

Table 1.1: Equivalence relations.

$E_1$	$\neg\neg p \Leftrightarrow p$		$E_3$	$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$
$E_2$	$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$		$E_5$	$p \wedge q \Leftrightarrow q \wedge p$
$E_4$	$p \vee q \Leftrightarrow q \vee p$		$E_7$	$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$
$E_6$	$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$		$E_9$	$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$
$E_8$	$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$		$E_{11}$	$p \wedge p \Leftrightarrow p$
$E_{10}$	$p \vee p \Leftrightarrow p$		$E_{13}$	$p \wedge \mathbf{T} \Leftrightarrow p$
$E_{12}$	$p \vee \mathbf{F} \Leftrightarrow p$		$E_{15}$	$p \wedge \mathbf{F} \Leftrightarrow \mathbf{F}$
$E_{14}$	$p \vee \mathbf{T} \Leftrightarrow \mathbf{T}$		$E_{17}$	$p \wedge \neg p \Leftrightarrow \mathbf{F}$
$E_{16}$	$p \vee \neg p \Leftrightarrow \mathbf{T}$		$E_{19}$	$p \wedge (p \vee q) \Leftrightarrow p$
$E_{18}$	$p \vee (p \wedge q) \Leftrightarrow p$		$E_{21}$	$\neg(p \rightarrow q) \Leftrightarrow p \wedge \neg q$
$E_{20}$	$p \rightarrow q \Leftrightarrow \neg p \vee q$		$E_{23}$	$\neg\Diamond p \Leftrightarrow \Box\neg p$
$E_{22}$	$\neg\Box p \Leftrightarrow \Diamond\neg p$		$E_{25}$	$\Box(p \wedge q) \Leftrightarrow \Box p \wedge \Box q$
$E_{24}$	$\Box(p \vee q) \Leftrightarrow \Box p \vee \Box q$		$E_{27}$	$\Diamond(p \wedge q) \Leftrightarrow \Diamond p \wedge \Diamond q$
$E_{26}$	$\Diamond(p \vee q) \Leftrightarrow \Diamond p \vee \Diamond q$			

Table 1.2: Implication relations.

$I_1$	$p \wedge q \Rightarrow p$	$I_2$	$p \wedge q \Rightarrow q$
$I_3$	$p \Rightarrow p \vee q$	$I_4$	$q \Rightarrow p \vee q$
$I_5$	$\neg p \Rightarrow p \rightarrow q$	$I_6$	$q \Rightarrow p \rightarrow q$
$I_7$	$\neg(p \rightarrow q) \Rightarrow p$	$I_8$	$\neg(p \rightarrow q) \Rightarrow \neg q$
$I_9$	$\Diamond(p \wedge q) \Rightarrow \Diamond p \wedge \Diamond q$	$I_{10}$	$\Box p \vee \Box q \Rightarrow \Box(p \vee q)$

$$A||B = \langle Q^A \times Q^B, \Sigma^A \cup \Sigma^B, \delta, \langle q_i^A, q_i^B \rangle, Q_m^A \times Q_m^B, (Q_x^A \times Q^B) \cup (Q^A \times Q_x^B) \rangle$$

$$\delta(\langle q^A, q^B \rangle, \sigma) = \begin{cases} \delta^A(q^A, \sigma) \times \delta^B(q^B, \sigma) & \sigma \in \Sigma^A \cap \Sigma^B \\ \delta^A(q^A, \sigma) \times \{q^B\} & \sigma \in \Sigma^A \setminus \Sigma^B \\ \{q^A\} \times \delta^B(q^B, \sigma) & \sigma \in \Sigma^B \setminus \Sigma^A \end{cases}$$

Table 1.3: CTL\* formulas and their equivalent  $\mu$ -calculus fixpoints

CTL* formula	Equivalent $\mu$ -calculus fixpoint
$\exists \Diamond p$	$\mu y. p \vee \exists \bigcirc y$
$\exists \Box p$	$\nu y. p \wedge \exists \bigcirc y$
$\exists p \mathcal{U} q$	$\mu y. q \vee (p \wedge \exists \bigcirc y)$
$\exists p \mathcal{W} q$	$\nu y. q \vee (p \wedge \exists \bigcirc y)$
$\exists (p \rightarrow \Diamond q)$	$\mu y. \neg p \vee q \vee \exists \bigcirc y$
$\exists \Box p \wedge \exists \Diamond q$	$\nu y. (p \wedge \exists \bigcirc y) \wedge \mu z. (q \vee \exists \bigcirc z)$
$\exists \Box \exists \Diamond p$	$\nu y. (\mu z. p \vee \exists \bigcirc z) \wedge \exists \bigcirc y$
$\exists \Box \Diamond p$	$\nu y. \mu z. (p \wedge \exists \bigcirc y) \vee \exists \bigcirc z$
$\exists \Diamond \Box p$	$\mu y. \nu z. (p \vee \exists \bigcirc y) \wedge \exists \bigcirc z$
$\exists \Box p \mathcal{U} q$	$\nu y. \mu z. (q \wedge \exists \bigcirc y) \vee (p \wedge \exists \bigcirc z)$

$$\begin{aligned}
\llbracket p \rrbracket &= \{x \mid p \in \lambda(x)\}, \\
\llbracket y \rrbracket &= Y \in 2^X, \\
\llbracket \neg \psi \rrbracket &= \sim \llbracket \psi \rrbracket = X \setminus \llbracket \psi \rrbracket, \\
\llbracket \psi_1 \vee \psi_2 \rrbracket &= \llbracket \psi_1 \rrbracket \cup \llbracket \psi_2 \rrbracket, \\
\llbracket \psi_1 \wedge \psi_2 \rrbracket &= \llbracket \psi_1 \rrbracket \cap \llbracket \psi_2 \rrbracket, \\
\llbracket \exists \bigcirc y \rrbracket &= \mathbf{Pre}^\exists(\llbracket y \rrbracket) = \{x \mid (\exists a \in \Sigma(x)) \delta(x, a) \subseteq \llbracket y \rrbracket\}, \\
\llbracket \forall \bigcirc y \rrbracket &= \mathbf{Pre}^\forall(\llbracket y \rrbracket) = \{x \mid (\forall a \in \Sigma(x)) \delta(x, a) \subseteq \llbracket y \rrbracket\}, \\
\llbracket \mu y. \psi \rrbracket &= \mu Y. \Psi(Y) = \bigcap \{Y \in 2^X \mid Y = \Psi(Y)\}, \\
\llbracket \nu y. \psi \rrbracket &= \nu Y. \Psi(Y) = \bigcup \{Y \in 2^X \mid Y = \Psi(Y)\}.
\end{aligned}$$

$$\begin{aligned}
J^*(x) &= \max_{a \in \Sigma(x)} [\rho(x, a) + \gamma J^*(\delta(x, a))] = \max_{a \in \Sigma(x)} Q(x, a) \\
Q(x, a) &= \rho(x, a) + \gamma J^*(\delta(x, a)) = \rho(x, a) + \gamma \max_{b \in \Sigma(\delta(x, a))} Q(\delta(x, a), b) \\
\mu(x) &= \arg \max_{a \in \Sigma(x)} Q(x, a).
\end{aligned}$$