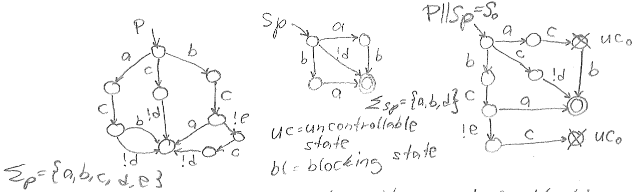
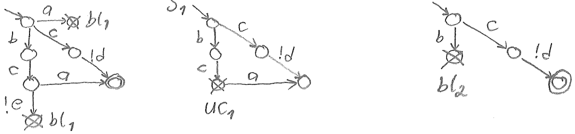
Supervisor synthesis: Example



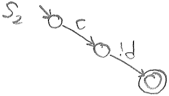
k=1 blocking

k=1 uncontrollable

k=2 blocking



k=2 uncontrollable



$S = S_3 = S_2$ = maximal permissive controllable and nonblocking supervisor

7. Supervisor Synthesis

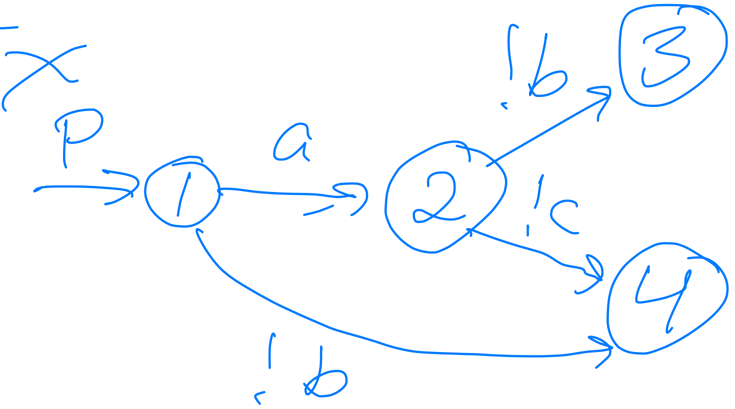
Generation of uncontrollable states in the synchronization

$S_0 = P \| Sp$

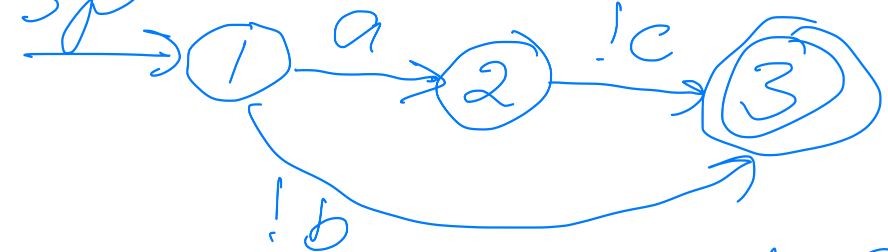
For all uncontrollable events $\sigma_u \in \Sigma_u \cap \Sigma^{sp}$, identify related plant transitions $q^p \xrightarrow{\sigma_u} q^{p'}$. Corresponding states $\langle q^p, q^{sp} \rangle \not\xrightarrow{\sigma_u}$ (no σ_u-transition from state $\langle q^p, q^{sp} \rangle$) without σ_u-transition are uncontrollable states.

Ex

P

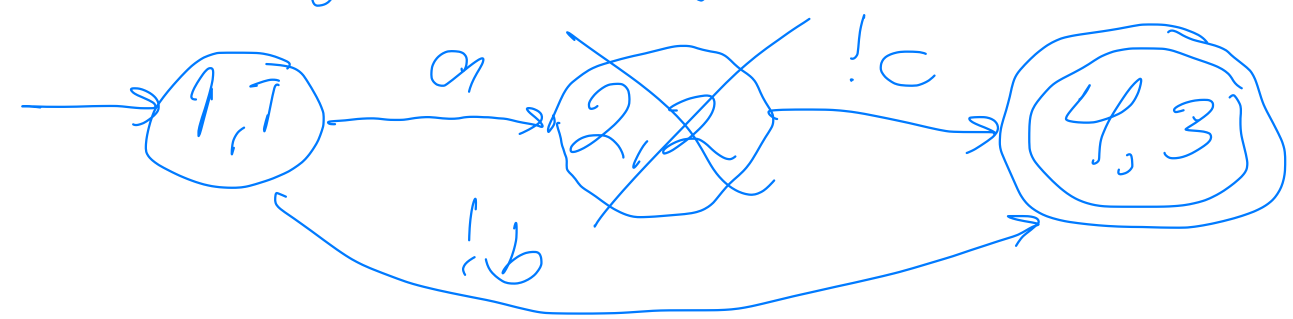


Sp

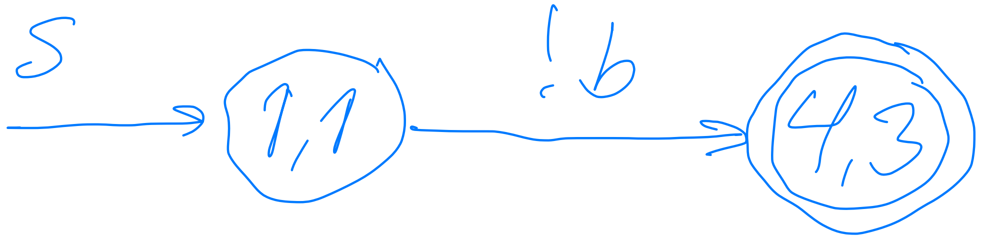


$S_0 = P \| Sp$

uncontrollable state in S_0



Since the transition $2 \xrightarrow{!b} 3$ exists in the plant P, while $(2,2) \not\xrightarrow{!b}$ does not exist in $S_0 = P \| Sp$ the state $(2,2)$ is uncontrollable.

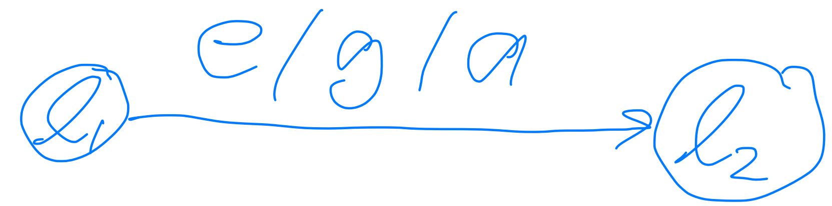


Alternative formulation of this supervisor S:

Event a is not allowed in state 1 in P.

8. Extended automata models

Extended finite automata (EFA)



l_1 and l_2 are locations

e = event label

g = guard expression e.g.

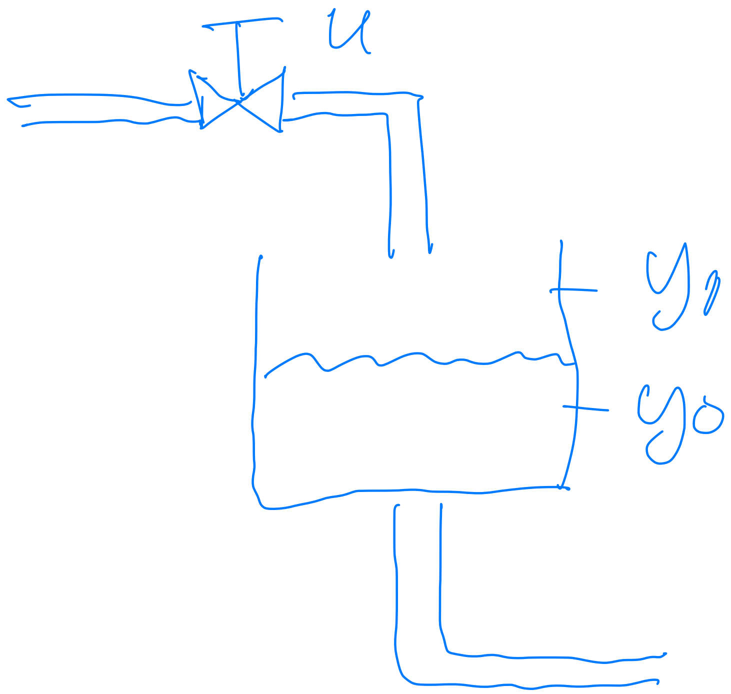
$$(v_1 = 0) \wedge (v_2 > 2)$$

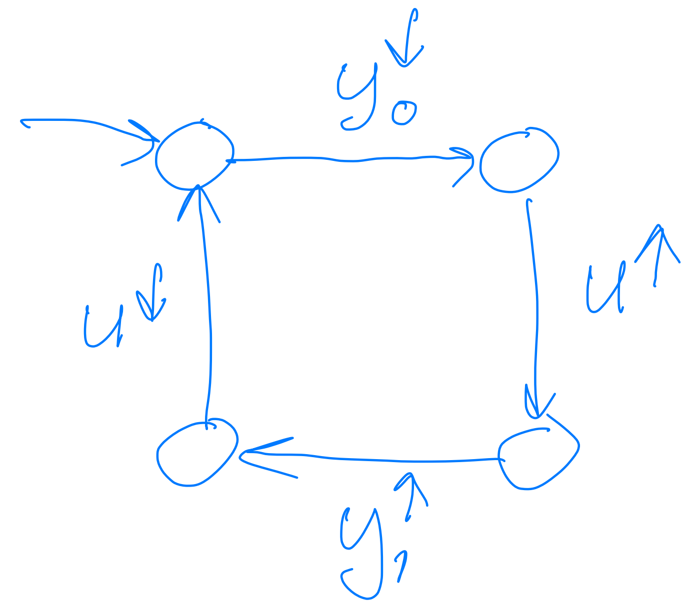
a = action function e.g.

$$v_1 := 1$$

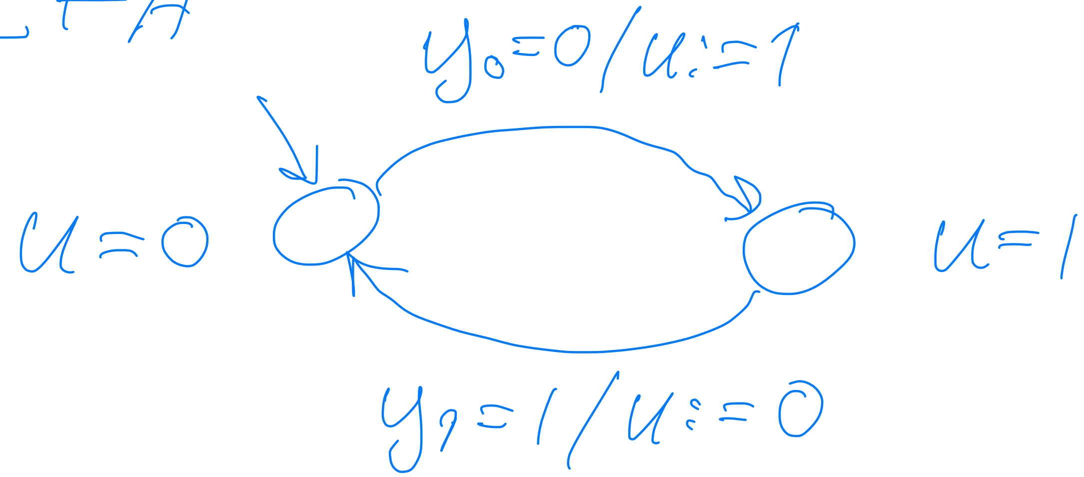
The total state is a combination of actual location and the value of the variables in that location.

Ex Tank process



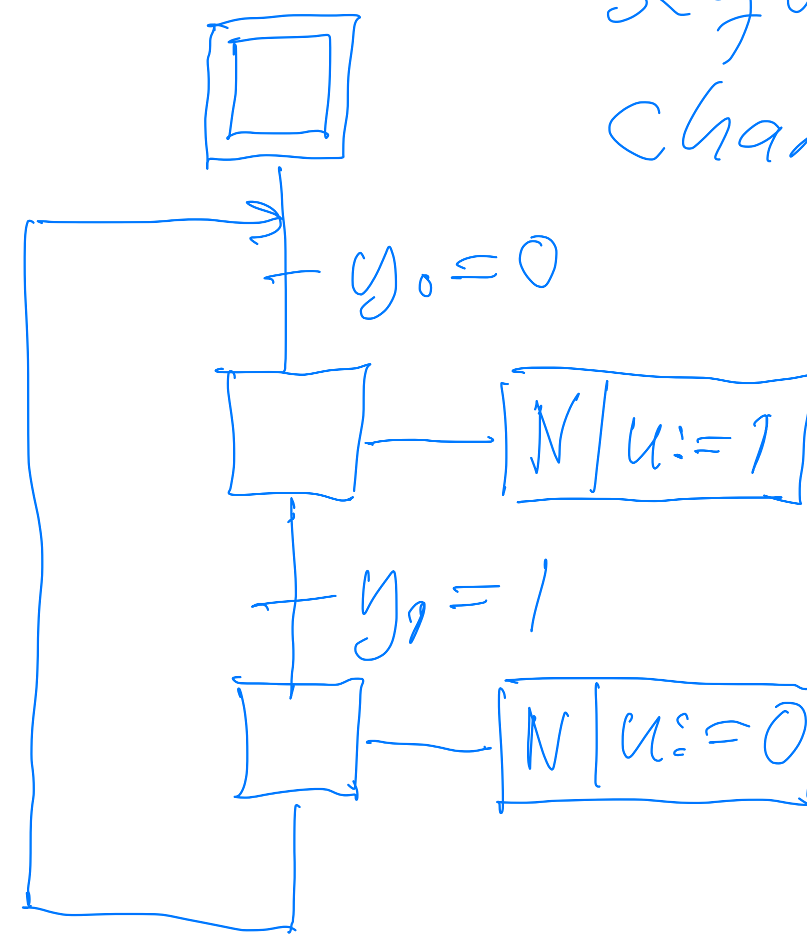


EFA



PLC - controller implementation

sequential function chart (SFC)



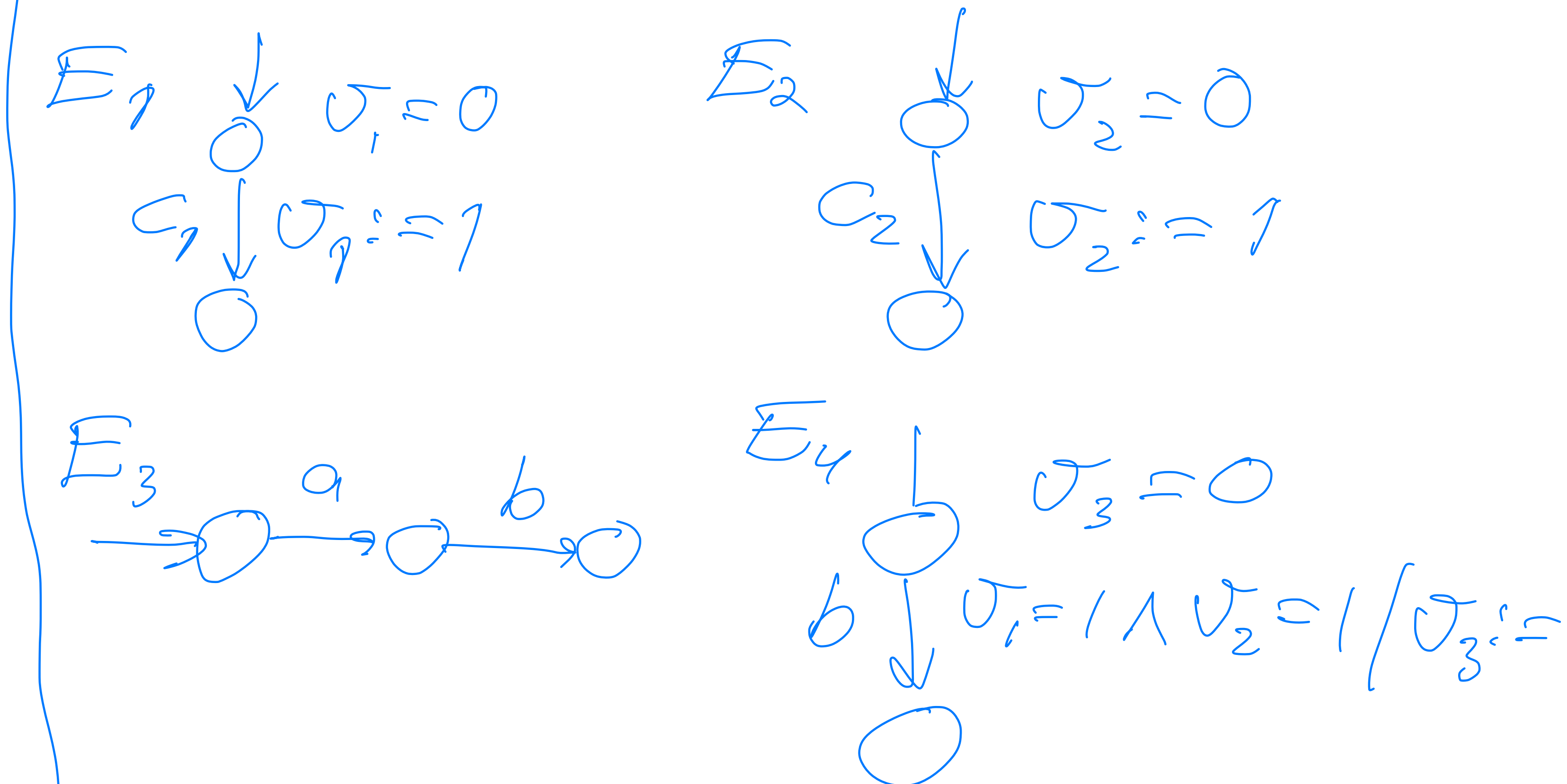
Synchronization of EFAs

$E_1 \| E_2$ For transitions with shared events the corresponding guards must be satisfied in both E_1 and E_2, and the actions should not be in conflict to execute the shared transition.

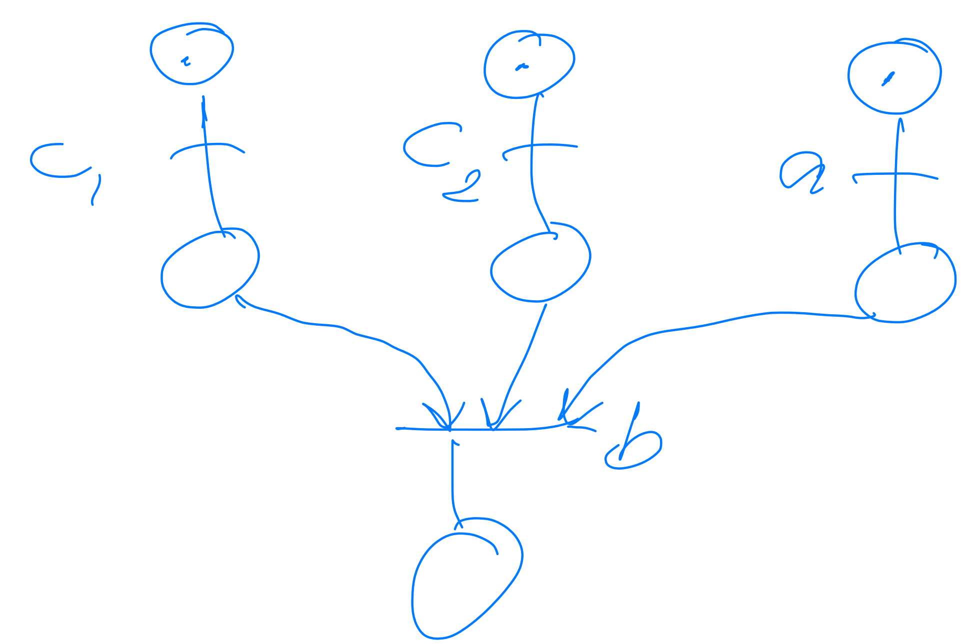
$u := 1$ in E_1 and $u := 2$ in E_2 $\Rightarrow$ conflict

while $u := 1$ in E_1 and $v := 2$ in E_2 $\Rightarrow$ no conflict

Ex Compare EFAs, Petri nets and automata

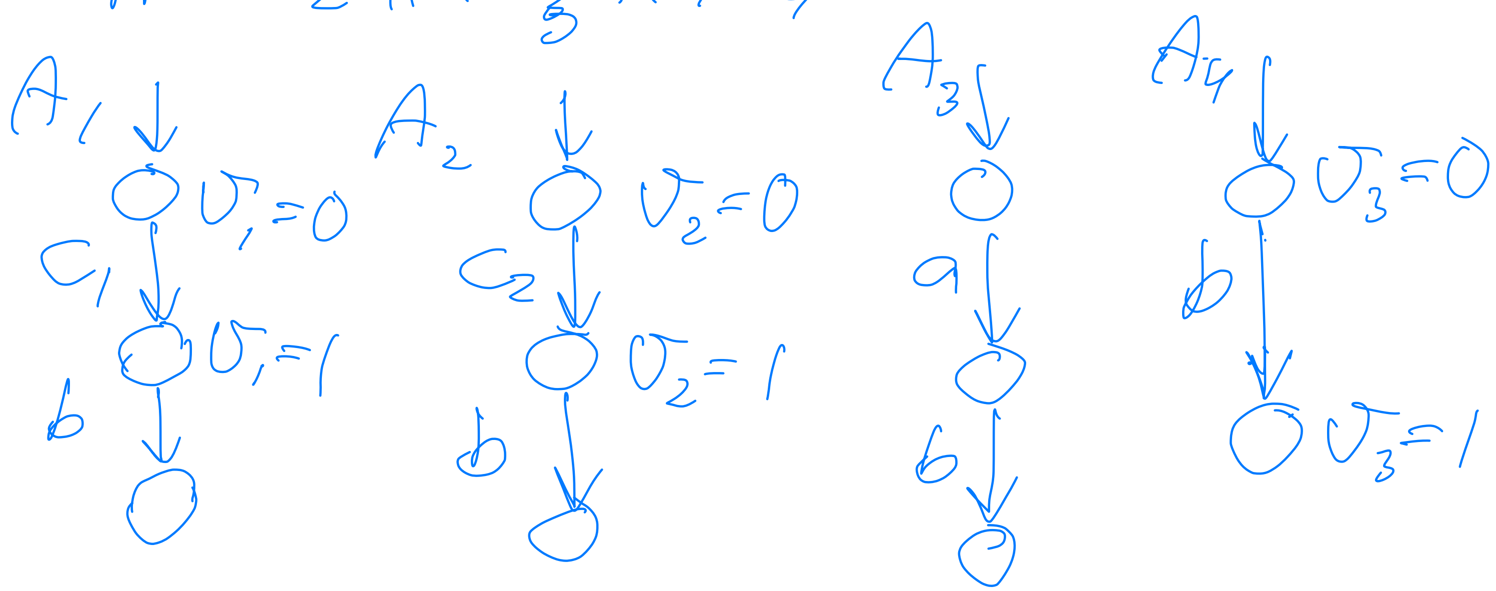


$E_1 \| E_2 \| E_3 \| E_4$

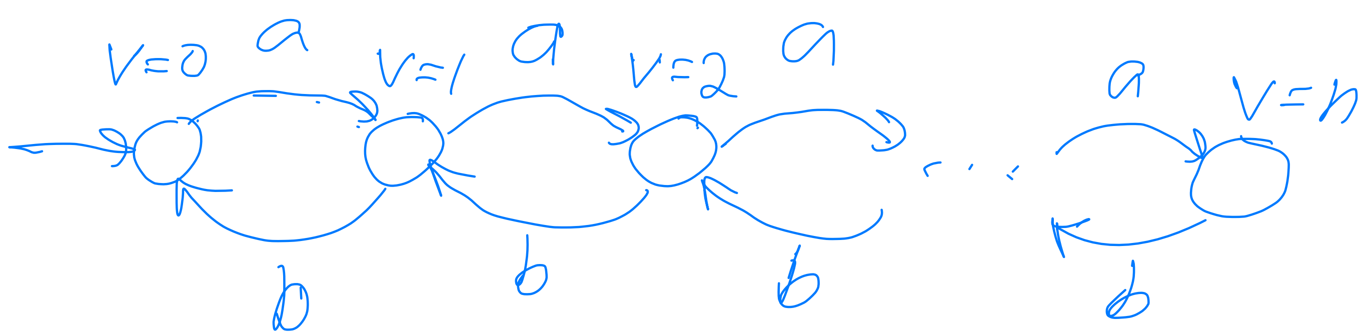


Corresponding automata

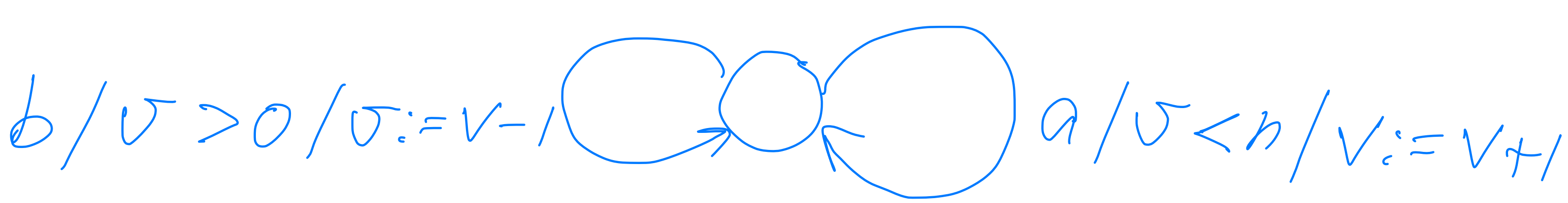
$A_1 \| A_2 \| A_3 \| A_4$



Ex Buffer

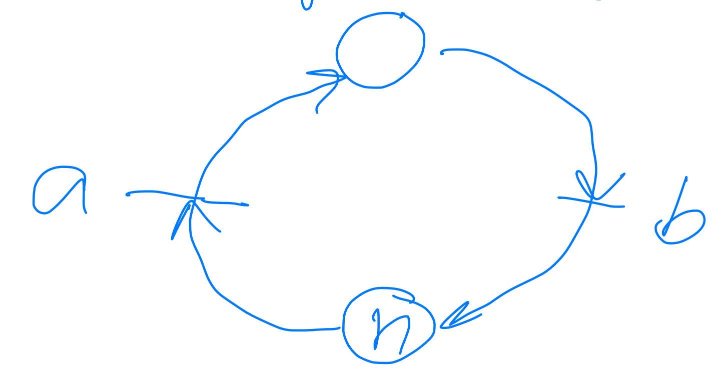


EFA



PN

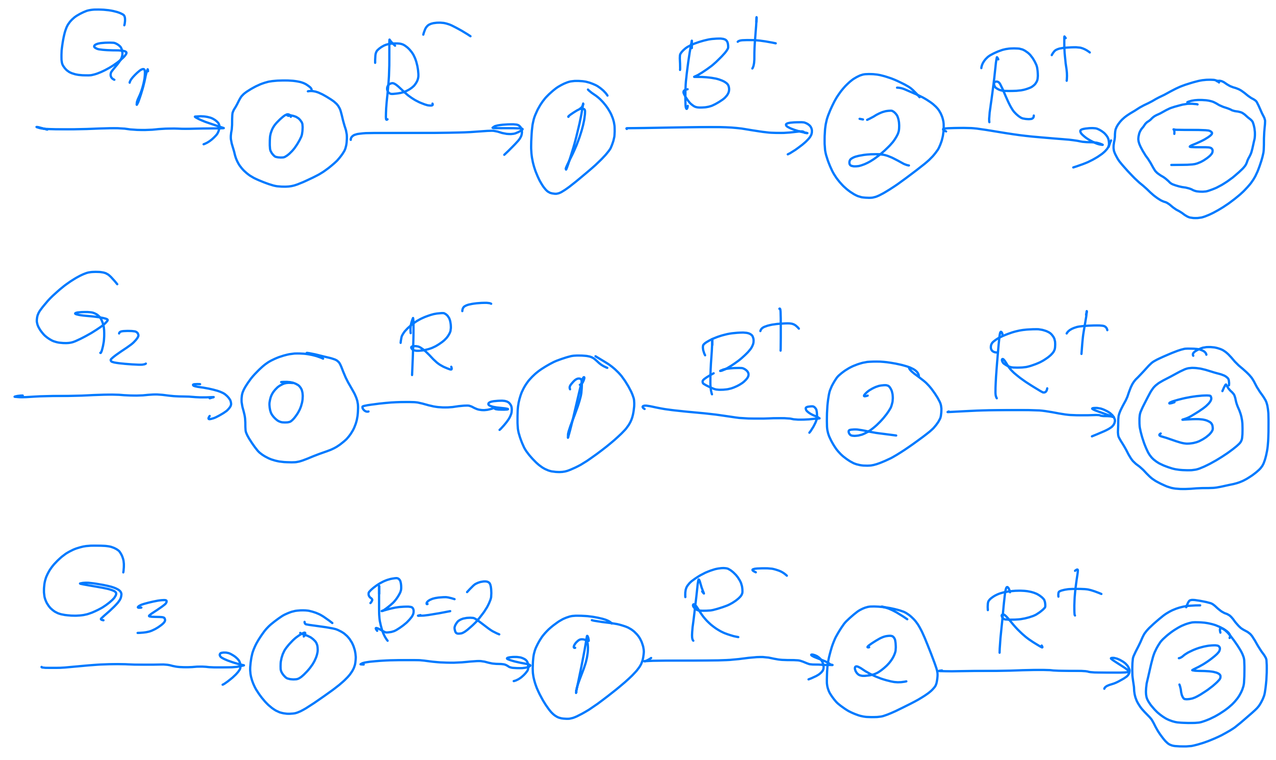
number of occupied places in the buffer



n tokens

number of empty places in the buffer

Ex



R^- means $R := R-1$

R^+ means $R := R+1$

Assume that the domains of R and B are $\{0, 1, 2\}$.

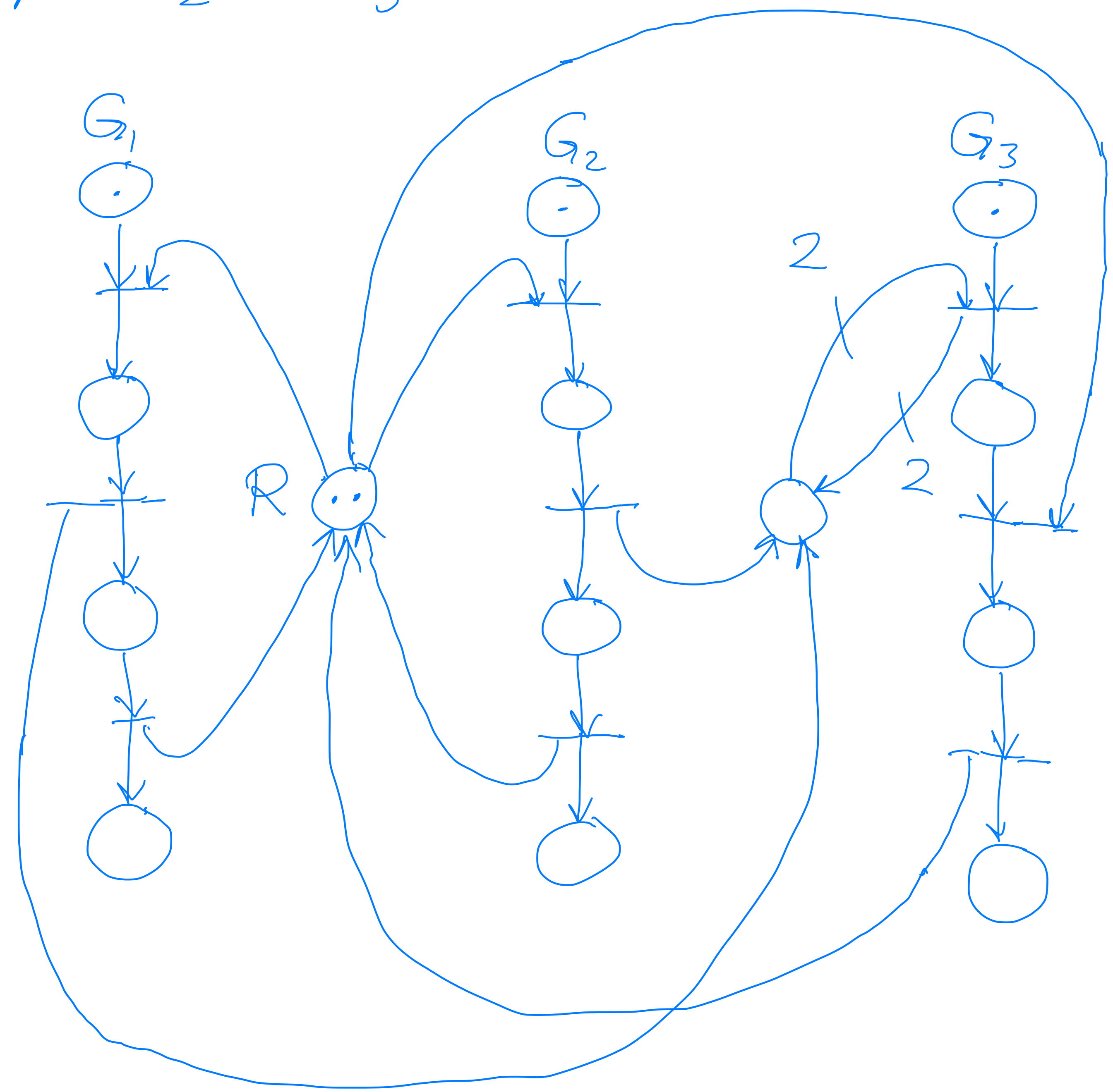
Implicitly this introduces the guards $0 \leq R \leq 2$

$0 \leq B \leq 2$

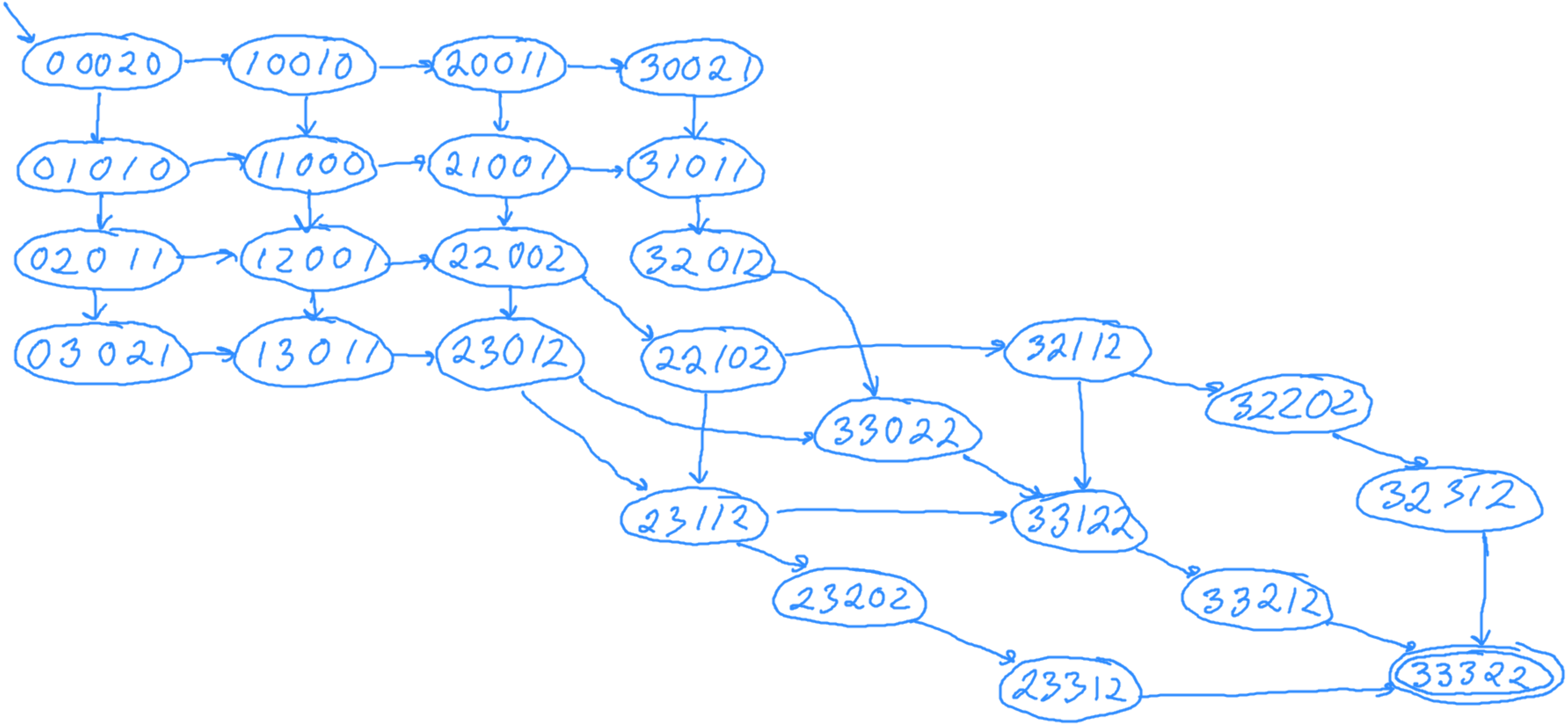
Initial values: $R=2$, $B=0$

corresponding PN for

$G_1 \,||\, G_2 \,||\, G_3$

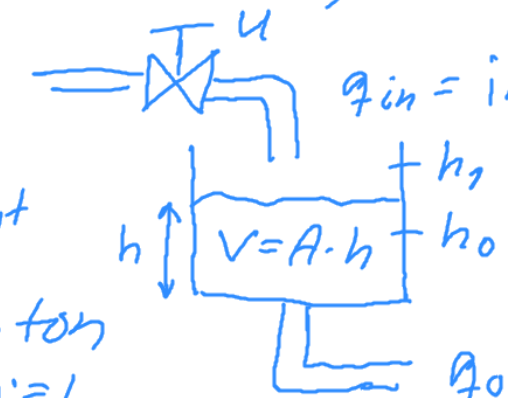


state (q_1, q_2, q_3, R, B)



Hybrid automata

Introduce continuous dynamics in terms of differential equations (state space model) in each location

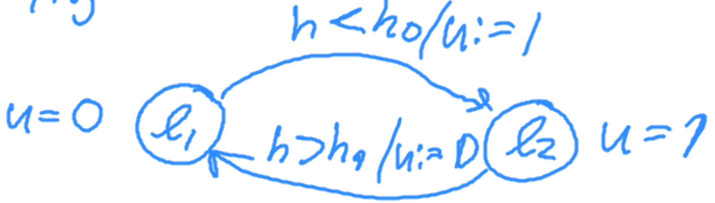


q_{in} = input flow

q_{out} = output flow

$$\frac{dV}{dt} = A\dot{h} = u \cdot q_{in} - q_{out}$$

Hybrid automaton

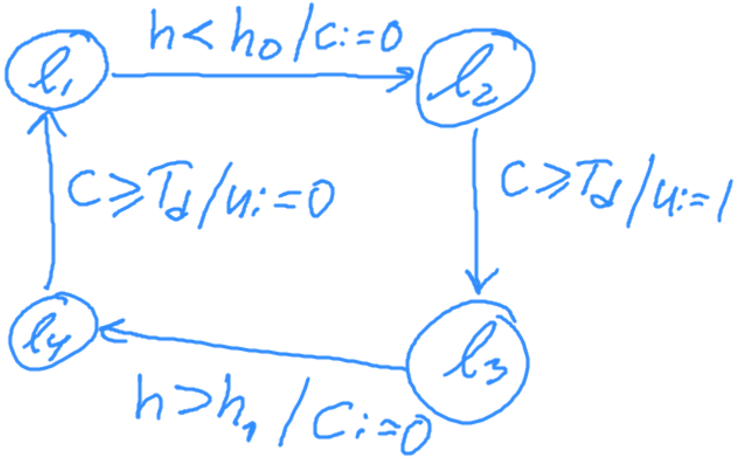


u = discrete control signal either 0 or 1.

Differential eq. valid in both locations l_1 and l_2

$$\dot{h} = \frac{1}{A}(u \cdot q_{in} - q_{out})$$

Extended model including a time delayed valve



In all locations we have two differential equations

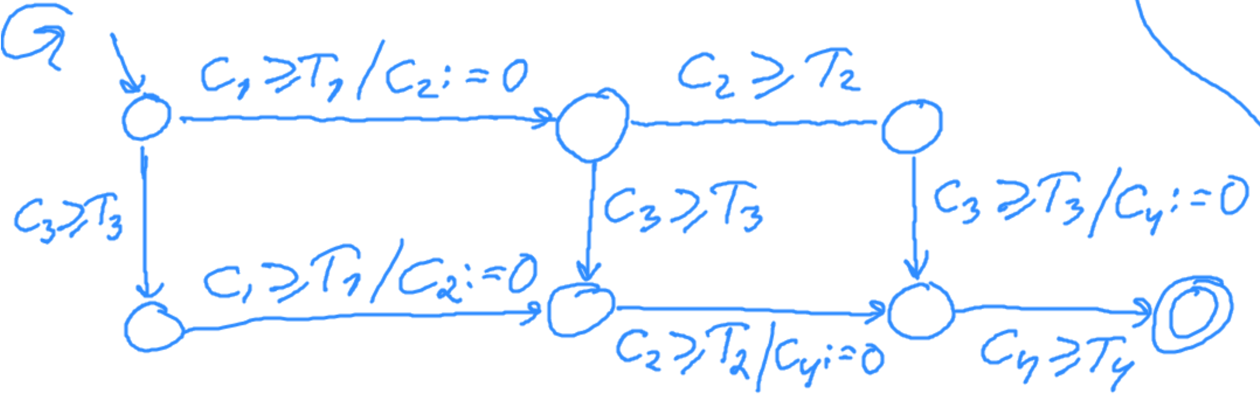
$$\begin{cases} \dot{h} = \frac{1}{A}(u \cdot q_{in} - q_{out}) \\ \dot{c} = 1 \end{cases}$$

Timed automaton

A system including four clocks $\dot{c}_i = 1 \quad i = 1, \dots, 4$

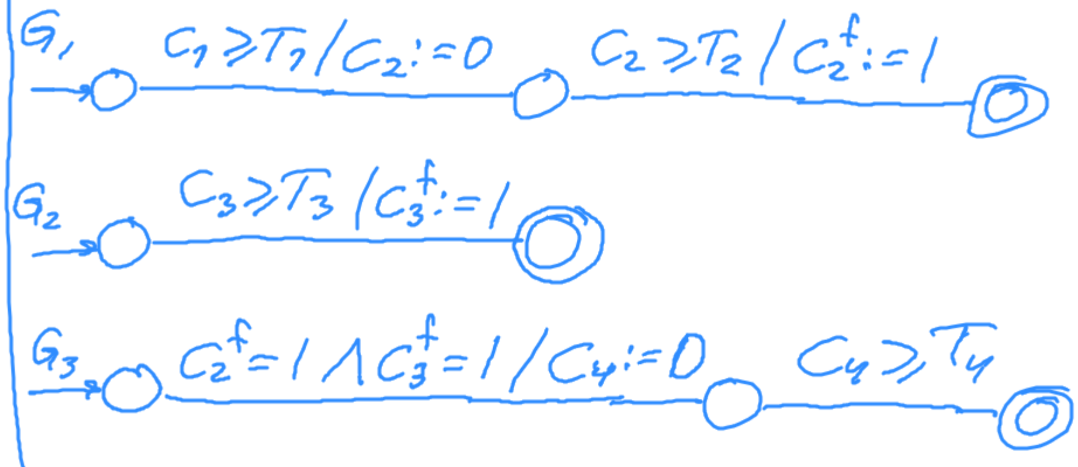
Every clock has a completion time condition $c_i \geq T_i$

Global clock model



We assume that in the initial state $c_1 = c_2 = c_3 = c_4 = 0$

Local clock models



$$G = G_1 \parallel G_2 \parallel G_3$$