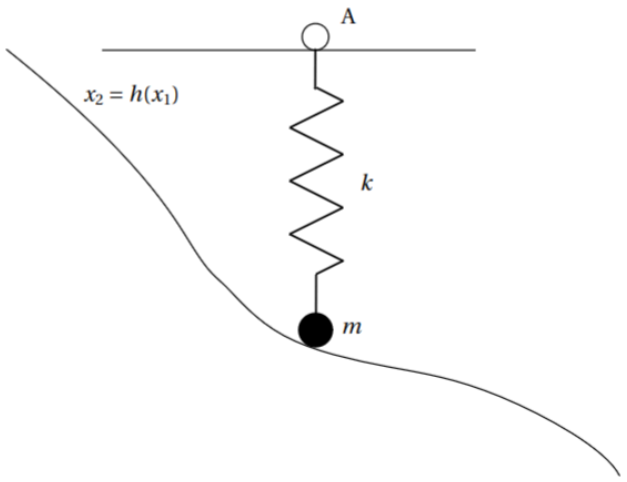
Consider the mechanical system depicted below. A ball with (point-like) mass m has the position (x_1, x_2), where x_1 is the horizontal and x_2 the vertical coordinate. The ball is gliding without friction along a rail that is described by the relation $x_2 = h(x_1)$. Further, the ball is attached to one end of a spring, having the spring constant k. The other end of the spring (A) is gliding without friction along a horizontal rail, so that the spring is always vertical.
The forces acting on the ball are thus the spring force (assuming the neutral position of the force corresponds to $x_2 = 0$), gravity g, and the normal force from the rail.



(a) (4 points) Determine the Lagrange function for the system. *Reminder:* Potential energy of a spring is defined as $\frac{1}{2}kp^2$ where k is spring constant and p denotes spring displacement.

(b) (4 points) Derive a dynamic model of the system.

Consider two masses of mass m (the black balls in Fig. 1) linked by a massless rigid rod of length L. The masses are gliding without friction on a surface of equation:

$$z = \frac{1}{2}x^2 + \frac{1}{2}y^2 \tag{1}$$

(a) (4 points) Write the Lagrange function describing the system, assuming gravity is the only external force.

(b) (4 points) Derive the equations of motion

(c) (3 points) What are the consistency conditions associated to the equations?

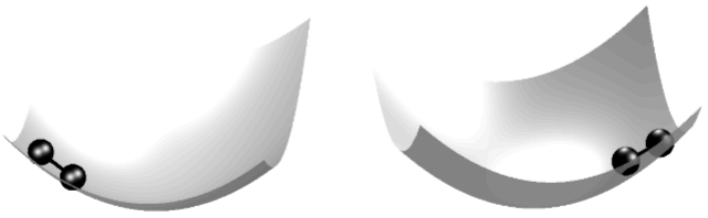


Figure 1: Illustration of the rollercoaster