

## ARX

$$y(t) + a_1 y(t-1) + \dots + a_n y(t-n) = b_1 u(t-1) + \dots + b_n u(t-n) + e(t)$$

$$y(t) = -a_1 y(t-1) - a_2 y(t-2) - \dots - a_n y(t-n) + b_1 u(t-1) + b_2 u(t-2) + \dots + b_n u(t-n) + e(t)$$

$$y(t) = b_1 u(t-1) + \dots + b_n u(t-n) - a_1 y(t-1) - \dots - a_n y(t-n) + e(t)$$

$$q^{-1} = \text{backshift operator} : q^{-1} y(t) = y(t-1)$$

$$\left. \begin{array}{l} A(q^{-1}), B(q^{-1}) \text{ are polynomials in } q^{-1} \text{ with coefficients } a_i \text{ and } b_j \end{array} \right\}$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} + \dots + a_n q^{-n}$$

$$B(q^{-1}) = b_1 q^{-1} + b_2 q^{-2} + \dots + b_n q^{-n}$$

$$y(t) = b_1 q^{-1} u(t) + \dots + b_n q^{-n} u(t) - a_1 q^{-1} y(t) - \dots - a_n q^{-n} y(t) + e(t)$$

$$y(t) + a_1 q^{-1} y(t) + \dots + a_n q^{-n} y(t) = b_1 q^{-1} u(t) + \dots + b_n q^{-n} u(t) + e(t)$$

$$\boxed{A(q, \theta) y(t) = B(q, \theta) u(t) + e(t)}$$

$$\hat{y}(t|t-1, \theta) = B(q, \theta) u(t) + (1 - A(q, \theta)) y(t)$$

$$\hat{y}(t|t-1, \theta) = - \sum_{i=1}^n a_i y(t-i) + \sum_{j=1}^n b_j u(t-j)$$

$$\theta = [a_1, \dots, a_n, b_1, \dots, b_n]^T$$

$$\varphi(t) = [-y(t-1), \dots, -y(t-n), u(t-1), \dots, u(t-n)]^T$$

$$y(t) = \theta^T \varphi(t) + e(t)$$

$$\hat{y}(t) = \theta^T \varphi(t) = \hat{y}(t|t-1, \theta)$$

$$\hat{\theta}_N = R_N^{-1} f_N$$