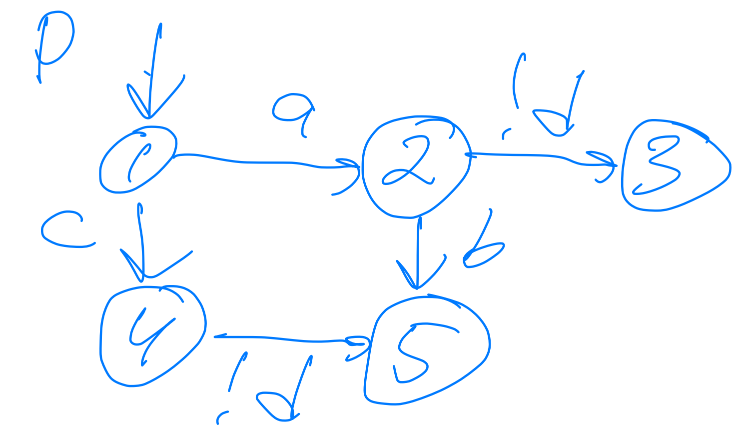
7. Continue with supervisor synthesis

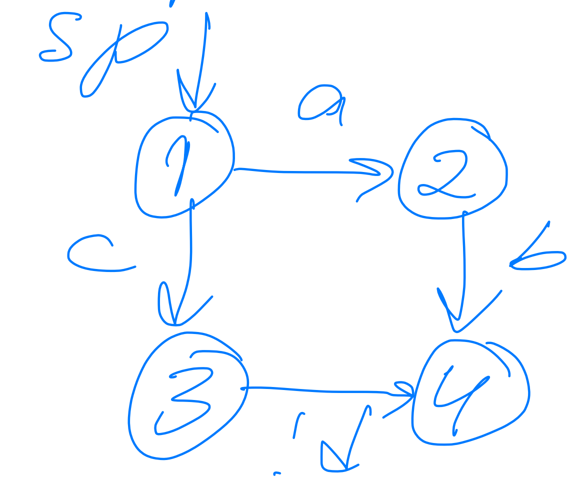
Controllable supervisor

$\Sigma^P = \Sigma_c \cup \Sigma_u$ where Σ_u includes the uncontrollable events. Remove all uncontrollable states by including them in the set of forbidden states Q_x. ($S_0 = P \| S_p$)

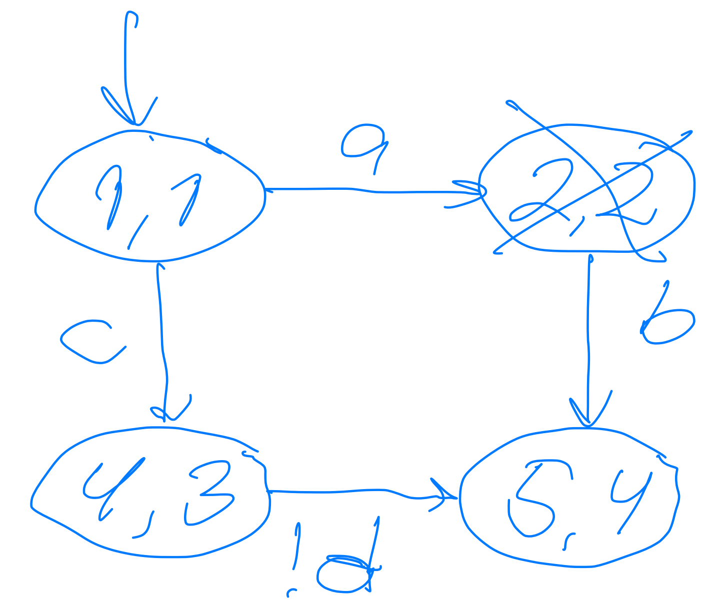
$$Q_x = \underbrace{(Q_x^{Sp} \times Q^P \cup Q^{Sp} \times Q_x^P)}_{\text{Explicitly forbidden states in } S_0} \cup \{q \in Q^{P\|Sp} \mid q \text{ is uncontrollable in } P\|Sp\}$$

Ex controllable supervisor

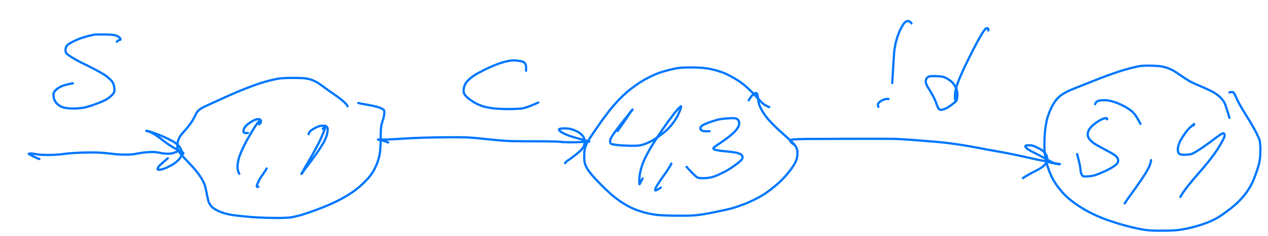




$S_0 = P \| S_p$



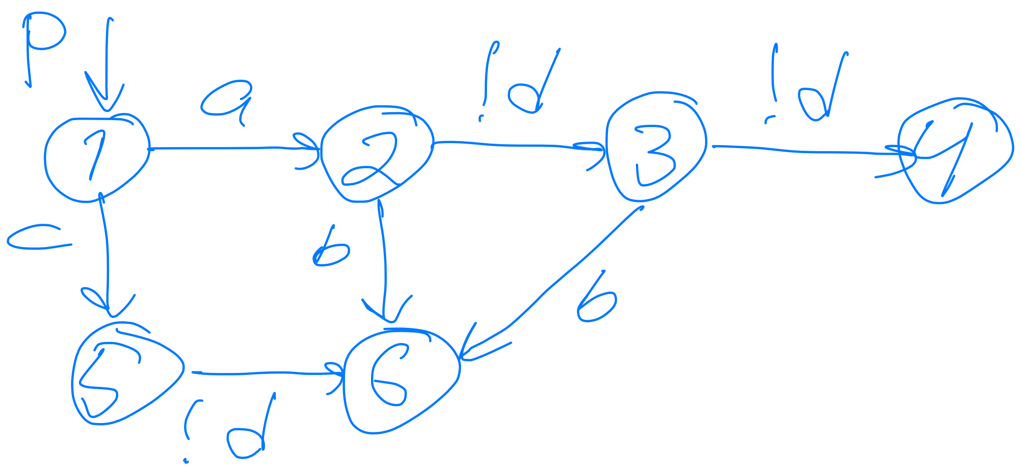
uncontrollable state



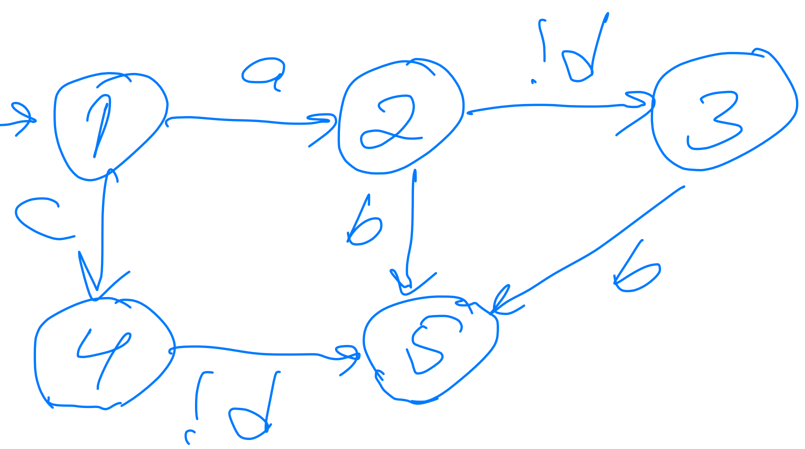
S = controllable supervisor

Extended uncontrollable states

Ex

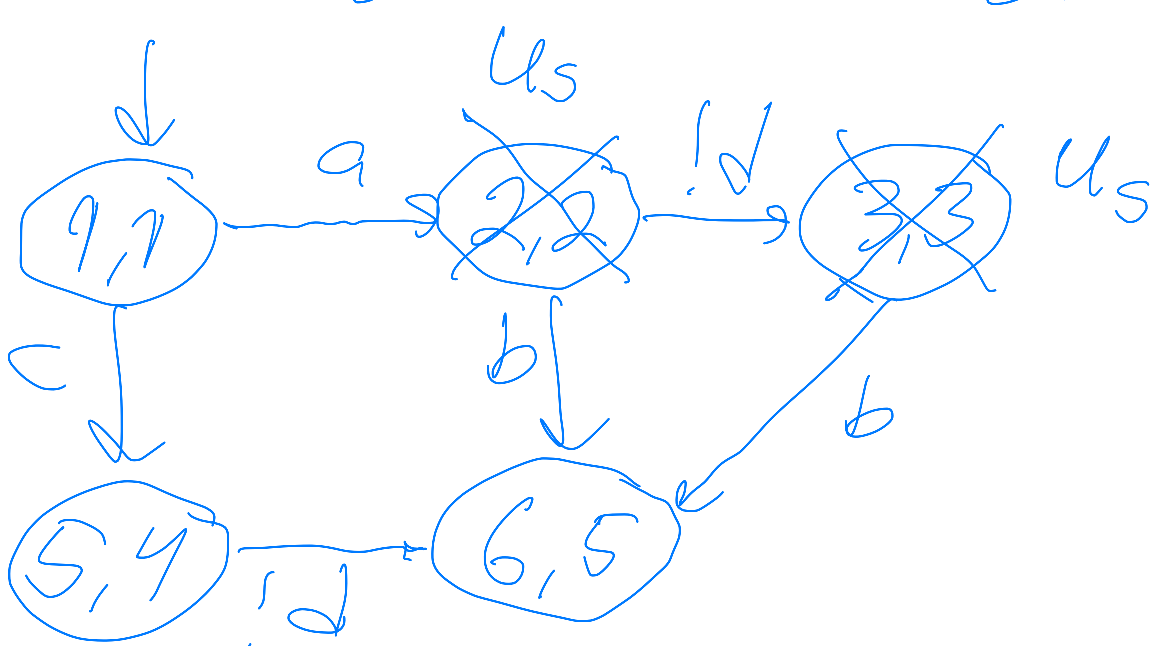


Sp



$S_o = P \| Sp$

u_s = uncontrollable state



controllable supervisor

S

state (2,2) in S_o is an <u>extended</u> uncontrollable state, since an uncontrollable state in S_o can be reached by an uncontrollable transition in S_o.

Extended uncontrollable states are avoided by going backward from the uncontrollable states, as long as there are uncontrollable transitions.

These states that also are forbidden are computed as

$Q_{ex} = \text{coreachable}(\Sigma_u^{S_0}, \delta^{S_0}, Q_x^{S_0}, \emptyset)$

extended forbidden states

Controllable and nonblocking supervisor

Theorem 7.1 Given a plant P and a total specification $P \| Sp$, there exists a nonblocking and controllable supervisor S, such that the closed loop system $P \| S \leq P \| Sp$

if and only if (iff) there exists a non-blocking and controllable supervisor $S' \leq S_0 = P \| Sp$

Algorithm 3

safestate_synthesis($Q, \Sigma, \Sigma_u, \delta, Q_m, Q_x$)

let $k := 0$, $X_0 = Q_x$ % X_k = forbidden states

repeat

$k := k + 1$

$Q' := \mathrm{coreachable}(\Sigma, \delta, Q_m, X_{k-1})$ % Nonblocking states

$X_k := Q \setminus Q'$ % Blocking and forbidden states

$X_k := \mathrm{coreachable}(\Sigma_u, \delta, X_k, \emptyset)$ % Extended forbidden states

until $X_k = X_{k-1}$

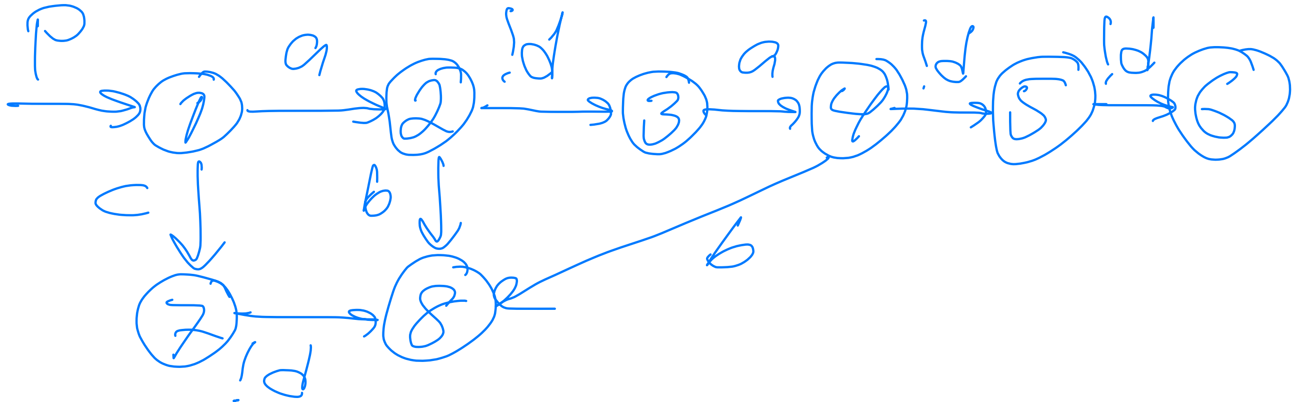
return $Q \setminus X_k$

Given $S_0 = P \| S_p$, the states Q^S of the controllable and nonblocking supervisor $S \leq S_0$ is generated as

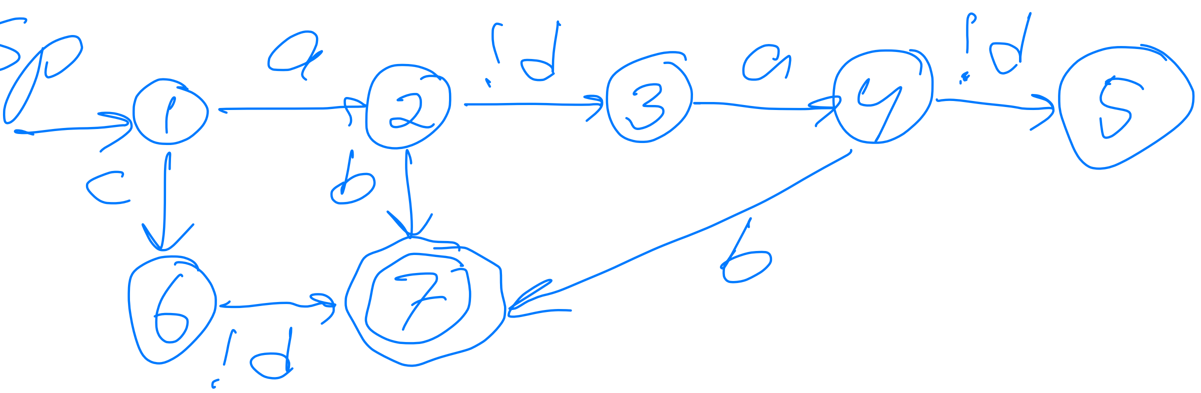
$$Q^S = \text{safestate\_synthesis}(Q^{S_0}, \Sigma^{S_0}, \Sigma_u^{S_0}, \delta^{S_0}, Q_m^{S_0}, Q_x^{S_0})$$

Ex

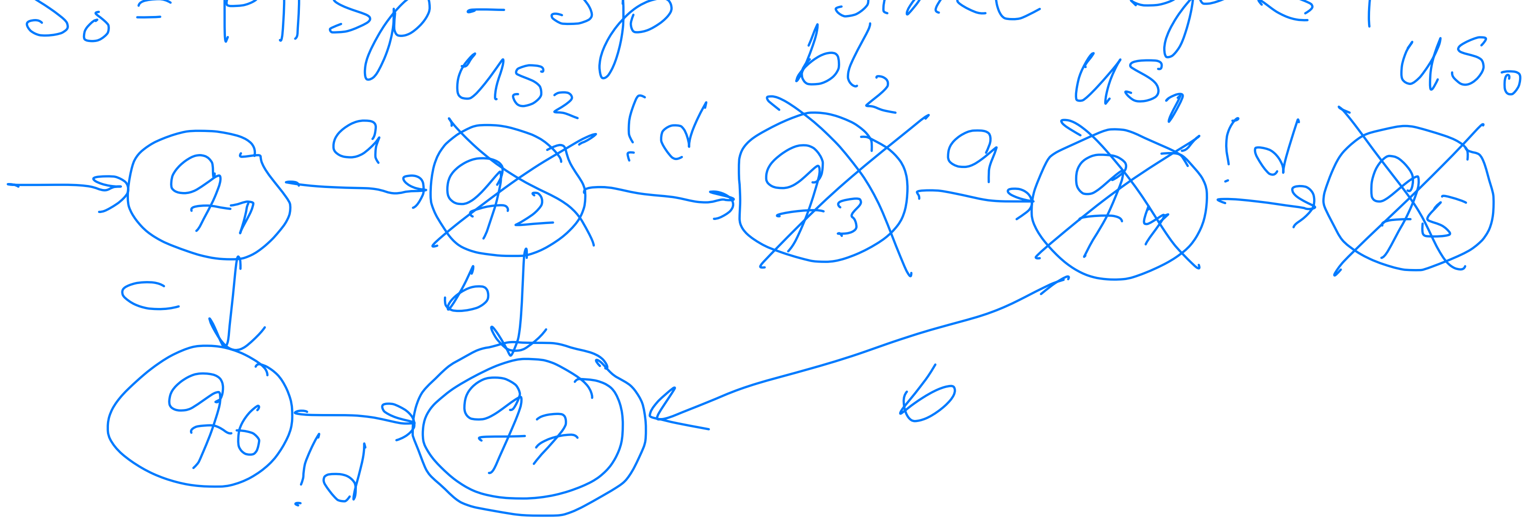
P



Sp



$S_0 = P \| Sp = Sp$ since $Sp \leq P$



us_k = uncontrollable state in iteration k

bl_k = blocking state in iteration k

$Q_x^{S_0} = X_0 = \{q_5\}$

$Q_m^{S_0} = \{q_7\}$ $\quad \Sigma_u^{S_0} = \{d\}$

Algorithm 3

$Q' := \text{coreachability}(\Sigma^{S_0}, \delta^{S_0}, Q_m^{S_0}, X_{k-1})$

$X_k := Q \setminus Q'$

$X_k := \text{coreachability}(\Sigma_u^{S_0}, \delta^{S_0}, X_k, \emptyset)$

$k = 1$

$Q' = \{q_1, q_2, q_3, q_4, q_6, q_7\}$

$Q \setminus Q' = \{q_5\}$

$X_1 = \{q_4, q_5\}$

$k = 2$

$Q' = \{q_1, q_2, q_6, q_7\}$

$Q \setminus Q' = \{q_3, q_4, q_5\}$

$X_2 = \{q_2, q_3, q_4, q_5\}$

$k = 3$

$Q' = \{q_1, q_6, q_7\}$

$Q \setminus Q' = \{q_2, q_3, q_4, q_5\}$

$X_3 = X_2 = Q \setminus Q'$ % fixed point

$Q^S = Q \setminus X_3 = \{q_1, q_6, q_7\}$

Nonblocking and controllable supervisor