1.1

a)

$$\dddot{y} + 5\ddot{y} + 2\dot{y} + 5y = 3u$$

$$\begin{aligned} x_1 &= y \\ x_2 &= \dot{y} \\ x_3 &= \ddot{y} \end{aligned} \Rightarrow \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -5x_3 - 2x_2 - 5x_1 + 3u \\ y &= x_1 \end{aligned}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -2 & -5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} u \qquad y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

b)

$$\dddot{y} + \ddot{y} + 4\dot{y} + 4y = 2\ddot{u} + \dot{u} + 2u$$

$$(s^3 + s^2 + 4s + 4)Y(s) = (2s^2 + s + 2)U(s)$$

$$Y(s) = \frac{(2s^2 + s + 2)}{s^3 + s^2 + 4s + 4} U(s)$$

$$Z(s) = \frac{U(s)}{s^3 + s^2 + 4s + 4} \Rightarrow Y(s) = (2s^2 + s + 2)Z(s)$$

$$y = 2\ddot{z} + \dot{z} + 2z$$

$$\Rightarrow U(s) = (s^3 + s^2 + 4s + 4)Z(s)$$

$$u = \dddot{z} + \ddot{z} + 4\dot{z} + 4z$$

$$\begin{aligned} x_1 &= z \\ x_2 &= \dot{z} \\ x_3 &= \ddot{z} \end{aligned} \rightarrow \begin{aligned} y &= 2x_3 + x_2 + 2x_1 \\ u &= \dot{x}_3 + x_3 + 4x_2 + 4x_1 \end{aligned}$$

if we rearrange it

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -4x_1 - 4x_2 - x_3 + u \\ y &= 2x_1 + x_2 + 2x_3 \end{aligned} \Rightarrow \dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -4 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 2 & 1 & 2 \end{bmatrix} u$$