Latent Variable Augmentation in Bayesian Inference

Applications for Gaussian Processes

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Zusammenfassung

Hier kommt der deutsche Abstrakt rein... ÜÖ sind ok.

Abstract

Put your abstract here...

Dedicated to ...

Acknowledgements

I would like to acknowledge the thousands of individuals who have coded for open-source projects for free. It is due to their efforts that scientific work with powerful tools is possible.

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Abbreviations

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1
Introduction

1.1 Following Bayes

- Bayes is awesome

1.2 The use of Gaussian Processes

- All these things you can do with Gaussian processes

1.3 The underestimated importance of representation

- Different representation lead to very different results, efficiency etc

2

Background

2.1 Probabilistic Bayesian Modeling

The Bayes' theorem is one of the simplest theorem in probabilites and its demonstration holds in one line, its implications are however more complex.

Let's give the very general modeling setting. We have a set of observed variables $\boldsymbol{x}$ and a set of latent (unobserved) variables $\boldsymbol{\theta}$. Given a prior distribution on $\boldsymbol{\theta}$, $p(\boldsymbol{\theta})$, and a likelihood function $p(\boldsymbol{x} \mid \boldsymbol{\theta})$ we are interested in the posterior distribution $p(\boldsymbol{\theta}|\boldsymbol{x})$ which is given by:

$$p(\boldsymbol{\theta}|\boldsymbol{x}) = \frac{p(\boldsymbol{x}|\boldsymbol{\theta})p(\boldsymbol{\theta})}{p(\boldsymbol{x})} = \frac{p(\boldsymbol{x}|\boldsymbol{\theta})p(\boldsymbol{\theta})}{\int p(\boldsymbol{x}|\boldsymbol{\theta})p(\boldsymbol{\theta})d\boldsymbol{\theta}} \tag{2.1}$$

The posterior is of interest for making prediction on previously unseen data. For example, in the example of logistic regression, we have

2.2 Gaussian Processes

GP! (**GP!**) are a class of non-parametric models to approximate functions. By definition, a **GP!** is a stochastic process where the joint distribution on any collection of variables X_t follows a (multivariate) Gaussian distribution. This Gaussian nature is what make them so attractive since operations on Gaussian variables tend to be easier and many calculus have closed-form solutions. The Gaussian distribution is to statistics what the harmonic oscillator is to physics. Although, **GP!** are defined to be a non-parametric model, one still needs to define how the covariance between each variable of the process is defined. One resorts to kernel functions [**NEED TO CITE THIS**].

2.3 Approximate Bayesian Inference

The posterior distribution in (2.1) cannot be computed in closed-form for non-trivial problems. To still be able to make predictions and render the model useful one can resort to different approximations. They can be generally sorted into two categories: sampling and variational inference.

2.3.1 Sampling

2.3.2 Variational Inference

3

Efficient Gaussian Process Classification Using Polya-Gamma Data Augmentation

4

Multi-Class Gaussian Process Classification Made Conjugate: Efficient Inference via Data Augmentation

5

Automated Augmented Conjugate Inference for Non-conjugate Gaussian Process Models

6

Variational Gaussian Particle Flow

7 Discussion

A

Appendix A