

$$y = \begin{bmatrix} y(1) \\ \vdots \\ y(N) \end{bmatrix}_{N \times 1} \quad \Phi = \begin{bmatrix} \phi^T(1) \\ \vdots \\ \phi^T(N) \end{bmatrix} = \begin{bmatrix} 1 & x(1) \\ 1 & x(2) \\ \vdots & \vdots \\ 1 & x(N) \end{bmatrix}_{N \times 2} \quad \frac{d(x^T a)}{dx} = \frac{d(a^T x)}{dx} = a^T$$

$$V_N = \frac{1}{2} (y - \Phi \theta)^T (y - \Phi \theta)$$

$$V_N = \frac{1}{2} (y^T y - y^T \Phi \theta - \underbrace{(\Phi \theta)^T y}_{\theta^T \Phi^T y} + \underbrace{(\Phi \theta)^T (\Phi \theta)}_{\theta^T \Phi^T \Phi \theta})$$

$$\frac{d}{d\theta} V_N = -\frac{1}{2} y^T \Phi - \frac{1}{2} (\Phi^T y)^T + \frac{1}{2} \cdot 2 \theta^T (\Phi^T \Phi)^T$$

$$= -\frac{1}{2} y^T \Phi - \frac{1}{2} y^T \Phi + \theta^T \Phi^T \Phi$$

$$= -y^T \Phi + \theta^T \Phi^T \Phi$$

$$\frac{d}{d\theta} V = -y^T \Phi + \theta^T \Phi^T \Phi = 0$$

$$(\theta^T \Phi^T \Phi)^T = (y^T \Phi)^T$$

$$(\Phi^T \Phi)^T \cdot \theta = \Phi^T y$$

$$\Phi^T \Phi \theta = \Phi^T y$$

$$\hat{\theta} = \underbrace{(\Phi^T \Phi)^{-1}}_{2 \times 2} \underbrace{\Phi^T}_{2 \times N} \underbrace{y}_{N \times 1}$$