GAME THEORY: Lecture 2

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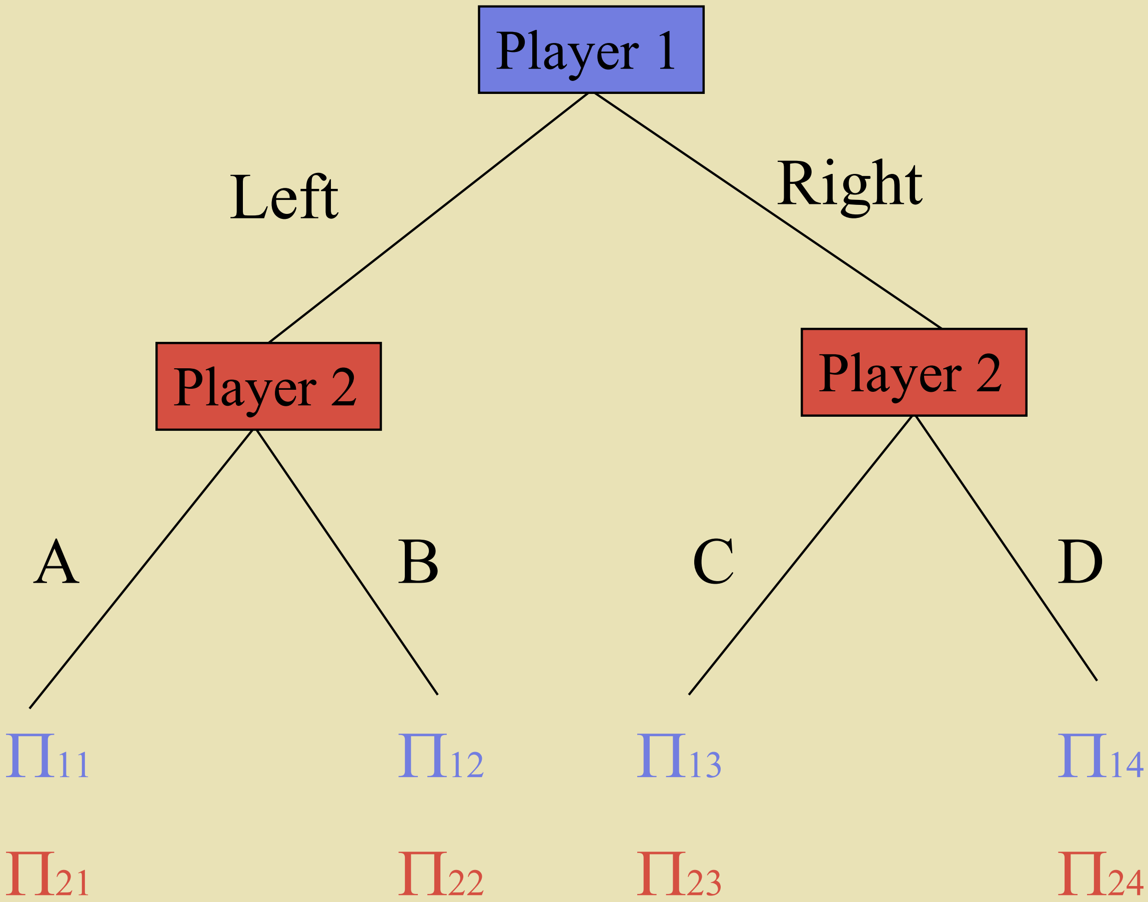
Extensive Form (Sequential move) Games

Dynamic Games of
Complete and Perfect
Information

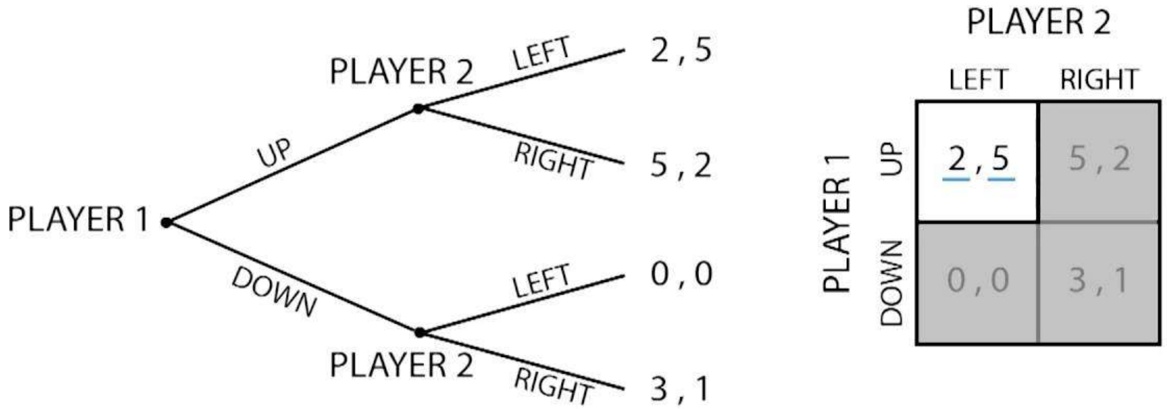
What is a Game Tree?



Assumptions in Dynamic Extensive Form Games

- All players are rational.
- Rationality is common knowledge
- Players move sequentially. (Therefore, also called sequential games)
- Players have complete and perfect information
  - Players can see the full game tree including the payoffs
  - Players can observe and recall all previous moves

Strategies in an Extensive Form Game



Actually player 2 has 4 strategies: (Left, Left), (Left, Right), (Right, Left), (Right, Right)

Strategies in an Extensive Form Game

Actually, player 2 has 4 strategies:

(Left, Left), (Left, Right), (Right, Left), (Right, Right)

A

	(Left, Left)	(Left, Right)	(Right, Left)	(Right, Right)
Up	2, 5	2, 5	5, 2	5, 2
Down	0, 0	3, 1	0, 0	3, 1

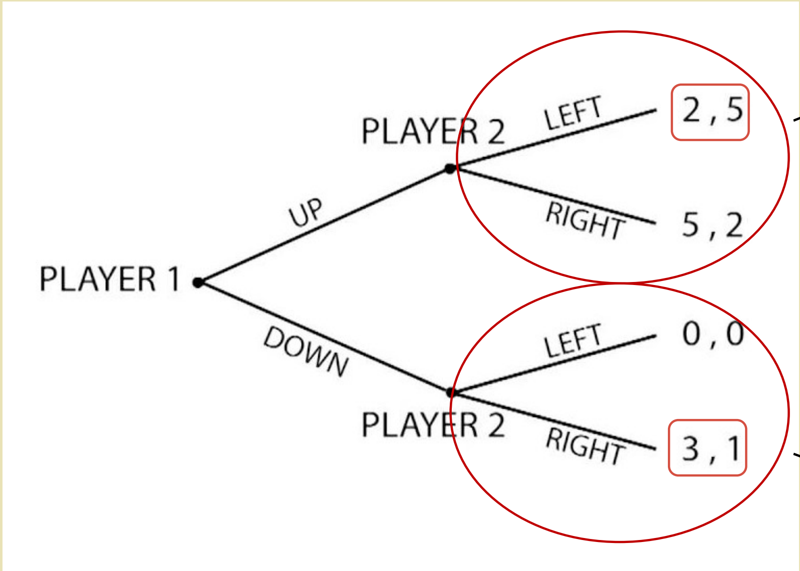
There are 2 NE! (Up, (Left, Left)) & (Down, (Left, Right))

First one involves an empty threat. How do we get rid of it?

Solution of an Extensive Form Game

- Subgame Perfect Equilibrium: For an equilibrium to be subgame perfect, it has to be a NE for all the subgames as well as for the entire game.
  - A subgame is a decision node from the original game along with the decision nodes and end nodes.
  - Backward induction is used to find SPE

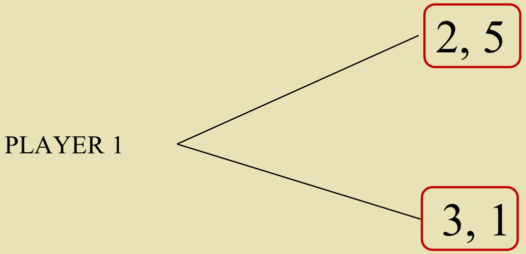
Start at the end nodes



If the game reaches this subgame, Player 2 will choose Left (since 5 > 2)

If the game reaches this subgame, Player 2 will choose Right (since 1 > 0)

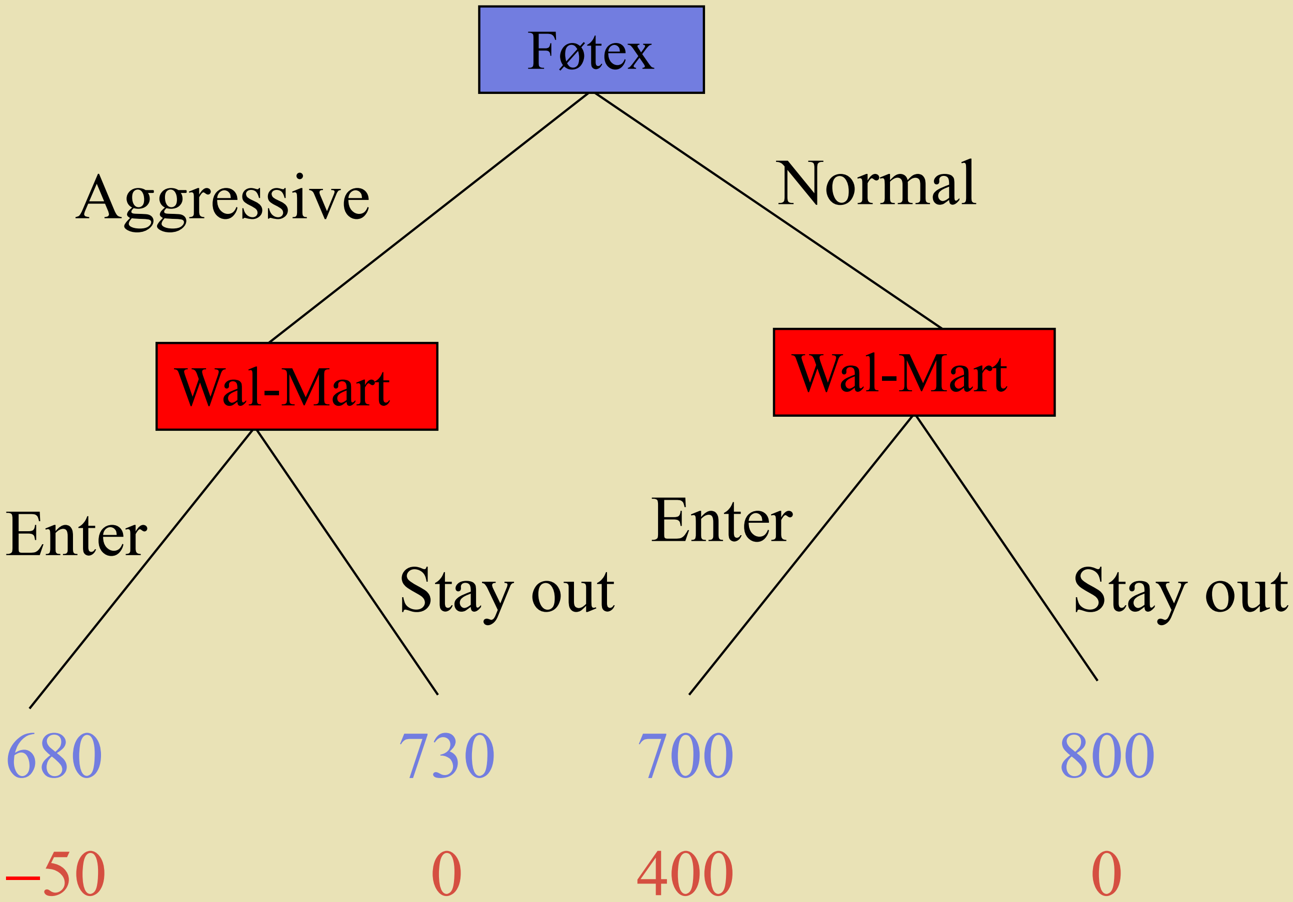
Trimming the tree:



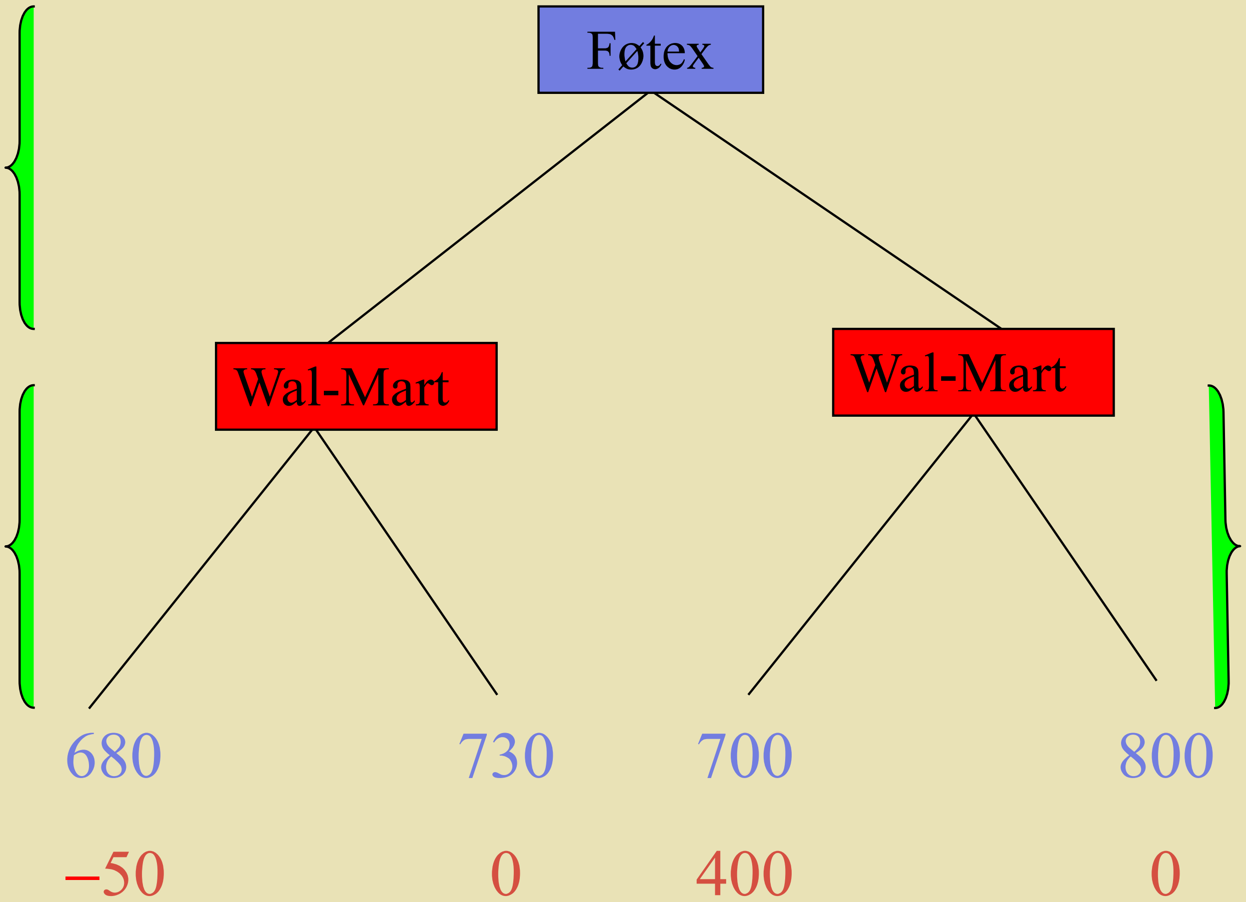
Player 1 will choose Down (since 3 > 2)

SPE: (Down, (Left, Right))

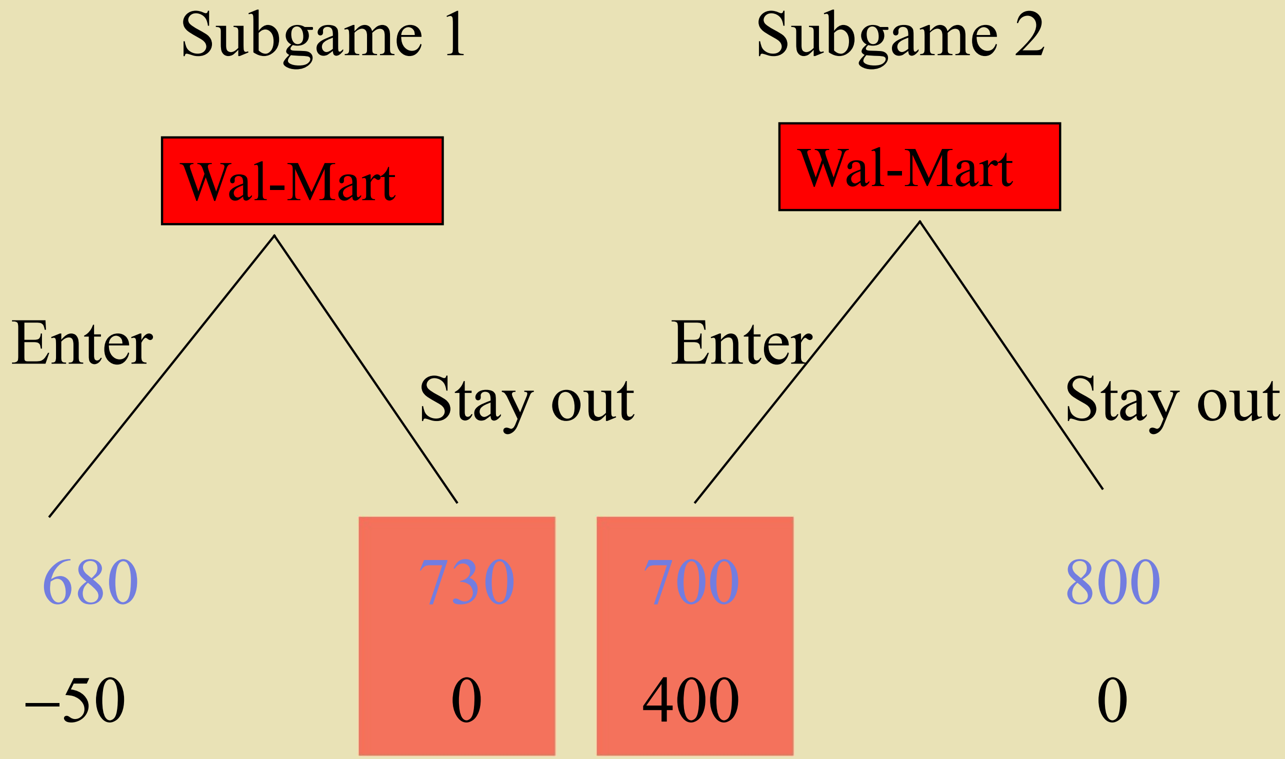
An Advertising Example



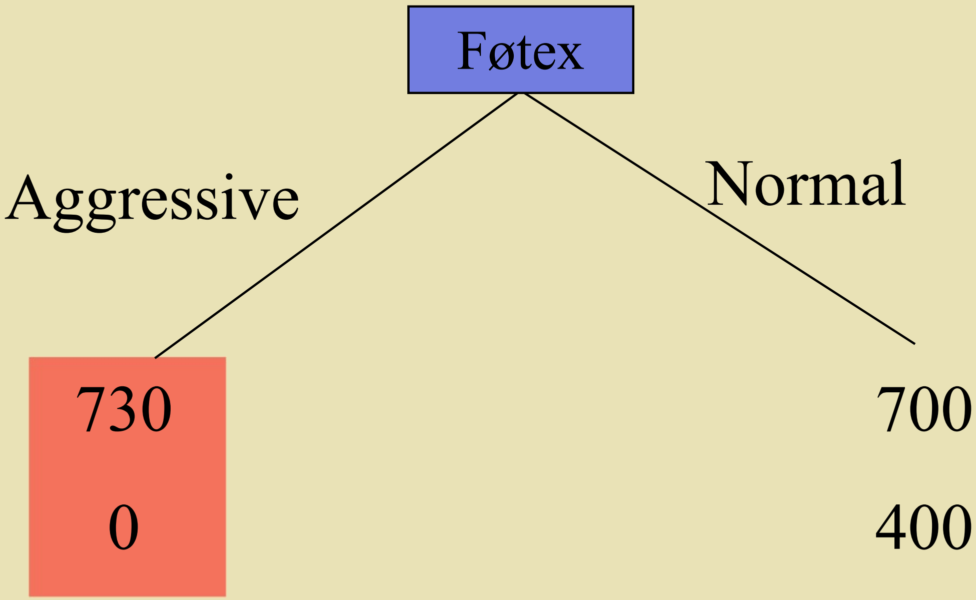
Advertising Example: 3 proper subgames



Solution of the Advertising Game



Solution of the Advertising Game (cont.)



SPE of the game is the strategy profile:

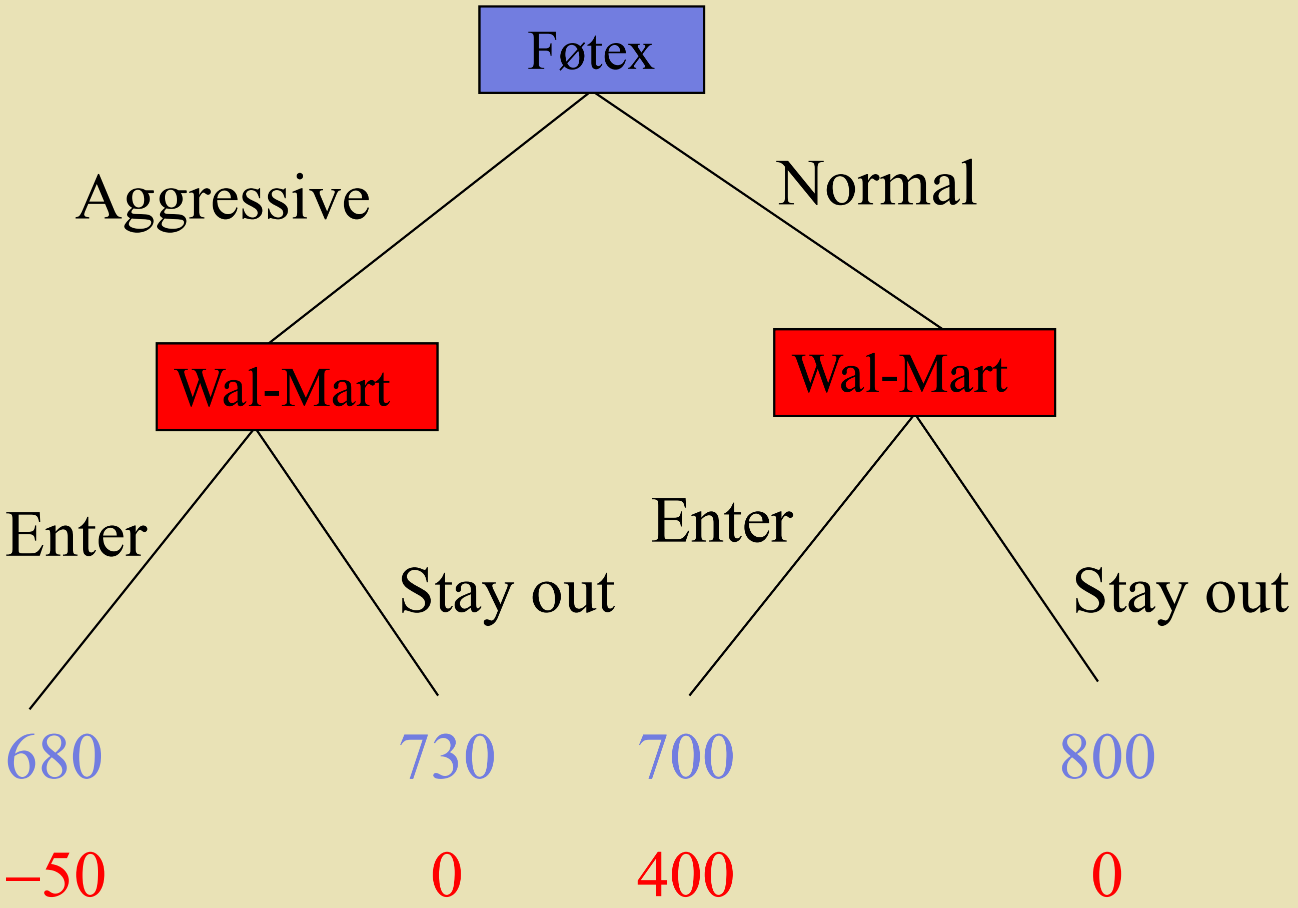
{aggressive, (stay out, enter)}

Properties of SPE

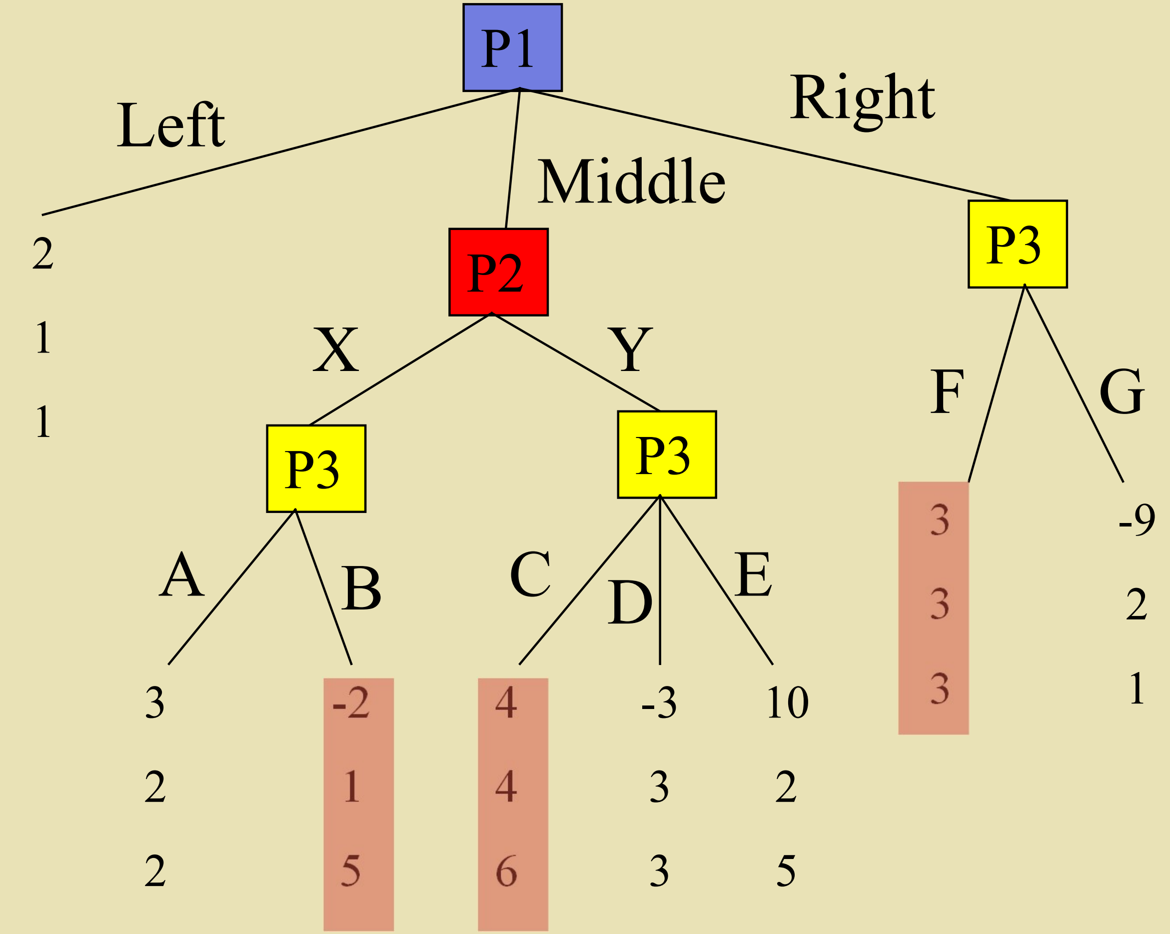
- The outcome that is selected by the backward induction procedure is always a NE of the game with perfect information.

  - ❖SPE is a stronger equilibrium concept than NE
  - ❖SPE eliminates NE that involve incredible threats.

Suppose WM threatens to enter no matter what Føtex does. Is this a credible threat?



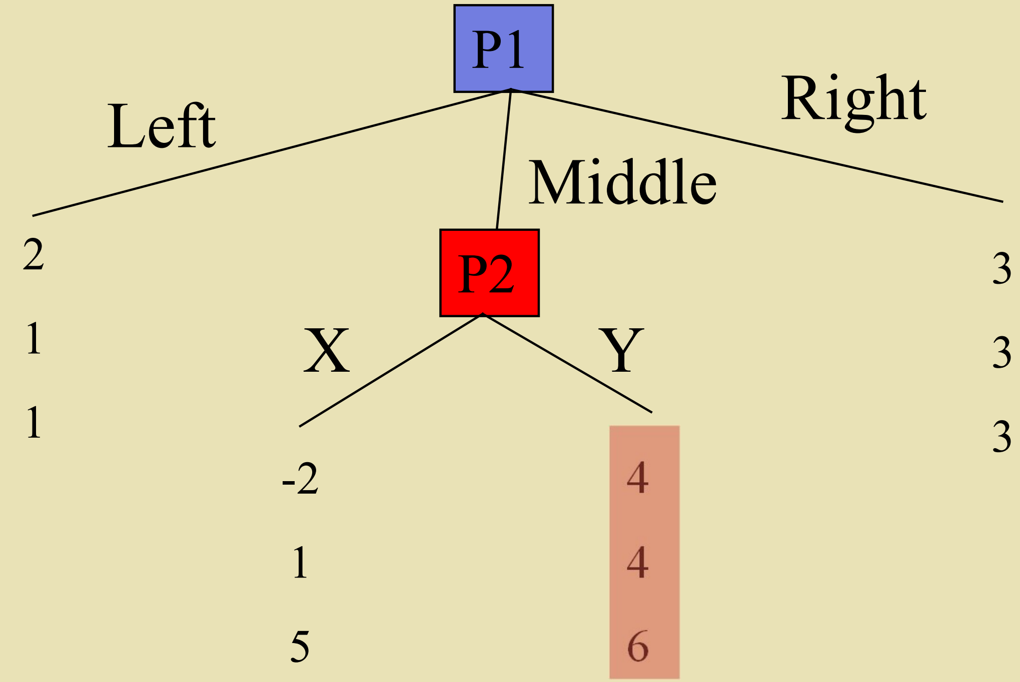
A 3 Player Sequential Game



Backwards Induction

- Obviously, Player 3's choices are B, C, and F in the three last period subgames.
- Eliminating the non-equilibrium strategies will make the game tree simpler.
- The game tree reduces to:

Reduced Game Tree

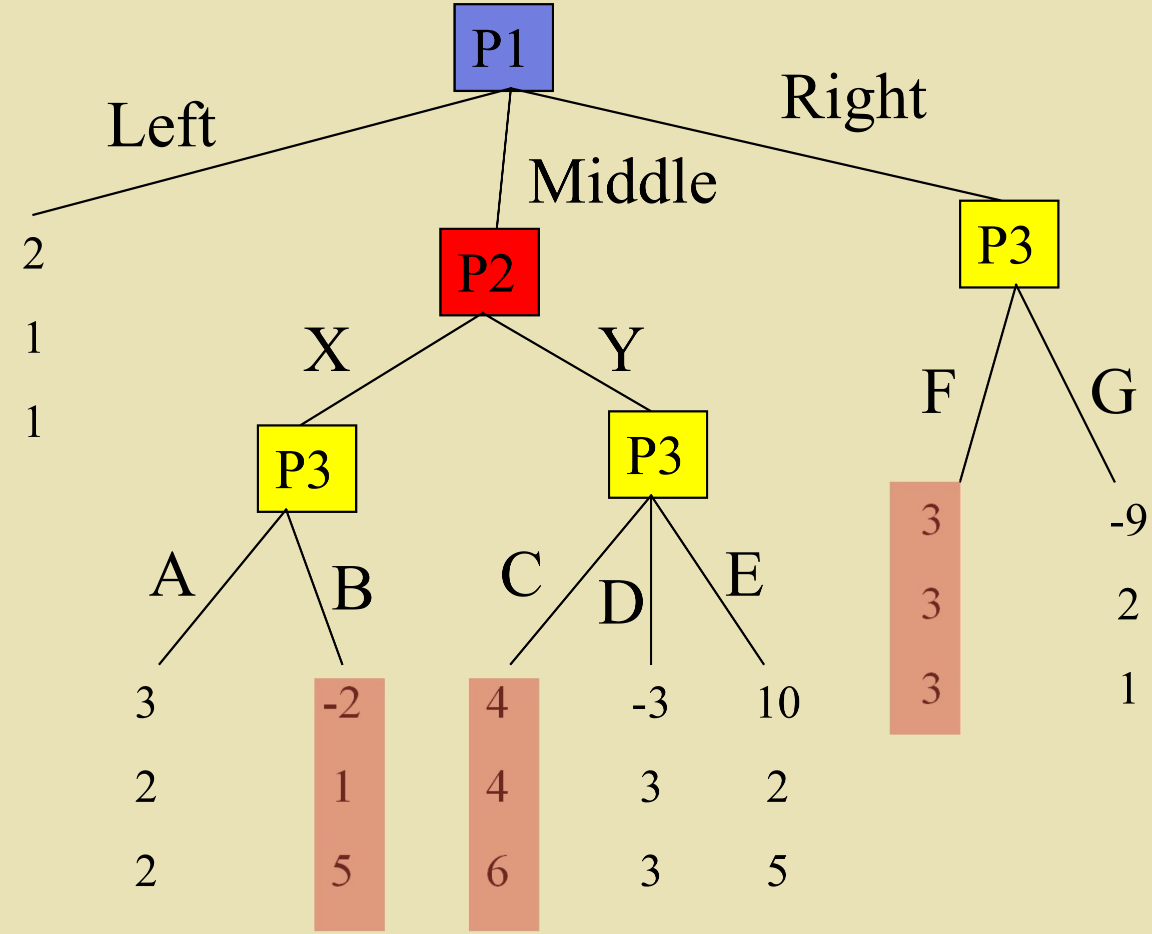


SPE is when player 1 plays middle, 2 plays Y, and 3 plays C.

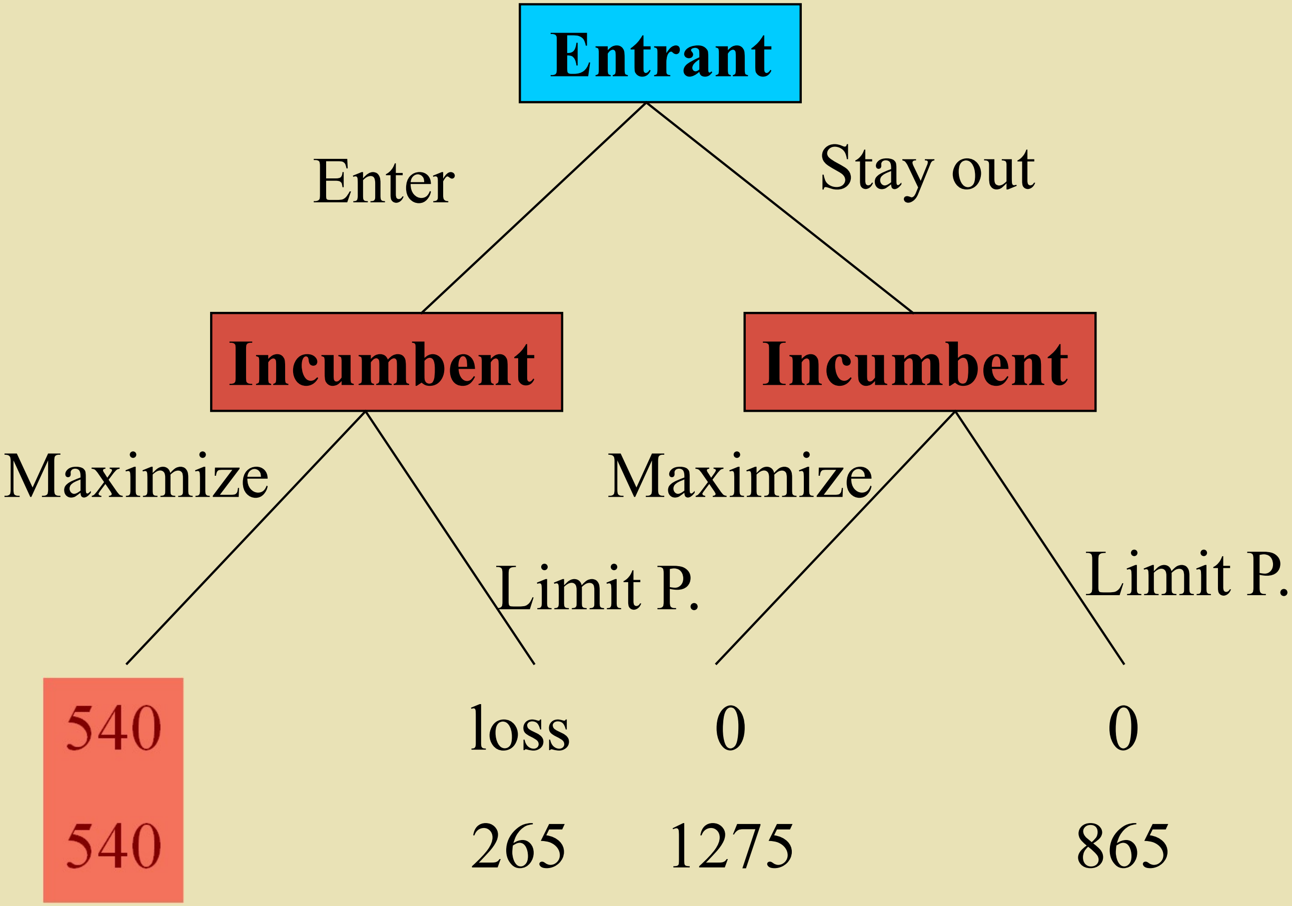
A Critique of SPE

- What do you think player 1 would do, if he is not certain whether player 2 is rational or not, but he is certain that player 3 is rational?
- What do you think player 1 would do, if he is not certain whether player 3 is rational or not, but he is certain that player 2 is rational?

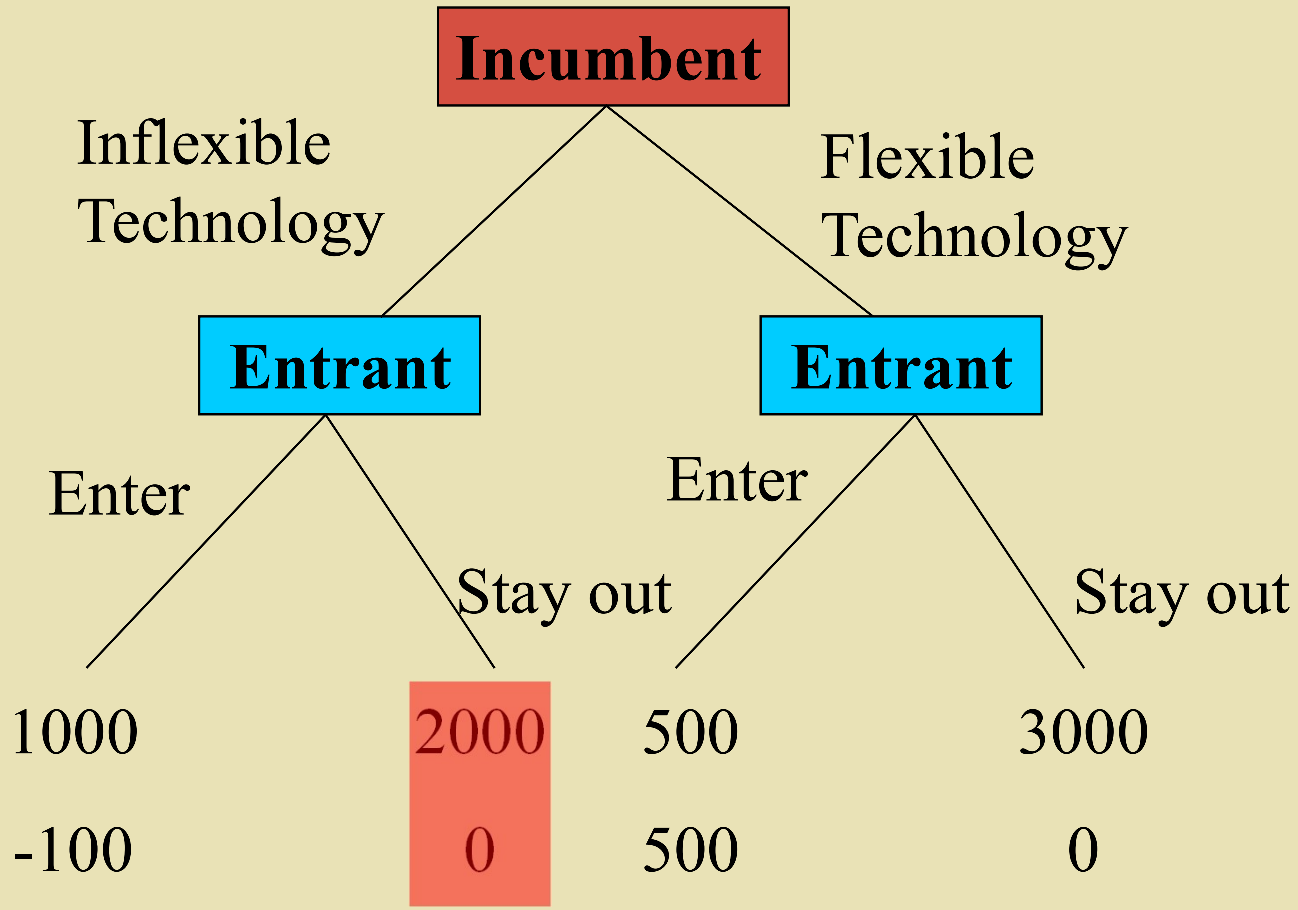
Doubts about rationality?



Limit Pricing Game



Commitment: changes the sequence



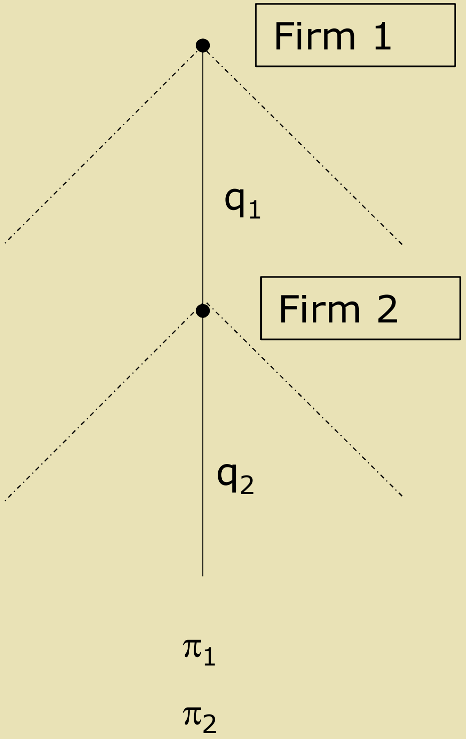
Stackelberg Duopoly

Two firms are competing in quantities. The leader (Firm 1) chooses a quantity to produce. Observing this choice, the follower (Firm 2) chooses a production level. They have identical cost functions, $C_i = cq_i$ and face the inverse demand function $P = a – Q$, where $Q = q_1 + q_2$. What output levels will they produce in (SP) equilibrium?

Game representation:

1. $I = \{\text{Firm 1, Firm 2}\}$
2. $Si = \{qi \mid qi \geq 0\}$, $i= 1, 2$
3. $ui(qi, qj) = [a – (qi+qj)]qi - cqi$, $i= 1, 2$
4. Firm 1 moves first, then Firm 2 chooses a quantity

Partial Game Tree



Backwards induction: Start at stage 2

Firm 2's maximization problem

Applying Backwards Induction

Stage 2:

Firm 2 takes q_1 as given and solves

Maximize $u_2 = (a- q_1 – q_2)q_2 –cq_2$

FOC: $a – q_1 – 2q_2 –c = 0$

$q_2 = (a-q_1-c)/2 = R_2(q_1)$ (Same as Cournot!)

Stage 1:

Firm 1 knows $R_2(q_1)$ will determine q_2 in the second stage, so it solves

Maximize $u_1 = (a- q_1 – q_2)q_1 –cq_1$

Applying Backwards Induction

Stage 1:

Firm 1 knows $R_2(q_1)$ will determine q_2 in the second stage, so it solves

Maximize $u_1 = (a- q_1 – [(a-q_1-c)/2])q_1 –cq_1$

FOC: Solving $\frac{\partial u_1}{\partial q_1} = 0$ for q_1 yields $q_1^* = (a-c)/2$

Then substituting what we find in $R_2(q_1^*)$ we find $q_2^* = (a-c)/4$

SPE: ((a-c)/2, (a-c)/4)

Stackelberg Duopoly

- Firm 2 produces less than it would if this was a Cournot game.
- Firm 1 produces more than it would in Cournot.
- Firm 2 know s more in Stackelberg than in Cournot. This extra information is not beneficial!
  - ❖ In multi-person optimization problems, more information does not necessarily lead to higher payoff.
- Firm 2 threatens to produce the Cournot output level no matter what Firm 1 does in the first stage. Should Firm 1 believe this?

Establishing Credibility

- Establish and use a reputation. Example: Do not negotiate with terrorists.
- Write contracts. Example: Supplier agrees to a punishment if he fails to deliver on time.
- Cut off communication. Example: Letter with no return address or will
- Burn bridges behind you. Example: Firm investing in inflexible technology.

Establishing Credibility (cont.)

- Leave the outcome to chance. Example: Automatic response to nuclear attacks (Dr. Strangelove).
- Move in small steps. Example: $1 million agreement versus 1000 sequential transactions limited to $1000.
- Develop credibility through teamwork. Example: Army requires soldiers to shoot deserters. Failing to kill a deserter gets the death sentence. If deserter kills, he is free of future charges.
- Employ negotiating agents. Example: Union leader negotiating a wage increase instead of the worker.

REPEATED GAMES

Repeated Normal Form Games

Prisoners' Dilemma Revisited

- Suppose that the two suspects play the same game every time they get caught.
- Can they coordinate their choices in order to get the best outcome for both of them?
- Finitely repeated game
- Infinitely repeated game

Prisoners' Dilemma: Stage game

Prisoner 2

Prisoner 1

	Confess	Don't Confess
Confess	-5, -5	0, -10
Don't Confess	-10, 0	-2, -2

Twice-repeated PD (backwards induction):

First stage payoff matrix after adding NE payoffs from the second stage

Prisoner 2

Prisoner 1

	Confess	**Don't Confess**
Confess	-10, -10	-5, -15
Don't Confess	-15, -5	-7, -7

N-times repeated PD

- Since in the last stage (n^{th} stage) the NE is (confess, confess) and all players know this, in all previous stages the same NE will prevail.
- In a finitely repeated (n times repeated game where $n \geq 2$) PD game, the cooperative outcome (don't confess, don't confess) cannot be enforced.

❖ If the stage game has a unique NE, then for any finite number of stages T, the repeated game has a unique SPE in which the NE of the stage game is played in every stage.

Infinitely Repeated PD

- When the game is played infinitely or players do not know when the game is going to end, the backward induction breaks down.
  - ❖One SPE is that stage game NE is played infinitely
  - ❖Following trigger strategies can enforce the cooperative outcome to be another SPE.
  - ❖Infinite stream of payoffs are computed using Present Value
- **Trigger strategy:** A player cooperates as long as the other players cooperate, but any defection from cooperation triggers the player to behave noncooperatively for a specified period of time (period of punishment).

Trigger Strategies

- **Grim strategy:** A trigger strategy in which the punishment period lasts till the end of the game.
  - **Grim strategy for PD game:** Play “don’t confess” in the first period. In period t, play “don’t confess” if the outcome was (don’t confess, don’t confess) in *all preceding t-1 periods*, and play “confess” otherwise.

Trigger Strategies (cont.)

- **Tit-for-tat (TFT):** A trigger strategy in which the punishment period lasts as long as the rival keeps on cheating (returning back to cooperative periods of game play is possible).
  - **TFT strategy for PD game:** Play "don't confess" in the first period. In period t, play "don't confess" if the *rival's most recent play* was (don't confess, don't confess), and play "confess" otherwise.

Infinitely Repeated Price Competition: Stage Game

Firm B

Firm A

	Low P	High P
Low P	20, 20	50, 15
High P	15, 50	30, 30

Grim Strategy in Pricing Game

Start playing HP. Keep playing HP as long as (HP, HP) was the outcome in all previous periods. Otherwise play LP forever.

- If d is the discount rate, present value of payoffs are:

Periods	t	t+1	t+2	…

Cooperation: $30 + 30d + 30\,d^2 + \ldots = 30\,(1+d+d^2+\ldots) = \dfrac{30}{1-d}$

Cheating : $50 + 20d + 20d^2 + \ldots = 50 + 20d(1+d+d^2+\ldots) = 50 + \dfrac{20d}{1-d}$

If $\dfrac{30}{1-d} \geq 50 + \dfrac{20d}{1-d}$ **OR d ≥ 2/3** then cooperation is subgame perfect!

TFT Strategy in Pricing Game

Start playing HP. Keep playing HP as long as (HP, HP) was the outcome of the last stage. Otherwise play LP until the other returns back to cooperation.

- If d is the discount rate, present value of payoffs are:

Cooperation: $\frac{30}{1-\mathrm{d}}$

Cheating once and going back to cooperation:

$$50 + 15d + 30d^2 + 30d^3 + \ldots = 50 + 15d + 30d^2(1+d+d^2+\ldots)$$

$$= 50 + 15d + \frac{30\mathrm{d}^2}{1-\mathrm{d}}$$

TFT Strategy in Pricing Game

If $\frac{30}{1-d} \geq 50 + 15d + \frac{30d^2}{1-d}$ then cooperation is subgame perfect!

$$0 \geq 4 - 8d + 3d^2$$

Solving roots yields:

$d \geq 2$ or $\mathbf{d \geq 2/3}$ (Note: discount rate above 1 is not plausible).

❖ **Folk theorem:** If the discount rate is large enough, an infinite number of SPE exist for infinitely repeated games, which involve higher payoffs than in the single period Nash outcome.

Axelrod’s Tournament

- Axelrod’s 4 rules for successful repeated PD game play:

  i) Don’t be envious

  ii) Don’t be the first to defect

  iii) Reciprocate both cooperation and defection

  iv) Don’t be too clever

If you want to read more about the tournament:
https://cs.stanford.edu/people/eroberts/courses/soco/projects/1998-99/game-theory/axelrod.html

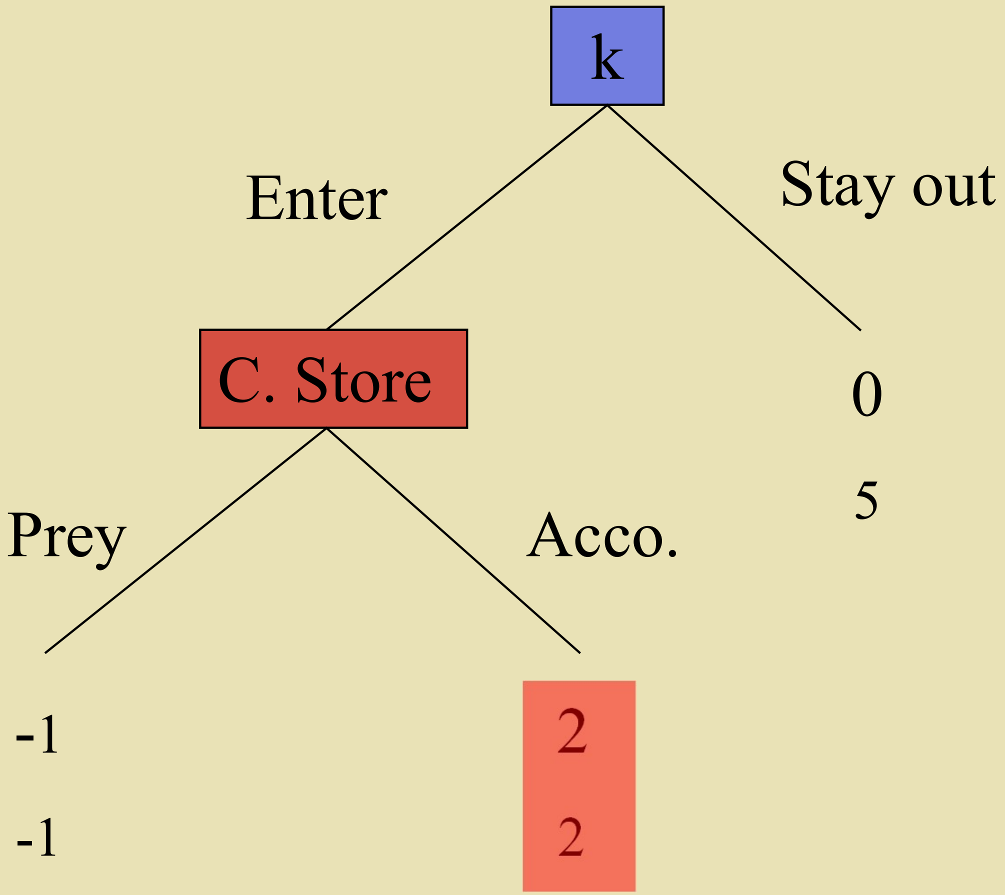
REPEATED GAMES (cont.)

Repeated Extensive Form Games

Chain Store Example

- A Chain store has branches in K towns.
- There is a potential entrant (k) in each town.
- Chain store has to decide between fighting or accommodating entry in each town.
- The rival in the next town can observe how the chain store behaved in previous towns.

Game Tree in Town K



SPE of Chain Store Game

- In the last town the entrant will solve SPE and choose to enter.
- In town K-1 the entrant will do the same, for that matter in all previous towns the outcome will be the same.
- Solution: In every town, entry will occur and the chain store will accommodate.

Chain Store Paradox

- The incumbent has an incentive to prey on the first entrant and hence to scare off the entrants in other towns by establishing a predatory reputation.
- However, the second potential entrant will not be impressed (expecting a rational behavior from the C. store.
- In that case, there is no incentive for the C. store to prey in the first town.

The Paradox

- The result is counterintuitive
- The result is due to strict reliance on backward induction.
- Infinitely repeated version: predatory behavior is an equilibrium (SPE) strategy.

A Critique of Backward Induction

- Longer chains of backward induction are more sensitive to small changes in the information structure of the game.
- Backward induction rules out any behavior that is contingent upon an event to which the theory assigns zero probability.

Managers May Choose

- Which game to play
- With whom to play
- Which strategies are available to each player
- What payoff each outcome will yield

More importantly

- Whether to play or not