Extra notes :

① Prenex Normal Form (PNF)

Before we get to PNF, see the following equivalence:

$$\forall x \left(\exists y (R(x,y)) \wedge \forall y (\sim S(x,y)) \rightarrow \sim (\exists y (R(x,y)) \wedge P)\right)$$
$$\equiv \forall x \left(\exists y_1 R(x,y_1) \wedge \forall y_2 (\sim S(x,y_2)) \rightarrow \sim (\exists y_3 (R(x,y_3)) \wedge P)\right)$$
$$\equiv \forall x \forall y_1 \exists y_2 \forall y_3 (\sim R(x,y_1) \vee S(x,y_2) \vee \sim R(x,y_3) \vee \sim P)$$

The last line is called a prenex normal form, which has all the quantifier in front. In this case, $\forall x \forall y_1 \exists y_2 \forall y_3$ is called the prefix and the $(\sim R(x,y_1) \vee \dots \vee \sim P)$ is called the matrix.

How to get PNF

① Eliminate all $\rightarrow$ and $\leftrightarrow$ using implication law
(i.e. $a \rightarrow b \equiv \sim a \vee b$, $a \leftrightarrow b \equiv (a \rightarrow b) \wedge (b \rightarrow a) \equiv \dots$)

② Move all negations inward:
(i.e. e.g. $\sim (\exists y\ P(x))$ becomes $\forall y\ \sim P(x)$)

③ Standardise variable: ⇒ See: ~~Extra Notes ②~~ α-equivalence on ~~their~~ λ-calculus.

$$\forall x (P(x) \wedge R(x)) \wedge (\exists x (Q(x)) \vee \exists x (S(x)))$$
becomes $$\forall x_1 (P(x_1) \wedge R(x_1)) \wedge (\exists x_2 (Q(x_2)) \vee \exists x_3 (S(x_3)))$$

④ Move all quantifier forward.

Example

Every child irritates his parent

Interpretation: $\forall x (Child(x) \rightarrow \forall y (Parent(y,x) \rightarrow Irritate(x,y)))$
$\equiv \forall x (\sim Child(x) \vee \forall y (Parent(y,x) \rightarrow Irritate(x,y)))$
$\equiv \forall x (\sim Child(x) \vee \forall y (\sim Parent(y,x) \vee Irritate(x,y)))$
$\equiv \forall x \forall y (\sim Child(x) \vee \sim Parent(y,x) \vee Irritate(x,y))$
↑ This is PNF
$\equiv \forall x \forall y ((Child(x) \wedge Parent(y,x)) \rightarrow Irritate(x,y))$

The teaching team will accept equivalent correct answers in an exam/assignment as long as it is not superfluous.