CS1231S : Materials from tutorials

Tutorial 1

D2 (a) $(p \wedge q) \rightarrow p$

(b) $((p \vee q) \wedge \sim p) \rightarrow q$

(c) $((p \rightarrow q) \wedge p) \rightarrow q$

(d) $(\sim p \rightarrow (q \wedge \sim q)) \rightarrow p$

* Related to 6b:

Prove $p \leftrightarrow q$

~~$\leftrightarrow$~~ $\equiv (\sim p \vee q) \wedge (p \vee \sim q)$

$\equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

(Can't quote this). unless in lecture.

D5 $((a \rightarrow x) \wedge (b \rightarrow y)) \rightarrow ((a \wedge b) \rightarrow (x \wedge y))$

* Note: the $\leftarrow$ way ~~is wrong~~. does not hold.

2 (b) $(\sim a \rightarrow \sim (b \vee \sim a)) \equiv a$

(d) $((p \wedge q) \rightarrow q) \equiv$ true

(e) $((p \rightarrow q) \rightarrow r) \equiv (p \wedge \sim q) \vee r$

4. $(12x - 7 = 29) \leftrightarrow (x = 3)$

6a. $(p \leftrightarrow q) \rightarrow (p \rightarrow q)$

(similarly, since $p \leftrightarrow q \equiv q \leftrightarrow p$, $\therefore (p \leftrightarrow q) \rightarrow (q \rightarrow p)$)

↑ can't quote, need explanation.

b. Also known as transitivity of biconditional

$((p \leftrightarrow q) \wedge (q \leftrightarrow r)) \rightarrow (p \leftrightarrow r)$

* Note: only applicable for $\rightarrow$, the $\leftarrow$ way does not hold.

7a. $(p \vee (q \wedge r)) \wedge \sim p \rightarrow q \wedge r$

9. The product of any two ~~odd~~ integers is an odd integer.

10. Let n be an integer.

$\therefore$ Odd$(n) \leftrightarrow$ Odd(n^2)

* Note: not exactly how it is written in Tut 1. ~~but~~

Tutorial 2

* Additional Notes:

$\forall x \in D,\ P(x) \equiv \forall x\ ((x \in D) \rightarrow P(x))$

$\exists x \in D,\ P(x) \equiv \exists x\ ((x \in D) \wedge P(x))$

Well-Formed Formulas (WFF): (see ~~notes~~ tut)

Bound variables, scope of quantifiers: (see tut).

Common mistake:

e.g. Aiken loves Dueet the reindeer

Given predicates Loves(x,y) and Reindeer(x),

CORRECT: Loves(Aiken, Dueet) $\wedge$ Reindeer(Dueet)

WRONG: Loves(Aiken, Reindeer(Dueet))

this is either true/false.

Q1. a) $\forall n \in \mathbb{Z}\ (6 \mid n \leftrightarrow 2 \mid n \wedge 3 \mid n)$

b) $\forall x\ (x \in \mathbb{Z} \rightarrow x \in \mathbb{Q})$ is True. The converse is not.

c) ~~$\forall x$~~ $\forall p, q \in \mathbb{Z}\ (\text{Even}(p) \wedge \text{Even}(q) \rightarrow \text{Even}(p+q))$

3 a) Integers are not closed under division.

b) $\mathbb{Q}$ is closed under addition.

c) $\mathbb{Q}$ is not closed under division

> consider division by 0.

4. <Unlikely to be used, but check this if you have sets $\{1,3,5,7,11,13\}$ and $\{0,2,4,6\}$ in a question>

7. $\forall x \in \mathbb{R}\ ((x^2 > x) \rightarrow (x < 0) \vee (x > 1))$.