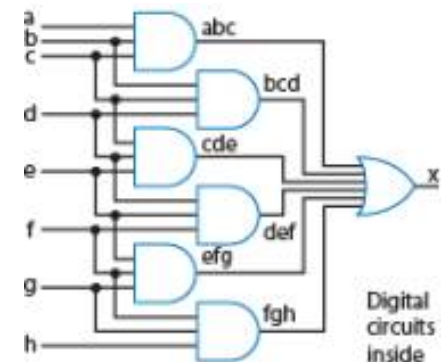


Topic 1

Introduction to Digital Design

Why Study Digital Design?

- Many elements of our lives are or are to become digital
 - Computer, camera, cell phone, TV, car...
- Electronic devices are made smaller and smarter
 - Enabled by shrinking and more capable chips
 - Supported by “embedded systems”
 - Providing better quality
- Solid understanding benefits ECE engineers
 - For computer engineer – fundamental
 - For electrical engineer – many times necessary
 - Even for software engineer – confident and insightful when aware of hardware resource issues

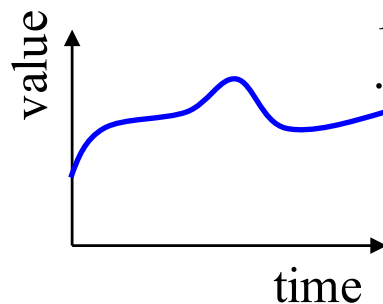
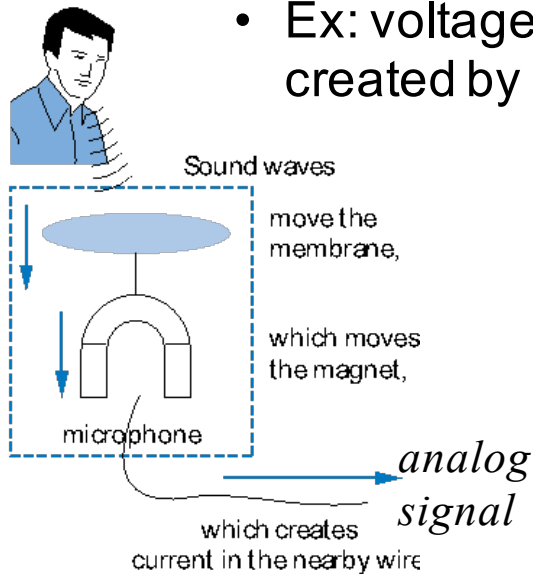


What Does “Digital” Mean?

- Analog signal

- Infinite possible values

- Ex: voltage on a wire created by microphone

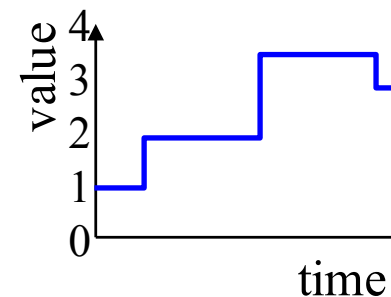
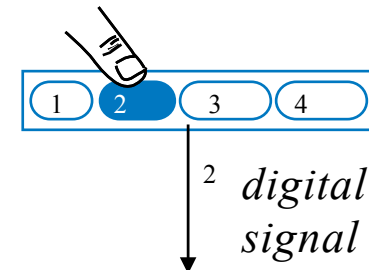


Possible values:
1.00, 1.01, 2.0000009,
... infinite possibilities

- Digital signal

- Finite possible values

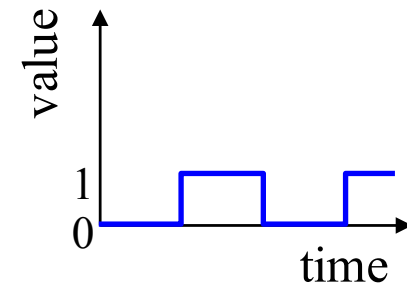
- Ex: button pressed on a keypad



Possible values:
0, 1, 2, 3, or 4.
That's it.

Digital Signals with Only Two Values: Binary

- **Binary** digital signal -- only *two* possible values
 - low voltage (e.g. 0V or -5V) and high voltage (e.g. 3.3V or 5V)
 - Typically represented as **0** and **1**, respectively
 - All values are represented as combinations of 0's and 1's, e.g. 1011, 11010
 - Called binary value or binary number
 - Each binary digit is a **bit**
 - We'll only consider binary digital signals
 - Although there are other types of digital signals
 - Binary is popular because
 - Transistors, the basic digital electric component, operate using *two* voltages
 - Storing/transmitting one of *two* values is easier than three or more

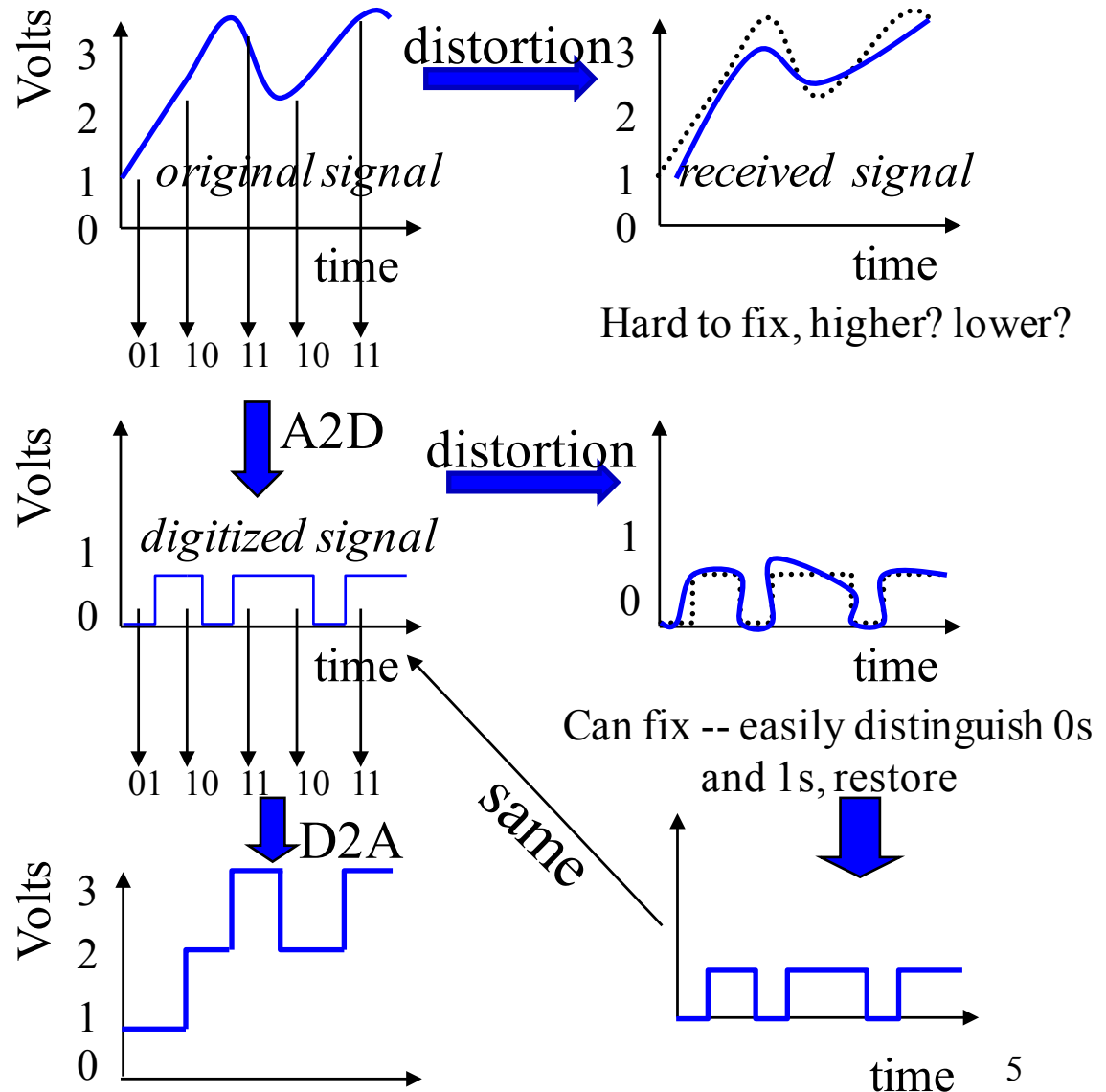


From Analog to Digital – Digitization

- Analog signal (e.g., audio) may lose quality
 - Voltage levels not saved/copied/transmitted perfectly
 - Hard to recover
- Digitized version:
 - “Sample” voltage at particular rate
 - Easy to distinguish 0s from 1s, thus easy to recover
 - Increase sample rate to improve quality

Example:
if only 4 sampled values
let binary representation be:

0 V:	“00”
1 V:	“01”
2 V:	“10”
3 V:	“11”



Resolution

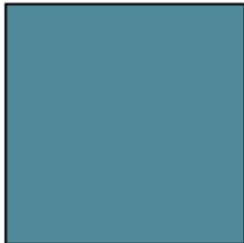


What happened?



(source: cntv.cn)

1 × 1



2 × 2



5 × 5



10 × 10



20 × 20



50 × 50

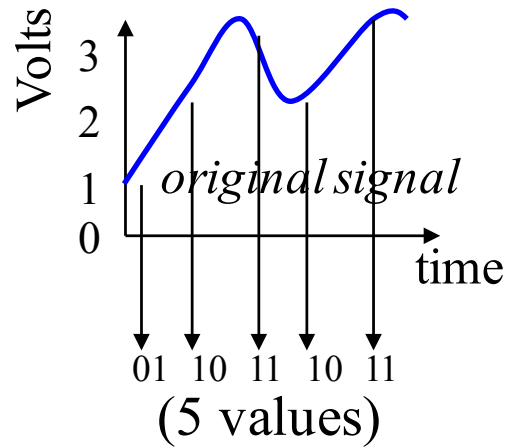


100 × 100

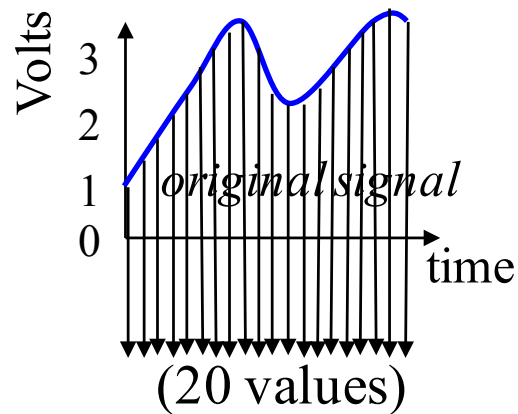
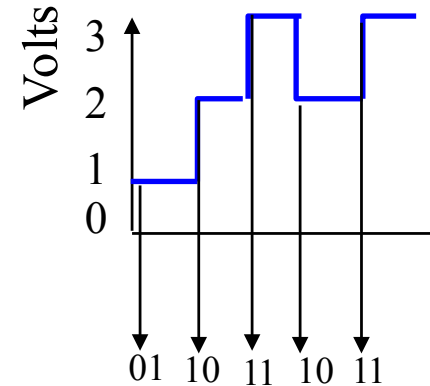


(source: wikipedia.org)

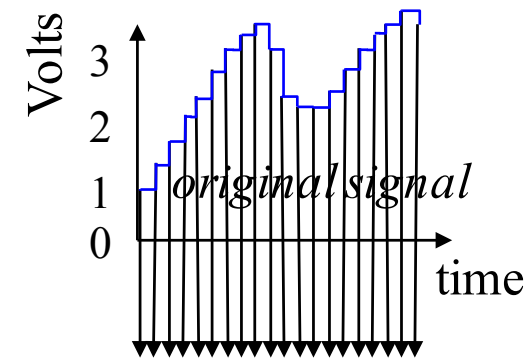
From Analog to Digital – Digitization



Restore
→

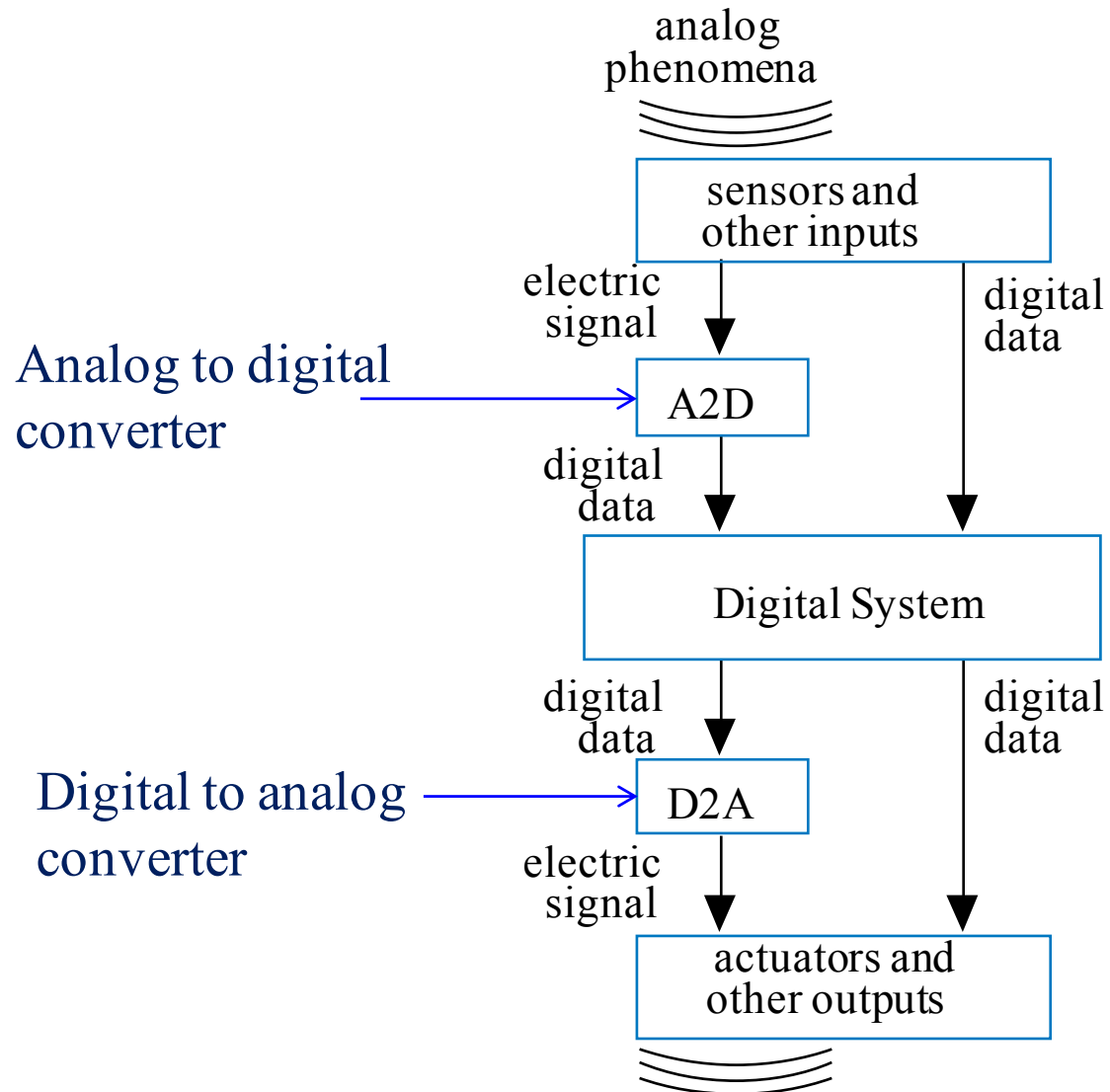


Restore
→

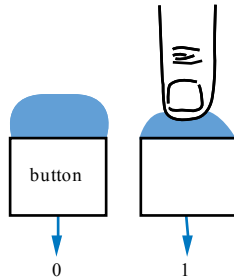


Higher resolution (sample rate)

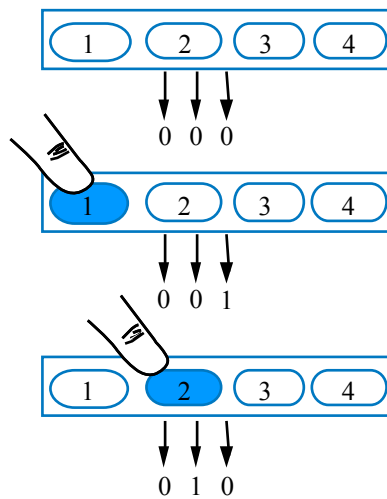
Typical Digital System



How Do We Encode Signal as Binary?



- Some signals are inherently binary
 - Example:
Button: not pressed (0), pressed (1)



- Some signals are inherently digital
 - But not binary digital
 - Outputs finite possible values
 - Each value maybe encoded in binary with multiple bits
 - More values need more bits to represent
 - e.g., multi-button input: encode “1”=001, “2”=010, ...

How Do We Encode Numbers with Bits

- Number systems: decimal, binary, octal, hexadecimal, ...
- Each position of a number is associated with a weight quantity
 - Base ten (*decimal*)

$$\begin{array}{ccccc} & & 5 & 2 & 3 \\ \hline & & 10^2 & 10^1 & 10^0 \end{array}$$

- Base two (*binary*)

$$\begin{array}{ccccc} & & 1 & 0 & 1 \\ \hline & & 2^2 & 2^1 & 2^0 \end{array}$$

Binary System

- The Binary System is a base 2 (modulo 2) number system:
 - 2 digits: 0 or 1
- Counting beyond 1 requires additional place
- In a binary number, each position has a decimal weight in power of 2, **10011.01**

	<u>1</u>	<u>0</u>	<u>0</u>	<u>1</u>	<u>1</u>	.	<u>0</u>	<u>1</u>
	$\times 16$	$\times 8$	$\times 4$	$\times 2$	$\times 1$		$\frac{1}{2}$	$\frac{1}{4}$
weight	(2^4)	(2^3)	(2^2)	(2^1)	(2^0)		(2^{-1})	(2^{-2})
position	4	3	2	1	0		-1	-2

Find Equivalent Decimal for Binary Numbers

- Example: Convert binary number 10011.01_2 to decimal

Number:	<u>1</u>	<u>0</u>	<u>0</u>	<u>1</u>	<u>1</u>	.	<u>0</u>	<u>1</u>
Position:	4	3	2	1	0		-1	-2
Weight:	2^4	2^3	2^2	2^1	2^0		2^{-1}	2^{-2}

$$\begin{aligned} & 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2} \\ = & 16 + 0 + 0 + 2 + 1 + 0 + 1/4 \\ = & 19.25_{10} = 10011.01_2 \end{aligned}$$

Encode Decimal as Binary Numbers: Subtraction Method (Easy for Humans)

- Subtraction method
 - To make the job easier (especially for big numbers), we can just subtract a selected binary weight from the (remaining) quantity
 - Then, we have a new remaining quantity, and we start again (from the present binary position)
 - Stop when remaining quantity is 0

Remaining quantity: 12

<u>32</u>	<u>16</u>	<u>8</u>	<u>4</u>	<u>2</u>	<u>1</u>	
<u>1</u>						32 is too much

<u>0</u>	<u>1</u>					16 is too much
<u>32</u>	<u>16</u>	<u>8</u>	<u>4</u>	<u>2</u>	<u>1</u>	

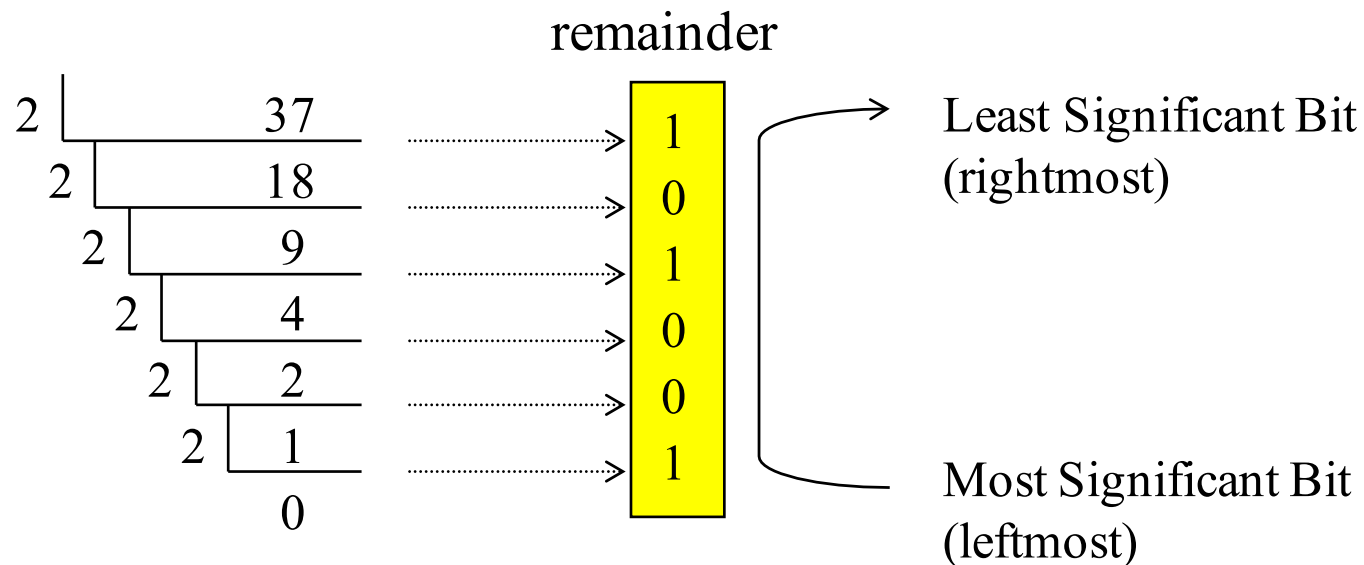
<u>0</u>	<u>0</u>	<u>1</u>				$\underline{12} - 8 = \underline{4}$
<u>32</u>	<u>16</u>	<u>8</u>	<u>4</u>	<u>2</u>	<u>1</u>	

<u>0</u>	<u>0</u>	<u>1</u>	<u>1</u>			$\underline{4} - 4 = \underline{0}$ DONE
<u>32</u>	<u>16</u>	<u>8</u>	<u>4</u>	<u>2</u>	<u>1</u>	

<u>0</u>	<u>0</u>	<u>1</u>	<u>1</u>	<u>0</u>	<u>0</u>	answer
<u>32</u>	<u>16</u>	<u>8</u>	<u>4</u>	<u>2</u>	<u>1</u>	

Encode Decimal in Binary Numbers: Division Method (Good for Computers)

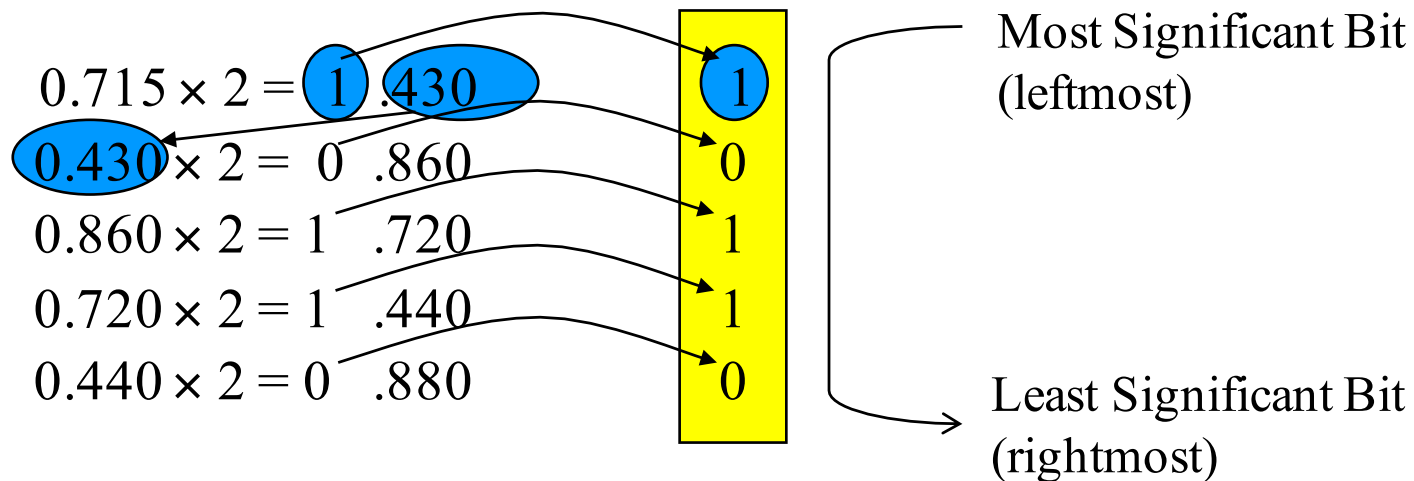
- Example: Convert decimal number 37 to binary
 - Repeated-division-by-base (here, base 2)



$$(37)_{10} = (100101)_2$$

Encode Fractional Decimal in Binary

- Example: Convert fractional part 0.715_{10} to binary
 - Repeated-multiplication-by-base (here, base 2)



$$(0.715)_{10} \approx (0.10110\dots)_2$$

Encode Numbers with Bits

- Bigger number needs more bits to encode
 - $37_{10} = 100101_2$ (6 bits)
 - $137_{10} = 10001001_2$ (8 bits)
 - $10307_{10} = 10100001000011_2$ (14 bits)
- N bits can represent 2^N non-negative integers
 - 0, 1, 2, ..., 2^N-1
 - Negative numbers will be discussed later

Encode Decimal Numbers by Binary Bits

	Binary				Decimal
	0	0	0	0	0
	0	0	0	1	1
	0	0	1	0	2
	0	0	1	1	3
	0	1	0	0	4
	0	1	0	1	5
	0	1	1	0	6
	0	1	1	1	7
.....	1	0	0	0	8
	1	0	0	1	9
	1	0	1	0	10
	1	0	1	1	11
	1	1	0	0	12
	1	1	0	1	13
	1	1	1	0	14
	1	1	1	1	15
					16

Hexadecimal System

- The Hexadecimal system is a base 16 (modulo 16) number system:
 - 16 digits: 0 1 2 3 4 5 6 7 8 9 A B C D E F
- Letters **A ~ F** represent decimal 10 through decimal 15
- Each position has a decimal weight in power of 16, e.g. **E3A**

	<u>E</u>	<u>3</u>	<u>A</u>
	×256	×16	×1
weight	(16²)	(16¹)	(16⁰)

$$(E3A)_{16} = E \times 256 + 3 \times 16 + A \times 1 = 3584 + 48 + 10 = 3642$$

Encode Decimal to Hexadecimal

- Example: Convert decimal number 58 to hexadecimal
 - Repeated-division-by-base (here, base 16)

$$\begin{array}{r|l} 16 & 58 \\ \hline 16 & 3 \\ \hline & 0 \end{array} \quad \begin{array}{l} \text{.....} \rightarrow 10 \text{ (A)} \\ \text{.....} \rightarrow 3 \end{array} \quad \begin{array}{l} \text{Least Significant Digit} \\ \text{Most Significant Digit} \end{array}$$

$$(58)_{10} = (3A)_{16}$$

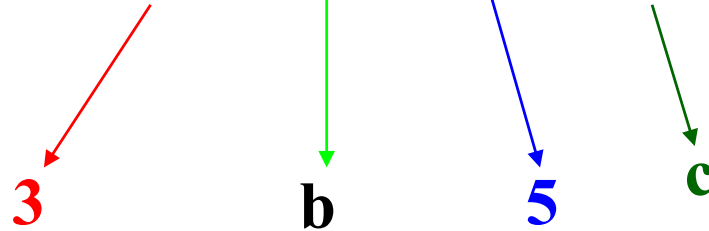
Summary

Binary				Decimal	Hexaecimal
0	0	0	0	0	0
0	0	0	1	1	1
0	0	1	0	2	2
0	0	1	1	3	3
0	1	0	0	4	4
0	1	0	1	5	5
0	1	1	0	6	6
0	1	1	1	7	7
1	0	0	0	8	8
1	0	0	1	9	9
1	0	1	0	10	A
1	0	1	1	11	b
1	1	0	0	12	C
1	1	0	1	13	d
1	1	1	0	14	E
1	1	1	1	15	F

Convert Binary to Hexadecimal

- Look for groups of 4 bits starting from the LSB
- Example: Convert **11** 1011 **0101.11** to hexadecimal:

$$\mathbf{11} \ 1011 \ \mathbf{0101.11} = (\mathbf{0011} \ 1011 \ \mathbf{0101.1100})_2$$



$$(\mathbf{11} \ 1011 \ \mathbf{0101.11})_2 = (\mathbf{3b5.c})_{16}$$

Binary				Decimal	Hexadecimal
0	0	0	0	0	0
0	0	0	1	1	1
0	0	1	0	2	2
0	0	1	1	3	3
0	1	0	0	4	4
0	1	0	1	5	5
0	1	1	0	6	6
0	1	1	1	7	7
1	0	0	0	8	8
1	0	0	1	9	9
1	0	1	0	10	A
1	0	1	1	11	b
1	1	0	0	12	C
1	1	0	1	13	d
1	1	1	0	14	E
1	1	1	1	15	F

Convert Hexadecimal to Binary

- Each digit is converted to 4 bits in binary
- Arrange the groups of 4 bits in the same order
- Example: convert $(3F7)_{16}$ to binary:

$$\begin{array}{c}
 (3 \text{ F } 7)_{16} \\
 \swarrow \quad \downarrow \quad \searrow \\
 (\text{0011} \quad 1111 \quad \text{0111})_2
 \end{array}$$

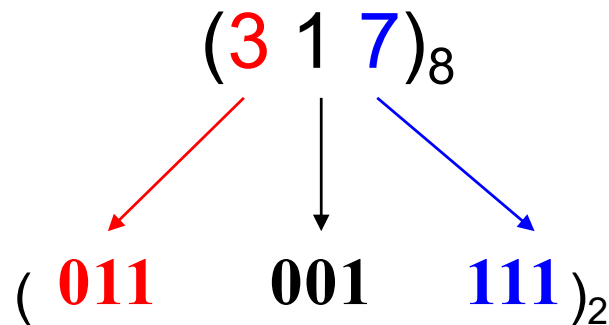
- Drop the initial 0's to simplify

$$(3F7)_{16} = (11 \text{ 1111 } 0111)_2$$

Binary	Decimal	Hexadecimal
0 0 0 0	0	0
0 0 0 1	1	1
0 0 1 0	2	2
0 0 1 1	3	3
0 1 0 0	4	4
0 1 0 1	5	5
0 1 1 0	6	6
0 1 1 1	7	7
1 0 0 0	8	8
1 0 0 1	9	9
1 0 1 0	10	A
1 0 1 1	11	b
1 1 0 0	12	C
1 1 0 1	13	d
1 1 1 0	14	E
1 1 1 1	15	F

Octal System

- The Octal number system is a base 8 (modulo 8) number system:
 - 8 digits: 0 1 2 3 4 5 6 7
- Each position has a decimal weight in power of 8
- Each octal digital corresponds to a 3-bit binary number



Convert Base-M System to Base-N System

- Decimal can always be used as the intermediate number system
- Generally, the rule “divide/multiply by the base of destination system” applies to all the number system conversions
 - Example, to convert a Hex number to base-3 number, just divide the Hex number by 3

Binary Arithmetic

- Example:

carry 11110111

$$\begin{array}{r} 10110101 \\ + 11010011 \\ \hline 110001000 \end{array}$$

A blue dashed vertical line with a downward arrow at the bottom indicates the carry path from the first column to the result.

borrow 11000010

$$\begin{array}{r} 10110101 \\ - 11010011 \\ \hline 11100010 \end{array}$$

Hexadecimal Arithmetic

- Example:

carry 111
 8F5A
 + 11BC

 A116

borrow 11
 8F5A
 - 11BC

 7D9E

How Do We Encode Text with Binary Bits

- A popular code: ASCII
(American Standard Code for Information Interchange)
 - 7- (or 8-) bit encoding of each letter, number, or symbol

Symbol	Encoding
R	1010010
S	1010011
T	1010100
L	1001100
N	1001110
E	1000101
0	0110000
.	0101110
<tab>	0001001

Symbol	Encoding
r	1110010
s	1110011
t	1110100
l	1101100
n	1101110
e	1100101
9	0111001
!	0100001
<space>	0100000

- Unicode: Increasingly popular 16-bit encoding
 - Encodes characters from various world languages

Question:
What does this ASCII bit sequence represent?

1010010 1000101 1010011 1010100

R E S T

ASCII Coding Chart

Decimal	Binary	Octal	Hex	ASCII	Decimal	Binary	Octal	Hex	ASCII
64	01000000	100	40	@	96	01100000	140	60	`
65	01000001	101	41	A	97	01100001	141	61	a
66	01000010	102	42	B	98	01100010	142	62	b
67	01000011	103	43	C	99	01100011	143	63	c
68	01000100	104	44	D	100	01100100	144	64	d
69	01000101	105	45	E	101	01100101	145	65	e
70	01000110	106	46	F	102	01100110	146	66	f
71	01000111	107	47	G	103	01100111	147	67	g
72	01001000	110	48	H	104	01101000	150	68	h
73	01001001	111	49	I	105	01101001	151	69	i
74	01001010	112	4A	J	106	01101010	152	6A	j
75	01001011	113	4B	K	107	01101011	153	6B	k
76	01001100	114	4C	L	108	01101100	154	6C	l
77	01001101	115	4D	M	109	01101101	155	6D	m

•
•
•

Signed Binary Numbers

- To represent negative numbers
 - Cannot use minus sign: binary systems work with only two values, 0 and 1
 - The left-most bit of a binary number represents the sign of a number – sign bit
 - Sign bit 0 indicates positive numbers
 - Sign bit 1 indicates negative number

Representation of Negative Numbers

- Negative numbers are represented by
 - Sign and magnitude
 - 1's complement code
 - 2's complement code
- Sign and magnitude
 - MSB is the sign bit: 0 \rightarrow positive, 1 \rightarrow negative
- 1's complement representation of $-N$
 - Negation of every bit of N
 - Example, 1's complement representation of -3
 - $N = 3 = 0011$
 - $-N = -3 = 1100$
- 2's complement representation of $-N$ is
 - Negation of every bit of N , then plus 1
 - Example, 2's complement representation of -3
 - $N = 3 = 0011$
 - $-N = -3 = 1100 + 1 = 1101$

Signed 2's Complement Number

- Signed numbers are represented as 2's complement numbers in computers
- Recognize a signed 2's complement number
 - Sign bit = 0, positive number, recognize as a regular binary number
 - $0101 = +5$;
 - Sign bit = 1, negative number, the magnitude of the number is obtained by 2's complement operation
 - 1011
 - Sign: negative number
 - Magnitude: 2's complement operation of $(1011) = 0100 + 1 = 0101 = 5$
 - So $1011 = -5$

Ranges of Signed 2's Complement Number

- In general the 2's complement values range from -2^{n-1} to $2^{n-1}-1$
- For $n = 4$, the 2's complement values range from -8 to 7
- For $n = 8$, the 2's complement values range from -128 to 127
- For $n = 16$, the 2's complement values range from -2^{15} to $2^{15}-1$
- Overflow
 - If an n -bit 2's complement number is greater than $2^{n-1}-1$ or less than -2^{n-1} , we say there is an overflow

Detecting Overflow: Method 1

- Overflow detection logic
 - Two numbers' sign bits are the same but are different from the result's sign bit
 - If the two numbers' sign bits are different, overflow is impossible
 - Adding a positive and negative can't exceed largest magnitude positive or negative
- 4-bit example

sign bits

$\begin{array}{r} \textcircled{0} \ 1 \ 1 \ 1 \\ + \textcircled{0} \ 0 \ 0 \ 1 \\ \hline \textcircled{1} \ 0 \ 0 \ 0 \\ \text{overflow} \\ \text{(a)} \end{array}$	$\begin{array}{r} \textcircled{1} \ 1 \ 1 \ 1 \\ + \textcircled{1} \ 0 \ 0 \ 0 \\ \hline \textcircled{0} \ 1 \ 1 \ 1 \\ \text{overflow} \\ \text{(b)} \end{array}$	$\begin{array}{r} \textcircled{1} \ 0 \ 0 \ 0 \\ + \textcircled{0} \ 1 \ 1 \ 1 \\ \hline \textcircled{1} \ 1 \ 1 \ 1 \\ \text{no overflow} \\ \text{(c)} \end{array}$
--	--	---

Binary Number Subtraction


- Using two's complement representation

$$A - B = A + (-B)$$

$$= A + (\text{two's complement of } B)$$

$$= A + \text{invert_bits}(B) + 1$$

- Example:

11000010		00111101
10110101		10110101
- 11010011		+ 00101100
-----		+ 1
11100010		-----
	<i>same</i>	11100010