OE

$$c(q) = D(q) = 1$$

$$y(t) = \frac{B(q,\theta)}{F(q,\theta)} u(t) + e(t)$$

$$\hat{y}(t|t-1,\theta) = \frac{B(q,\theta)}{F(q,\theta)} u(t)$$

- $$F(q,\theta)\hat{y}(t|t-1,\theta) = B(q,\theta) u(t)$$

- $$\hat{y}(t|t-1,\theta) = (1 - F(q,\theta))\underbrace{\hat{y}(t|t-1,\theta)}_{\substack{\text{delayed (past) values} \\ \text{of prediction}}} + B(q,\theta) u(t)$$

$$\hat{y}(t|t-1,\theta) = \theta^T \underset{\sim}{\varphi}(t,\theta)$$

$\quad\quad\quad\quad\quad\quad \hookrightarrow$ depends on θ via $\hat{y}$

prediction is nonlinear in θ

ARMAX

$$F(q) = D(q) = A(q)$$

$$y(t) = \frac{B(q,\theta)}{A(q,\theta)} u(t) + \frac{C(q,\theta)}{A(q,\theta)} e(t)$$

$$A(q,\theta) y(t) = B(q,\theta) u(t) + C(q,\theta) e(t)$$

$$\hat{y}(t|t-1,\theta) = \frac{B(q,\theta)}{C(q,\theta)} u(t) + \frac{C(q,\theta) - A(q,\theta)}{C(q,\theta)} y(t)$$

$$\hat{y}(t|t-1,\theta) = B(q,\theta) u(t) + \left(C(q,\theta) - A(q,\theta) \right) y(t) + \left(1 - C(q,\theta)\right) \hat{y}(t|t-1,\theta)$$

$$\hat{y}(t|t-1,\theta) = B(q,\theta) u(t) + (1 - A(q,\theta)) y(t) + (C-1)\underbrace{\left(y(t) - \hat{y}(t|t-1,\theta) \right)}_{}$$

$$\underbrace{y(t) - \hat{y}(t|t-1,\theta)}_{\varepsilon(t|\theta)}$$

prediction
error

As in $\theta\varepsilon$) for ARMAX:

$$\hat{y}(t|t-1,\theta) = \theta^T \underbrace{\varphi(t,\theta)}_{}$$

$\hookrightarrow$ depends on θ via $\hat{y}$

prediction is nonlinear in θ.