Our “toss a coin and draw a ball” Example

- What are the events?
- What are the probabilities?
- What is the random variable?
- Why do we care about the expected value?

Flip a Coin

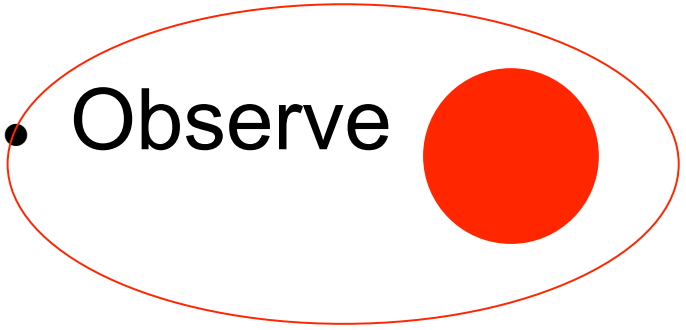
"Head"

"Tail"

EVENTS!

The Challenge

- Try again…
- Observe

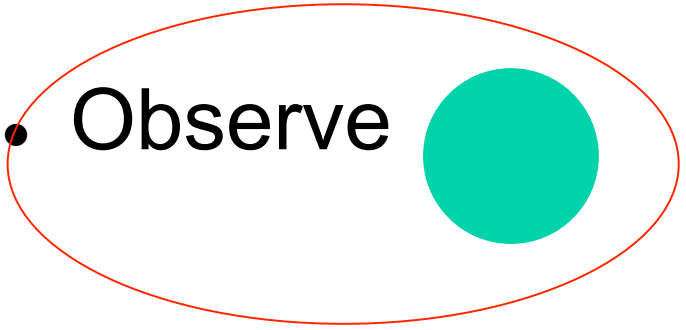


EVENT!

- Was it “head” or “tail”?

The Challenge

- And again…
- Observe



EVENT!

- Was it “head” or “tail”?

Our Previous Example

- What are the events?
- What are the probabilities?
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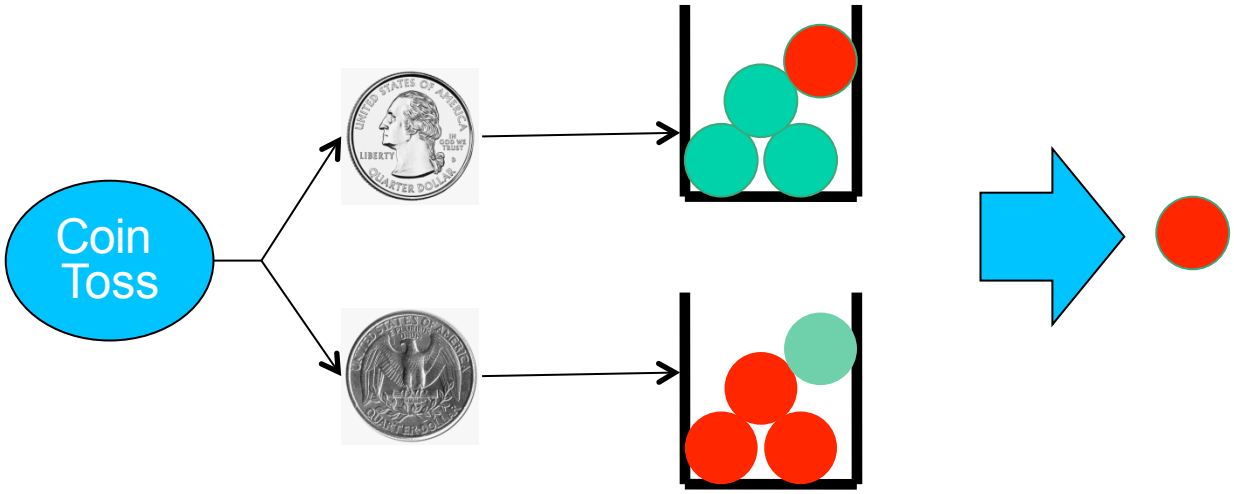
Probabilities

- What is the probability of observing ● ?

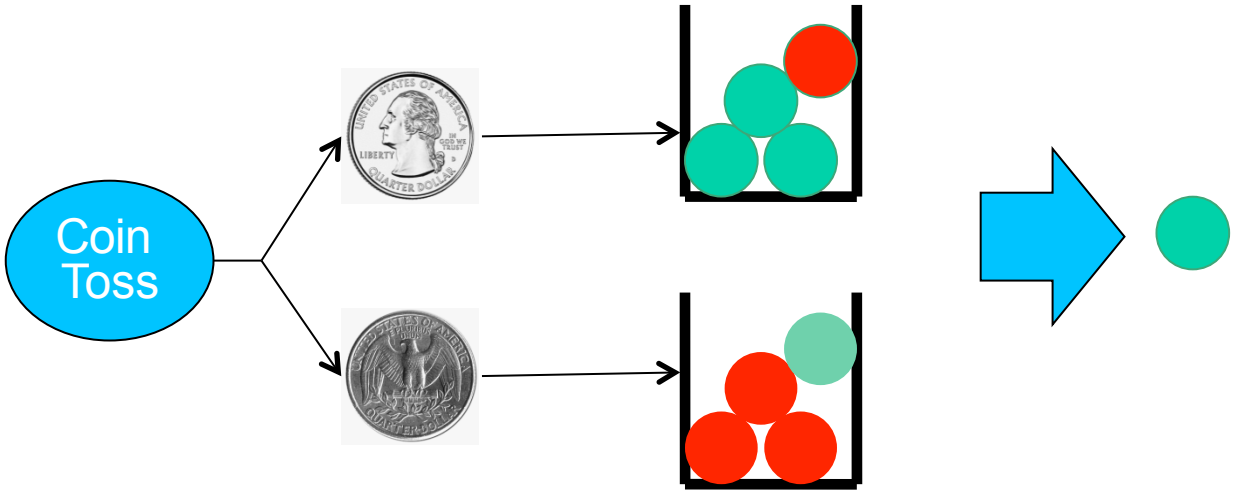


$$P(\text{red}) = P(\text{red}\,|\,\text{head})P(\text{head}) + P(\text{red}\,|\,\text{tail})P(\text{tail})$$

$$= \frac{1}{4}\frac{1}{2} + \frac{3}{4}\frac{1}{2} = \frac{1}{2}.$$

Probability (● observed)

Similarly,



$$P(\text{green}) = \frac{1}{4}\frac{1}{2} + \frac{3}{4}\frac{1}{2} = \frac{1}{2}$$

Our Previous Example

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Keeping Score

- Suppose *A* pays *B* $1 for every correct call, and *B* pays *A* $1 for every incorrect call.

- After, say, 100 calls by *B*, how much money do you think *B* will have?

Expected Value

"Suppose *A* pays *B* \$1 for every correct call, and *B* pays *A* \$1 for every incorrect call."

Define x as the amount of money that B has.

From the rule, x is given by

$$x = \begin{cases} +1 & \text{if the decision is correct} \\ -1 & \text{if the decision is incorrect} \end{cases}.$$

What is $E[x]$?