

Duration models / Exam of June 16, 2016

2 hours – no documents allowed / no calculator

Some properties of the Makeham model

The quality of drafting, justifications as well as presentation will be taken into account in the notation.

One is interested, within the framework of mortality modelling, in the following model:

$$h(x) = \alpha + \beta \exp(\gamma x)$$

with h the hazard function of the model.

Question n°1 (2 points): Point out the definition and the principal properties of the hazard function. What is the link with the survival function?

Question n°2 (2 points): Compute the survival function of the Makeham model.

Question n°3 (3 points): Show that with this model, if q_x is small, there exist a and b such that $\ln(q_{x+1} - q_x) \approx a + b \times x$.

Question n°4 (2 points): Use the previous expression to build an estimator of β and γ , using a non-parametric estimator of the q_x 's.

Question n°5 (2 points): Compute an approximation of the expectancy of a Makeham random variable.

Question n°6 (4 points): Recall, within a general framework, the expression of log-likelihood according to S and h . Without giving demonstration of this expression, you will provide an intuitive justification of the terms associated with each observation. How does this expression have to be modified in the presence of left truncation? You will point out what is left truncation.

Question n°7 (2.5 points): Could you describe another way of estimating the parameters using a discrete model?

Question n°8 (2.5 points): Describe the model of Brass and explain why it is more flexible than the Makeham one.