Math Camp Exercises - Day 2

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1. Let: $\mathbf{a} = \begin{pmatrix} 4 \\ 1 \\ -5 \\ 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ 5 \\ 5 \\ 2 \end{pmatrix}$, $\mathbf{c} = (1, 1, 3)$, $\mathbf{d} = (6, 4, 2)$ and $\mathbf{e} = (10, 20, 30, 40)^T$.

   Calculate each of the following, indicating that it is not possible if there is a calculation you cannot perform.

   - $\mathbf{a} + \mathbf{b}$
   - $\mathbf{b} - \mathbf{e}$
   - $\mathbf{b} + \mathbf{c}$
   - $6\mathbf{a}$
   - $||\mathbf{a} - \mathbf{b}||$
   - $\mathbf{c} \cdot \mathbf{d}$
   - $\mathbf{e} \cdot (\mathbf{a} + \mathbf{b})$

2. Identify the following matrices as diagonal, identity, square, symmetric, triangular or none of the above (note all that apply).

   - $A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 0 & 5 & 3 \end{bmatrix}$
   - $B = \begin{bmatrix} 1 & 2 & 5 & 7 \\ 2 & 3 & 6 & -3 \end{bmatrix}$
   - $C = \begin{bmatrix} 10 & 5 & 2 \\ 5 & -3 & 7 \\ 2 & 7 & -9 \end{bmatrix}$

3. Write down the transpose of matrices A through C from the previous problem.

4. Given the following matrices, perform the calculations below.

   $A = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 5 & 2 \end{bmatrix}$; $B = \begin{bmatrix} 2 & 4 \\ 2 & 8 \\ -5 & 2 \end{bmatrix}$; $C = \begin{bmatrix} 5 & 0 \\ 3 & 6 \end{bmatrix}$

- $A + B$
- B^T
- $A - B^T$
- $3A$
- AC
- AB
- BA
- $(AB)^T$
- $B^T A^T$
- Trace(C)

5. Find the determinants and, if they exist, the inverses of the following matrices:
   - $A = \begin{bmatrix} 6 & 8 \\ 2 & 7 \end{bmatrix}$
   - $B = \begin{bmatrix} 18 & 6 \\ 3 & 1 \end{bmatrix}$
   - $C = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 6 & 4 \\ 1 & 5 & 2 \end{bmatrix}$
   - $D = \begin{bmatrix} 12 & -7 & 2 \\ 3 & 6 & -5 \end{bmatrix}$
6. Solve the following system of equations:

$$5x + 4y - 3z = 4 \quad (1)$$
$$2x + 2y - z = 3 \quad (2)$$
$$3x - 2y + 2z = 5 \quad (3)$$

7. Let $A = \begin{pmatrix} 5 & 0 \\ 3 & 6 \end{pmatrix}$ and $C = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$. Solve $Ax = c$ for x using matrix inversion.