Forelesning 11

Vi kan finne de korteste veiene fra hver node etter tur, men mange av delproblemene vil overlappe – om vi har mange nok kanter lønner det seg å bruke dynamisk programmering med dekomponeringen «Skal vi innom k eller ikke?»

Pensum

☐ Kap. 25. All-pairs shortest paths: Innledning, 25.2 og 25.3

Læringsmål

[K$_1$] Forstå *forgjengerstrukturen* for *alle-til-alle*-varianten av korteste vei-problemet

Operasjoner: PRINT-ALL-PAIRS-SHORTEST-PATH

[K$_2$] Forstå FLOYD-WARSHALL

[K$_3$] Forstå TRANSITIVE-CLOSURE

[K$_4$] Forstå JOHNSON

Forelesningen filmes

Forelesning 11

Korteste vei fra alle til alle

1. **Johnsons algoritme**

2. **Transitiv lukning**

3. **Floyd-Warshall**

1:3

Johnsons algoritme

Fra 1977.

Efficient Algorithms for Shortest Paths in Sparse Networks

DONALD B. JOHNSON

The Pennsylvania State University, University Park, Pennsylvania

ABSTRACT   Algorithms for finding shortest paths are presented which are faster than algorithms previously known on networks which are relatively sparse in arcs Known results which the results of this paper extend are surveyed briefly and analyzed A new implementation for priority queues is employed, and a class of "arc set partition" algorithms is introduced For the single source problem on networks with nonnegative arcs a running

Input: En vektet, rettet graf $G = (V, E)$ uten negative sykler, der $V = \{1, \ldots, n\}$, og vektene er gitt av matrisen $W = (w_{ij})$.

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Output: En $n \times n$-matrise D = (d_{ij}) med avstander, dvs., $d_{ij} = \delta(i, j)$.

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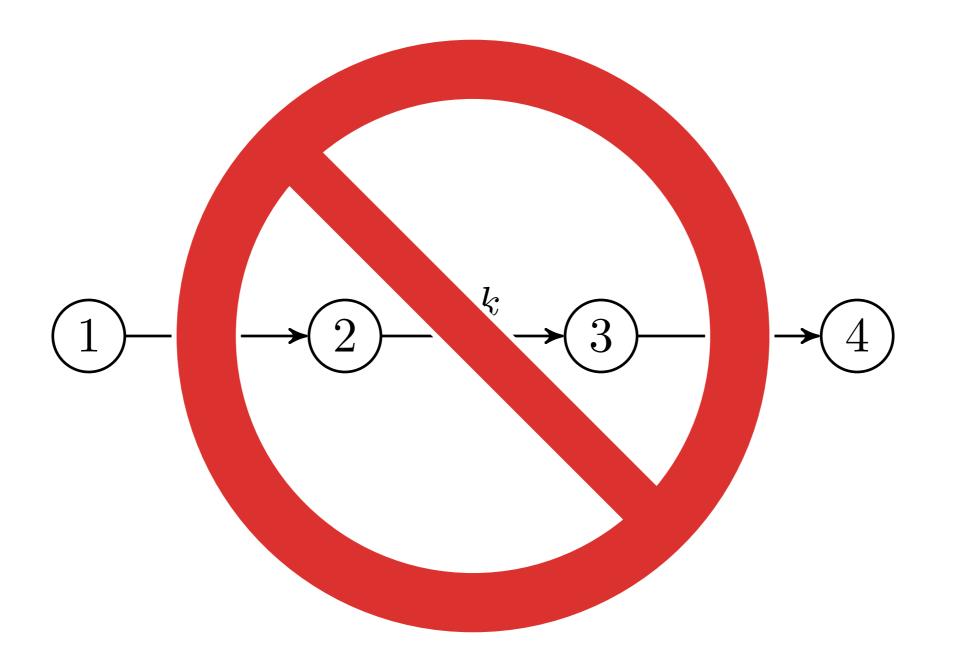
Vi returnerer også en forgjengermatrise $\Pi = (\pi_{ij})$

For spinkle grafer: Dijkstra fra hver node!

Men ... hva om vi har negative kanter?
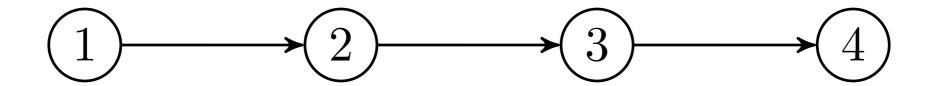
Fast økning: Stier med mange kanter taper på det

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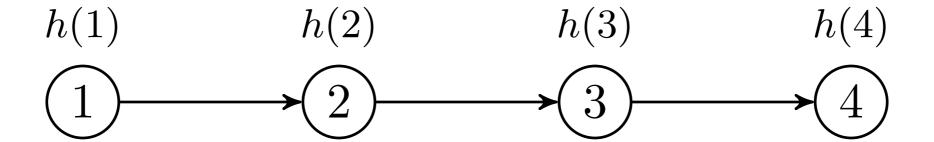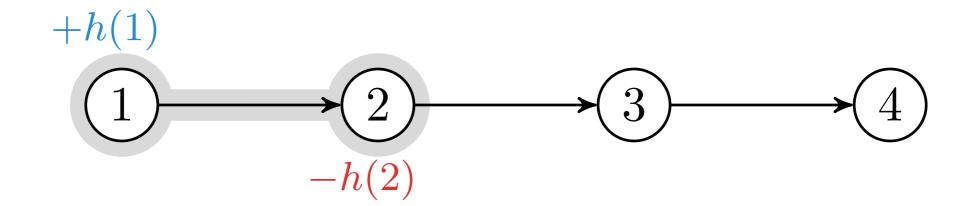
Vi kan tillate oss en <u>teleskopsum</u> ...
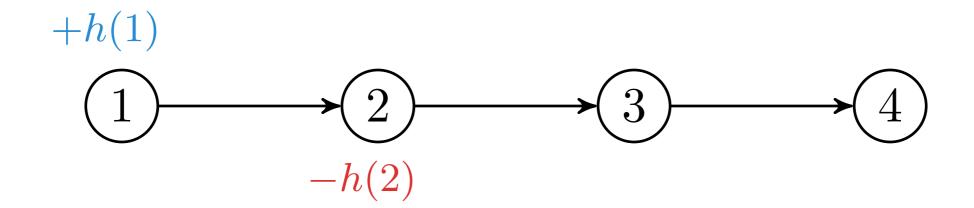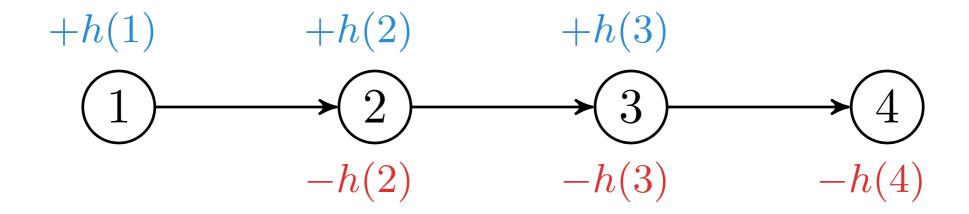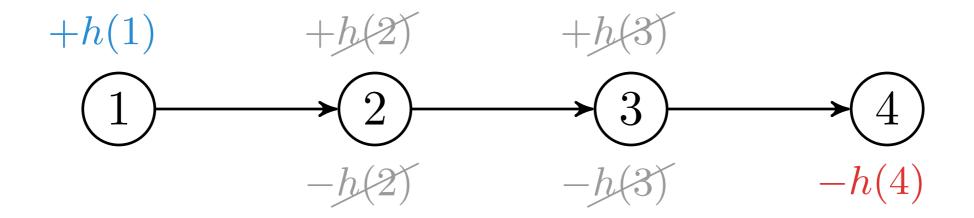
$h(1)$ $h(2)$ $h(3)$ $h(4)$

$1 \rightarrow 2 \rightarrow 3 \rightarrow 4$

Hver node tilordnes en verdi

Vekten $w(u, v)$ økes med differansen $h(u) - h(v)$

$+h(1)$

$-h(2)$

Vekten $w(u, v)$ økes med <u>differansen</u> $h(u) - h(v)$

$+h(1)$

$-h(2)$

Positive og negative ledd opphever hverandre...

Positive og negative ledd opphever hverandre . . .

Positive og negative ledd opphever hverandre...

$+h(1)$

$$1 \longrightarrow 2 \longrightarrow 3 \longrightarrow 4$$

$-h(4)$

Unntatt første og siste ledd

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$+h(1)$

$-h(4)$
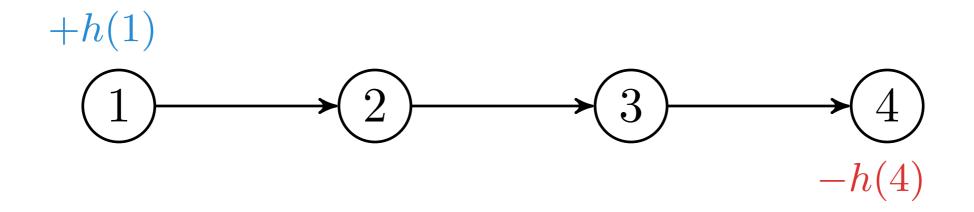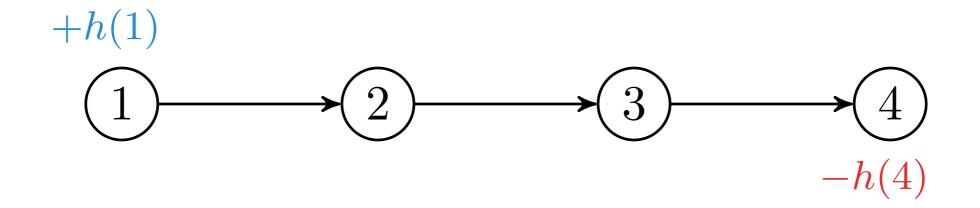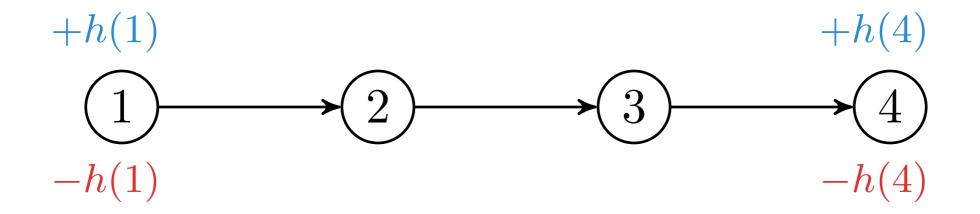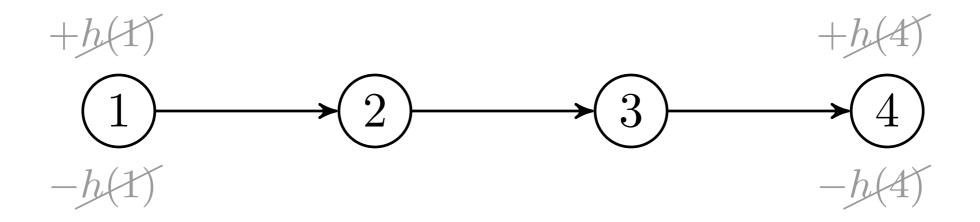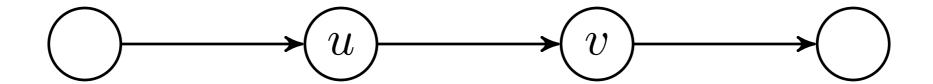
Men disse deles av alle stier mellom disse nodene!

Rekonstruér lengden ved å trekke fra differansen deres

$+h(1)$ $+h(4)$

(1) → (2) → (3) → (4)

$-h(1)$ $-h(4)$

Rekonstruér lengden ved å trekke fra differansen deres

$+h(1)$

$-h(1)$

$+h(4)$

$-h(4)$

Rekonstruér lengden ved å trekke fra differansen deres
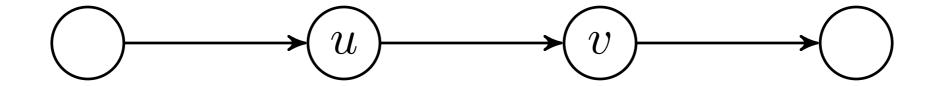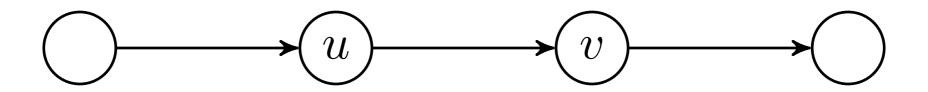
Hvordan sikrer vi ikke-negative vekter?
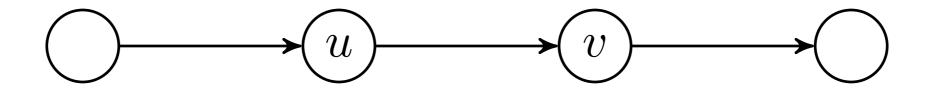
Vi må ha $w(u, v) + h(u) - h(v) \geqslant 0$, dvs. $w(u, v) + h(u) \geqslant h(v)$
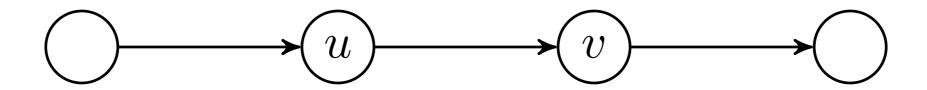
$$w(u,v) + h(u) \geqslant h(v)$$

Vi må ha $w(u,v) + h(u) - h(v) \geqslant 0$, dvs. $w(u,v) + h(u) \geqslant h(v)$

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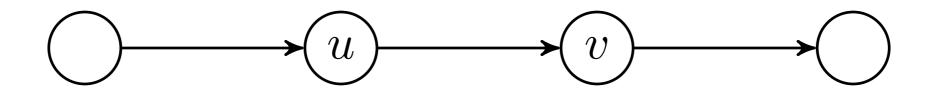
Fra én-til-alle: $\delta(s, v) \leqslant \delta(s, u) + w(u, v)$. Kan la $h(v) = \delta(s, v)$!
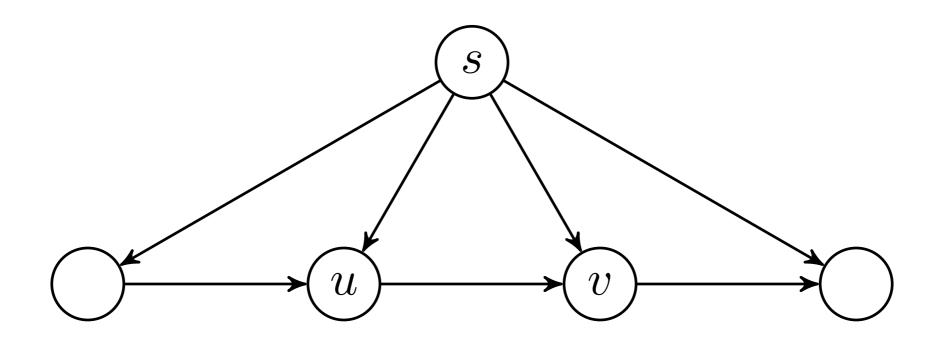
$$w(u,v) + h(u) \geqslant h(v)$$

$$w(u,v) + \delta(s,u) \geqslant \delta(s,v)$$

Men hva er s? Vi må sikre at vi når alle...

$$w(u,v) + h(u) \geqslant h(v)$$

$$w(u,v) + \delta(s,u) \geqslant \delta(s,v)$$

Vi kan legge til en ny node!

$$w(u, v) + h(u) \geqslant h(v)$$

$$w(u, v) + \delta(s, u) \geqslant \delta(s, v)$$
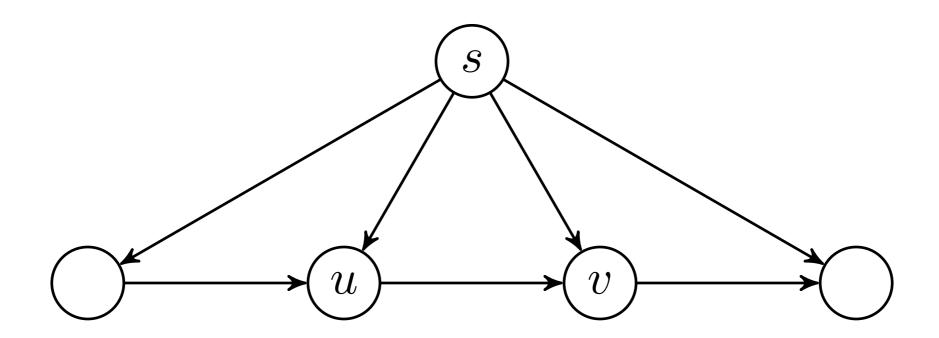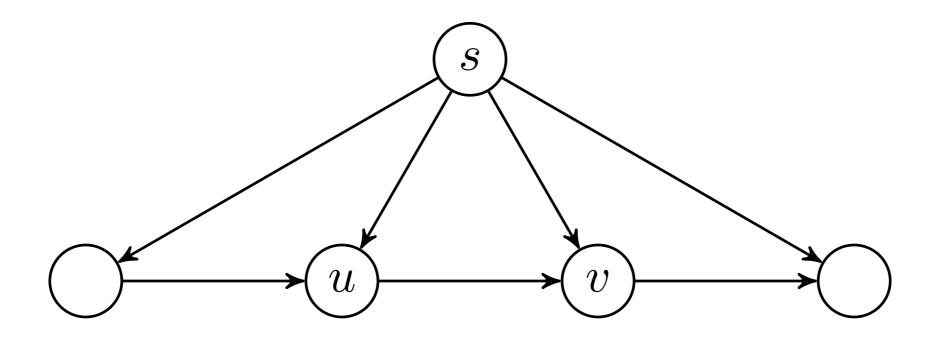
Vi kan legge til en ny node!

$$w(u,v) + h(u) \geqslant h(v)$$

$$w(u,v) + \delta(s,u) \geqslant \delta(s,v)$$

Kantene fra s kan f.eks. få vekt 0

$$w(u, v) + h(u) \geqslant h(v)$$

$$w(u, v) + \delta(s, u) \geqslant \delta(s, v)$$

(Merk: Vi verken innfører eller fjerner negative sykler)

Forenkling: Antar at vi ikke har negative sykler, heller enn å sjekke for det – dvs., jeg ignorerer returverdien fra Bellman-Ford. Vil normalt avbryte om den er false (som de gjør i boka).

$$\textsc{Johnson}(\textrm{G}, w)$$

G graf
w vekting

Alle til alle. Tillater negative vekter

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Johnson(G, w)
1 construct G' with start node s

Skal brukes til forarbeide med Bellman-Ford

Johnson(G, w)
1 construct G$'$ with start node s
2 Bellman-Ford(G$'$, w, s)

G	graf
w	vekting

Forarbeide: Gir oss info vi trenger til h

Johnson(G, w)
  1 construct G' with start node s
  2 Bellman-Ford(G', w, s)
  3 **for** each vertex $v \in G.V$

G	graf
w	vekting
v	node

(Cormen et al. bruker her G', men s er overflødig)

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Johnson(G, w)
  1 construct G$'$ with start node s
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  3 **for** each vertex $v \in$ G.V
  4      $h(v) = v.d$

G	graf
w	vekting
v	node
h	$\delta(s, v)$

Her er $v.d = \delta(s, v)$, beregnet av Bellman-Ford

Johnson(G, w)
1   construct G′ with start node s
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3   **for** each vertex $v \in$ G.V
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5   **for** each edge $(u, v) \in$ G.E

G	graf
w	vekting
u	node
v	node
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(Cormen et al. bruker her G′, men s er overflødig)

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G	graf
w	vekting
u	node
v	node
h	$\delta(s, v)$
$\hat{w}$	ny vekting

$$h(v) \leqslant h(u) + w(u, v) \implies \hat{w}(u, v) \geqslant 0$$

Johnson(G, w)

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7   let D $= (d_{uv})$ be a new $n \times n$ matrix

G	graf
w	vekting
u	node
v	node
h	$\delta(s, v)$
$\hat{w}$	ny vekting
d_{uv}	$\delta(u, v)$

Resultat: Alle til alle med Dijkstra

$\textsc{Johnson}(\text{G}, w)$
1   construct G' with start node s
2   $\textsc{Bellman-Ford}(\text{G}', w, s)$
3   **for** each vertex $v \in \text{G.V}$
4       $h(v) = v.d$
5   **for** each edge $(u, v) \in \text{G.E}$
6       $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$
7   let $\text{D} = (d_{uv})$ be a new $n \times n$ matrix
8   **for** each vertex $u \in \text{G.V}$

G	graf
w	vekting
u	node
v	node
h	$\delta(s, v)$
$\hat{w}$	ny vekting
d_{uv}	$\delta(u, v)$

For hver u, finn $\delta(u, \cdot)$ for $\hat{w}$ med $\textsc{Dijkstra}$

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7 let $D = (d_{uv})$ be a new $n \times n$ matrix

8 **for** each vertex $u \in G.V$

9   Dijkstra$(G, \hat{w}, u)$

G	graf
w	vekting
u	node
v	node
h	$\delta(s, v)$
$\hat{w}$	ny vekting
d_{uv}	$\delta(u, v)$

Lovlig, fordi $\hat{w}$ er ikke-negativ

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8 **for** each vertex $u \in$ G.V
9   DIJKSTRA(G, $\hat{w}$, u)
10   **for** each vertex $v \in$ G.V

G	graf
w	vekting
u	node
v	node
h	$\delta(s, v)$
$\hat{w}$	ny vekting
d_{uv}	$\delta(u, v)$

For hver v, finn $d_{uv} = \delta(u, v)$ for w fra $\delta(u, v)$ for $\hat{w}$

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v	node
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$\hat{w}$	ny vekting
d_{uv}	$\delta(u, v)$

Teleskopsum med $-h(x) + h(x)$ for x mellom u og v

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10   **for** each vertex $v \in \mathrm{G.V}$

11          $d_{uv} = v.d + h(v) - h(u)$

12 **return** D

G	graf
w	vekting
u	node
v	node
h	$\delta(s, v)$
$\hat{w}$	ny vekting
d_{uv}	$\delta(u, v)$

Rett fordi $w(p) \leqslant w(q) \iff \hat{w}(p) \leqslant \hat{w}(q)$ for stier $u \overset{p,q}{\rightsquigarrow} v$
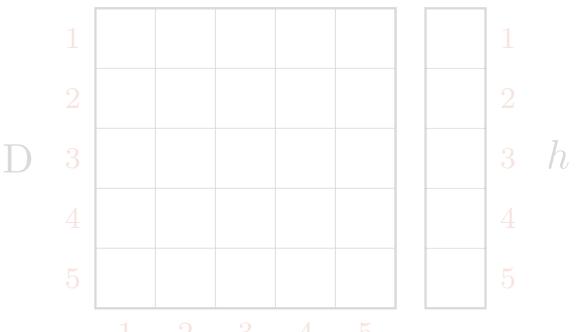
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Merk: s er ikke tegnet inn

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JOHNSON(G, w)

1   construct G$'$ with start node s

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7   let D $= (d_{uv})$ be a new $n \times n$ matrix

8   **for** each vertex $u \in$ G.V

9   DIJKSTRA(G, $\hat{w}$, u)

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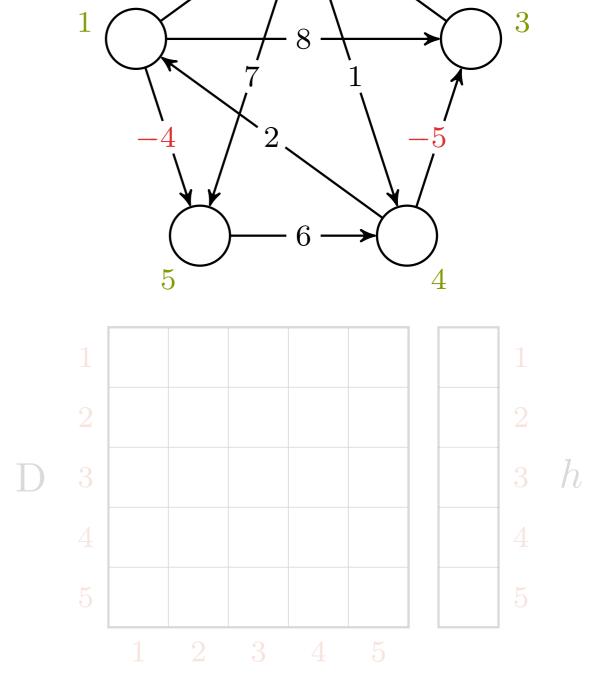11 $d_{uv} = v.d + h(v) - h(u)$
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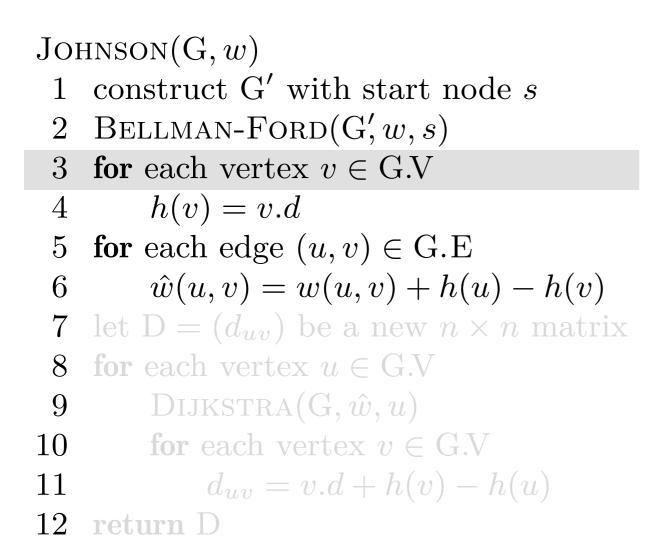11 $d_{uv} = v.d + h(v) - h(u)$
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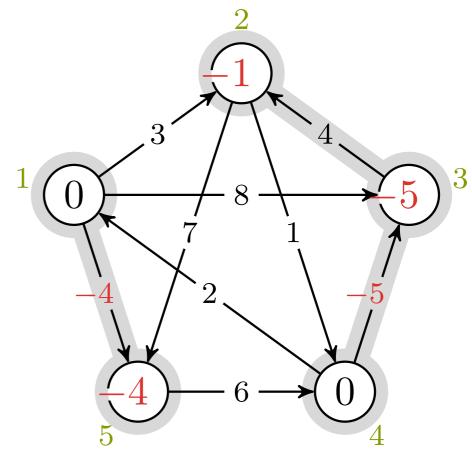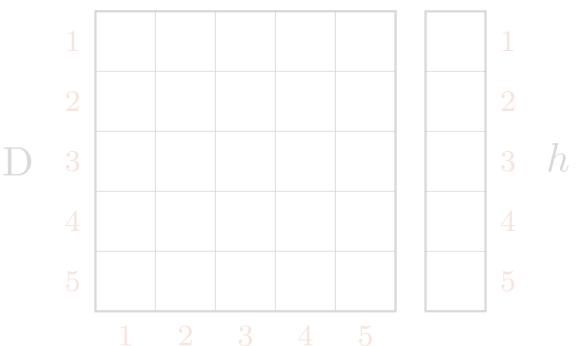
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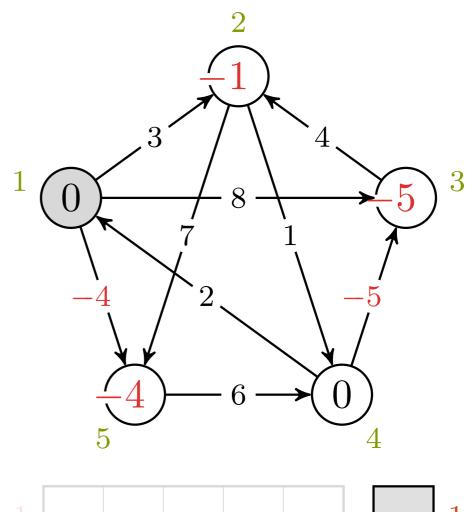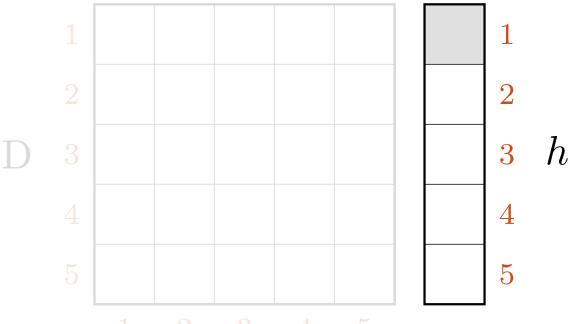7   let D $= (d_{uv})$ be a new $n \times n$ matrix
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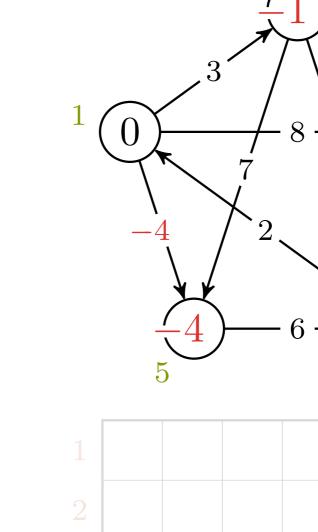9   DIJKSTRA(G, $\hat{w}, u$)
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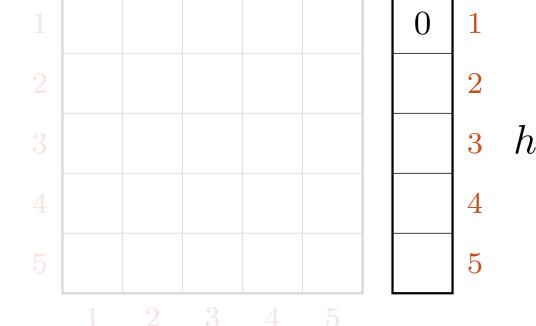10   **for** each vertex $v \in G.V$
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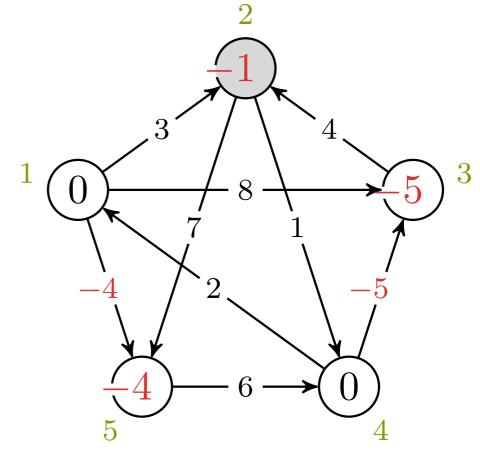7 let $D = (d_{uv})$ be a new $n \times n$ matrix
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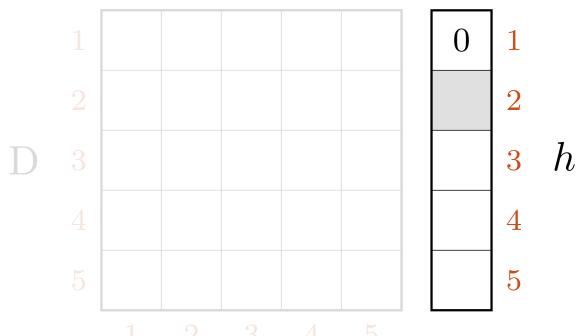 8   **for** each vertex $u \in G.V$
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h

	1
0	1
−1	2
−5	3
0	4
−4	5

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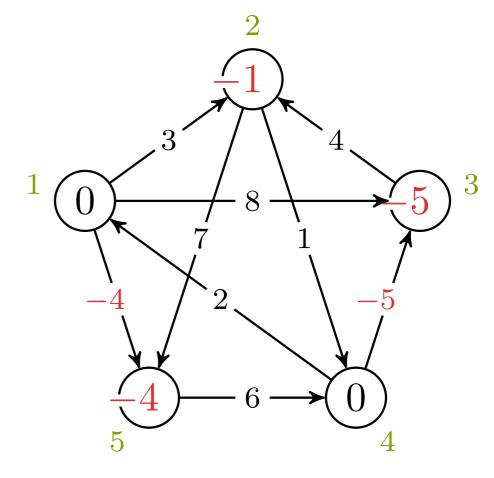7 let D $= (d_{uv})$ be a new $n \times n$ matrix
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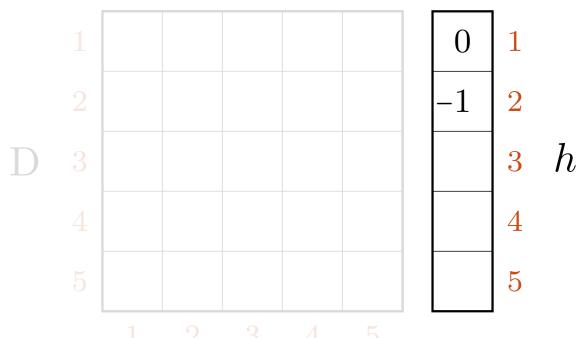7   let D $= (d_{uv})$ be a new $n \times n$ matrix
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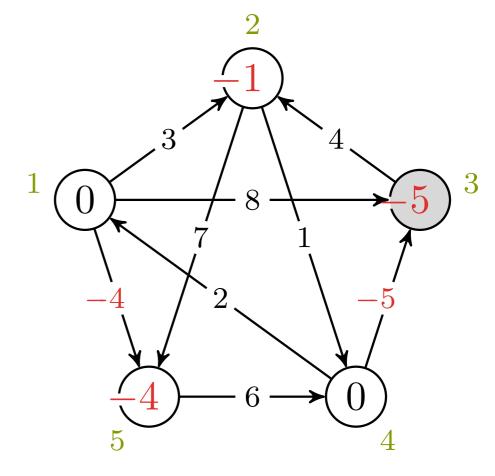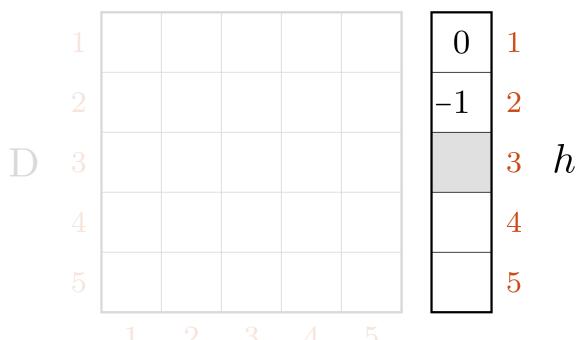7   let D $= (d_{uv})$ be a new $n \times n$ matrix
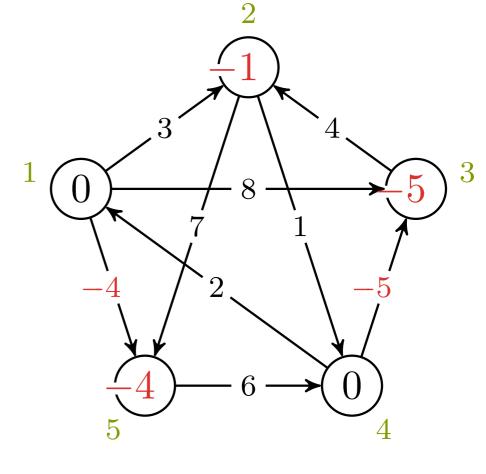8   **for** each vertex $u \in$ G.V
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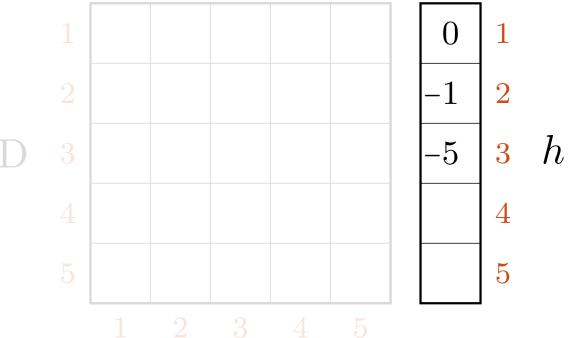10   **for** each vertex $v \in$ G.V
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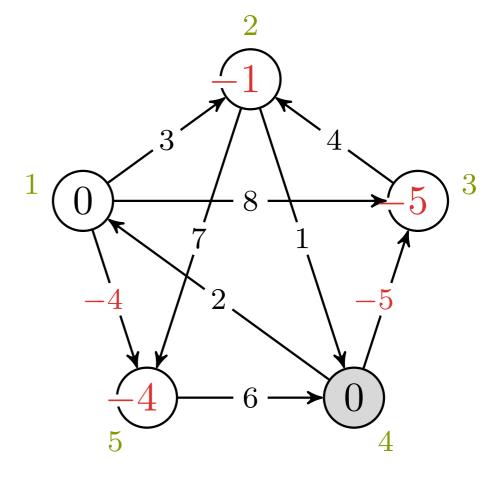7 let $D = (d_{uv})$ be a new $n \times n$ matrix
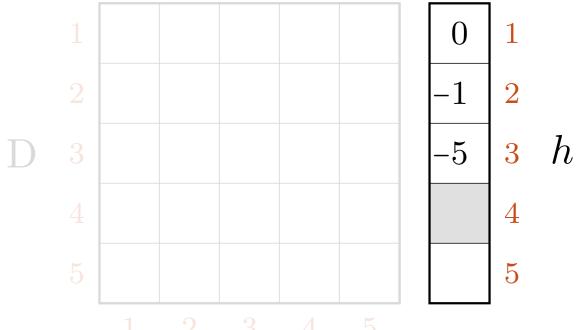8 **for** each vertex $u \in G.V$
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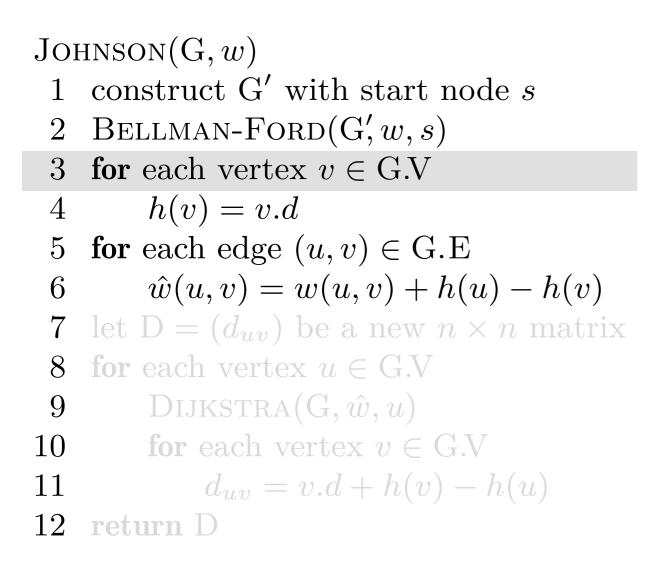8 **for** each vertex $u \in G.V$
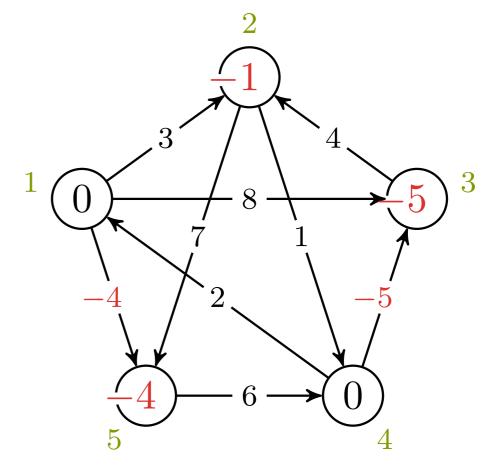9 $\text{Dijkstra}(G, \hat{w}, u)$
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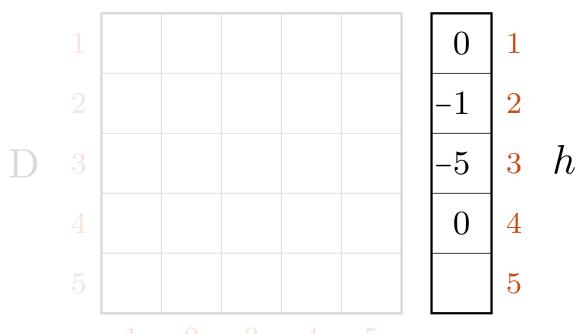10   **for** each vertex $v \in G.V$
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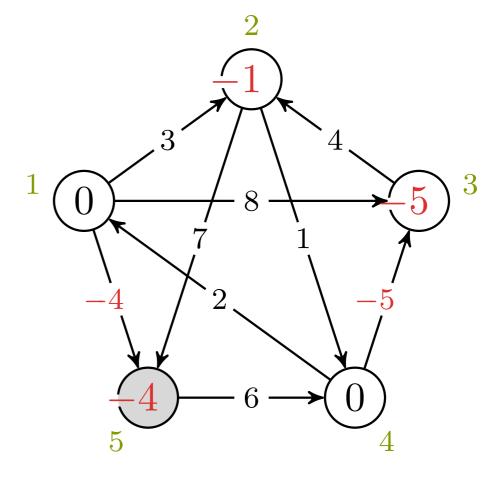7 let D $= (d_{uv})$ be a new $n \times n$ matrix
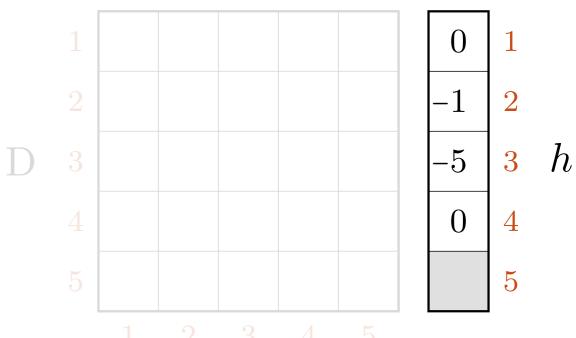8 **for** each vertex $u \in$ G.V
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D

	1	2	3	4	5
1	0	1			
2					
3					
4					
5					

h

1	0
2	−1
3	−5
4	0
5	−4

mlh@ntnu.no

Johnson(G, w)
1 construct G′ with start node s
2 Bellman-Ford(G′, w, s)
3 for each vertex $v \in$ G.V
4      $h(v) = v.d$
5 for each edge $(u, v) \in$ G.E
6      $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$
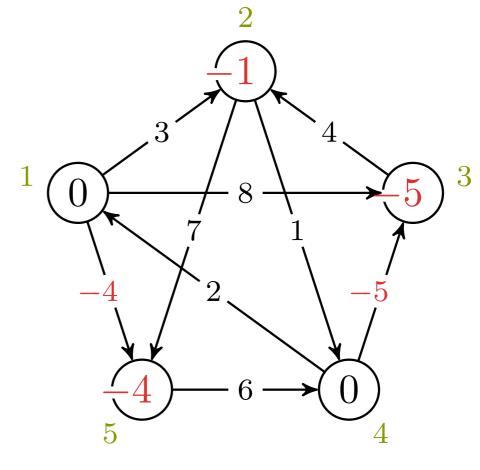7 let D = (d_{uv}) be a new $n \times n$ matrix
8 **for** each vertex $u \in$ G.V
9   Dijkstra(G, $\hat{w}$, u)
10   **for** each vertex $v \in$ G.V
11         $d_{uv} = v.d + h(v) - h(u)$
12 **return** D

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$\text{Johnson}(G, w)$
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7   let $D = (d_{uv})$ be a new $n \times n$ matrix
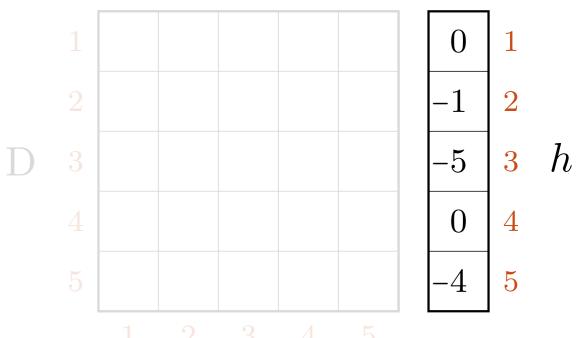8   **for** each vertex $u \in G.V$
9       $\text{Dijkstra}(G, \hat{w}, u)$
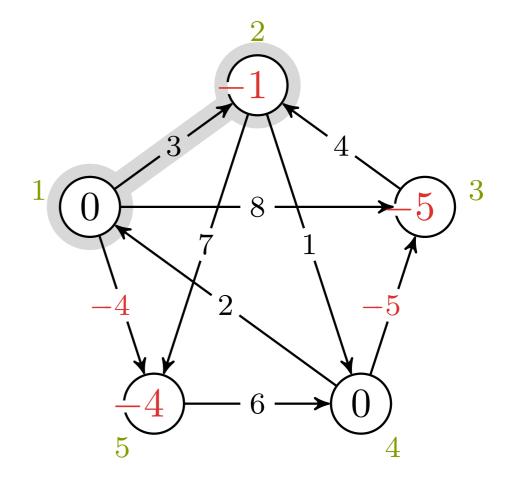10   **for** each vertex $v \in G.V$
11              $d_{uv} = v.d + h(v) - h(u)$
12 **return** D

	1	2	3	4	5
1	0	1	-3		
2					
3					
4					
5					

D

0	1
-1	2
-5	3
0	4
-4	5

h

$\text{JOHNSON}(G, w)$
1 construct G' with start node s
2 $\text{BELLMAN-FORD}(G', w, s)$
3 **for** each vertex $v \in G.V$
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8 **for** each vertex $u \in G.V$
9      $\text{DIJKSTRA}(G, \hat{w}, u)$
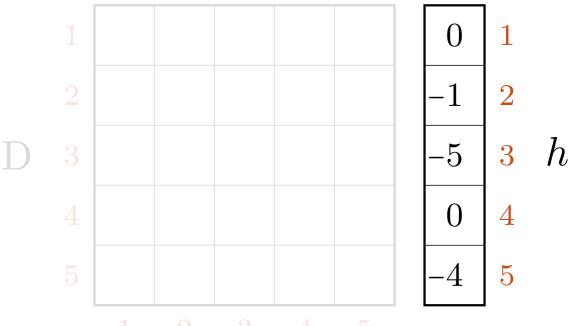10   **for** each vertex $v \in G.V$
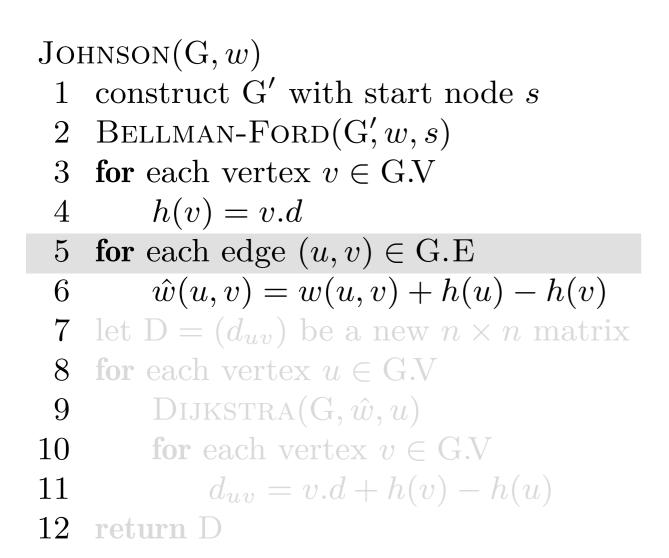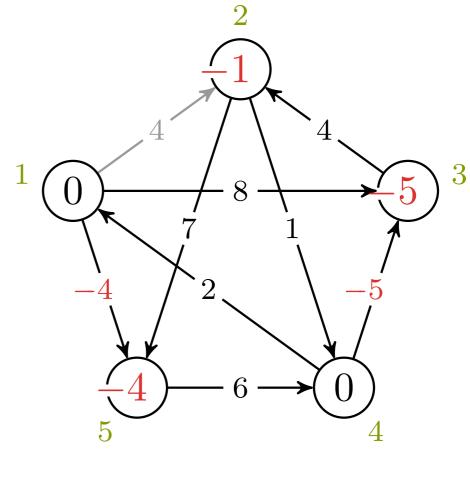11 $d_{uv} = v.d + h(v) - h(u)$
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D

	1	2	3	4	5
1	0	1	-3		
2					
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5					

	h
0	1
-1	2
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0	4
-4	5

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1 construct G' with start node s
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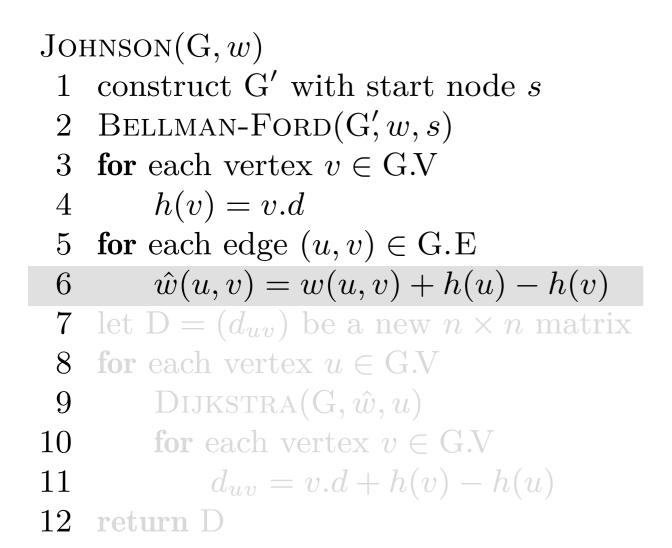5 **for** each edge $(u, v) \in G.E$
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12 **return** D

D

	1	2	3	4	5
1	0	1	-3	2	
2					
3					
4					
5					

0	1
-1	2
-5	3
0	4
-4	5

h

$\textsc{Johnson}(\textrm{G}, w)$
1 construct $\textrm{G}'$ with start node s
2 $\textsc{Bellman-Ford}(\textrm{G}', w, s)$
3 **for** each vertex $v \in \textrm{G.V}$
4      $h(v) = v.d$
5 **for** each edge $(u, v) \in \textrm{G.E}$
6      $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$
7 let $\textrm{D} = (d_{uv})$ be a new $n \times n$ matrix
8 **for** each vertex $u \in \textrm{G.V}$
9      $\textsc{Dijkstra}(\textrm{G}, \hat{w}, u)$
10   **for** each vertex $v \in \textrm{G.V}$
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	1	2	3	4	5
1	0	1	-3	2	
2					
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4					
5					

D

h	
0	1
-1	2
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0	4
-4	5

JOHNSON(G, w)
1 construct G′ with start node s
2 BELLMAN-FORD(G′, w, s)
3 for each vertex $v \in$ G.V
4     $h(v) = v.d$
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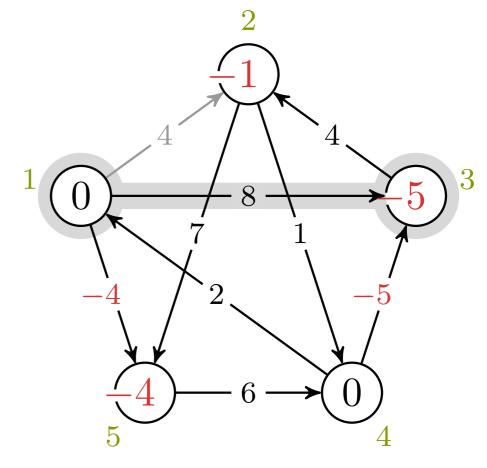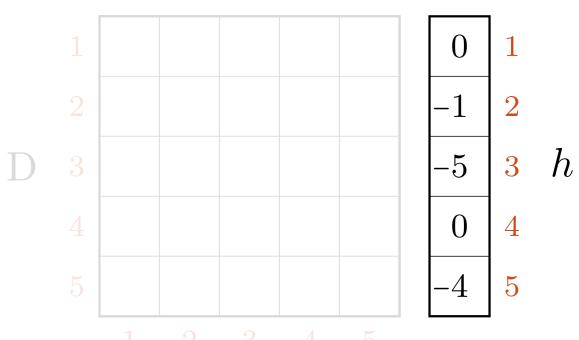6 $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$
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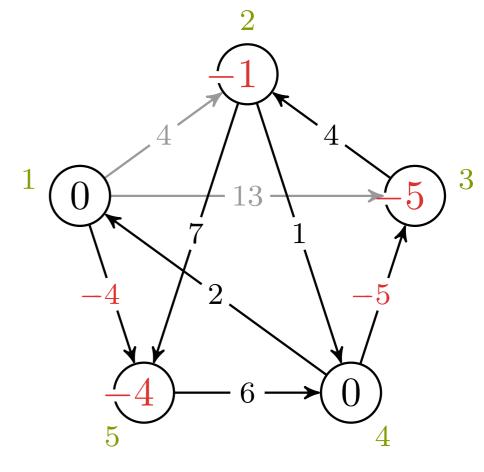7   let D $= (d_{uv})$ be a new $n \times n$ matrix
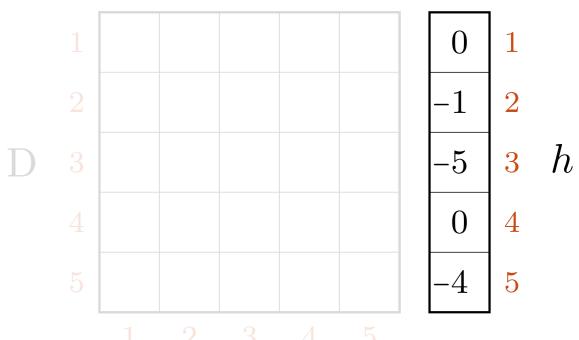8   **for** each vertex $u \in$ G.V
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	1	2	3	4	5
1	0	1	-3	2	-4
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0	1
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Johnson(G, w)

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8 **for** each vertex $u \in$ G.V
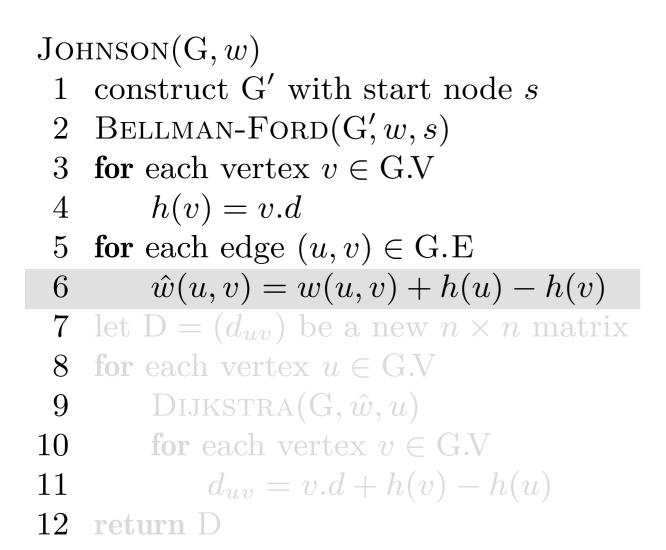9   Dijkstra(G, $\hat{w}, u$)
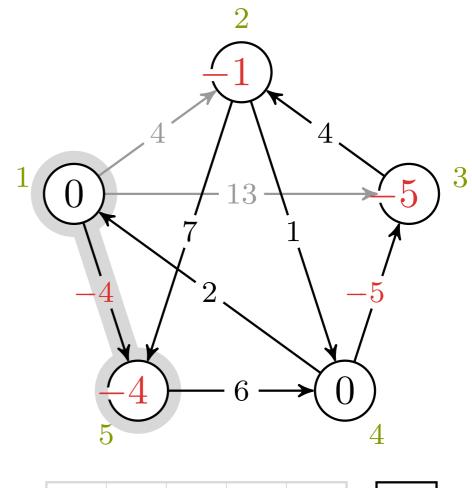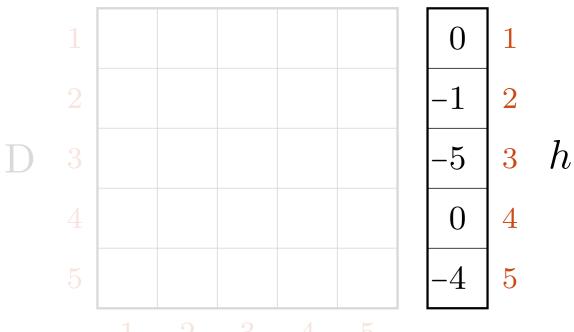10   **for** each vertex $v \in$ G.V
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	1	2	3	4	5
1	0	1	-3	2	-4
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h	
0	1
-1	2
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Johnson(G, w)
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h

0	1
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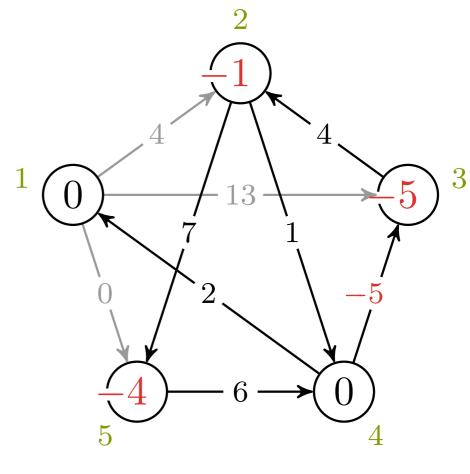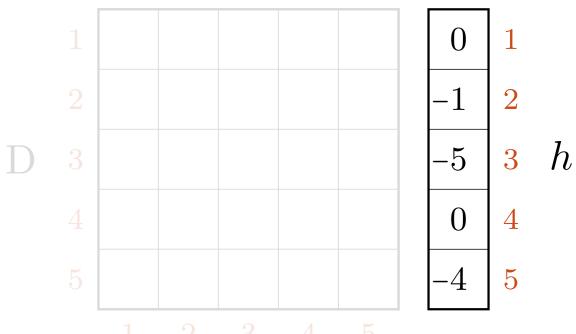 7 let D $= (d_{uv})$ be a new $n \times n$ matrix
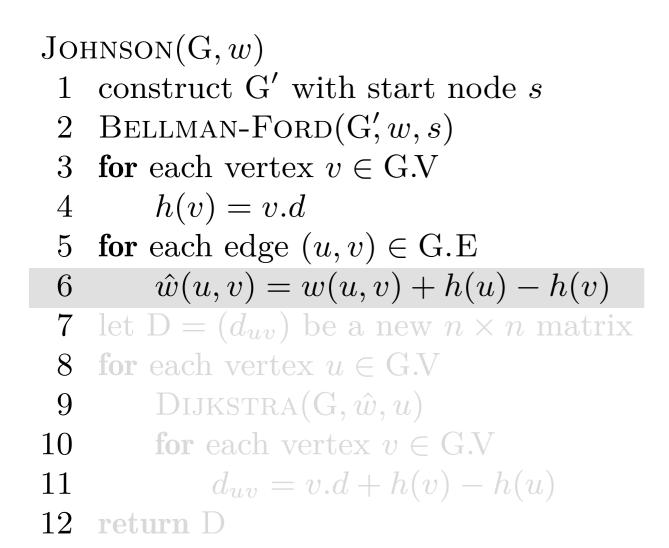 8 **for** each vertex $u \in$ G.V
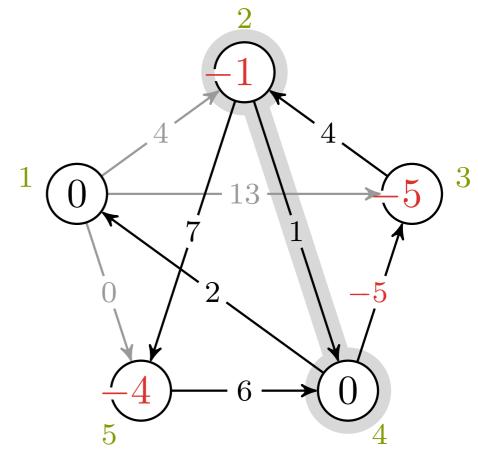 9   DIJKSTRA(G, $\hat{w}$, u)
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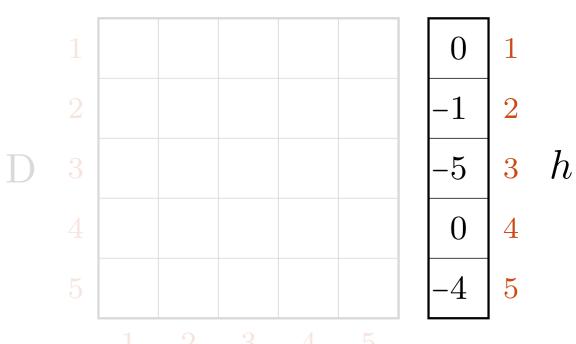11 $d_{uv} = v.d + h(v) - h(u)$
12 **return** D

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3				
3					
4					
5					

h

0	1
-1	2
-5	3
0	4
-4	5

94

$\textsc{Johnson}(G, w)$
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1	0	1	-3	2	-4
2	3				
3					
4					
5					

0	1
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-4	5

h

95

$\textsc{Johnson}(\text{G}, w)$
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	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0			
3					
4					
5					

h	
0	1
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-5	3
0	4
-4	5

$\text{J{\small OHNSON}}(G, w)$
1   construct G' with start node s
2   $\text{B{\small ELLMAN}-F{\small ORD}}(G', w, s)$
3   **for** each vertex $v \in G.V$
4       $h(v) = v.d$
5   **for** each edge $(u, v) \in G.E$
6       $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$
7   let $D = (d_{uv})$ be a new $n \times n$ matrix
8   **for** each vertex $u \in G.V$
9       $\text{D{\small IJKSTRA}}(G, \hat{w}, u)$
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1	0	1	-3	2	-4
2	3	0			
3					
4					
5					

D

h	
0	1
-1	2
-5	3
0	4
-4	5

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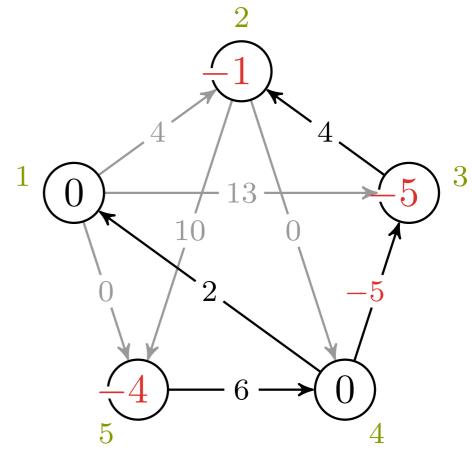7 let D $= (d_{uv})$ be a new $n \times n$ matrix
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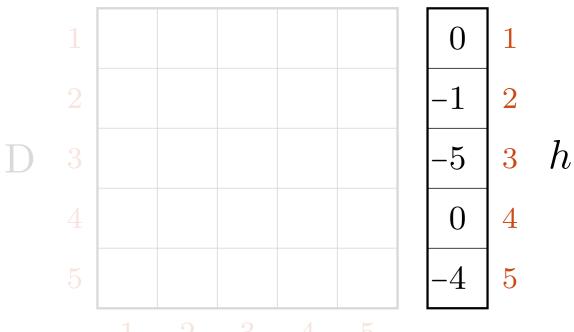8   **for** each vertex $u \in G.V$
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D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4		
3					
4					
5					

0	1
-1	2
-5	3
0	4
-4	5

h

99

$\textsc{Johnson}(G, w)$
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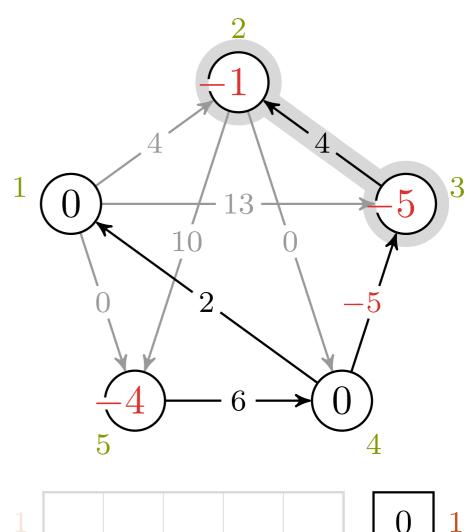7 let $D = (d_{uv})$ be a new $n \times n$ matrix
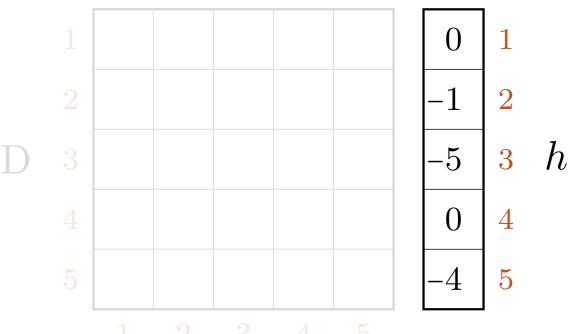8 **for** each vertex $u \in G.V$
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D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	
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4					
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h

0	1
-1	2
-5	3
0	4
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$\textsc{Johnson}(G, w)$
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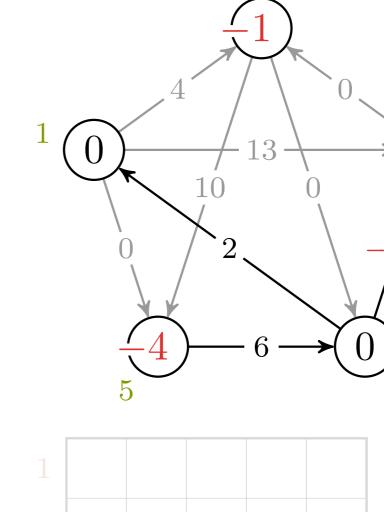 8 **for** each vertex $u \in G.V$
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D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	
3					
4					
5					

h

0	**1**
-1	**2**
-5	**3**
0	**4**
-4	**5**

JOHNSON(G, w)

1 construct G′ with start node s
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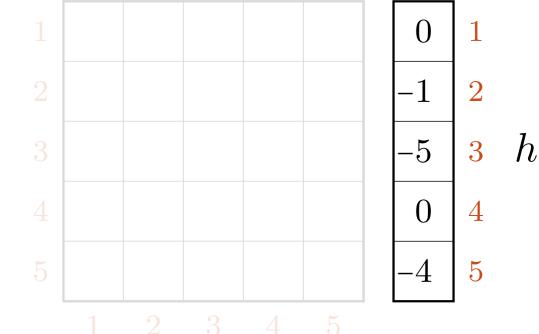10   for each vertex $v \in$ G.V
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	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3					
4					
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h	
0	1
-1	2
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0	4
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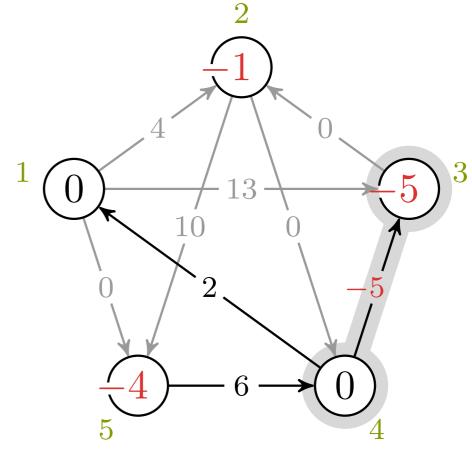7 let $D = (d_{uv})$ be a new $n \times n$ matrix
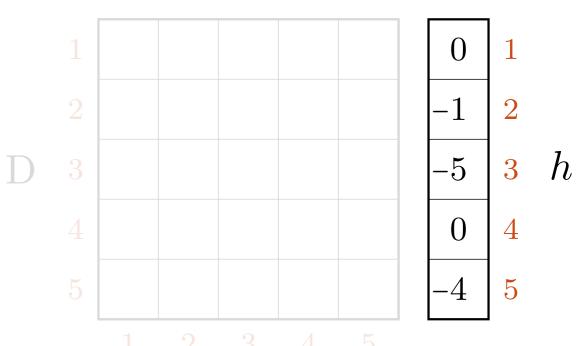8 **for** each vertex $u \in G.V$
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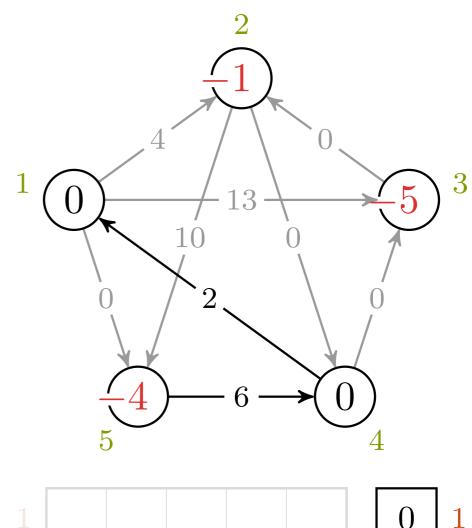 7   let $D = (d_{uv})$ be a new $n \times n$ matrix
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 5 for each edge $(u, v) \in$ G.E
 6      $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$
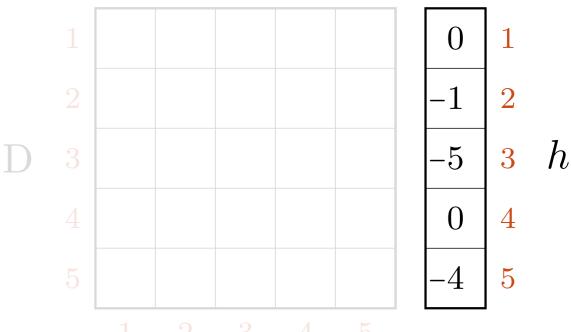 7 let D $= (d_{uv})$ be a new $n \times n$ matrix
 8 for each vertex $u \in$ G.V
 9   DIJKSTRA(G, $\hat{w}$, u)
10   for each vertex $v \in$ G.V
11          $d_{uv} = v.d + h(v) - h(u)$
12 return D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3					
4					
5					

D

h	
0	1
-1	2
-5	3
0	4
-4	5

JOHNSON(G, w)
1   construct G′ with start node s
2   BELLMAN-FORD(G′, w, s)
3   **for** each vertex $v \in$ G.V
4       $h(v) = v.d$
5   **for** each edge $(u, v) \in$ G.E
6       $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$
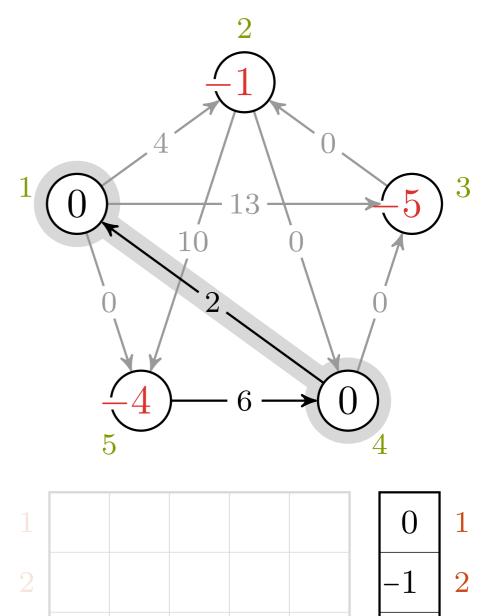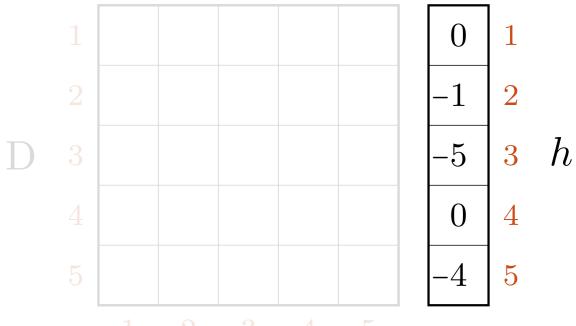7   let D $= (d_{uv})$ be a new $n \times n$ matrix
8   **for** each vertex $u \in$ G.V
9   DIJKSTRA(G, $\hat{w}, u$)
10   **for** each vertex $v \in$ G.V
11          $d_{uv} = v.d + h(v) - h(u)$
12 **return** D

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7				
4					
5					

h	
0	1
-1	2
-5	3
0	4
-4	5

106

JOHNSON(G, w)
1   construct G′ with start node s
2   BELLMAN-FORD(G′, w, s)
3   for each vertex $v \in$ G.V
4      $h(v) = v.d$
5   for each edge $(u, v) \in$ G.E
6      $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$
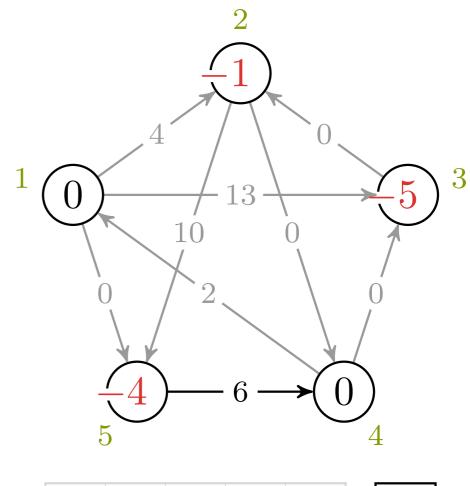7   let D = (d_{uv}) be a new $n \times n$ matrix
8   **for** each vertex $u \in$ G.V
9   DIJKSTRA(G, $\hat{w}$, u)
10   **for** each vertex $v \in$ G.V
11       $d_{uv} = v.d + h(v) - h(u)$
12 **return** D

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7				
4					
5					

h

0	1
-1	2
-5	3
0	4
-4	5

107

JOHNSON(G, w)
1   construct G′ with start node s
2   BELLMAN-FORD(G′, w, s)
3   **for** each vertex $v \in$ G.V
4       $h(v) = v.d$
5   **for** each edge $(u, v) \in$ G.E
6       $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$
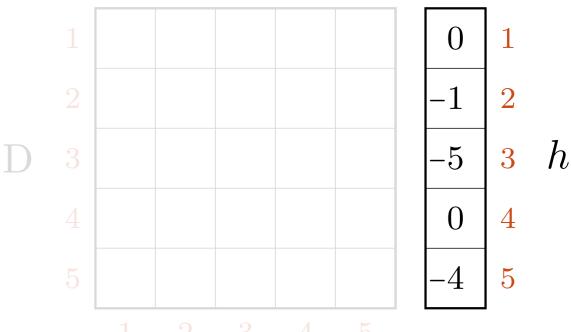7   let D $= (d_{uv})$ be a new $n \times n$ matrix
8   **for** each vertex $u \in$ G.V
9   DIJKSTRA(G, $\hat{w}, u$)
10   **for** each vertex $v \in$ G.V
11              $d_{uv} = v.d + h(v) - h(u)$
12 **return** D

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4			
4					
5					

h

0	1
-1	2
-5	3
0	4
-4	5

108

JOHNSON(G, w)
1 construct G′ with start node s
2 BELLMAN-FORD(G′, w, s)
3 **for** each vertex $v \in$ G.V
4     $h(v) = v.d$
5 **for** each edge $(u, v) \in$ G.E
6     $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$
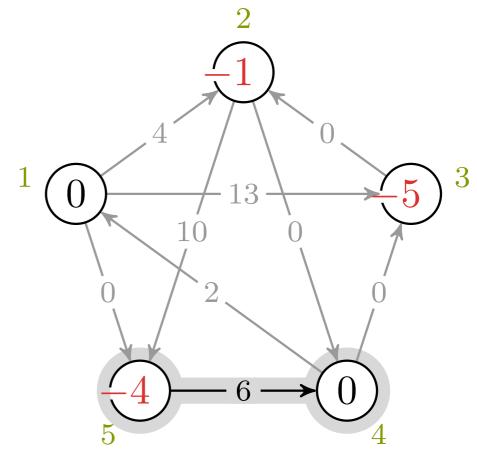7 let D $= (d_{uv})$ be a new $n \times n$ matrix
8 **for** each vertex $u \in$ G.V
9   DIJKSTRA(G, $\hat{w}$, u)
10   **for** each vertex $v \in$ G.V
11       $d_{uv} = v.d + h(v) - h(u)$
12 **return** D

$\textsc{Johnson}(G, w)$

1 construct G' with start node s

2 $\textsc{Bellman-Ford}(G', w, s)$

3 **for** each vertex $v \in G.V$

4      $h(v) = v.d$

5 **for** each edge $(u, v) \in G.E$

6      $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$

7 let $D = (d_{uv})$ be a new $n \times n$ matrix
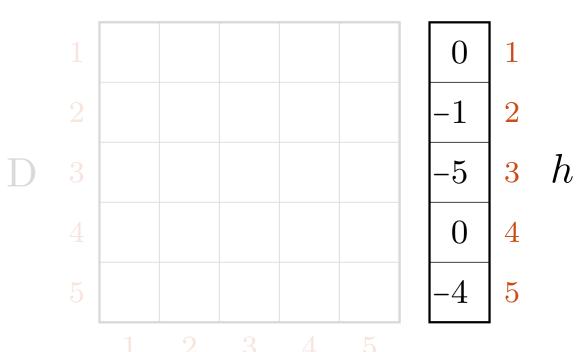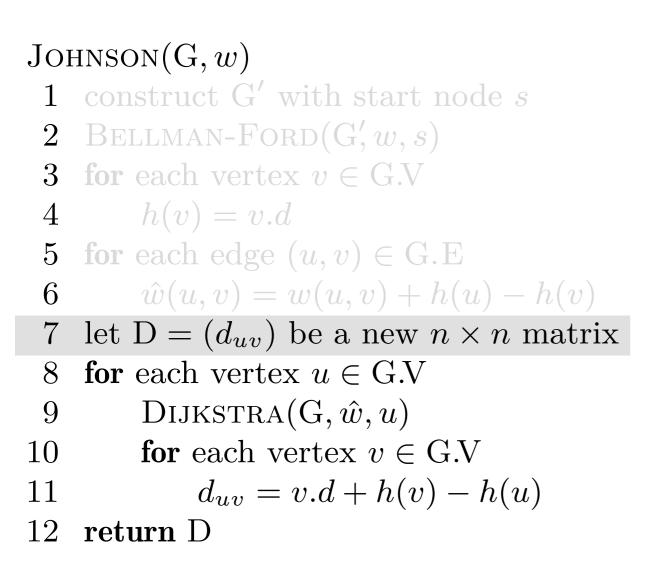
8 **for** each vertex $u \in G.V$

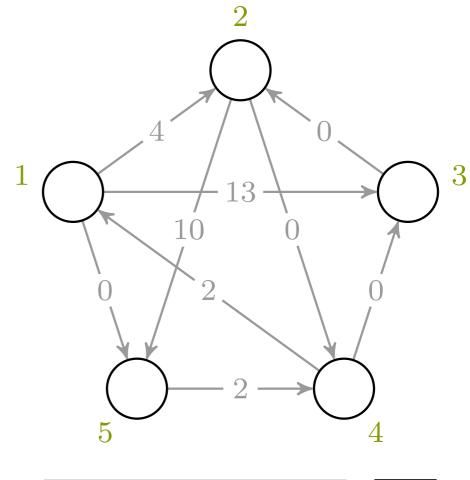9 $\textsc{Dijkstra}(G, \hat{w}, u)$

10   **for** each vertex $v \in G.V$

11         $d_{uv} = v.d + h(v) - h(u)$

12 **return** D

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0		
4					
5					

h

0	1
-1	2
-5	3
0	4
-4	5

110

JOHNSON(G, w)
 1 construct G′ with start node s
 2 BELLMAN-FORD(G′, w, s)
 3 **for** each vertex $v \in$ G.V
 4      $h(v) = v.d$
 5 **for** each edge $(u, v) \in$ G.E
 6      $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$
 7 let D = (d_{uv}) be a new $n \times n$ matrix
 8 **for** each vertex $u \in$ G.V
 9   DIJKSTRA(G, $\hat{w}$, u)
10   **for** each vertex $v \in$ G.V
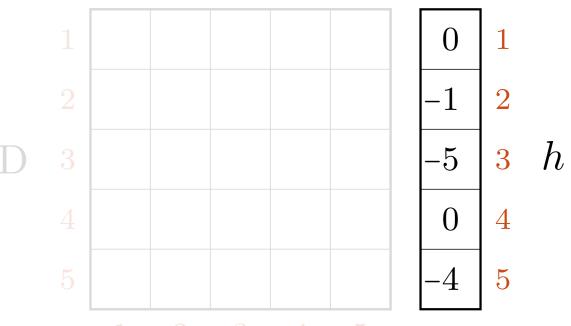11 $d_{uv} = v.d + h(v) - h(u)$
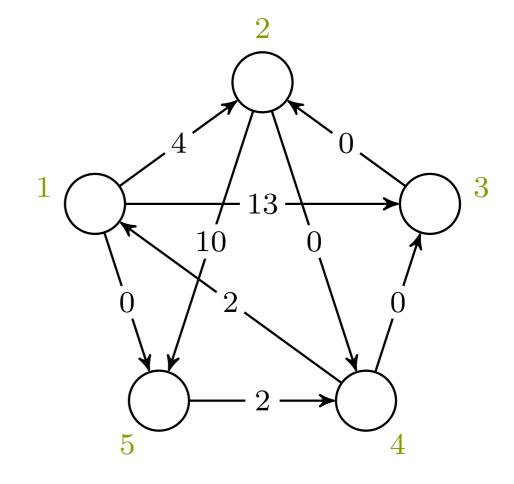12 **return** D

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0		
4					
5					

h

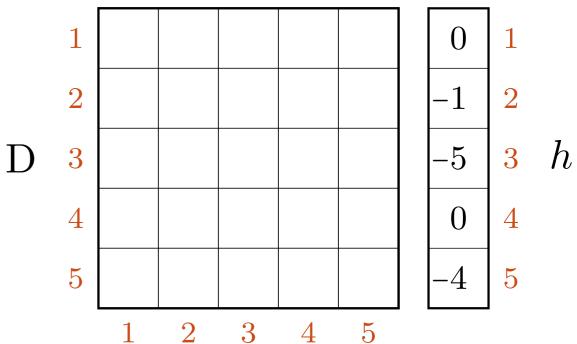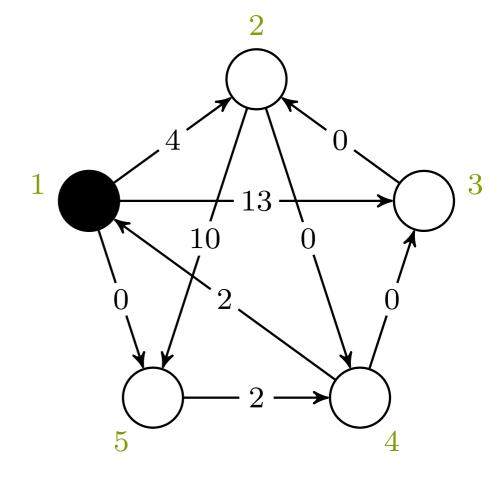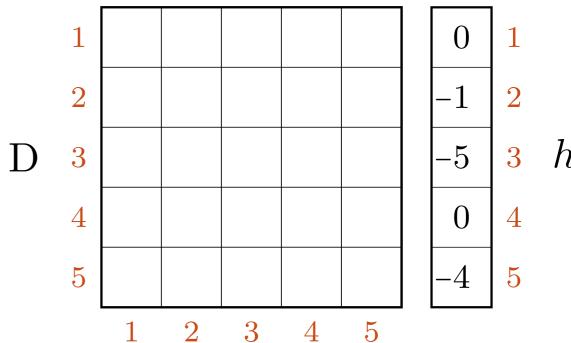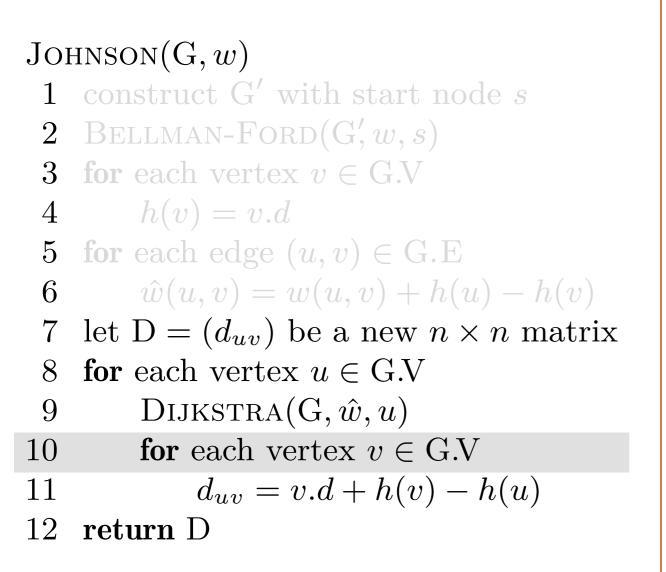|	
1	0
2	-1
3	-5
4	0
5	-4

JOHNSON(G, w)
1 construct G' with start node s
2 BELLMAN-FORD(G', w, s)
3 **for** each vertex $v \in$ G.V
4      $h(v) = v.d$
5 **for** each edge $(u, v) \in$ G.E
6      $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$
7 let D = (d_{uv}) be a new $n \times n$ matrix
8 **for** each vertex $u \in$ G.V
9   DIJKSTRA(G, $\hat{w}$, u)
10   **for** each vertex $v \in$ G.V
11          $d_{uv} = v.d + h(v) - h(u)$
12 **return** D

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	
4					
5					

h

0	1
-1	2
-5	3
0	4
-4	5

$\textsc{Johnson}(\mathrm{G}, w)$

1 construct G' with start node s
2 $\textsc{Bellman-Ford}(\mathrm{G}', w, s)$
3 **for** each vertex $v \in \mathrm{G.V}$
4      $h(v) = v.d$
5 **for** each edge $(u, v) \in \mathrm{G.E}$
6      $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$
7 let $\mathrm{D} = (d_{uv})$ be a new $n \times n$ matrix
8 **for** each vertex $u \in \mathrm{G.V}$
9      $\textsc{Dijkstra}(\mathrm{G}, \hat{w}, u)$
10   **for** each vertex $v \in \mathrm{G.V}$
11         $d_{uv} = v.d + h(v) - h(u)$
12 **return** D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	
4					
5					

D

h

0	**1**
-1	**2**
-5	**3**
0	**4**
-4	**5**

$\text{JOHNSON}(G, w)$
1   construct G' with start node s
2   $\text{BELLMAN-FORD}(G', w, s)$
3   **for** each vertex $v \in G.V$
4       $h(v) = v.d$
5   **for** each edge $(u, v) \in G.E$
6       $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$
7   let $D = (d_{uv})$ be a new $n \times n$ matrix
8   **for** each vertex $u \in G.V$
9       $\text{DIJKSTRA}(G, \hat{w}, u)$
10   **for** each vertex $v \in G.V$
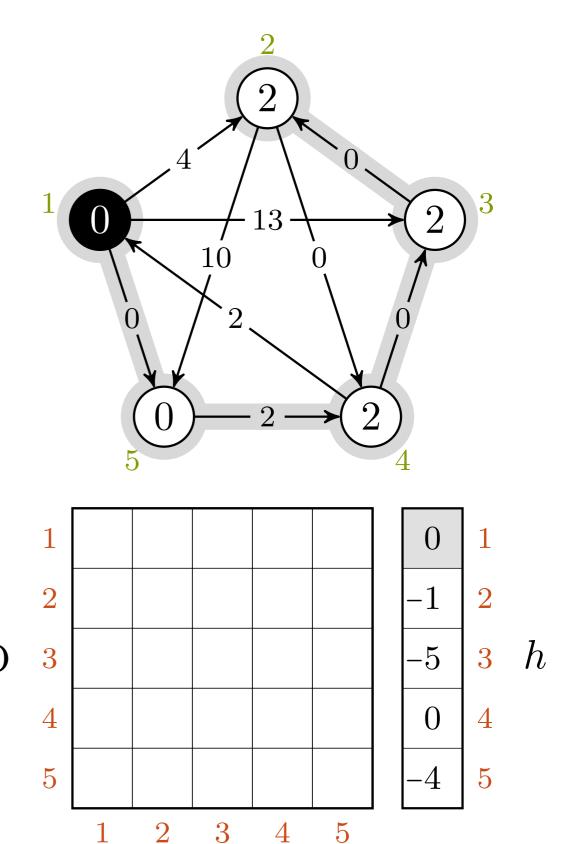11 $d_{uv} = v.d + h(v) - h(u)$
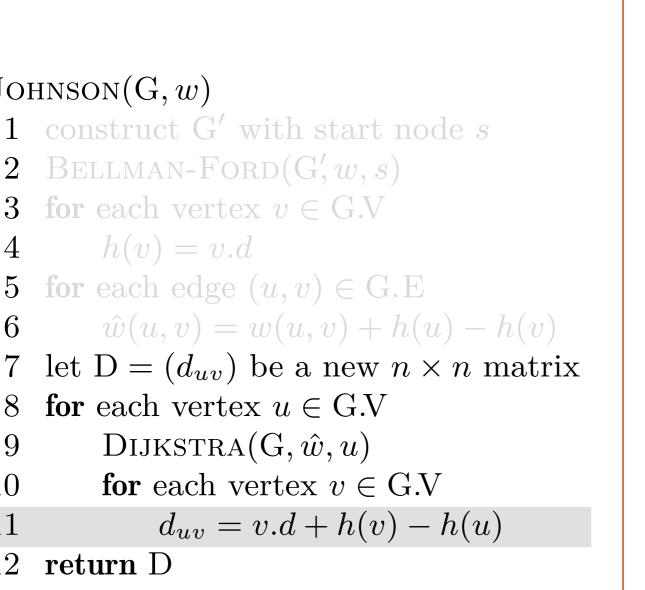12 **return** D

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4					
5					

h

0	1
-1	2
-5	3
0	4
-4	5

114

JOHNSON(G, w)
1   construct G′ with start node s
2   BELLMAN-FORD(G′, w, s)
3   **for** each vertex $v \in$ G.V
4       $h(v) = v.d$
5   **for** each edge $(u, v) \in$ G.E
6       $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$
7   let D $= (d_{uv})$ be a new $n \times n$ matrix
8   **for** each vertex $u \in$ G.V
9   DIJKSTRA(G, $\hat{w}, u$)
10   **for** each vertex $v \in$ G.V
11          $d_{uv} = v.d + h(v) - h(u)$
12 **return** D

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4					
5					

h

0	1
-1	2
-5	3
0	4
-4	5

115

mlh@ntnu.no

Johnson(G, w)
1 construct G′ with start node s
2 Bellman-Ford(G′, w, s)
3 for each vertex $v \in$ G.V
4     $h(v) = v.d$
5 for each edge $(u, v) \in$ G.E
6     $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$
7 let D $= (d_{uv})$ be a new $n \times n$ matrix
8 **for** each vertex $u \in$ G.V
9   Dijkstra(G, $\hat{w}$, u)
10   **for** each vertex $v \in$ G.V
11        $d_{uv} = v.d + h(v) - h(u)$
12 **return** D

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4					
5					

h	
0	1
-1	2
-5	3
0	4
-4	5

JOHNSON(G, w)
 1 construct G$'$ with start node s
 2 BELLMAN-FORD(G$'$, w, s)
 3 **for** each vertex $v \in$ G.V
 4      $h(v) = v.d$
 5 **for** each edge $(u, v) \in$ G.E
 6      $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$
 7 let D $= (d_{uv})$ be a new $n \times n$ matrix
 8 **for** each vertex $u \in$ G.V
 9   DIJKSTRA(G, $\hat{w}$, u)
10   **for** each vertex $v \in$ G.V
11          $d_{uv} = v.d + h(v) - h(u)$
12 **return** D

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4					
5					

h

0	1
-1	2
-5	3
0	4
-4	5

$\textsc{Johnson}(G, w)$
1 construct G' with start node s
2 $\textsc{Bellman-Ford}(G', w, s)$
3 **for** each vertex $v \in G.V$
4     $h(v) = v.d$
5 **for** each edge $(u, v) \in G.E$
6     $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$
7 let $D = (d_{uv})$ be a new $n \times n$ matrix
8 **for** each vertex $u \in G.V$
9     $\textsc{Dijkstra}(G, \hat{w}, u)$
10   **for** each vertex $v \in G.V$
11       $d_{uv} = v.d + h(v) - h(u)$
12 **return** D

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2				
5					

0	1
-1	2
-5	3
0	4
-4	5

h

$\textsc{Johnson}(G, w)$
1 construct G' with start node s
2 $\textsc{Bellman-Ford}(G', w, s)$
3 **for** each vertex $v \in G.V$
4      $h(v) = v.d$
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6      $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$
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8 **for** each vertex $u \in G.V$
9      $\textsc{Dijkstra}(G, \hat{w}, u)$
10   **for** each vertex $v \in G.V$
11         $d_{uv} = v.d + h(v) - h(u)$
12 **return** D

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2				
5					

h

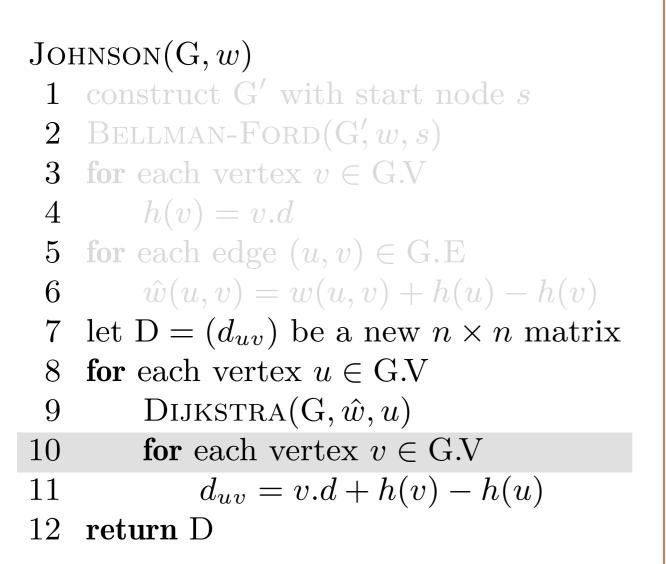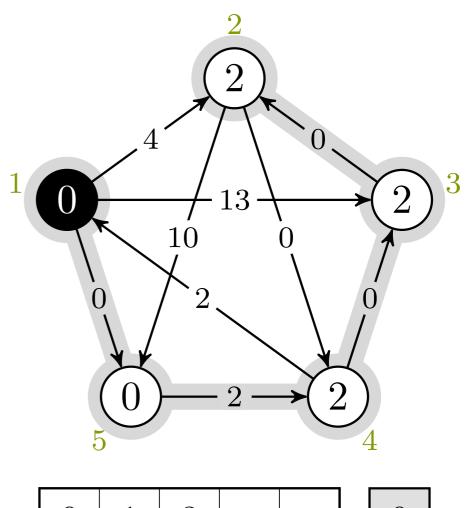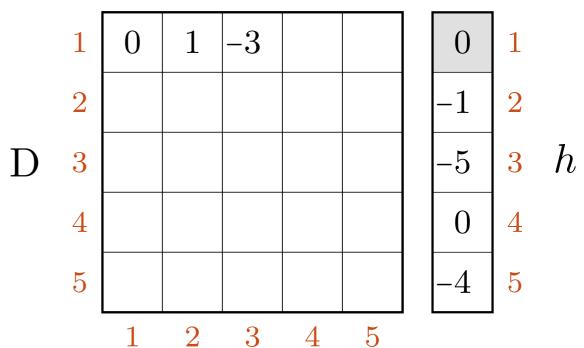|	
0	1
-1	2
-5	3
0	4
-4	5

$\textsc{Johnson}(G, w)$
1 construct G' with start node s
2 $\textsc{Bellman-Ford}(G', w, s)$
3 for each vertex $v \in G.V$
4      $h(v) = v.d$
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8 **for** each vertex $u \in G.V$
9      $\textsc{Dijkstra}(G, \hat{w}, u)$
10   **for** each vertex $v \in G.V$
11         $d_{uv} = v.d + h(v) - h(u)$
12 **return** D

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1			
5					

0	1
-1	2
-5	3
0	4
-4	5

h

$\text{Johnson}(G, w)$
1   construct G' with start node s
2   $\text{Bellman-Ford}(G', w, s)$
3   **for** each vertex $v \in G.V$
4      $h(v) = v.d$
5   **for** each edge $(u, v) \in G.E$
6      $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$
7   let $D = (d_{uv})$ be a new $n \times n$ matrix
8   **for** each vertex $u \in G.V$
9      $\text{Dijkstra}(G, \hat{w}, u)$
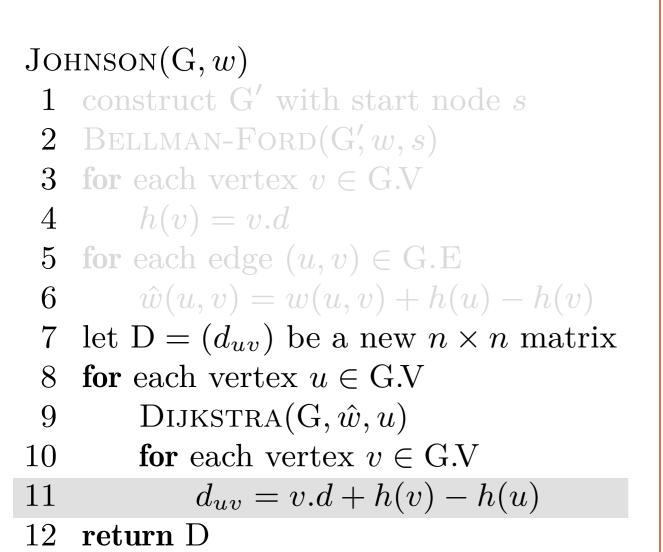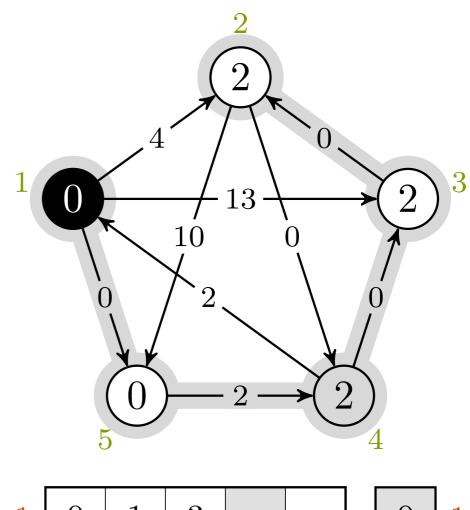10   **for** each vertex $v \in G.V$
11       $d_{uv} = v.d + h(v) - h(u)$
12 **return** D

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1			
5					

h

0	1
-1	2
-5	3
0	4
-4	5

JOHNSON(G, w)
1 construct G′ with start node s
2 BELLMAN-FORD(G′, w, s)
3 **for** each vertex $v \in$ G.V
4      $h(v) = v.d$
5 **for** each edge $(u, v) \in$ G.E
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8 **for** each vertex $u \in$ G.V
9   DIJKSTRA(G, $\hat{w}$, u)
10   **for** each vertex $v \in$ G.V
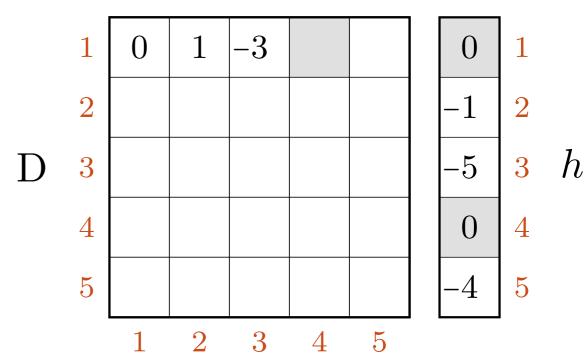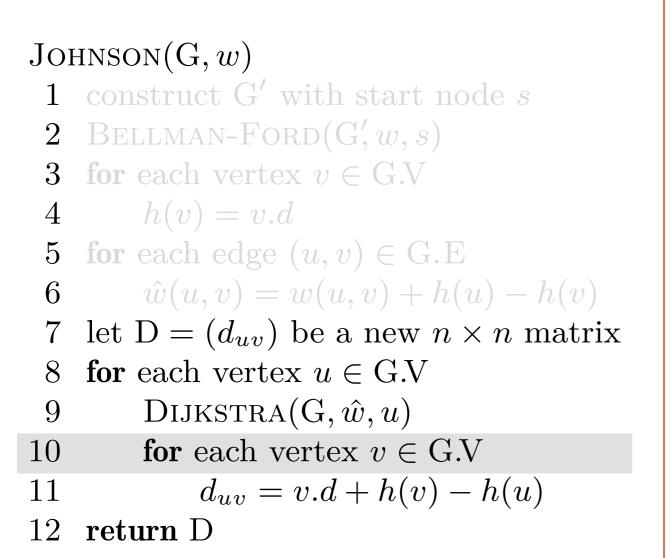11 $d_{uv} = v.d + h(v) - h(u)$
12 **return** D

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5		
5					

0	1
-1	2
-5	3
0	4
-4	5

h

JOHNSON(G, w)
1   construct G′ with start node s
2   BELLMAN-FORD(G′, w, s)
3   **for** each vertex $v \in$ G.V
4       $h(v) = v.d$
5   **for** each edge $(u, v) \in$ G.E
6       $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$
7   let D = (d_{uv}) be a new $n \times n$ matrix
8   **for** each vertex $u \in$ G.V
9   DIJKSTRA(G, $\hat{w}$, u)
10   **for** each vertex $v \in$ G.V
11          $d_{uv} = v.d + h(v) - h(u)$
12 **return** D

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5		
5					

0	1
-1	2
-5	3
0	4
-4	5

h

$\text{Johnson}(G, w)$

1 construct G′ with start node s
2 $\text{Bellman-Ford}(G', w, s)$
3 **for** each vertex $v \in G.V$
4      $h(v) = v.d$
5 **for** each edge $(u, v) \in G.E$
6      $\hat{w}(u, v) = w(u, v) + h(u) - h(v)$
7 let $D = (d_{uv})$ be a new $n \times n$ matrix
8 **for** each vertex $u \in G.V$
9      $\text{Dijkstra}(G, \hat{w}, u)$
10   **for** each vertex $v \in G.V$
11         $d_{uv} = v.d + h(v) - h(u)$
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D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	
5					

h

0	1
-1	2
-5	3
0	4
-4	5

$\textsc{Johnson}(G, w)$
 1   construct G' with start node s
 2   $\textsc{Bellman-Ford}(G', w, s)$
 3   **for** each vertex $v \in G.V$
 4       $h(v) = v.d$
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11           $d_{uv} = v.d + h(v) - h(u)$
12   **return** D

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	
5					

h

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-1	2
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0	4
-4	5

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126

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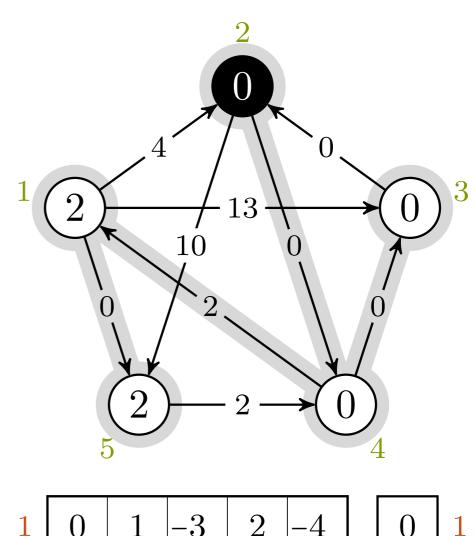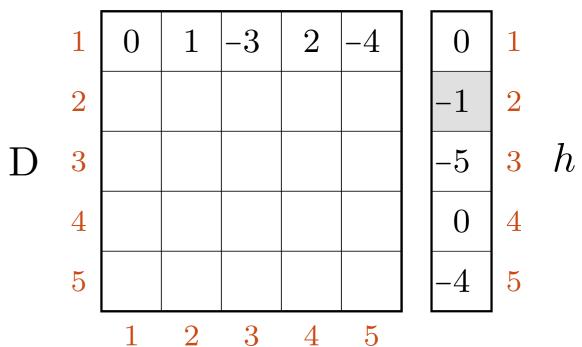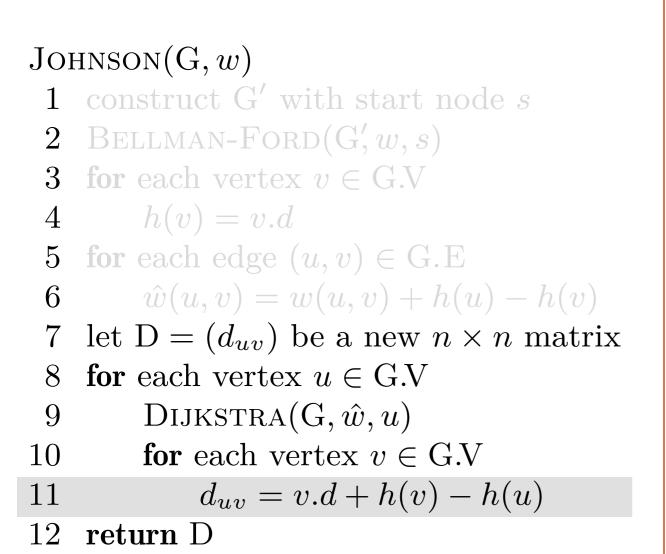11 $d_{uv} = v.d + h(v) - h(u)$
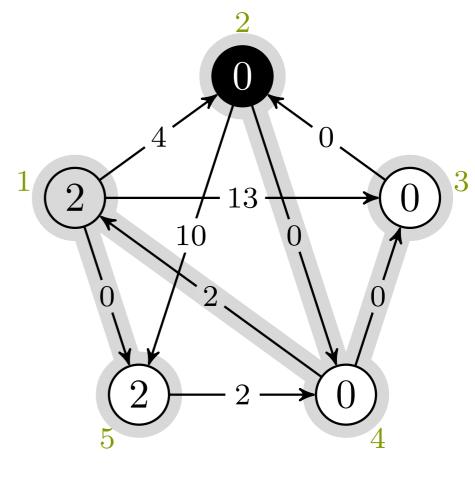12 **return** D

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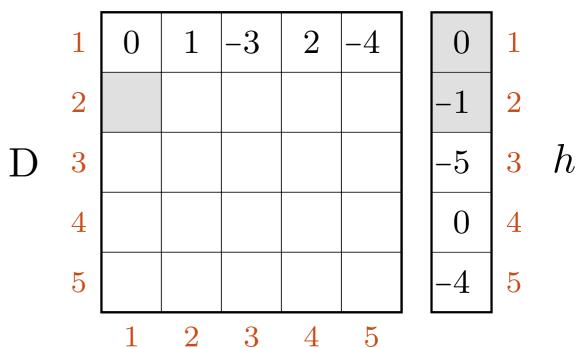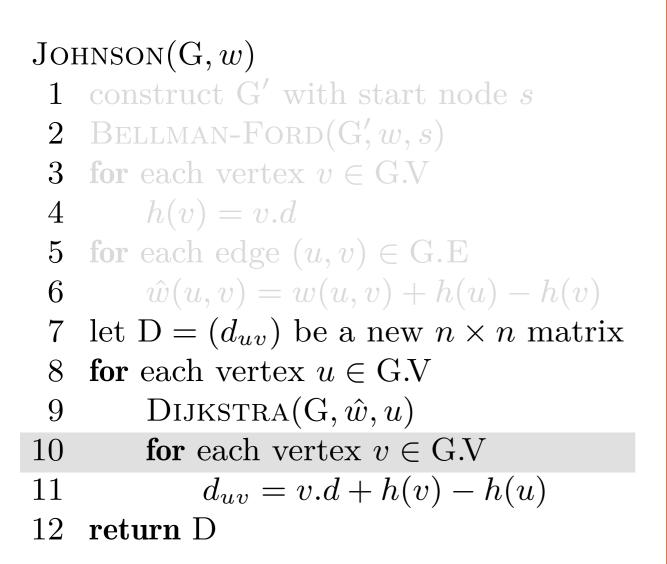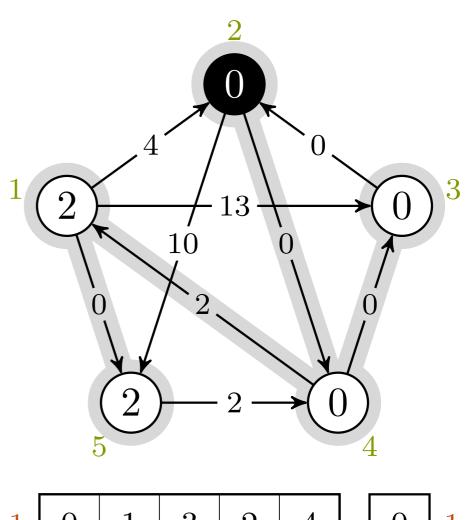11 $d_{uv} = v.d + h(v) - h(u)$
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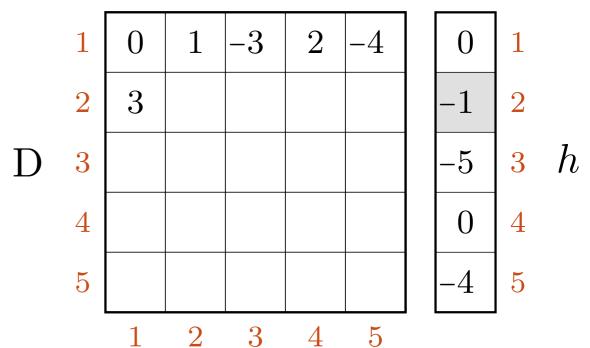12 **return** D

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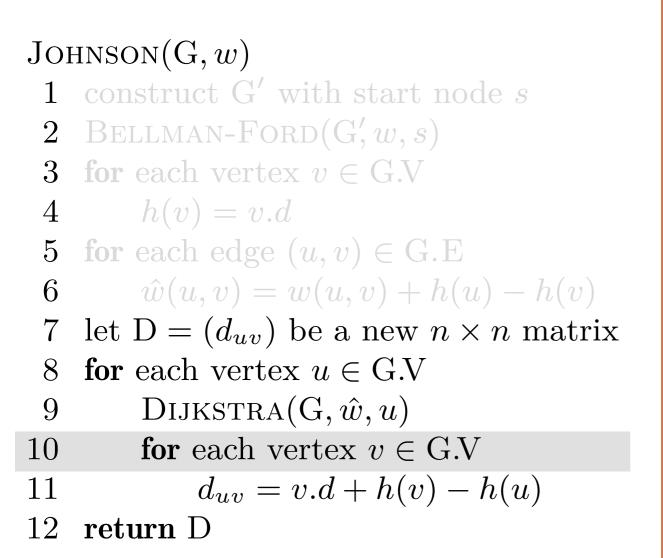| h	
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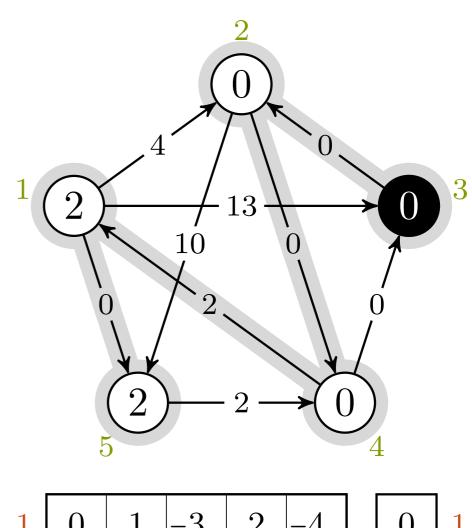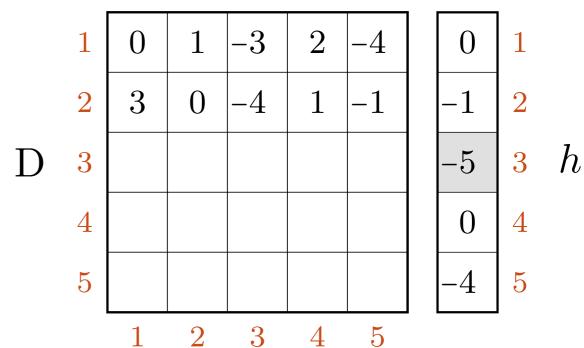11 $d_{uv} = v.d + h(v) - h(u)$
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2:3

Transitiv lukning

A Theorem on Boolean Matrices*

STEPHEN WARSHALL†

Computer Associates, Inc., Woburn, Massachusetts

Given two boolean matrices A and B, we define the boolean product $A \wedge B$ as that matrix whose (i, j)th entry is $\vee_k(a_{ik} \wedge b_{kj})$.

We define the boolean sum $A \vee B$ as that matrix whose (i, j)th entry is $a_{ij} \vee b_{ij}$.

The use of boolean matrices to represent program topology (Prosser [1], and Marimont [2], for example) has led to interest in algorithms for transforming the $d \times d$ boolean matrix M to the $d \times d$ boolean matrix M' given by:

$$M' = \bigvee_{i=1}^{d} M^i \quad \text{where we define } M^1 = M \text{ and } M^{i+1}$$

The convenience of describing th

products has appar

times

Fra 1960 (publisert 1962).
Bernhard Roy publiserte
samme resultat separat i 1959.

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Input: En rettet graf $G = (V, E)$.

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Output: En rettet graf $G^* = (V, E^*)$ der $(i, j) \in E^*$ hvis og bare hvis det finnes en sti fra i til j i G.

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Vi kommer til å returnere en nabomatrise T for G^*.

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Dekomponering: Hva blir «koordinatene» til delproblemene?

i

Det finnes en vei fra $i \ldots$
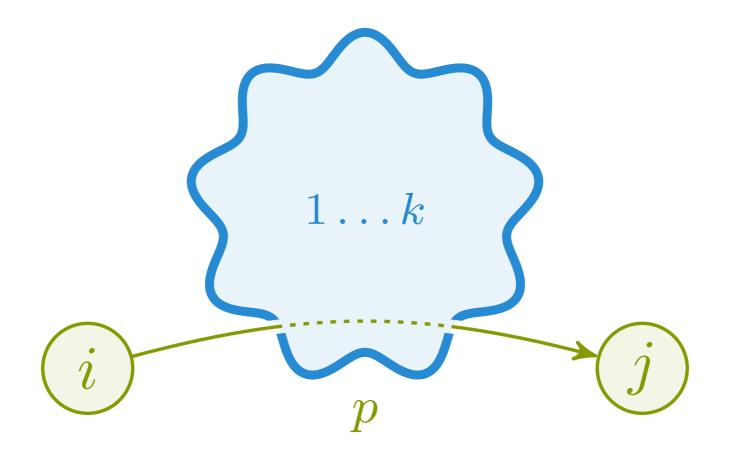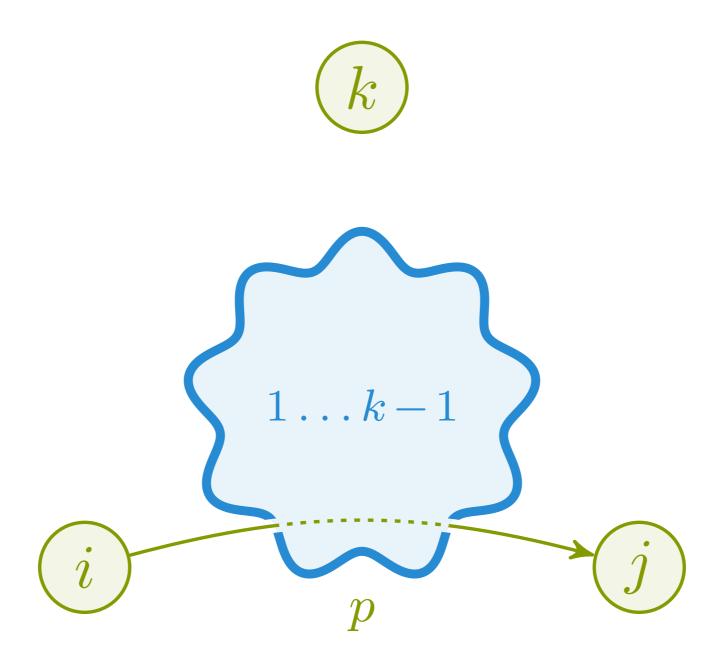
$$i \; j$$

Det finnes en vei fra i til $j \ldots$

$$i \, j \, k$$

$$i\,j\,k$$

Det finnes en vei fra i til j via noder fra $\{1\ldots k\}$

$$t_{ij}^{(k)}$$

Det finnes en vei fra i til j via noder fra $\{1 \ldots k\}$
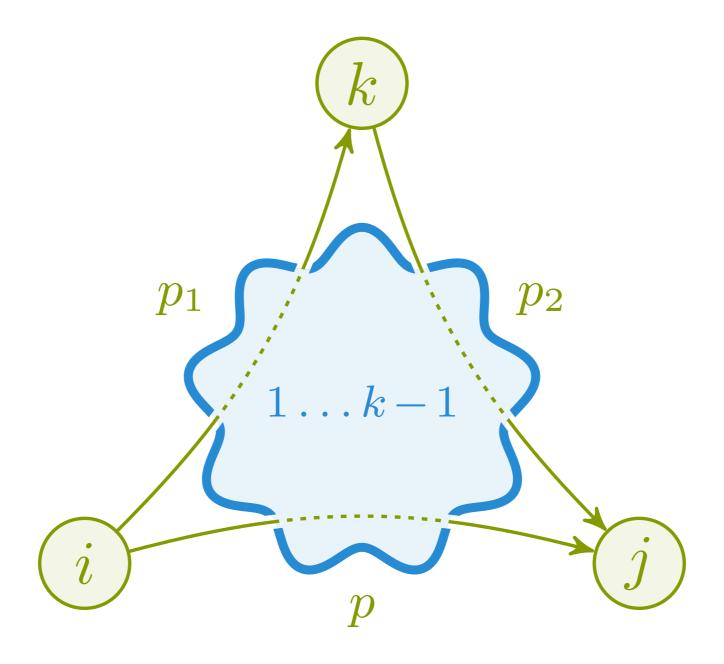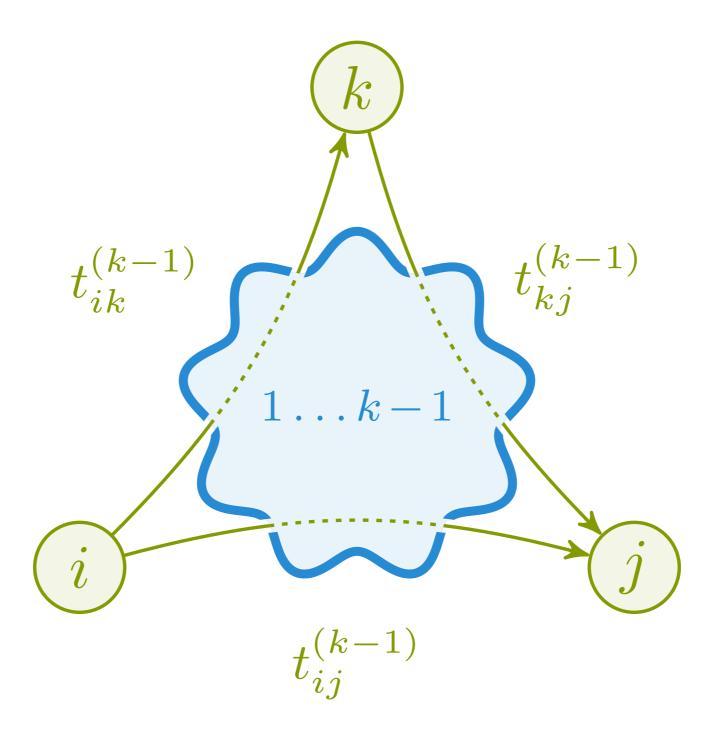
$$t_{ij}^{(k)} = \text{det går en vei fra } i \text{ til } j \text{ via noder fra } \{1 \ldots k\}$$
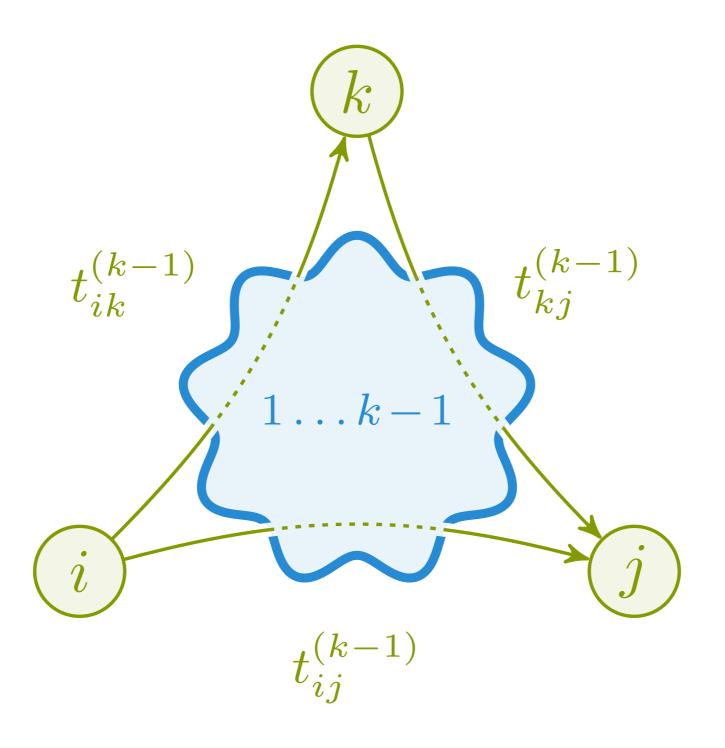
Som for ryggsekkproblemet: Skal k være med eller ikke?

De mulige stiene p, p_1 og p_2 går kun via noder fra $\{1 \ldots k-1\}$

$$t_{ij}^{(k)} = t_{ij}^{(k-1)} \vee \left(t_{ik}^{(k-1)} \wedge t_{kj}^{(k-1)} \right)$$

$$
t_{ij}^{(0)} = \begin{cases} 0 & \text{if } i \neq j \text{ and } (i,j) \notin \mathrm{E}\,, \\ 1 & \text{if } i = j \text{ or } (i,j) \in \mathrm{E}\,. \end{cases}
$$

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Problemet er at iterasjonene 1…k «blandes», så vi kan risikere at noen av del-stiene allerede går innom k – så kanskje vi går innom k mer enn én gang? I så fall har vi en sykel … men om det finnes en sti *med* en sykel, så finnes det også en sti *uten* en sykel!

Det er trygt å bruke én tabell. Hvorfor?

Transitive-Closure(G)

G   graf

For hver node i og j: Finnes det en sti $i \rightsquigarrow j$?

TRANSITIVE-CLOSURE(G)
1 $n = |G.V|$

G   graf
n ant. noder

$\textsc{Transitive-Closure}(G)$

1   $n = |G.V|$

2   let $T^{(0)} = (t_{ij}^{(0)})$ be a new $n \times n$ matrix

$\begin{array}{ll} G & \text{graf} \\ n & \text{ant. noder} \\ t_{ij}^{(k)} & i \overset{1 \cdots k}{\leadsto} j \, ? \end{array}$

Finnes en sti $i \leadsto j$ som ikke går via andre noder?

mlh@ntnu.no

TRANSITIVE-CLOSURE(G)

1   $n = |\text{G.V}|$

2   let $\text{T}^{(0)} = (t_{ij}^{(0)})$ be a new $n \times n$ matrix

3   **for** $i = 1$ **to** n

G   graf

n ant. noder

$t_{ij}^{(k)}$ $i \overset{1 \cdots k}{\rightsquigarrow} j$?

For hver mulig startnode...

Transitive-Closure(G)

1 $n = |\text{G.V}|$

2 let $\text{T}^{(0)} = \left(t_{ij}^{(0)}\right)$ be a new $n \times n$ matrix

3 **for** $i = 1$ **to** n

4   **for** $j = 1$ **to** n

G   graf

G   graf

n ant. noder

$t_{ij}^{(k)}$ $i \overset{1 \cdots k}{\rightsquigarrow} j$?

For hver mulig sluttnode...

TRANSITIVE-CLOSURE(G)
1   $n = |G.V|$
2   let $T^{(0)} = (t_{ij}^{(0)})$ be a new $n \times n$ matrix
3   **for** $i = 1$ **to** n
4   **for** $j = 1$ **to** n
5   **if** $i == j$ or $(i, j) \in G.E$

G   graf
n ant. noder
$t_{ij}^{(k)}$ $i \overset{1\cdots k}{\rightsquigarrow} j$?

Samme node eller sammenkoblet med kant?

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TRANSITIVE-CLOSURE(G)

1 $n = |\text{G.V}|$

2 let $\text{T}^{(0)} = (t_{ij}^{(0)})$ be a new $n \times n$ matrix

3 **for** $i = 1$ **to** n

4   **for** $j = 1$ **to** n

5   **if** $i == j$ or $(i, j) \in \text{G.E}$

6              $t_{ij}^{(0)} = 1$

G   graf
n ant. noder
$t_{ij}^{(k)}$ $i \overset{1\cdots k}{\rightsquigarrow} j$?

Da finnes det en sti $i \rightsquigarrow j$ som ikke går via andre noder

$\textsc{Transitive-Closure}(G)$

1 $n = |G.V|$

2 let $T^{(0)} = (t_{ij}^{(0)})$ be a new $n \times n$ matrix

3 **for** $i = 1$ **to** n

4   **for** $j = 1$ **to** n

5   **if** $i == j$ or $(i, j) \in G.E$

6              $t_{ij}^{(0)} = 1$

7   **else** $t_{ij}^{(0)} = 0$

G graf

n ant. noder

$t_{ij}^{(k)}$ $i \overset{1 \cdots k}{\rightsquigarrow} j$?

Ellers finnes ingen slik sti

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Transitive-Closure(G)
1   $n = |\text{G.V}|$
2   let $\text{T}^{(0)} = (t_{ij}^{(0)})$ be a new $n \times n$ matrix
3   **for** $i = 1$ **to** n
4   **for** $j = 1$ **to** n
5   **if** $i == j$ or $(i, j) \in \text{G.E}$
6               $t_{ij}^{(0)} = 1$
7   **else** $t_{ij}^{(0)} = 0$
8   $\ldots$

G   graf
n ant. noder
$t_{ij}^{(k)}$ $i \overset{1 \cdots k}{\rightsquigarrow} j$?

Grunntilfelle på plass

TRANSITIVE-CLOSURE(G)

G graf

n ant. noder

$t_{ij}^{(k)}$ $i \overset{1\cdots k}{\rightsquigarrow} j$?

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Transitive-Closure(G)
  7   . . .

G   graf
n ant. noder
$t_{ij}^{(k)}$ $i \overset{1\cdots k}{\rightsquigarrow} j$?

TRANSITIVE-CLOSURE(G)

  7 ...

  8 **for** $k = 1$ **to** n

G graf

n ant. noder

$t_{ij}^{(k)}$ $i \overset{1 \cdots k}{\leadsto} j$?

Vi får nå lov til å gå innom node k

TRANSITIVE-CLOSURE(G)

7 ...

8 **for** $k = 1$ **to** n

9   let $T^{(k)} = (t_{ij}^{(k)})$ be a new $n \times n$ matrix

G graf

n ant. noder

$t_{ij}^{(k)}$ $i \overset{1\cdots k}{\rightsquigarrow} j$?

Finnes en sti $i \rightsquigarrow j$ som kun får gå innom $\{1, \ldots, k\}$?

Transitive-Closure(G)

7   ...
8   **for** $k = 1$ **to** n
9   let $T^{(k)} = (t_{ij}^{(k)})$ be a new $n \times n$ matrix
10   **for** $i = 1$ **to** n

G graf
n ant. noder
$t_{ij}^{(k)}$ $i \overset{1\cdots k}{\rightsquigarrow} j$?

For hver mulig startnode...

TRANSITIVE-CLOSURE(G)

7 . . .

8 **for** $k = 1$ **to** n

9   let $T^{(k)} = \left(t_{ij}^{(k)}\right)$ be a new $n \times n$ matrix

10   **for** $i = 1$ **to** n

11   **for** $j = 1$ **to** n

G   graf

n ant. noder

$t_{ij}^{(k)}$ $i \overset{1\cdots k}{\rightsquigarrow} j$?

For hver mulig sluttnode . . .

TRANSITIVE-CLOSURE(G)

  7 ...

  8 **for** $k = 1$ **to** n

  9   let $\mathrm{T}^{(k)} = (t_{ij}^{(k)})$ be a new $n \times n$ matrix

10   **for** $i = 1$ **to** n

11   **for** $j = 1$ **to** n

12              $t_{ij}^{(k)} = t_{ij}^{(k-1)} \vee (t_{ik}^{(k-1)} \wedge t_{kj}^{(k-1)})$

G graf
n ant. noder
$t_{ij}^{(k)}$ $i \overset{1\cdots k}{\rightsquigarrow} j$?

Finnes $i \rightsquigarrow j$ eller $i \rightsquigarrow k \rightsquigarrow j$, om vi kun får gå innom $\{1, \ldots, k-1\}$?

TRANSITIVE-CLOSURE(G)

7 ...

8 **for** $k = 1$ **to** n

9 let $T^{(k)} = (t_{ij}^{(k)})$ be a new $n \times n$ matrix

10 **for** $i = 1$ **to** n

11   **for** $j = 1$ **to** n

12    $t_{ij}^{(k)} = t_{ij}^{(k-1)} \vee (t_{ik}^{(k-1)} \wedge t_{kj}^{(k-1)})$

13 **return** $T^{(n)}$

G graf

n ant. noder

$t_{ij}^{(k)}$ $i \overset{1 \cdots k}{\rightsquigarrow} j$?

Finnes det en sti $i \rightsquigarrow j$ som får gå innom $\{1, \ldots, n\}$, dvs. alle?

$$\textsc{Transitive-Closure}'(\textsc{G})$$

Forenklet utgave: Bruker bare én tabell, T

Transitive-Closure′(G)

1   $n = |G.V|$

Transitive-Closure$'$(G)
1   $n = |\text{G.V}|$
2   initialize T

Samme node (I er identitetsmatrise) eller nabo (A er nabomatrise)

Transitive-Closure$'$(G)
1 $n = |\text{G.V}|$
2 initialize T
3 **for** $k = 1$ **to** n

Vi får nå lov til å gå innom node k

Transitive-Closure$'$(G)
1 $n = |\text{G.V}|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n

For hver mulig startnode...

Transitive-Closure$'$(G)

1 $n = |\text{G.V}|$

2 initialize T

3 **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

For hver mulig sluttnode...

Transitive-Closure$'$(G)
1 $n = |\text{G.V}|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                 $t_{ij} = t_{ij} \lor (t_{ik} \land t_{kj})$

Enten allerede en sti $i \rightsquigarrow j$ eller en ny sti $i \rightsquigarrow k \rightsquigarrow j$?

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TRANSITIVE-CLOSURE$'$(G)
1 $n = |\text{G.V}|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                 $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

Finnes det en sti $i \rightsquigarrow j$?

Transitive-Closure′(G)

1 $n = |\text{G.V}|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		
2	1	1		1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j = 1, 1, 1$

TRANSITIVE-CLOSURE$'$(G)

1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
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5   **for** $j = 1$ **to** n
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7 **return** T

	1	2	3	4	5
1	1		1		
2	1	1		1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j \;=\; 1, 1, 2$

Transitive-Closure′(G)
1   $n = |G.V|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                  $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7   **return** T

	1	2	3	4	5
1	1		1		
2	1	1		1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j \;=\; 1, 1, 3$

Transitive-Closure'(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1	■	
2	1	1		1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j \ = \ 1, 1, 4$

Transitive-Closure$'$(G)

1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		■
2	1	1		1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j = 1, 1, 5$

Transitive-Closure′(G)

1 $n = |\text{G.V}|$

2 initialize T

3 **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$

7 **return** T

	1	2	3	4	5
1	1		1		
2	1	1		1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j = 1, 2, 1$

Transitive-Closure$'$(G)
1   $n = |\text{G.V}|$
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4   **for** $i = 1$ **to** n
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4				1	
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$k, i, j = 1, 2, 2$

Transitive-Closure'(G)
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	1	2	3	4	5
1	1		1		
2	1	1	■	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j = 1, 2, 3$

Transitive-Closure′(G)
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4				1	
5		1	1	1	1

$k, i, j \ = \ 1, 2, 4$

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Transitive-Closure$'$(G)
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1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j \ = \ 1, 2, 5$

Transitive-Closure′(G)
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2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6               $t_{ij} = t_{ij} \lor (t_{ik} \land t_{kj})$
7   **return** T

	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j = 1, 3, 1$

Transitive-Closure'(G)
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	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j = 1, 3, 2$

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TRANSITIVE-CLOSURE′(G)
1   $n = |\text{G.V}|$
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5   **for** $j = 1$ **to** n
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	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j \; = \; 1, 3, 3$

Transitive-Closure'(G)
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1	1		1		
2	1	1	1	1	
3			1	■	1
4				1	
5		1	1	1	1

$k, i, j \;=\; 1, 3, 4$

TRANSITIVE-CLOSURE′(G)

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	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j \ = \ 1, 3, 5$

Transitive-Closure$'$(G)

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3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
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	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4	■			1	
5		1	1	1	1

$k, i, j = 1, 4, 1$

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Transitive-Closure'(G)
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	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j = 1, 4, 2$

Transitive-Closure′(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
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2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j = 1, 4, 3$

Transitive-Closure'(G)
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6              $t_{ij} = t_{ij} \lor (t_{ik} \land t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j = 1, 4, 4$

Transitive-Closure$'$(G)

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3			1		1
4				1	
5		1	1	1	1

$k, i, j \;=\; 1, 4, 5$

TRANSITIVE-CLOSURE′(G)
1 $n = |\text{G.V}|$
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3 **for** $k = 1$ **to** n
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4				1	
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$k, i, j = 1, 5, 1$

Transitive-Closure′(G)
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3   **for** $k = 1$ **to** n
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	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j = 1, 5, 2$

Transitive-Closure$'$(G)
1 $n = |$G.V$|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                 $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j = 1, 5, 3$

Transitive-Closure′(G)
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3 **for** $k = 1$ **to** n
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5   **for** $j = 1$ **to** n
6                 $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
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	1	2	3	4	5
1	1		1		
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3			1		1
4				1	
5		1	1	1	1

$k, i, j = 1, 5, 4$

TRANSITIVE-CLOSURE$'$(G)
1 $n = |$G.V$|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
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6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
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	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j \ = \ 1, 5, 5$

Transitive-Closure′(G)

1   $n = |G.V|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
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	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j = 2, 1, 1$

Transitive-Closure'(G)
1 $n = |\text{G.V}|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                 $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1	■	1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j = 2, 1, 2$

Transitive-Closure′(G)
1   $n = |\text{G.V}|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                  $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7   **return** T

	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j = 2, 1, 3$

Transitive-Closure′(G)
1   $n = |G.V|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                           $t_{ij} = t_{ij} \lor (t_{ik} \land t_{kj})$
7   **return** T

	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j = 2, 1, 4$

Transitive-Closure$'$(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j \ = \ 2, 1, 5$

Transitive-Closure′(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                    $t_{ij} = t_{ij} \lor (t_{ik} \land t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j \ = \ 2, 2, 1$

Transitive-Closure'(G)
1   $n = |G.V|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                  $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7   **return** T

	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j = 2, 2, 2$

Transitive-Closure'(G)
1   $n = |G.V|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                        $t_{ij} = t_{ij} \lor (t_{ik} \land t_{kj})$
7   **return** T

	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j = 2, 2, 3$

TRANSITIVE-CLOSURE′(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j \ = \ 2, 2, 4$

Transitive-Closure′(G)
1 $n = |\text{G.V}|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j = 2, 2, 5$

TRANSITIVE-CLOSURE'(G)
1   $n = |G.V|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                  $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7   **return** T

	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j = 2, 3, 1$

Transitive-Closure′(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j \; = \; 2, 3, 2$

Transitive-Closure′(G)
1   $n = |\text{G.V}|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6               $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
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	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j = 2, 3, 3$

Transitive-Closure′(G)
1   $n = |\text{G.V}|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                           $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7   **return** T

	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1	1	1
4				1	
5		1	1	1	1

$k, i, j \ = \ 2, 3, 4$

$\textsc{Transitive-Closure}'(\text{G})$

1 $n = |\text{G.V}|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
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	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j \ = \ 2, 3, 5$

Transitive-Closure′(G)
1   $n = |G.V|$
2   initialize T
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6                           $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
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	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j \; = \; 2, 4, 1$

TRANSITIVE-CLOSURE$'$(G)
1   $n = |\text{G.V}|$
2   initialize T
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5   **for** $j = 1$ **to** n
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	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4		■		1	
5		1	1	1	1

$k, i, j = 2, 4, 2$

Transitive-Closure′(G)
1   $n = |G.V|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6               $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
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	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j \ = \ 2, 4, 3$

Transitive-Closure′(G)

1   $n = |\text{G.V}|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                           $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
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	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j \; = \; 2, 4, 4$

Transitive-Closure$'$(G)
1 $n = |\text{G.V}|$
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4   **for** $i = 1$ **to** n
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	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j \;=\; 2, 4, 5$

Transitive-Closure$'$(G)

1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
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	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5		1	1	1	1

$k, i, j \ = \ 2, 5, 1$

Transitive-Closure$'$(G)
1   $n = |$G.V$|$
2   initialize T
3   **for** $k = 1$ **to** n
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6                  $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7   **return** T

	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j \; = \; 2, 5, 2$

Transitive-Closure′(G)

1 $n = |G.V|$

2 initialize T

3 **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$

7 **return** T

	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 2, 5, 3$

TRANSITIVE-CLOSURE′(G)
1   $n = |G.V|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6               $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7   **return** T

	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
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$k, i, j \ = \ 2, 5, 4$

Transitive-Closure$'$(G)

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4				1	
5	1	1	1	1	1

$k, i, j = 2, 5, 5$

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Transitive-Closure′(G)

1   $n = |\text{G.V}|$
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$k, i, j \;=\; 3, 1, 1$

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Transitive-Closure′(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
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5   **for** $j = 1$ **to** n
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	1	2	3	4	5
1	1		1		
2	1	1	1	1	
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j \ = \ 3, 1, 2$

Transitive-Closure'(G)
1 $n = |\text{G.V}|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		
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4				1	
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$k, i, j = 3, 1, 3$

Transitive-Closure'(G)
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Transitive-Closure′(G)
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$k, i, j \ = \ 3, 1, 5$

Transitive-Closure'(G)
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2	1	1	1	1	
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j \ = \ 3, 2, 1$

Transitive-Closure′(G)
1 $n = |G.V|$
2 initialize T
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5   **for** $j = 1$ **to** n
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$k, i, j = 3, 2, 2$

Transitive-Closure′(G)
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4				1	
5	1	1	1	1	1

$k, i, j = 3, 2, 3$

TRANSITIVE-CLOSURE′(G)

1   $n = |G.V|$
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3   **for** $k = 1$ **to** n
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3			1	1	1
4				1	
5	1	1	1	1	1

$k, i, j \; = \; 3, 2, 4$

Transitive-Closure$'$(G)
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	1	2	3	4	5
1	1		1		1
2	1	1	1	1	
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 3, 2, 5$

Transitive-Closure′(G)

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2   initialize T
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	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j \ = \ 3, 3, 1$

Transitive-Closure′(G)
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3 **for** $k = 1$ **to** n
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	1	2	3	4	5
1	1		1		1
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3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 3, 3, 2$

Transitive-Closure′(G)
1 $n = |\text{G.V}|$
2 initialize T
3 **for** $k = 1$ **to** n
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	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 3, 3, 3$

TRANSITIVE-CLOSURE$'$(G)

1   $n = |G.V|$
2   initialize T
3   **for** $k = 1$ **to** n
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	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1	■	1
4				1	
5	1	1	1	1	1

$k, i, j = 3, 3, 4$

Transitive-Closure′(G)
1   $n = |G.V|$
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	1	2	3	4	5
1	1		1		1
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3			1		1
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$k, i, j \; = \; 3, 3, 5$

Transitive-Closure$'$(G)

1   $n = |\text{G.V}|$
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3   **for** $k = 1$ **to** n
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	1	2	3	4	5
1	1		1		1
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3			1		1
4				1	
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$k, i, j \;=\; 3, 4, 1$

Transitive-Closure$'$(G)
1 $n = |$G.V$|$
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3 **for** $k = 1$ **to** n
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	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j \ = \ 3, 4, 2$

Transitive-Closure′(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
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5   **for** $j = 1$ **to** n
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	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j \;=\; 3, 4, 3$

TRANSITIVE-CLOSURE$'$(G)
1   $n = |$G.V$|$
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3			1		1
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$k, i, j \;=\; 3, 4, 4$

Transitive-Closure′(G)
1 $n = |G.V|$
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	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 3, 4, 5$

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Transitive-Closure′(G)
1 $n = |G.V|$
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5   **for** $j = 1$ **to** n
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7 **return** T

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2	1	1	1	1	1
3			1		1
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5	1	1	1	1	1

$k, i, j = 3, 5, 1$

Transitive-Closure'(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 3, 5, 2$

TRANSITIVE-CLOSURE$'$(G)

1 $n = |\text{G.V}|$

2 initialize T

3 **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$

7 **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j \ = \ 3, 5, 3$

TRANSITIVE-CLOSURE′(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 3, 5, 4$

TRANSITIVE-CLOSURE$'$(G)

1   $n = |\text{G.V}|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6               $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7   **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j \; = \; 3, 5, 5$

Transitive-Closure′(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 4, 1, 1$

Transitive-Closure'(G)

1 $n = |G.V|$

2 initialize T

3 **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$

7 **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 4, 1, 2$

TRANSITIVE-CLOSURE'(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
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	1	2	3	4	5
1	1		1		1
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3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 4, 1, 3$

Transitive-Closure′(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1	■	1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 4, 1, 4$

Transitive-Closure′(G)

1   $n = |G.V|$
2   initialize T
3   **for** $k = 1$ **to** n
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	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j \ = \ 4, 1, 5$

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TRANSITIVE-CLOSURE$'$(G)
1   $n = |G.V|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6               $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7   **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j \ = \ 4, 2, 1$

Transitive-Closure$'$(G)
1 $n = |$G.V$|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 4, 2, 2$

Transitive-Closure′(G)
1   $n = |G.V|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                  $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7   **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 4, 2, 3$

TRANSITIVE-CLOSURE′(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
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	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 4, 2, 4$

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TRANSITIVE-CLOSURE'(G)

1 $n = |\text{G.V}|$

2 initialize T

3 **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$

7 **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j \;=\; 4, 2, 5$

Transitive-Closure$'$(G)

1 $n = |G.V|$

2 initialize T

3 **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

6            $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$

7 **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j \ = \ 4, 3, 1$

TRANSITIVE-CLOSURE$'$(G)
1   $n = |$G.V$|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                  $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7   **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j \; = \; 4, 3, 2$

Transitive-Closure$'$(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 4, 3, 3$

T<small>RANSITIVE</small>-C<small>LOSURE</small>'(G)
1 $\quad n = |G.V|$
2 $\quad$ initialize T
3 $\quad$ **for** $k = 1$ **to** n
4 $\qquad$ **for** $i = 1$ **to** n
5 $\qquad\quad$ **for** $j = 1$ **to** n
6 $\qquad\qquad t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 $\quad$ **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1	■	1
4				1	
5	1	1	1	1	1

$k, i, j \;=\; 4, 3, 4$

Transitive-Closure$'$(G)
1 $n = |\text{G.V}|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 4, 3, 5$

Transitive-Closure$'$(G)
1   $n = |$G.V$|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                     $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7   **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 4, 4, 1$

TRANSITIVE-CLOSURE′(G)
1   $n = |G.V|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6               $t_{ij} = t_{ij} \lor (t_{ik} \land t_{kj})$
7   **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4		■		1	
5	1	1	1	1	1

$k, i, j \ = \ 4, 4, 2$

Transitive-Closure′(G)
1 $n = |\text{G.V}|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4			■	1	
5	1	1	1	1	1

$k, i, j = 4, 4, 3$

Transitive-Closure$'$(G)
1   $n = |G.V|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                        $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7   **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j \ = \ 4, 4, 4$

Transitive-Closure′(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j \ = \ 4, 4, 5$

Transitive-Closure'(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \lor (t_{ik} \land t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 4, 5, 1$

Transitive-Closure′(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                 $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 4, 5, 2$

Transitive-Closure′(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                       $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j \ = \ 4, 5, 3$

Transitive-Closure′(G)
1 $n = |\text{G.V}|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j \; = \; 4, 5, 4$

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Transitive-Closure'(G)
1 $n = |\text{G.V}|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                 $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j \ = \ 4, 5, 5$

Transitive-Closure'(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \lor (t_{ik} \land t_{kj})$
7 **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 5, 1, 1$

TRANSITIVE-CLOSURE′(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
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7 **return** T

	1	2	3	4	5
1	1		1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j \;=\; 5, 1, 2$

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Transitive-Closure$'$(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
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	1	2	3	4	5
1	1	1	1		1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j \ = \ 5, 1, 3$

Transitive-Closure′(G)

1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1	1	1	■	1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 5, 1, 4$

Transitive-Closure′(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
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4				1	
5	1	1	1	1	1

$k, i, j = 5, 1, 5$

Transitive-Closure′(G)
1   $n = |\text{G.V}|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
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	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j \;=\; 5, 2, 1$

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Transitive-Closure′(G)
1 $n = |\text{G.V}|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                    $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
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1	1	1	1	1	1
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3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 5, 2, 2$

Transitive-Closure′(G)
1   $n = |G.V|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                           $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7   **return** T

	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 5, 2, 3$

Transitive-Closure$'$(G)

1 $n = |\text{G.V}|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
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	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j = 5, 2, 4$

Transitive-Closure$'$(G)
1   $n = |\text{G.V}|$
2   initialize T
3   **for** $k = 1$ **to** n
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5   **for** $j = 1$ **to** n
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	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j \;=\; 5, 2, 5$

Transitive-Closure'(G)
1 $n = |\text{G.V}|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                    $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3			1		1
4				1	
5	1	1	1	1	1

$k, i, j \; = \; 5, 3, 1$

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Transitive-Closure′(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	■	1		1
4				1	
5	1	1	1	1	1

$k, i, j = 5, 3, 2$

Transitive-Closure$'$(G)
1   $n = |\text{G.V}|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                     $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7   **return** T

	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1		1
4				1	
5	1	1	1	1	1

$k, i, j \ = \ 5, 3, 3$

Transitive-Closure′(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \lor (t_{ik} \land t_{kj})$
7 **return** T

	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	■	1
4				1	
5	1	1	1	1	1

$k, i, j = 5, 3, 4$

Transitive-Closure$'$(G)
1 $n = |$G.V$|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	1	1
4				1	
5	1	1	1	1	1

$k, i, j = 5, 3, 5$

Transitive-Closure′(G)
1   $n = |G.V|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                  $t_{ij} = t_{ij} \lor (t_{ik} \land t_{kj})$
7   **return** T

	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	1	1
4				1	
5	1	1	1	1	1

$k, i, j = 5, 4, 1$

Transitive-Closure′(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \lor (t_{ik} \land t_{kj})$
7 **return** T

	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	1	1
4				1	
5	1	1	1	1	1

$k, i, j = 5, 4, 2$

Transitive-Closure′(G)
1   $n = |\text{G.V}|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6               $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7   **return** T

	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	1	1
4			■	1	
5	1	1	1	1	1

$k, i, j \;=\; 5, 4, 3$

Transitive-Closure′(G)

1 $n = |\text{G.V}|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	1	1
4				1	
5	1	1	1	1	1

$k, i, j = 5, 4, 4$

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Transitive-Closure'(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                    $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	1	1
4				1	
5	1	1	1	1	1

$k, i, j = 5, 4, 5$

Transitive-Closure′(G)
1 $n = |\text{G.V}|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                 $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	1	1
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5	1	1	1	1	1

$k, i, j = 5, 5, 1$

Transitive-Closure$'$(G)
1 $n = |\text{G.V}|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	1	1
4				1	
5	1	1	1	1	1

$k, i, j = 5, 5, 2$

Transitive-Closure′(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                 $t_{ij} = t_{ij} \lor (t_{ik} \land t_{kj})$
7 **return** T

	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	1	1
4				1	
5	1	1	1	1	1

$k, i, j = 5, 5, 3$

Transitive-Closure′(G)

1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
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3	1	1	1	1	1
4				1	
5	1	1	1	1	1

$k, i, j = 5, 5, 4$

Transitive-Closure′(G)
1   $n = |G.V|$
2   initialize T
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6                           $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7   **return** T

	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	1	1
4				1	
5	1	1	1	1	1

$k, i, j \;=\; 5, 5, 5$

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Transitive-Closure'(G)
1 $n = |G.V|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6              $t_{ij} = t_{ij} \vee (t_{ik} \wedge t_{kj})$
7 **return** T

	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	1	1
4				1	
5	1	1	1	1	1

$k, i, j \; = \; 5, 5, 5$

Transitive-Closure$'$(G)
1 $n = |$G.V$|$
2 initialize T
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
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	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	1	1
4				1	
5	1	1	1	1	1

$k, i, j = 5, 5, 5$

Korteste vei › Transitiv lukning › **Kjøretid**

init › $\Theta(n^2)$
for $i = 1$ **to** n
$\qquad$ **for** $j = 1$ **to** n
$\qquad\qquad$ sett $t_{ij}^{(0)}$ › $\mathrm{O}(1)$
for $k = 1$ **to** n
$\qquad$ evt. ny matrise › $\Theta(n^2)$
$\qquad$ **for** $i = 1$ **to** n
$\qquad\qquad$ **for** $j = 1$ **to** n
$\qquad\qquad\qquad$ sett $t_{ij}^{(k)}$ › $\mathrm{O}(1)$
return › $\mathrm{O}(1)$

init › $\Theta(n^2)$
for $i = 1$ **to** n
    for $j = 1$ **to** n
        sett $t_{ij}^{(0)}$ › $O(1)$
for $k = 1$ **to** n
    evt. ny matrise › $\Theta(n^2)$
    for $i = 1$ **to** n
        for $j = 1$ **to** n
            sett $t_{ij}^{(k)}$ › $O(1)$
return › $O(1)$

Totalt: $\Theta(n^3)$

m [j, k] :=
end ancestor

ALGORITHM 97
SHORTEST PATH
ROBERT W. FLOYD
Armour Research Foundation, Chicago, Ill.

procedure shortest path (m, n); **value** n; **integ**
comment Initially $m[i, j]$ is the length of a d
point i of a network to point j. If no direct link
initially 1010. At completion, $m [i, j]$ is the leng
path from i to j. If none exists, $m [i, j]$ is 1010.
SHALL, S. A theorem on Boolean matrices. J, A
begin
integer i, j, k; **real** inf, s; inf := 1010;
for i := 1 **step** 1 **until** n **do**
for j := 1 **step** 1 **until** n **do**
if m [j, i] < inf **then**
for k := 1 **step** 1 **until** n **do**
if m [i, k] < inf **then**
begin s := m [j, i] + m [i, k];
if s < m [j, k] **then** m [j, k] := s
end
end shortest path

Contributions to this departme
stated in the Algorithms Depar
(*Communications*, February, 1960
notation should be used (see Co
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Floyd-Warshall

Fra hver node til alle andre?

Fra hver node til alle andre?

› DIJKSTRA med tabell: $O(V^3 + VE)$

Fra hver node til alle andre?

› DIJKSTRA med tabell: $O(V^3 + VE)$

› ...med binærhaug: $\quad O(VE \lg V)$

Fra hver node til alle andre?

› $\textsc{Dijkstra}$ med tabell: $O(V^3 + VE)$

› . . . med binærhaug: $O(VE \lg V)$

› . . . med Fib.-haug: $O(V^2 \lg V + VE)$

Fra hver node til alle andre?

> › Dijkstra med tabell: $O(V^3 + VE)$
>
> › ...med binærhaug: $O(VE \lg V)$
>
> › ...med Fib.-haug: $O(V^2 \lg V + VE)$
>
> › Bellman-Ford: $\Theta(V^2 E)$

Fra hver node til alle andre?

- › DIJKSTRA med tabell: $O(V^3 + VE)$
- › ...med binærhaug: $O(VE \lg V)$
- › ...med Fib.-haug: $O(V^2 \lg V + VE)$
- › BELLMAN-FORD: $\Theta(V^2 E)$

Målsetting:

Fra hver node til alle andre?

› DIJKSTRA med tabell: $O(V^3 + VE)$

› ...med binærhaug: $O(VE \lg V)$

› ...med Fib.-haug: $O(V^2 \lg V + VE)$

› BELLMAN-FORD: $\Theta(V^2 E)$

Målsetting:

› Vi vil tillate negative kanter

Fra hver node til alle andre?

› DIJKSTRA med tabell: $O(V^3 + VE)$

› ...med binærhaug: $O(VE \lg V)$

› ...med Fib.-haug: $O(V^2 \lg V + VE)$

› BELLMAN-FORD: $\Theta(V^2 E)$

Målsetting:

› Vi vil tillate negative kanter

› Vi vil ha lavere asymptotisk kjøretid ...

Fra hver node til alle andre?

› DIJKSTRA med tabell: $O(V^3 + VE)$

› . . . med binærhaug: $O(VE \lg V)$

› . . . med Fib.-haug: $O(V^2 \lg V + VE)$

› BELLMAN-FORD: $\Theta(V^2 E)$

Målsetting:

› Vi vil tillate negative kanter

› Vi vil ha lavere asymptotisk kjøretid . . .

› . . . og vil ha lavere konstantledd

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Fra hver node til alle andre?

›   DIJKSTRA med tabell: $O(V^3 + VE)$

›   . . . med binærhaug: $O(VE \lg V)$

›   . . . med Fib.-haug: $O(V^2 \lg V + VE)$

›   BELLMAN-FORD: $\Theta(V^2 E)$

Målsetting:

›   Vi vil tillate negative kanter

›   Vi vil ha lavere asymptotisk kjøretid . . .

›   . . . og vil ha lavere konstantledd

Nok en gang: Hva blir «koordinatene» til delproblemene?

$$i\,j\,k$$

Korteste vei fra i til j via noder fra $\{1\ldots k\}$

$$d_{ij}^{(k)}$$

Korteste vei fra i til j via noder fra $\{1 \ldots k\}$

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$$\pi_{ij}^{(k)}$$

Forgjengeren til j om vi starter i i og går via noder fra $\{1\dots k\}$
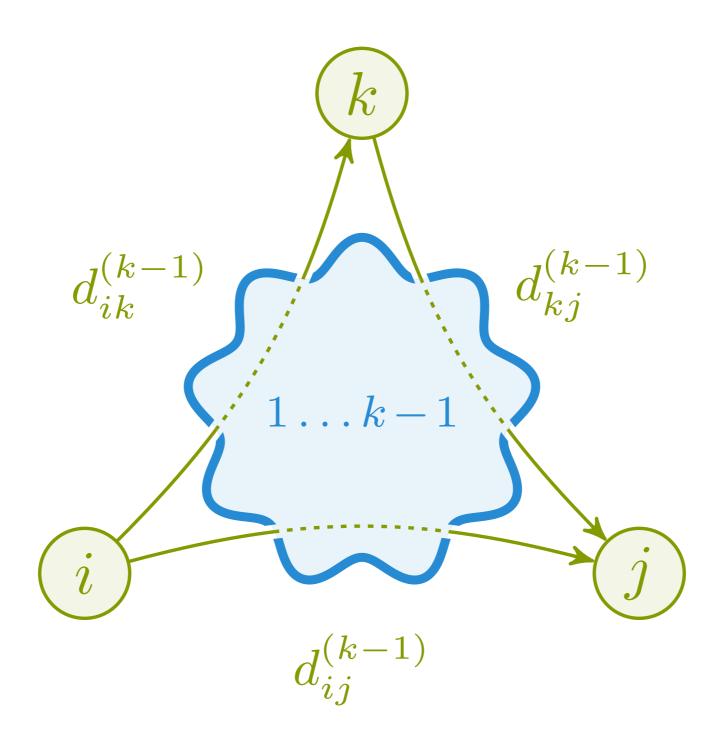
$$d_{ij}^{(k)} = \text{korteste vei fra } i \text{ til } j \text{ via noder fra } \{1 \dots k\}$$
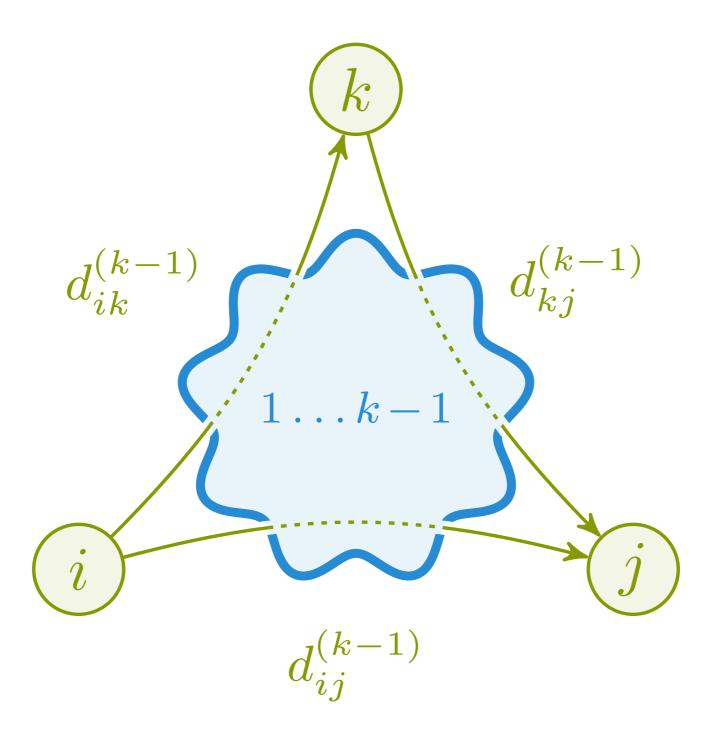
Som for ryggsekkproblemet: Skal k være med eller ikke?

Stiene p, p_1 og p_2 går kun via noder fra $\{1 \ldots k-1\}$

$$d_{ij}^{(k)} \text{ kan enten være } d_{ij}^{(k-1)} \text{ eller } d_{ik}^{(k-1)} + d_{kj}^{(k-1)}$$

$$d_{ij}^{(k)} = \min\left(d_{ij}^{(k-1)},\ d_{ik}^{(k-1)} + d_{kj}^{(k-1)}\right)$$

$$d_{ij}^{(k)} = \begin{cases} w_{ij} & \text{if } k = 0\,, \\ \min\left(d_{ij}^{(k-1)},\, d_{ik}^{(k-1)} + d_{kj}^{(k-1)}\right) & \text{if } k \geq 1\,. \end{cases}$$

$$\pi_{ij}^{(0)} = \begin{cases} \text{NIL} & \text{if } i = j \text{ or } w_{ij} = \infty\,, \\ i & \text{if } i \neq j \text{ and } w_{ij} < \infty\,. \end{cases}$$

$$
\pi_{ij}^{(k)} = \left\{ \quad\quad\quad \text{if } d_{ij}^{(k-1)} \leq d_{ik}^{(k-1)} + d_{kj}^{(k-1)} \ ,
\right.
$$

$$
\pi_{ij}^{(k)} = \begin{cases} \pi_{ij}^{(k-1)} & \text{if } d_{ij}^{(k-1)} \leq d_{ik}^{(k-1)} + d_{kj}^{(k-1)} \end{cases},
$$

$$
\pi_{ij}^{(k)} = \begin{cases} \pi_{ij}^{(k-1)} & \text{if } d_{ij}^{(k-1)} \leq d_{ik}^{(k-1)} + d_{kj}^{(k-1)} \ , \\ & \text{if } d_{ij}^{(k-1)} > d_{ik}^{(k-1)} + d_{kj}^{(k-1)} \ . \end{cases}
$$

$$
\pi_{ij}^{(k)} = \begin{cases} \pi_{ij}^{(k-1)} & \text{if } d_{ij}^{(k-1)} \leq d_{ik}^{(k-1)} + d_{kj}^{(k-1)} \ , \\ \pi_{kj}^{(k-1)} & \text{if } d_{ij}^{(k-1)} > d_{ik}^{(k-1)} + d_{kj}^{(k-1)} \ . \end{cases}
$$

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$$
d_{ij}^{(k)} = \begin{cases} w_{ij} & \text{if } k = 0\,, \\ \min\left(d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)}\right) & \text{if } k \geq 1\,. \end{cases}
$$

$$
d_{ij}^{(k)} = \begin{cases} w_{ij} & \text{if } k = 0\,, \\ \min\left(d_{ij}^{(k-1)},\, d_{ik}^{(k-1)} + d_{kj}^{(k-1)}\right) & \text{if } k \geq 1\,. \end{cases}
$$

Som før: Vi kan ha gått innom k i én av delstiene, siden vi blander iterasjoner – men vi antar at det ikke er noen negative sykler, og en positiv sykel vil aldri lønne seg (og vil dermed ikke bli med).

Vi kan bruke én tabell igjen (se oppgave 25.2-4)

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$$\pi_{ij}^{(k)} = \begin{cases} \pi_{ij}^{(k-1)} & \text{if } d_{ij}^{(k-1)} \leq d_{ik}^{(k-1)} + d_{kj}^{(k-1)} \;, \\ \pi_{kj}^{(k-1)} & \text{if } d_{ij}^{(k-1)} > d_{ik}^{(k-1)} + d_{kj}^{(k-1)} \;. \end{cases}$$

Fordi denne bare baserer seg på valget vi gjør for d.

Også her holder det med én tabell

Floyd-Warshall(W)

For hver node i og j: Hva er korteste vei $i \rightsquigarrow j$?

Floyd-Warshall(W)
1   $n = W.rows$

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Floyd-Warshall(W)

1 $n = $ W.$rows$

2 $D^{(0)} = $ W

Korteste vei $i \rightsquigarrow j$ som ikke går via andre $= w(i,j)$

Floyd-Warshall(W)
1   $n = \text{W}.rows$
2   $\text{D}^{(0)} = \text{W}$
3   **for** $k = 1$ **to** n

Vi får nå lov til å gå innom node k

$\textsc{Floyd-Warshall}(\text{W})$

1 $n = \text{W}.rows$

2 $\text{D}^{(0)} = \text{W}$

3 **for** $k = 1$ **to** n

4   let $\text{D}^{(k)} = (d_{ij}^{(k)})$ be a new $n \times n$ matrix

Hva er korteste vei $i \rightsquigarrow j$ som kun får gå innom $\{1, \ldots, k\}$?

$\textsc{Floyd-Warshall}(W)$

1 $n = W.rows$

2 $D^{(0)} = W$

3 **for** $k = 1$ **to** n

4   let $D^{(k)} = (d_{ij}^{(k)})$ be a new $n \times n$ matrix

5   **for** $i = 1$ **to** n

For hver mulig startnode...

Floyd-Warshall(W)

1 $n = \text{W}.rows$

2 $\text{D}^{(0)} = \text{W}$

3 **for** $k = 1$ **to** n

4   let $\text{D}^{(k)} = (d_{ij}^{(k)})$ be a new $n \times n$ matrix

5   **for** $i = 1$ **to** n

6   **for** $j = 1$ **to** n

For hver mulig sluttnode...

Floyd-Warshall(W)

1 $n = \mathrm{W}.rows$

2 $\mathrm{D}^{(0)} = \mathrm{W}$

3 **for** $k = 1$ **to** n

4   let $\mathrm{D}^{(k)} = (d_{ij}^{(k)})$ be a new $n \times n$ matrix

5   **for** $i = 1$ **to** n

6   **for** $j = 1$ **to** n

7                  $d_{ij}^{(k)} = \min\left(d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)}\right)$

Er det raskere å gå innom k (og ellers fortsatt kun $\{1, \dots, k-1\}$)?

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FLOYD-WARSHALL(W)

1 $n = \text{W}.rows$

2 $\text{D}^{(0)} = \text{W}$

3 **for** $k = 1$ **to** n

4   let $\text{D}^{(k)} = \left(d_{ij}^{(k)}\right)$ be a new $n \times n$ matrix

5   **for** $i = 1$ **to** n

6   **for** $j = 1$ **to** n

7                  $d_{ij}^{(k)} = \min\left(d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)}\right)$

8 **return** $\text{D}^{(n)}$

Korteste vei $i \rightsquigarrow j$ som får gå innom $\{1, \ldots, n\}$, dvs. alle

$$\textsc{Floyd-Warshall}'(W)$$

Forenklet: Bruker bare D heller enn $D^{(k)}$ (se 25.2-4, uten Π)

$\textsc{Floyd-Warshall}'(\textrm{W})$

1 $n = \textrm{W}.rows$

$\textsc{Floyd-Warshall}'(\text{W})$
1 $n = \text{W}.rows$
2 initialize D and Π

Ikke via noen: $d_{ij} = w(i, j)$; $\pi_{ij} = i$ når $i \neq j$ og $w(i, j) < \infty$

Floyd-Warshall$'$(W)

1   $n = $ W.$rows$

2   initialize D and Π

3   **for** $k = 1$ **to** n

Vi får nå lov til å gå innom node k

Floyd-Warshall′(W)
1   $n = \text{W}.rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n

For hver mulig startnode...

$\textsc{Floyd-Warshall}'(\text{W})$

1 $n = \text{W}.rows$

2 initialize D and Π

3 **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

For hver mulig sluttnode ...

$\textsc{Floyd-Warshall}'(\text{W})$

1 $n = \text{W}.rows$

2 initialize D and Π

3 **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

6   **if** $d_{ij} > d_{ik} + d_{kj}$

Er det raskere å gå innom k (og ellers fortsatt kun $\{1, \ldots, k-1\}$)?

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Floyd-Warshall$'$(W)

1   $n = \text{W}.rows$

2   initialize D and Π

3   **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

6   **if** $d_{ij} > d_{ik} + d_{kj}$

7                       $d_{ij} = d_{ik} + d_{kj}$

Oppdatér avstanden

Floyd-Warshall$'$(W)
1 $n = \mathrm{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$

Husk valget: Forgengeren til i om vi kommer fra j

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Floyd-Warshall$'$(W)

1 $n = \text{W}.rows$

2 initialize D and Π

3 **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

6   **if** $d_{ij} > d_{ik} + d_{kj}$

7                          $d_{ij} = d_{ik} + d_{kj}$

8                          $\pi_{ij} = \pi_{kj}$

9 **return** D, Π

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Floyd-Warshall$'$(W)
1   $n = $ W.*rows*
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \ = \ 1, 1, 1$

D

	1	2	3	4	5
1	0	3	8	∞	−4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	∞	−5	0	∞
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4		4		
5				5	

Floyd-Warshall$'$(W)
1 $n = \mathrm{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 1, 1, 2$

D

	1	2	3	4	5
1	0	3	8	∞	–4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	∞	–5	0	∞
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4		4		
5				5	

Floyd-Warshall′(W)
1   $n = $ W.$rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                       $d_{ij} = d_{ik} + d_{kj}$
8                       $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \ = \ 1, 1, 3$

D

	1	2	3	4	5
1	0	3	8	∞	–4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	∞	–5	0	∞
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4		4		
5				5	

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$\textsc{Floyd-Warshall}'(W)$

1 $n = W.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \; = \; 1, 1, 4$

D

	1	2	3	4	5
1	0	3	8	∞	-4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	∞	-5	0	∞
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4		4		
5				5	

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$\textsc{Floyd-Warshall}'(\text{W})$

1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \; = \; 1, 1, 5$

D

	1	2	3	4	5
1	0	3	8	∞	–4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	∞	–5	0	∞
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4		4		
5				5	

Floyd-Warshall′(W)
1 $n = \text{W.}rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 1, 2, 1$

D

	1	2	3	4	5
1	0	3	8	∞	–4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	∞	–5	0	∞
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4		4		
5				5	

Floyd-Warshall'(W)
1 $n = \mathrm{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                      $d_{ij} = d_{ik} + d_{kj}$
8                      $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \;=\; 1, 2, 2$

D

	1	2	3	4	5
1	0	3	8	∞	−4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	∞	−5	0	∞
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4		4		
5				5	

$\textsc{Floyd-Warshall}'(\text{W})$

1 $n = \text{W}.rows$

2 initialize D and Π

3 **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

6   **if** $d_{ij} > d_{ik} + d_{kj}$

7                  $d_{ij} = d_{ik} + d_{kj}$

8                  $\pi_{ij} = \pi_{kj}$

9 **return** D, Π

$k, i, j \ = \ 1, 2, 3$

D

	1	2	3	4	5
1	0	3	8	∞	–4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	∞	–5	0	∞
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4		4		
5				5	

$\text{Floyd-Warshall}'(\text{W})$

1 $n = \text{W}.rows$

2 initialize D and Π

3 **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

6   **if** $d_{ij} > d_{ik} + d_{kj}$

7                  $d_{ij} = d_{ik} + d_{kj}$

8                  $\pi_{ij} = \pi_{kj}$

9 **return** D, Π

$k, i, j = 1, 2, 4$

D

	1	2	3	4	5
1	0	3	8	∞	−4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	∞	−5	0	∞
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4		4		
5				5	

Floyd-Warshall$'$(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \ = \ 1, 2, 5$

D

	1	2	3	4	5
1	0	3	8	∞	-4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	∞	-5	0	∞
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4		4		
5				5	

Floyd-Warshall′(W)
1 $n = W.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 1, 3, 1$

D

	1	2	3	4	5
1	0	3	8	∞	−4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	∞	−5	0	∞
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4		4		
5				5	

$\textsc{Floyd-Warshall}'(W)$

1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \ = \ 1, 3, 2$

D

	1	2	3	4	5
1	0	3	8	∞	–4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	∞	–5	0	∞
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4		4		
5				5	

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Floyd-Warshall$'$(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 1, 3, 3$

D

	1	2	3	4	5
1	0	3	8	∞	−4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	∞	−5	0	∞
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4		4		
5				5	

Floyd-Warshall′(W)

1   $n = \text{W}.rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \;=\; 1, 3, 4$

D

	1	2	3	4	5
1	0	3	8	∞	–4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	∞	–5	0	∞
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4		4		
5				5	

Floyd-Warshall′(W)
1   $n = $ W.$rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                       $d_{ij} = d_{ik} + d_{kj}$
8                       $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j = 1, 3, 5$

D

	1	2	3	4	5
1	0	3	8	∞	-4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	∞	-5	0	∞
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4		4		
5				5	

Floyd-Warshall$'$(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 1, 4, 1$

D

	1	2	3	4	5
1	0	3	8	∞	-4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	∞	-5	0	∞
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4		4		
5				5	

Floyd-Warshall'(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                          $d_{ij} = d_{ik} + d_{kj}$
8                          $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \ = \ 1, 4, 2$

D

	1	2	3	4	5
1	0	3	8	∞	–4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	∞	–5	0	∞
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4		4		
5				5	

Floyd-Warshall'(W)
1 $n = $ W.*rows*
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \;=\; 1, 4, 2$

D

	1	2	3	4	5
1	0	3	8	∞	–4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	∞	–5	0	∞
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4		4		
5				5	

Floyd-Warshall$'$(W)

1 $n = \text{W}.rows$

2 initialize D and Π

3 **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

6   **if** $d_{ij} > d_{ik} + d_{kj}$

7                  $d_{ij} = d_{ik} + d_{kj}$

8                  $\pi_{ij} = \pi_{kj}$

9 **return** D, Π

$k, i, j \; = \; 1, 4, 2$

D

	1	2	3	4	5
1	0	3	8	∞	–4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	–5	0	∞
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4		4		
5				5	

Floyd-Warshall$'$(W)
1   $n = \text{W.}rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                           $d_{ij} = d_{ik} + d_{kj}$
8                           $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \; = \; 1, 4, 3$

D

	1	2	3	4	5
1	0	3	8	∞	-4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	-5	0	∞
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4	1	4		
5				5	

$\textsc{Floyd-Warshall}'(W)$

1 $n = W.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \; = \; 1, 4, 4$

D

	1	2	3	4	5
1	0	3	8	∞	-4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	-5	0	∞
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4	1	4		
5				5	

Floyd-Warshall'(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                      $d_{ij} = d_{ik} + d_{kj}$
8                      $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \ = \ 1, 4, 5$

D

	1	2	3	4	5
1	0	3	8	∞	–4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	–5	0	∞
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4	1	4		
5				5	

Floyd-Warshall′(W)
1 $n = $ W.$rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7              $d_{ij} = d_{ik} + d_{kj}$
8              $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \ = \ 1, 4, 5$

D

	1	2	3	4	5
1	0	3	8	∞	-4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	-5	0	∞
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4	1	4		
5				5	

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$$\text{Floyd-Warshall}'(W)$$

1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                 $d_{ij} = d_{ik} + d_{kj}$
8                 $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 1, 4, 5$

D

	1	2	3	4	5
1	0	3	8	∞	–4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4	1	4		
5				5	

Floyd-Warshall′(W)
1 $n = \text{W}.rows$
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3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 1, 5, 1$

D

	1	2	3	4	5
1	0	3	8	∞	−4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4	1	4		1
5				5	

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Floyd-Warshall'(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 1, 5, 2$

D

	1	2	3	4	5
1	0	3	8	∞	–4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4	1	4		1
5				5	

Floyd-Warshall'(W)
1 $n = $ W.$rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \ = \ 1, 5, 3$

D

	1	2	3	4	5
1	0	3	8	∞	–4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4	1	4		1
5				5	

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Floyd-Warshall'(W)
1 $n = $ W.*rows*
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \; = \; 1, 5, 4$

D

	1	2	3	4	5
1	0	3	8	∞	–4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4	1	4		1
5				5	

Floyd-Warshall$'$(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                          $d_{ij} = d_{ik} + d_{kj}$
8                          $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 1, 5, 5$

D

	1	2	3	4	5
1	0	3	8	∞	-4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4	1	4		1
5				5	

Floyd-Warshall′(W)
1 $n = W.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 2, 1, 1$

D

	1	2	3	4	5
1	0	3	8	∞	–4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4	1	4		1
5				5	

$$\text{D}$$

	1	2	3	4	5
1	0	3	8	∞	–4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	–5	0	–2
5	∞	∞	∞	6	0

FLOYD-WARSHALL$'$(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$$\Pi$$

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4	1	4		1
5				5	

$k, i, j = 2, 1, 2$

Floyd-Warshall$'$(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \;=\; 2, 1, 3$

D

	1	2	3	4	5
1	0	3	8	∞	–4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4	1	4		1
5				5	

Floyd-Warshall'(W)
1 $n = W.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

D

	1	2	3	4	5
1	0	3	8	∞	−4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4	1	4		1
5				5	

$k, i, j = 2, 1, 4$

Floyd-Warshall'(W)
1 $n = W.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \ = \ 2, 1, 4$

D

	1	2	3	4	5
1	0	3	8	∞	–4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4	1	4		1
5				5	

Floyd-Warshall$'$(W)
1   $n = $ W.*rows*
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \ = \ 2, 1, 4$

D

	1	2	3	4	5
1	0	3	8	4	–4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1		1
2				2	2
3		3			
4	4	1	4		1
5				5	

Floyd-Warshall$'$(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \;=\; 2, 1, 5$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3			
4	4	1	4		1
5				5	

Floyd-Warshall$'$(W)
1   $n = \text{W.}rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                       $d_{ij} = d_{ik} + d_{kj}$
8                       $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \ = \ 2, 2, 1$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3			
4	4	1	4		1
5				5	

Floyd-Warshall$'$(W)
1   $n = \text{W}.rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j = 2, 2, 2$

D

	1	2	3	4	5
1	0	3	8	4	–4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3			
4	4	1	4		1
5				5	

$\textsc{Floyd-Warshall}'(\text{W})$

1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \ = \ 2, 2, 3$

D

	1	2	3	4	5
1	0	3	8	4	–4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3			
4	4	1	4		1
5				5	

Floyd-Warshall$'$(W)
1   $n = \text{W.}rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \ = \ 2, 2, 4$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3			
4	4	1	4		1
5				5	

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & 3 & 8 & 4 & -4 \\
2 & \infty & 0 & \infty & 1 & 7 \\
3 & \infty & 4 & 0 & \infty & \infty \\
4 & 2 & 5 & -5 & 0 & -2 \\
5 & \infty & \infty & \infty & 6 & 0 \\
\end{array}
$$

D

$\textsc{Floyd-Warshall}'(\text{W})$

1 $n = \text{W.}rows$

2 initialize D and Π

3 **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

6   **if** $d_{ij} > d_{ik} + d_{kj}$

7                  $d_{ij} = d_{ik} + d_{kj}$

8                  $\pi_{ij} = \pi_{kj}$

9 **return** D, Π

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & & 1 & 1 & 2 & 1 \\
2 & & & & 2 & 2 \\
3 & & 3 & & & \\
4 & 4 & 1 & 4 & & 1 \\
5 & & & & 5 & \\
\end{array}
$$

Π

$k, i, j = 2, 2, 5$

Floyd-Warshall$'$(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \;=\; 2, 3, 1$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3			
4	4	1	4		1
5				5	

Floyd-Warshall$'$(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 2, 3, 2$

D

	1	2	3	4	5
1	0	3	8	4	−4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3			
4	4	1	4		1
5				5	

$\textsc{Floyd-Warshall}'(\text{W})$

1 $n = \text{W}.rows$

2 initialize D and Π

3 **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

6   **if** $d_{ij} > d_{ik} + d_{kj}$

7                 $d_{ij} = d_{ik} + d_{kj}$

8                 $\pi_{ij} = \pi_{kj}$

9 **return** D, Π

$k, i, j \ = \ 2, 3, 3$

D

	1	2	3	4	5
1	0	3	8	4	–4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3			
4	4	1	4		1
5				5	

Floyd-Warshall$'$(W)
1   $n = \text{W}.rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                       $d_{ij} = d_{ik} + d_{kj}$
8                       $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

D

	1	2	3	4	5
1	0	3	8	4	−4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3			
4	4	1	4		1
5				5	

$k, i, j \; = \; 2, 3, 4$

FLOYD-WARSHALL$'$(W)
1 $n = $ W.*rows*
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                              $d_{ij} = d_{ik} + d_{kj}$
8                              $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \; = \; 2, 3, 4$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3			
4	4	1	4		1
5				5	

Floyd-Warshall′(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 2, 3, 4$

D

	1	2	3	4	5
1	0	3	8	4	−4
2	∞	0	∞	1	7
3	∞	4	0	5	∞
4	2	5	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3			
4	4	1	4		1
5				5	

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Floyd-Warshall'(W)
1 $n = W.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 2, 3, 5$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	∞
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	
4	4	1	4		1
5				5	

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FLOYD-WARSHALL$'$(W)

1 $n = \text{W}.rows$

2 initialize D and Π

3 **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

6   **if** $d_{ij} > d_{ik} + d_{kj}$

7                  $d_{ij} = d_{ik} + d_{kj}$

8                  $\pi_{ij} = \pi_{kj}$

9 **return** D, Π

$k, i, j \; = \; 2, 3, 5$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	∞
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	
4	4	1	4		1
5				5	

FLOYD-WARSHALL$'$(W)

1 $n = \text{W}.rows$

2 initialize D and Π

3 **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

6   **if** $d_{ij} > d_{ik} + d_{kj}$

7                  $d_{ij} = d_{ik} + d_{kj}$

8                  $\pi_{ij} = \pi_{kj}$

9 **return** D, Π

$k, i, j = 2, 3, 5$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	
4	4	1	4		1
5				5	

Floyd-Warshall'(W)
1 $n = $ W.*rows*
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \; = \; 2, 4, 1$

D

	1	2	3	4	5
1	0	3	8	4	−4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

Floyd-Warshall$'$(W)
1   $n = \text{W}.rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                           $d_{ij} = d_{ik} + d_{kj}$
8                           $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \; = \; 2, 4, 2$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

Floyd-Warshall′(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 2, 4, 3$

D

	1	2	3	4	5
1	0	3	8	4	−4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

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Floyd-Warshall′(W)
1 $n = $ W.$rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7              $d_{ij} = d_{ik} + d_{kj}$
8              $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \; = \; 2, 4, 4$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

FLOYD-WARSHALL′(W)
1   $n = W.rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                       $d_{ij} = d_{ik} + d_{kj}$
8                       $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j = 2, 4, 5$

D

	1	2	3	4	5
1	0	3	8	4	−4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

Floyd-Warshall'(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 2, 5, 1$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

$$\text{D}$$

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

$$\Pi$$

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

Floyd-Warshall$'$(W)

1 $n = \text{W.}rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \ = \ 2, 5, 2$

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Floyd-Warshall$'$(W)

1 $n = $ W.*rows*
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \ = \ 2, 5, 3$

D

	1	2	3	4	5
1	0	3	8	4	−4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

$\textsc{Floyd-Warshall}'(\text{W})$
1   $n = \text{W}.rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \ = \ 2, 5, 4$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

Floyd-Warshall′(W)
1   $n = $ W.$rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \; = \; 2, 5, 5$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

Floyd-Warshall′(W)
1 $n = W.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 3, 1, 1$

D

	1	2	3	4	5
1	0	3	8	4	−4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

$\textsc{Floyd-Warshall}'(\text{W})$
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \;=\; 3, 1, 2$

D

	1	2	3	4	5
1	0	3	8	4	−4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

$\textsc{Floyd-Warshall}'(\text{W})$

1 $n = \text{W}.rows$

2 initialize D and Π

3 **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

6   **if** $d_{ij} > d_{ik} + d_{kj}$

7                  $d_{ij} = d_{ik} + d_{kj}$

8                  $\pi_{ij} = \pi_{kj}$

9 **return** D, Π

$k, i, j \ = \ 3, 1, 3$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

Floyd-Warshall$'$(W)
1 $n = $ W.$rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \ = \ 3, 1, 4$

D

	1	2	3	4	5
1	0	3	8	4	–4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

$\textsc{Floyd-Warshall}'(\text{W})$
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \;=\; 3, 1, 5$

D

	1	2	3	4	5
1	0	3	8	4	–4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

Floyd-Warshall$'$(W)
1 $n = $ W.*rows*
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                      $d_{ij} = d_{ik} + d_{kj}$
8                      $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \ = \ 3, 2, 1$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

FLOYD-WARSHALL$'$(W)

1 $n = $ W.*rows*

2 initialize D and Π

3 **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

6   **if** $d_{ij} > d_{ik} + d_{kj}$

7                  $d_{ij} = d_{ik} + d_{kj}$

8                  $\pi_{ij} = \pi_{kj}$

9 **return** D, Π

$k, i, j \ = \ 3, 2, 2$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

Floyd-Warshall$'$(W)
1 $n = $ W.*rows*
2 initialize D and Π
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4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \; = \; 3, 2, 3$

D

	1	2	3	4	5
1	0	3	8	4	–4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

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$\textsc{Floyd-Warshall}'(\text{W})$

1   $n = \text{W}.rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \ = \ 3, 2, 4$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

$$\text{D}$$

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

$$\Pi$$

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

Floyd-Warshall$'$(W)

1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \ = \ 3, 2, 5$

Floyd-Warshall$'$(W)
1   $n = \text{W}.rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j = 3, 3, 1$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

Floyd-Warshall$'$(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \; = \; 3, 3, 2$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

$$\text{D}$$

	1	2	3	4	5
1	0	3	8	4	–4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	–5	0	–2
5	∞	∞	∞	6	0

Floyd-Warshall$'$(W)
1 $n = \text{W}.rows$
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6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$$\Pi$$

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

$k, i, j \; = \; 3, 3, 3$

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Floyd-Warshall$'$(W)
1   $n = \text{W}.rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \ = \ 3, 3, 4$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

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Floyd-Warshall'(W)
1 $n = \text{W.}rows$
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3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 3, 3, 5$

D

	1	2	3	4	5
1	0	3	8	4	–4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

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Floyd-Warshall$'$(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 3, 4, 1$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

Floyd-Warshall$'$(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                          $d_{ij} = d_{ik} + d_{kj}$
8                          $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

D

	1	2	3	4	5
1	0	3	8	4	−4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

$k, i, j \; = \; 3, 4, 2$

Floyd-Warshall'(W)
1 $n = W.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 3, 4, 2$

D

	1	2	3	4	5
1	0	3	8	4	−4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

Floyd-Warshall′(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \ = \ 3, 4, 2$

D

	1	2	3	4	5
1	0	3	8	4	−4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	−1	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	1	4		1
5				5	

Floyd-Warshall'(W)
1 $n = \mathrm{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \ = \ 3, 4, 3$

D

	1	2	3	4	5
1	0	3	8	4	−4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	−1	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	3	4		1
5				5	

Floyd-Warshall$'$(W)
1   $n = $ W.$rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                           $d_{ij} = d_{ik} + d_{kj}$
8                           $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \; = \; 3, 4, 4$

D

	1	2	3	4	5
1	0	3	8	4	–4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	–1	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	3	4		1
5				5	

Floyd-Warshall$'$(W)
1 $n = $ W.*rows*
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \ = \ 3, 4, 5$

D

	1	2	3	4	5
1	0	3	8	4	−4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	−1	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	3	4		1
5				5	

Floyd-Warshall$'$(W)

1   $n = $ W.*rows*

2   initialize D and Π

3   **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

6   **if** $d_{ij} > d_{ik} + d_{kj}$

7                   $d_{ij} = d_{ik} + d_{kj}$

8                   $\pi_{ij} = \pi_{kj}$

9   **return** D, Π

$k, i, j \ = \ 3, 5, 1$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	-1	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	3	4		1
5				5	

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Floyd-Warshall'(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 3, 5, 2$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	-1	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	3	4		1
5				5	

Floyd-Warshall′(W)
1 $n = W.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 3, 5, 3$

D

	1	2	3	4	5
1	0	3	8	4	−4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	−1	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	3	4		1
5				5	

FLOYD-WARSHALL′(W)

1 $n = W.rows$

2 initialize D and Π

3 **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

6   **if** $d_{ij} > d_{ik} + d_{kj}$

7                  $d_{ij} = d_{ik} + d_{kj}$

8                  $\pi_{ij} = \pi_{kj}$

9 **return** D, Π

$k, i, j \; = \; 3, 5, 4$

D

	1	2	3	4	5
1	0	3	8	4	–4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	–1	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	3	4		1
5				5	

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Floyd-Warshall$'$(W)
1   $n = \mathrm{W}.rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                                   $d_{ij} = d_{ik} + d_{kj}$
8                                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \;=\; 3, 5, 5$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	-1	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	3	4		1
5				5	

Floyd-Warshall$'$(W)
1   $n = \mathrm{W}.rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                       $d_{ij} = d_{ik} + d_{kj}$
8                       $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \ = \ 4, 1, 1$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	-1	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	3	4		1
5				5	

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Floyd-Warshall′(W)
1 $n = \text{W.}rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 4, 1, 2$

D

	1	2	3	4	5
1	0	3	8	4	−4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	−1	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	3	4		1
5				5	

Floyd-Warshall′(W)

1 $n = W.rows$

2 initialize D and Π

3 **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

6   **if** $d_{ij} > d_{ik} + d_{kj}$

7                  $d_{ij} = d_{ik} + d_{kj}$

8                  $\pi_{ij} = \pi_{kj}$

9 **return** D, Π

$k, i, j \;=\; 4, 1, 3$

D

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	-1	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	3	4		1
5				5	

Floyd-Warshall′(W)
1   $n = W.rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j = 4, 1, 3$

D

	1	2	3	4	5
1	0	3	8	4	–4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	–1	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	3	4		1
5				5	

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FLOYD-WARSHALL$'$(W)

1 $n = $ W.$rows$

2 initialize D and Π

3 **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

6   **if** $d_{ij} > d_{ik} + d_{kj}$

7                  $d_{ij} = d_{ik} + d_{kj}$

8                  $\pi_{ij} = \pi_{kj}$

9 **return** D, Π

$k, i, j \; = \; 4, 1, 3$

D

	1	2	3	4	5
1	0	3	–1	4	–4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	–1	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	1	2	1
2				2	2
3		3		2	2
4	4	3	4		1
5				5	

Floyd-Warshall′(W)
1   $n = W.rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \; = \; 4, 1, 4$

D

	1	2	3	4	5
1	0	3	−1	4	−4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	−1	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2				2	2
3		3		2	2
4	4	3	4		1
5				5	

Floyd-Warshall′(W)
1 $n = $ W.$rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \ = \ 4, 1, 5$

D

	1	2	3	4	5
1	0	3	–1	4	–4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	–1	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2				2	2
3		3		2	2
4	4	3	4		1
5				5	

$\textsc{Floyd-Warshall}'(W)$

1 $n = W.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 4, 2, 1$

D

	1	2	3	4	5
1	0	3	−1	4	−4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	−1	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2				2	2
3		3		2	2
4	4	3	4		1
5				5	

Floyd-Warshall'(W)
1 $n = $ W.$rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                          $d_{ij} = d_{ik} + d_{kj}$
8                          $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \; = \; 4, 2, 1$

D

	1	2	3	4	5
1	0	3	–1	4	–4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	–1	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2				2	2
3		3		2	2
4	4	3	4		1
5				5	

Floyd-Warshall'(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \;=\; 4, 2, 1$

D

	1	2	3	4	5
1	0	3	−1	4	−4
2	3	0	∞	1	7
3	∞	4	0	5	11
4	2	−1	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2				2	2
3		3		2	2
4	4	3	4		1
5				5	

Floyd-Warshall$'$(W)
1 $n = $ W.*rows*
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \; = \; 4, 2, 2$

D

	1	2	3	4	5
1	0	3	–1	4	–4
2	3	0	∞	1	7
3	∞	4	0	5	11
4	2	–1	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4			2	2
3		3		2	2
4	4	3	4		1
5				5	

Floyd-Warshall$'$(W)

1 $n = $ W.$rows$

2 initialize D and Π

3 **for** $k = 1$ **to** n

4   **for** $i = 1$ **to** n

5   **for** $j = 1$ **to** n

6   **if** $d_{ij} > d_{ik} + d_{kj}$

7                  $d_{ij} = d_{ik} + d_{kj}$

8                  $\pi_{ij} = \pi_{kj}$

9 **return** D, Π

$k, i, j = 4, 2, 3$

D

	1	2	3	4	5
1	0	3	–1	4	–4
2	3	0	∞	1	7
3	∞	4	0	5	11
4	2	–1	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4			2	2
3		3		2	2
4	4	3	4		1
5				5	

Floyd-Warshall$'$(W)
1   $n = $ W.*rows*
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \;=\; 4, 2, 3$

D

	1	2	3	4	5
1	0	3	–1	4	–4
2	3	0	∞	1	7
3	∞	4	0	5	11
4	2	–1	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4			2	2
3		3		2	2
4	4	3	4		1
5				5	

FLOYD-WARSHALL′(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \; = \; 4, 2, 3$

D

	1	2	3	4	5
1	0	3	−1	4	−4
2	3	0	−4	1	7
3	∞	4	0	5	11
4	2	−1	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4			2	2
3		3		2	2
4	4	3	4		1
5				5	

$\text{Floyd-Warshall}'(\text{W})$
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 4, 2, 4$

D

	1	2	3	4	5
1	0	3	−1	4	−4
2	3	0	−4	1	7
3	∞	4	0	5	11
4	2	−1	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	2
3		3		2	2
4	4	3	4		1
5				5	

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$\textsc{Floyd-Warshall}'(W)$

1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \;=\; 4, 2, 5$

D

	1	2	3	4	5
1	0	3	–1	4	–4
2	3	0	–4	1	7
3	∞	4	0	5	11
4	2	–1	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	2
3		3		2	2
4	4	3	4		1
5				5	

FLOYD-WARSHALL$'$(W)
1   $n = $ W.*rows*
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \ = \ 4, 2, 5$

D

	1	2	3	4	5
1	0	3	−1	4	−4
2	3	0	−4	1	7
3	∞	4	0	5	11
4	2	−1	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	2
3		3		2	2
4	4	3	4		1
5				5	

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FLOYD-WARSHALL$'$(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                          $d_{ij} = d_{ik} + d_{kj}$
8                          $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \;=\; 4, 2, 5$

D

	1	2	3	4	5
1	0	3	−1	4	−4
2	3	0	−4	1	−1
3	∞	4	0	5	11
4	2	−1	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	2
3		3		2	2
4	4	3	4		1
5				5	

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$$\text{Floyd-Warshall}'(W)$$

1 $n = W.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 4, 3, 1$

D

	1	2	3	4	5
1	0	3	−1	4	−4
2	3	0	−4	1	−1
3	∞	4	0	5	11
4	2	−1	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3		3		2	2
4	4	3	4		1
5				5	

FLOYD-WARSHALL$'$(W)
1   $n = \text{W}.rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                       $d_{ij} = d_{ik} + d_{kj}$
8                       $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \ = \ 4, 3, 1$

D

	1	2	3	4	5
1	0	3	–1	4	–4
2	3	0	–4	1	–1
3	∞	4	0	5	11
4	2	–1	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3		3		2	2
4	4	3	4		1
5				5	

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Floyd-Warshall$'$(W)
1   $n = \text{W.}rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \ = \ 4, 3, 1$

D

	1	2	3	4	5
1	0	3	–1	4	–4
2	3	0	–4	1	–1
3	7	4	0	5	11
4	2	–1	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3		3		2	2
4	4	3	4		1
5				5	

Floyd-Warshall′(W)
1 $n = $ W.*rows*
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                          $d_{ij} = d_{ik} + d_{kj}$
8                          $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 4, 3, 2$

D

	1	2	3	4	5
1	0	3	–1	4	–4
2	3	0	–4	1	–1
3	7	4	0	5	11
4	2	–1	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	2
4	4	3	4		1
5				5	

478

FLOYD-WARSHALL′(W)
1 $n = $ W.*rows*
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                      $d_{ij} = d_{ik} + d_{kj}$
8                      $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

D

	1	2	3	4	5
1	0	3	–1	4	–4
2	3	0	–4	1	–1
3	7	4	0	5	11
4	2	–1	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	2
4	4	3	4		1
5				5	

$k, i, j = 4, 3, 3$

Floyd-Warshall$'$(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 4, 3, 4$

D

	1	2	3	4	5
1	0	3	–1	4	–4
2	3	0	–4	1	–1
3	7	4	0	5	11
4	2	–1	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	2
4	4	3	4		1
5				5	

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Floyd-Warshall$'$(W)
1 $n = $ W.$rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \; = \; 4, 3, 5$

D

	1	2	3	4	5
1	0	3	-1	4	-4
2	3	0	-4	1	-1
3	7	4	0	5	11
4	2	-1	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	2
4	4	3	4		1
5				5	

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$\textsc{Floyd-Warshall}'(W)$

1 $n = W.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \ = \ 4, 3, 5$

D

	1	2	3	4	5
1	0	3	–1	4	–4
2	3	0	–4	1	–1
3	7	4	0	5	11
4	2	–1	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	2
4	4	3	4		1
5				5	

$\textsc{Floyd-Warshall}'(\text{W})$
1   $n = \text{W}.rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                           $d_{ij} = d_{ik} + d_{kj}$
8                           $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \; = \; 4, 3, 5$

D

	1	2	3	4	5
1	0	3	−1	4	−4
2	3	0	−4	1	−1
3	7	4	0	5	3
4	2	−1	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	2
4	4	3	4		1
5				5	

Floyd-Warshall′(W)
1   $n = $ W.*rows*
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j = 4, 4, 1$

D

	1	2	3	4	5
1	0	3	–1	4	–4
2	3	0	–4	1	–1
3	7	4	0	5	3
4	2	–1	–5	0	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5				5	

FLOYD-WARSHALL'(W)
1   $n = $ W.$rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \; = \; 4, 4, 2$

D

	1	2	3	4	5
1	0	3	−1	4	−4
2	3	0	−4	1	−1
3	7	4	0	5	3
4	2	−1	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5				5	

Floyd-Warshall'(W)
1 $n = \text{W.}rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 4, 4, 3$

D

	1	2	3	4	5
1	0	3	−1	4	−4
2	3	0	−4	1	−1
3	7	4	0	5	3
4	2	−1	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5				5	

FLOYD-WARSHALL′(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 4, 4, 4$

D

	1	2	3	4	5
1	0	3	–1	4	–4
2	3	0	–4	1	–1
3	7	4	0	5	3
4	2	–1	–5	**0**	–2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5				5	

Floyd-Warshall$'$(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 4, 4, 5$

D

	1	2	3	4	5
1	0	3	−1	4	−4
2	3	0	−4	1	−1
3	7	4	0	5	3
4	2	−1	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5				5	

$\textsc{Floyd-Warshall}'(\text{W})$
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 4, 5, 1$

D

	1	2	3	4	5
1	0	3	-1	4	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5				5	

Floyd-Warshall′(W)
1 $n = W.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 4, 5, 1$

D

	1	2	3	4	5
1	0	3	−1	4	−4
2	3	0	−4	1	−1
3	7	4	0	5	3
4	2	−1	−5	0	−2
5	∞	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5				5	

Floyd-Warshall'(W)
1 $n = \text{W.}rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 4, 5, 1$

D

	1	2	3	4	5
1	0	3	−1	4	−4
2	3	0	−4	1	−1
3	7	4	0	5	3
4	2	−1	−5	0	−2
5	8	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5				5	

Floyd-Warshall′(W)
1 $n = $ W.$rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \ = \ 4, 5, 2$

D

	1	2	3	4	5
1	0	3	–1	4	–4
2	3	0	–4	1	–1
3	7	4	0	5	3
4	2	–1	–5	0	–2
5	8	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4			5	

Floyd-Warshall'(W)
1   $n = $ W.$rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j = 4, 5, 2$

D

	1	2	3	4	5
1	0	3	–1	4	–4
2	3	0	–4	1	–1
3	7	4	0	5	3
4	2	–1	–5	0	–2
5	8	∞	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4			5	

FLOYD-WARSHALL′(W)
1   $n = \text{W}.rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j = 4, 5, 2$

D

	1	2	3	4	5
1	0	3	–1	4	–4
2	3	0	–4	1	–1
3	7	4	0	5	3
4	2	–1	–5	0	–2
5	8	5	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4			5	

Floyd-Warshall$'$(W)
1   $n = $ W.*rows*
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                           $d_{ij} = d_{ik} + d_{kj}$
8                           $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \; = \; 4, 5, 3$

D

	1	2	3	4	5
1	0	3	-1	4	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3		5	

Floyd-Warshall′(W)
1 $n = W.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 4, 5, 3$

D

	1	2	3	4	5
1	0	3	–1	4	–4
2	3	0	–4	1	–1
3	7	4	0	5	3
4	2	–1	–5	0	–2
5	8	5	∞	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3		5	

Floyd-Warshall$'$(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 4, 5, 3$

D

	1	2	3	4	5
1	0	3	–1	4	–4
2	3	0	–4	1	–1
3	7	4	0	5	3
4	2	–1	–5	0	–2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3		5	

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Floyd-Warshall'(W)
1 $n = W.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 4, 5, 4$

D

	1	2	3	4	5
1	0	3	–1	4	–4
2	3	0	–4	1	–1
3	7	4	0	5	3
4	2	–1	–5	0	–2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall$'$(W)
1 $n = $ W.*rows*
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \; = \; 4, 5, 5$

D

	1	2	3	4	5
1	0	3	−1	4	−4
2	3	0	−4	1	−1
3	7	4	0	5	3
4	2	−1	−5	0	−2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

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Floyd-Warshall'(W)
1   $n = \text{W}.rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j = 5, 1, 1$

D

	1	2	3	4	5
1	0	3	−1	4	−4
2	3	0	−4	1	−1
3	7	4	0	5	3
4	2	−1	−5	0	−2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall′(W)
1   $n = \text{W}.rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j = 5, 1, 2$

D

	1	2	3	4	5
1	0	3	–1	4	–4
2	3	0	–4	1	–1
3	7	4	0	5	3
4	2	–1	–5	0	–2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall′(W)
1   $n = \text{W}.rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \ = \ 5, 1, 2$

D

	1	2	3	4	5
1	0	3	–1	4	–4
2	3	0	–4	1	–1
3	7	4	0	5	3
4	2	–1	–5	0	–2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall$'$(W)
1   $n = $ W.$rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                   $d_{ij} = d_{ik} + d_{kj}$
8                   $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j \; = \; 5, 1, 2$

D

	1	2	3	4	5
1	0	1	−1	4	−4
2	3	0	−4	1	−1
3	7	4	0	5	3
4	2	−1	−5	0	−2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		1	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall′(W)
1   $n = \text{W}.rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                           $d_{ij} = d_{ik} + d_{kj}$
8                           $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j = 5, 1, 3$

D

	1	2	3	4	5
1	0	1	-1	4	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall′(W)
1 $n = $ W.*rows*
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \;=\; 5, 1, 3$

D

	1	2	3	4	5
1	0	1	–1	4	–4
2	3	0	–4	1	–1
3	7	4	0	5	3
4	2	–1	–5	0	–2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall'(W)
1 $n = W.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 5, 1, 3$

D

	1	2	3	4	5
1	0	1	-3	4	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

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FLOYD-WARSHALL′(W)
1 $n = $ W.*rows*
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 5, 1, 4$

D

	1	2	3	4	5
1	0	1	-3	4	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall'(W)
1 $n = $ W.*rows*
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \ = \ 5, 1, 4$

D

	1	2	3	4	5
1	0	1	-3	4	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall'(W)
1   $n = $ W.$rows$
2   initialize D and Π
3   **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
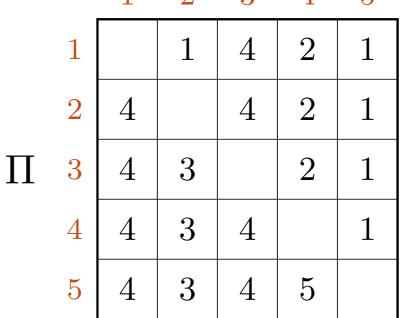6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                           $d_{ij} = d_{ik} + d_{kj}$
8                           $\pi_{ij} = \pi_{kj}$
9   **return** D, Π

$k, i, j = 5, 1, 4$

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	2	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall'(W)
1 $n = $ W.*rows*
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5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \;=\; 5, 1, 5$

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall′(W)
1 $n = \text{W.}rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \; = \; 5, 2, 1$

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall'(W)
1 $n = \text{W.}rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \ = \ 5, 2, 2$

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall′(W)
1 $n = \mathrm{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 5, 2, 3$

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall′(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 5, 2, 4$

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall′(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 5, 2, 5$

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

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Floyd-Warshall'(W)
1 $n = W.rows$
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5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 5, 3, 1$

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall$'$(W)
1 $n = $ W.$rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 5, 3, 2$

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall$'$(W)
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4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 5, 3, 3$

D

	1	2	3	4	5
1	0	1	–3	2	–4
2	3	0	–4	1	–1
3	7	4	0	5	3
4	2	–1	–5	0	–2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall'(W)
1 $n = \mathrm{W}.rows$
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3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 5, 3, 4$

D

	1	2	3	4	5
1	0	1	−3	2	−4
2	3	0	−4	1	−1
3	7	4	0	5	3
4	2	−1	−5	0	−2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall′(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 5, 3, 5$

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall'(W)
1 $n = \text{W.}rows$
2 initialize D and Π
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5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 5, 4, 1$

D

	1	2	3	4	5
1	0	1	–3	2	–4
2	3	0	–4	1	–1
3	7	4	0	5	3
4	2	–1	–5	0	–2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall'(W)
1 $n = W.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 5, 4, 2$

D

	1	2	3	4	5
1	0	1	–3	2	–4
2	3	0	–4	1	–1
3	7	4	0	5	3
4	2	–1	–5	0	–2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall′(W)
1 $n = \text{W}.rows$
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6   **if** $d_{ij} > d_{ik} + d_{kj}$
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8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \;=\; 5, 4, 3$

D

	1	2	3	4	5
1	0	1	–3	2	–4
2	3	0	–4	1	–1
3	7	4	0	5	3
4	2	–1	–5	0	–2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall$'$(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 5, 4, 4$

D

	1	2	3	4	5
1	0	1	–3	2	–4
2	3	0	–4	1	–1
3	7	4	0	5	3
4	2	–1	–5	0	–2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall$'$(W)
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6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 5, 4, 5$

D

	1	2	3	4	5
1	0	1	–3	2	–4
2	3	0	–4	1	–1
3	7	4	0	5	3
4	2	–1	–5	0	–2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall′(W)
1 $n = W.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 5, 5, 1$

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

mlh@ntnu.no

Floyd-Warshall'(W)
1 $n = $ W.*rows*
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j \; = \; 5, 5, 2$

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

FLOYD-WARSHALL$'$(W)
1 $n = \mathrm{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 5, 5, 3$

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall′(W)
1 $n = $ W.*rows*
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 5, 5, 4$

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall$'$(W)
1 $n = \text{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 5, 5, 5$

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall'(W)
1 $n = \text{W.}rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 5, 5, 5$

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Floyd-Warshall'(W)
1 $n = \mathrm{W}.rows$
2 initialize D and Π
3 **for** $k = 1$ **to** n
4   **for** $i = 1$ **to** n
5   **for** $j = 1$ **to** n
6   **if** $d_{ij} > d_{ik} + d_{kj}$
7                  $d_{ij} = d_{ik} + d_{kj}$
8                  $\pi_{ij} = \pi_{kj}$
9 **return** D, Π

$k, i, j = 5, 5, 5$

D

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Print-All-Pairs-Shortest-Path(Π, i, j)

Hvilke noder er i stien fra i til j?

Print-All-Pairs-Shortest-Path(Π, i, j)
1   **if** $i == j$

Samme node? Kommer ingenstedsfra . . .

PRINT-ALL-PAIRS-SHORTEST-PATH(Π, i, j)
1   **if** $i == j$
2   print i

...så bare skriv ut noden

Print-All-Pairs-Shortest-Path(Π, i, j)
1   **if** $i == j$
2   print i
3   **elseif** $\pi_{ij} ==$ NIL

Hvis vi ellers ikke kom fra noe sted . . .

Print-All-Pairs-Shortest-Path(Π, i, j)

1   **if** $i == j$

2   print i

3   **elseif** $\pi_{ij} ==$ NIL

4   print "no path from" i "to" j "exists"

. . . så finnes ingen sti!

PRINT-ALL-PAIRS-SHORTEST-PATH(Π, i, j)
1 **if** $i == j$
2   print i
3 **elseif** $\pi_{ij} ==$ NIL
4   print "no path from" i "to" j "exists"
5 **else** PRINT-ALL-PAIRS-SHORTEST-PATH(Π, i, π_{ij})

Ellers kom vi fra π_{ij}. Skriv veien dit først...

PRINT-ALL-PAIRS-SHORTEST-PATH(Π, i, j)
1 **if** $i == j$
2   print i
3 **elseif** $\pi_{ij} ==$ NIL
4   print "no path from" i "to" j "exists"
5 **else** PRINT-ALL-PAIRS-SHORTEST-PATH(Π, i, π_{ij})
6   print j

. . . og deretter sluttnoden

PRINT-PATH(Π, i, j)
1 **if** $i == j$
2   print i
3 **elseif** $\pi_{ij} ==$ NIL
4   print "no such path"
5 **else** PRINT-PATH(Π, i, π_{ij})
6   print j

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

$i, j = 1, 2$

Print-Path(Π, i, j)
1 **if** $i == j$
2   print i
3 **elseif** $\pi_{ij} ==$ NIL
4   print "no such path"
5 **else** Print-Path(Π, i, π_{ij})
6   print j

$$\Pi$$

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

$i, j = 1, 2$

PRINT-PATH(Π, i, j)
1 **if** $i == j$
2   print i
3 **elseif** $\pi_{ij} ==$ NIL
4   print "no such path"
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6   print j

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

$i, j = 1, 2$

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2   print i
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6   print j

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

$i, j \;=\; 1, 2 \;›\; 1, 3$

PRINT-PATH(Π, i, j)
1   **if** $i == j$
2   print i
3   **elseif** $\pi_{ij} ==$ NIL
4   print "no such path"
5   **else** PRINT-PATH(Π, i, π_{ij})
6   print j

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

$i, j \;=\; 1, 2 \;›\; 1, 3$

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Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

$i, j = 1, 2 › 1, 3$

PRINT-PATH(Π, i, j)
1   **if** $i == j$
2   print i
3   **elseif** $\pi_{ij} ==$ NIL
4   print "no such path"
5   **else** PRINT-PATH(Π, i, π_{ij})
6   print j

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Π

$i, j \;=\; 1, 2 \;›\; 1, 3 \;›\; 1, 4$

Print-Path(Π, i, j)
1   **if** $i == j$
2   print i
3   **elseif** $\pi_{ij} ==$ NIL
4   print "no such path"
5   **else** Print-Path(Π, i, π_{ij})
6   print j

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Π

$i, j = 1, 2 › 1, 3 › 1, 4$

Print-Path(Π, i, j)

1 **if** $i == j$
2   print i
3 **elseif** $\pi_{ij} ==$ NIL
4   print "no such path"
5 **else** Print-Path(Π, i, π_{ij})
6   print j

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

$i, j \;=\; 1, 2 \,›\, 1, 3 \,›\, 1, 4$

Print-Path(Π, i, j)
1 **if** $i == j$
2   print i
3 **elseif** π_{ij} == NIL
4   print "no such path"
5 **else** Print-Path(Π, i, π_{ij})
6   print j

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Π

$i, j \;=\; 1,2 \rightarrow 1,3 \rightarrow 1,4 \rightarrow \mathbf{1,5}$

PRINT-PATH(Π, i, j)
1   **if** $i == j$
2   print i
3   **elseif** $\pi_{ij} ==$ NIL
4   print "no such path"
5   **else** PRINT-PATH(Π, i, π_{ij})
6   print j

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

$i, j \; = \; 1,2 \; › \; 1,3 \; › \; 1,4 \; › \; \mathbf{1,5}$

PRINT-PATH(Π, i, j)
1 **if** $i == j$
2   print i
3 **elseif** π_{ij} == NIL
4   print "no such path"
5 **else** PRINT-PATH(Π, i, π_{ij})
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	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

Π

$i, j \; = \; 1, 2 \, › \, 1, 3 \, › \, 1, 4 \, › \, \mathbf{1, 5}$

PRINT-PATH(Π, i, j)
1 **if** $i == j$
2   print i
3 **elseif** $\pi_{ij} ==$ NIL
4   print "no such path"
5 **else** PRINT-PATH(Π, i, π_{ij})
6   print j

$$\Pi$$

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

$i, j = 1,2 › 1,3 › 1,4 › 1,5 › \mathbf{1,1}$

PRINT-PATH(Π, i, j)
1 **if** $i == j$
2   print i
3 **elseif** π_{ij} == NIL
4   print "no such path"
5 **else** PRINT-PATH(Π, i, π_{ij})
6   print j

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

$i, j \ = \ 1, 2 \,›\, 1, 3 \,›\, 1, 4 \,›\, 1, 5 \,›\, \mathbf{1, 1}$

PRINT-PATH(Π, i, j)
1 **if** $i == j$
2   print i
3 **elseif** $\pi_{ij} ==$ NIL
4   print "no such path"
5 **else** PRINT-PATH(Π, i, π_{ij})
6   print j

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

$i, j = 1, 2 › 1, 3 › 1, 4 › \mathbf{1, 5}$

1

PRINT-PATH(Π, i, j)
1 **if** $i == j$
2   print i
3 **elseif** $\pi_{ij} ==$ NIL
4   print "no such path"
5 **else** PRINT-PATH(Π, i, π_{ij})
6   print j

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

$i, j \; = \; 1, 2 \; › \; 1, 3 \; › \; 1, 4$

1 5

PRINT-PATH(Π, i, j)
1 **if** $i == j$
2   print i
3 **elseif** π_{ij} == NIL
4   print "no such path"
5 **else** PRINT-PATH(Π, i, π_{ij})
6   print j

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

$i, j \ = \ 1, 2 \, › \, 1, 3$

1 5 4

PRINT-PATH(Π, i, j)
1 **if** $i == j$
2   print i
3 **elseif** $\pi_{ij} ==$ NIL
4   print "no such path"
5 **else** PRINT-PATH(Π, i, π_{ij})
6   print j

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

$i, j = 1, 2$

1 5 4 3

PRINT-PATH(Π, i, j)
1 **if** $i == j$
2   print i
3 **elseif** $\pi_{ij} ==$ NIL
4   print "no such path"
5 **else** PRINT-PATH(Π, i, π_{ij})
6   print j

Π

	1	2	3	4	5
1		3	4	5	1
2	4		4	2	1
3	4	3		2	1
4	4	3	4		1
5	4	3	4	5	

$i, j = 1, 2$

1 5 4 3 2

Korteste vei › Floyd-Warshall › Kjøretid

$\text{init} \rightarrow \Theta(n^2)$
for $k = 1$ **to** n
    for $i = 1$ **to** n
        for $j = 1$ **to** n
            $\min \rightarrow O(1)$
return $\rightarrow O(1)$

init › $\Theta(n^2)$
for $k = 1$ **to** n
    for $i = 1$ **to** n
        for $j = 1$ **to** n
            min › $\mathrm{O}(1)$
return › $\mathrm{O}(1)$

Totalt: $\Theta(n^3)$

1. **Johnsons algoritme**

2. **Transitiv lukning**

3. **Floyd-Warshall**