0	1	1
0	0	1	0	0	1	1
0	0	1	1	0	1	1
0	1	0	0	0	0	1
0	1	0	1	1	1	1
0	1	1	0	0	1	1
0	1	1	1	0	1	1

K	*L*	*M*	*N*	*X*	*Y*	*Z*
1	0	0	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	1	1	0
1	0	1	1	0	1	0
1	1	0	0	0	0	0
1	1	0	1	0	0	0
1	1	1	0	0	0	0
1	1	1	1	1	1	0



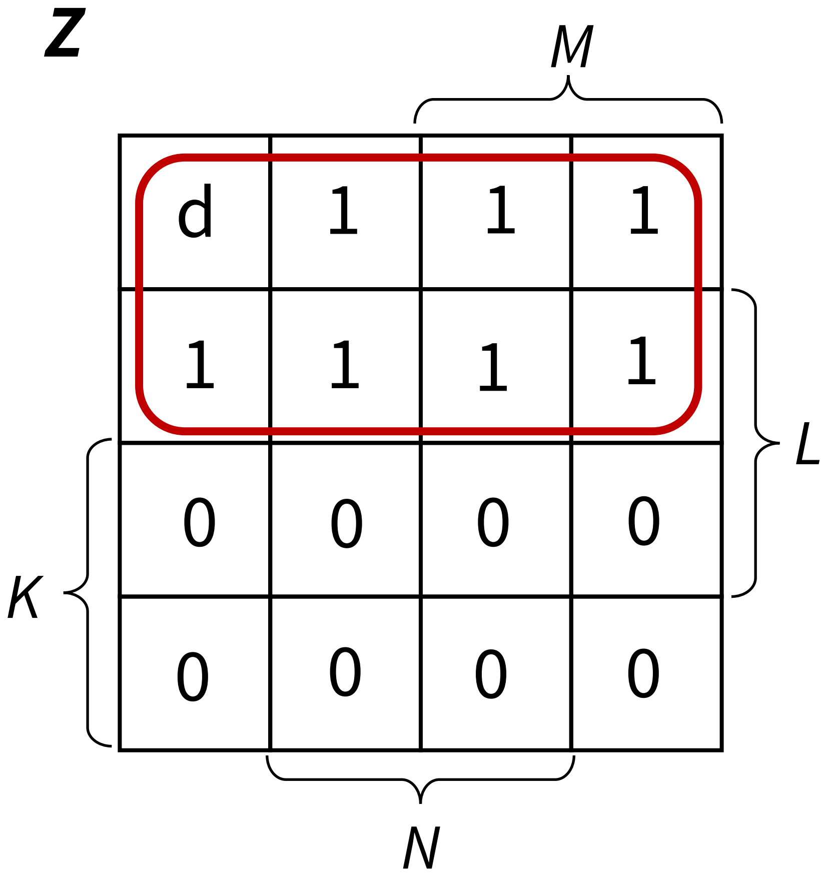
$Y = M \cdot N + K' \cdot N + K' \cdot M + L' \cdot M$

$KLMN = 0000 \rightarrow Y = 0$

Q4. Circuits with don't-cares

K	L	M	N	X	Y	Z
0	0	0	0	d	d	d
0	0	0	1	0	1	1
0	0	1	0	0	1	1
0	0	1	1	0	1	1
0	1	0	0	0	0	1
0	1	0	1	1	1	1
0	1	1	0	0	1	1
0	1	1	1	0	1	1

K	L	M	N	X	Y	Z
1	0	0	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	1	1	0
1	0	1	1	0	1	0
1	1	0	0	0	0	0
1	1	0	1	0	0	0
1	1	1	0	0	0	0
1	1	1	1	1	1	0



$Z = K'$

$KLMN = 0000 \rightarrow Z = 1$

End of Tutorial 6

- Slides uploaded on github.com/theodoreleebrant/TA-2425S1
- Email: theo@comp.nus.edu.sg
- Anonymous feedback:
  bit.ly/feedback-theodore
  (or scan on the right)

(Also reminder for me to take attendance)