

Statistical finance: Exam

2 hours-No document allowed

11 January 2021

- 1) What is the market portfolio? In the CAPM model, at equilibrium, what is the portfolio of the agents made of? Give the CAPM equation for the expected return of an asset. How can one test statistically for the validity of the CAPM?
- 2) Define the Ridge estimator and provide its closed form formula (without proof). What is the interest of Ridge estimator compared to ordinary least squares? What does Lasso mean? Define the Lasso estimator. What is the interest of Lasso estimator compared to Ridge? How do we choose the regularization parameter (λ) of the Ridge and Lasso estimators? Give one example of situation where Ridge or Lasso estimators are useful in finance.
- 3) In the PCA of a cloud of data points in \mathbb{R}^p given by the $n \times p$ array X , how does one build the best sub-vector space of \mathbb{R}^p with dimension k to project the array?
- 4) State the Marcenko-Pastur theorem (no need to give the density of the Marcenko-Pastur law, just call it L). How can this result be used in finance?
- 5) In the Black-Scholes model, how does the option price evolve with the drift? Connect this with statistical estimation of the drift from historical data.
- 6) You work for a brokerage company, with access to an accurate database about past transactions, order flows and order books. Your mission is to build an algorithm enabling you to buy in 8 hours 600.000 Hermes stocks and 300.000 LVMH stocks. Which mathematical tools, models and statistical methods do you use to build it (give mathematical formulas when needed)? How do you implement and calibrate the models? What do you measure on data in the construction process? How do you run some testing procedures? (2 pages maximum!)
- 7) How do you assess the quality of the tick value for a given asset?
- 8) From a time series of prices (high frequency data over several years), how would you show that volatility is rough? What are the advantages of rough models compared to classical models used for options? What are the microstructural foundations of rough volatility? Summarize how this can be shown mathematically.