TD 3

Exercice 1 . On a $\varphi(C(u,v)) = \varphi(u) + \varphi(v)$

1) On remarque que

$$\frac{\partial^2 \varphi(C(u,v))}{\partial u \partial v} = \frac{\partial(\varphi'(v))}{\partial u} = 0 \quad (1)$$

Puis $$\frac{\partial \varphi(C(u,v))}{\partial u} = \frac{\partial C(u,v)}{\partial u} \varphi'(C(u,v))$$

$$\Rightarrow \frac{\partial C(u,v)}{\partial (u)} = \frac{\varphi'(u)}{\varphi'(C(u,v))}$$

De même $$\frac{\partial (C(u,v))}{\partial v} = \frac{\varphi'(v)}{\varphi'(C(u,v))}$$

$$\frac{\partial^2 \varphi(C(u,v))}{\partial u \partial v} = \frac{\partial^2 C(u,v)}{\partial u \partial v} \varphi'(C(u,v)) + \frac{\partial C(u,v)}{\partial u} \frac{\partial C(u,v)}{\partial v} \varphi''(C(u,v))$$

$$\Rightarrow c(u,v) = \frac{-\varphi''(C(u,v))\, \varphi'(u)\, \varphi'(v)}{[\varphi'(C(u,v))]^3}$$

2) On utilise $I = 4 \iint_{[0,1]^2} C(u,v)\, dC(u,v) - 1$

On utilise un changement de variable.

Calcul de la matrice jacobienne :

$$\begin{vmatrix} \frac{\partial v_1}{\partial u_1} & \frac{\partial v_1}{\partial u_2} \\ \frac{\partial v_2}{\partial u_1} & \frac{\partial v_2}{\partial u_2} \end{vmatrix} = \begin{vmatrix} & \\ & 0 \end{vmatrix}$$

$$\det = -\frac{\partial v_1}{\partial u_2} = -\frac{\varphi'(u_2)}{\varphi'(v_1)}$$

Et $\varphi(v_1) = \varphi(u_1) + \varphi(u_2)$

$= \varphi(v_2) + \varphi(u_2)$

$\varphi(u_1) \geq \varphi(u_2)$ $\quad \boxed{V_1 \leq V_2}$

$$I = \int_0^1 \int_{V_1}^1 -V_1 \frac{\varphi''(v_1)\varphi'(v_2)\varphi'(u_2)}{[\varphi'(v_1)]^3} \frac{1}{|\det J|} \, dv_2 \, dv_1$$

$$= \int_0^1 \int_{V_1}^1 -V_1 \frac{\varphi'(v_1)\varphi'(v_2)}{[\varphi'(v_1)]^2} \, dv_2 \, dv_1$$

$$= \int_0^1 \frac{V_1 \varphi''(v_1)\varphi(v_1)}{\varphi'(v_1)^2} \, dv_1$$

$$\frac{d\left[\frac{t\,\varphi(t)}{\varphi'(t)}\right]}{dt} = \frac{\varphi(t)}{\varphi'(t)} + t - \frac{\varphi''(t)\, t\, \varphi(t)}{\varphi'(t)^2}$$

$$\left[\frac{t\,\varphi(t)}{\varphi(t)}\right]_0^1 = \int_0^1 \frac{\varphi(t)}{\varphi'(t)}\,dt + \int_0^1 t\,dt - \int_0^1 \frac{\varphi''(t)\, t\, \varphi(t)}{\varphi(t)^2}$$

$$\int_0^1 \frac{\varphi''(t)\, t\, \varphi(t)}{\varphi'(t)^2}\,dt = \int_0^1 \frac{\varphi(t)}{\varphi'(t)}\,dt + \frac{1}{2}$$

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