Probability Theory Concepts

- Event
  - A subset of the set of all possible outcomes
- Probability
  - A measure defined for each event

- Random variable
- Expectation, Expected value

Our Example

- Events
  - Head, tail
  - Red ball, green ball

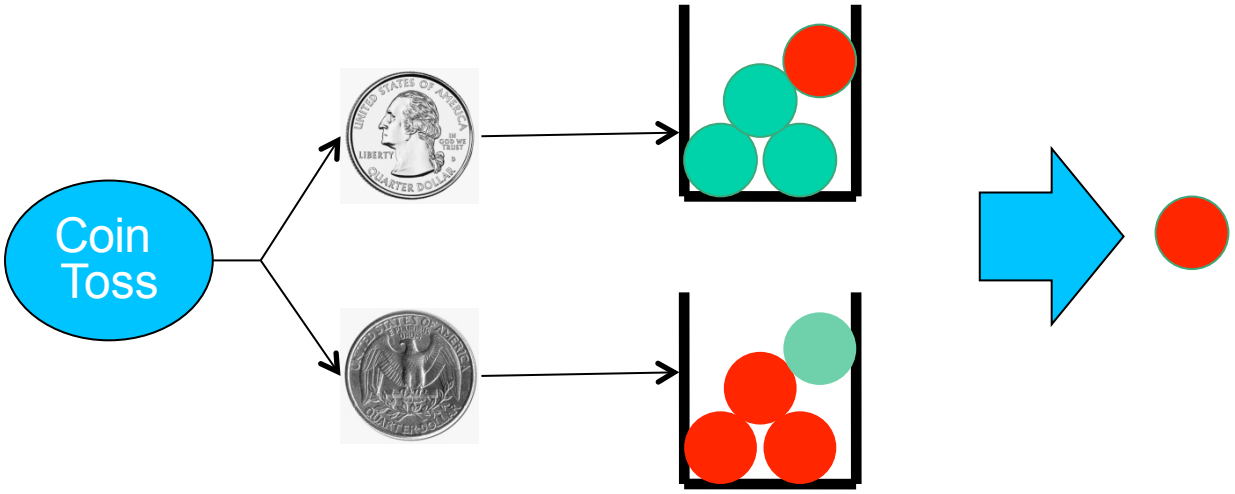
Assume the coin is fair,

$$P(\text{head}) = P(\text{tail}) = \frac{1}{2}.$$

Our Example

- Events
  - Red ball, green ball

Defining the probability of drawing a red ball or green ball is not so straightforward.



Random Variable

- A real-valued function defined on an event
  - assign a real-value to an event
  - has no relationship with how likely an event is to occur
  - assignment of the value has to do with the problem

Example

- Event: “Coin toss results in a tail”
- Random variable: Assign “1” if the coin toss results in a tail

Expected Value

- Expected value of a random variable is the sum of all values taken on by the random variable weighted by the probability of the associated event

- There is no expected value of an event; one only talks about the expected value of a random variable

Coin Toss Example

- Events:
  - “head”
  - “tail”
- Probabilities

Coin Toss Example

- Note that we will not ask “What is the expected value of the coin toss?”
- Random variable x defined as follows

  x(“head”) = 1

  x(“tail”) = 0
- What is the expected value of x?

Expected Value

- Expected value of a random variable is the sum of all values taken on by the random variable weighted by the probability of the associated event

Coin Toss Example:

$$\begin{aligned} E[x] &= 1 \times P(x = 1) + 0 \times P(x = 0) \\ &= 1 \times P(\text{Head}) \\ &= 1 \times \frac{1}{2} = \frac{1}{2}. \end{aligned}$$