$$p = \frac{e^{rT} - e^{-\sigma\sqrt{T/n}}}{e^{\sigma\sqrt{T/n}} - e^{-\sigma\sqrt{T/n}}} \qquad p^* = \frac{pu}{pu + (1-p)d} = \frac{\left(e^{rT/n} - e^{-\sigma\sqrt{T/n}}\right)e^{\sigma\sqrt{T/n}}}{\left(e^{rT/n} - e^{-\sigma\sqrt{T/n}}\right)e^{\sigma\sqrt{T/n}} + \left(e^{\sigma\sqrt{T/n}} - e^{rT/n}\right)e^{-\sigma\sqrt{T/n}}}$$

$$= \frac{\left(e^{rT/n} - e^{-\sigma\sqrt{T/n}}\right)e^{\sigma\sqrt{T/n}}}{\left(e^{\sigma\sqrt{T/n}} - e^{-\sigma\sqrt{T/n}}\right)e^{rT/n}}$$

Mq $\sqrt{n}\left(p^* - \frac{1}{2}\right) \xrightarrow[n\to\infty]{} \frac{\left(r + \frac{\sigma^2}{2}\right)\sqrt{T}}{2\sigma}$

$$\sqrt{n}\left(p^* - \frac{1}{2}\right) \underset{\substack{\uparrow \\ n \in \mathcal{V}(\infty)}}{=} \sqrt{n}\left[\frac{\left(1 + r\,T/n - \left(1 - \sigma\sqrt{T/n}\right)\right)\left(1 + \sigma\sqrt{T/n}\right)}{\left(1 + \sigma\sqrt{T/n} - \left(1 - \sigma\sqrt{T/n}\right)\right) \times \left(1 + rT/n\right)} - \frac{1}{2}\right]$$

$$= \sqrt{n}\left[\frac{\left(r\,T/n + \sigma\sqrt{T/n}\right)\left(1 + \sigma\sqrt{T/n}\right)}{2\sigma\sqrt{T/n}\left(1 + rT/n\right)} - \frac{1}{2}\right]$$

$$= \sqrt{n}\left[\frac{\left(r\sqrt{T/n} + \sigma\right)\left(1 + \sigma\sqrt{T/n}\right)}{2\sigma\left(1 + r\,T/n\right)} - \frac{1}{2}\right]$$

$$\frac{r\sqrt{T/n} + r\sigma\,T/n + \sigma + \sigma^2\sqrt{T/n}}{2\sigma\left(1 + rT/n\right)} - \frac{1}{2}$$

$$r\sqrt{T/n} + \cancel{r\sigma\,T/n} \cancel{+\sigma} + \sigma^2\sqrt{T/n} \cancel{- \sigma\left(1 + rT/n\right)}$$

$$r\sqrt{T/n}$$

$$= \sqrt{n}\left[\frac{r\sqrt{T/n} + \sigma^2\sqrt{T/n}}{2\sigma\left(1 + rT/n\right)}\right]$$

$$p^* = pu \times \frac{1}{pu + (1-p)d}$$

$\frac{R-d}{u-d}$ $\quad u-d-R+d$

$$= \frac{\left(e^{rT/m} - e^{-\sigma\sqrt{T/m}}\right)}{\left(\cancel{e^{\sigma\sqrt{T}}} - e^{-\sigma\sqrt{T}}\right)} \times e^{\sigma\sqrt{T/m}} \times \frac{1}{\frac{\left(e^{rT/m} - e^{-\sigma\sqrt{T/m}}\right)}{\left(\cancel{e^{\sigma\sqrt{T}} - e^{-\sigma\sqrt{T}}}\right)} \times e^{\sigma\sqrt{T/m}} + \frac{\left(e^{\sigma\sqrt{T}} - e^{rT/m}\right)}{\left(\cancel{e^{\sigma\sqrt{T}} - e^{-\sigma\sqrt{T}}}\right)} \times e^{-\sigma\sqrt{T}}}$$

$$= \frac{\left(e^{rT/m} - e^{-\sigma\sqrt{T/m}}\right)e^{\sigma\sqrt{T}}}{\left(e^{rT/m} - e^{-\sigma\sqrt{T}}\right)e^{\sigma\sqrt{T}} + \left(e^{\sigma\sqrt{T}} - e^{rT/m}\right)e^{-\sigma\sqrt{T}}} = \frac{\left(e^{rT/m} - e^{-\sigma\sqrt{T/m}}\right)e^{\sigma\sqrt{T}}}{e^{r}\left(e^{\sigma} - e^{-\sigma}\right)}$$

$$e^{r}e^{\sigma} - 1 + 1 - e^{r}e^{-\sigma}$$

$$\left(e^{r} - e^{-\sigma}\right)e^{\sigma}$$

$$e^{r}\left(1 - e^{-\sigma - r}\right)e^{\sigma}$$

2) $$\sqrt{m}\left(p^* - \tfrac{1}{2}\right) = \sqrt{m}\left[\frac{\left(e^{rT/m} - e^{-\sigma\sqrt{T/m}}\right)e^{\sigma\sqrt{T/m}}}{\left(e^{\sigma\sqrt{T/m}} - e^{-\sigma\sqrt{T/m}}\right)e^{rT/m}} - \frac{1}{2}\right]$$

$$= \sqrt{m}\left[\frac{\left(e^{\sigma\sqrt{T/m}} - e^{-rT/m}\right)}{\left(e^{\sigma\sqrt{T/m}} - e^{-\sigma\sqrt{T/m}}\right)} - \frac{1}{2}\right]$$

$$\underset{m \in \mathcal{V}(\infty)}{=} \sqrt{m}\left[\frac{\left(1 + \sigma\sqrt{T/m} + \frac{\sigma^2}{2}T/m - (1 - rT/m)\right)}{\left(1 + \sigma\sqrt{T/m} - (1 - \sigma\sqrt{T/m})\right)} - \frac{1}{2}\right]$$

$$= \sqrt{m}\left[\frac{rT/m + \sigma\sqrt{T/m} + \frac{\sigma^2}{2}T/m}{2\sigma\sqrt{T/m}} - \frac{1}{2}\right]$$

$$= \sqrt{m}\left[\frac{r\sqrt{T/m} + \sigma + \frac{\sigma^2}{2}\sqrt{T/m} - \sigma}{2\sigma}\right]$$

$$= \frac{\left(r + \frac{\sigma^2}{2}\right)\sqrt{T}}{2\sigma}$$

D'où $\sqrt{m}\left(p^* - \frac{1}{2}\right) \xrightarrow[m \to \infty]{} \frac{\left(r + \frac{\sigma^2}{2}\right)\sqrt{T}}{2\sigma}$