2023 Marchés de taux

N°4 Obligation à taux fixe

Maturity = 2Y

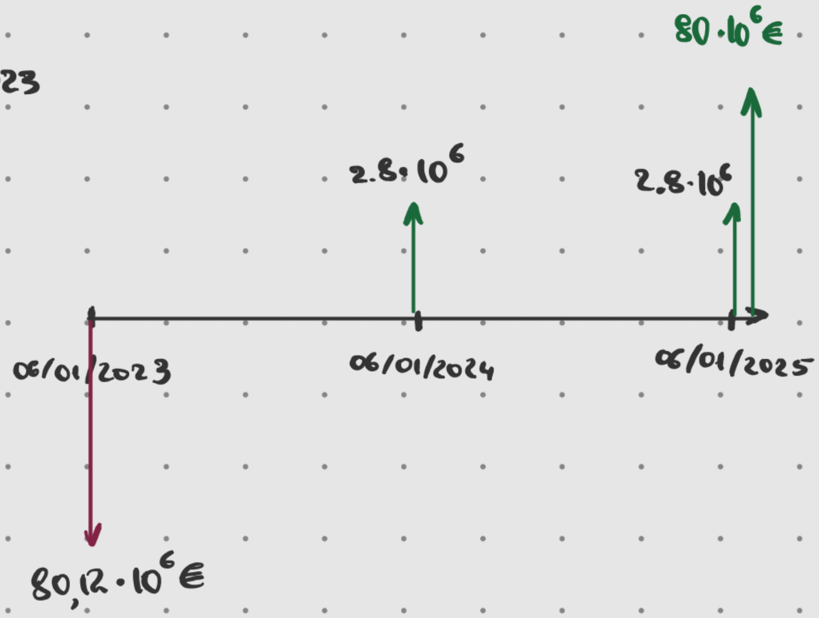
Coupon an. 3,5%

$N = 80 \cdot 10^6$ €

Date d'émission 06/01/2023

Prix d'émission = 100,15

Les flux



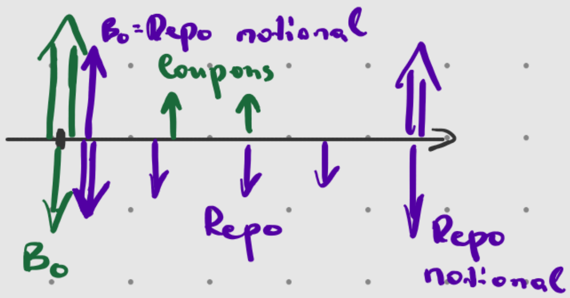
N°5 FWD price

- Sell the bond 3 months later at the price specified today (K)

Cash & Carry strategy to calculate fwd price

Two strategies of equal PVs

Contract to buy at the price K



- Buy today, finance carry & recieve coupons

- Wait an buy at K at the maturity (PV = PV(Fwd))

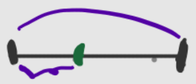
$$PV(\text{Fwd}) = \text{Spot} + PV(\underbrace{\text{Cost of carry}}_{\text{repo payments}}) - PV(\underbrace{\text{Products}}_{\text{Coupons}})$$

For bond sell the payments are invers but the price should be the same (no-arbitrage argument)

N°6 Enter an IRS with the same maturity

Complete missing Fwd rates by interpolating the Euribor futures

$$08/01/2024 = 21/12/2023 + \alpha\,(\underbrace{21/03/2024 - 21/12/2023}_{3M\,=\,91})$$



$$\alpha = \frac{18}{91}$$

$$\text{Fwd}(08/01/2024) = \text{Fwd}(21/12/2023) + \frac{18}{91}\big(\text{Fwd}(21/03/2024) - \text{Fwd}(21/12/2023)\big) = 96.85344$$

$\rightarrow$ 1 − Fwd ≈ 3.1466

$$'08/04/2024' = '21/03/2024' + \frac{18}{91}\cdot('20/06/2024' - '21/03/2024')$$

$$1 - \text{Fwd} = 1 - \left(96.545 + \frac{18}{91}(96.7042 - 96.545)\right) \approx 3.4235$$

N°7. Fixed rate for 2Y swap Annual 30/360

$t = T_0$, $B(t, T_0)$

$$\text{Swap rate} = \frac{P(T_0) - P(T_M)}{\sum_{k=1}^{M} P(T_k)(T_k - T_{k-1})} = \frac{1 - 0.93915}{(90\cdot 0.99453 + \dots)/360} = \frac{0.06085}{1.93260722} \approx 0.031482$$

N°8 • YTM for fixed coupond bond

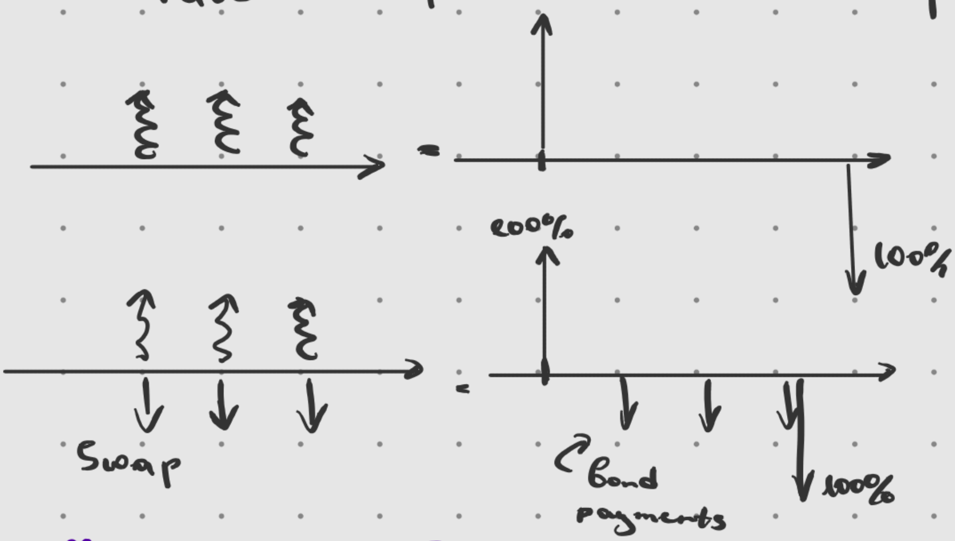
YTM = rate Y such that

$$\underset{\text{market price}}{B} = \sum_{i=1}^{N} \frac{\overset{\text{coupon}}{C}}{(1+Y)^i} + \frac{\overset{\text{notional}}{N}}{(1+Y)^N}$$

- YTM = 3,42% This rate vs swap rate?

Swap rate = Par rate → coupon rate such that price = 100%

B = 100%.



What does the difference represents?

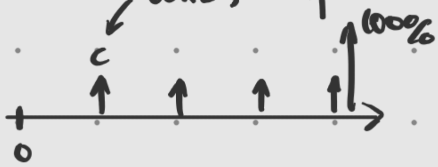
~~If YTM − R_{swap} ≥ 0 → market price of the bond ≤ 100%.~~

<u>Bonus</u> Asset swap fee = Full Bond price − Par value

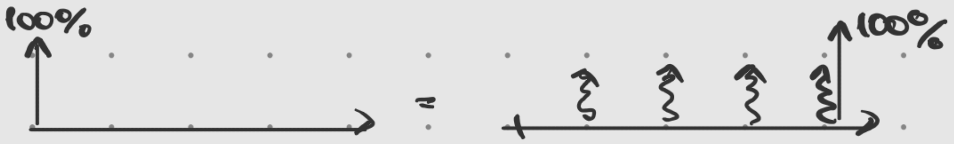
Calculate the fee

bond's coupon rate

Bond =

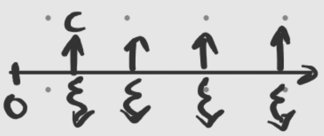


Par value =



Fee =

= Bond − Par value =



= PV of swap with fixed = rate equal to coupon

Discount factor

$$= PV_{fixed} - PV_{floating} = C \cdot \sum_{k=1}^{M} P(T_k)(T_k - T_{k-1}) - P(T_0) + P(T_N)$$

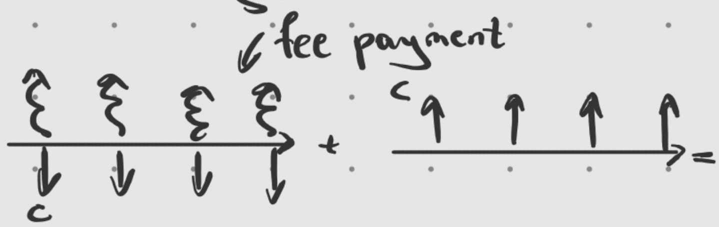
2Y swap : pay fix, receive float

Fixed: Asset swap fee + Bond coupon

Calculate the PV of the fixed leg

Calculate the asset swap margin X to add to forward Euribors in order to equate the legs.

PV_{fixed} = fee + fixed coupon =



$$= P(T_0) - P(T_m) = 1 - 0.93915 = 0.06085$$

" PV_{float} $\Rightarrow$ $\underline{X = 0}$

$\boxed{2022}$

N°5. FRA = forward rate agreement: OTC contract:

fixed rate F will be applied between T and $T+\delta$

with the notional N,

$$\text{Payoff at } T = \frac{(F - F_{fixing})\delta}{1 + F_{fixing}\delta} \cdot N$$

$\delta = 3M$ $\quad F = -0.4\%$ $\quad N = 500 \cdot 10^6$ €

$T = 04/04/2022 \rightarrow$ 3M Euribor forward $(T, T+\delta)$ by interpolation

Bonus Notional $= 75 \cdot 10^6$

$$PVBP = PV(S + 1b.p.) - PV(S) = 1b.p. \sum_{k=1}^{m} P(T_k)(T_k - T_{k-1}) \cdot N$$

How do hedge? Futures on the Bond:

$$\text{Number of Futures} = \frac{PVBP_{swap}}{PVBP_{future}}$$

Limitations:
- Changes in spread between swap & Bond
- Skew risk if $T_{swap} \neq T_{bond}$

Lozere 2023

Q1 Cobb-Duglas production function: $F(K,L) = K^{\alpha} L^{1-\alpha}$

1) Real wage of equilibrium?

2) The real cost of capital?

$$Y = A \cdot K^{\alpha} \cdot L^{1-\alpha}$$

cost of capital

Profit maximization: $Y \cdot p - wL - c \cdot K \longrightarrow \max_{K,L}$

$$\begin{cases} \partial_L Y = \dfrac{w}{P} \rightarrow w^* = P(1-\alpha)\left(\dfrac{K}{L}\right)^{\alpha} \\ \partial_K Y = \dfrac{c}{P} \rightarrow c^* = p\alpha\left(\dfrac{L}{K}\right)^{1-\alpha} \end{cases}$$

At equilibrium

Validité historique:
- Law of diminishing returns
- Scaling property
- However, it's oversimplified

Q2 IS-LM

Linear derivation of IS-LM

IS curve: $Y = C + I + G + \underbrace{NX}_{X - M}$

$C = A + bY \qquad G, X$ exogenous

$I = E - er$

$M = F + fY$

$$Y = A + E + G + X - F - er + (b - f)Y$$

$$Y = \frac{1}{1-b+f}\left(A + E + G + X - F - er\right) \quad \downarrow \text{ in } r$$

LM curve

$$\frac{M_s}{P} = L_d(r + \pi^e, Y) = \frac{L_{d1}}{P} + \frac{L_{d2}}{P} = \alpha Y - \beta r$$

($\frac{L_{d1}}{P}$: transact money dem; βr: speculative money dem.)

$$Y = \frac{1}{\alpha}\left(\frac{M_s}{P} + \beta r\right) \quad \nearrow \text{ in } r$$

Equilibrium $\quad \frac{1}{\alpha}\left(\frac{M_s}{P} + \beta r\right) = \frac{1}{1-b+f}\left(A + E + G + X - F - er\right)$

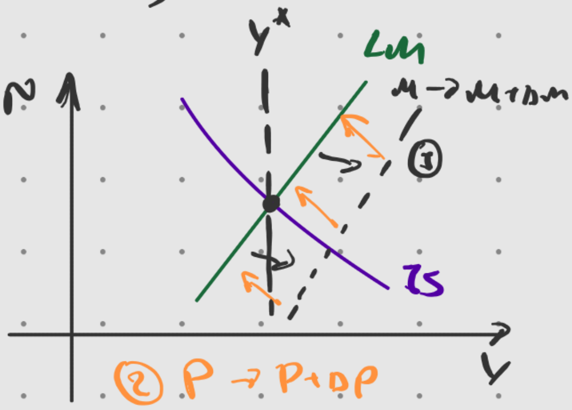
$$\frac{\beta/\alpha + e}{1-b+f} \cdot r = \frac{1}{1-b+f}\left(A + E + G + X - F - \frac{M_s}{\alpha P}(1-b+f)\right)$$

$$r^* = \frac{1}{\beta/\alpha + e}\left(A + E + G + X - F - \frac{M_s}{\alpha P}(1-b+f)\right)$$

$$Y\left(\frac{\beta/\alpha\,(1-b+f) + e}{1-b+f}\right) = \frac{\beta/\alpha}{1-b+f}\left(A + E + G + X - F\right) + \frac{e/\alpha}{1-b+f}\left(\frac{M_s}{P}\right) =$$

$$= \frac{\beta/\alpha}{1-b+f}\left(A + E + G + X - F + \frac{e}{\beta}\frac{M_s}{P}\right)$$

$$Y^* = \frac{1}{(1-b+f) + \frac{e\alpha}{\beta}}\left(A + E + G + X - F + \frac{e}{\beta}\frac{M_s}{P}\right)$$



The prices increase → → return to the equilib.

2. Biggs-Mayer-Pick macromodel

$$Y_t = Y_{c,t} + Y_{n,t}$$
$$Y_{c,t} = C_t \qquad Y_{n,t} = I_t$$

$$C_t = A K_t$$

depreciation coef.

$$K_t = (1-\delta)K_{t-1} + I_t$$

I_t is financed by debt D_t

r-rate of debt $\quad r = A - \delta \;\rightarrow\; C_t = \underbrace{rK_t}_{\text{to pay interest}} + \underbrace{\delta K_t}_{\text{to repay debt}}$

$\Rightarrow D_t = (1-\delta)K_t + I_t \;\rightarrow\; D_t = K_t$

debt growth

$$Y_t = C_t + I_t = (r+\delta)D_t + D_t - (1-\delta)D_{t-1} = (r+2\delta)D_t + (1-\delta)\Delta D_t$$

GDP relative growth

~ second derivat "credit impulse"

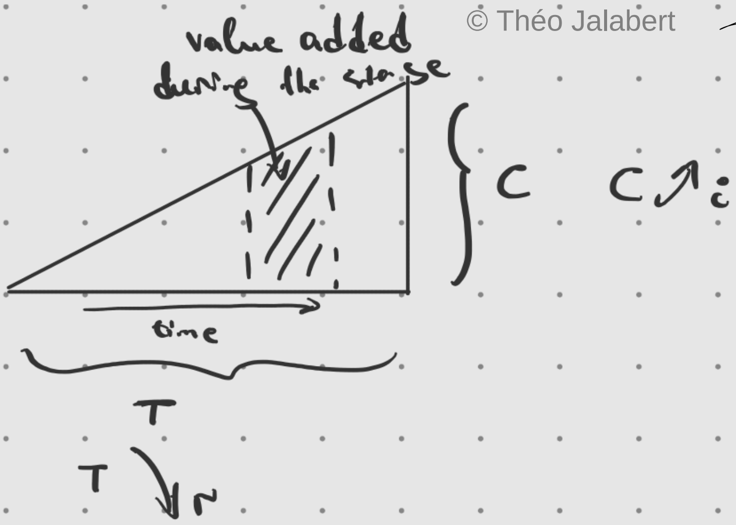
$$y = \frac{Y_t - Y_{t-1}}{Y_{t-1}} = \frac{(r+2\delta)\Delta D_t}{Y_{t-1}} + \frac{(1-\delta)\Delta\Delta D_t}{Y_{t-1}}$$

for reasonable r, δ it is more significant

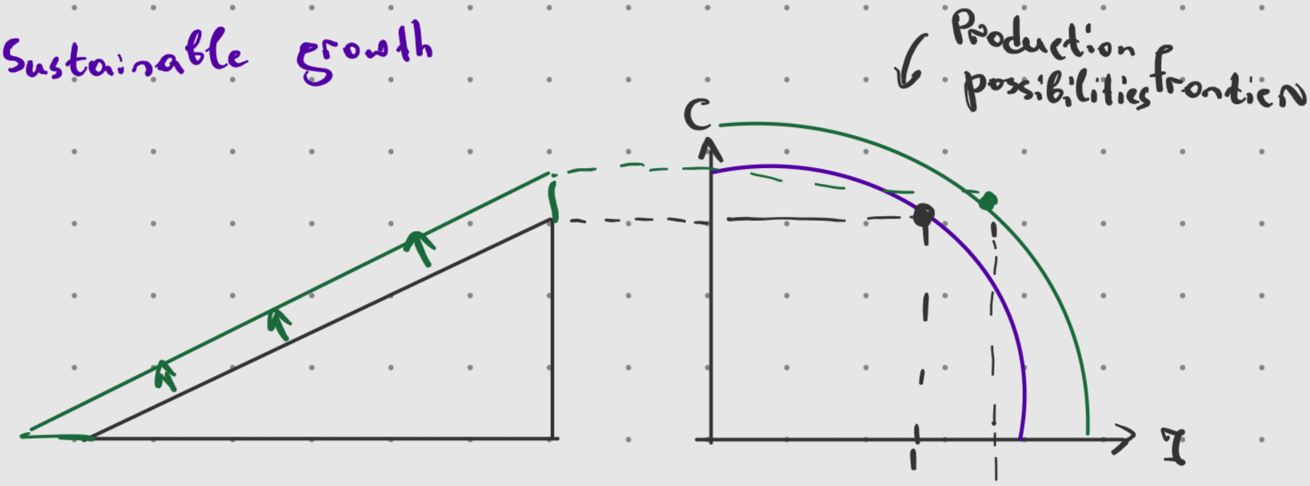
Effectiveness: $\Delta\Delta D_t$ provides a good signal to predict y due to the lag in the time series

Bonus Diagrammatical Austrian modelization

Hayekian triangle

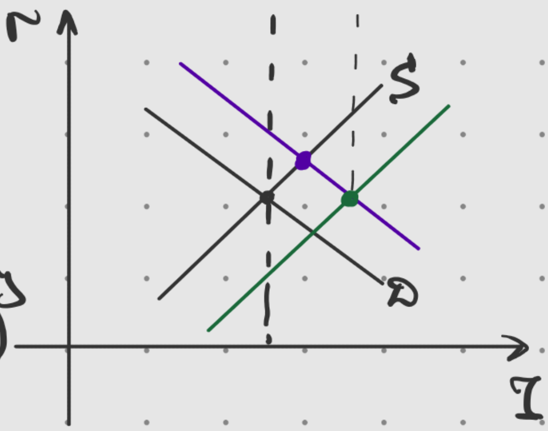


Sustainable growth



Demand = intention to participate in economy r
production, plant, human cap

Supply: Willingness to lend money
(retain earnings, equity purch)

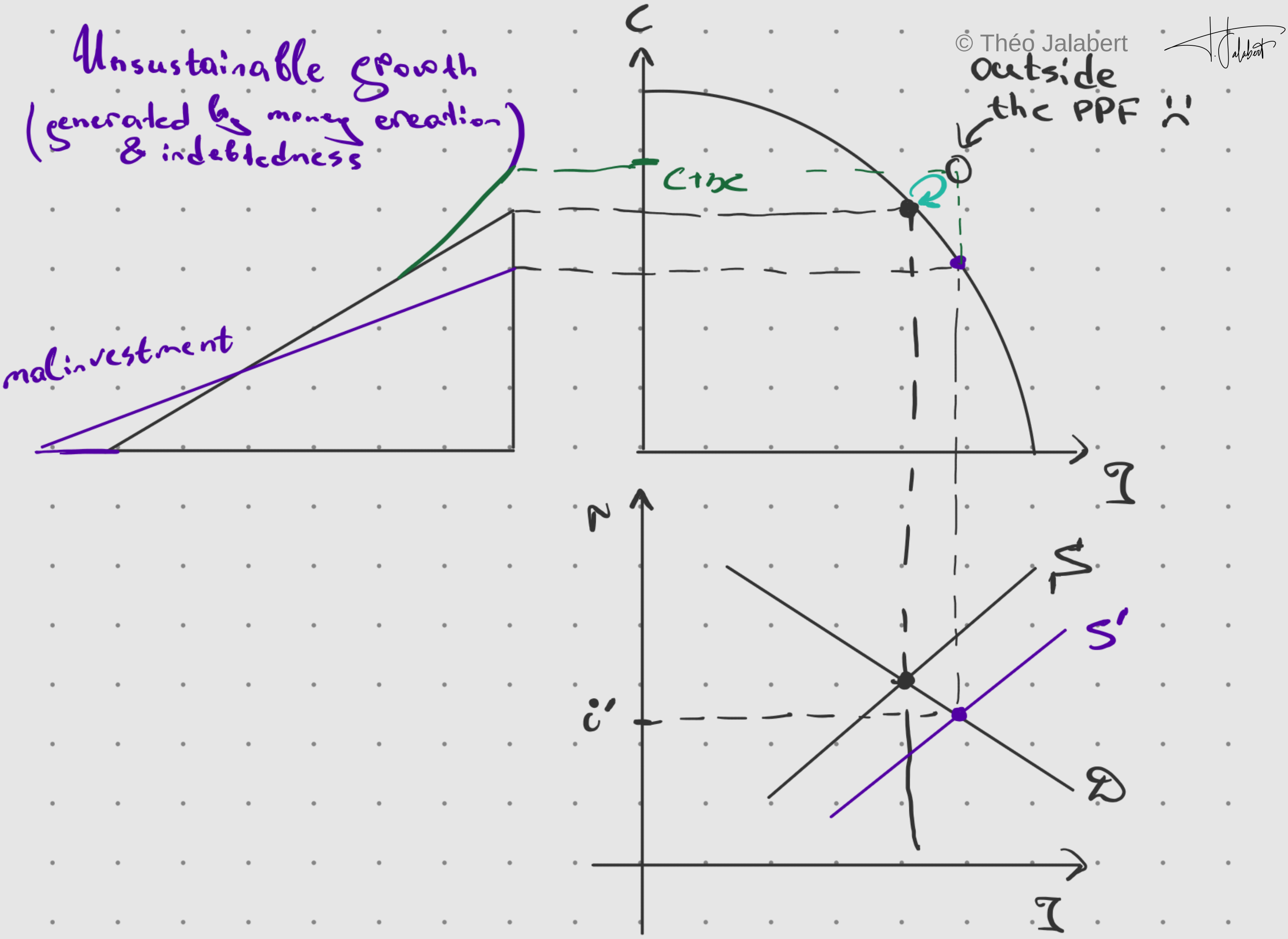


1) Rising productivity → rising demand → rising r

2) Rise of r and revenue → S ↗

3) C, I ↗ → higher and more stable PPF.

Unsustainable growth
(generated by money creation & indebtedness)



1) $M_s \nearrow$ → Savings $S \nearrow S'$ → $I \nearrow I + \Delta I$

2) C is not decreased (ΔI financed by ΔM). False expectation of BOOM ⇒ overconsumption

3) Bust: I & C have to return to initial equilibrium, but the time could be very long. System won't come back and could overshoot ⇒ Hayekian secondary deflation

YTM vs. Par yield

$$B = \sum \frac{C}{(1+YTM)^k} + \frac{1}{(1+YTM)^n}$$

par yield

$$1 = \sum c_m P(T_k) + P(T_m)$$

If coupon $c < c_m \rightarrow B < 1$

Proposition Bond issued at par has a YTM that is equal to the coupon yield (par yield)

$$\triangleright \quad 1 = \sum_{k=1}^{N} \frac{c_m}{(1+YTM)^k} + \frac{1}{(1+YTM)^N} = c_m \frac{1 - \frac{1}{(1+YTM)^N}}{1 - \frac{1}{1+YTM}} + \frac{1}{(1+YTM)^N} =$$

$$\left\{ \sum_{k=1}^{N} q^k = \frac{1-q^N}{1-q} \cdot q \right\}$$

$$= \frac{c_m}{YTM} \cdot \left(1 - \frac{1}{(1+YTM)^N}\right) + \frac{1}{(1+YTM)^N} = 1 \longrightarrow \boxed{c_m = YTM} \quad \square$$

In the general case $\quad \dfrac{C}{YTM} = \dfrac{B - (1+YTM)^{-1}}{1-(1+YTM)^{-1}}$

We conclude that if current coupon yield is higher than YTM then $B > 1$. At par, they will coincide.