Petri net

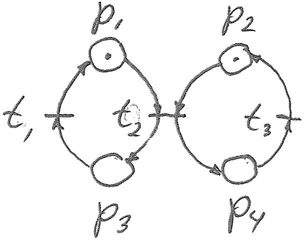
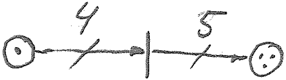


Figure 3.8 A Petri net with four places and three transitions.

Petri net with weighted arcs

N_1

N_2



N_1: 2 arcs from input place and 3 arcs to output place

N_2: 4-weighted arc from input place and 5-weighted arc to output place

Synchronized Petri nets

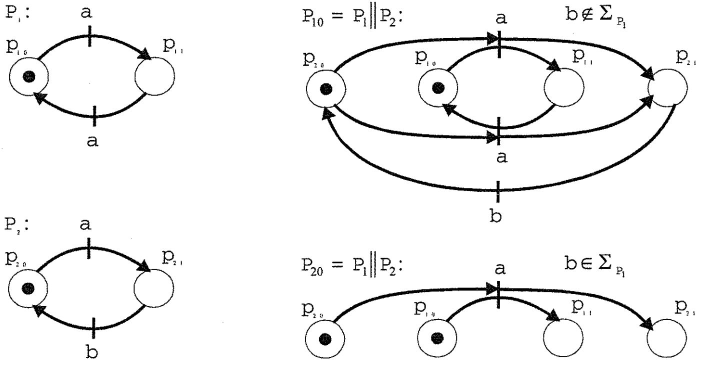
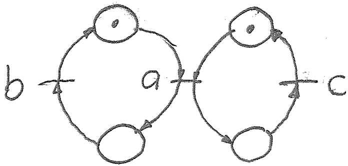


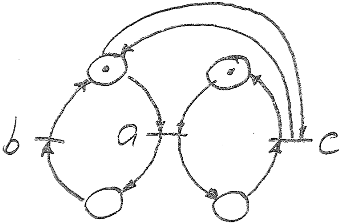
Figure 3.11 Synchronized Petri nets.

Controlled Petri net by adding arcs

N_1



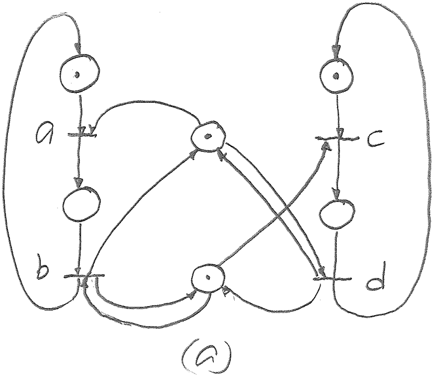
N_2

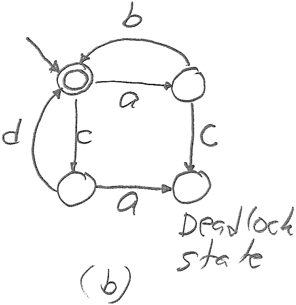


$$\mathcal{L}(N_1) = \overline{(a(bc + cb))^*}$$

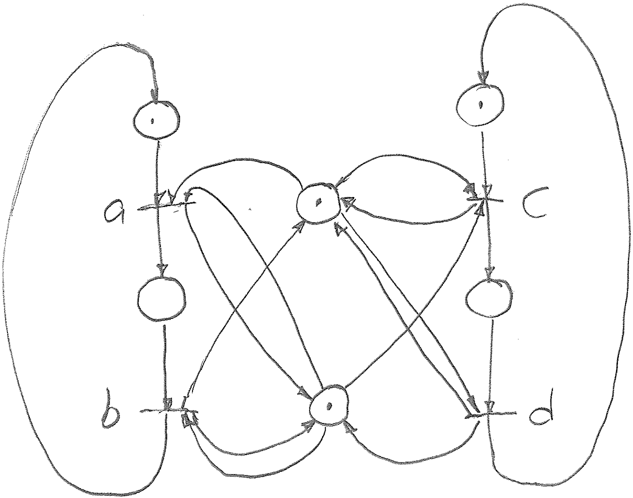
$$\mathcal{L}(N_2) = \overline{(abc)^*}$$

Petri net including deadlock state



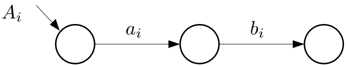


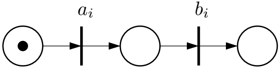
Controlled Petri net without deadlock state



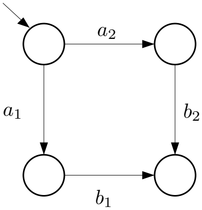
Automata and corresponding Petri nets

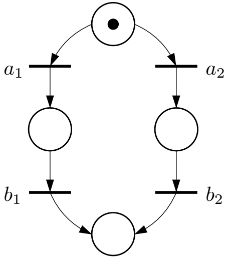
Straight sequences





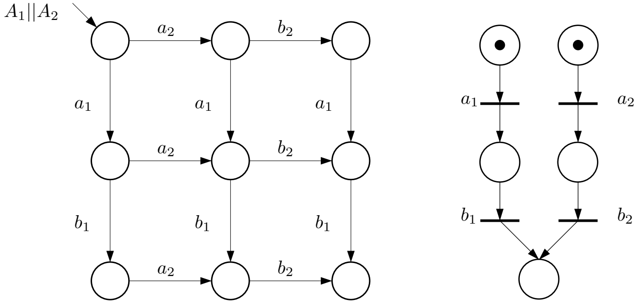
Alternative sequences





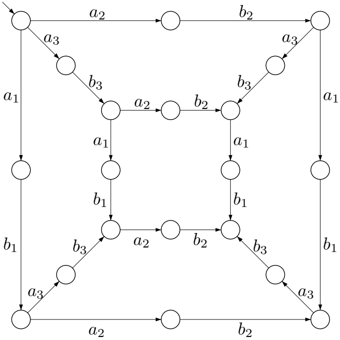
Automaton and corresponding Petri net

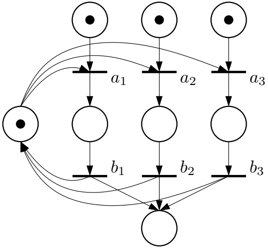
Concurrent sequences



Automaton and corresponding Petri net

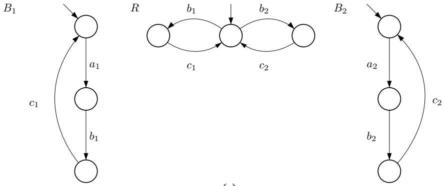
Arbitrary order



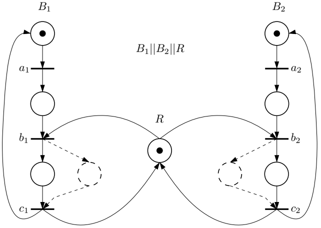


Automaton and corresponding Petri net

Mutual exclusion (shared resource R)



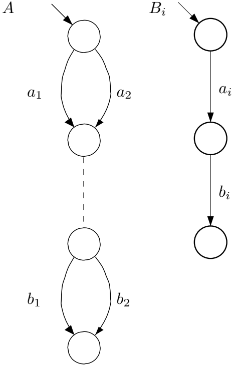
(a)

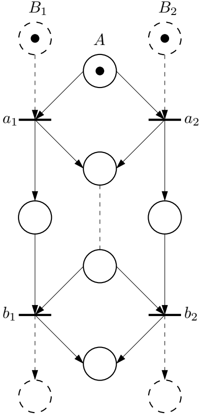


(b)

Automaton and corresponding Petri net

Alternative sequences in A with memory in B_1 and B_2





4. Modelling and Specification

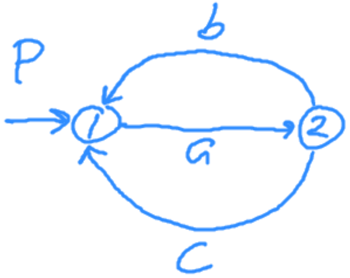
P = plant = $\langle Q^P, \Sigma^P, \delta^P, q_i^P \rangle$

Sp = specification =

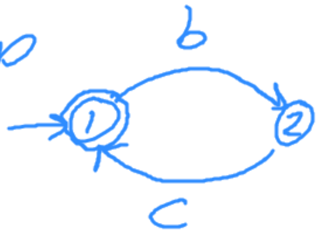
$= \langle Q^{Sp}, \Sigma^{Sp}, \delta^{Sp}, q_i^{Sp}, Q_m^{Sp}, Q_x^{Sp} \rangle$

Note that all states in P are assumed to be marked.

Ex P



Sp



$P \| Sp$

Total specification

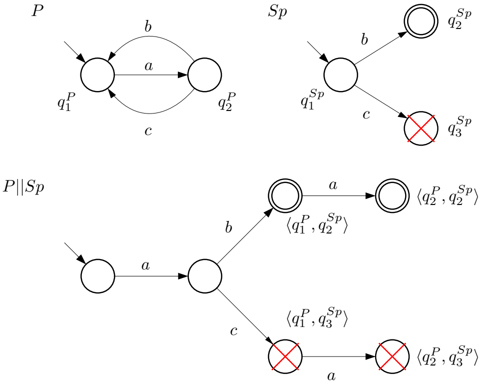
$S_0 = P \| Sp$ is called the total specification. This is indeed a first candidate for a supervisor S that can control (restrict) the plant P such that the local specification Sp will be satisfied.

Note that if $\Sigma^{Sp} \subseteq \Sigma^P \Rightarrow$

$$\Sigma^{S_0} = \Sigma^{P \| Sp} = \Sigma^P \cup \Sigma^{Sp} = \Sigma^P$$

This assumption simplifies the synthesis of the supervisor S.

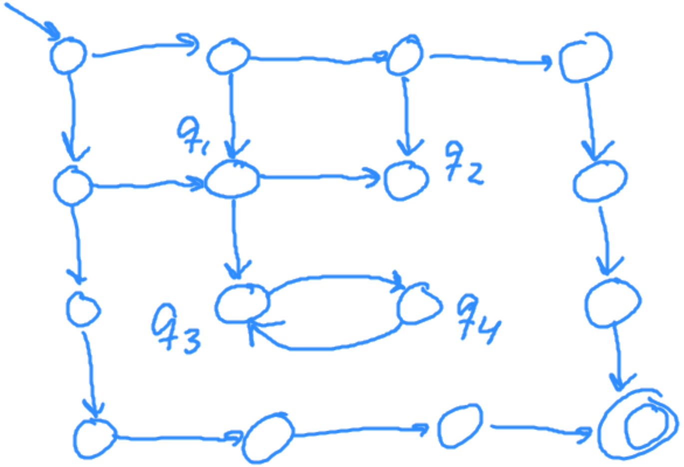
Synchronous composition including forbidden states



6. Verification

Nonblocking

Ex $S_0 = P \| Sp$



$\{q_1, q_2, q_3, q_4\}$ = set of blocking states

q_2 = deadlock state = nonmarked state without output transitions

$\{q_3, q_4\}$ = set of livelock states = = states where a loop is executed but without the possibility to reach a marked state.

Blocking states are all states from which a marked state cannot be reached. This includes both deadlock and livelock states.

Nonblocking system

$$A = \langle Q, \Sigma, \delta, q_i, Q_m, Q_x \rangle$$

A is nonblocking if for every reachable state $q = \delta(q_i, s)$ where $s \in L(A)$

it is always possible to reach at least one marked state $q_m \in Q_m$

$q_m = \delta(q, s_2)$ where $s_1 s_2 \in L(A)$

Reachability Algorithm

Algorithm 1 Reachability$(\Sigma, \delta, q_i, Q_x)$

let $k := 0$ $Q_0 := \{q_i\} \setminus Q_x$

repeat

$k := k+1$

$Q_k := Q_{k-1} \cup \{q' \mid q' = \delta(q, \sigma), q \in Q_{k-1}, \sigma \in \Sigma\}$

until $Q_k = Q_{k-1}$

return Q_k = set of reachable states