ISFA – Pricing

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Outline: Pricing

Goal: Price a reinsurance structure

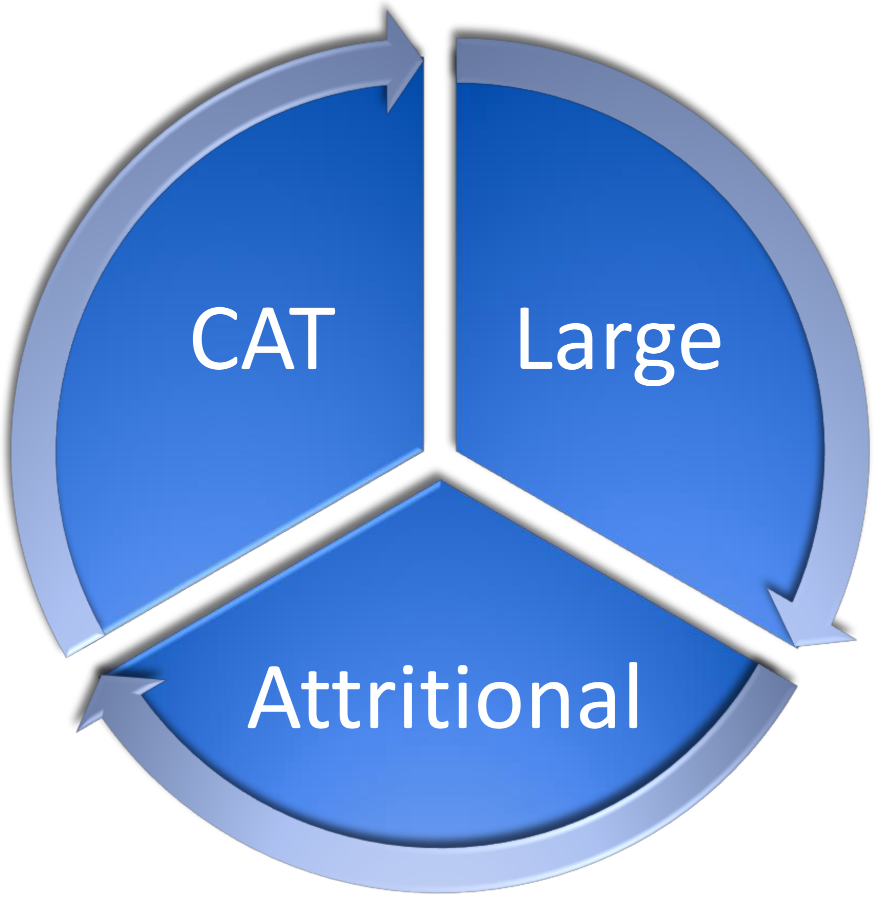
Fundamental point: the CLA split

Pricing methods

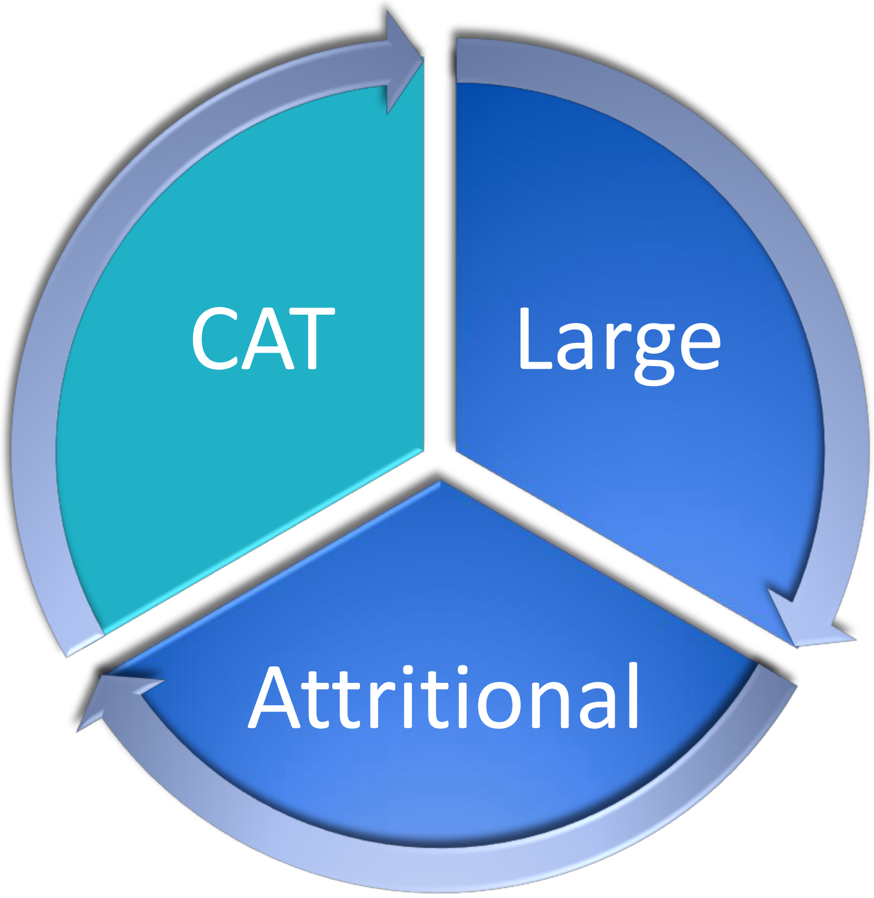
- Standard methods
  - Burning Cost
  - Collective Risk Model (for mention)
  - Exposure rating
- Alternative methods
  - Monte Carlo simulations

More about Exposure Pricing

CLAims split



CLAims split



CAT losses

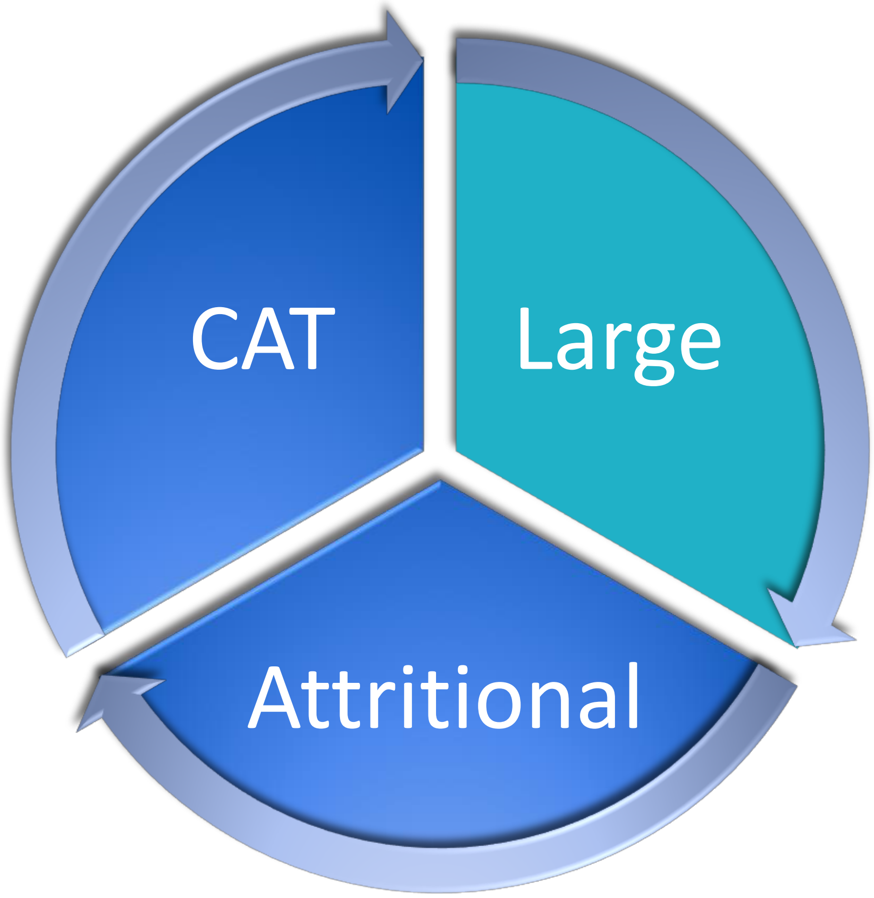
Very extreme events: very low frequency/very high severity process

CAT events for all LoB?

Examples:

1. Property: Windstorm, Earthquake, Flood,…
2. Marine: Windstorm, Tsunami, Hurricane,…
3. Life & Accident: Terrorism, Pandemy, CATNAT?
4. Casualty: Terrorism?, Crashes?, Other?
5. …

CLA split



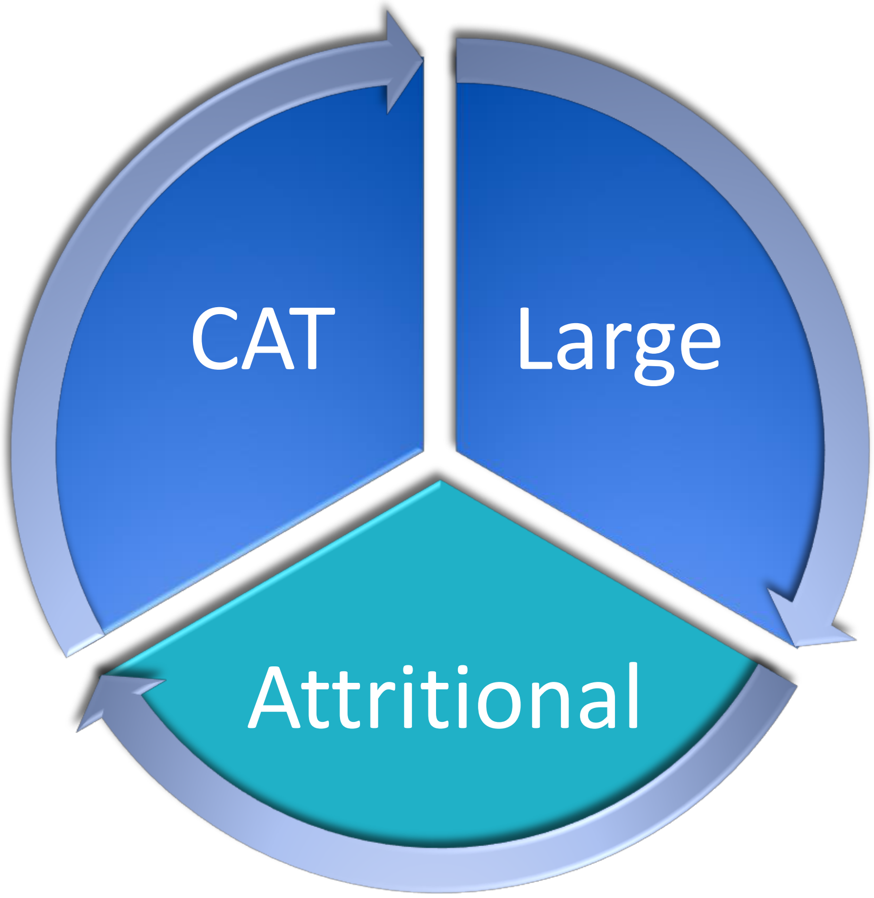
Large losses

Extreme value theory:

Describe the unusual rather than the usual

- What is a large loss?

CLA split



Attritional losses

- Represent the mass of « small » claims

$$A(t) = Total(t) - Large(t) - CAT(t)$$

→ Function of the large loss threshold

- Often expressed on a loss ratio basis (LR)

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Class of business

Short tail (almost no development)

- Property
  - Fire and related covers
  - Hail, …
- Motor own damage
- …

Long tail (long development)

- Motor third party liability
- General liability
  - Professional - product – environmental
  - Medical …
- Workers' compensation
- Marine
- …

➢ Different way to quote LT & ST business

Goal

Estimate the pure reinsurance premium P_R for an XL: C XS D

Methods:

1. Burning cost method
2. Collective risk model
3. Others

➔ Estimate the distribution of N and X

Calculation

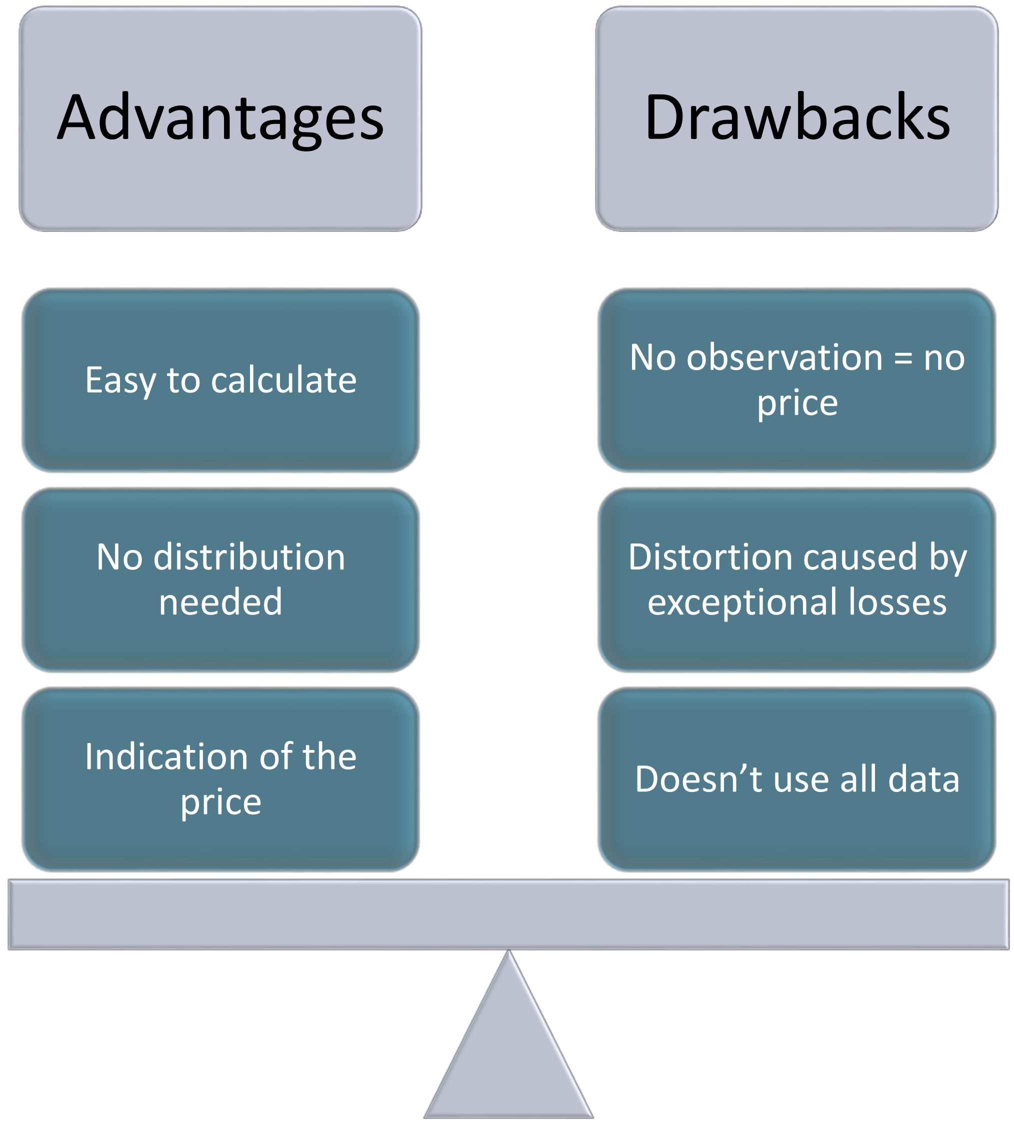
$$BC = \frac{\text{Observed XL Losses}}{\text{Premium Income}}$$

Example

Consider a XL treaty :
10.000 xs 10.000

Year	Premium Income	Loss	XL Loss	BC
2009	100.000	5000	0	7%
		12.000	2.000	
		15.000	5.000	
2010	110.000	8.000	0	0%
		8.000	0	
2011	120.000	15.000	5.000	12,5%
		30.000	10.000	
		7.000	0	
2012	130.000	5.000	0	5,4%
		17.000	7.000	
Total	460.000		29.000	6,3%

Burning Cost



Goal

Estimate the pure reinsurance premium P_R for an XL: C XS D

Methods:

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→ Estimate the distribution of N and X

Collective risk model

- Let $X_1 \dots X_n$ be iid realizations of X : the claim amounts
- Let N be a random number independent of the $X_1 \dots X_n$
- We define:
  - $S = X_1 + \dots + X_N$ the aggregate claims fgu
  - $R_i = \min(C; \max(0 ; X_i - P))$ the XL reinsurer's liability
- Then $S_R = R_1 + \dots + R_n$: the XL reinsurer's aggregate liability

Results

$$E[S] = E[N]E[X]$$

$$V[S] = E[N].V[X] + V[N]E[X]^2$$

$$F_S(t) = \sum_{n=0}^{\infty} p(n).F_x^{*n}(t)$$

Application to XL treaty

Mathematical expectation principle:

$$P_R = E[S_R]$$
$$= E[N].E[\min(C, \max(0, X\text{-}D))]$$

- All we need to know is the distribution of N and X
- In some cases, we also need the distribution of S_R

Poisson-Pareto process

$$S(t) = \sum_{i=1}^{N(t)} X_i$$

Where $\{N(t): t \geq 0\}$ is Poisson process with rate λ and $\{X_i: i \geq 1\}\ i.i.d. \sim Pareto(A, \alpha)$ independent of $\{N(t): t \geq 0\}$

Frequency-Severity approach:

- Frequency: Poisson $P(N(t) = k) = e^{-\lambda} \frac{\lambda^k}{k!}$
- Severity: Pareto $P(X \leq x) = \begin{cases} 1 - \left(\frac{x}{A}\right)^{-\alpha} & \text{if } x > A \\ 0 & \text{if } x \leq A \end{cases}$

Pareto properties

- Moments: $E[X^r] < \infty$ if α>r

$$E[X] = \frac{\alpha A}{\alpha - 1}$$

$$V[X] = \frac{\alpha A^2}{(\alpha - 1).(\alpha - 2)}$$

- Conditioning: for any B>A we have

$$X\left|X > A \approx Pareto(A,\alpha) \Rightarrow X\right|X > B \approx Pareto(B,\alpha)$$

- Mean excess function

$$e(t) = E[X - t|X > t] = \frac{t}{\alpha - 1}$$

- MLE

$$\hat{\alpha} = \frac{m - 1}{\sum_{i=1}^{m} \log(\frac{X_{m-j+1,m}}{A})}$$

Pareto fitting procedure

Determination of A:

Suppose we have a sample $X_1, \dots, X_m$ then the ME function can be estimated by

$$\hat{e}_m(t) = \frac{\sum_{i=1}^{m} X_i 1_{(t,\infty)}(X_i)}{\sum_{i=1}^{m} 1_{(t,\infty)}(X_i)} - t$$

In practice, one often takes $t = X_{m-k,m}$

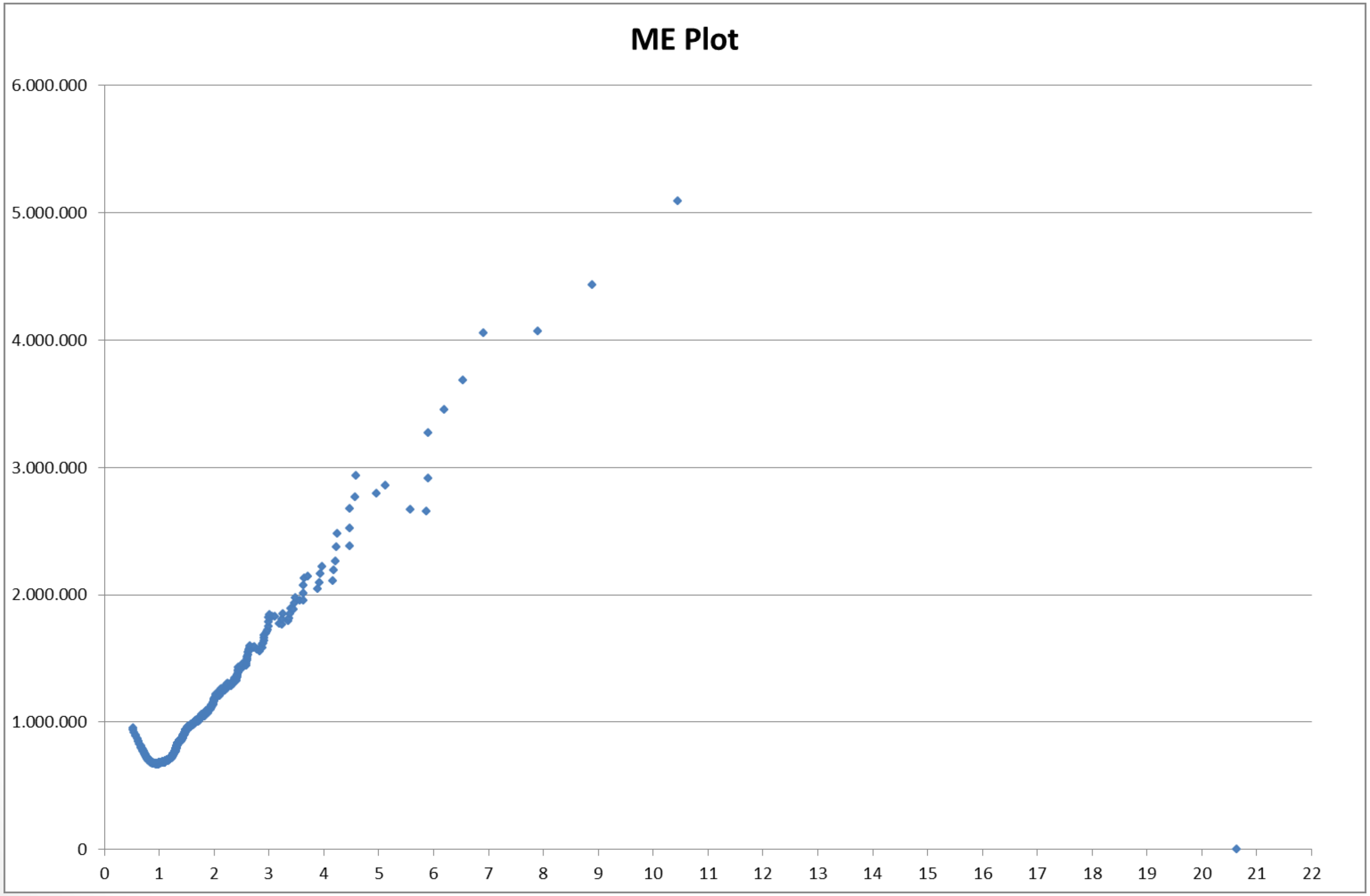
i.e. we have

$$\hat{e}_m\left(X_{m-k,m}\right) = \frac{1}{k}\sum_{j=1}^{k} X_{m-j+1,m} - X_{m-k,m}$$

ME plot: $\left(X_{m-k,m}, \hat{e}_m\left(X_{m-k,m}\right)\right) \qquad k = 1, \dots, m-1$

Pareto fitting procedure

ME example:



Pareto fitting procedure

Determination of α:

Suppose we have a sample $X_1, \dots, X_m$ then the MLE of α above the attachment point $X_{m-k,m}$ is

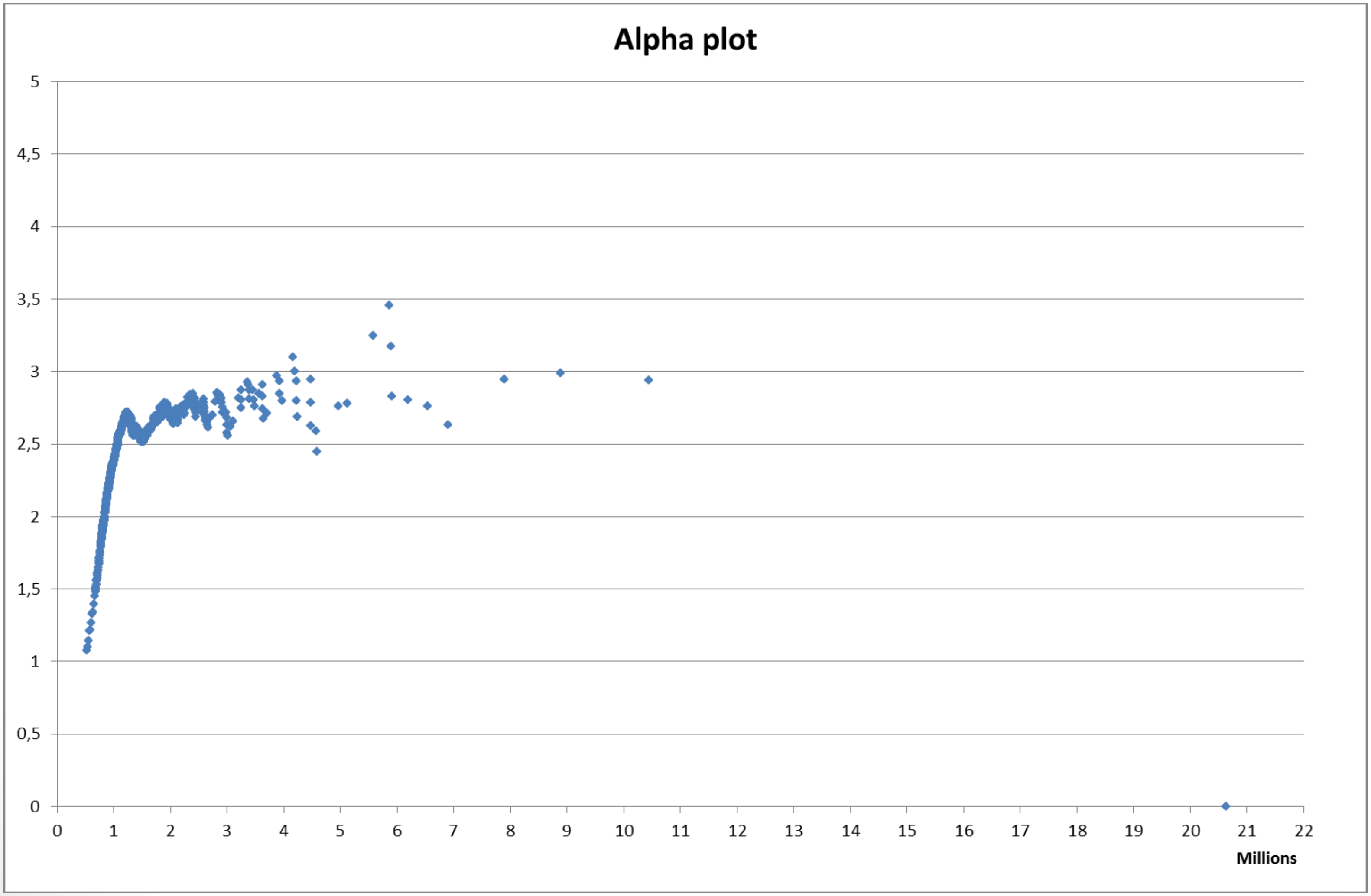
$$\hat{\alpha}_{k,m} = \frac{k}{\sum_{j=1}^{k} \ln\left(\frac{X_{m-j+1,m}}{X_{m-k,m}}\right)}$$

In practice as A is unknown, we perform an α-plot:

$$\left(X_{m-k,m}, \hat{\alpha}_{k,m}\right) \qquad k = 1, \dots, m-1$$

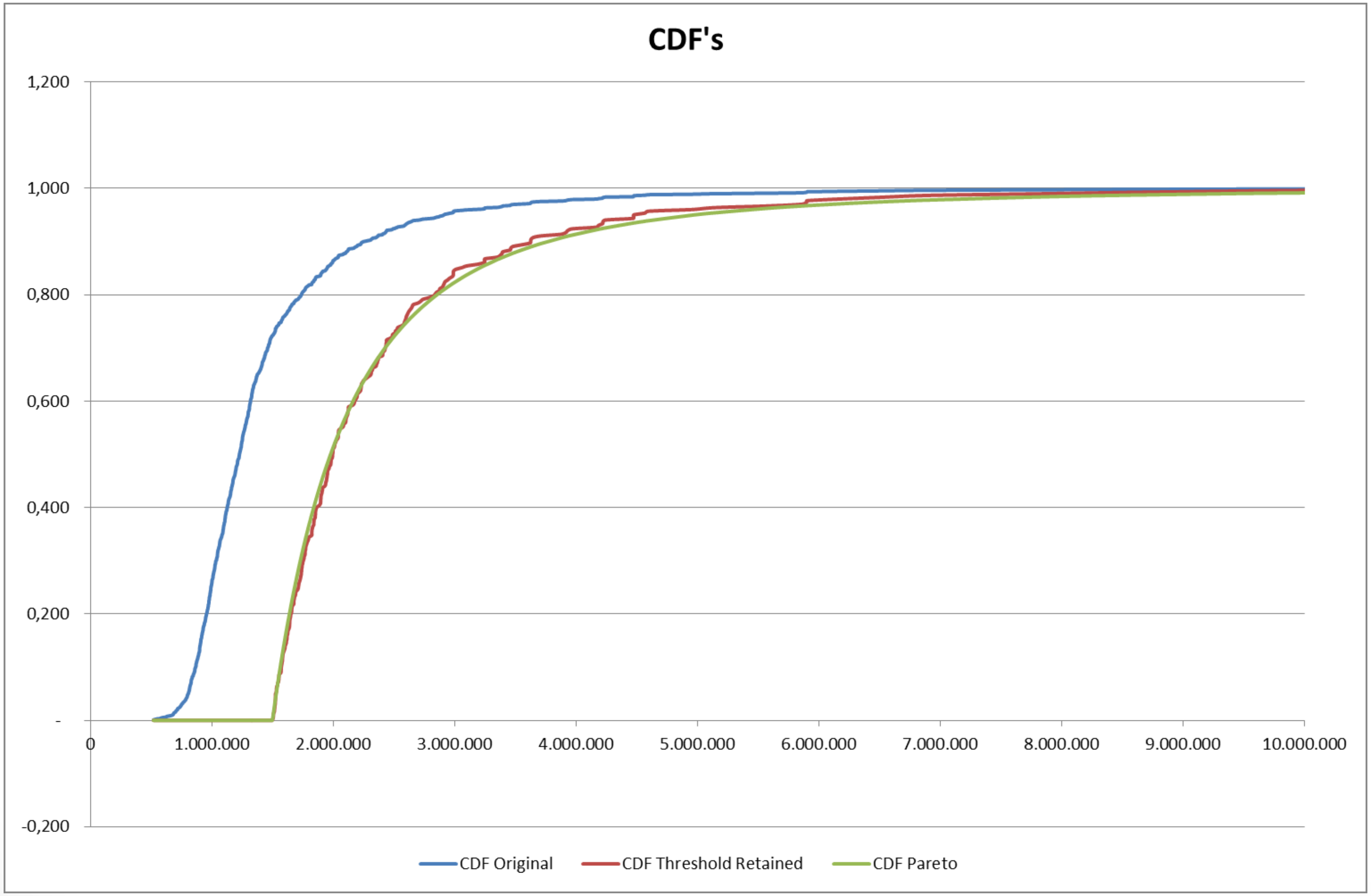
Pareto fitting procedure

Alpha plot example:



Pareto fitting procedure

CDF graph check



Possible generalizations

- Pareto distributions are a specific case of GPD (really ?!)
- Fitting a GPD distribution may be (a bit) less straightforward
- Mean-Excess plot flat/decreasing ?
- Empirical + GPD distribution (in line with EVT)
- Piece-wise distributions ?

Practical point

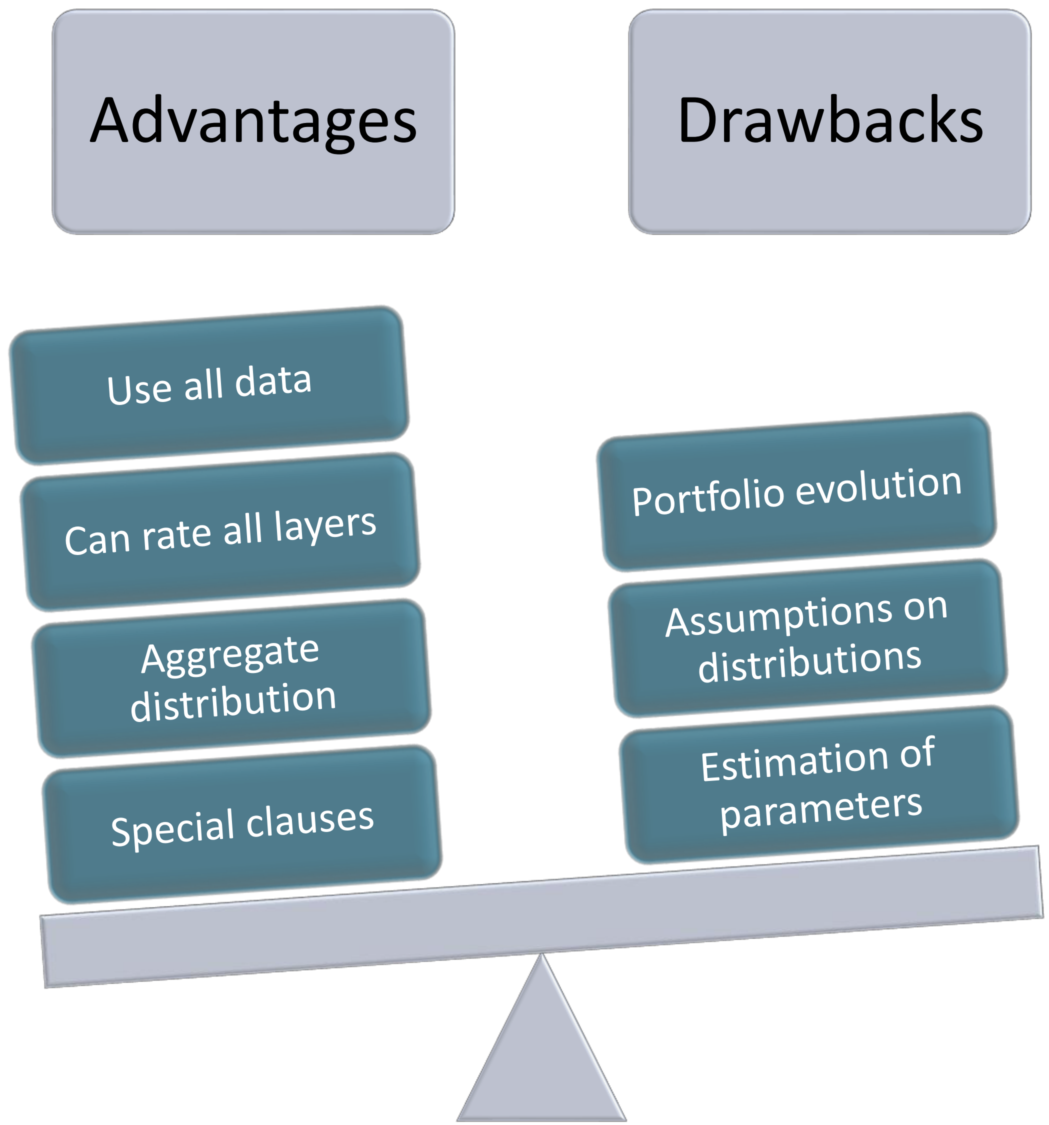
- You'll have FGU data

- You have to fit a distribution of the extreme … For your Large claims

  - Indexation of all claims together
  - Computation of the severity of Large Losses above a certain threshold
  - Losses below this threshold are to be considered as attritional losses, and treated as such !!!

- For your Large Losses, you'll have to determine an adequate Threshold

Collective risk model



Goal

Estimate the pure reinsurance premium P_R for an XL C XS D

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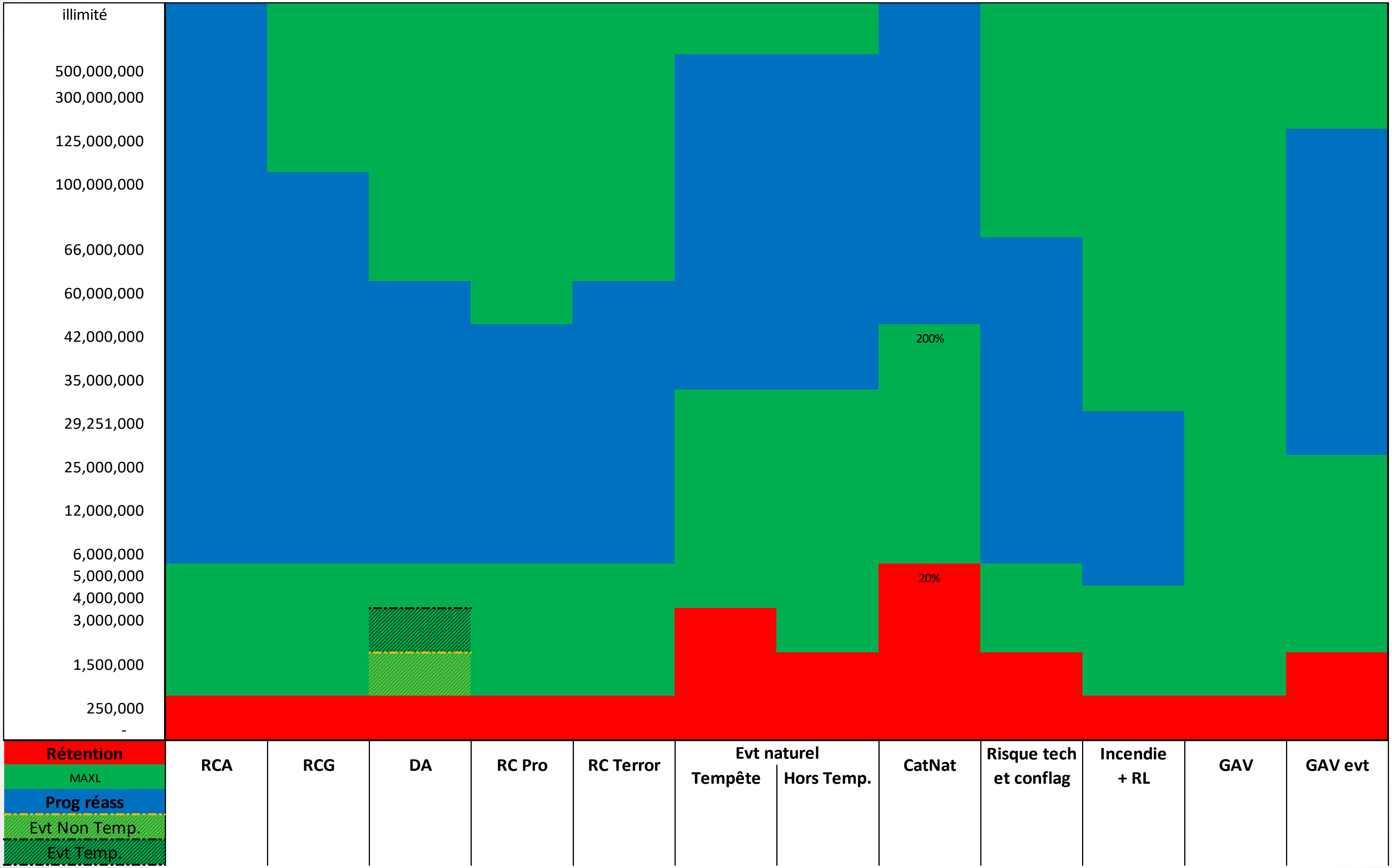
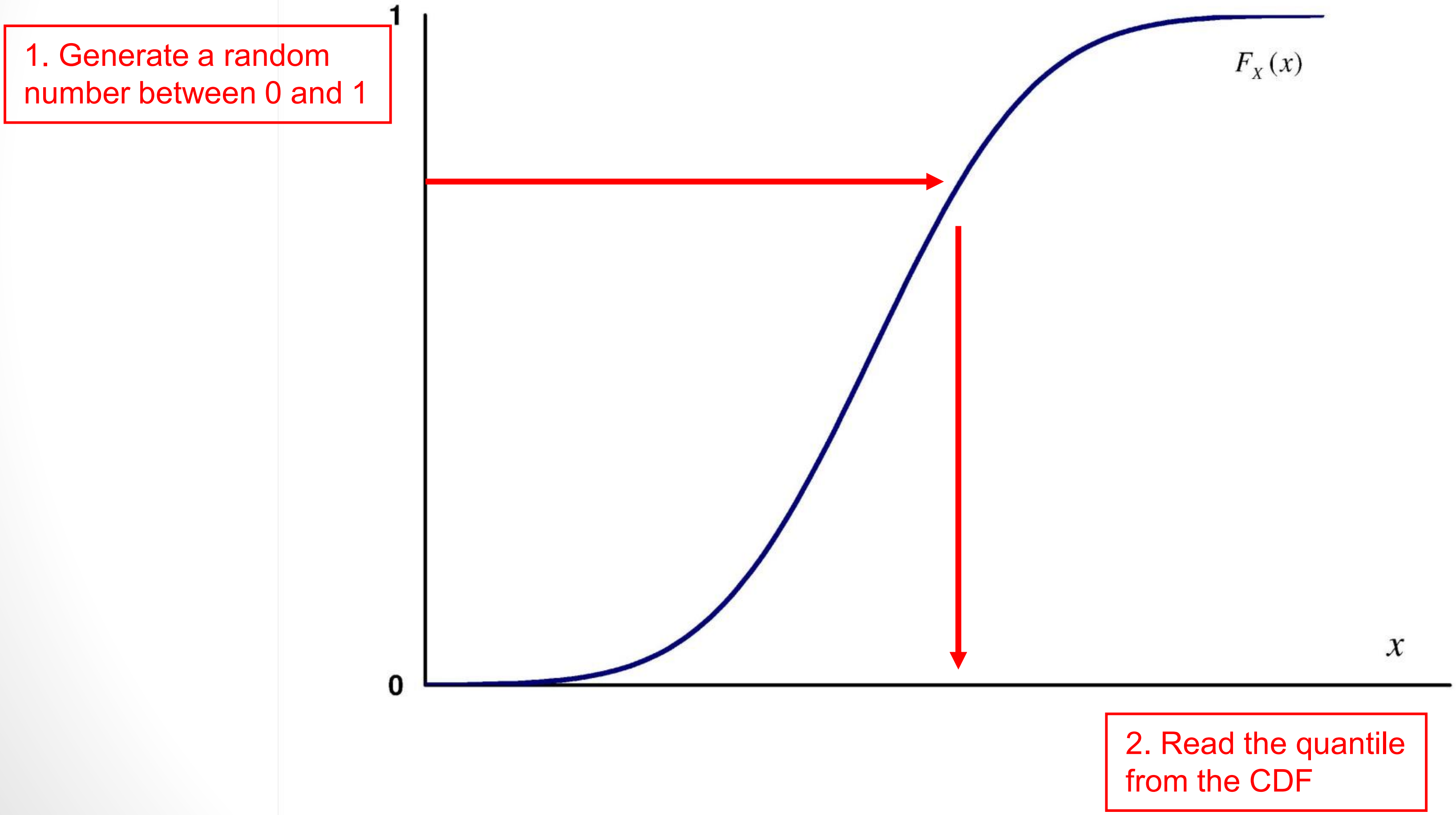
➢ Non standard approach, or very specific lines of business

Non standard approach - Example



- Monte Carlo simulation

Monte Carlo simulations



Questions ?