Day Count Convention

- $\frac{Act}{360}$: $\frac{\text{Nb jours exact}}{360}$

- $\frac{30}{360}$: $\frac{\text{Nb jours exacts} < 30 \text{ sinon mois de 30 jours}}{360}$

- $\frac{30E}{360}$: On ne compte pas les 31

- $\frac{Act}{Act}$ = $\frac{\text{Nb jours exacts}}{365 \text{ ou } 366}$

- $\frac{Act}{365}$: $\frac{\text{Nb jours exacts}}{365}$.

Par Yield : $r_{PY} = f^{-1}(N)$

Yield To Maturity : $r_{YTM} = f^{-1}(B_t)$

Exemple

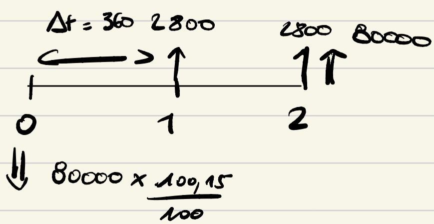
$\frac{A}{360}$ $\begin{cases} N(1+R\Delta t) \\ NR\Delta t \end{cases}$

pour 94 jours
taux Euribor 3M : 0,218 %.

$$N \times \frac{0,218}{100} \times \frac{94}{360}$$

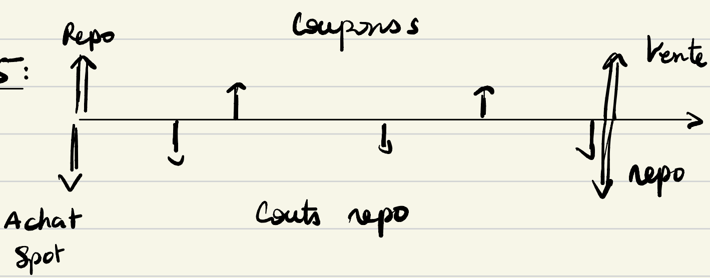
Question 4

taux fixe 2 ans 3,5 % base bond basis, annuel
nominal N = 80 k€
prix d'émission : 100,15 %



Prix payé : N × 100,15 %
reçu :
- $\frac{3,5}{100} \times N \times \frac{\Delta t}{360} = 2800$
- 80000

Question 5 :



prix spot + couts repo − coupons = prix forward.
+ coupon couru

Question 6

$$FWD_{interpole} = FWD_{inf} + \frac{FWD_{sup} - FWD_{inf}}{Nb_{sup} - Nb_{inf}} \cdot Nb_{int} - Nb_{inf}$$

$$= 3{,}5711 + \frac{3{,}2717 - 3{,}5711}{89+90+90+92-92} \left(92+90-92\right)$$

$$= 3{,}4709$$

$$= 3{,}5711 + \frac{3{,}2717 - 3{,}5711}{89+90+90+92-92} \left(92+90+90-92\right)$$

$$= 3{,}3707.$$

Question 7 :

$$TSW = \frac{B(0,T_1) - B(t,T_N)}{\sum \frac{\delta_i}{360} B(t,T_i)} = \frac{\sum_i FWD_i \cdot DF_i \cdot \Delta t_i}{\sum_i DF_i \cdot \Delta t_i}.$$

Question 8 :

- $\underbrace{YTM}_{\text{taux fixe}} = f^{-1}\left(\underbrace{B(t,T_N)}_{\text{prix de marché}}\right)$

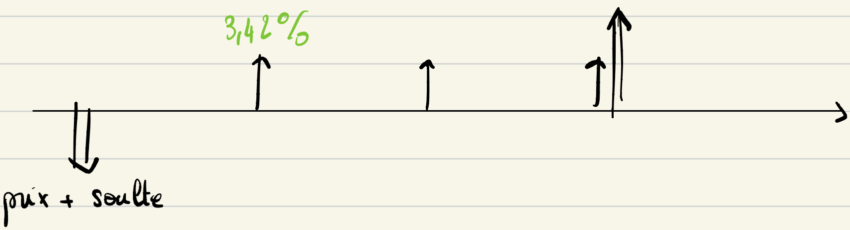
taux fixe qui explique le prix de marché.

- le différence entre peut être expliquée par le spread de risque de crédit entre l'émetteur de l'obligation et du participant du swap.

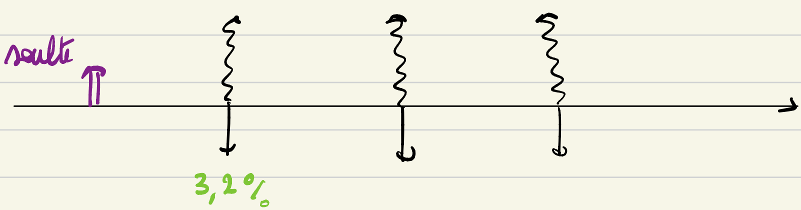
- Aussi la liquidité, les swaps sont plus liquide que les obligations

Question Bonus :

YTM = 3,42%



TSWAP = 3,2%



Question FRA :

Contrat OTC spécifiant qu'un taux déterminé (F) sera appliqué à un prêt / emprunt pour une date future.

- Règlement = $\dfrac{(F - F_{constaté})\,\Delta t \times N}{(1 + F_{constaté} \times \Delta t)}$

Application

Question 5

$$\text{reg} = \frac{500\,000\,000 \times \left(-\frac{0,40}{100} + \frac{0,384}{100}\right) 90}{1 - 0,384 \times 90}$$

on regarde le forward associé pour la période souhaitée pour $-0,384\,\%$

$$= -11,002 \cdot 10^{6}$$

Question 7 ASSET SWAP

$$FWD_{int} = FWD_{inf} + \frac{FWD_{sup} - FWD_{inf}}{\Delta\{t_{sup} - t_{inf}\}} \Delta\{t_{int} - t_{inf}\}$$

1. $$FWD^{1}_{int} = -0,39043 + \frac{-0,36405 + 0,39043}{92 + 88 + 90}(92 + 88)$$

$$= -0,3728$$

2.

Question 8 Asset Swap

$$\text{TSWAP} = \frac{\sum_i FWD_i \frac{\Delta t_i}{360} DF_i}{\sum_i \Delta t_i \, DF_i}$$

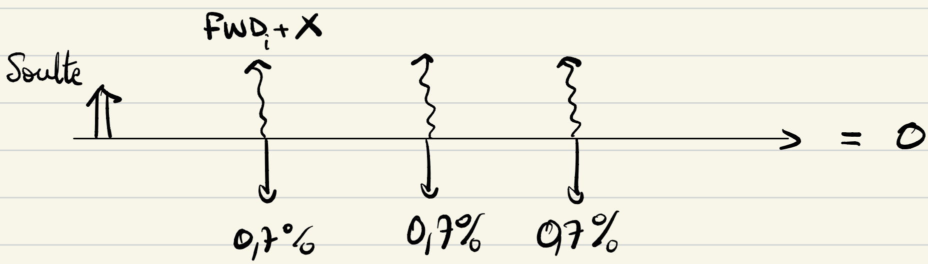
- Taux négatifs $\Rightarrow$ $DF > 1$

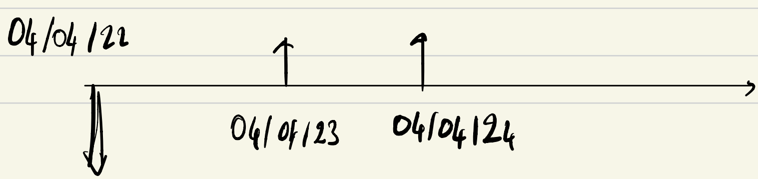
Question 9 :

Oblig 2 ans coupon annuel 0,7 %
$N = 75 \cdot 10^6$ €
prix emission 101

emission 04/04/2022

Soulte : $75 \cdot 10^6$ € $(101 - 100)$





$$\text{Soulte} + \underbrace{(FWD_0 + X)}_{04/04/2023} DF_0 + \underbrace{(FWD_1 + X)}_{04/04/24} DF_1 = r\,(DF_0 + DF_1)$$

$$\text{Soulte} + FWD_0\,DF_0 + FWD_1\,DF_1 + X\,(DF_0 + DF_1) = r\,(DF_0 + DF_1)$$

$$X\,(DF_0 + DF_1) = r\,(DF_0 + DF_1) - \text{Soulte} - FWD_0\,DF_0 - FWD_1\,DF_1.$$

$$X = r - \frac{1}{(DF_0 + DF_1)}\left[\text{Soulte} + FWD_0\,DF_0 + FWD_1\,DF_1\right]$$

<u>Bonus</u> :

$$PVBP = PV(S + 1bp) - PV(S)$$

$$= \sum_{k=1}^{N} (S + 1bp)\,(T_k - T_{k-1})\,P\,T_k - \sum_{i=1}^{N} L(T_{i-1}, T_i)\,(T_i - T_{i-1})\,P(T_i)$$

$$- \sum_{k=1}^{M} S \times (T_k - T_{k-1}) \times P(T_k)$$

$$= 1\,bp \times \underbrace{\sum_{k=1}^{M} (T_k - T_{k-1}) \times P(T_k)}_{\text{jambe fixe.}}$$

On peut hedger un swap avec des futures.

Partie Lozève

$$X - IM$$

$$Y = \sum VA = C + \underline{I} + G + \widetilde{NX}$$

Savings: $Y_d = Y - T + T_n$: Disposable Income
$\uparrow$ Taxes $\uparrow$ Transfers

$$Y_d = C + S_{HH} \leftarrow \text{Households savings.}$$

$$\boxed{S_{HH} = Y + T_n - T - C}$$

personal savings : $\dfrac{S_{HH}}{Y_d}$

$$I + G + NX$$

$$S_{Gov} = T - (G + T_n)$$

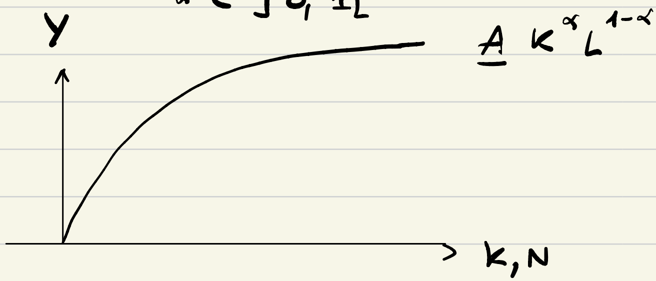
$$S = S_{HH} + S_{Gov} = Y + \cancel{T_n} - \cancel{T} - C + T - G - \cancel{T_n} = Y - C - G$$
$$= I + NX$$

$$\boxed{S = I + NX}$$

Cobb - Douglas Production Fonction:

$$Y = \underset{\text{productivity factor}}{A}\ \underset{\text{cost of capital}}{K^{\alpha}}\ \underset{\text{labor}}{L^{1-\alpha}}$$

$\alpha \in]0, 1[$



- $$MPN = \frac{dY}{dL} = (1-\alpha)\, A \left(\frac{K}{L}\right)^{\alpha}$$
- $$MPK = \frac{dY}{dK} = \alpha\, A \left(\frac{L}{K}\right)^{1-\alpha}$$

- Elasticities :

Labor :
$$\eta_N = \frac{L}{Y}\frac{dY}{dL} = \frac{L}{AK^{\alpha}L^{1-\alpha}} (1-\alpha) A \left(\frac{K}{L}\right)^{\alpha}$$
$$= (1-\alpha) \frac{L}{L} \cdot \frac{A}{A} \left(\frac{L}{K}\right)^{\alpha} \left(\frac{K}{L}\right)^{\alpha}$$
$$= 1-\alpha$$

Capital :
$$\eta_K = \frac{K}{Y}\frac{dY}{dK} = \frac{\cancel{K}}{\cancel{A}\, \cancel{K^{\alpha}}\, \cancel{L^{1-\alpha}}}\, \alpha\, \cancel{A} \left(\frac{\cancel{L}}{\cancel{K}}\right)^{1-\alpha} = \alpha.$$

$$Y = A\, f(K, L)$$

$$f(\lambda K, L) < \lambda\, f(K, L)$$

$$f(K, \lambda L) < \lambda\, f(K, L)$$

$$f(\lambda K, \lambda L) = \lambda\, f(K, L)$$

- **Markets Equilibrium :**

$MPL = \frac{W}{P}$ = real wage

$MPK = \frac{c}{P}$: real cost of capital

If:

- $MPL > \frac{W}{P} \Rightarrow \nearrow L$ to $\searrow MPL$ so it reaches $\frac{W}{P}$.
- $MPK > \frac{c}{P} \Rightarrow \nearrow K$ to $\searrow MPK$ so it reaches $\frac{c}{P}$

IS - LM Model :

$$Y = \underset{\text{conso}}{C} + \underset{\text{investiss}}{I} + \overset{\text{Gouv}}{G} + X - \underset{\text{importation}}{M}$$

Courbe I-S :

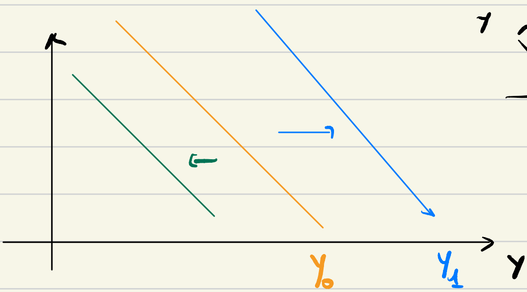
$$\begin{cases} C = A + bY \\ I = E - er \\ M = F + fY \end{cases}$$

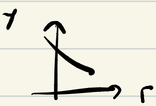
Dérivation linéaire car modèle de court terme

$$Y = A + bY + E - er + G + X - F - fY$$

$$Y(1 - b + f) = A + E + G + X - F - er$$

$$\boxed{Y = \frac{A + E + G + X - F}{1 - b + f} - \frac{e}{1 - b + f}\, r}$$: Courbe IS





Si A ↗, $Y_0 \to Y_1$

Si E ↘

Courbe LM :

$$\frac{L_d}{P} = \boxed{\frac{M_s}{P} = \alpha Y - \beta r}$$

offre de la banque centrale

→ demande de monnaie spec : les taux ↗, demande oblig ↗.

demande monétaire transactionnelle

$$Y = \frac{M_s}{\alpha P} + \frac{\beta}{\alpha} r.$$

Equilibre IS-LM :

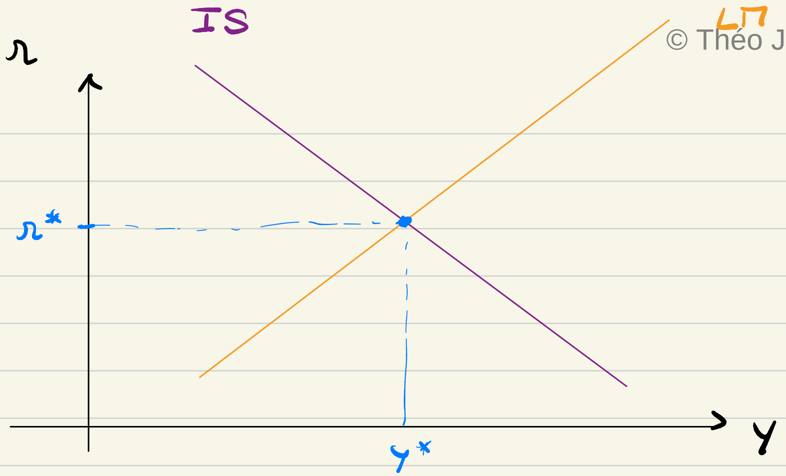
$$r^* = \frac{\alpha}{\beta} Y^* - \frac{M_s}{\beta P}$$

$$Y^* = \frac{1}{1+b-f}(A+E+G+M-F) - \frac{e}{1+b-f} \cdot \left[\frac{\alpha}{\beta} Y^* - \frac{M_s}{\beta P}\right]$$

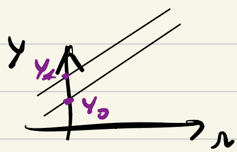
$$Y^* = \frac{1}{\left[1 + \frac{e \frac{\alpha}{\beta}}{1+b-f}\right]} \cdot \left[\frac{1}{1+b-f}(A+E+G+M-F) + \frac{e}{(1+b-f)\beta} \frac{M_s}{P}\right]$$

$$Y^* = \frac{1}{\left(1+b-f+e\frac{\alpha}{\beta}\right)}(A+E+G+M-F) + \frac{e}{\beta(1+b-f)\left[1+\frac{e\frac{\alpha}{\beta}}{1+b-f}\right]} \frac{M_s}{P}.$$

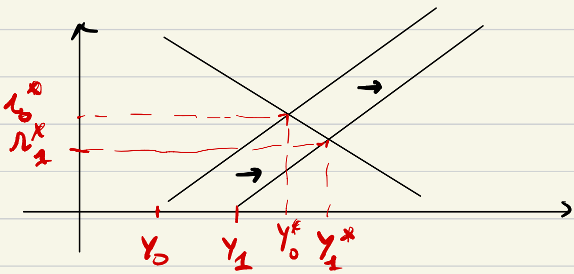
$$= \frac{1}{1+b-f+e\frac{\alpha}{\beta}}(A+E+G+M-F) + \frac{e\frac{M_s}{P}}{\left[\beta(1+b-f)+e\alpha\right]}$$



Graphical Derivations



- Si $Y_0 \to Y_1$

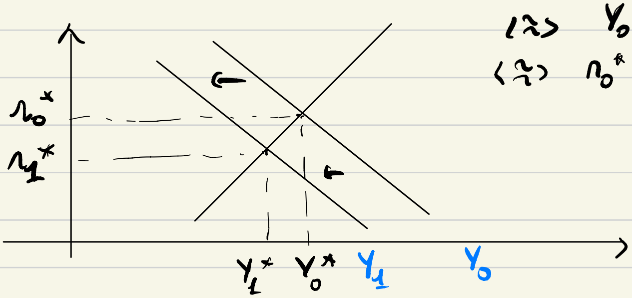


$Y_0^* \nearrow Y_1^*$ & $r_0^* \searrow r_1^*$

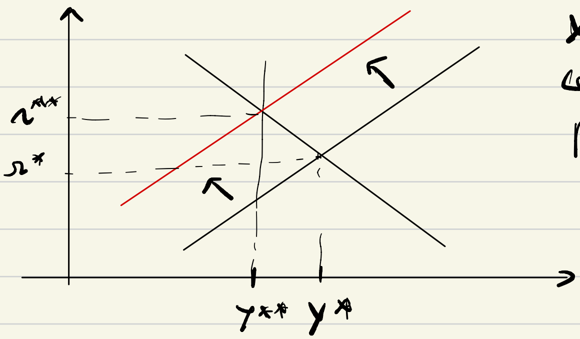
- Si $S \searrow \Leftrightarrow I \searrow \Leftrightarrow E \searrow \Leftrightarrow Y_0 \searrow Y_1$

$\Leftrightarrow Y_0^* \searrow Y_1^*$

$\Leftrightarrow r_0^* \searrow r_1^*$



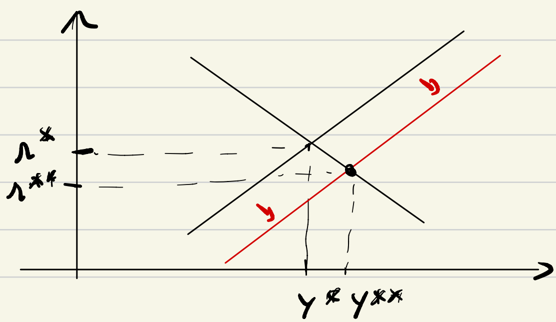
TEMPORARY DECREASE IN A :



$Y^* \searrow Y^{**} \Leftrightarrow p \nearrow$ car les compagnies ne peuvent plus produire autant

$\Leftrightarrow LM \nwarrow$

MONEY POLICIES



$M_s \nearrow$

Dans le <u>court terme</u> $M_s \nearrow \Rightarrow Y^* \nearrow Y^{**}$

$\Rightarrow r^* \searrow r^{**}$

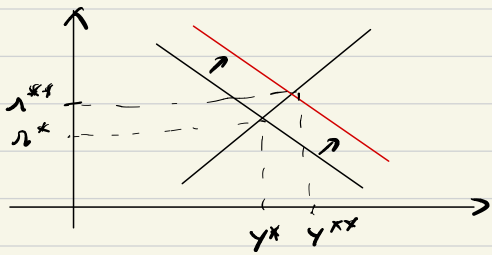
<u>Long terme</u> : $r^* \searrow r^{**} \Rightarrow I, C \nearrow$

$\Rightarrow p \nearrow$

$\Rightarrow M_s \searrow \Rightarrow LM$ revient à la normale

Fiscal Policy

$G \nearrow \Rightarrow Y^{IS} \nearrow \Rightarrow Y^{*} \nearrow Y^{**} \Rightarrow r^{*} \nearrow r^{**}$ Short term

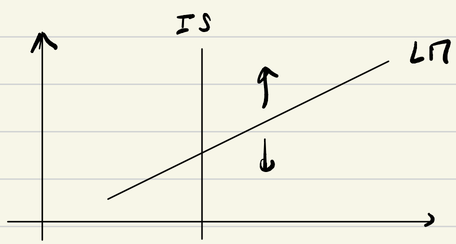


Long run : $I, C \searrow \Rightarrow P \nearrow \Rightarrow M_S \searrow$
$\Rightarrow Y^{**} \searrow Y^{*}$

When money policy doesn't work

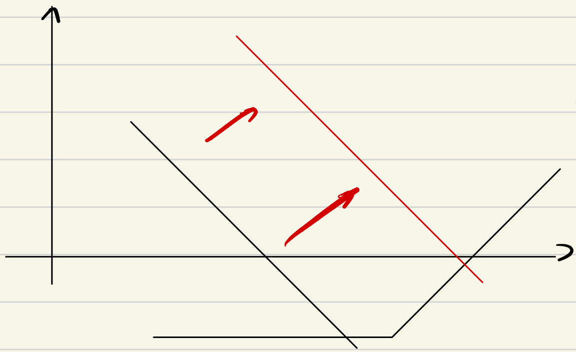
IS vertical
LM horizontal → liquidity trap

Vertical IS :



r^{*} & Y^{*} stay unchanged.

HORIZONTAL LM



Aucune relance monnaitaire ne fonctionne, il faut des relances gouvernementale

Limites de IS-LM

- prixs et inflation ne reflétés
- modèle statique
- n'explique pas les crises.

Biggs - Mayer Model

- Debt matters
- Credit impulse and not credit growth

$$Y = C + I = GDP$$

$$\Delta GDP = \Delta C + \Delta I$$

Hypothesis : $I = \Delta D$ car investissement financé par la dette

$$\Delta GDP = \Delta C + \Delta\Delta D$$

In details :

$$Y = Y_{c,t} + Y_{d,t} = C_t + I_t$$

$Y_{c,t} = C_t = A K_t$ ← plutôt les machines

$Y_{d,t} = I_t = f(N_t)$ ← plutôt les hommes

$K_t = (1-\delta) K_{t-1} + Y_{d,t}$ δ : depreciation rate of capital

$$r = A - \delta$$

$$D_t = (1-\delta) D_{t-1} + Y_{d,t} \iff Y_{d,t} = \Delta D_t + \delta D_{t-1}$$

$$Y_{c,t} = A K_t = (r+\delta) D_t$$

$$Y_t = \Delta D_t + \delta D_{t-1} + (r+\delta) D_t$$

$$= D_t - D_{t-1} + \delta D_{t-1} + (r+\delta) D_t$$

$$= D_t - (1-\delta) D_{t-1} + (r+\delta) D_t$$

$$= (1-\delta) \Delta D_t + (r+2\delta) D_t$$

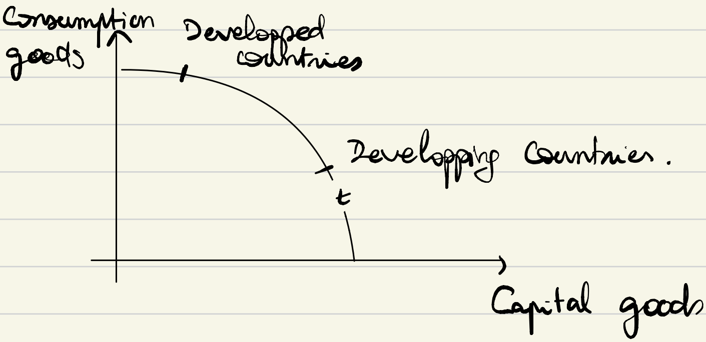
$$Y_{t-1} = (1-\delta) \Delta D_{t-1} + (r+2\delta) D_{t-1}$$

$$\boxed{Y_t - Y_{t-1} = (1-\delta) \Delta\Delta D_t + (r+2\delta) \Delta D_t}$$

$$1-\delta > (r+2\delta)$$

DIAGRAMMATICAL AUSTRIAN MODELIZATION

ARBITRAGE CURVE

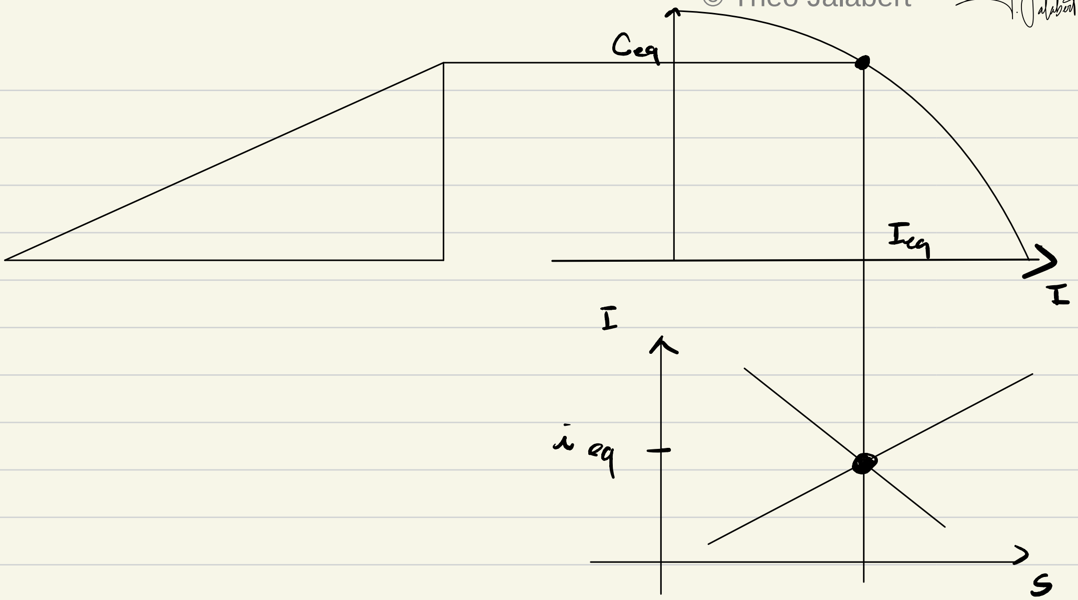


HAYEKIAN TRIANGLE

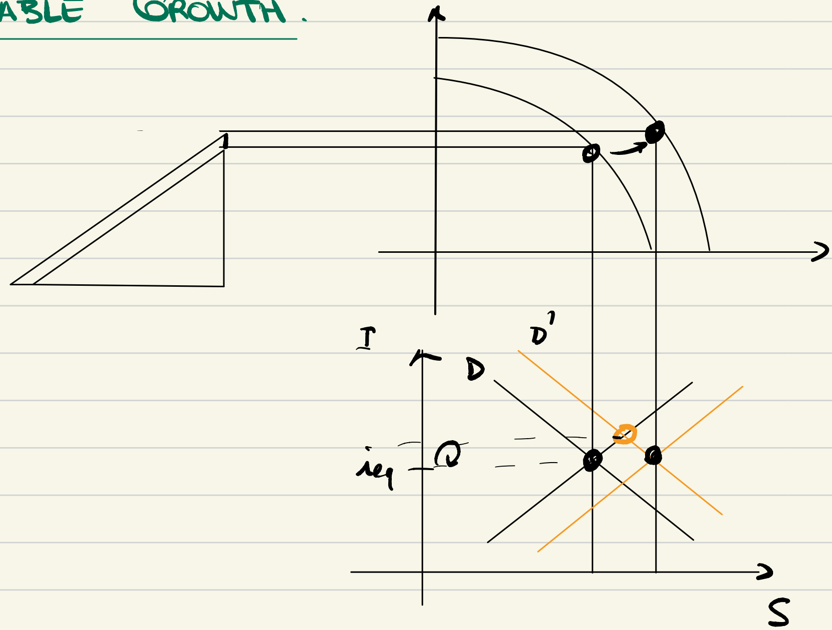
comme $I = f(i)$ mais $\frac{dI}{di} < 0$.

$C = g(i)$ mais $\frac{dC}{di} > 0$

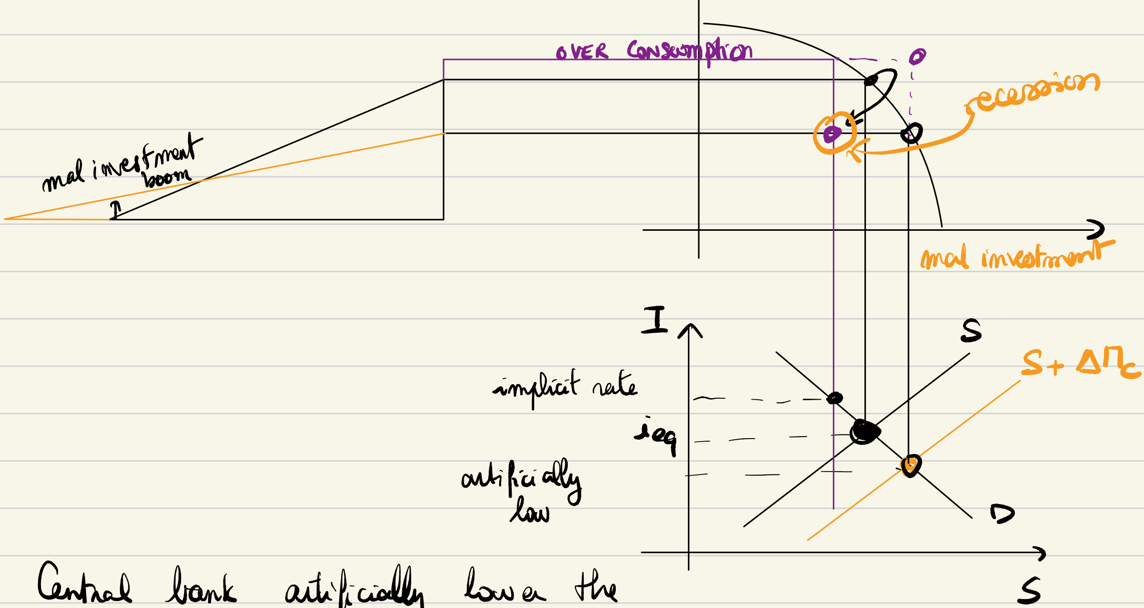
Production time T : $\frac{dT}{di} < 0$



Sustainable Growth.



UNSUSTAINABLE GROWTH



Central bank artificially lower the real interest rate ⇒ bad signal in the economy.