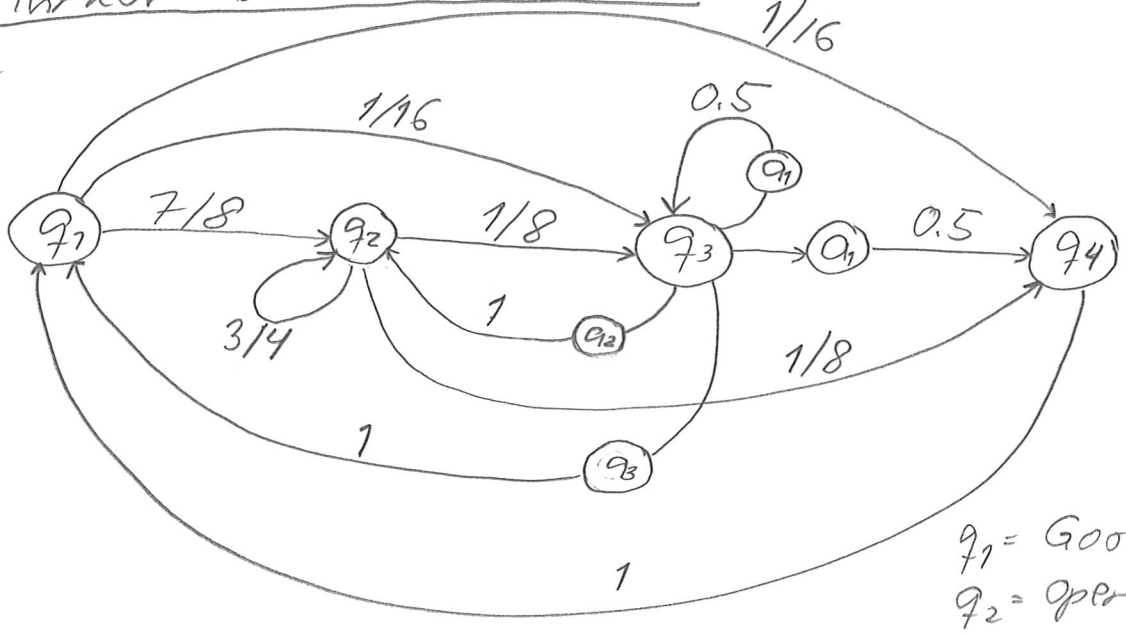
Markov Decision Process

Ex



q_1 = Good as new
q_2 = Operable - minor deterioration
q_3 = Operable - major deterioration
q_4 = Inoperable - output of unacceptable quality

$Q = \{q_1, q_2, q_3, q_4\}$ states

$A = \{a_1, a_2, a_3\}$ actions

$$P = \begin{bmatrix} 0 & 7/8 & 1/16 & 1/16 \\ 0 & 3/4 & 1/8 & 1/8 \\ p_{31} & p_{32} & p_{33} & p_{34} \\ 1 & 0 & 0 & 0 \end{bmatrix} \qquad p = p \cdot P$$

Action: $a_1 \Rightarrow p_{31} = p_{32} = 0,\ p_{33} = 0.5,\ p_{34} = 0.5$

$a_2 \Rightarrow p_{32} = 1,\ p_{31} = p_{33} = p_{34} = 0$

$a_3 \Rightarrow p_{31} = 1,\ p_{32} = p_{33} = p_{34} = 0$

State probability

$p(a_1) = \frac{1}{13}(2\ 7\ 2\ 2)$

$p(a_2) = \frac{1}{21}(2\ 15\ 2\ 2)$

$p(a_3) = \frac{1}{11}(2\ 7\ 1\ 1)$

Cost

$C(a_1) = (0\ 1\ 3\ 6)$

$C(a_2) = (0\ 1\ 4\ 6)$

$C(a_3) = (0\ 1\ 6\ 6)$

1 minor loss
3 major loss
4 repair machine
6 replace machine

Action a_2 $\quad p_{32}=1 \quad p_{31}=p_{33}=p_{34}=0$

$$\underbrace{[p_1\ p_2\ p_3\ p_4]}_{p} = \underbrace{[p_1\ p_2\ p_3\ p_4]}_{p} \underbrace{\begin{bmatrix} 0 & \frac{7}{8} & \frac{1}{16} & \frac{1}{16} \\ 0 & \frac{3}{4} & \frac{1}{8} & \frac{1}{8} \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{TP} =$$

$$= [p_4 \quad \underbrace{\tfrac{7}{8}p_1 + \tfrac{3}{4}p_2 + p_3}_{=p_2} \quad \underbrace{\tfrac{p_1}{16} + \tfrac{p_2}{8}}_{=p_3} \quad \underbrace{\tfrac{p_1}{16} + \tfrac{p_2}{8}}_{=p_4}]$$

$p_1 = p_4 = p$

$p_3 = p_4 = p$

$p_1 + p_2 + p_3 + p_4 = 1 \Rightarrow$

$p_2 = 1 - p_1 - p_3 - p_4 = 1 - p - p - p =$

$= 1 - 3p$

$\frac{7}{8}p_1 + \frac{3}{4}p_2 + p_3 = \frac{7}{8}p + \frac{3}{4}(1-3p) + p = p_2 = 1-3p$

$\frac{7}{8}p - \frac{9}{4}p + p + 3p = 1 - \frac{3}{4} = \frac{1}{4}$

$\underbrace{(7 - 18 + 8 + 24)}_{21}p = 2 \quad \Rightarrow \quad p_1 = p_3 = p_4 = p = \frac{2}{21}$

$p_2 = 1 - 3p = \frac{15}{21}$

$p(a_2)$ = state probability for action 2 = $\frac{1}{21}[2\ 15\ 2\ 2]$

$$V(a_1) = p(a_1)\, C^T(a_1) = \frac{7\cdot1 + 2\cdot3 + 2\cdot6}{13} = \frac{25}{13} = 1.92$$

$$V(a_2) = p(a_2)\, C^T(a_2) = \frac{15\cdot1 + 2\cdot4 + 2\cdot6}{21} = \frac{35}{21} = 1.67$$

$$V(a_3) = p(a_3)\, C^T(a_3) = \frac{7\cdot1 + 1\cdot6 + 1\cdot6}{11} = \frac{19}{11} = 1.73$$

$$\min_{a \in \{a_1, a_2, a_3\}} V(a) = V(a_2) = 1.67$$

$\therefore$ Policy a_2 gives the lowest cost including minor loss in g_2, repair cost in g_3 and (machine) replacement in g_4.