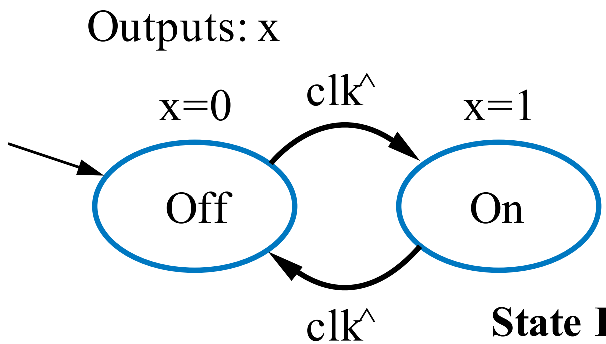
Topic 10

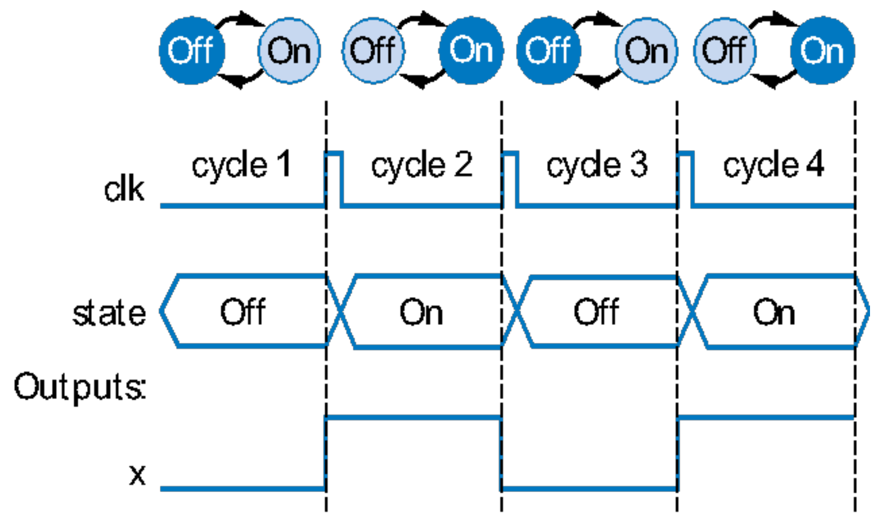
Finite State Machine

Describing Behavior of Sequential Circuit

- Finite-State Machine (FSM)
  - A way to describe **desired behavior** of a sequential circuit
  - Consists of a set of states, transitions between states, and maybe inputs and outputs
    - ***present state***: currently happening
    - ***next state***: next to happen
  - Example: Toggle output x every clock cycle
    - Two states: “Off” and “On”
    - Corresponding outputs: x=0 or 1
    - No input
    - Transition from Off to On, or On to Off, on rising clock edge
    - Arrow with no starting state points to initial state

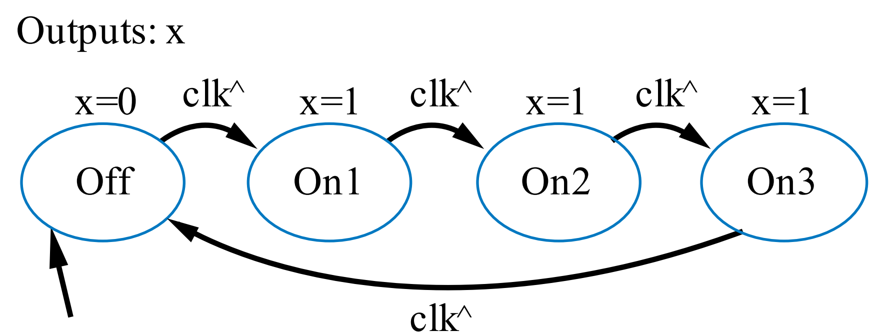


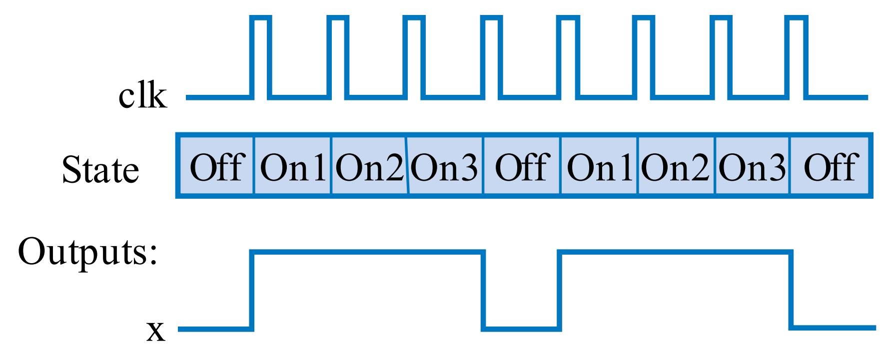
State Diagram



Example: Output Special Pattern

- Want a circuit to output
  0, 1, 1, 1, 0, 1, 1, 1, ...
  – One bit at a time
  – Each bit for one clock cycle
- Can be described as FSM
  – Four states
    - Each state corresponds to an output, 0, 1, 1, 1
    - Then repeat
  – Transition on rising clock edge to next state

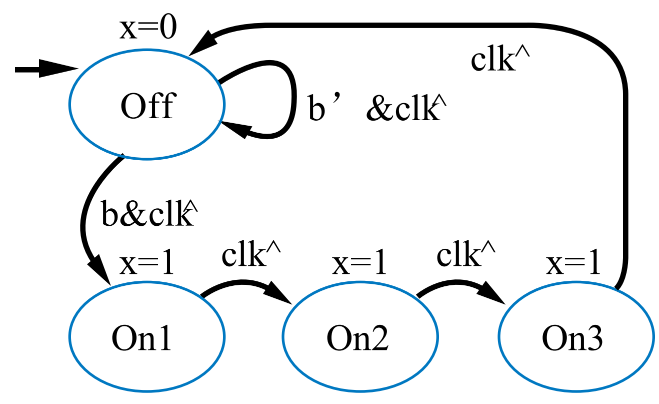


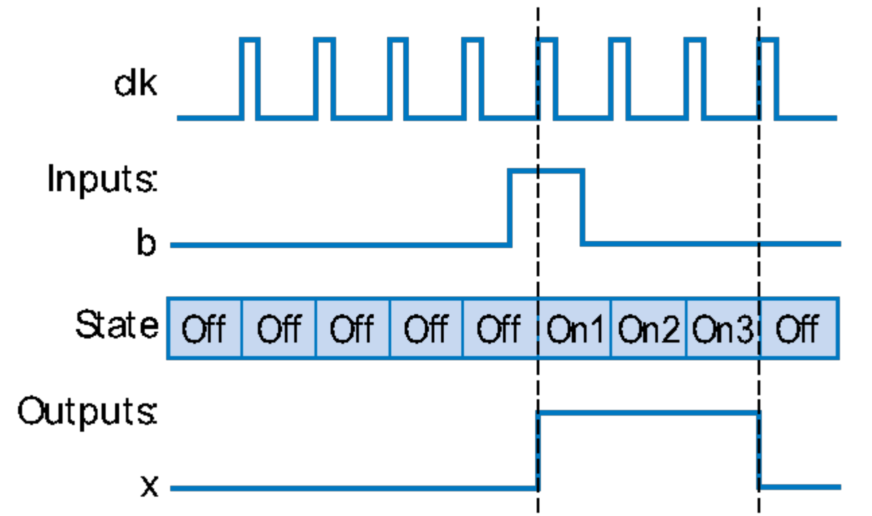


Example: FSM with Input

- b is a push button, output x to stay on for exactly 3 clock cycles no matter how long b is pushed
- Wait in “Off” state while b is 0 (b’)
- When b is 1 (and rising clock edge), transition to On1
  - Sets x=1
  - On next two clock edges, transition to On2, then On3, which also set x=1
- So x=1 for three cycles after button pressed
- Potential issue: if button b stays on, what will happen?

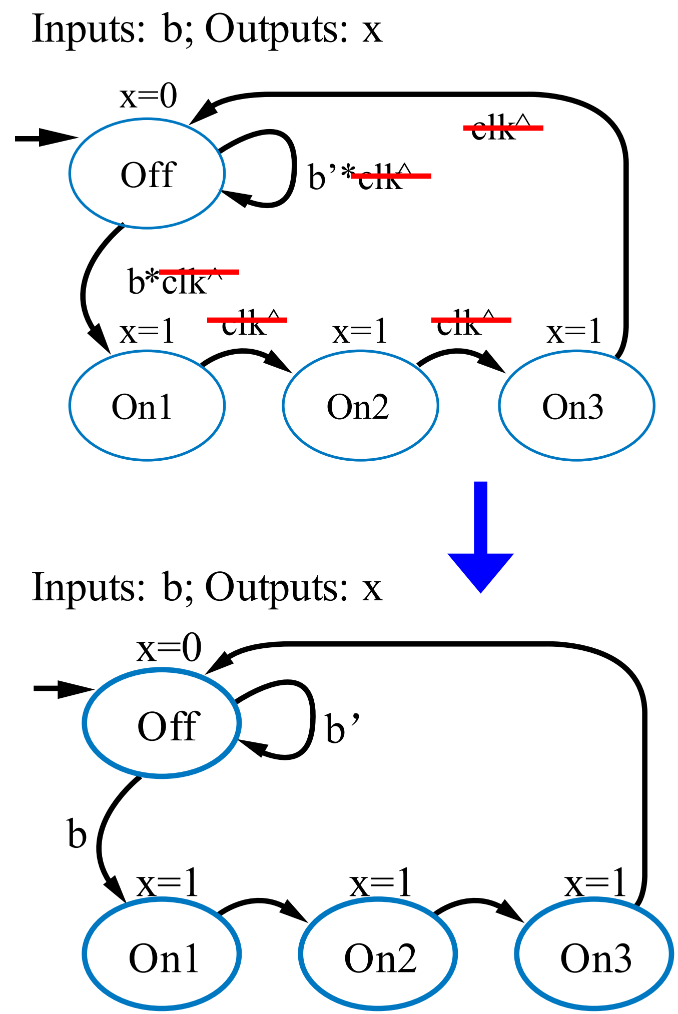
Inputs: b; Outputs: x





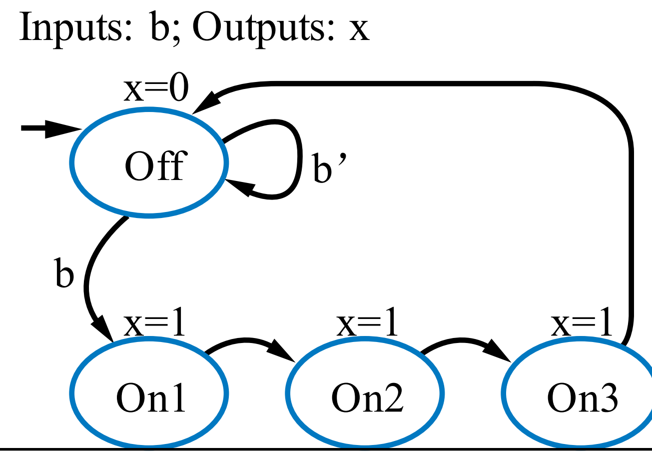
State Diagram Simplification: Clock Implicit

- ***Synchronous*** FSMs – FSM behaviors synchronized to active edge of clock
  - Asynchronous FSMs -- less common, advanced topic
- Make implicit – all state transitions are triggered by rising edge of clock
- Make implicit – Unlabeled path means transition is triggered only by clock, inputs don't matter



FSM Definition

- FSM consists of
  - Set of states
    - Ex: {Off, On1, On2, On3}
  - Set of inputs, set of outputs
    - Ex: Inputs: {b}, Outputs: {x}
  - Initial state
    - Ex: “Off”
  - Set of transitions
    - Describes next states
    - Ex: Has 5 transitions
  - Set of actions (outputs)
    - Sets outputs while in states
    - Ex: x=0, x=1, x=1, and x=1



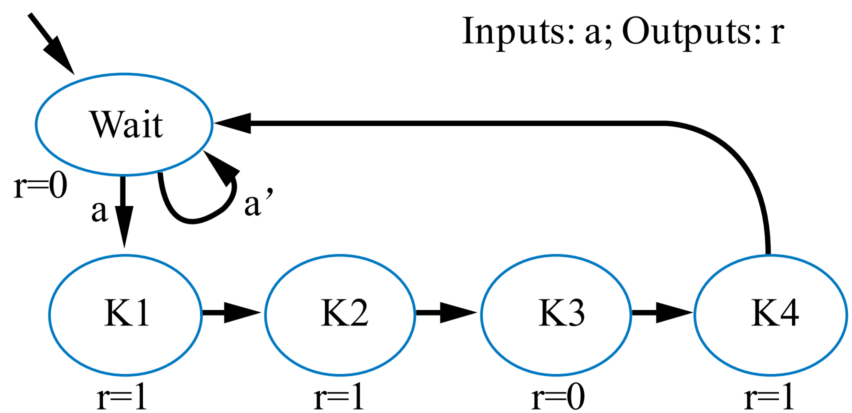
FSM can be represented graphically, known as ***state diagram***

	Inputs			Outputs		
	s1	s0	b	x	n1	n0
Off	0	0	0	0	0	0
	0	0	1	0	0	1
On1	0	1	0	1	1	0
	0	1	1	1	1	0
On2	1	0	0	1	1	1
	1	0	1	1	1	1
On3	1	1	0	1	0	0
	1	1	1	1	0	0

FSM can also be represented in tabular form, known as ***state table***

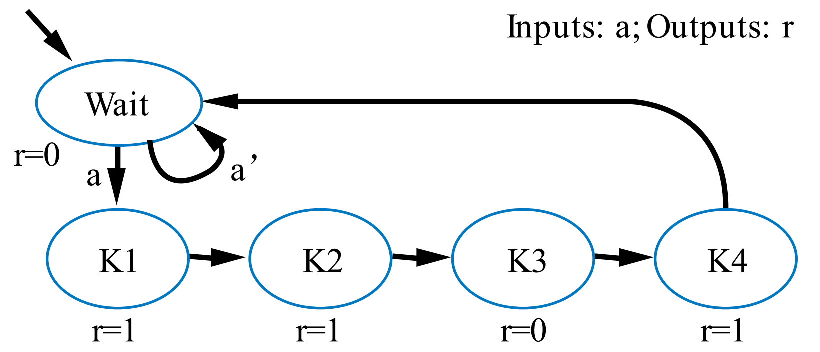
Example: Secure Car Key

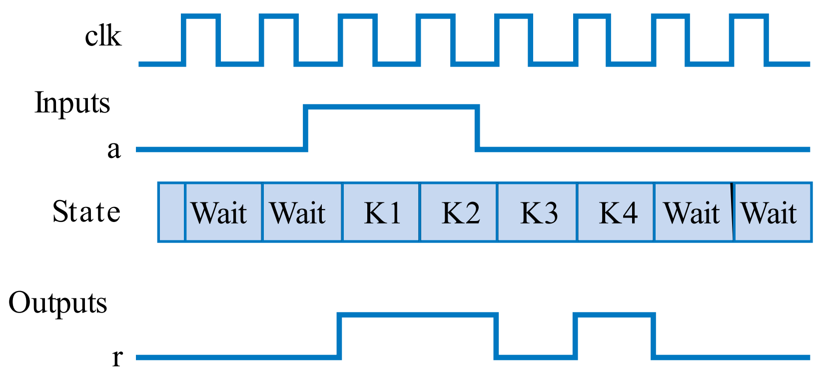
- All new car keys contain a tiny computer chip
  - When car starts, car's computer (under engine hood) requests identifier from key
  - Key transmits identifier
    - If not, computer shuts off car
- FSM
  - Wait until computer requests ID (a=1)
  - Transmit ID (in this case, 1101)

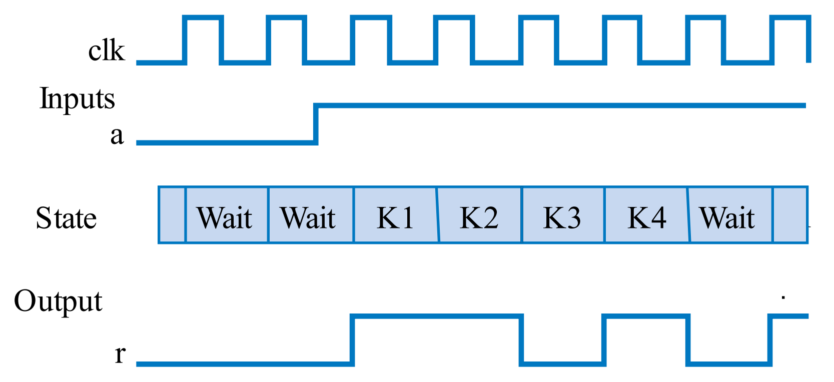


FSM Example: Secure Car Key (cont.)

- Timing diagrams show states and output values for different input waveforms

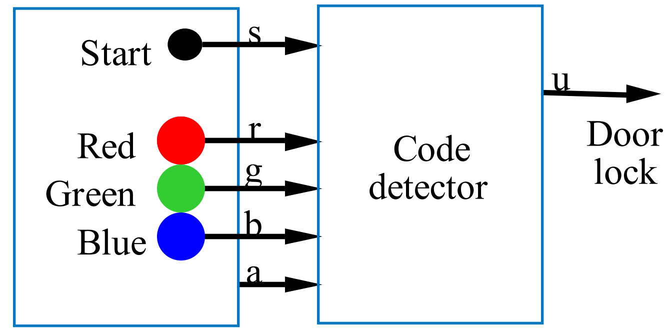


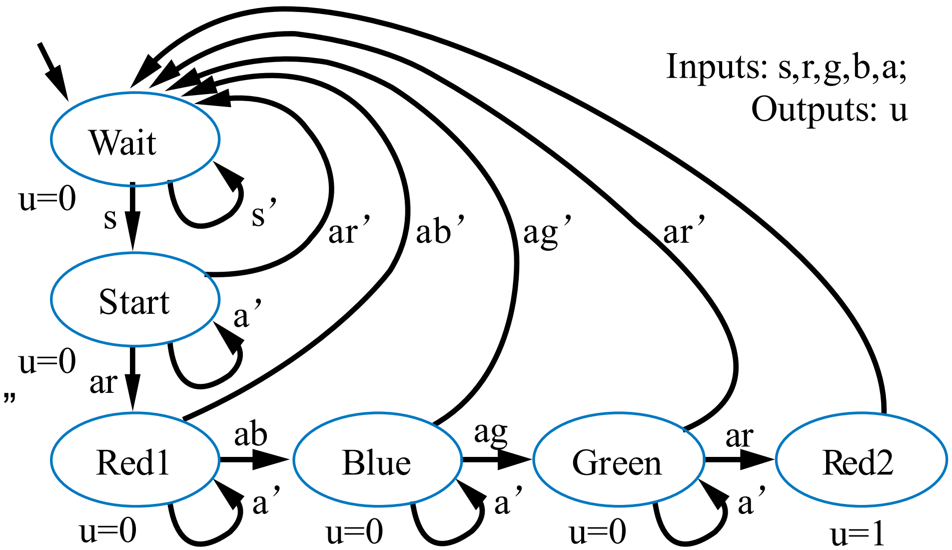




Example: Digital Lock

- Unlock door (u=1) only when buttons pressed in sequence:
  - start, then red, blue, green, red
- Input buttons: *s, r, g, b*
- Input *a* indicates that some color button pressed
- FSM
  - Wait for start (s=1) in “Wait”
  - Once started, go to “Start”, then
    - If see red, go to “Red1”
    - Then, if see blue, go to “Blue”
    - Then, if see green, go to “Green”
    - Then, if see red, go to “Red2”, and u=1
    - Wrong button at any step, return to “Wait”, without opening door

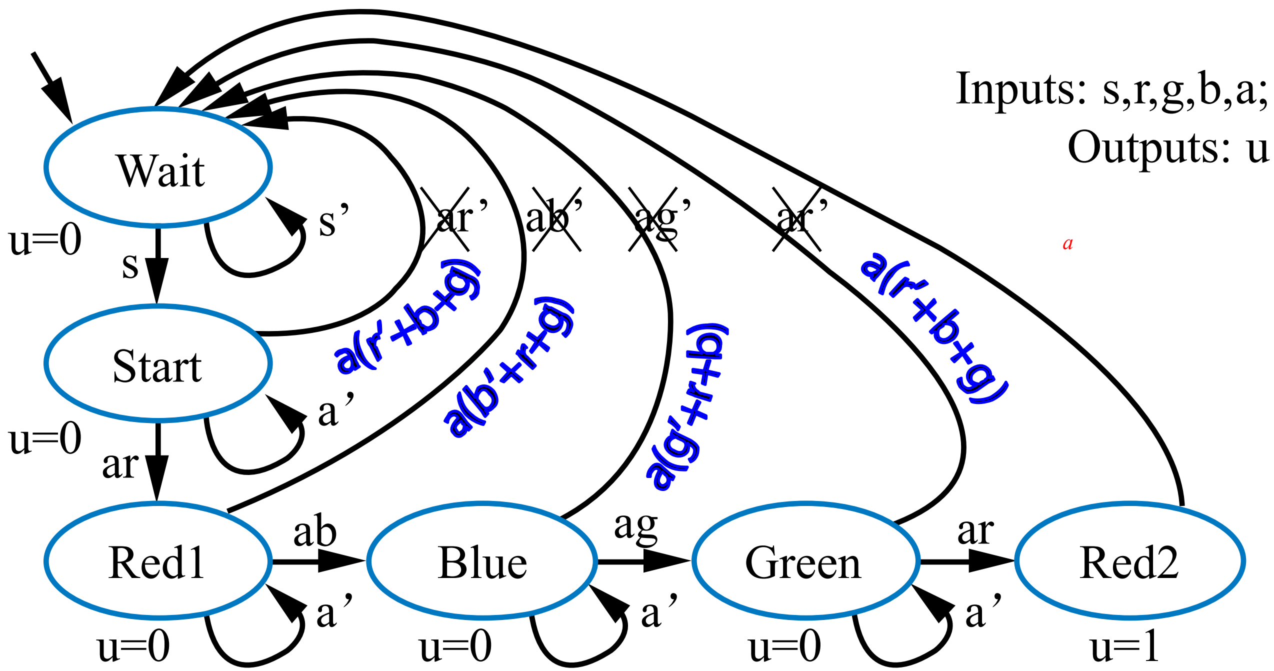




Q: Can you trick this FSM to open the door, without knowing the code?

A: Yes, hold all buttons simultaneously

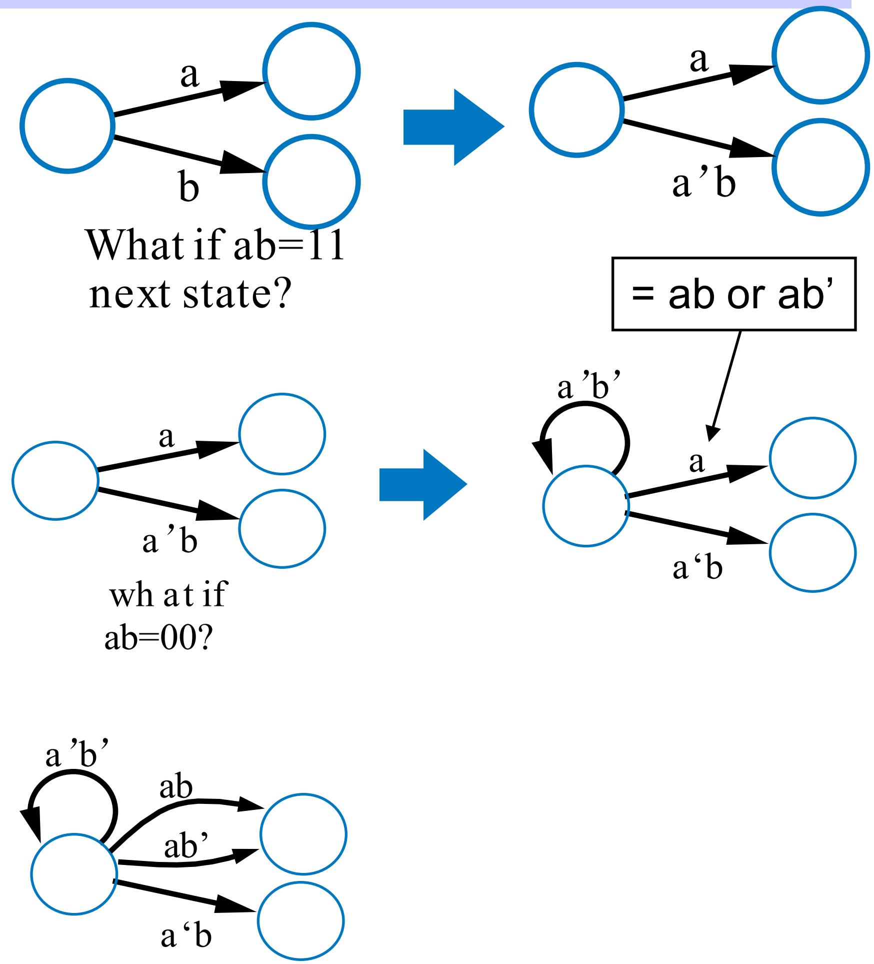
Improve FSM for Code Detector



- New transition conditions detect if wrong button pressed, returns to "Wait"

Common State Transition Property

- *Only* one condition should be true, among all transitions leaving a state
- *One* condition must be true
  - For any input combination
- All conditions must be considered when leaving a state



Pitfall is Common

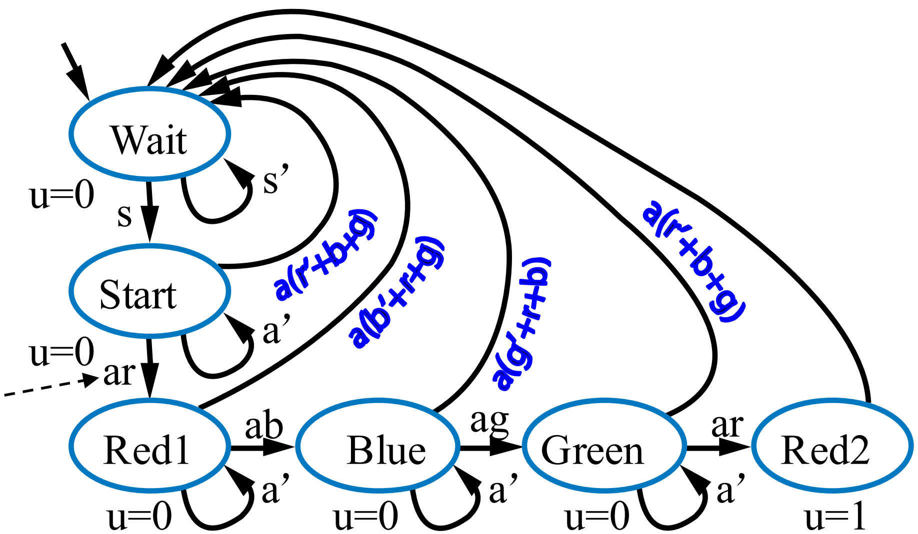
- Recall code detector FSM
  - Do the transitions obey the two required transition properties?

    NO!

  - How would it go wrong?

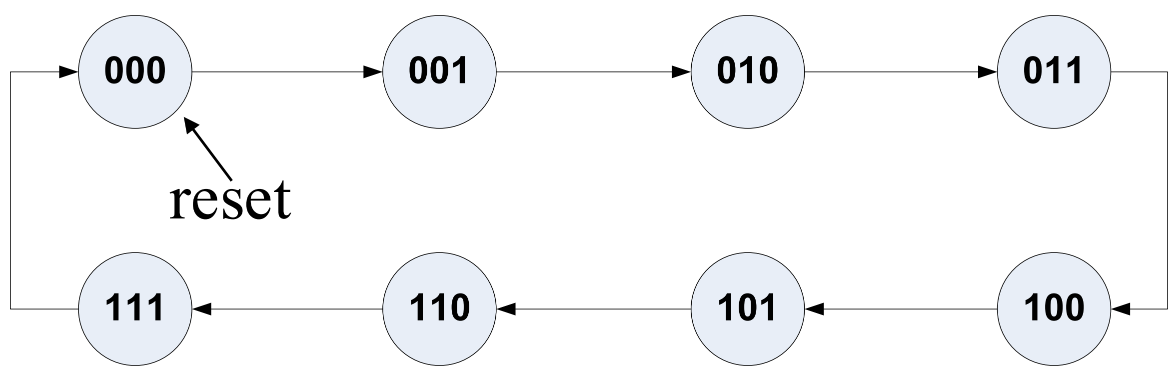
    E.g. arbg = 1111
    How to solve?

Answer: ar should be arb'g'
(likewise for ab, ag, ar)



FSM Example: Synchronous Binary Counter

- FSM that counts binary numbers – counter
- An n-bit binary counter can count in binary from 0 up to 2^n-1 and repeat
- An n-bit binary counter consists of n flip-flops
- All the flip-flops are synchronized to the same clock – synchronous counter
- May be implemented by different type of flip-flops
- Example: a 3-bit binary counter can count through this sequence



Synchronous Binary Counter Design

- An FSM (without external inputs)
  - State Table

Present State			Next State		
Q2	Q1	Q0	$Q2^+$	$Q1^+$	$Q0^+$
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	0	1
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	0

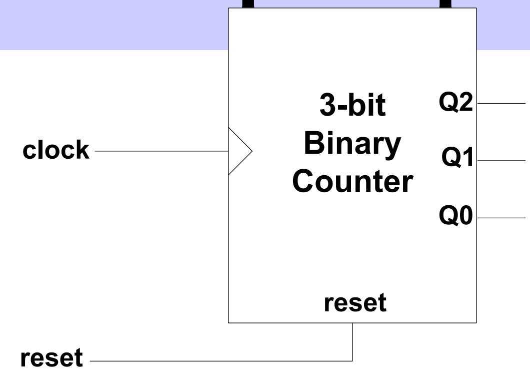
Counter Implemented with D Flip-Flop

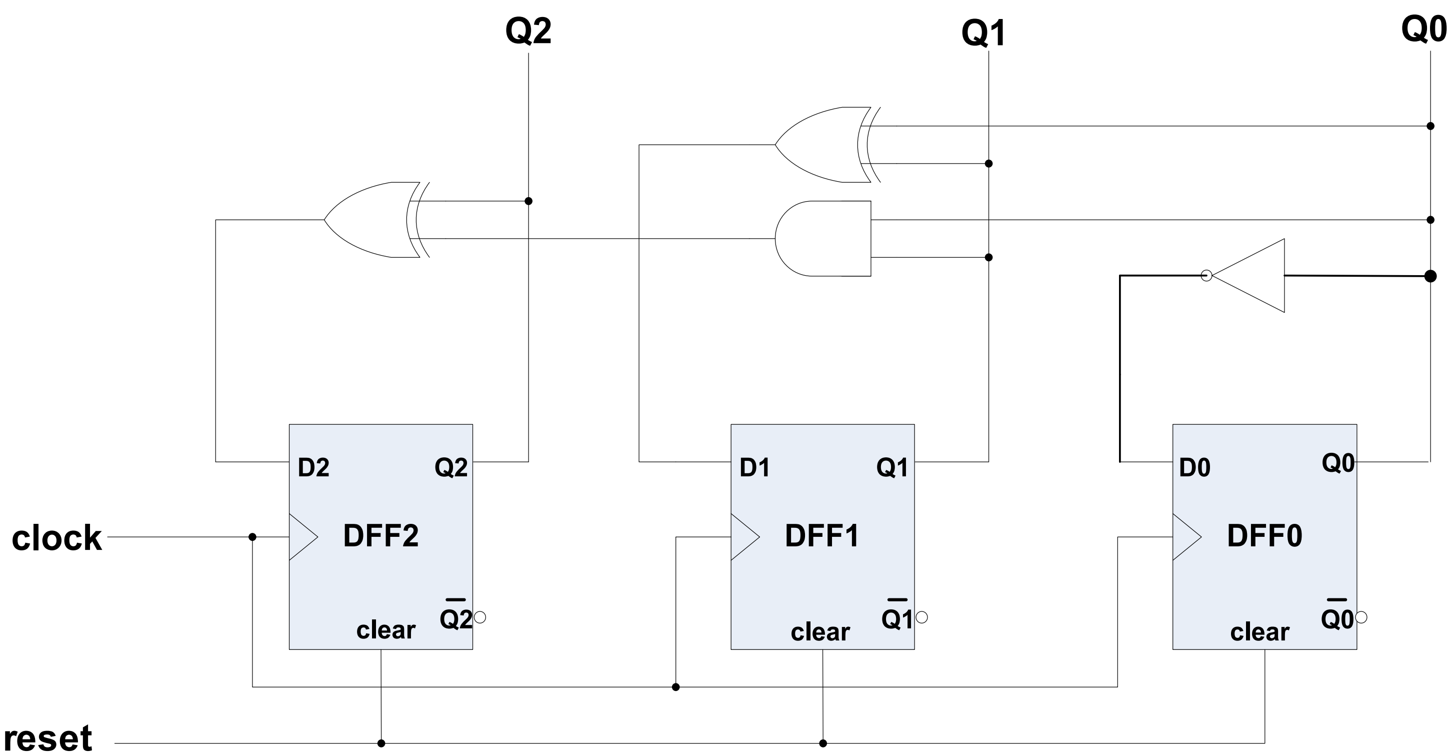
- Use D flip flops to hold values：**$Q^+ = D$ upon active edge**
- Next state equations

Present State			Next State			D flip flop input		
Q2	Q1	Q0	$Q2^+$	$Q1^+$	$Q0^+$	D2	D1	D0
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	0
0	1	0	0	1	1	0	1	1
0	1	1	1	0	0	1	0	0
1	0	0	1	0	1	1	0	1
1	0	1	1	1	0	1	1	0
1	1	0	1	1	1	1	1	1
1	1	1	0	0	0	0	0	0

Synchronous Binary Counter with D Flip-Flop

- 3-bit binary counter by D FF





Implement FSM with T Flip-Flop

- Characteristic table and equation of T-FF

T	Q^+	
0	Q	No Change
1	Q'	Complement

$$Q^+ = T \oplus Q$$

- Excitation table of T-FF

Q	Q^+		T
0	0	No Change	0
0	1	Toggle	1
1	0	Toggle	1
1	1	No Change	0

Q	Q^+		T
X	Q	No Change	0
X	Q'	Toggle	1

Implement FSM with T Flip-Flop

- State Table with T input

Intermediate columns

Present State			Next State			T Input		
Q2	Q1	Q0	$Q2^+$	$Q1^+$	$Q0^+$	T2	T1	T0
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	0	1	1	0	0	1
0	1	1	1	0	0	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	1	1	0	0	1
1	1	1	0	0	0	1	1	1

Implement FSM with T Flip-Flop

- T input equations in terms of Qs can be found from the state table

T2

Q2 \ Q1Q0	00	01	11	10
0	0	0	1	0
1	0	0	1	0

T2 = Q1Q0

T1

Q2 \ Q1Q0	00	01	11	10
0	0	1	1	0
1	0	1	1	0

T1 = Q0

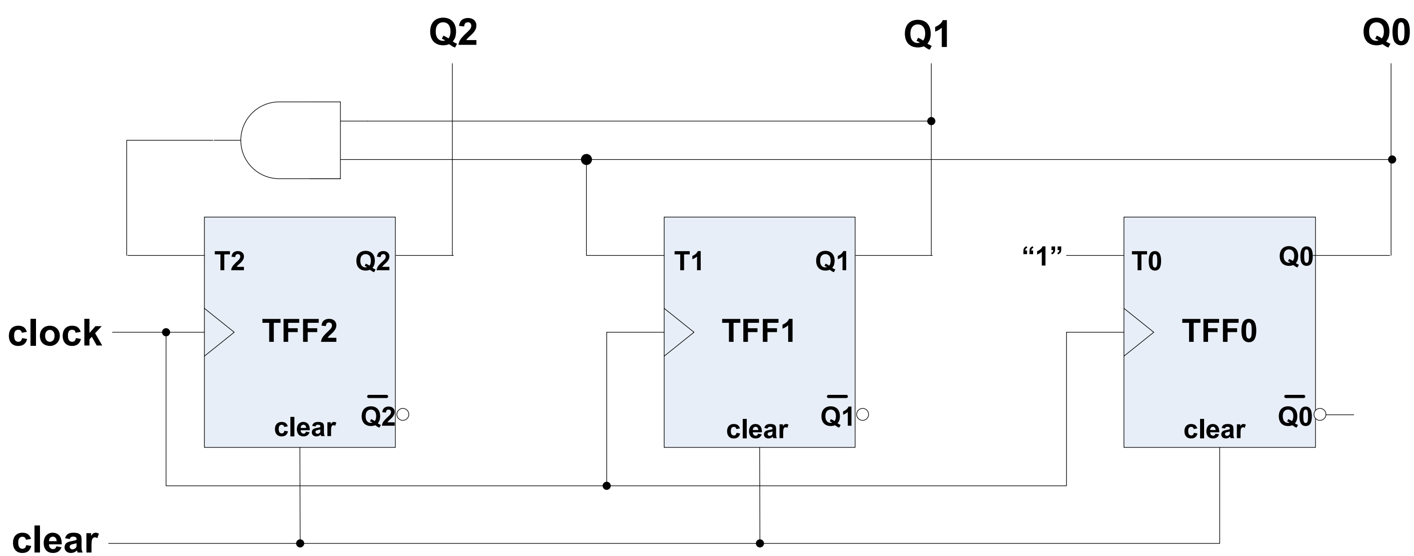
T0

Q2 \ Q1Q0	00	01	11	10
0	1	1	1	1
1	1	1	1	1

T0 = 1

Implement FSM with T Flip-Flop

- 3-bit binary counter by T FF



Implement FSM with JK Flip-Flop

- Characteristic table and equation of JK-FF

J	K	Q^+	Action
0	0	Q	Hold
0	1	0	Reset
1	0	1	Set
1	1	Q'	Toggle

$$Q^+ = JQ' + K'Q$$

- Excitation table of JK-FF

Q	Q^+	Action	J	K	J	K
0	0	Reset/Hold	0/0	1/0	0	X
0	1	Set/Toggle	1/1	0/1	1	X
1	0	Reset/Toggle	0/1	1/1	X	1
1	1	Set/Hold	1/0	0/0	X	0

Implement FSM with JK Flip-Flop

- State Table with JK input

Intermediate columns

Present State			Next State			JK Inputs					
Q2	Q1	Q0	$Q2^+$	$Q1^+$	$Q0^+$	J2	K2	J1	K1	J0	K0
0	0	0	0	0	1	0	X	0	X	1	X
0	0	1	0	1	0	0	X	1	X	X	1
0	1	0	0	1	1	0	X	X	0	1	X
0	1	1	1	0	0	1	X	X	1	X	1
1	0	0	1	0	1	X	0	0	X	1	X
1	0	1	1	1	0	X	0	1	X	X	1
1	1	0	1	1	1	X	0	X	0	1	X
1	1	1	0	0	0	X	1	X	1	X	1

Implement FSM with JK Flip-Flop

- J-K Input equations

J2

Q2 \ Q1Q0	00	01	11	10
0	0	0	1	0
1	X	X	X	X

J2 = Q1Q0

J1

Q2 \ Q1Q0	00	01	11	10
0	0	1	X	X
1	0	1	X	X

J1 = Q0

J0

Q2 \ Q1Q0	00	01	11	10
0	1	X	X	1
1	1	X	X	1

J0 = 1

K2

Q2 \ Q1Q0	00	01	11	10
0	X	X	X	X
1	0	0	1	0

K2 = Q1Q0 = J2

K1

Q2 \ Q1Q0	00	01	11	10
0	X	X	1	0
1	X	X	1	0

K1=Q0 = J1

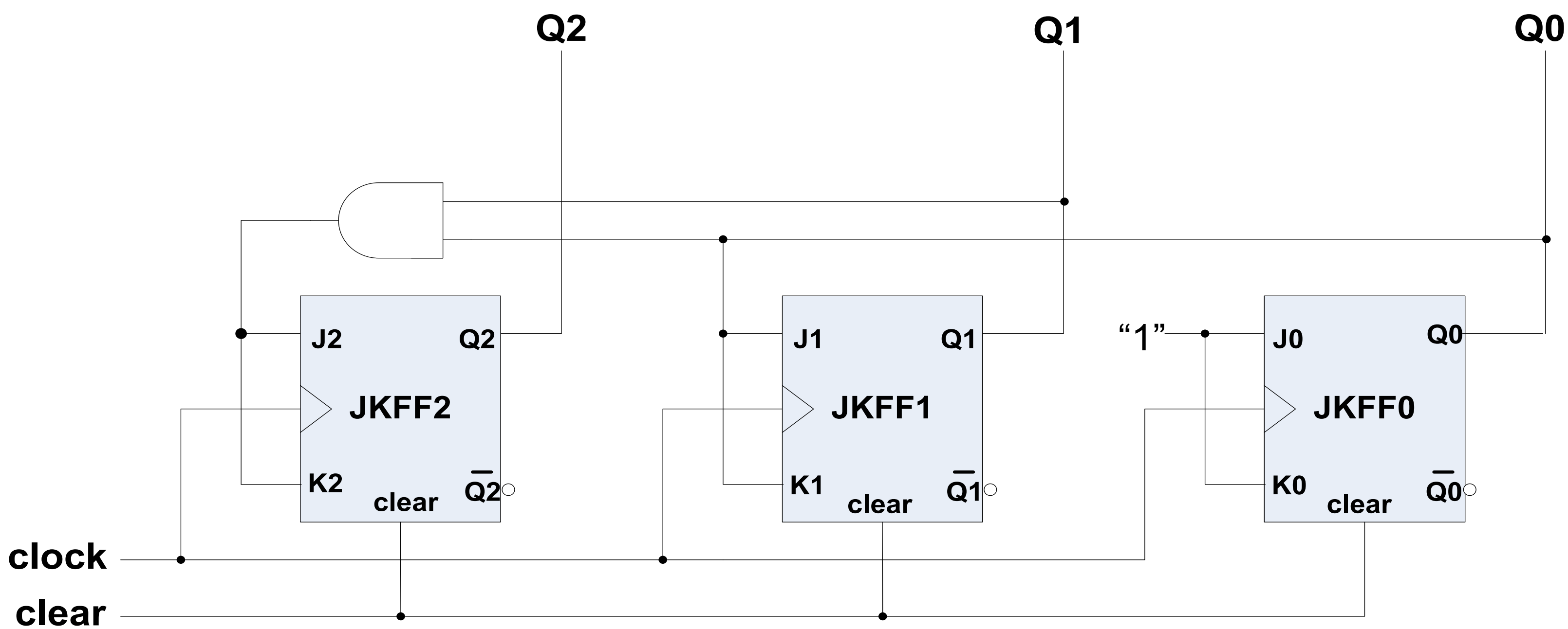
K0

Q2 \ Q1Q0	00	01	11	10
0	X	1	1	X
1	X	1	1	X

K0=1= J0

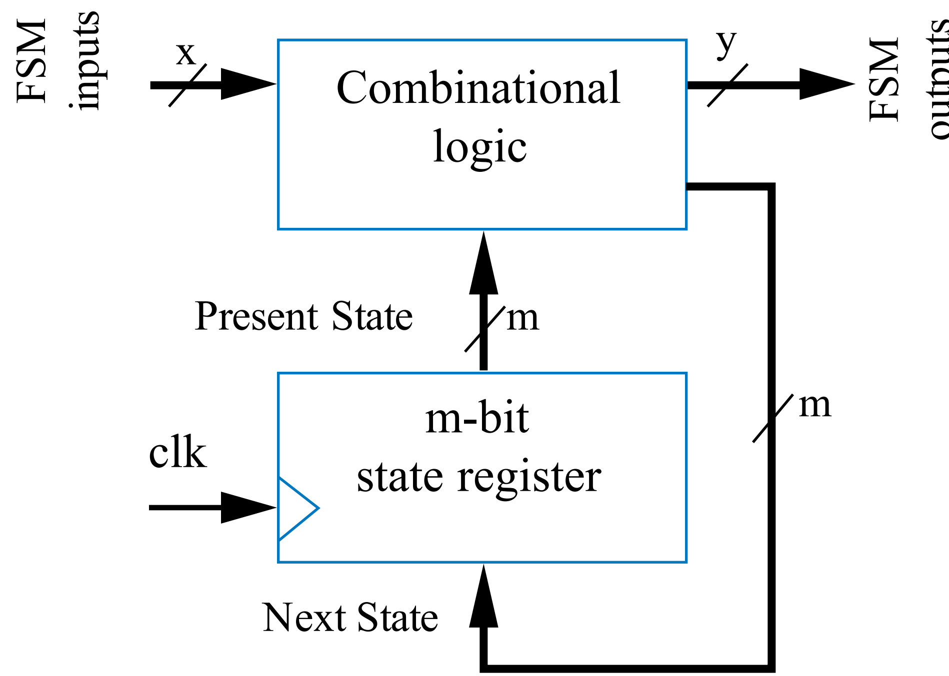
Implement FSM with JK Flip-Flop

- Circuit diagram



Standard FSM Architecture

- How to design sequential circuit?
  - Design as FSM
  - Use standard architecture
    - **State register** -- to store the present state
    - **Combinational logic** -- to compute outputs, and next state
  - Known as ***controller***



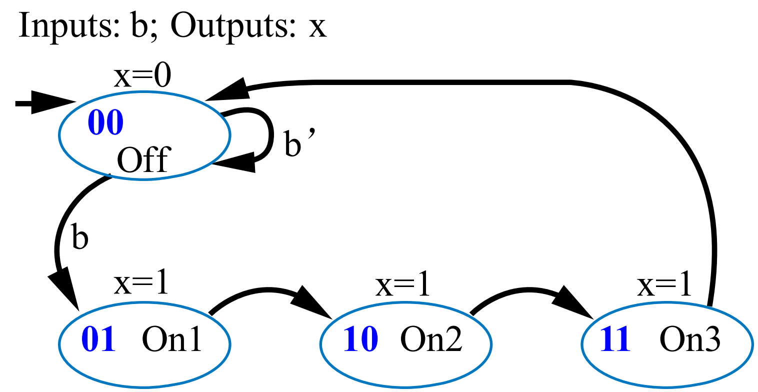
FSM (Controller) Design

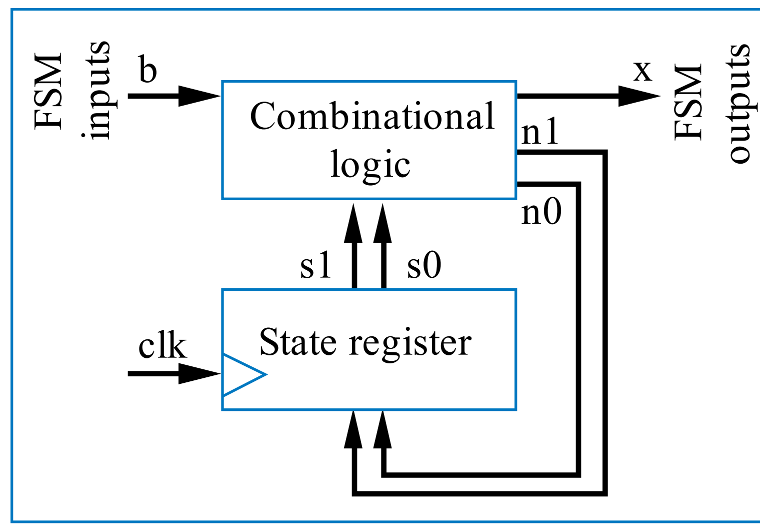
- Five step FSM design process

	Step	Description
Step 1	*Capture the FSM*	Create an FSM that describes the desired behavior of the controller.
Step 2	*Create the architecture*	Create the standard architecture by using a state register of appropriate width, and combinational logic with inputs being the state register bits and the FSM inputs and outputs being the next state bits and the FSM outputs.
Step 3	*Encode the states*	Assign a unique binary number to each state. Each binary number representing a state is known as an ***encoding***. Any encoding will do as long as each state has a unique encoding.
Step 4	*Create the state table*	Create a truth table for the combinational logic such that the logic will generate the correct FSM outputs and next state signals. Ordering the inputs with state bits first makes this truth table describe the state behavior, so the table is a state table.
Step 5	*Implement the combinational logic*	Implement the combinational logic using any method.

FSM Design Example: Push Button

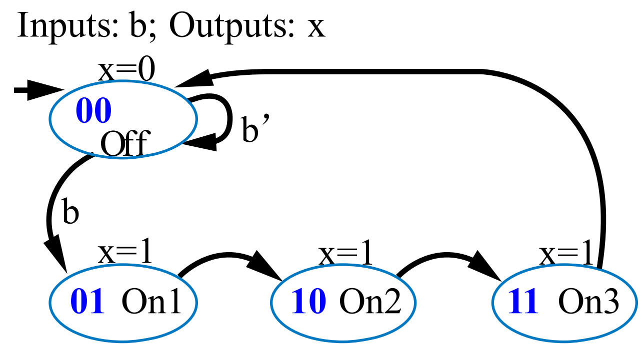
- Step 1: Capture the FSM
  - Already done
- Step 2: Create architecture
  - 2-bit state register (for 4 states)
  - Input b, output x
  - Present state signals (s1, s0)
  - Next state signals (n1, n0)
- Step 3: Encode the states
  - Any encoding with unique representation for each state will work





FSM Design Example: Push Button (cont.)

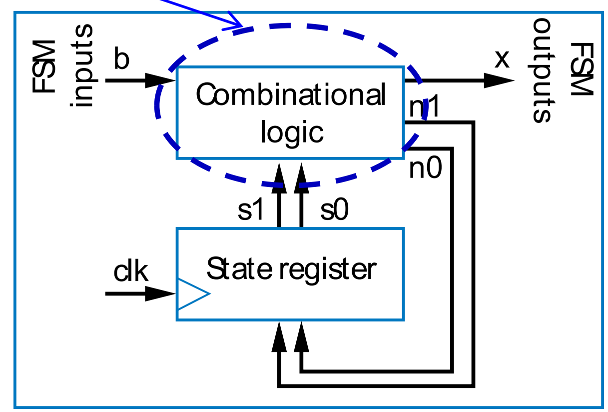
- Step 4: Create state table



	Inputs			Outputs		
	Present state		b	x	Next state	
Off	0	0	0	0	0	0
	0	0	1	0	0	1
On1	0	1	0	1	1	0
	0	1	1	1	1	0
On2	1	0	0	1	1	1
	1	0	1	1	1	1
On3	1	1	0	1	0	0
	1	1	1	1	0	0

Combinational Logic Inputs

Combinational Logic Outputs



FSM Design Example: Push Button (cont.)

- Step 5: Implement combinational logic

	Inputs			Outputs		
	s1	s0	b	x	n1	n0
Off	0	0	0	0	0	0
	0	0	1	0	0	1
On1	0	1	0	1	1	0
	0	1	1	1	1	0
On2	1	0	0	1	1	1
	1	0	1	1	1	1
On3	1	1	0	1	0	0
	1	1	1	1	0	0

x = s1 + s0

x s1 \ s0 b	00	01	11	10
0	**0**	**0**	**1**	**1**
1	**1**	**1**	**1**	**1**

n1 = s1’s0 + s1s0’

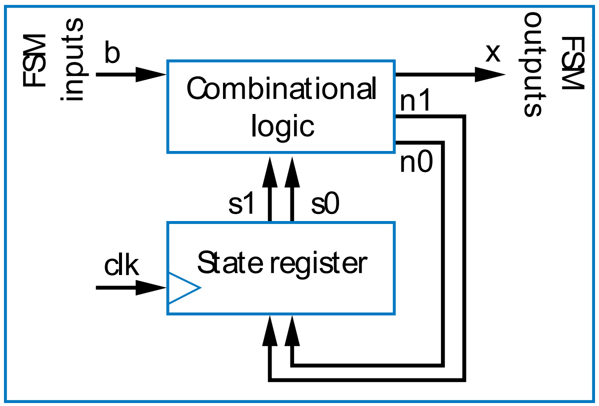
n1 s1 \ s0 b	00	01	11	10
0	**0**	**0**	**1**	**1**
1	**1**	**1**	**0**	**0**

n0 = s0’b + s1s0’

n0 s1 \ s0 b	00	01	11	10
0	**0**	**1**	**0**	**0**
1	**1**	**1**	**0**	**0**

FSM Design Example: Push Button (cont.)

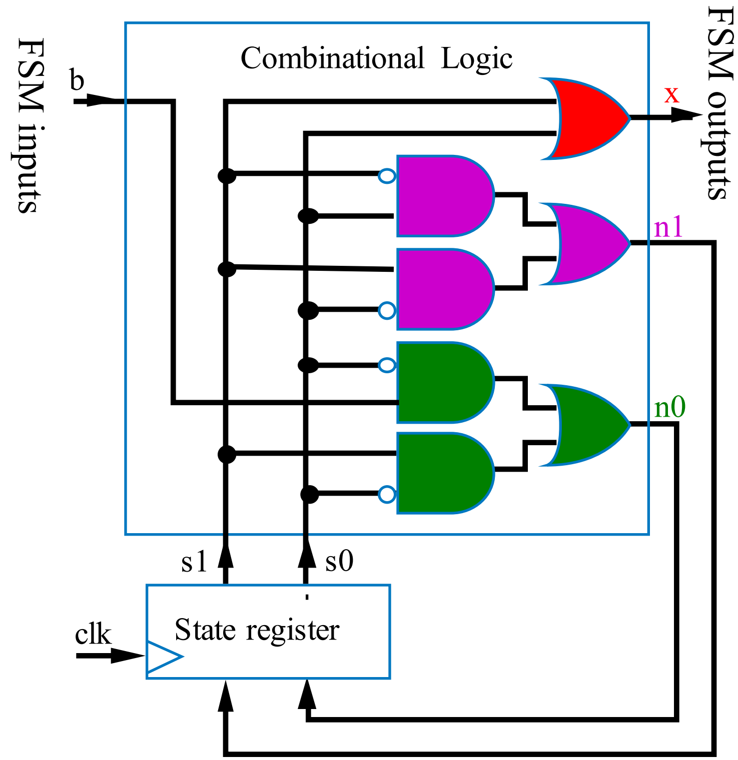
- Step 5: Implement combinational logic (cont)



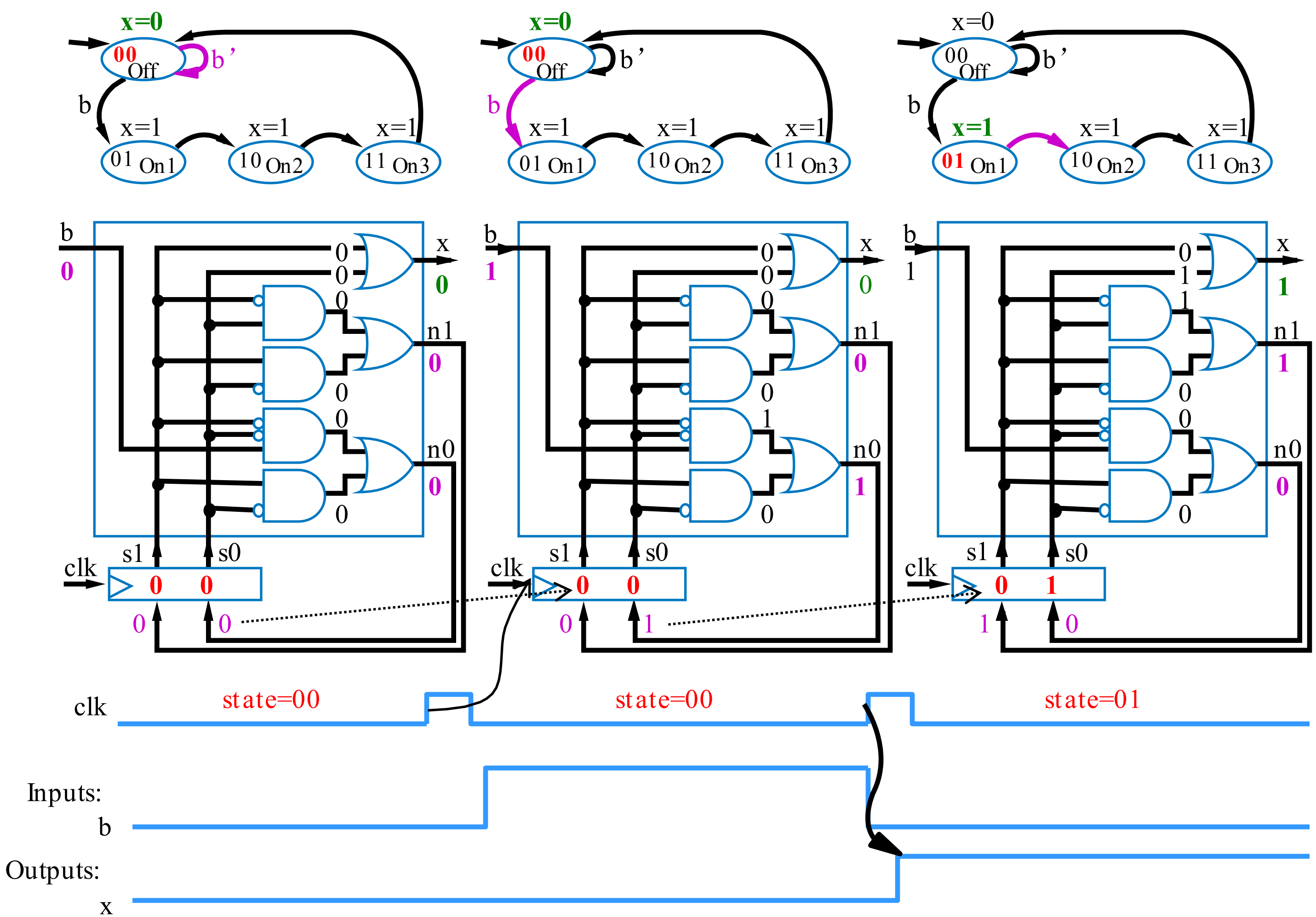
$x = s1 + s0$

$n1 = s1's0 + s1s0'$

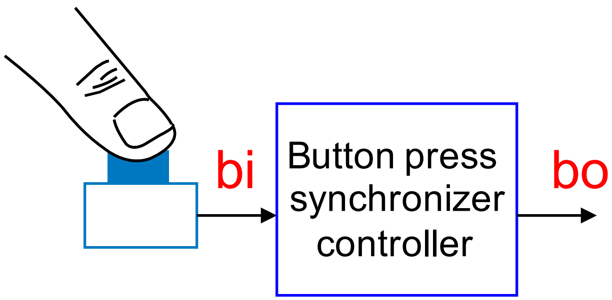
$n0 = s0'b + s1s0'$

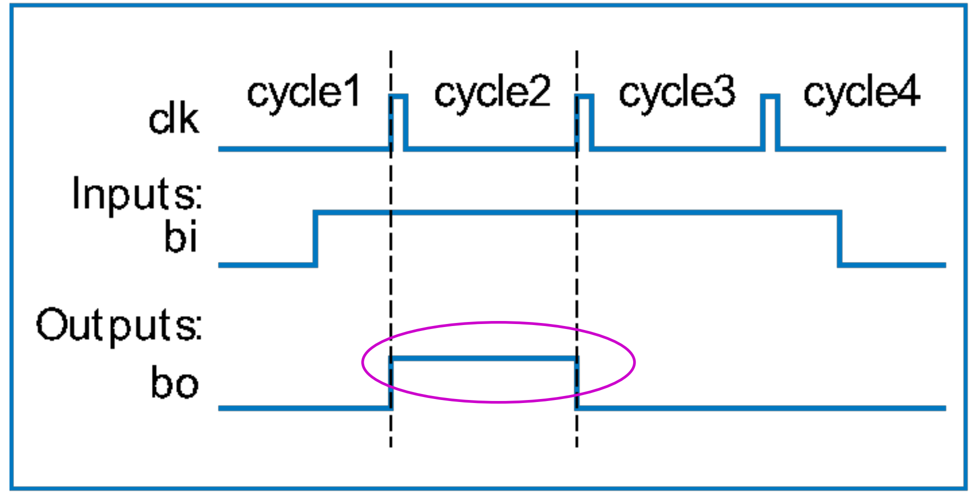


Understanding the Controller's Behavior



FSM Design Example:
Button Press Synchronizer

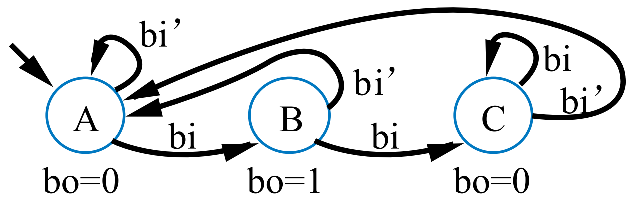




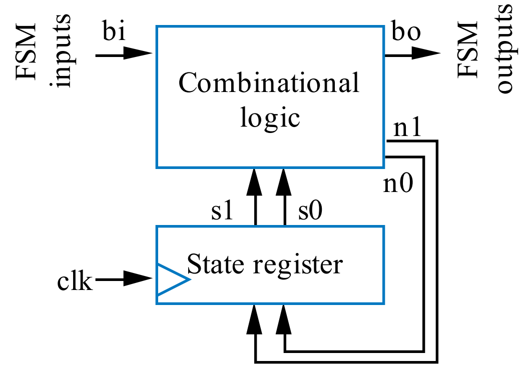
- Want simple sequential circuit that converts button press to single clock cycle duration, regardless of length of time that button actually pressed

FSM Design Example:
Button Press Synchronizer (cont.)

FSM inputs: bi; FSM outputs: bo

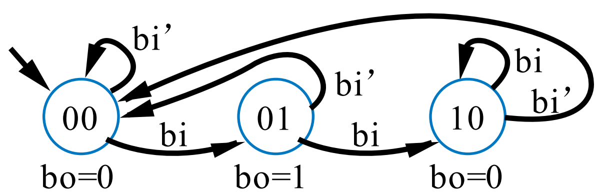


Step 1: FSM



Step 2: Create architecture

FSM inputs: bi; FSM outputs: bo



Step 3: Encode states

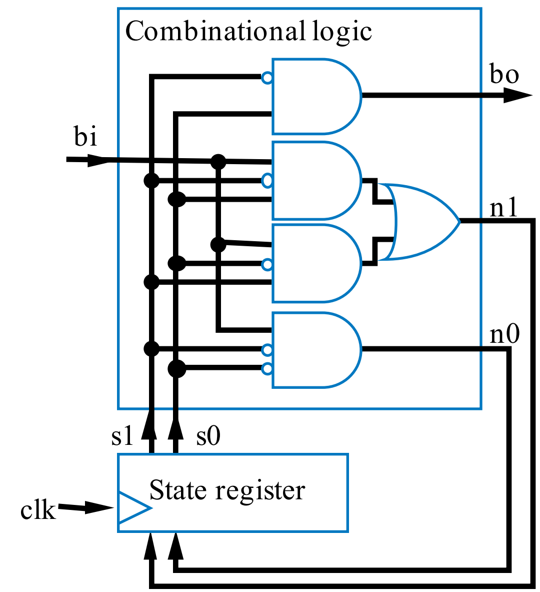
Combinational logic

	Inputs			Outputs		
	s1	s0	bi	n1	n0	bo
A	0	0	0	0	0	0
	0	0	1	0	1	0
B	0	1	0	0	0	1
	0	1	1	1	0	1
C	1	0	0	0	0	0
	1	0	1	1	0	0
unused	1	1	0	0	0	0
	1	1	1	0	0	0

Step 4: State table

may be ‘x’

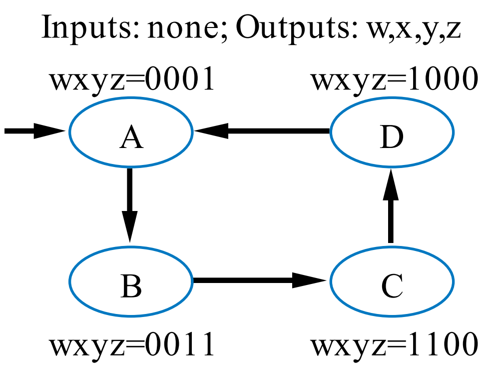
n1 = s1’s0bi + s1s0bi
n0 = s1’s0’bi
bo = s1’s0bi’ + s1’s0bi = s1s0



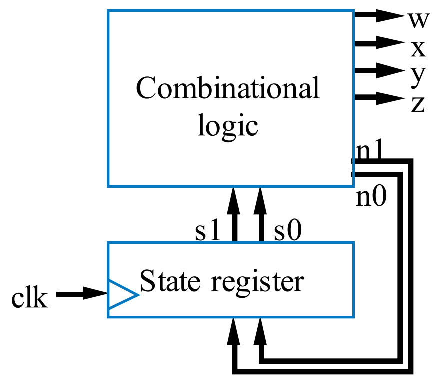
Step 5: Create combinational circuit

FSM Example: Sequence Generator

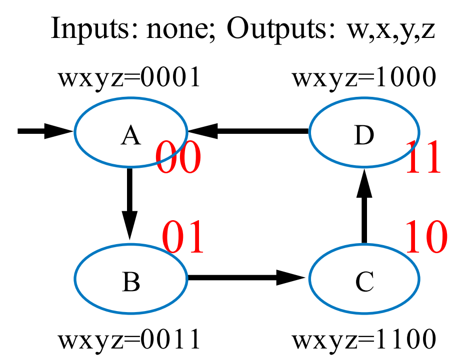
- Want generate sequence 0001, 0011, 1100, 1000, (repeat)
  - Each value for one clock cycle



Step 1: Create FSM



Step 2: Create architecture

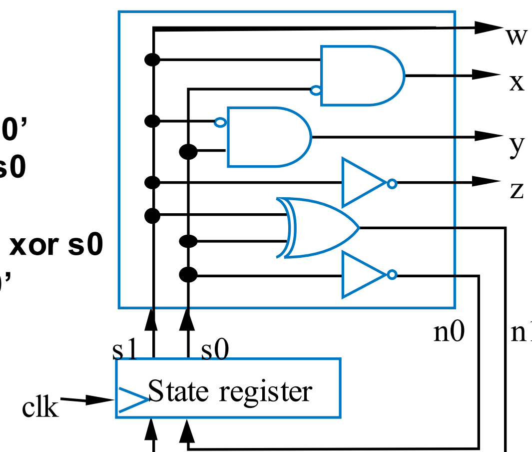


Step 3: Encode states

	Inputs		Outputs					
	s1	s0	w	x	y	z	n1	n0
A	0	0	0	0	0	1	0	1
B	0	1	0	0	1	1	1	0
C	1	0	1	1	0	0	1	1
D	1	1	1	0	0	0	0	0

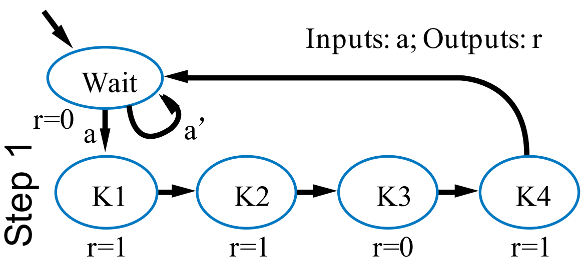
Step 4: Create state table

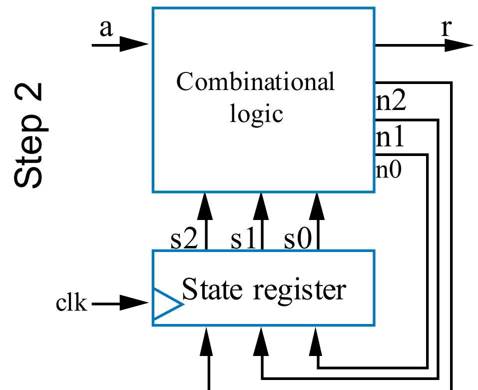
w = s1
x = s1s0'
y = s1's0
z = s1'
n1 = s1 xor s0
n0 = s0'

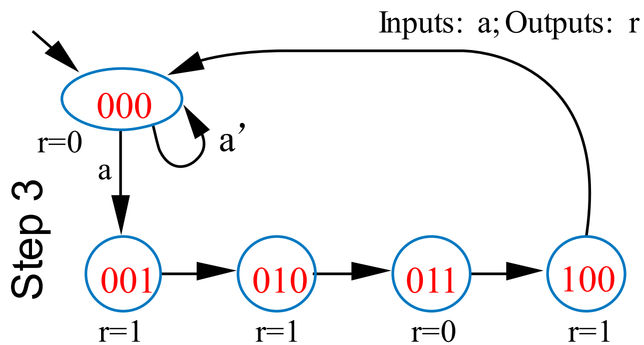


Step 5: Create combinational circuit

FSM Example: Secure Car Key







We omit Step 5

	Inputs				Outputs			
	s2	s1	s0	a	r	n2	n1	n0
Wait	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	1
K1	0	0	1	0	1	0	1	0
	0	0	1	1	1	0	1	0
K2	0	1	0	0	1	0	1	1
	0	1	0	1	1	0	1	1
K3	0	1	1	0	0	1	0	0
	0	1	1	1	0	1	0	0
K4	1	0	0	0	1	0	0	0
	1	0	0	1	1	0	0	0
Unused	1	0	1	0	0	0	0	0
	1	0	1	1	0	0	0	0
	1	1	0	0	0	0	0	0
	1	1	0	1	0	0	0	0
	1	1	1	0	0	0	0	0
	1	1	1	1	0	0	0	0

Step 4

may be 'x'