9. Temporal logic

For a transition system G with state set X and initial state set I, G satisfies a temporal logic formula ψ, written as $G \models \psi$, if ψ holds in all initial states of G, i.e. if

$$I \subseteq \Psi = [\![\psi]\!] = \{x \in X \mid x \in \psi\}$$

The set $[\![\psi]\!]$ can be determined by μ-calculus.

Ex

Evaluate the nonblocking condition $[\![\forall\Box\exists\Diamond p]\!]$, where the state label $\{p\}$ determines a marked state.

Since $\exists\Diamond p \equiv \mu z.\, p \vee \exists\bigcirc z$

$\forall\Box q \equiv \nu y.\, q \wedge \forall\bigcirc y$

$\forall\Box\exists\Diamond p \equiv \nu y.\, (\mu z.\, p \vee \exists\bigcirc z) \wedge \forall\bigcirc y$

$[\![\mu z.\, p \vee \exists\bigcirc z]\!] = \mu z.\, \Psi(z)$

where $\Psi(z) = [\![p]\!] \cup Pre^{\exists}(z) = z$

Least fixed point:

$z_{i+1} = \Psi(z_i)$, $z_0 = \emptyset$

$Z_1 = \Psi(Z_0) = [\![p]\!] \cup Pre^{\exists}(\emptyset) =$

$= \{2\} \cup \emptyset = \{2\}$

$Z_2 = [\![p]\!] \cup Pre^{\exists}(Z_1) =$

$\{2\} \cup \{0\} = \{0, 2\}$

$Z_3 = \{2\} \cup Pre^{\exists}(\{0,2\}) =$

$= \{2\} \cup \{0\} = \{0,2\} = Z_2 =$

$=$ fixed point Z^{ω}

$[\![\forall \Box \exists \Diamond p]\!] = \nu Y. \Psi(Y)$

where $\Psi(Y) = [\![\mu Z.\, p \vee \exists \bigcirc Z]\!]$

$\wedge\, Pre^{\forall}(Y) = Z^{\omega} \wedge Pre^{\forall}(Y) = Y$

Greatest fixed point:

$Y_{i+1} = \Psi(Y_i)$, $Y_0 = \Sigma$

$Y_1 = Z^{\omega} \cap Pre^{\forall}(\Sigma) = \{0,2\} \cap \{0,1,2\} = \{0,2\}$

$Y_2 = Z^{\omega} \cap Pre^{\forall}(Y_1) =$

$= \{0,2\} \cap \{2\}$ (0 is excluded since its target state 1 is not included in $Y_1 = \{0,2\}$) $= \{2\}$

$Y_3 = Y_2 =$ fixed point Y^{ω}.

$I = \{0\} \not\subseteq [\![\forall \Box \exists \Diamond p]\!] = Y^{\omega} = \{2\}$

$\Rightarrow G \not\models \forall \Box \exists \Diamond p$

Remove state $\Rightarrow$

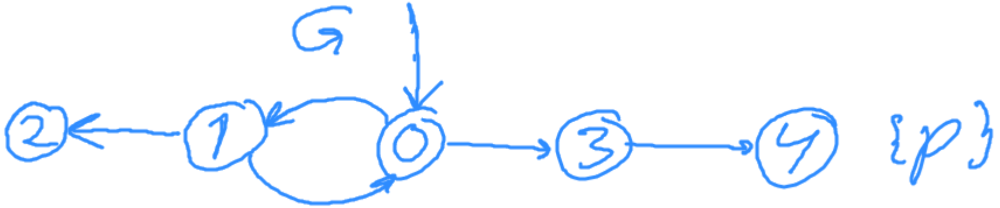
$Z^{\omega} = \{0,2\}$ $\quad Y_1 = Y_2 = Y^{\omega} = \{0,2\}$

$\Rightarrow I \subseteq Y^{\omega}$ and $G \models \forall \Box \exists \Diamond p$.

Correct since state 2 is a blocking state.

Removing this blocking state
$\Rightarrow$ G is nonblocking and satisfies the nonblocking condition $\forall\Box\exists\Diamond p$

Ex



Evaluate again the nonblocking condition $\forall\Box\exists\Diamond p$

$[\![\exists\Diamond p]\!] = \mu Z.\, \Psi(Z)$ where

$\Psi(Z) = [\![p]\!] \cup Pre^{\exists}(Z) = Z$

Least fixed point iteration:

$Z_0 = \emptyset$, $Z_1 = [\![p]\!] \cup Pre^{\exists}(\emptyset) = \{4\} \cup \emptyset$

$Z_2 = [\![p]\!] \cup Pre^{\exists}(\{4\}) =$
$= \{4\} \cup \{3\} = \{3,4\}$

$Z_3 = [\![p]\!] \cup Pre^{\exists}\{3,4\} = \{4\} \cup \{0,3\} =$
$= \{0,3,4\}$

$Z_4 = \{4\} \cup \{1,0,3\} = \{0,1,3,4\}$

$Z_5 = Z_4 =$ fixed point $Z^{\omega} = \{0,1,3,4\}$

$[\![\forall\Box\exists\Diamond p]\!] = \nu Y.\, Z^{\omega} \cap Pre^{\forall}(Y)$

Greatest fixed point iteration:

$Y_0 = \mathbb{X}$, $Y_1 = Z^{\omega} \cap Pre^{\forall}(\mathbb{X}) =$
$= Z^{\omega} \cap \mathbb{X} = Z^{\omega} = \{0,1,3,4\}$

$Y_2 = Z^{\omega} \cap Pre^{\forall}(\underbrace{\{0,1,3,4\}}_{Y_1}) =$

$= Z^w \cap \{0,3,4\} = ($state 1 excluded since its target state is not included in Y_1 in $\mathrm{Pre}^{\forall}(Y_1)) = \{0,3,4\}$

$Y_3 = Z^w \cap \mathrm{Pre}^{\forall}(\underbrace{\{0,3,4\}}_{Y_2}) = \{3,4\}$

since the target state 1 of state 0 not included in Y_2

$Y_4 = Y_3 =$ fixed point $Y^w = \{3,4\}$

$I = \{0\} \not\subseteq Y^w = [\![\forall\Box\exists\Diamond p]\!] \Rightarrow$

$G \not\models \forall\Box\exists\Diamond p$

10. Reinforcement learning (RL)

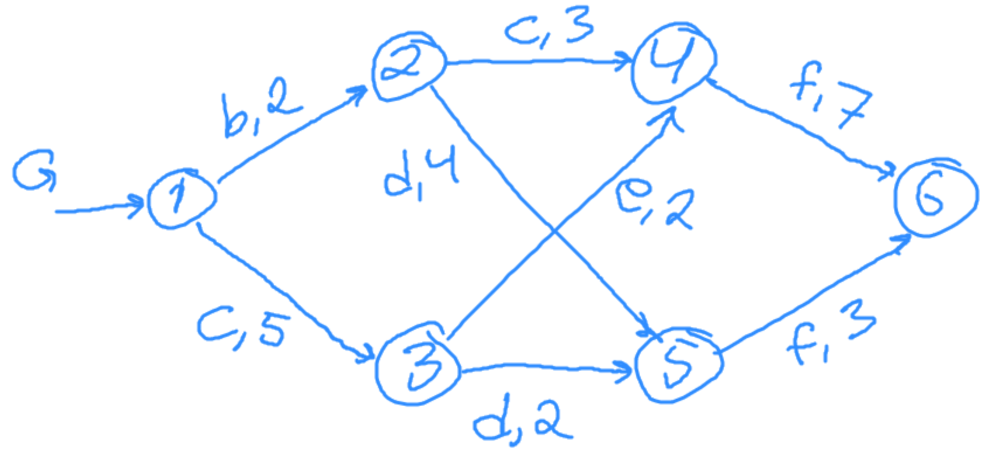
RL = optimization method where actions are sent to the plant and resulting rewards are evaluated such that optimal actions are selected after an initial learning phase.

Currently a very popular method within modern AI and machine learning.

AlphaGo is a popular program where its success is based on RL.

The optimization in RL is a minor reformulation of Dynamic Programming

Rewards are then introduced on the transitions in an automaton.

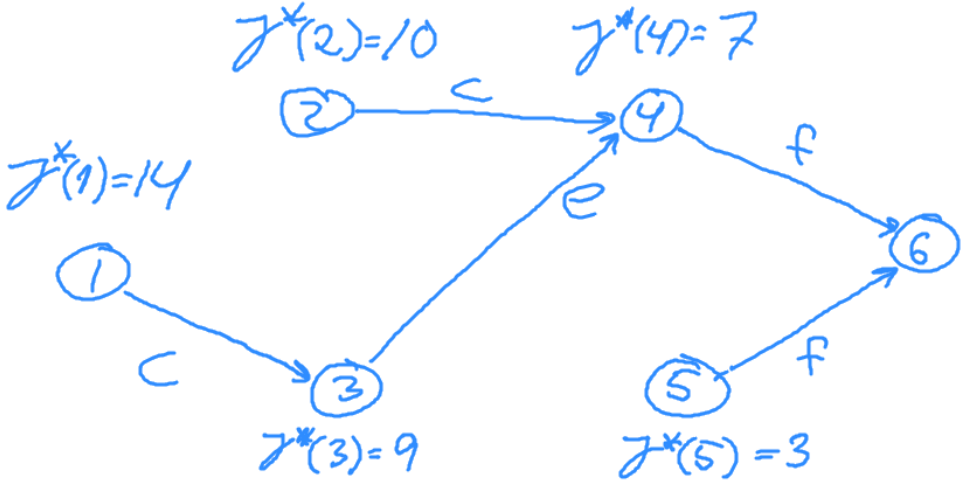


Reward function $\rho : X \times \Sigma \rightarrow \mathbb{R}$

$\rho(1, b) = 2, \ \rho(1, c) = 5, \ \rho(2, c) = 3, \ \ldots$

optimization

$$J^*(x_k) = \max_{\{a_k, \ldots, a_{N-1}\}} \sum_{i=k}^{N-1} \rho(x_i, a_i)$$



Optimal actions: $a(1) = c$, $a(2) = c$
$a(3) = e$, $a(4) = f$, $a(5) = f$