MASTER 1, Time Series

Homework

Due date: February 28 at 3.30 pm

Maximum number of students per group: 3

Problem I: Take a prince index, the inflation, the GDP, the short-term interest rate, and the daily consumption of electricity of a developed country. You could take them from FRED or Mark Watson's website or any other source.

1. Study their time series properties: plot, descriptive statistics, ACF and PACF; do the same work for the first difference. Comment the results.
2. Find the best AR(p) process for each of the variables.
3. Provide 1-step and 3-step ahead forecasts for each variable.

Problem II: Take your favorite stock price; take daily data, at least 4 years of observations.

1. Explain why it is better to work with (daily) log-returns.
2. Provide descriptive statistics (mean, variance, skewness, kurtosis, ACF, PACF) of the log-returns.
3. Find the best AR model of the data.
4. Provide diagnostics.
5. Define the variable $z_t = 1$ when the return is positive, and $z_t = 0$. Do the same work (that you did for the return) for the process z_t.

Problem III: Predictive regressions

Consider the following model

$$y_t = \beta x_{t-1} + u_t$$

$$x_t = \rho x_{t-1} + v_t$$

$$Cov\left([u_t, v_t]^\top, [u_t, v_t]\right) = \begin{bmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma_v^2 \end{bmatrix}.$$

1. Simulate the first design provided in the attached first Table (third panel). The Table is from the paper by Stambaugh (1999, Predictive Regressions, *Journal of Financial Economics*). For each design, simulate 1,000 samples. Compute the OLS estimator of β. Comment the results. Change ρ to 0.5 and 0; likewise, change σ_{uv} to 0; comment the results. Change the sample size (double the size); comment the results.

2. Do again the same regressions when one consider two-step and four-step ahead forecasts. Comment the results of the OLS estimators of the regression's slope. Compute the standard deviation of the OLS estimators by the naive method and by the Newey-West method. Compare them with the the true analytical formula (you have to compute it). Comment the results.

3. Go the the web-site of Robert Shiller (Yale University). He posted several data. Use the data he used in his book *Irrational Exuberance*. Do the same type of regressions as in the attached second table (from the book of Campbell, Lo, and MacKinlay) . You need to take real prices (and not nominal ones). It might be useful to read the appendix of Hodrick (1992, *Review of Financial Studies*) to help you in using the right variables.

Table 1
Finite-sample properties of $\hat{\beta}$

The table reports finite-sample properties of the ordinary least squares (OLS) estimator $\hat{\beta}$ in the regression

$$y_t = \alpha + \beta x_{t-1} + u_t.$$

The sampling properties are computed under the assumption that x_t obeys the process

$$x_t = \theta + \rho x_{t-1} + v_t,$$

where $\rho^2 < 1$ and $[u_t \; v_t]'$ is distributed N(0, Σ), identically and independently across t. The true bias and higher-order moments depend on ρ and Σ (with distinct elements σ_u^2, σ_v^2, and σ_{uv}). For each sample period, those parameters are set equal to the estimates obtained when y_t is the continuously compounded return in month t on the value-weighted NYSE portfolio, in excess of the one-month T-bill return, and x_t is the dividend–price ratio on the value-weighted NYSE portfolio at the end of month t. The moments in the standard setting are conditioned on $x_0, \ldots, x_{T-1}$ and ignore any dependence of u_t on those values. The p-values are associated with a test of $\beta = 0$ versus $\beta > 0$

	Sample period			
	1927–1996	1927–1951	1952–1996	1977–1996
A. True properties				
Bias	0.07	0.18	0.18	0.42
Standard deviation	0.16	0.33	0.27	0.45
Skewness	0.71	0.83	0.98	1.29
Kurtosis	3.84	4.14	4.62	5.83
p-value for $\beta = 0$	0.17	0.42	0.15	0.64
B. Properties in the standard regression setting				
Bias	0	0	0	0
Standard deviation	0.14	0.27	0.20	0.30
Skewness	0	0	0	0
Kurtosis	3	3	3	3
p-value for $\beta = 0$	0.06	0.22	0.02	0.26
C. Sample characteristics and parameter values				
$\hat{\beta}$	0.21	0.21	0.44	0.19
T	840	300	540	240
ρ	0.972	0.948	0.980	0.987
$\sigma_u^2 \times 10^4$	30.05	54.46	16.42	17.50
$\sigma_v^2 \times 10^4$	0.108	0.247	0.029	0.033
$\sigma_{uv} \times 10^4$	− 1.621	− 3.360	− 0.651	− 0.715

Eq. (3), and Σ is set equal to the sample covariance matrix of the least-squares residuals from (1) and (3). Those values for ρ and Σ, as well as the sample size T and the realized sample value of $\hat{\beta}$, are given in Part C of Table 1. Part B of the table reports the corresponding moments and p-values implied by the standard regression model. The standard deviations in Part B depend on σ_u^2 and are

Table 7.1. *Long-horizon regressions of log stock returns on the log dividend-price ratio.*

$$r_{t+1} + \cdots + r_{t+K} = \beta(K)(d_t - p_t) + \eta_{t+K,K}$$

	Forecast Horizon (K)					
	1	3	12	24	36	48
1927 to 1994						
$\hat{\beta}(K)$	0.012	0.044	0.191	0.383	0.528	0.654
$R^2(K)$	0.004	0.015	0.068	0.144	0.209	0.267
$t(\hat{\beta}(K))$	1.221	1.400	2.079	4.113	4.631	3.943
1927 to 1951						
$\hat{\beta}(K)$	0.015	0.059	0.274	0.629	0.880	1.050
$R^2(K)$	0.003	0.014	0.074	0.207	0.322	0.424
$t(\hat{\beta}(K))$	0.660	0.844	1.677	4.521	2.967	3.783
1952 to 1994						
$\hat{\beta}(K)$	0.024	0.079	0.329	0.601	0.776	0.863
$R^2(K)$	0.015	0.047	0.190	0.344	0.428	0.432
$t(\hat{\beta}(K))$	2.733	3.055	3.228	3.225	3.315	3.561

r is the log real return on a value-weighted index of NYSE, AMEX, and NASDAQ stocks. $(d-p)$ is the log ratio of dividends over the last year to the current price. Regressions are estimated by OLS, with Hansen and Hodrick (1980) standard errors, calculated from equation (A.3.3) in the Appendix setting autocovariances beyond lag $K-1$ to zero. Newey and West (1987) standard errors with $q = K-1$ or $q = 2(K-1)$ are very similar and typically are slightly smaller than those reported in the table.