PSS 1

Exercises: 1.2, 1.3, 1.4, 2.2

General Background

- x: states
  y: outputs
  u: inputs / commands

State Space Models

- $$\left.\begin{aligned} \dot{x} &= f(x,u) \\ y &= h(x,u) \end{aligned}\right\} \text{time invariant}$$

- $$\left.\begin{aligned} \dot{x} &= Ax+Bu \\ y &= Cx+Du \end{aligned}\right\} \text{Linear Time Invariant (LTI)}$$

External Models

- $$y^{(n)} + \dots + a_2\ddot{y} + a_1\dot{y} + a_0 y = b_m u^{(m)} + \dots + b_1\dot{u} + b_0 u \; \} \text{ In Time Domain}$$

  - s-transform
  $$\hookrightarrow (s^n + \dots + a_2 s^2 + a_1 s + a_0)\,Y(s) = (s^m b_m + \dots + s b_1 + b_0)\,U(s)$$

- $$\left. G(s) = \frac{Y(s)}{U(s)} \right\} \text{Transfer Function (TF)}$$

TF from LTI system

- $$G(s) = C(sI-A)^{-1}B + D$$

$$\hookrightarrow \dot{x} = Ax+Bu \Rightarrow sx = Ax+Bu \Rightarrow (sI-A)x = Bu \Rightarrow x = (sI-A)^{-1}Bu$$

$$y = Cx+Du \xrightarrow[\text{plug } x]{} y = C(sI-A)^{-1}Bu + Du$$

$$y = (\underbrace{C(sI-A)^{-1}B+D}_{\text{TF}})\,u$$

1.2

Find poles and zeros

a) $$G(s) = \frac{5s+4}{s^2+5s+4} = \frac{5\left(s+\frac{4}{5}\right)}{(s+4)(s+1)} \begin{array}{l} \rightarrow \text{zeros} = -4/5 \\ \rightarrow \text{poles} = -4, -1 \end{array}$$

b) $$\dot{x}_1 = -x_1 - 1/3\, u$$
$$\dot{x}_2 = -4x_2 + 16/3\, u$$
$$y = x_1 + x_2$$

$$\dot{x} = \underbrace{\begin{bmatrix} -1 & 0 \\ 0 & -4 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} -1/3 \\ 16/3 \end{bmatrix}}_{B} u$$

$$y = \underbrace{\begin{bmatrix} 1 & 1 \end{bmatrix}}_{C} x \qquad D = 0$$

TF:

$$C(sI-A)^{-1}B + D$$

$$sI - A = \begin{bmatrix} s+1 & 0 \\ 0 & s+4 \end{bmatrix} \qquad (sI-A)^{-1} = \begin{bmatrix} 1/s+1 & 0 \\ 0 & 1/s+4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1/s+1 & 0 \\ 0 & 1/s+4 \end{bmatrix} \begin{bmatrix} -1/3 \\ 16/3 \end{bmatrix} = -\frac{1}{3} \cdot \frac{1}{s+1} + \frac{16}{3} \cdot \frac{1}{s+4} = \frac{5s+4}{s^2+5s+4}$$

same TF as A

$\Downarrow$

sames poles and zeros

1. 3

Linearization Recap (Book 1.28-1.32)

1.30 a)

$$\Delta \dot{x} = A \Delta x + B \Delta u$$
$$\Delta y = C \Delta x + D \Delta u$$

$$A = \left.\frac{\partial f}{\partial x}\right|_{x_0, u_0} \qquad B = \left.\frac{\partial f}{\partial u}\right|_{x_0, u_0}$$

$$C = \left.\frac{\partial h}{\partial x}\right|_{x_0, u_0} \qquad D = \left.\frac{\partial h}{\partial u}\right|_{x_0, u_0}$$

$$\dot{x}_1 = x_1^2 + x_2 \qquad x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad u_0 = 0$$
$$\dot{x}_2 = u$$
$$y = x_1$$

$$A = \begin{bmatrix} 2x_1 & 1 \\ 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Insert x_0, u_0

$$\Delta \dot{x} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A} \Delta x + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} \Delta u$$

$$\Delta y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \Delta x$$

1.4

$\dot{x}_1 = x_2$

$\dot{x}_2 = -x_1 - 2x_2 + u$

$y = 0.5x_1 + 0.5x_2$

Find TF?

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$M^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$ad - bc \neq 0$

$\boxed{\text{Reminder}}$

$$G(s) = C(sI-A)^{-1}B$$

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}$$

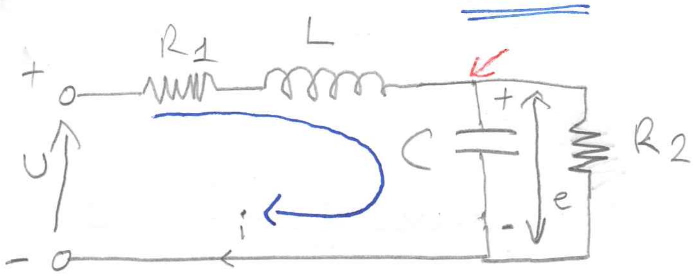
$$sI - A = \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \qquad (sI-A)^{-1} = \frac{1}{s^2+2s+1}\begin{bmatrix} s+2 & +1 \\ -1 & s \end{bmatrix}$$

$$G(s) = \frac{1}{s^2+2s+1}\begin{bmatrix} 0.5 & 0.5 \end{bmatrix}\begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{(s+1)^2}\begin{bmatrix} 0.5 & 0.5 \end{bmatrix}\begin{bmatrix} 1 \\ s \end{bmatrix} = \frac{1}{(s+1)^2} \cdot \frac{s+1}{2}$$

$$= \frac{1}{2(s+1)}$$

2.2



$V = i \cdot R$ } for R

$V = L\dot{i}$ } for L

$i = C\dot{V}$ } for C

Reminder

Determine differential equations

→ Applying Kirchhoff's law

→ $V = e + L\dot{i} + R_1 i$

→ $i = C\dot{e} + \frac{e}{R_2}$

$$\dot{i} = \frac{V - e - R_1 i}{L} = -\frac{R_1}{L} i - \frac{1}{L} e + \frac{1}{L} V$$

$$\dot{e} = \frac{i - \frac{e}{R_2}}{C} = \frac{i}{C} - \frac{e}{R_2 C} = \frac{1}{C} i - \frac{1}{R_2 C} e$$

$$x = \begin{bmatrix} i \\ e \end{bmatrix} \qquad u = V$$

$$\dot{x} = \begin{bmatrix} -R_1/L & -1/L \\ 1/C & -1/R_2 C \end{bmatrix} x + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} u$$