Lecture 4 practice exercises

Question 1:

For each scenario below, classify it as a priori probability, statistical probability, or estimates, and justify your decision in 1–2 sentences.

a) Probability of getting heads with a fair coin

b) Predicting quarterly defect rates on a mature assembly line with rich historical data.

Solution 1:

a) Probability of getting heads with a fair coin is an a priori probability. By symmetry, the two possible outcomes (heads, tails) are equally likely and mutually exclusive, so the probability can be deduced logically as 1/2 without relying on empirical trials.
b) Predicting quarterly defect rates on a mature assembly line with rich historical data is a statistical probability. Probabilities are inferred from the observed frequency of past defects under stable production conditions, making empirical data the basis for estimating future likelihoods.

Question 2:

Explain why R&D expenditure can be seen as the purchase of a growth option. In your answer, distinguish between:

- how irreversibility differs for fixed versus R&D investments, and
- why greater uncertainty may increase, rather than reduce, R&D spending.

Solution 2:

R&D expenditure can be seen as the purchase of a growth option, as spending today buys the right, but not the obligation, to undertake profitable follow-on projects if technical success and market conditions turn out favorably.

Irreversibility is high for fixed investments, where large sunk costs create an option value of waiting. By contrast, while R&D also involves sunk costs (see for instance the initial upfront costs (m^{α}) in the model, which might be research labour hours and equipment), projects are more modular and can be staged, with the possibility of abandonment. This staging limits downside risk and reduces the effective irreversibility relative to fixed capital.

Early investment gives firms access to a growth option (call option). The value of this call option increases with uncertainty. Greater uncertainty can increase R&D spending because R&D generates options with convex payoffs: the downside is capped at the research costs, while the upside can be very large if success is achieved. At the same time firms face first-mover pressures in R&D winner-takes-all environments, which makes delaying investment costly.

Question 3:

Define scenario analysis. What is its purpose and how does it differ from sensitivity analysis?

Solution 3:

Scenario analysis is a structured approach to exploring alternative, plausible "what-if" futures by combining multiple assumptions into internally consistent narratives. It is not a prediction, but a way to test how strategies or decisions might perform under different states of the world.

The purpose of scenario analysis is to help decision-makers understand how a strategy or decision might perform under different hypothetical future states. It should help them anticipate potential risks and opportunities, identify robust strategies, and prepare for uncertainty by considering a range of qualitatively different futures.

It differs from sensitivity analysis in that sensitivity analysis varies one or two parameters at a time around a base case to gauge its impact, while scenario analysis changes multiple variables simultaneously to construct coherent future worlds.

See also the lecture for more details.