



SI 630

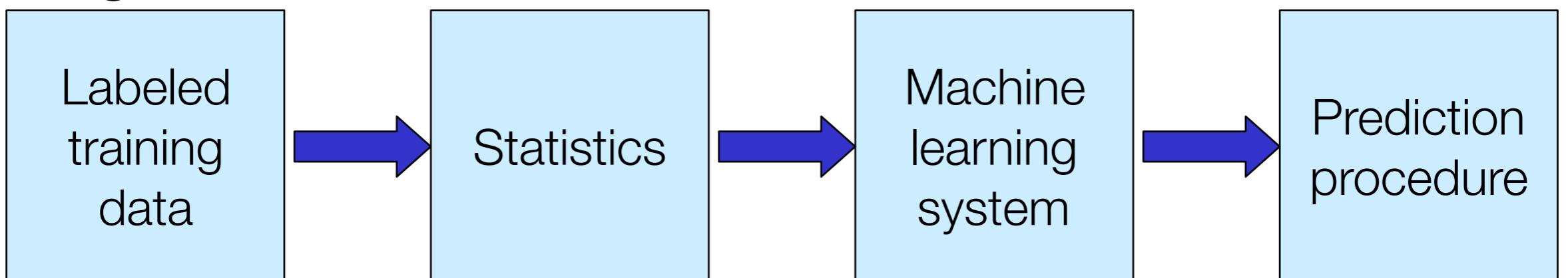
Natural Language Processing: Algorithms and People

Lecture 7: Unsupervised Learning
Feb. 19, 2020



Supervised Learning

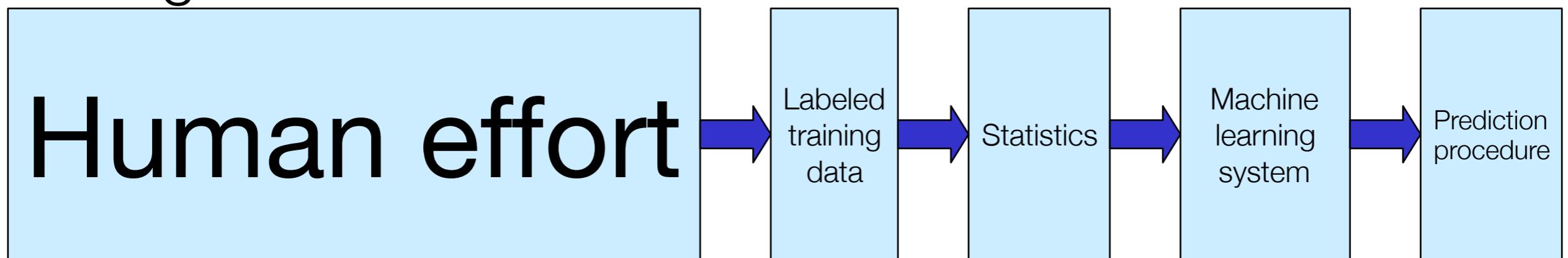
Training:



- Standard statistical systems use a **supervised** paradigm.

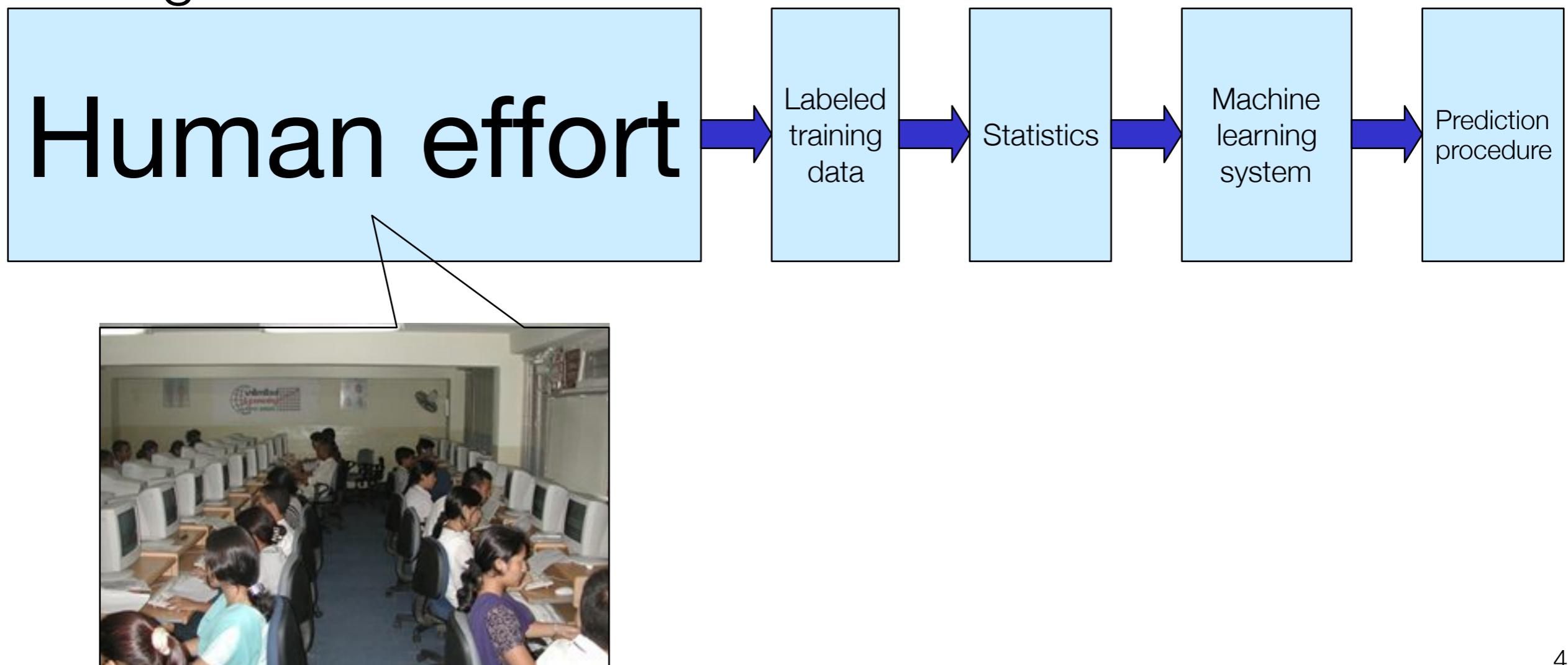
The real story: Annotating labeled data is labor-intensive!!!

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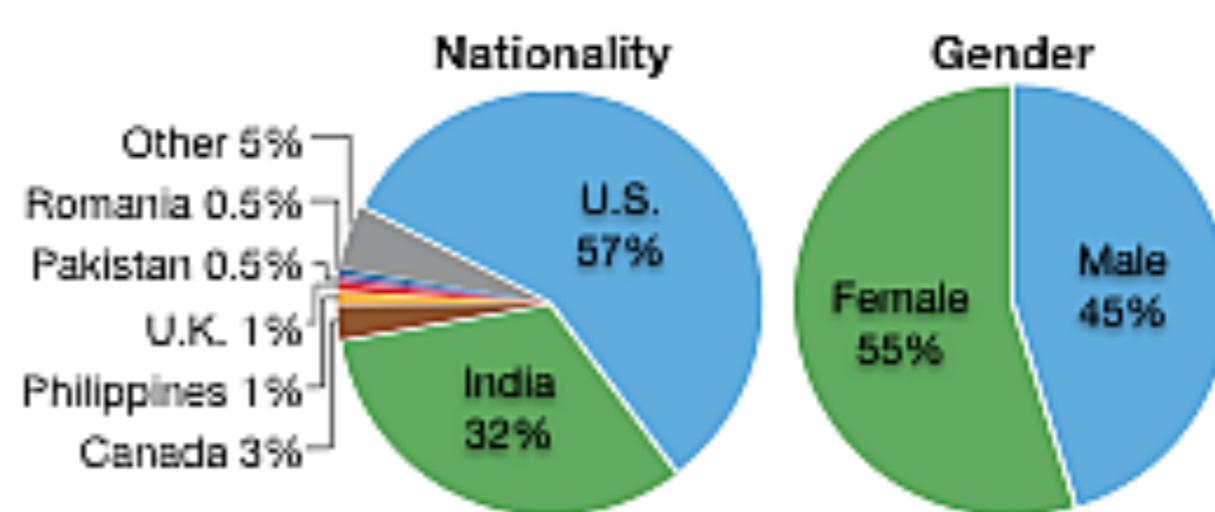
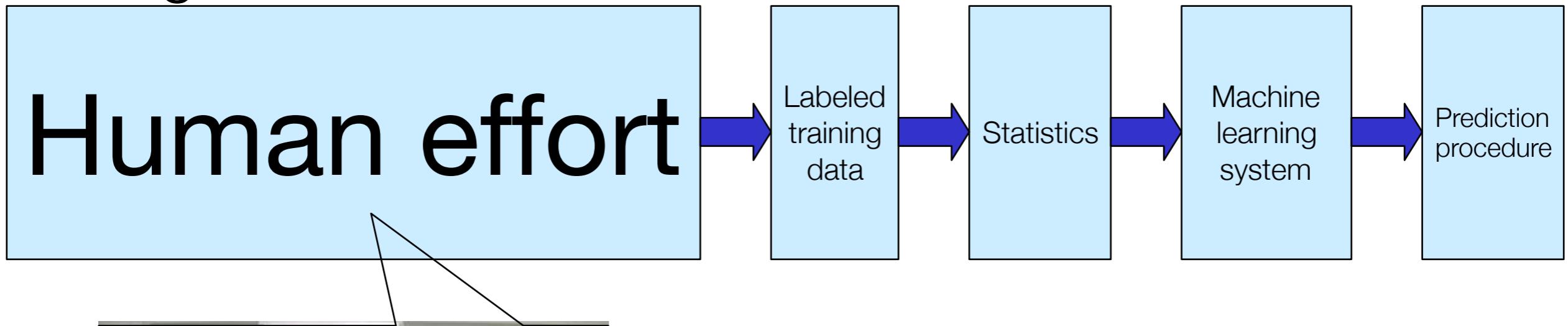
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Ross et al. (2011)

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- Reliance on training data also means that moving to a new language, domain, or even genre can be difficult.
- But unlabeled data is cheap!
- It would be nice to use the unlabeled data directly to learn the labelings you want in your model.
- Today we'll look at methods for doing this, which is called unsupervised learning

Motivating Example

Motivating Example



Donald J. Trump  @realDonaldTrump

The long anticipated release of the [#JFKFiles](#) will take place tomorrow. So interesting!

7:56 PM - Oct 25, 2017

19,252 replies 34,465 retweets 121,513 likes

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This is a screenshot of a tweet from Donald J. Trump's official Twitter account (@realDonaldTrump). The tweet, posted at 7:56 PM on October 25, 2017, discusses the release of the #JFKFiles. It has received 19,252 replies, 34,465 retweets, and 121,513 likes. The tweet includes a small profile picture of Donald Trump and a blue checkmark indicating verification. A 'Follow' button is visible in the top right corner of the tweet card.

- JFK Assassination Records - 2017 Additional Documents Release. July 24, 2017: 3,810 documents
 - October 26, 2017: 2,891 documents
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- 337,626 pages of documents!
How can we make sense of this quickly as a journalist?



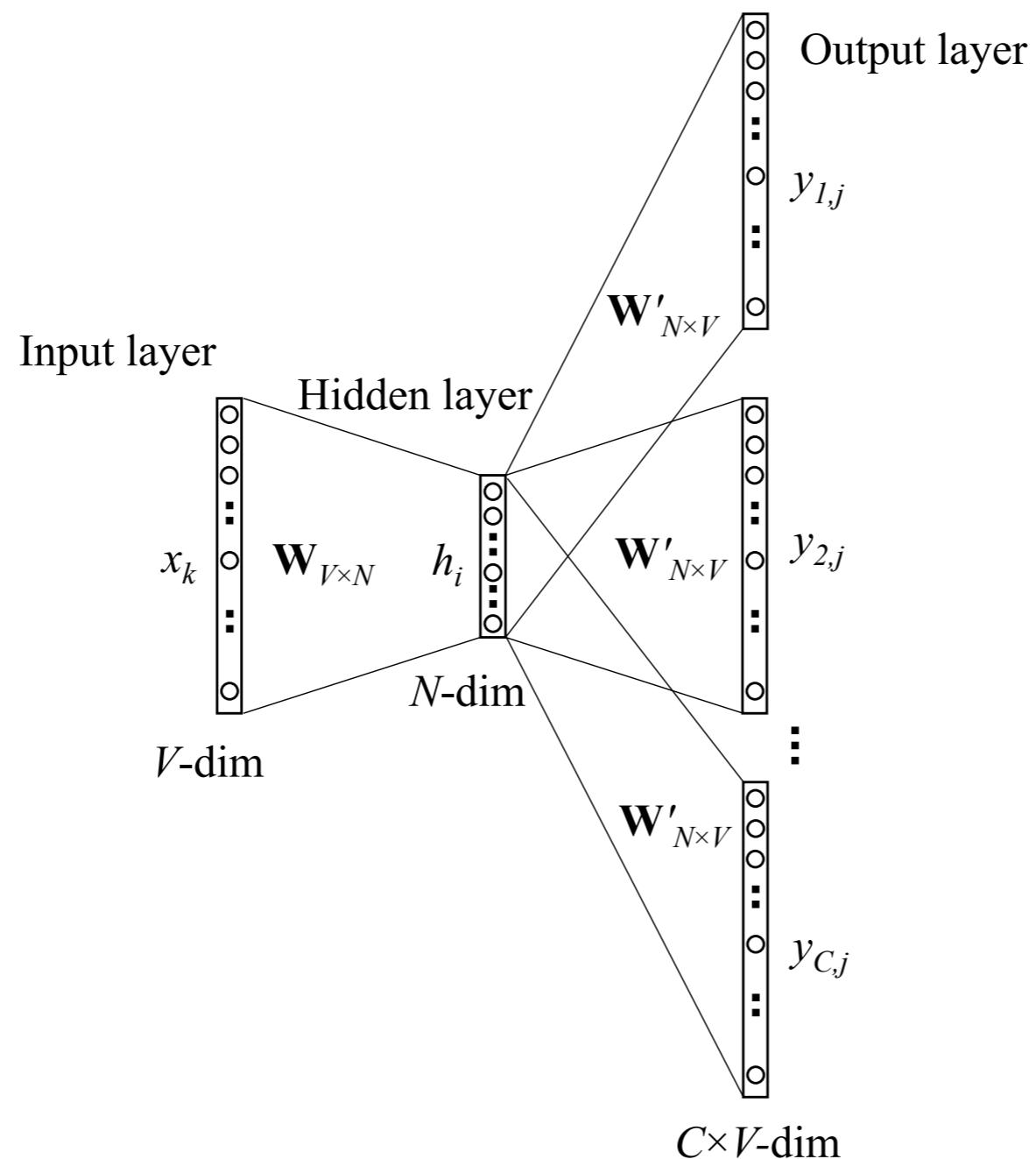
Today's plan

- Look at three kinds of unsupervised learning for NLP
- Start with words and move to documents



Brown Clustering

We know how to learn word meaning



What if we wanted to learn categories of words?

“You shall know a word by the company it keeps”

[Firth 1957]

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everyone likes	_____	
a bottle of	_____	is on the table
	_____	makes you drunk
a cocktail with	_____	and seltzer

Brown clustering combines the Distributional Hypothesis with Language Modeling!

- An **agglomerative clustering algorithm** that clusters words based on which words precede or follow them
- These word clusters can be turned into a kind of vector
- We'll give a very brief sketch here.

Brown Clustering

- Similar to a language model but the **basic unit is a word cluster**
- Intuition is that similar words appear in similar contexts
- Recap: Bigram Language Model:

$$\prod_i^n P(w_i \mid w_{i-1}) \times P(STOP \mid w_n)$$

Maximum likelihood estimate $\frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$

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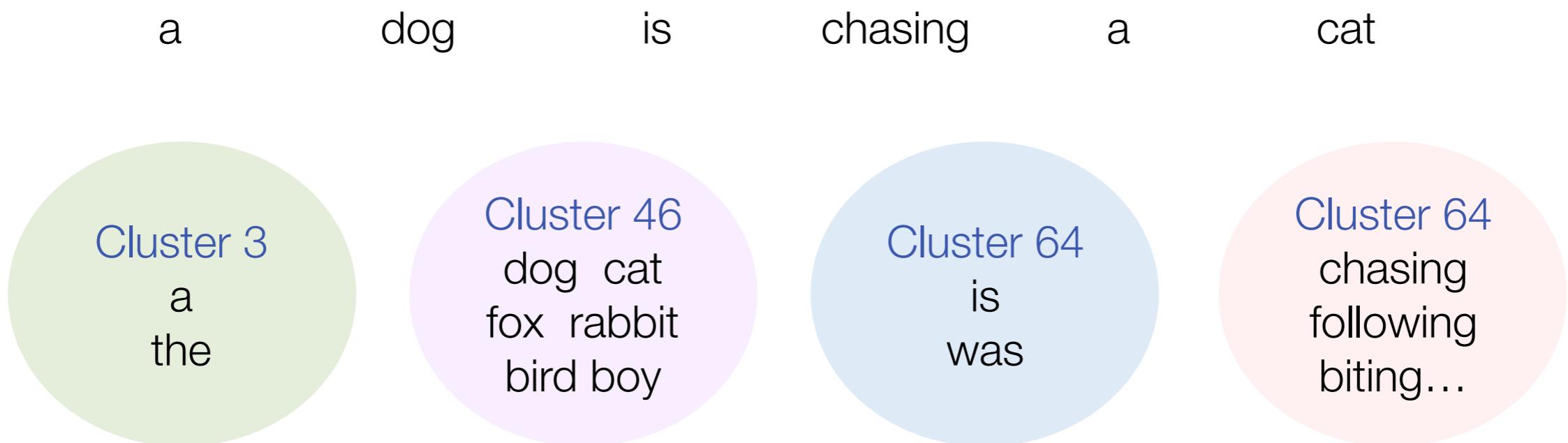
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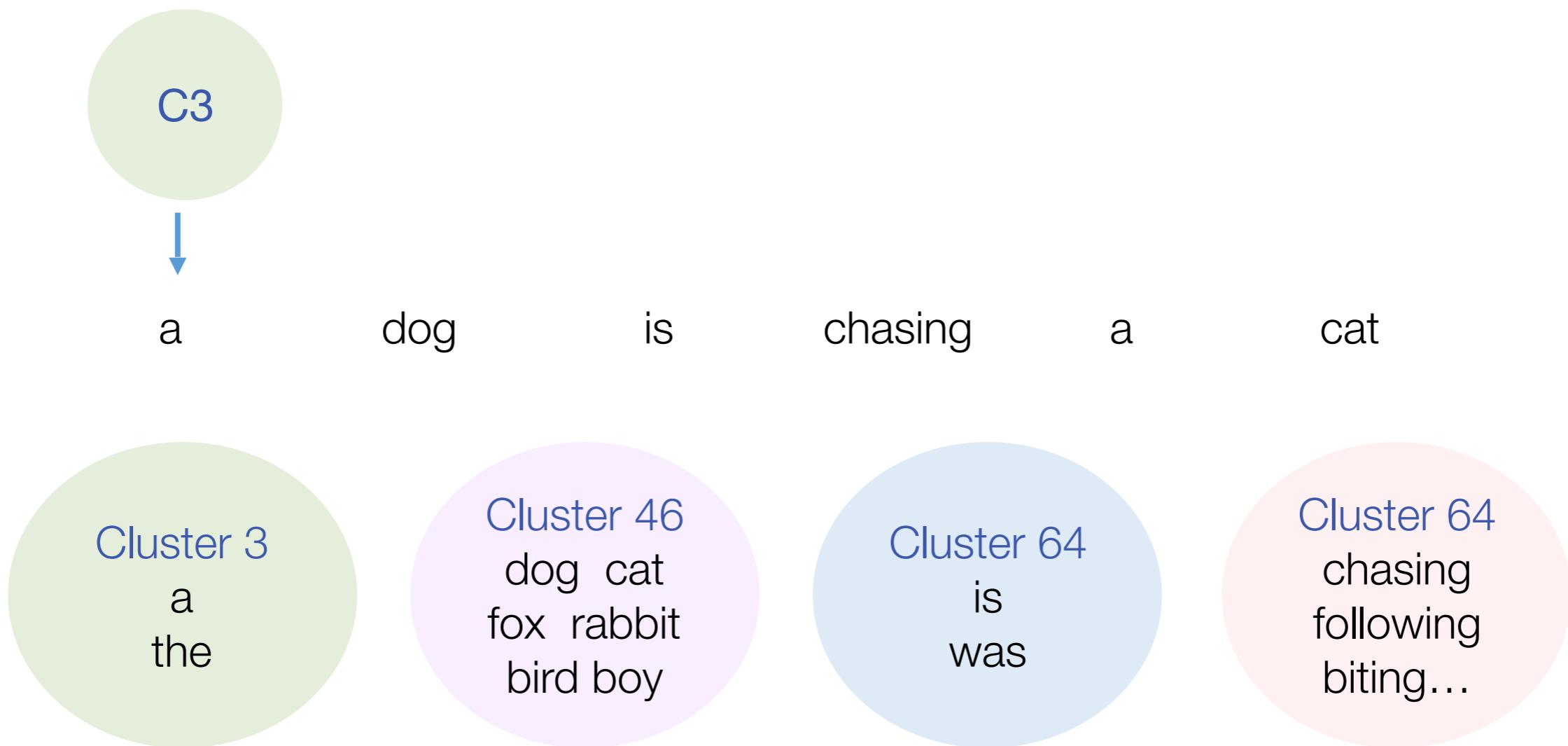
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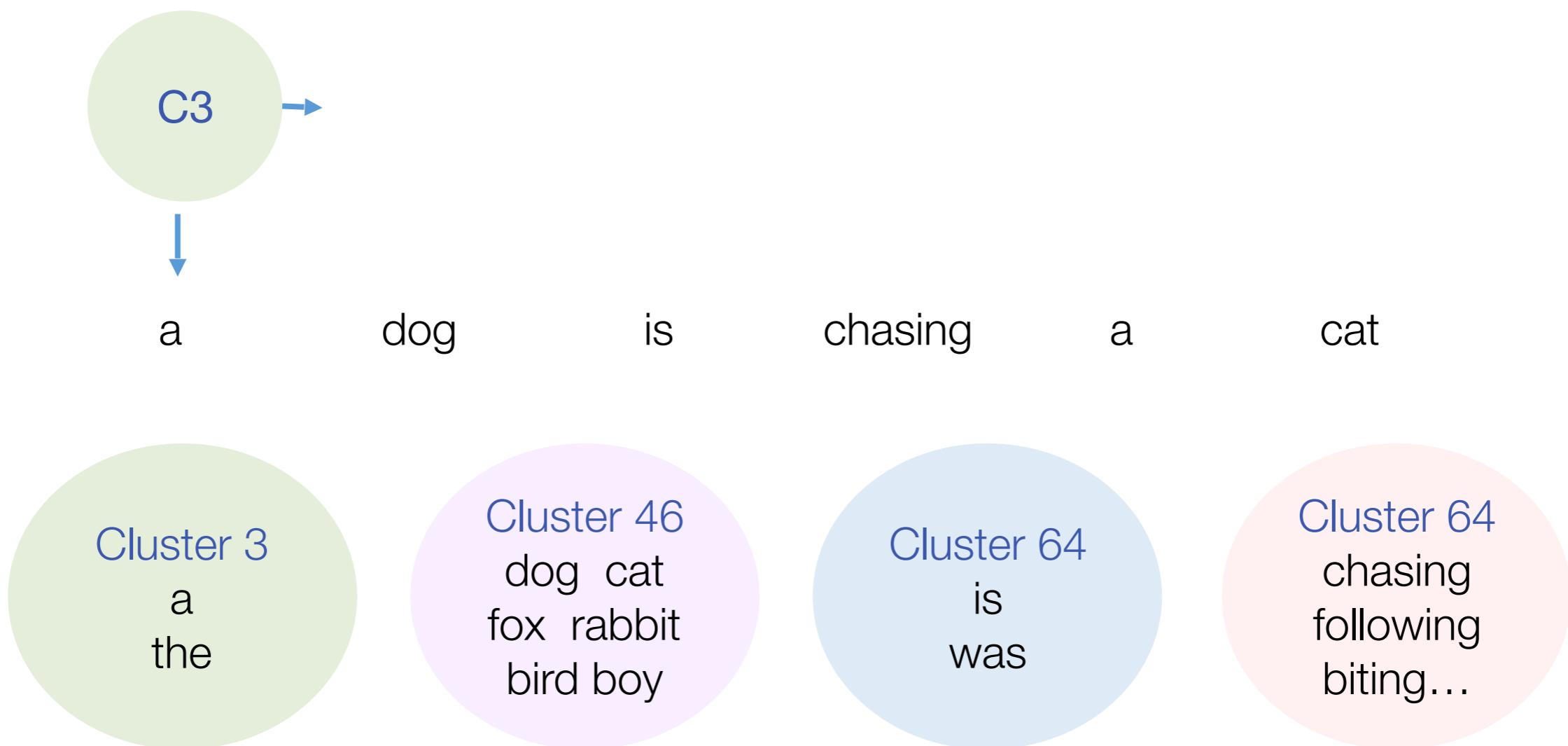
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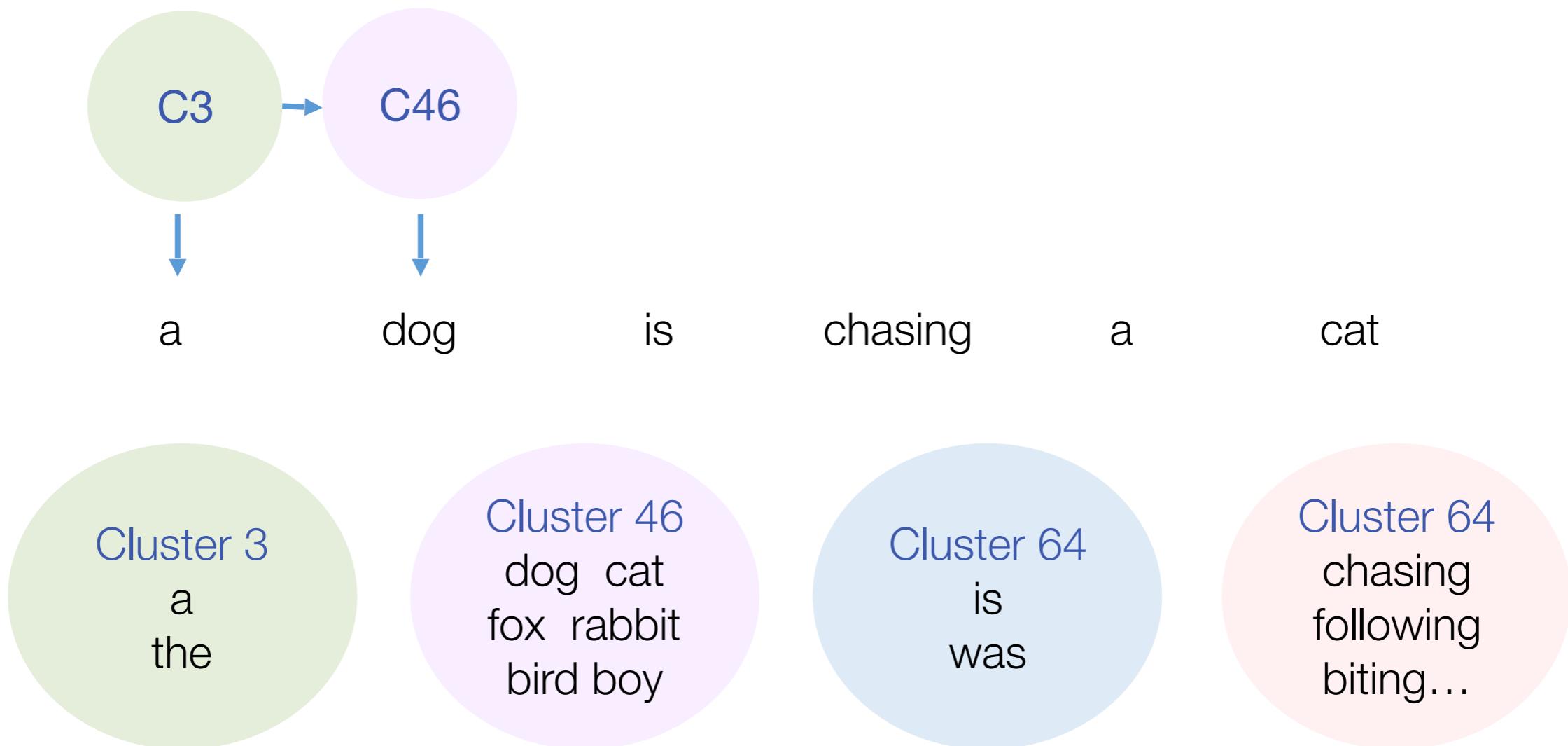
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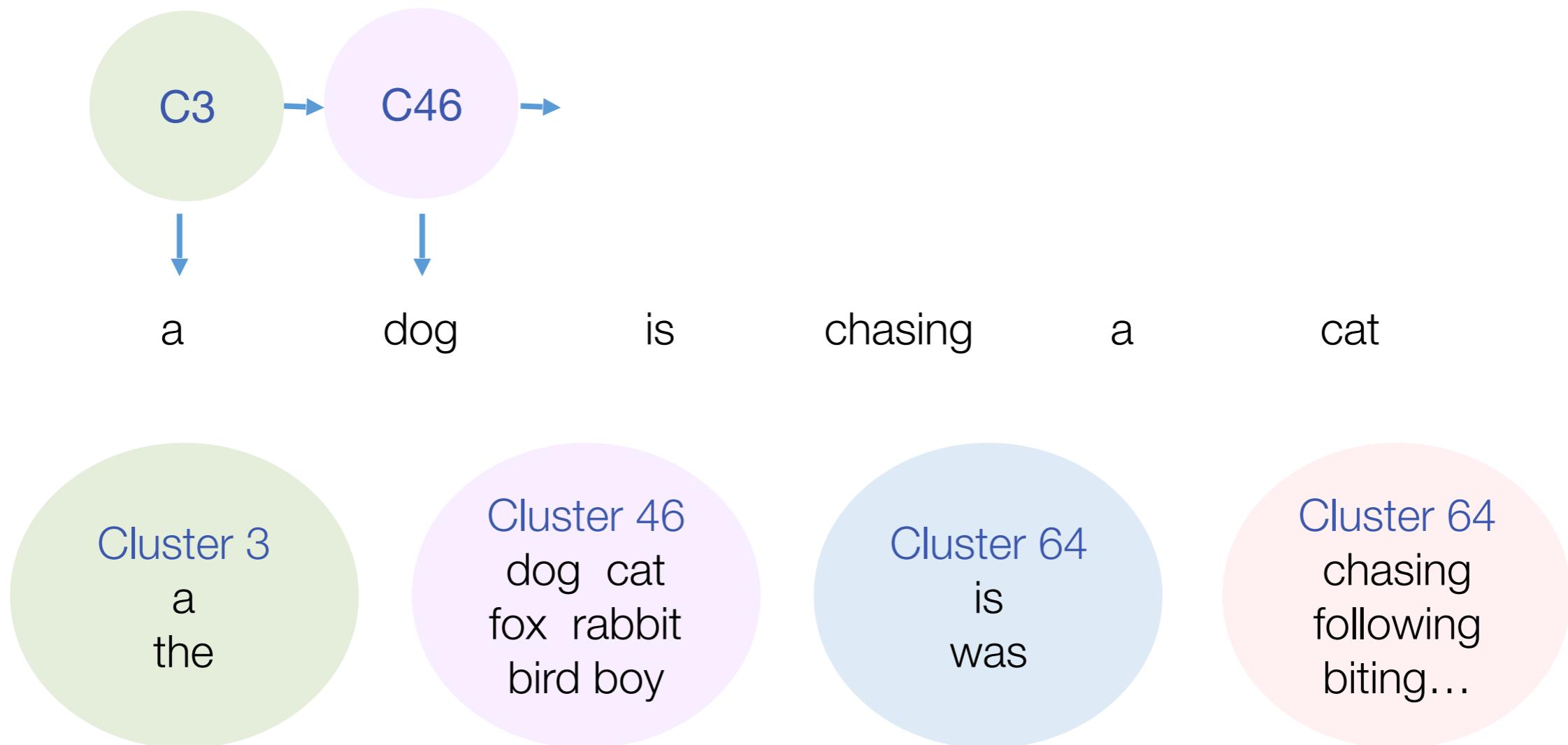
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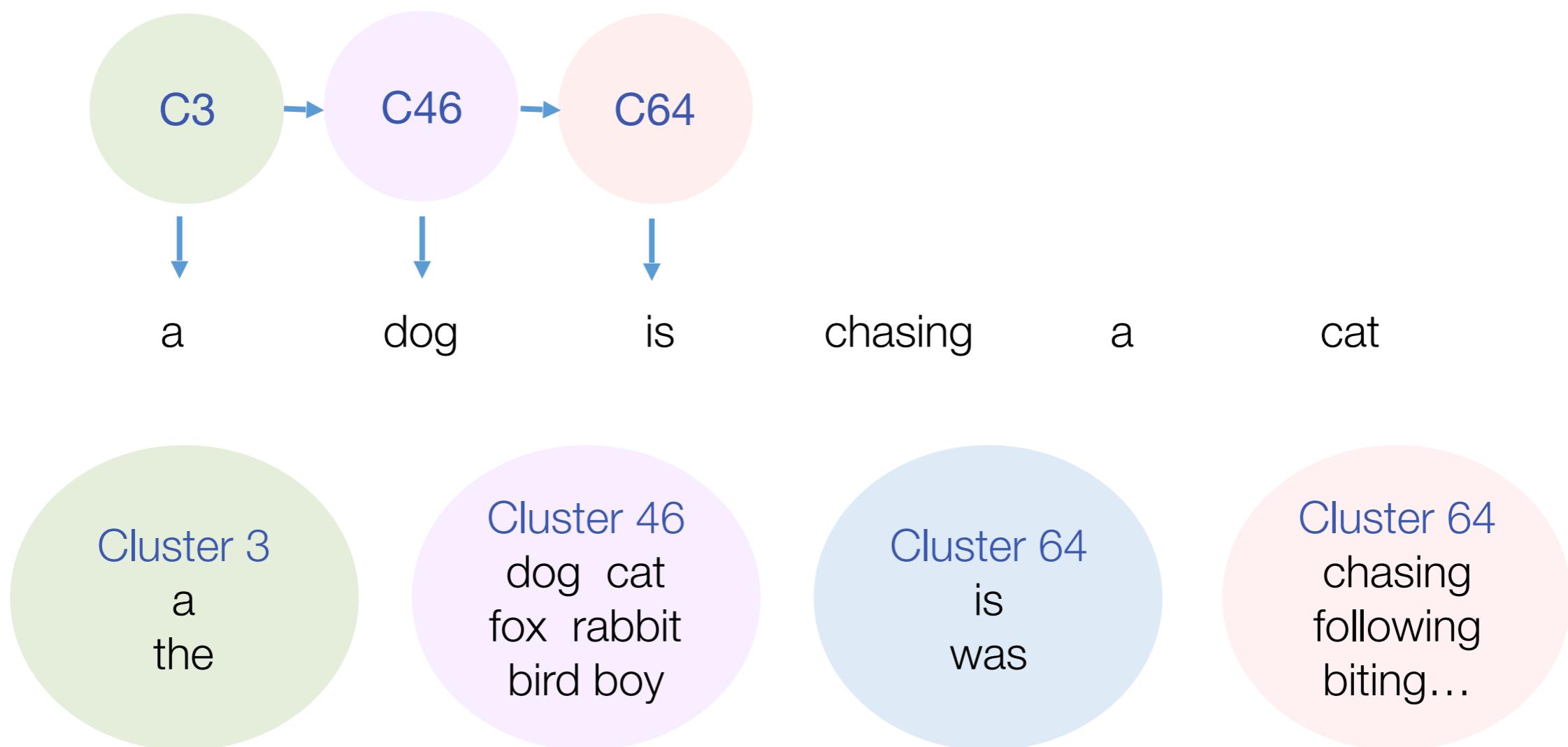
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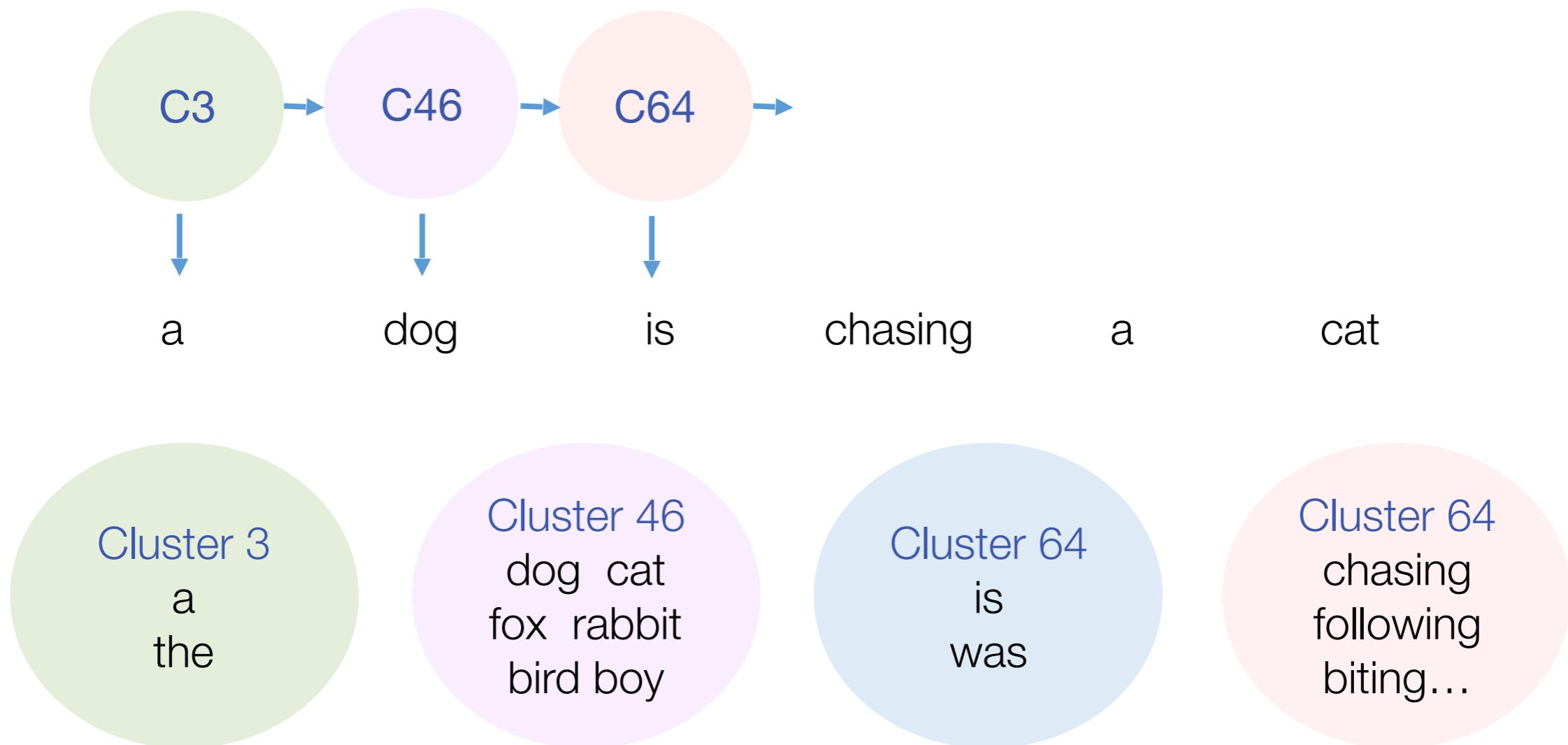
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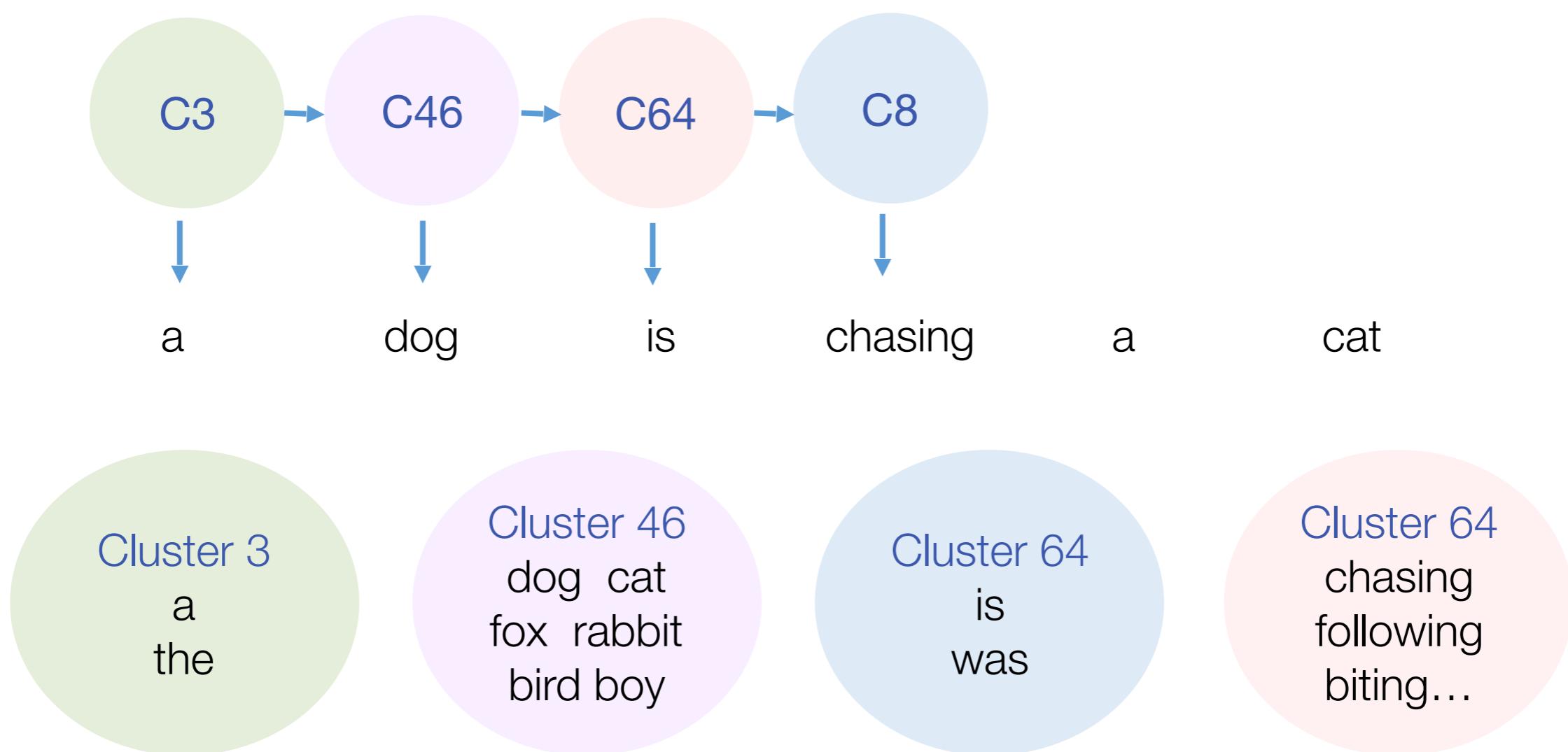
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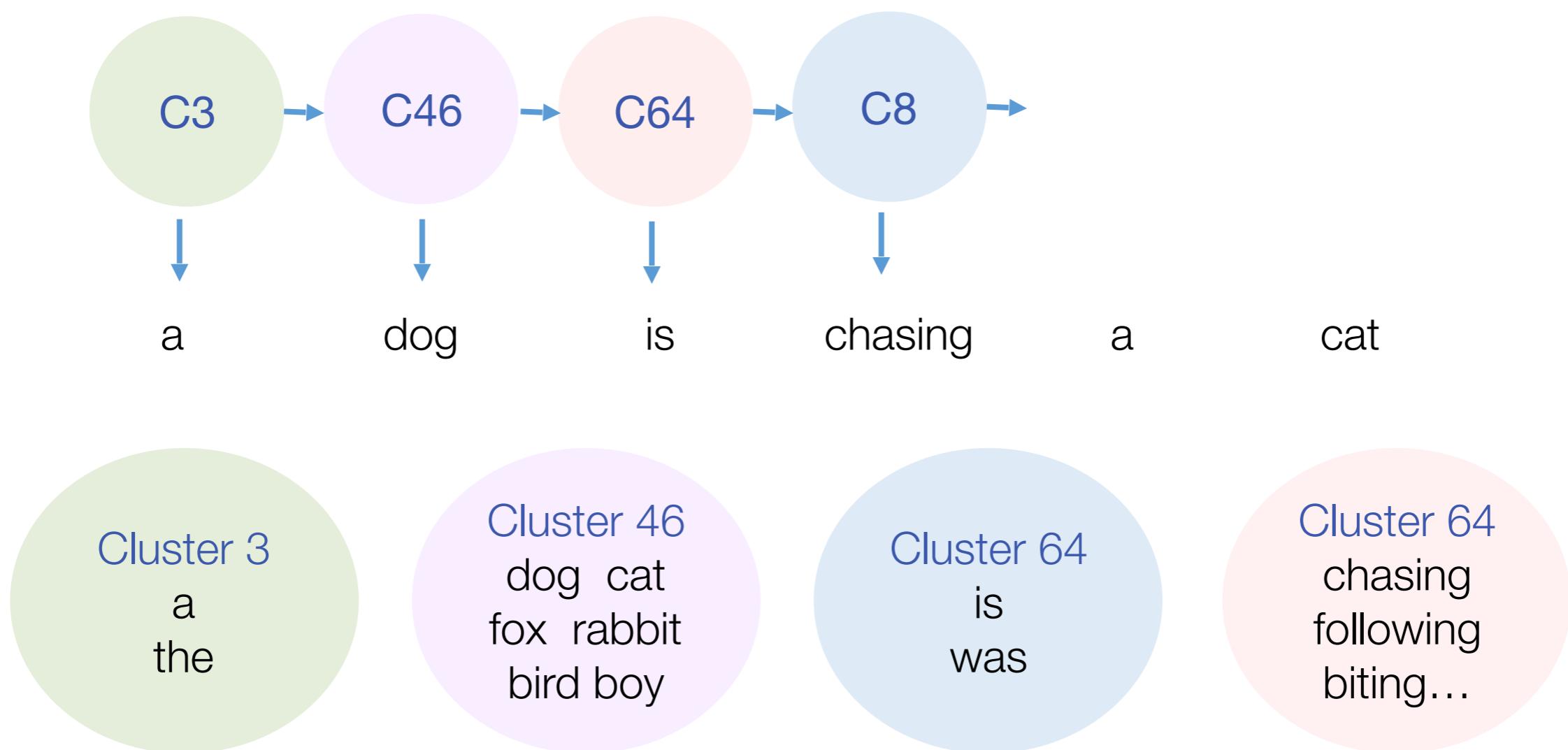
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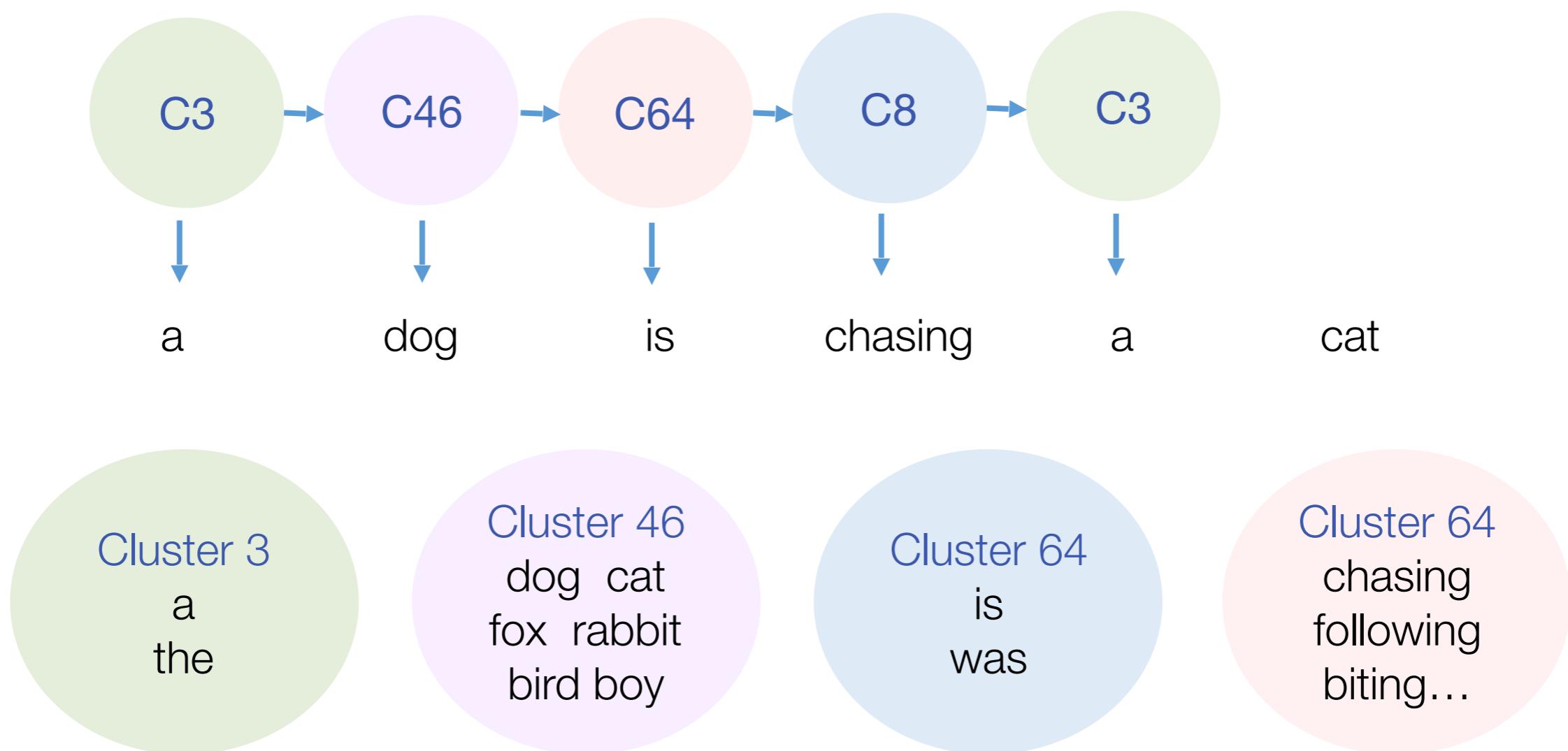
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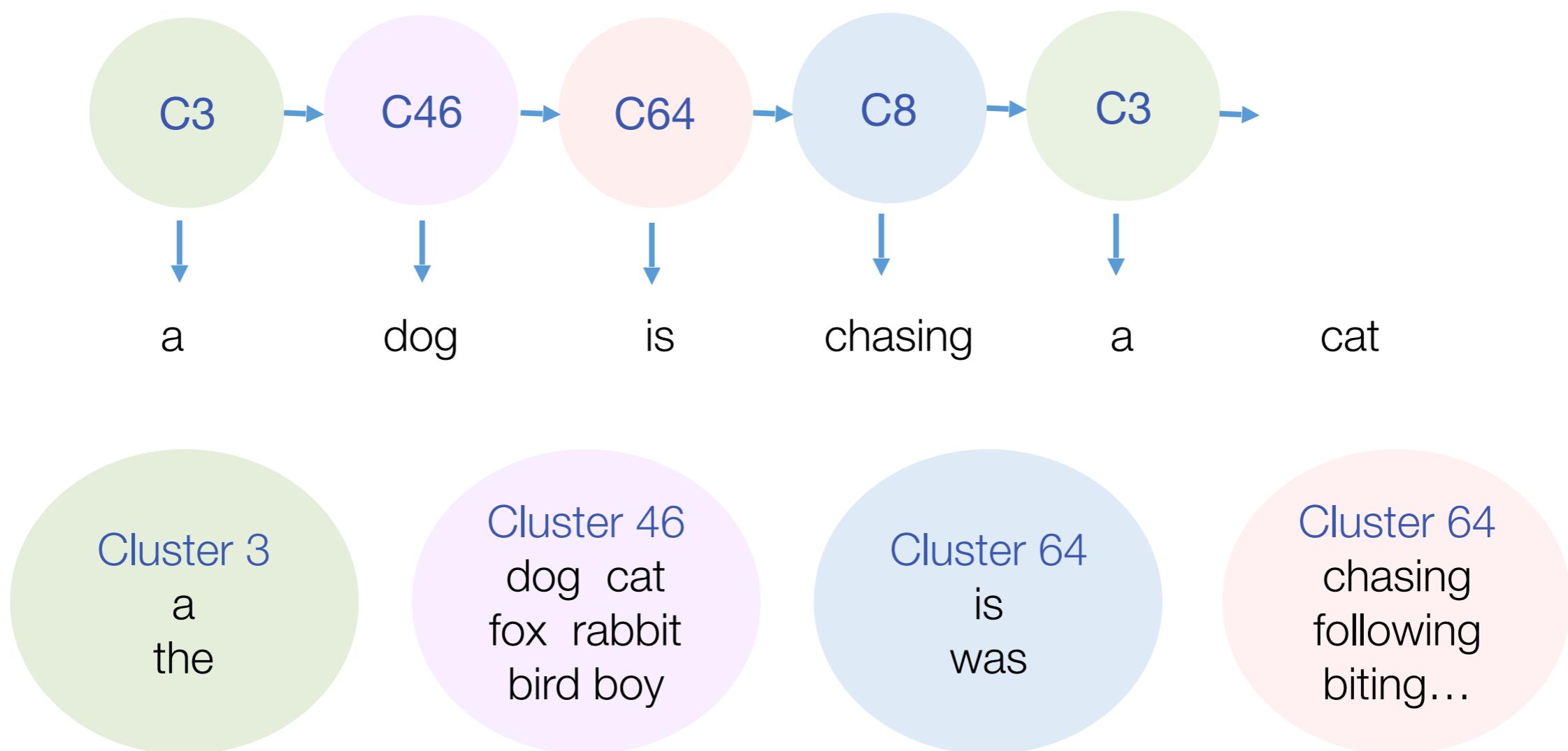
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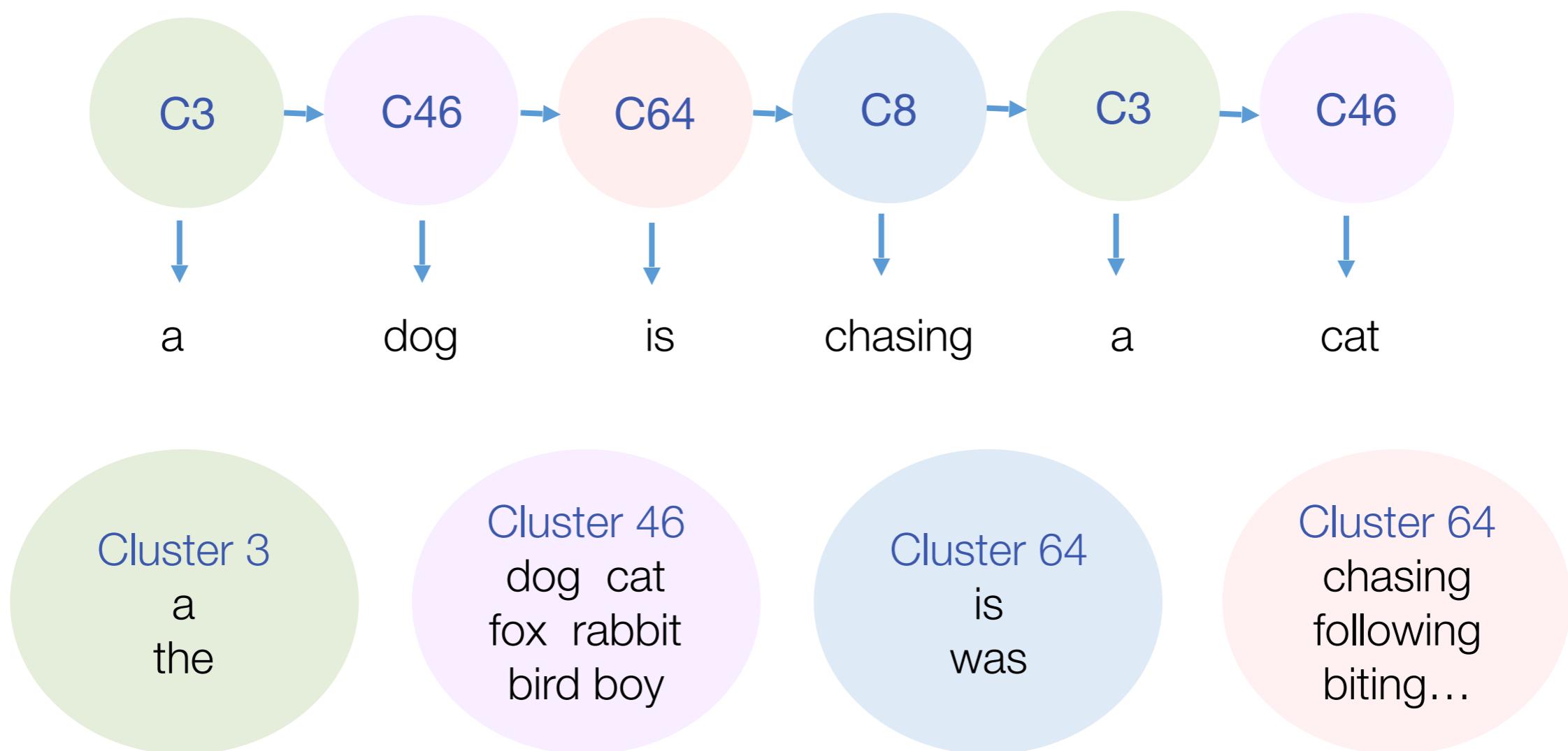
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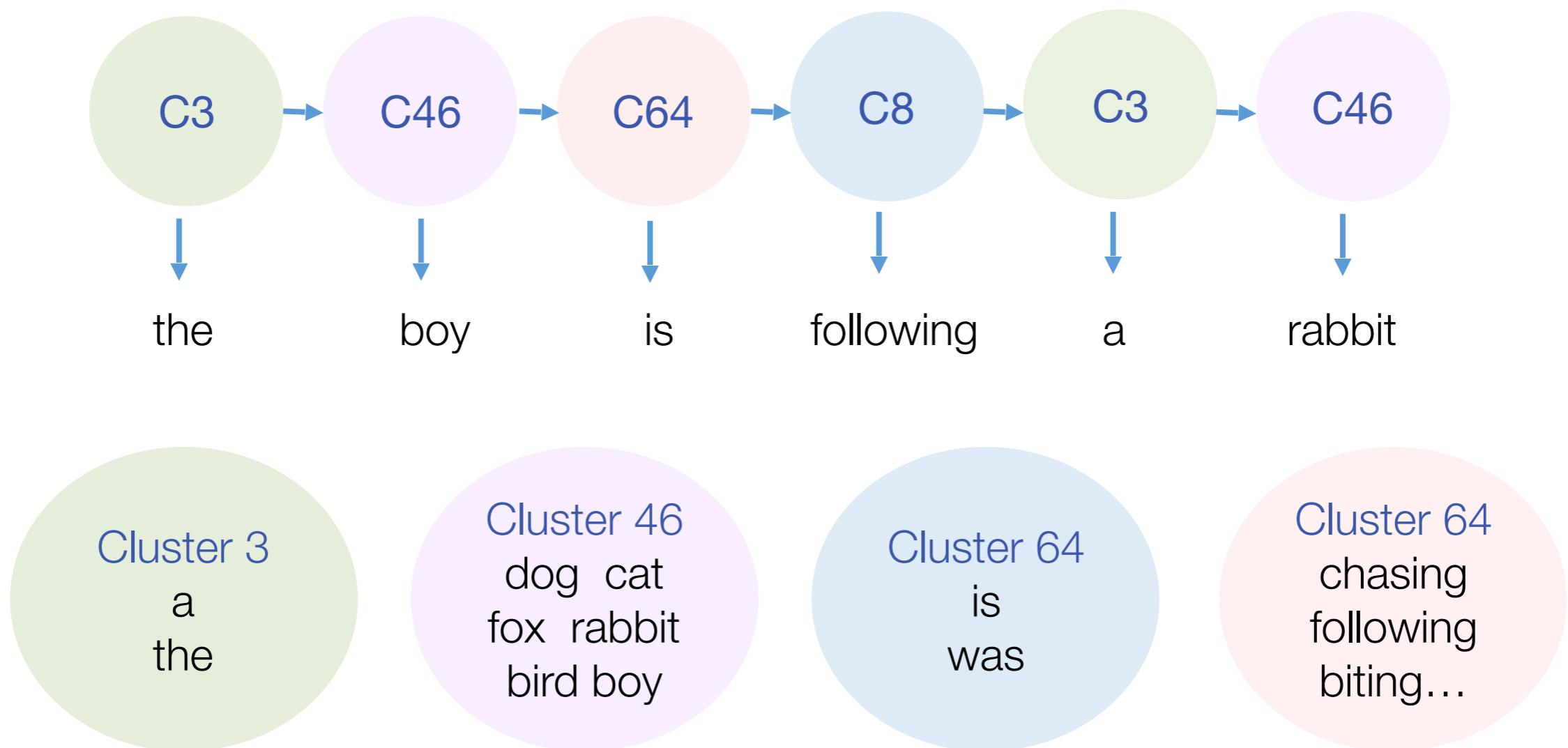
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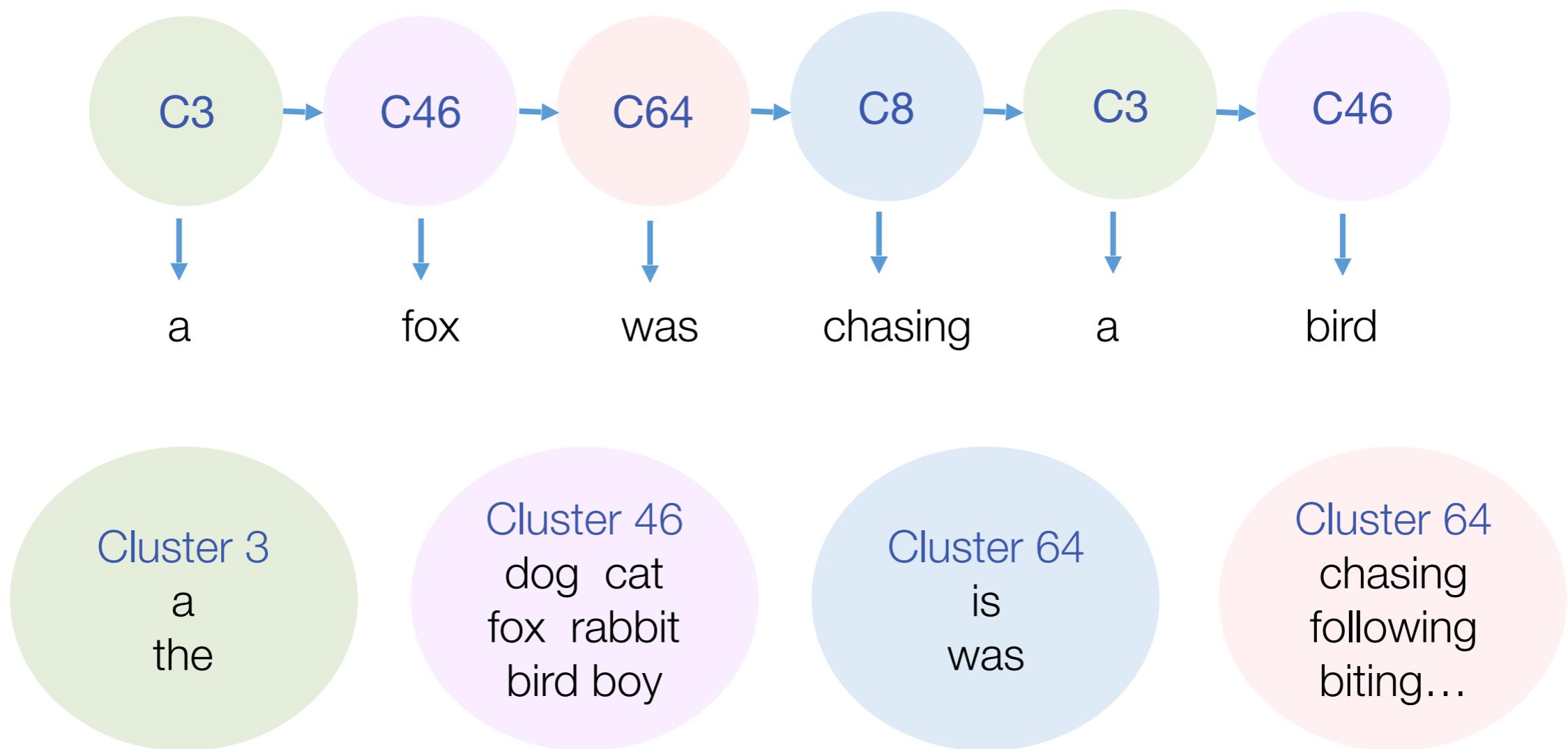
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- Assume every word belongs to a cluster
 - “the boy is following a rabbit”



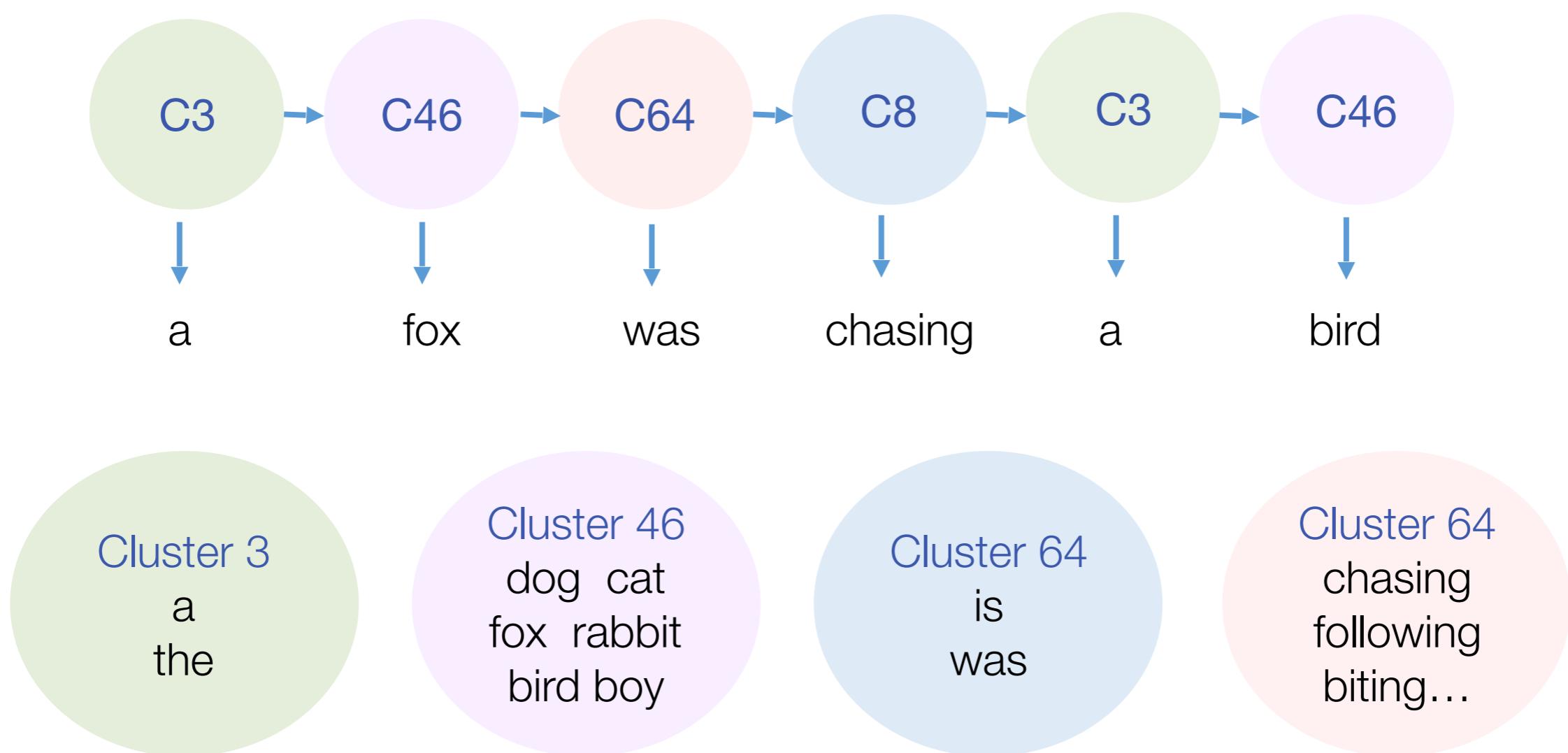
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- Assume every word belongs to a cluster
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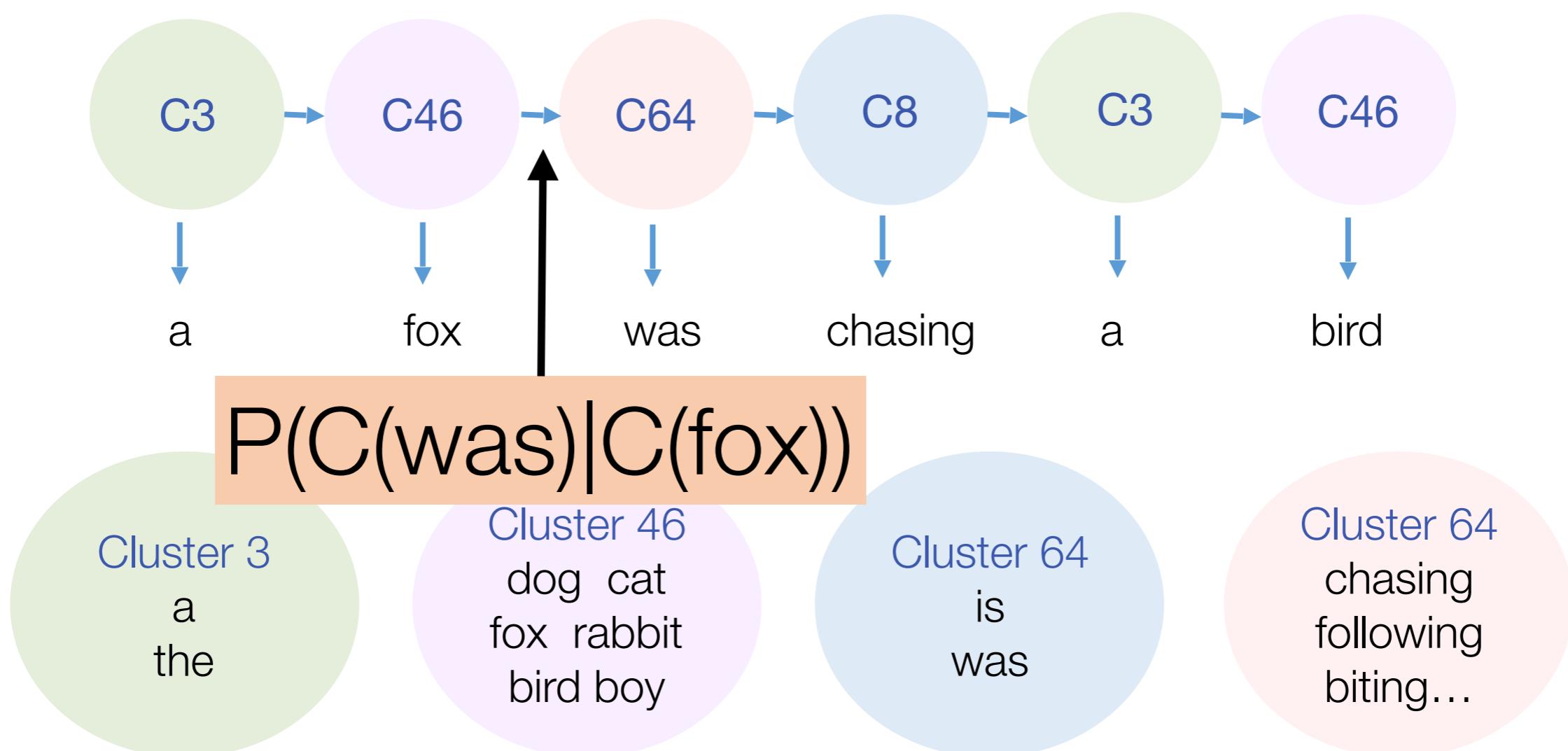
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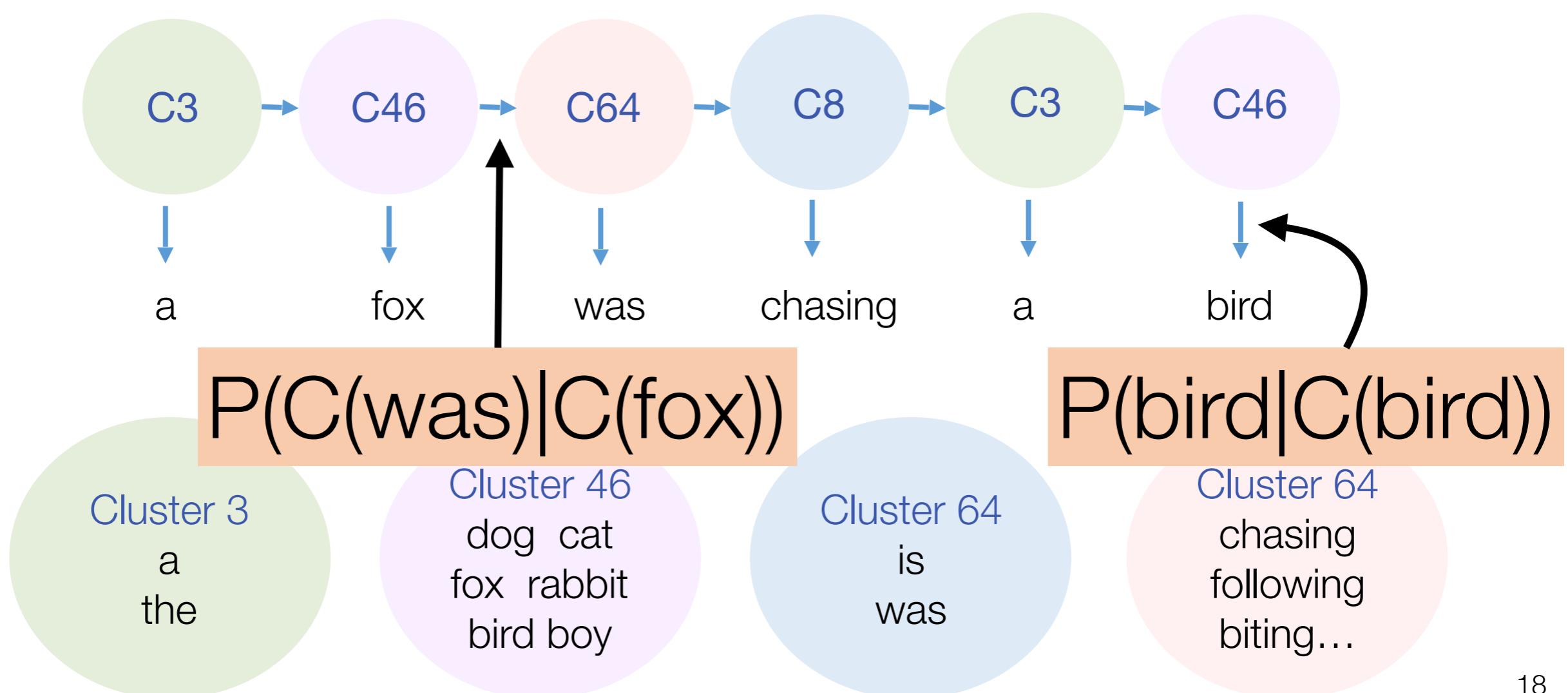
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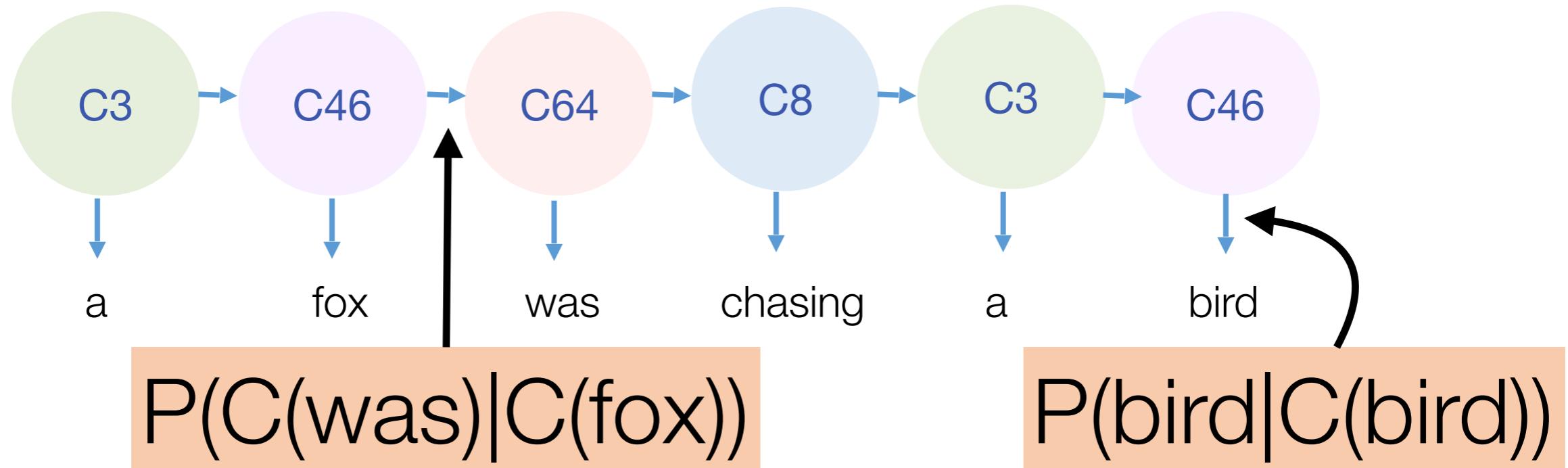
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How do we measure the probability of this sequence?

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 - Before: $P(\text{“a”}|\text{START}) \dots P(\text{“cat”}|\text{“a”})$
 - Now= $P(C_3|\text{START}) P(\text{“a”}|C_3) P(C_{46}|C_3) \dots P(\text{“bird”}|C_{46})$



Class-based language model

- Suppose each word was in some class c_i :

$$P(w_i | w_{i-1}) = P(c_i | c_{i-1})P(w_i | c_i)$$

$$P(\text{corpus} | C) = \prod_{i=1}^n P(c_i | c_{i-1})P(w_i | c_i)$$

Where do the classes come from?

Brown clustering algorithm

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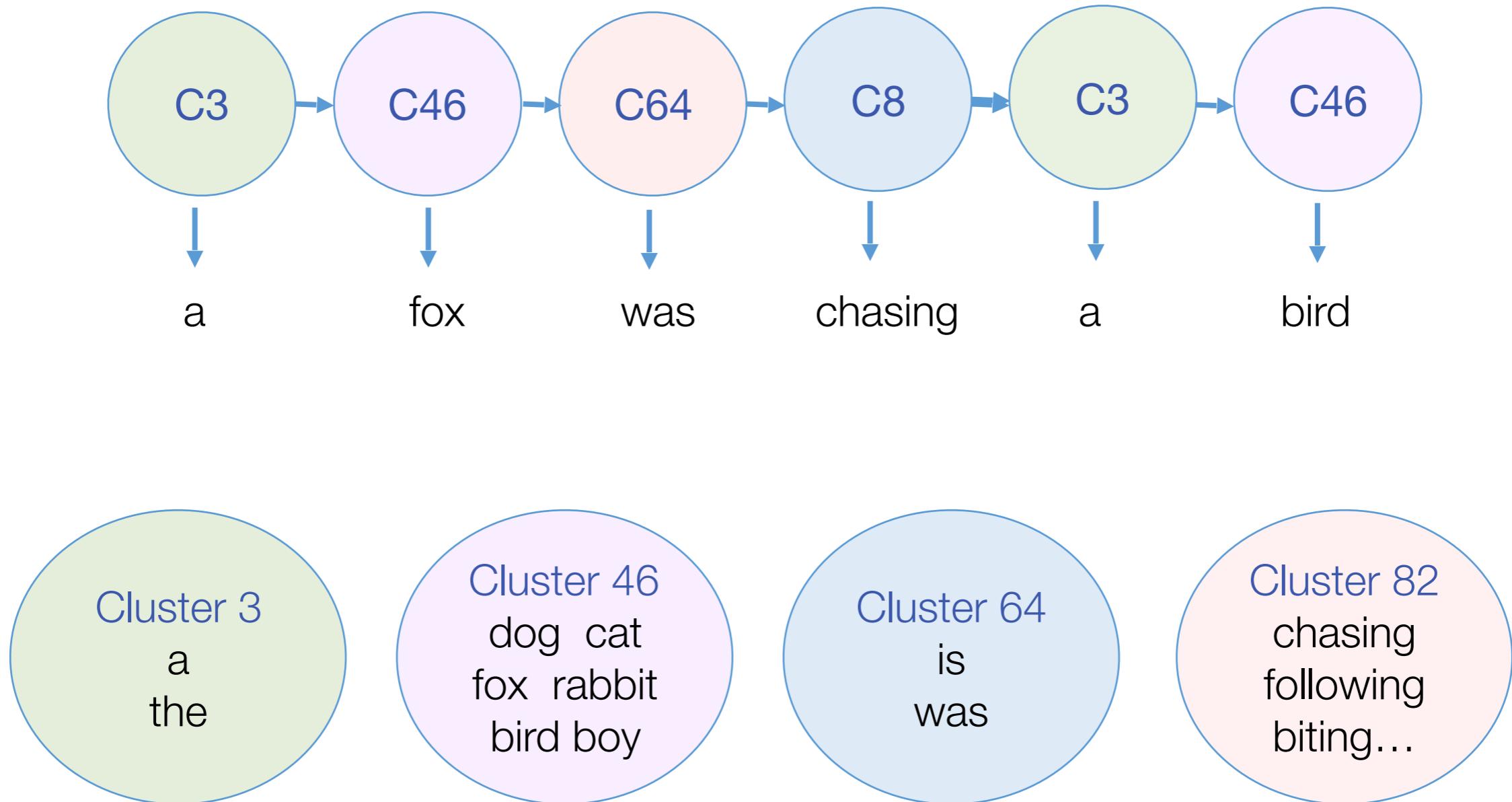
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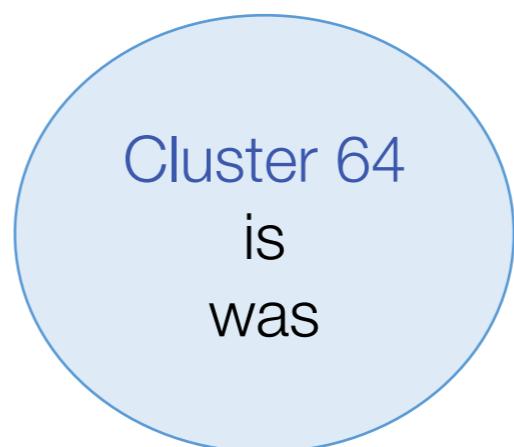
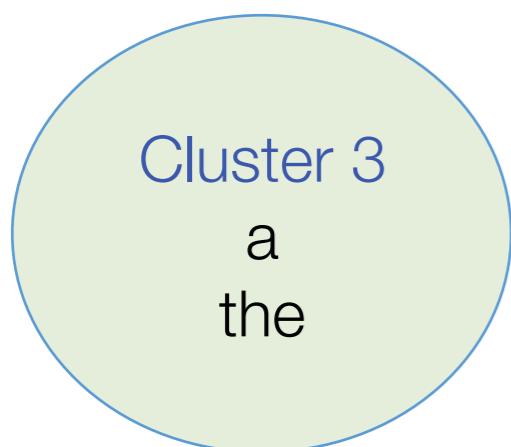
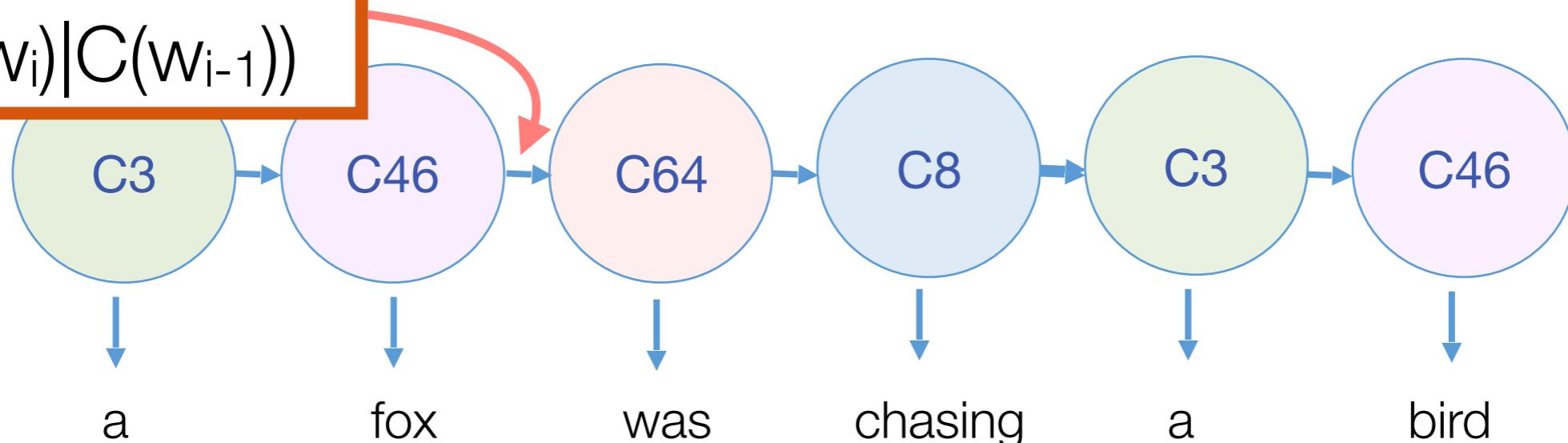
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- Clustering proceeds until all words are in one big cluster.

Model Parameters



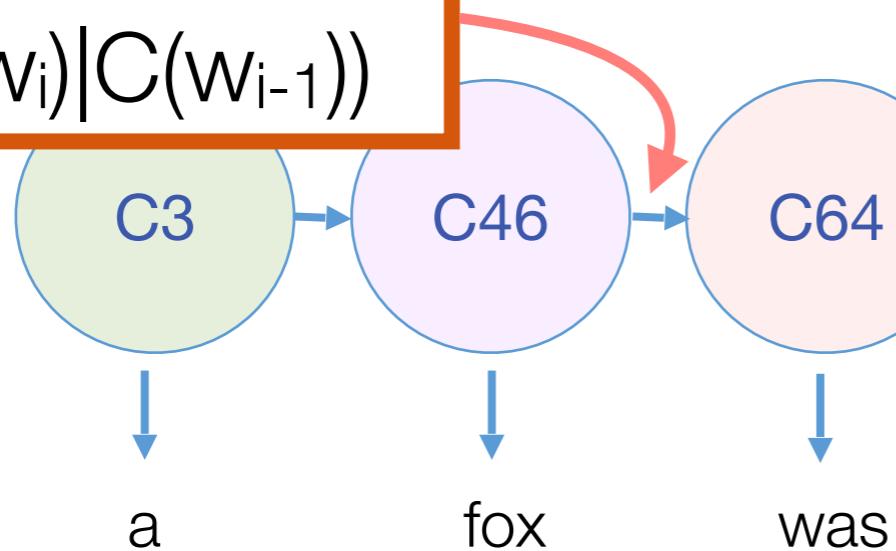
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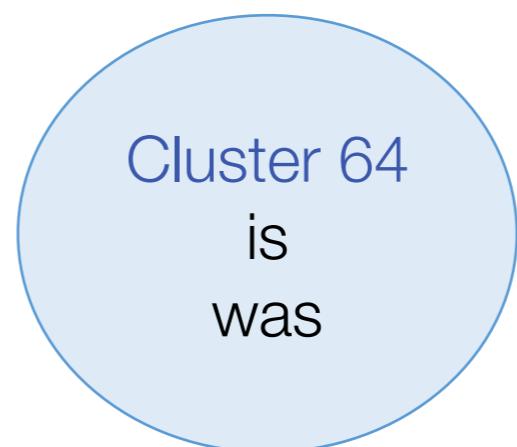
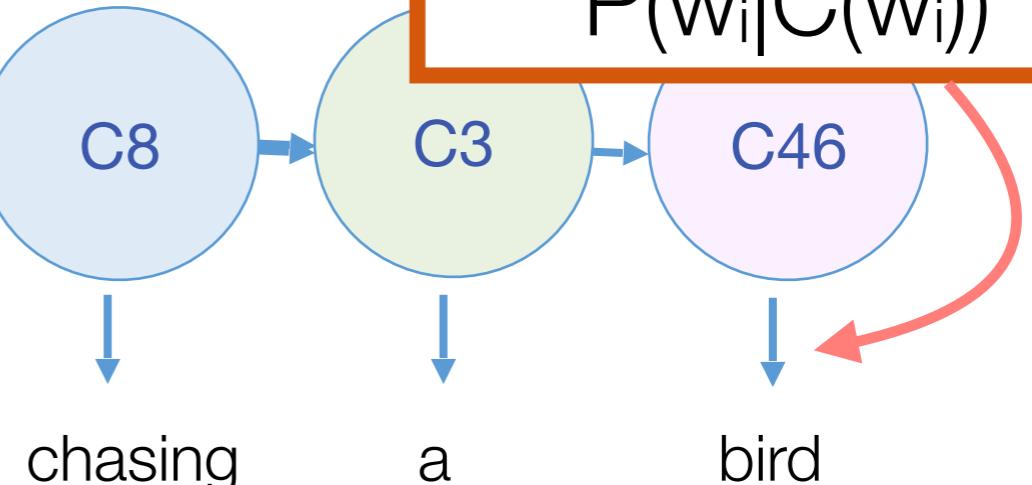


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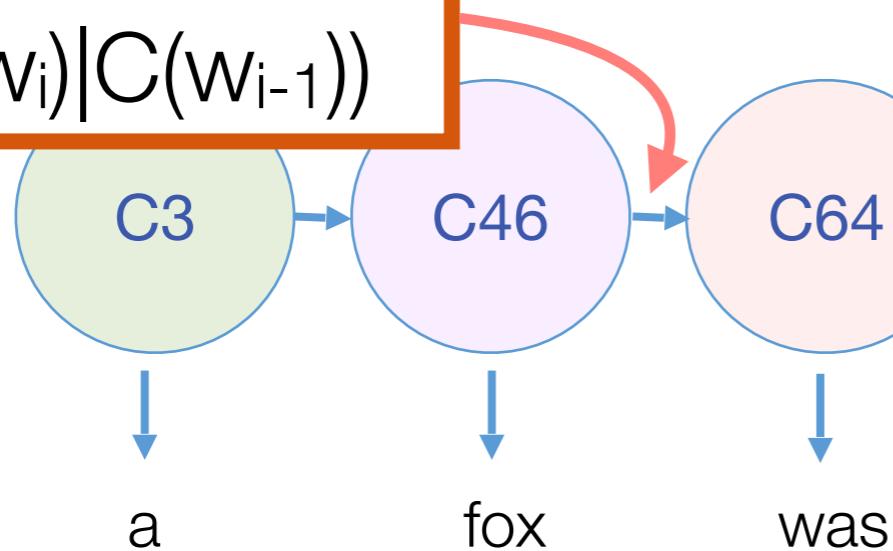


Parameter Set 2:
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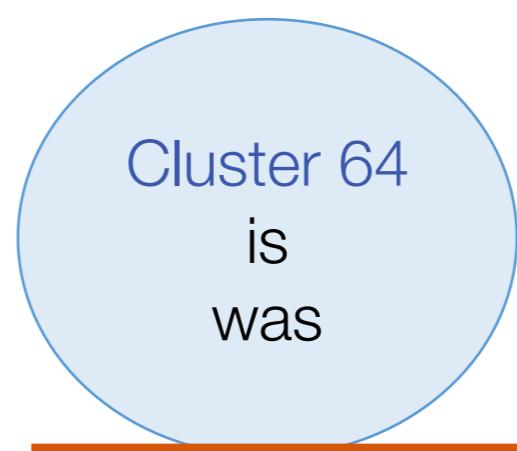
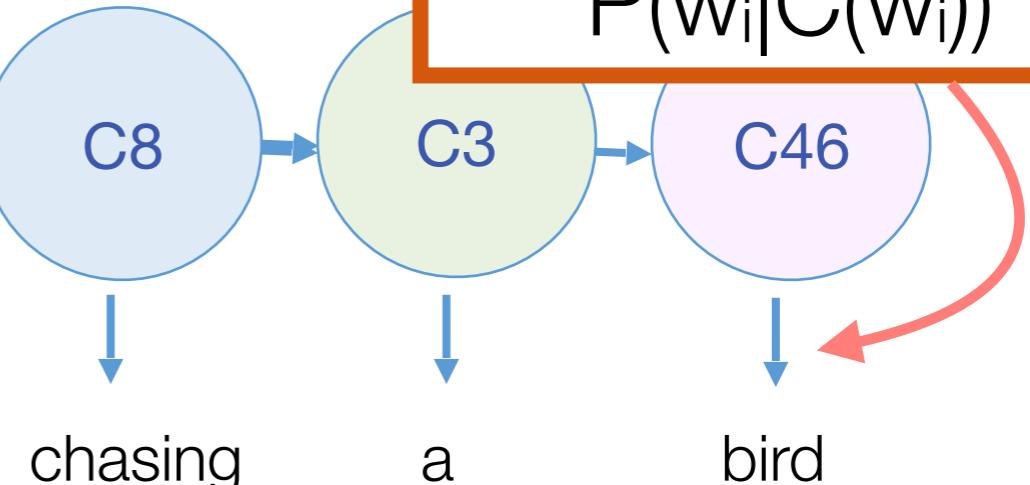


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Parameter Set 2:
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Parameter Set 3:
 $C(w_i)$

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- Maximizing $\text{LL}(\theta, C)$ can be done by alternatively updating θ and C
- Maximizing $\text{LL}(\theta, C)$ is “how much do the current classes and transition probabilities explain the data I see”

Maximizing θ for $LL(\theta, C)$

- Count things!
 - $P(C_i|C_{i-1}) = (\#(C_{i-1}, C_i)) / (\#C_i)$
 - $P(w|C) = (\#(w, C)) / (\#C)$

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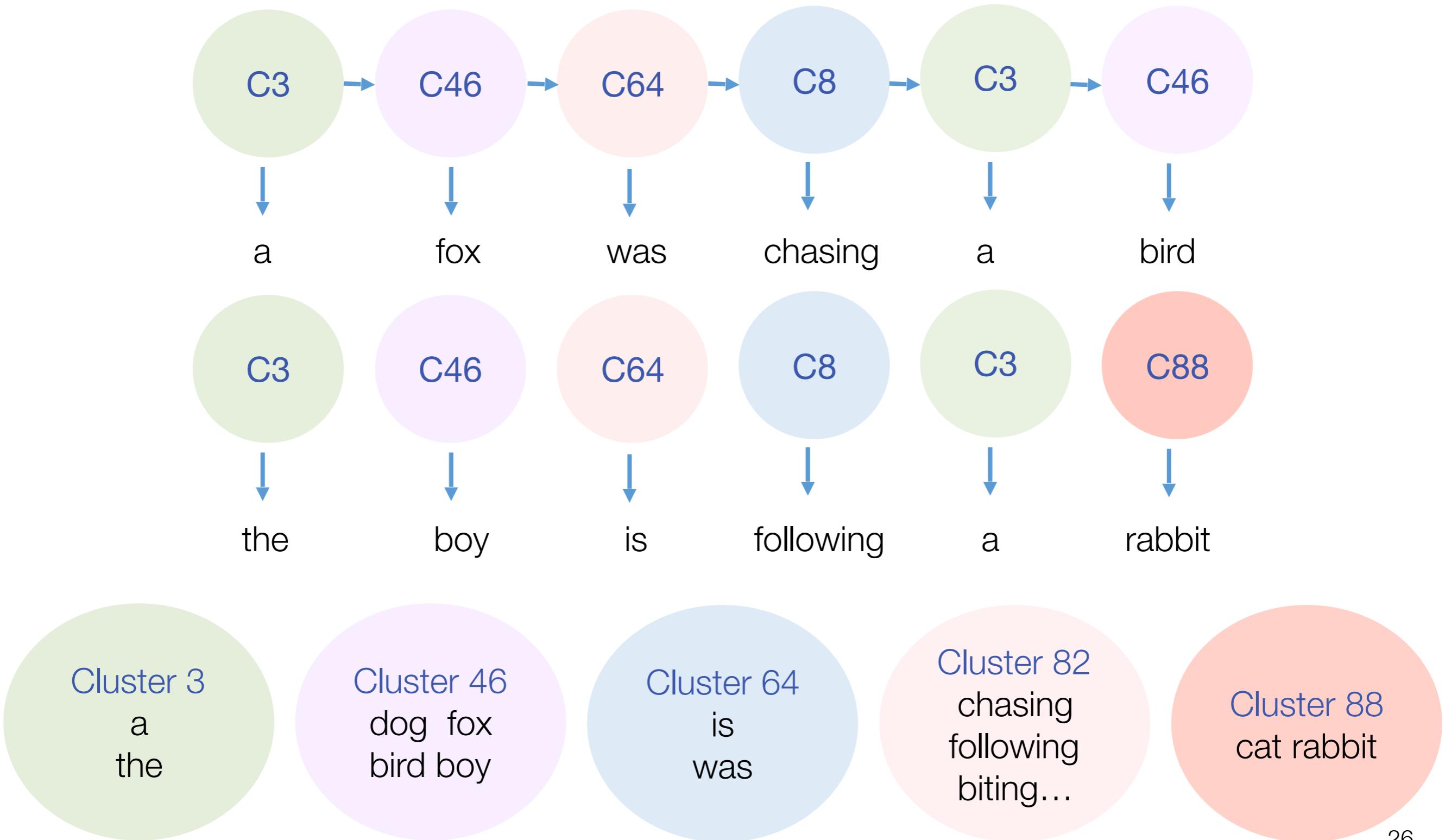
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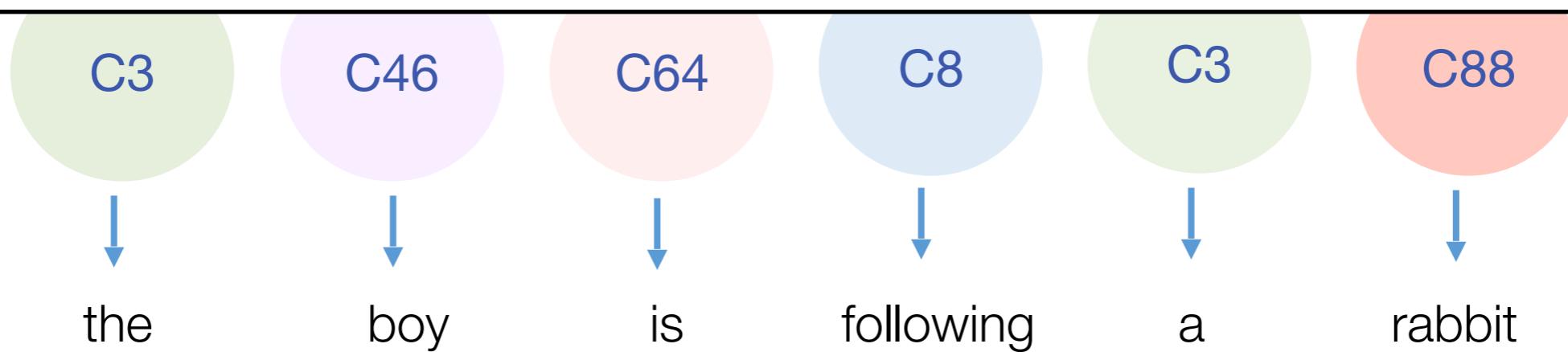
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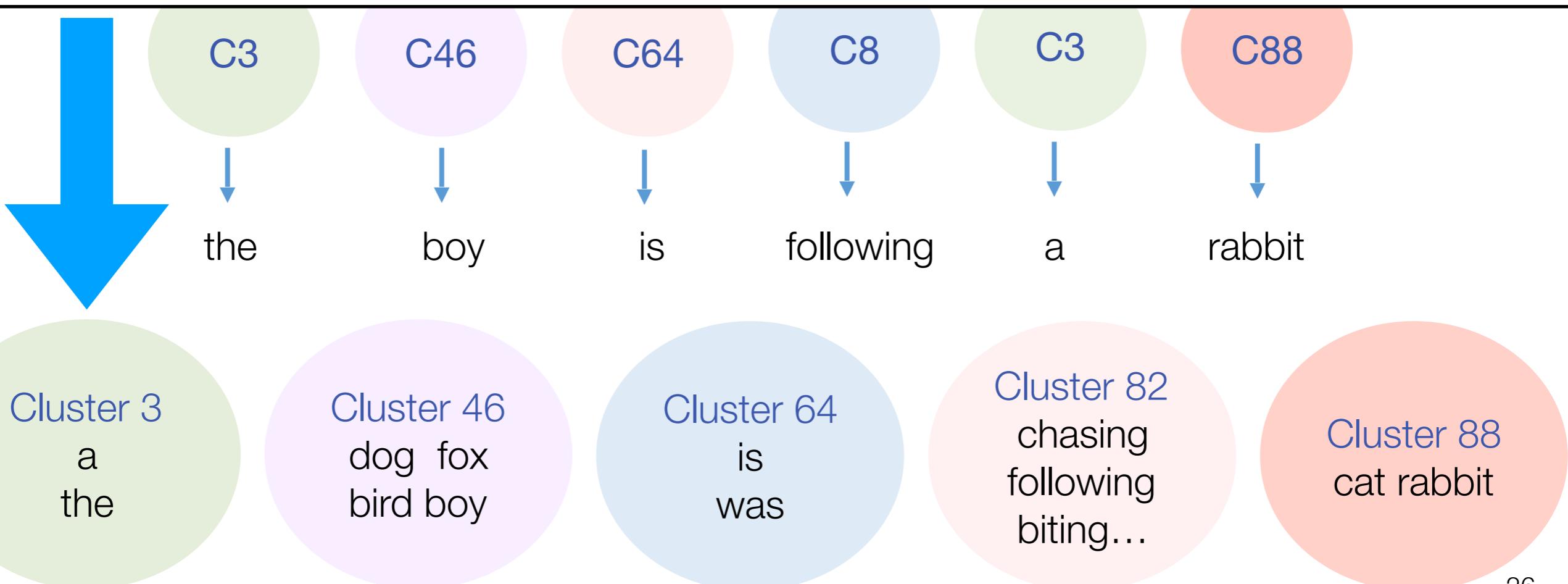
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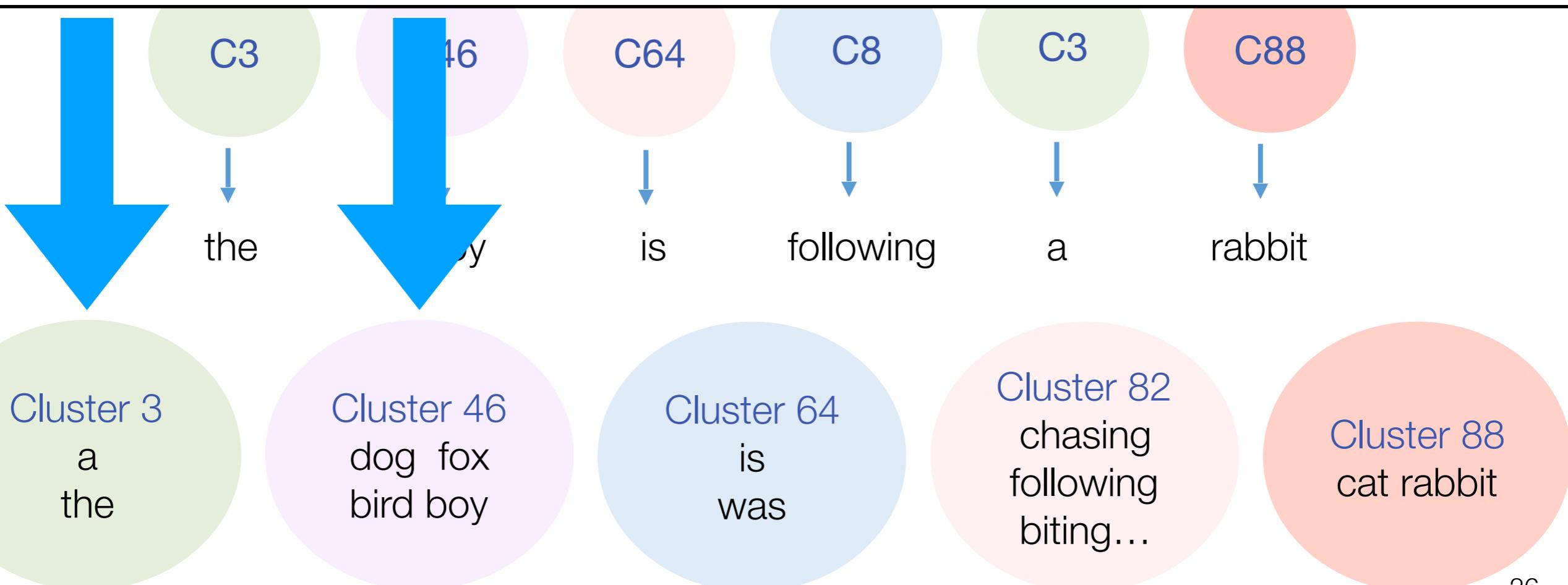
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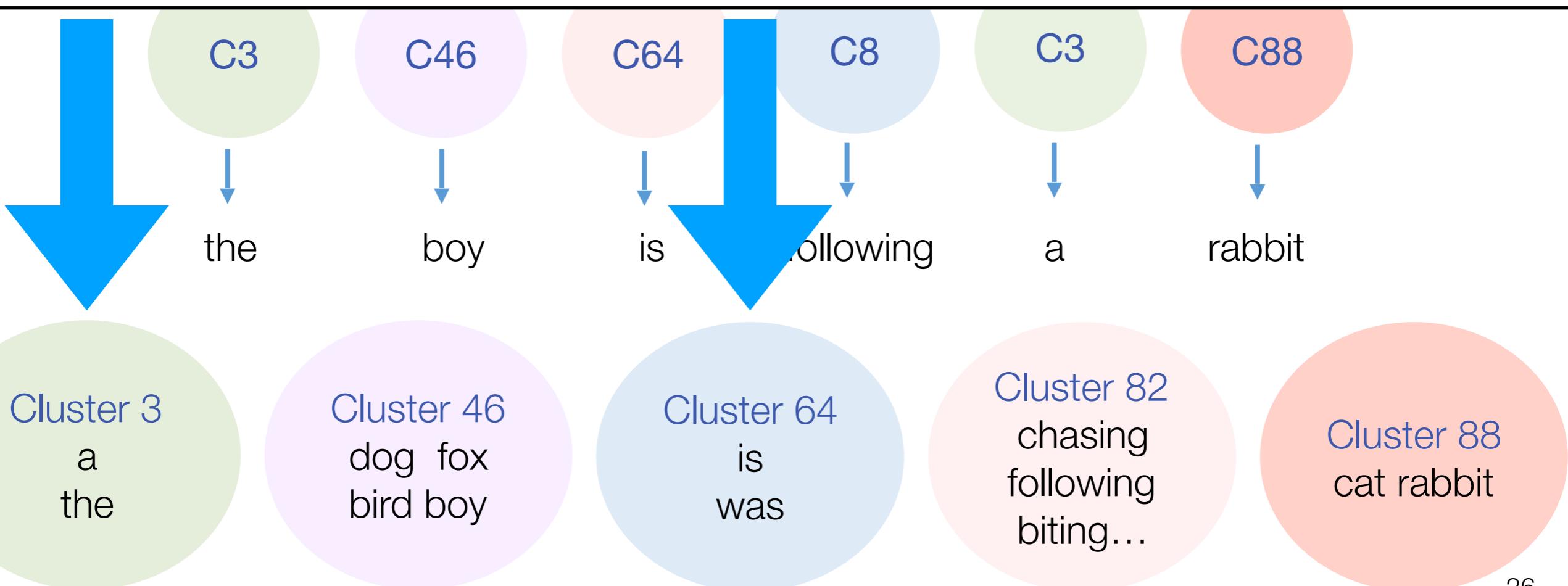
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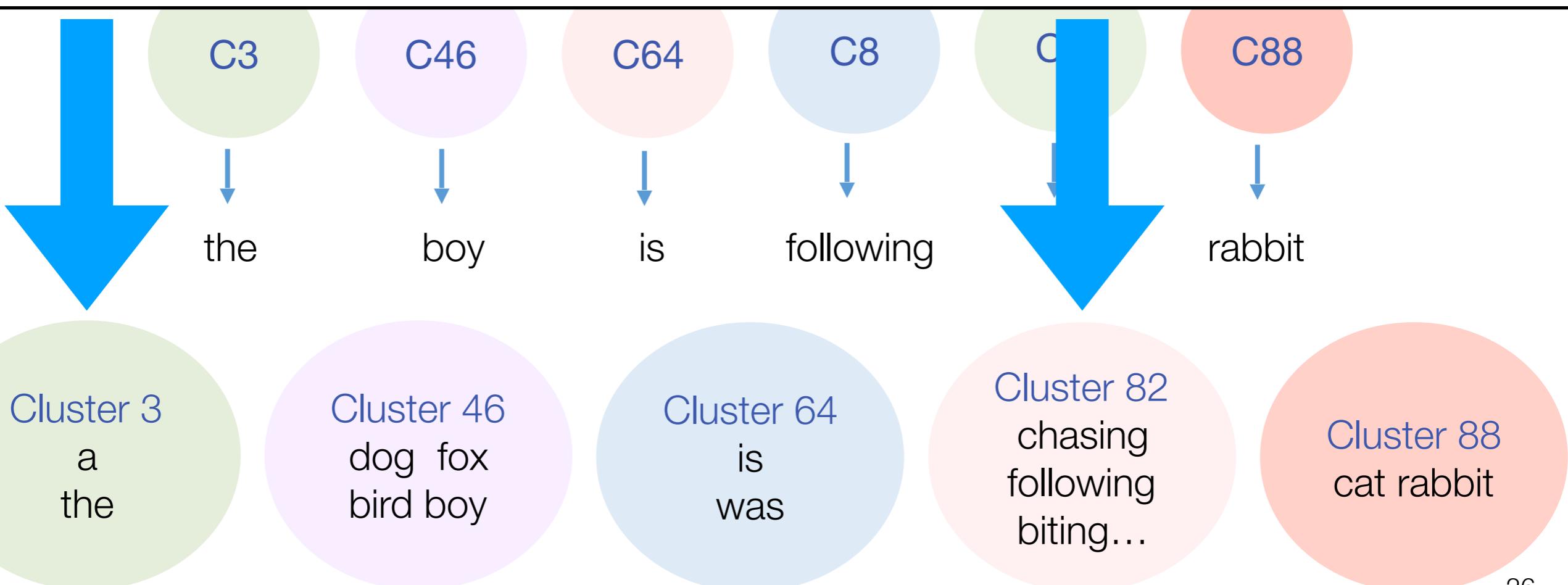
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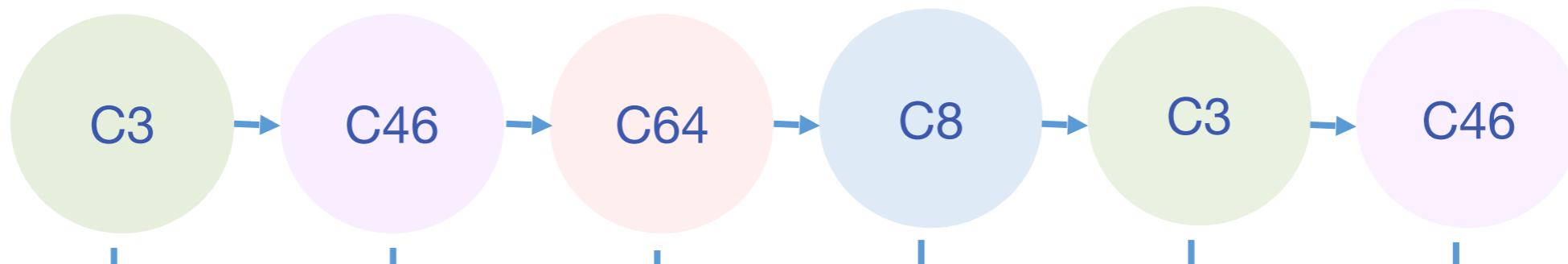
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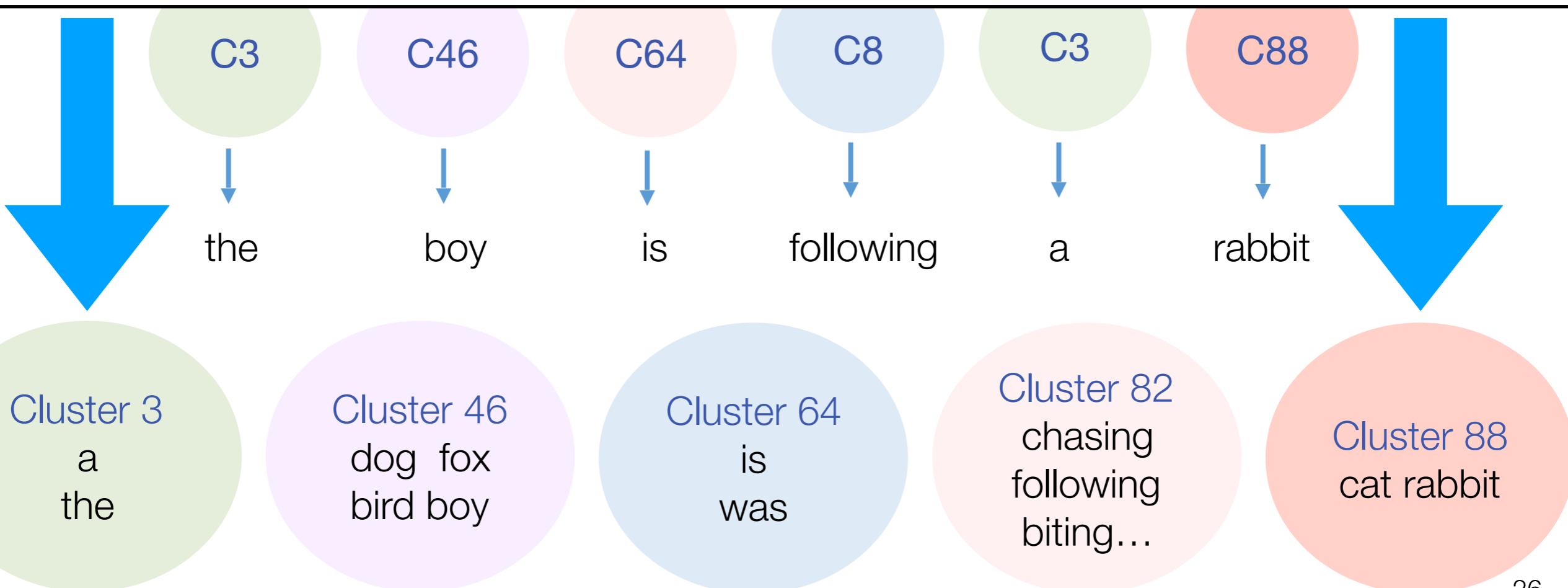
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 - (Can be improved to $O(|V|^3)$)

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 c_1, \dots, c_m
- For $i = (m+1) \dots |V|$
 - Create a new cluster c_{m+1} (so we have $m+1$ clusters)
 - Each step, chose to merge the two clusters that maximize $LL(\theta, C)$, merge them so we now have m clusters

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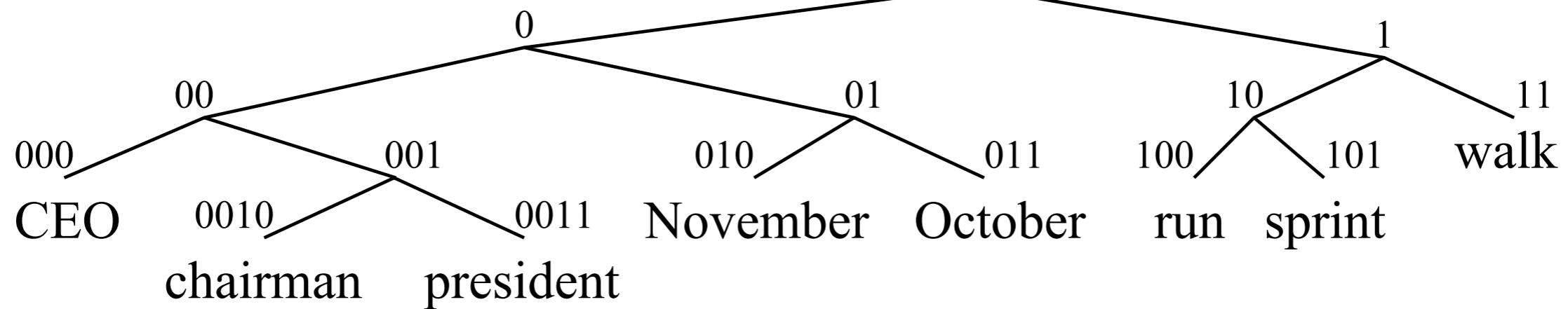
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- Put each of the top m most-frequent words in its own cluster
 c_1, \dots, c_m
- For $i = (m+1) \dots |V|$
 - Create a new cluster c_{m+1} (so we have $m+1$ clusters)
 - Each step, chose to merge the two clusters that maximize $LL(\theta, C)$, merge them so we now have m clusters
- Carry out $m-1$ final merges to get a full hierarchy

Algorithm 2 for Brown Clustering

- m : a hyper-parameter ; start by sorting words by frequency
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 c_1, \dots, c_m
- For $i = (m+1) \dots |V|$
 - Create a new cluster c_{m+1} (so we have $m+1$ clusters)
 - Each step, chose to merge the two clusters that maximize $LL(\theta, C)$, merge them so we now have m clusters
- Carry out $m-1$ final merges to get a full hierarchy
- Cost? $O(|V|m^2 + n)$ where n is the # of words in corpus

Brown Clusters as vectors

- By tracing the order in which clusters are merged, the model builds a binary tree from bottom to top.
- Each word represented by binary string = path from root to leaf
- Each intermediate node is a cluster
- Chairman is 0010, “months” = 01, and verbs = 1



Brown cluster examples

Friday Monday Thursday Wednesday Tuesday Saturday Sunday weekends Sundays Saturdays
June March July April January December October November September August
pressure temperature permeability density porosity stress velocity viscosity gravity tension
anyone someone anybody somebody
had hadn't hath would've could've should've must've might've
asking telling wondering instructing informing kidding reminding bothering thanking depositing
mother wife father son husband brother daughter sister boss uncle
great big vast sudden mere sheer gigantic lifelong scant colossal
down backwards ashore sideways southward northward overboard aloft downwards adrift

Example clusters

Friday Monday Thursday Wednesday Tuesday Saturday Sunday weekends Sundays Saturdays
June March July April January December October November September August
people guys folks fellows CEOs chaps doubters commies unfortunates blokes
down backwards ashore sideways southward northward overboard aloft downwards adrift
water gas coal liquid acid sand carbon steam shale iron
great big vast sudden mere sheer gigantic lifelong scant colossal
man woman boy girl lawyer doctor guy farmer teacher citizen
American Indian European Japanese German African Catholic Israeli Italian Arab
pressure temperature permeability density porosity stress velocity viscosity gravity tension
mother wife father son husband brother daughter sister boss uncle
machine device controller processor CPU printer spindle subsystem compiler plotter
John George James Bob Robert Paul William Jim David Mike
anyone someone anybody somebody
feet miles pounds degrees inches barrels tons acres meters bytes
director chief professor commissioner commander treasurer founder superintendent dean cus-
todian
liberal conservative parliamentary royal progressive Tory provisional separatist federalist PQ
had hadn't hath would've could've should've must've might've
asking telling wondering instructing informing kidding reminding bothering thanking depositing
that tha theat
head body hands eyes voice arm seat eye hair mouth

lawyer	1000001101000
newspaperman	100000110100100
stewardess	100000110100101
toxicologist	10000011010011
slang	1000001101010
babysitter	100000110101100
conspirator	1000001101011010
womanizer	1000001101011011
mailman	10000011010111
salesman	100000110110000
bookkeeper	1000001101100010
troubleshooter	10000011011000110
bouncer	10000011011000111
technician	1000001101100100
janitor	1000001101100101
saleswoman	1000001101100110
...	
Nike	1011011100100101011100
Maytag	10110111001001010111010
Generali	10110111001001010111011
Gap	10110111001001010111110
Harley-Davidson	10110111001001010111110
Enfield	101101110010010101111110
genus	101101110010010101111111
Microsoft	101101110010010111000
Ventrifex	1011011100100101110010
Tractebel	10110111001001011100110
Synopsys	10110111001001011100111
WordPerfect	10110111001001011101000
....	
John	1011100100000000000
Consuelo	101110010000000001
Jeffrey	101110010000000010
Kenneth	10111001000000001100
Phillip	101110010000000011010
WILLIAM	101110010000000011011
Timothy	101110010000000011110
Terrence	1011100100000000111110
Jerald	1011100100000000111111
Harold	10111001000000001100
Frederic	1011100100000000101
Wendell	101110010000000011

Example Hierarchy

from Miller (2004)

Works really well in social people,
especially with lots of spelling variation

Cluster ID	Constituent word types
00111001	can cn cann caan cannn ckan shalll ccan caaan cannnn caaaaan
00101111001	ii id ion iv ll iii ud wd uma ul idnt provoking hed 1+1 ididnt hast ine 2+2 idw #thingsblackpeopledo iiii #onlywhitepeople dost doan uon apt-get
0111101011110	hoping wishing considering wishin contemplating dreading regretting hopin hopeing considerin suspecting regreting wishn comtemplating hopen



K-Means Clustering

What is Clustering?

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- Cluster: a collection of data objects
 - Similar to one another within the same cluster
 - Dissimilar to the objects in other clusters

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 - no predefined classes

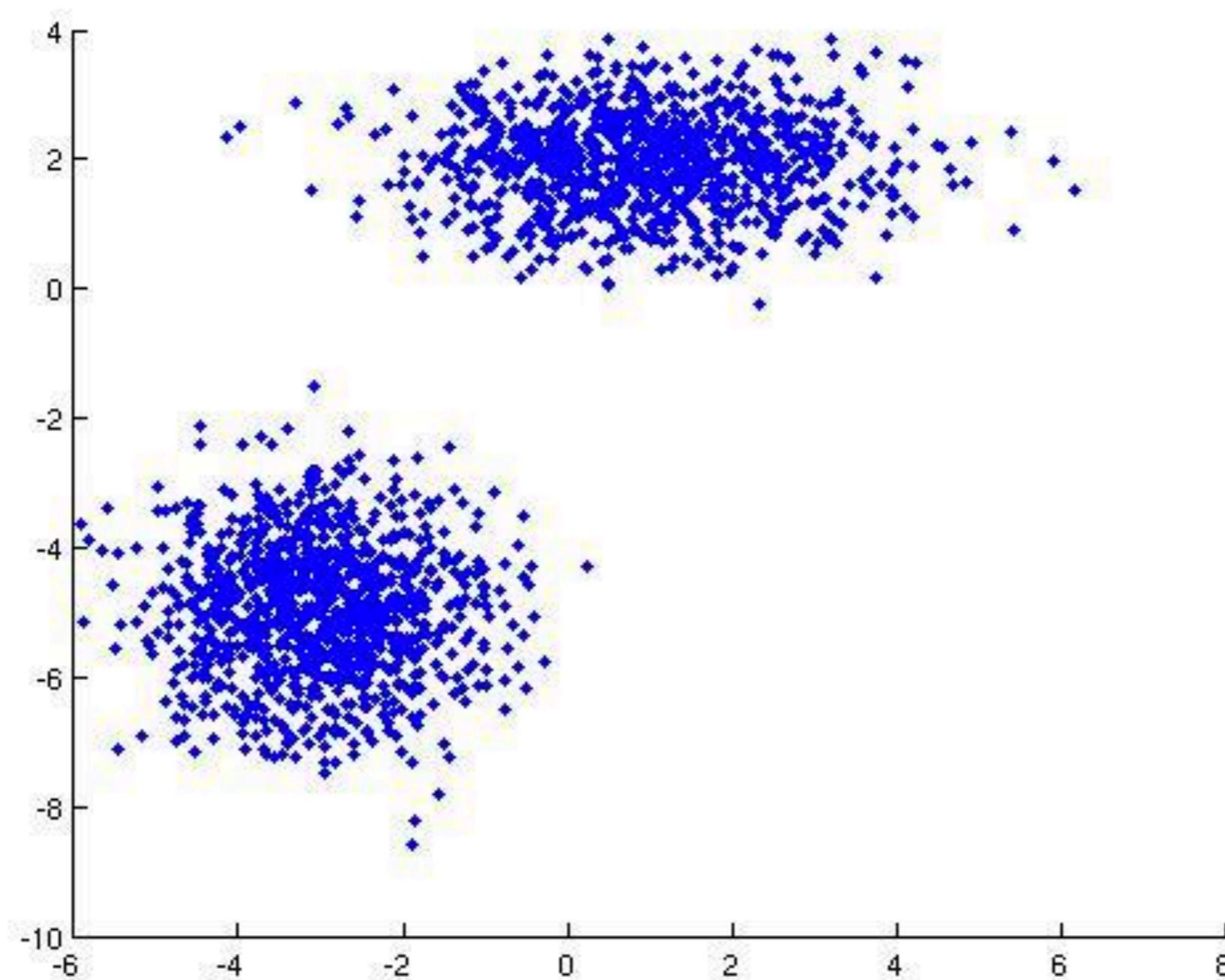
What is Clustering?

- Cluster: a collection of data objects
 - Similar to one another within the same cluster
 - Dissimilar to the objects in other clusters
- Clustering is **unsupervised classification**:
 - no predefined classes
- Typical applications
 - As a **stand-alone tool** to get insight into data distribution
 - As a **preprocessing step** for other NLP algorithms

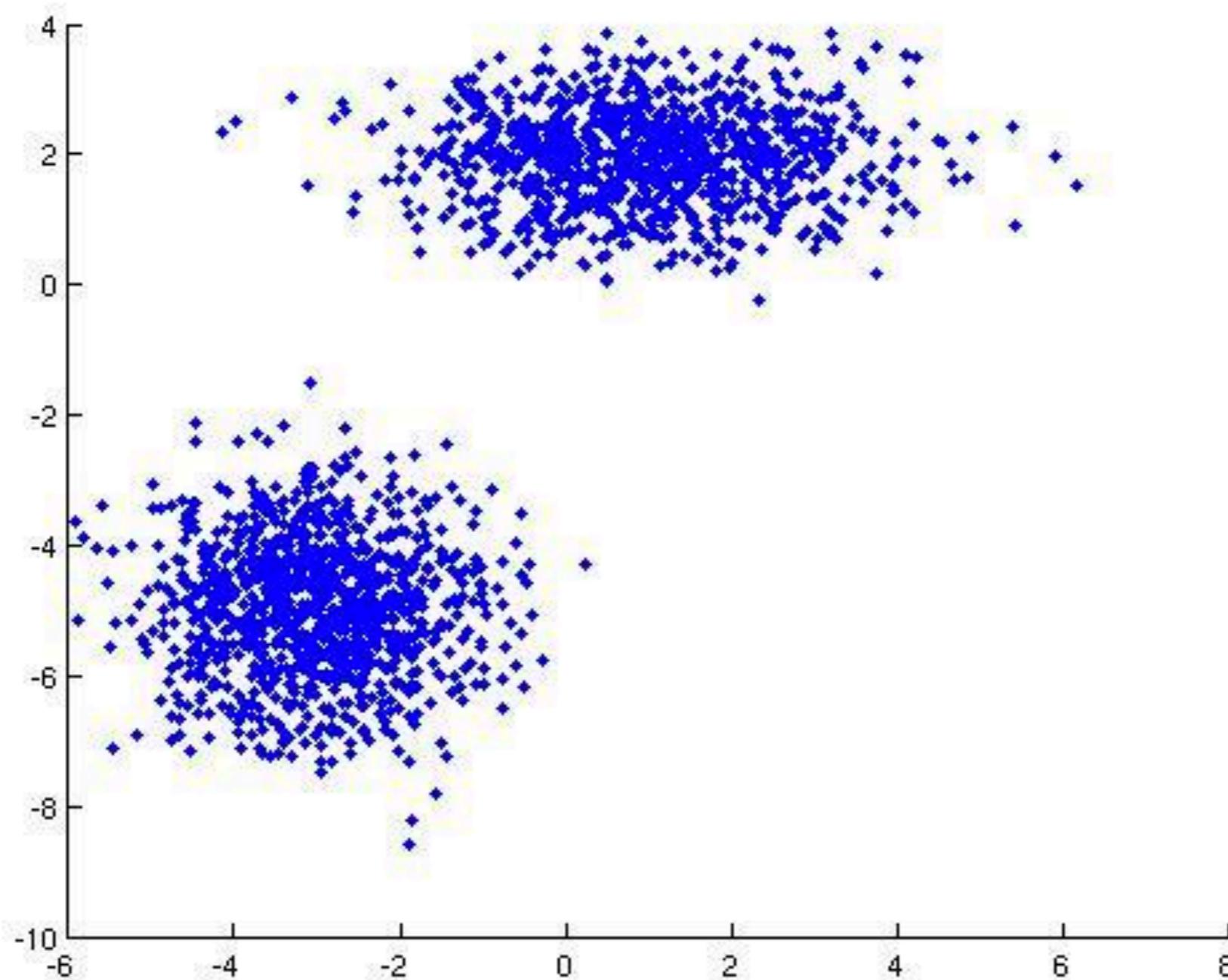
Clustering in NLP

- Most linguistic items can be represented as vectors, e.g., your word2vec vectors
- We compare linguistic items using similarity metrics — often cosine similarity
- This lets us apply standard clustering methods to text
 - but we often visualize algorithms in 2D as points

How many clusters are there here?

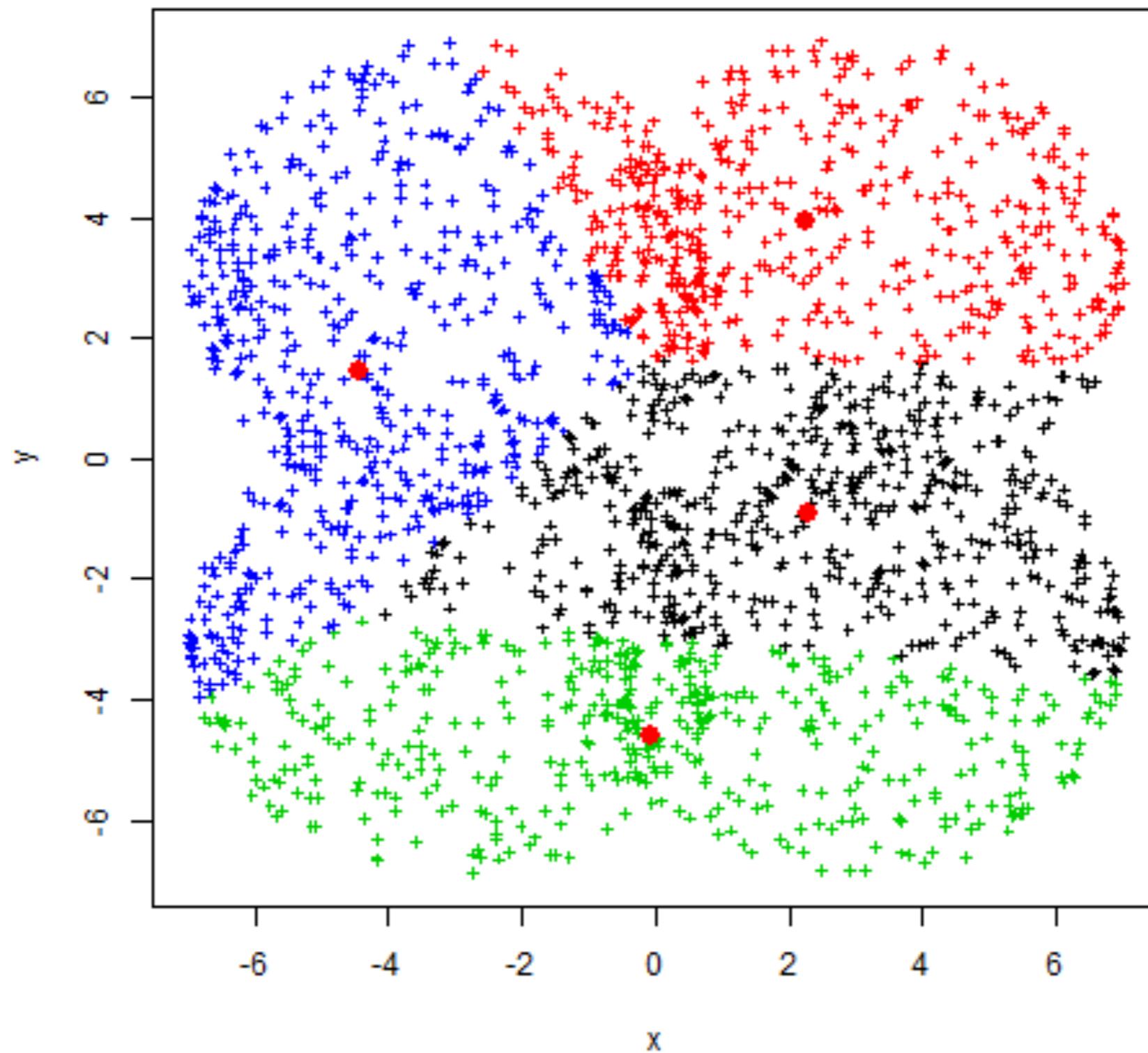


If I told you there were two clusters,
how would we find the center?



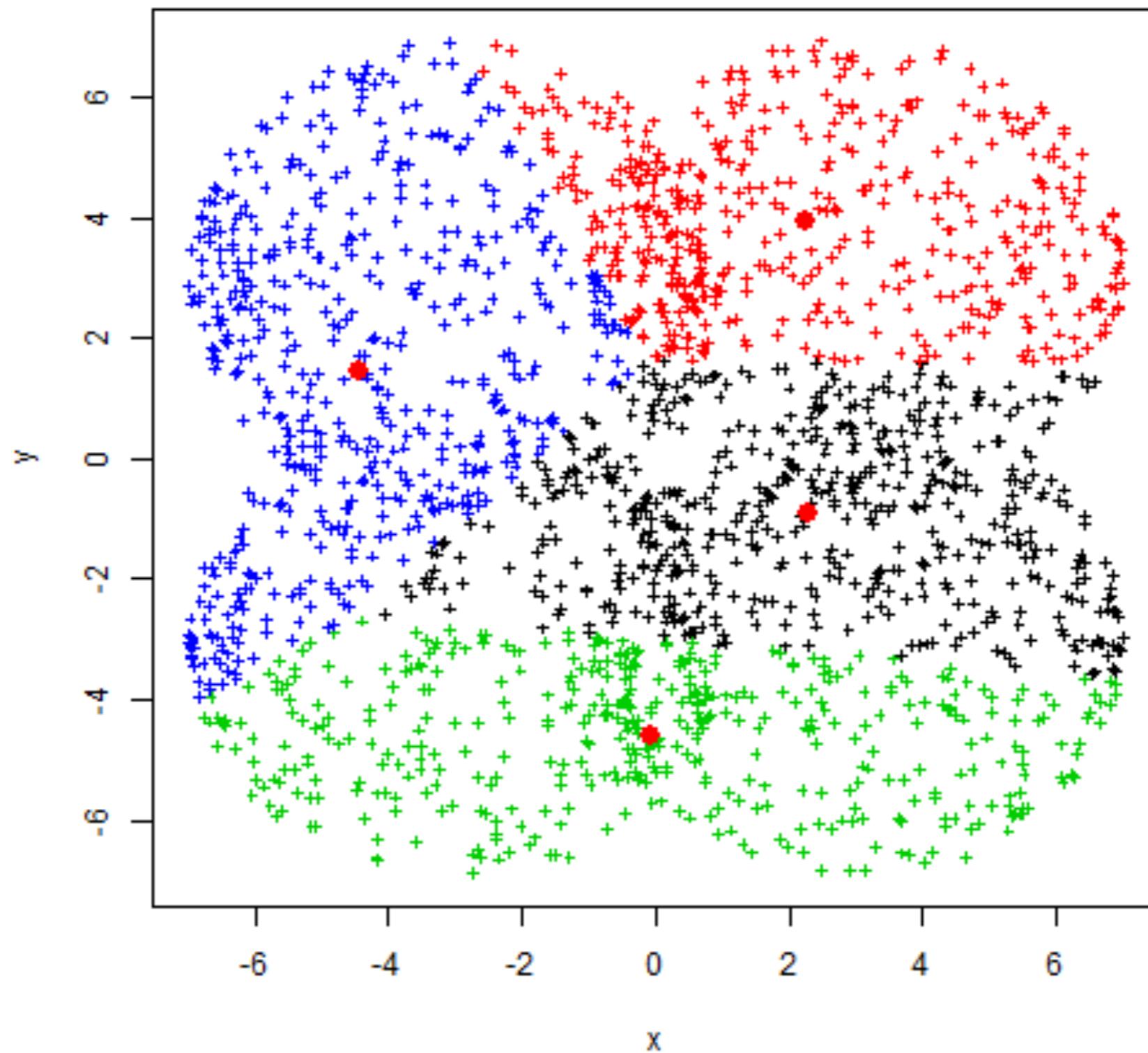
K-Means

K Means Clustering



K-Means

K Means Clustering



K-Means

- Groups data into K clusters and attempts to group data points to minimize the distance to their central mean.
- Algorithm works by iterating between two stages until the data points converge.

Problem Formulation

- Given a data set of $\{\mathbf{x}_1, \dots, \mathbf{x}_N\}$ which consists of N random instances of a random D -dimensional variable \mathbf{x} .
- Introduce a set of K prototype vectors, μ_k where $k=1, \dots, K$ and μ_k corresponds to the mean of the k^{th} cluster (also known as a **centroid**).

Problem Formulation

- Goal is to find a grouping of data points and prototype vectors that minimizes the *distance* of each data point.
 - In spatial cases, we define distance as the sum of squares
 - In text, we often define distance as cosine distance

Problem Formulation (cont.)

- This can be formalized by introducing an indicator variable for each data point:
 - r_{nk} is $\{0,1\}$, and $k=1,\dots,K$
 - The indicator is 1 if the data point is in the cluster
- Our objective function becomes:

$$J = \sum_{n=1}^N \sum_{k=1}^K r_{nk} \|x_n - \mu_k\|^2$$

How K-Means works

- Algorithm initializes the K prototype vectors to K distinct random data points.
- Cycles between two stages until convergence is reached.
- Convergence: since there are only a finite set of possible assignments.

How K-Means works

$$r_{nk} = \begin{cases} 1 & \text{if } k = \operatorname{argmin}_j \|x_n - \mu_k\|^2 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_k = \frac{\sum_n r_{nk} x_n}{\sum_n r_{nk}}$$

How K-Means works

$$r_{nk} = \begin{cases} 1 & \text{if } k = \operatorname{argmin}_j \|x_n - \mu_k\|^2 \\ 0 & \text{otherwise} \end{cases}$$

Is this cluster's centroid the closest?

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How K-Means works

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of items in the cluster

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Is this cluster's centroid the closest?

- 2. Update μ_k : The sum of all vectors in the cluster

$$\mu_k = \frac{\sum_n r_{nk} x_n}{\sum_n r_{nk}}$$

of items in the cluster

K-Means

Initialization example

- Pick K cluster centers (note the unfortunate choice)



K-Means

Initialization example

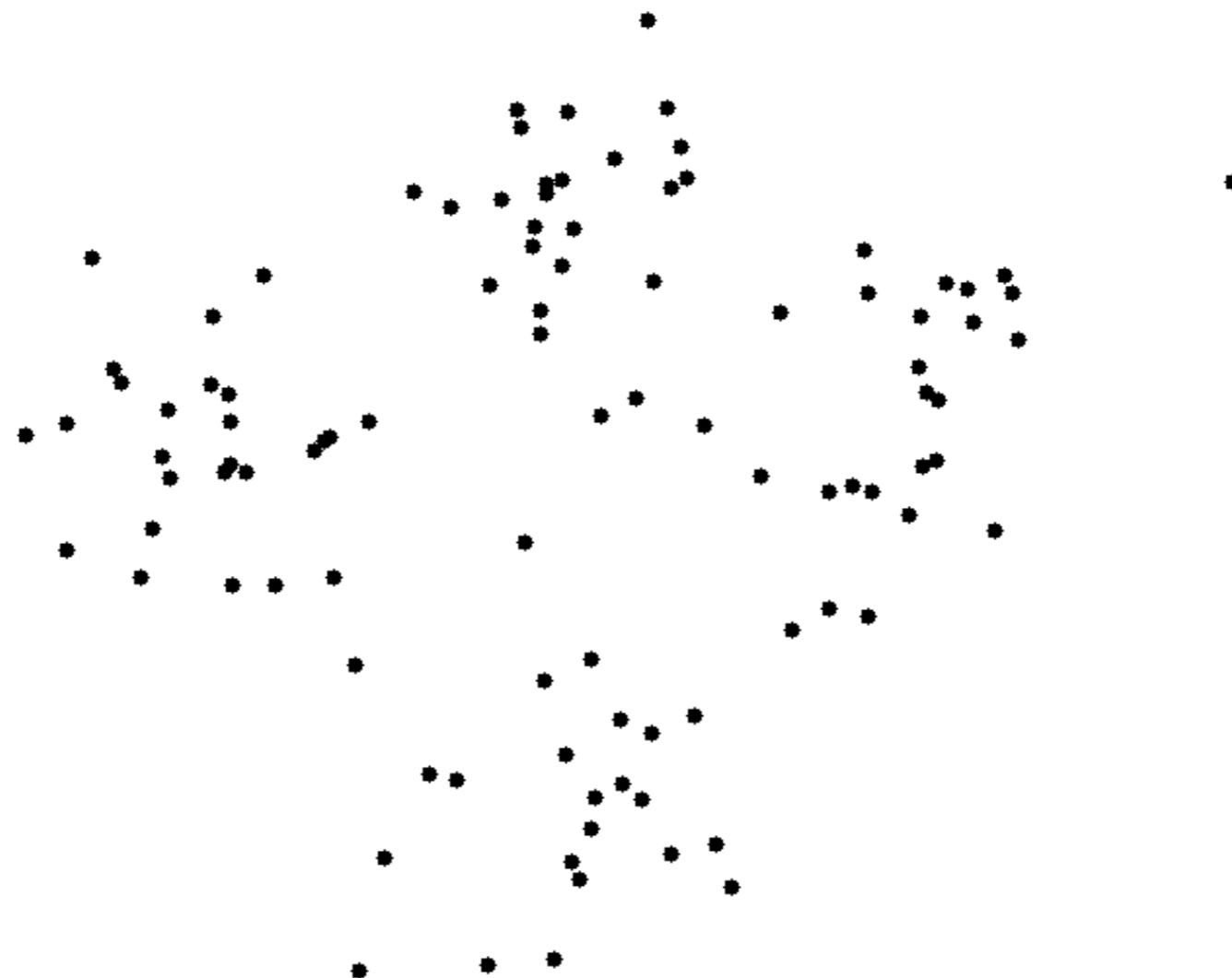
- Pick K cluster centers (note the unfortunate choice)



K-Means

Initialization example

- Pick K cluster centers (random choice)



K-Means

Initialization example

- Pick K cluster centers (random choice)



K-Means++

K-Means with smart initial seeding

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- For each data point x , compute $D(x)$, the distance between x and the nearest center that has already been chosen.

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- Choose one center uniformly at random from among the data points.
- For each data point x , compute $D(x)$, the distance between x and the nearest center that has already been chosen.
- Choose one new data point at random as a new center, using a weighted probability distribution where a point x is chosen with probability proportional to $D(x)^2$.

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- Repeat Steps 2 and 3 until k centers have been chosen.

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- Choose one new data point at random as a new center, using a weighted probability distribution where a point x is chosen with probability proportional to $D(x)^2$.
- Repeat Steps 2 and 3 until k centers have been chosen.
- Now that the initial centers have been chosen, proceed using standard k -means.

K-Means++

- This seeding method yields considerable improvement in the final error of k -means
- Takes more time to initialize
- Once initialized, K-Means converges quickly
- Usually faster than K-Means
- 1000 times less prone to error than K-Means

Pros and Cons of K-Means

$$J = \sum_{n=1}^N \sum_{k=1}^K r_{nk} \|x_n - \mu_k\|^2$$

- Convergence: J may converge to a local minima and not the global minimum. May have to repeat algorithm multiple times.
- With a large data set, the distance calculations can be slow.
- K is an input parameter. If K is inappropriately chosen it may yield poor results.

Local Minima

- K-Means might not find the best possible assignments and centers.
- Consider points 0, 20, 32.
 - K-means can converge to centers at 10, 32.
 - Or to centers at 0, 26.
- Heuristic solutions
 - Start with many random starting points and pick the best solution.





EM: Expectation Maximization

Soft Clustering

- Clustering typically assumes that each instance is given a “hard” assignment to exactly one cluster.

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Soft Clustering

- Clustering typically assumes that each instance is given a “hard” assignment to exactly one cluster.
- Does not allow uncertainty in class membership or for an instance to belong to more than one cluster.
- **Soft clustering** gives probabilities that an instance belongs to each of a set of clusters.
- Each instance is assigned a probability distribution across a set of discovered categories (probabilities of all categories must sum to 1).

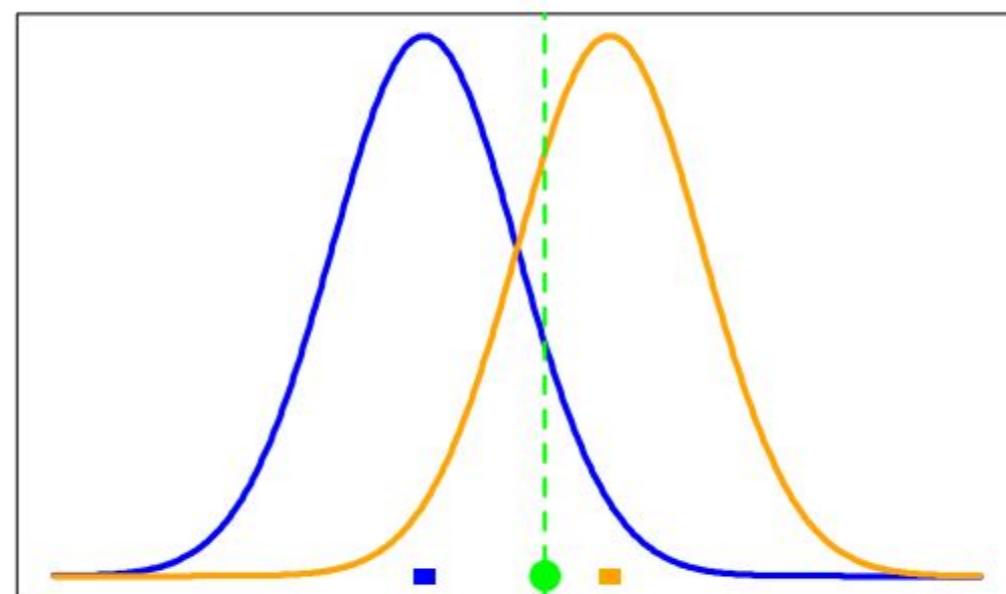
EM

- Soft Assignments
 - A point is partially assigned to all clusters.
- Uses a probabilistic formulation
- Tends to work better than K-Means

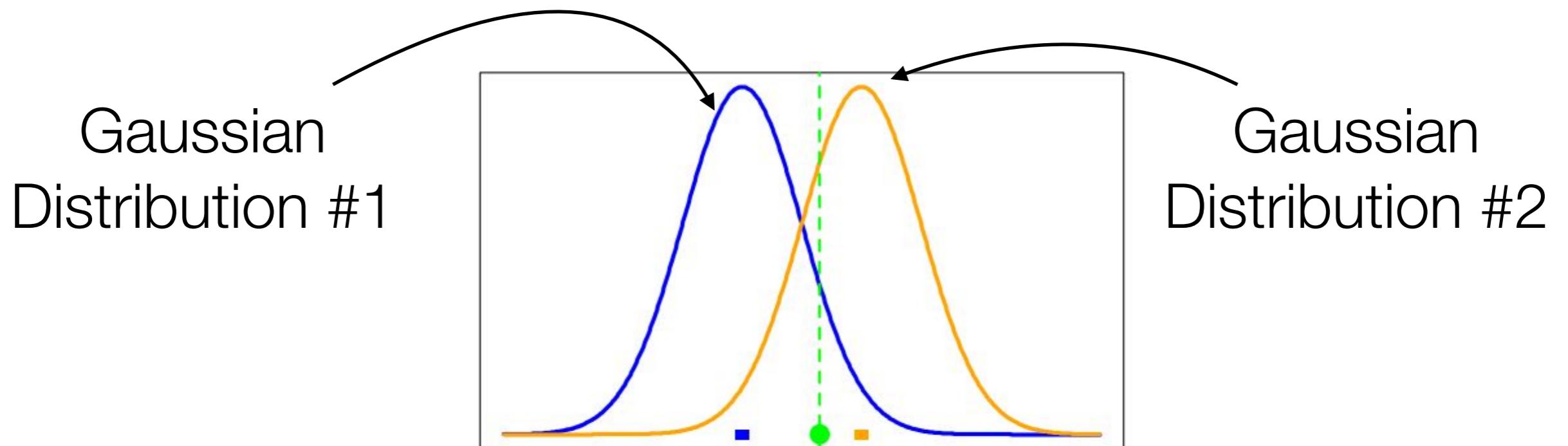
Mixture of Gaussians

- A gaussian specifies a distribution over points according to two parameters: mean m and variance σ
- $g(x; m, \sigma)$ is the probability of a point x based on a Gaussian Distribution with mean m and variance σ

Intuition



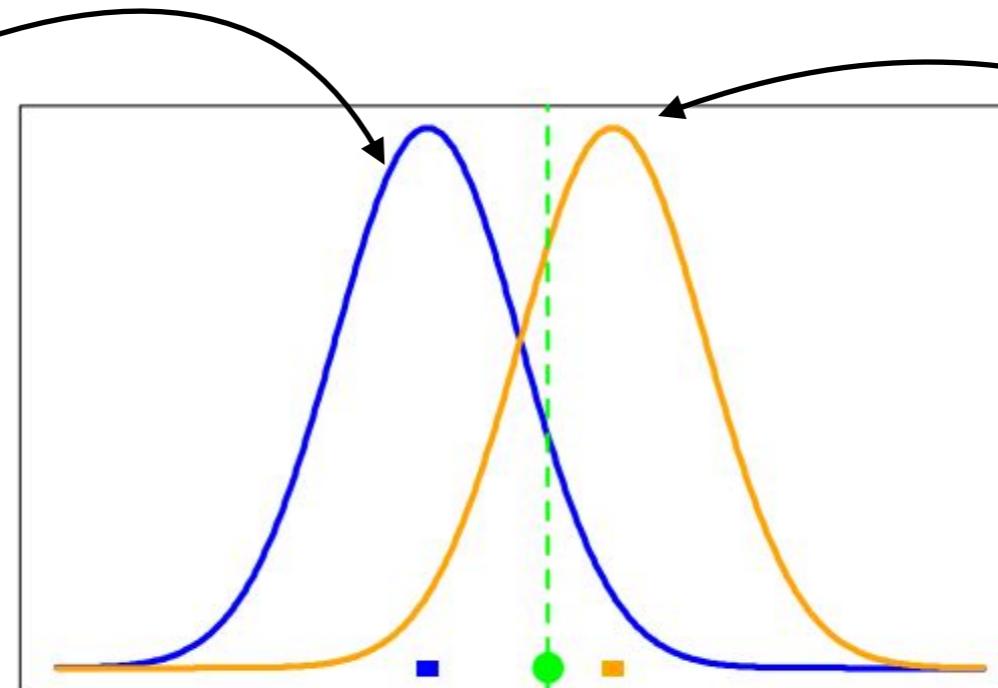
Intuition



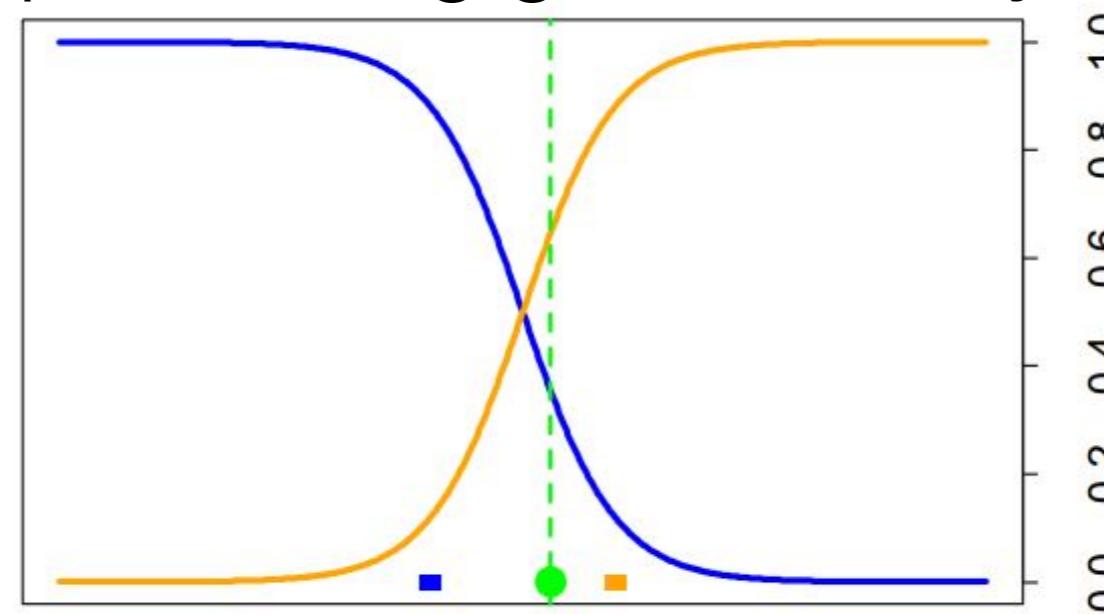
Intuition

Gaussian
Distribution #1

Gaussian
Distribution #2



Probability of a point being generated by each distribution



EM Intuition Building

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EM Intuition Building

- Suppose we have two coins A and B
- Assume bias of A is θ_1 , e.g., $p(\text{heads}|A)$
- Assume bias of B is θ_2 , e.g., $p(\text{heads}|B)$
- We want to find θ_1 and θ_2 by performing a bunch of trials (coin flips)

EM Intuition Building: Experiment #1

- We do 5 trials by picking a coin at random
- Then we flip the chosen coin 10 times

EM Intuition Building: Experiment #1

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- Then we flip the chosen coin 10 times

Trial	Results
A	H T T T H H T H T H

EM Intuition Building: Experiment #1

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Trial	Results
A	H T T T H H T H T H
B	H H H H T H H H H H

EM Intuition Building: Experiment #1

- We do 5 trials by picking a coin at random
- Then we flip the chosen coin 10 times

Trial	Results
A	H T T T H H T H T H
B	H H H H T H H H H H
B	H T H H H H H T H H

EM Intuition Building: Experiment #1

- We do 5 trials by picking a coin at random
- Then we flip the chosen coin 10 times

Trial	Results
A	H T T T H H T H T H
B	H H H H T H H H H H
B	H T H H H H H T H H
A	H T H T T T H H T T

EM Intuition Building: Experiment #1

- We do 5 trials by picking a coin at random
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Trial	Results
A	H T T T H H T H T H
B	H H H H T H H H H H
B	H T H H H H H T H H
A	H T H T T T H H T T
B	T H H H T H H H T H

EM Intuition Building: Experiment #1

- We do 5 trials by picking a coin at random
- Then we flip the chosen coin 10 times

Trial	Results	
A	H T T T H H T H T H	$\theta_1 =$
B	H H H H T H H H H H	
B	H T H H H H H T H H	
A	H T H T T T H H T T	$\theta_2 =$
B	T H H H T H H H T H	

EM Intuition Building: Experiment #1

- We do 5 trials by picking a coin at random
- Then we flip the chosen coin 10 times

Trial	Results
A	H T T T H H T H T H
B	H H H H T H H H H H
B	H T H H H H H T H H
A	H T H T T H H T T T
B	T H H H T H H H T H

$$\theta_1 = \frac{\text{\# A heads}}{\text{total \# of A flips}}$$

$$\theta_2 = \frac{\text{\# B heads}}{\text{total \# of B flips}}$$

EM Intuition Building: Experiment #1

Trial	Results
A	H T T T H H T H T H
B	H H H H T H H H H H
B	H T H H H H H T H H
A	H T H T T T H H T T
B	T H H H T H H H T H

EM Intuition Building: Experiment #1

Trial	Results									
A	H	T	T	T	H	H	T	H	T	H
B	H	H	H	H	T	H	H	H	H	H
B	H	T	H	H	H	H	H	T	H	H
A	H	T	H	T	T	T	H	H	T	T
B	T	H	H	H	T	H	H	H	T	H

A	B
5 H, 5 T	
	9 H, 1 T
	8 H, 2 T
4 H, 6 T	
	7 H, 3 T

EM Intuition Building: Experiment #1

Trial

A
B
B
A
B

Results

H T T T H H T H T H

H H H H T H H H H H

H T H H H H H T H H

H T H T T H H T T T

T H H H T H H H T H

A	B
5 H, 5 T	
	9 H, 1 T
	8 H, 2 T
4 H, 6 T	
	7 H, 3 T
24 H, 6 T	9 H, 11 T

EM Intuition Building:

Experiment #1

Trial

A
B
B
A
B

Results

H T T T H H T H T H

H H H H T H H H H H

H T H H H H H T H H

H T H T T H H T T T

T H H H T H H H H T H

A	B
5 H, 5 T	
	9 H, 1 T
	8 H, 2 T
4 H, 6 T	
	7 H, 3 T

24 H, 6 T 9 H, 11 T

$$\theta_1 = \frac{24}{24 + 6} = 0.8$$

$$\theta_2 = \frac{9}{9 + 11} = 0.45$$

EM Intuition Building: Experiment #2

Trial	Results
?	H T T T H H T H T H
?	H H H H T H H H H H
?	H T H H H H H T H H
?	H T H T T T H H T T
?	T H H H T H H H T H

Challenge: What if we don't know which code was flipped during each trial? (The coin name becomes a **hidden variable**)

EM Intuition Building: Experiment #2

Trial	Results									
?	H	T	T	T	H	H	T	H	T	H
?	H	H	H	H	T	H	H	H	H	H
?	H	T	H	H	H	H	H	T	H	H
?	H	T	H	T	T	T	H	H	T	T
?	T	H	H	H	T	H	H	H	T	H

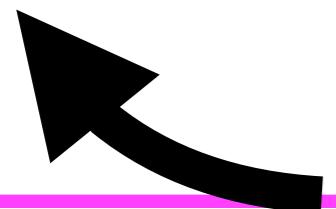
EM Intuition Building: Experiment #2

Trial	Results									
?	H	T	T	T	H	H	T	H	T	H
?	H	H	H	H	T	H	H	H	H	H
?	H	T	H	H	H	H	H	T	H	H
?	H	T	H	T	T	T	H	H	T	T
?	T	H	H	H	T	H	H	H	T	H

Step 1: $\hat{\theta}_A^{(0)} = 0.6$
initialize
parameters $\hat{\theta}_B^{(0)} = 0.5$

EM Intuition Building: Experiment #2

Trial	Results									
?	H	T	T	T	H	H	T	H	T	H
?	H	H	H	H	T	H	H	H	H	H
?	H	T	H	H	H	H	H	T	H	H
?	H	T	H	T	T	T	H	H	T	T
?	T	H	H	H	T	H	H	H	T	H



Step 1: $\hat{\theta}_A^{(0)} = 0.6$
initialize
parameters $\hat{\theta}_B^{(0)} = 0.5$

EM Intuition Building: Experiment #2

2: E-Step

The diagram illustrates the EM algorithm's iterative process. A curved arrow labeled "2: E-Step" points from the "Results" section to the "Trial" section. Below this, another arrow points from the "Trial" section back up to the "Results" section.

Trial	Results	$p(x A)$	$p(x B)$
?	H T T T H H T H T H		
?	H H H H T H H H H H		
?	H T H H H H H T H H		
?	H T H T T T H H T T		
?	T H H H T H H H T H		

Step 1: $\hat{\theta}_A^{(0)} = 0.6$
initialize
parameters $\hat{\theta}_B^{(0)} = 0.5$

EM Intuition Building: Experiment #2

2: E-Step

The diagram shows a curved arrow pointing from the text "Step 1: initialize parameters" to the "Results" section. A straight arrow points from the "Results" section to the "2: E-Step" label.

Trial	Results	$p(x A)$	$p(x B)$
?	H T T T H H T H T H	$0.45 \times A, 0.55 \times B$	
?	H H H H T H H H H H		
?	H T H H H H H T H H		
?	H T H T T T H H T T		
?	T H H H T H H H T H		

Step 1: $\hat{\theta}_A^{(0)} = 0.6$
initialize
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EM Intuition Building: Experiment #2

2: E-Step

Trial	Results	$p(x A)$	$p(x B)$	A	B
?	H T T T H H T H T H	$0.45 \times A, 0.55 \times B$		$\approx 2.2H, 2.2T$	$\approx 2.8H, 2.8T$
?	H H H H T H H H H H				
?	H T H H H H H T H H				
?	H T H T T T H H T T				
?	T H H H T H H H T H				

Step 1: $\hat{\theta}_A^{(0)} = 0.6$
initialize
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EM Intuition Building: Experiment #2

2: E-Step

Trial	Results	$p(x A)$	$p(x B)$	A	B
?	H T T T H H T H T H	$0.45 \times A, 0.55 \times B$		$\approx 2.2H, 2.2T$	$\approx 2.8H, 2.8T$
?	H H H H T H H H H H	$0.80 \times A, 0.20 \times B$			
?	H T H H H H H T H H				
?	H T H T T T H H T T				
?	T H H H T H H H T H				

Step 1: $\hat{\theta}_A^{(0)} = 0.6$
initialize
parameters $\hat{\theta}_B^{(0)} = 0.5$

EM Intuition Building: Experiment #2

2: E-Step

Trial	Results	$p(x A)$	$p(x B)$	A	B
?	H T T T H H T H T H	$0.45 \times A, 0.55 \times B$		$\approx 2.2H, 2.2T$	$\approx 2.8H, 2.8T$
?	H H H H T H H H H H	$0.80 \times A, 0.20 \times B$		$\approx 7.2H, 0.8T$	$\approx 1.8H, 0.2T$
?	H T H H H H H T H H				
?	H T H T T T H H T T				
?	T H H H T H H H T H				

Step 1: $\hat{\theta}_A^{(0)} = 0.6$
initialize
parameters $\hat{\theta}_B^{(0)} = 0.5$

EM Intuition Building: Experiment #2

2: E-Step

Trial	Results	$p(x A)$	$p(x B)$	A	B
?	H T T T H H T H T H	$0.45 \times A, 0.55 \times B$		$\approx 2.2H, 2.2T$	$\approx 2.8H, 2.8T$
?	H H H H T H H H H H	$0.80 \times A, 0.20 \times B$		$\approx 7.2H, 0.8T$	$\approx 1.8H, 0.2T$
?	H T H H H H H T H H	$0.73 \times A, 0.27 \times B$		$\approx 5.9H, 1.5T$	$\approx 2.1H, 0.5T$
?	H T H T T H H T T T				
?	T H H H T H H H T H				

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?	H T H H H H H T H H	$0.73 \times A, 0.27 \times B$		$\approx 5.9H, 1.5T$	$\approx 2.1H, 0.5T$
?	H T H T T H H T T T	$0.35 \times A, 0.65 \times B$		$\approx 1.4H, 2.1T$	$\approx 2.6H, 3.9T$
?	T H H H T H H H T H				

Step 1: $\hat{\theta}_A^{(0)} = 0.6$
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EM Intuition Building: Experiment #2

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EM Intuition Building: Experiment #2

2: E-Step

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				$\approx 21.3H, 8.6T$	$\approx 11.7H, 8.4T$

Step 1: $\hat{\theta}_A^{(0)} = 0.6$
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EM Intuition Building: Experiment #2

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 initialize
 parameters $\hat{\theta}_B^{(0)} = 0.5$

3: M-Step

EM Intuition Building: Experiment #2

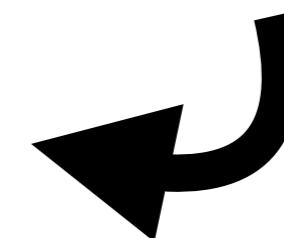
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Step 1: $\hat{\theta}_A^{(0)} = 0.6$
 initialize
 parameters $\hat{\theta}_B^{(0)} = 0.5$

$$\hat{\theta}_A^{(1)} \approx \frac{21.3}{21.3 + 8.6} \approx$$

$$\hat{\theta}_B^{(1)} \approx \frac{11.7}{11.7 + 8.4} \approx$$



3: M-Step

EM Intuition Building: Experiment #2

2: E-Step

Trial	Results	$p(x A)$	$p(x B)$	A	B
?	H T T T H H T H T H	$0.45 \times A, 0.55 \times B$		$\approx 2.2H, 2.2T$	$\approx 2.8H, 2.8T$
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$$\hat{\theta}_A^{(1)} \approx \frac{21.3}{21.3 + 8.6} \approx 0.71$$

$$\hat{\theta}_B^{(1)} \approx \frac{11.7}{11.7 + 8.4} \approx 0.58$$

3: M-Step

EM Intuition Building: Experiment #2

2: E-Step

Trial	Results	$p(x A)$	$p(x B)$	A	B
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$$\hat{\theta}_B^{(1)} \approx \frac{11.7}{11.7 + 8.4} \approx 0.58$$

3: M-Step

4: Finish

$\hat{\theta}_A^{(10)} \approx 0.80$
 $\hat{\theta}_B^{(10)} \approx 0.52$

What happens if we have more than two coins? A mixture of K Gaussians

- A distribution generated by randomly selecting one of K Gaussians, then randomly draw a point from that distribution.
- Gaussian k with a probability of p_k

$$p(x; m, \sigma) = \sum_{k=1}^K p_k g(x; m_k, \sigma_k)$$

- Goal: find p_k , σ_k , m_k that maximize the probability of all our data points x .

What happens if we have more than two coins? A mixture of K Gaussians

- A distribution generated by randomly selecting one of K Gaussians, then randomly draw a point from that distribution.
- Gaussian k with a probability of p_k

probability of a x being generated from a gaussian with mean m and variance σ

$$p(x; m, \sigma) = \sum_{k=1}^K p_k g(x; m_k, \sigma_k)$$

- Goal: find p_k , σ_k , m_k that maximize the probability of all our data points x .

Back to EM

- Iterative Algorithm
- Goal: group some primitives together
- Chicken and Egg problem:
 - Items in group -> Description of the group
 - Description of the group -> Items in group

Brace Yourselves..



EM

- Iterative Algorithm: E Step and M Step
- E Step:
- Compute the probability that point n is generated by distribution k

$$p^{(i)}(k|n) = \frac{p_k^{(i)} g(x_n; m_k^{(i)}, \sigma_k^{(i)})}{\sum_{m=1}^K p_k^{(i)} g(x_n; m_k^{(i)}, \sigma_k^{(i)})}$$

EM

- M Step:

$$m_k^{(i+1)} = \frac{\sum_{n=1}^N p^{(i)}(k|n)x_n}{\sum_{n=1}^N p^{(i)}(k|n)}$$

$$\sigma_k^{(i+1)} = \sqrt{\frac{1}{D} \frac{\sum_{n=1}^N p^{(i)}(k|n) ||x_n - m_k^{(i+1)}||^2}{\sum_{n=1}^N p^{(i)}(k|n)}}$$

$$p_k^{(i+1)} = \frac{1}{N} \sum_{n=1}^N p^{(i)}(k|n)$$

EM

- Converges to a locally optimal solution
- Each step increases the probability of the points given the distributions.
- Can get stuck in local optima
(less than K-Means)

EM and K-means

EM and K-means

- Notice the similarity between EM for Normal mixtures and K-means.
 - The expectation step is the assignment.
 - The maximization step is the update of centers

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EM and K-means

- Notice the similarity between EM for Normal mixtures and K-means.
 - The expectation step is the assignment.
 - The maximization step is the update of centers
- K-means is a simplified EM.
- K-means makes a hard decision while EM makes a soft decision when updating the parameters of the model.

Ethics, Society, and Computing (ESC)

Undergraduate Student Mixer

Free food, games, and conversation.
No previous ESC experience necessary.

Monday, February 24, 2020
4:30–6:30 p.m., drop-ins welcome
South Lounge, Michigan Union

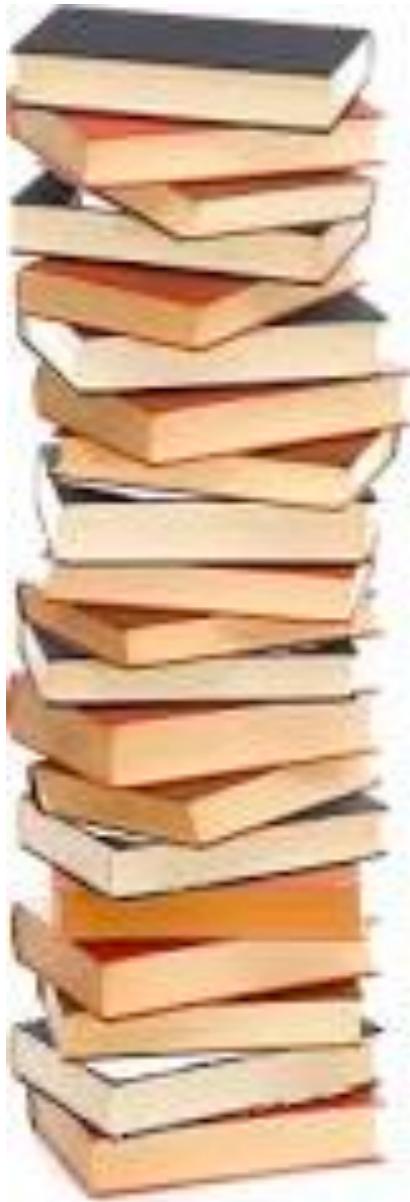


esc.umich.edu



Topic Modeling: Latent Dirichlet Allocation

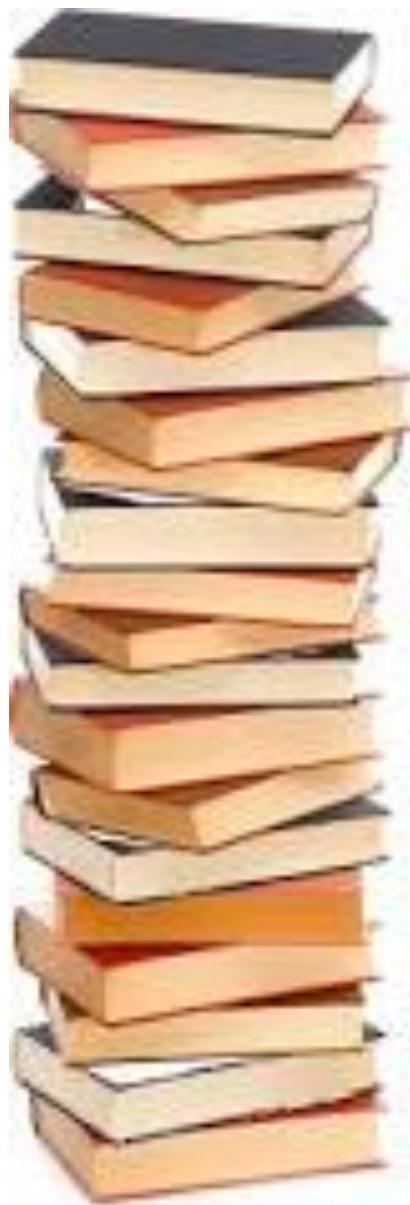
Why Use Topic Models?



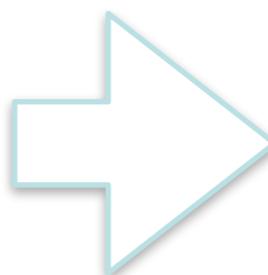
- Often you have a large collection of text
 - Decades of books
 - Newly released gov't docs
 - Wikipedia
- We want to know what happens in it
- But it's infeasible to read it all!
- Topic models provide an automatic way to get high-level themes from a corpus

Topic Modeling at a high level

An input corpus



How many topics? k



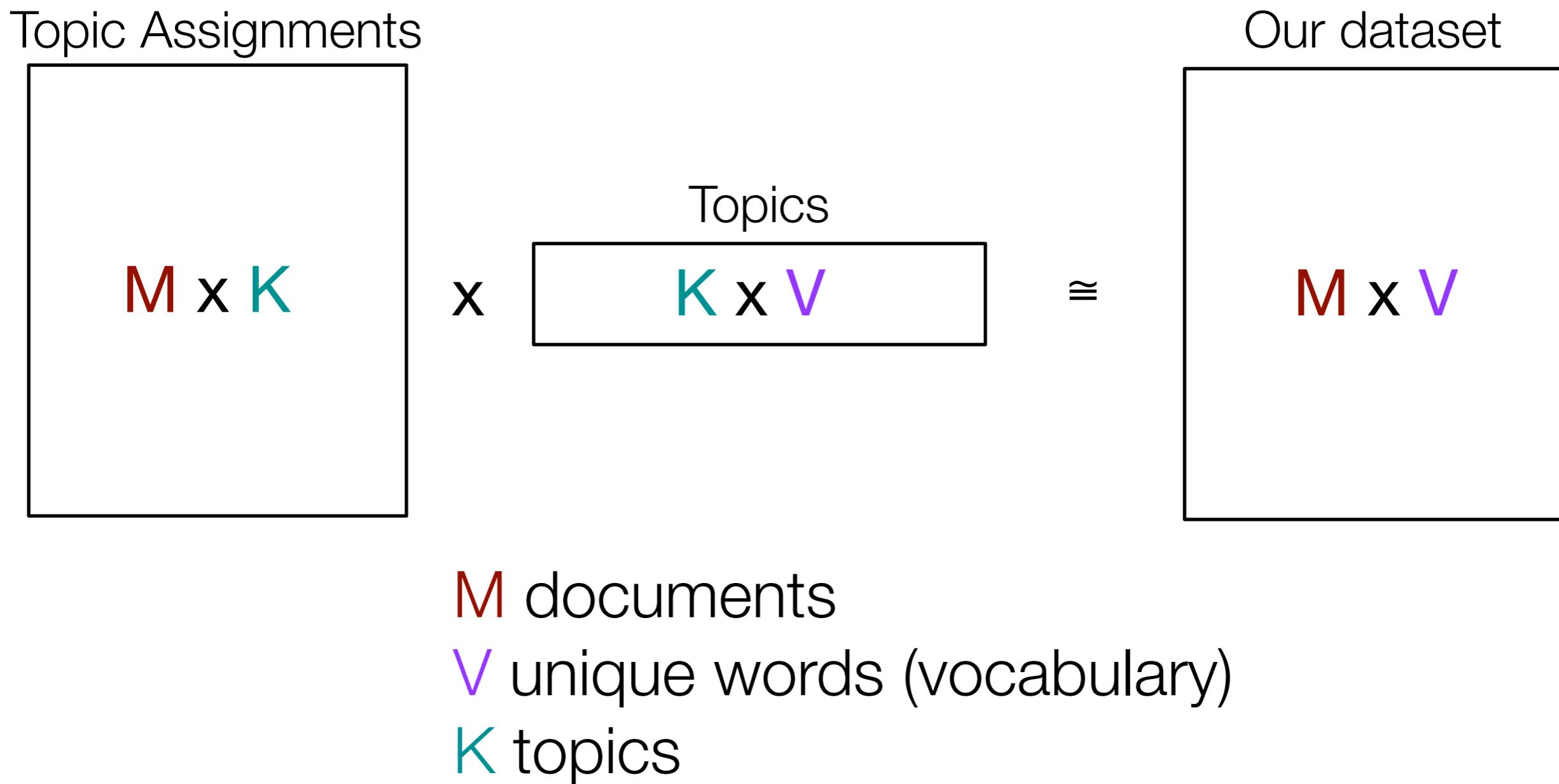
- Each doc's topics
- Each topic's words

Example: Topics

human genome	evolutionary	disease	computer models
dna	species	bacteria	information
genetic genes	organisms	diseases	data
sequence	life	resistance	computers
gene	origin	bacterial	system
molecular sequencing	biology	new	network
map	groups	strains	systems
information	phylogenetic	control	model
genetics	living	infectious	parallel
mapping	diversity	malaria	methods
project	group	parasite	networks
sequences	new	parasites	software
	two	united	new
	common	tuberculosis	simulations

- Example from David Blei's slides

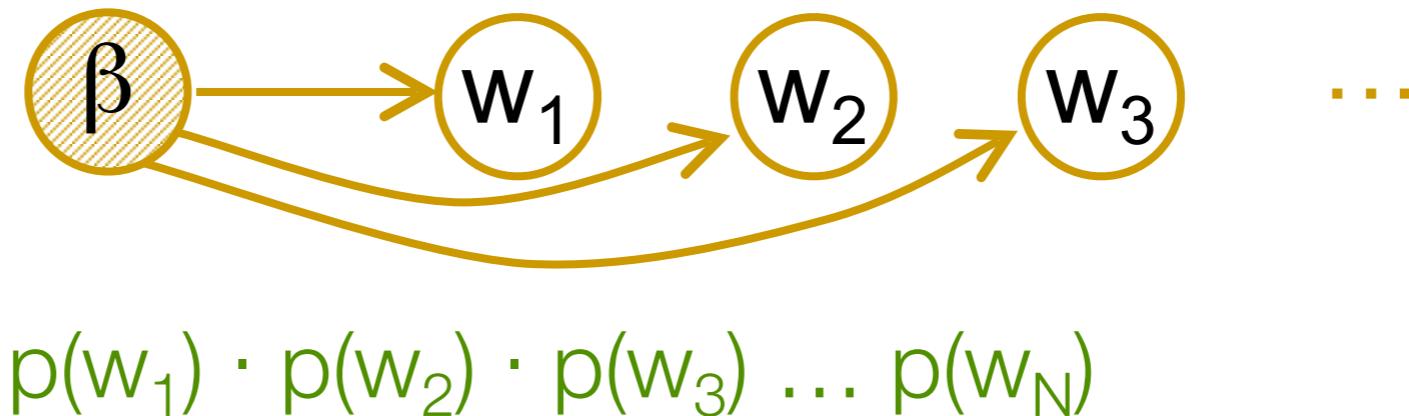
Topic Modeling is a kind of Matrix Factorization



Demo Time!

<https://mimno.infosci.cornell.edu/jsLDA/j lda.html>

Consider a unigram model for generating text



We can explicitly show model's parameters β

β is a **vector** that says which unigrams are likely

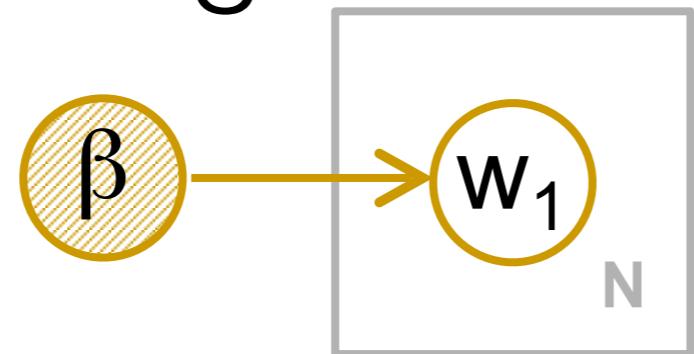


$$p(\beta) \cdot p(w_1 | \beta) \cdot p(w_2 | \beta) \cdot p(w_3 | \beta) \dots p(w_N | \beta)$$

The arrows show that each word is generated from the **same** parameters, β

Plate notation simplifies diagram

β is a **vector** that says which unigrams are likely

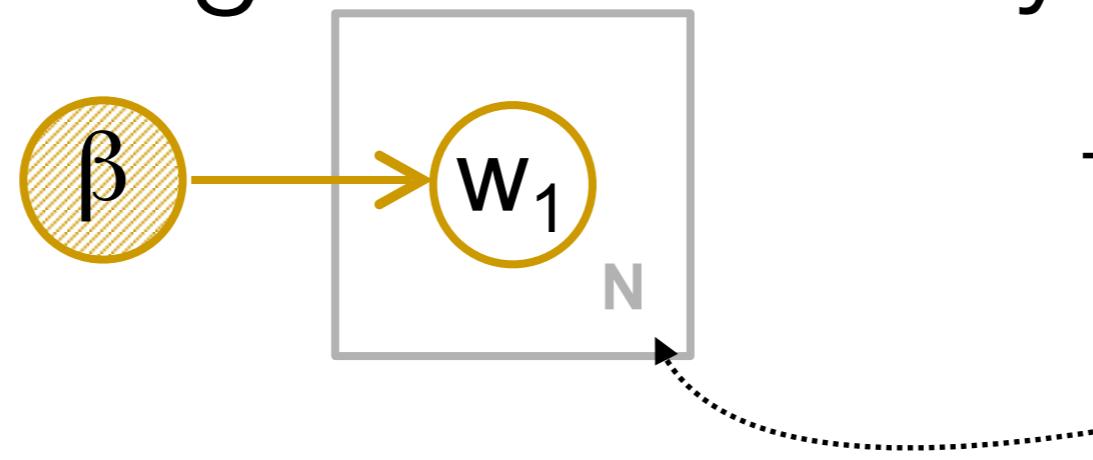


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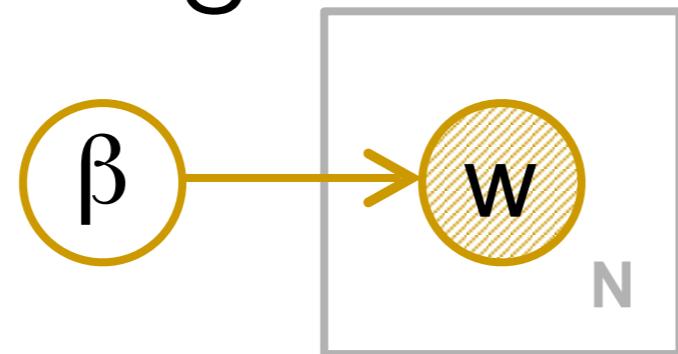
The N says how many things are generated

$$p(\beta) \cdot p(w_1 | \beta) \cdot p(w_2 | \beta) \cdot p(w_3 | \beta) \dots$$

The arrows show that each word is generated from the **same** parameters, β

Learn β from observed words

β is a **vector** that says which unigrams are likely



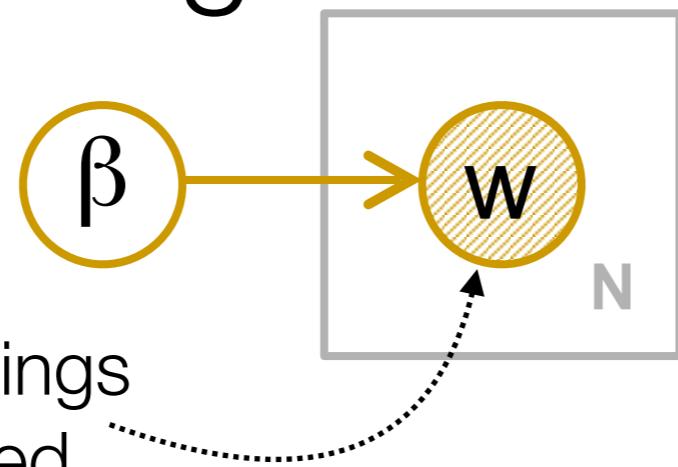
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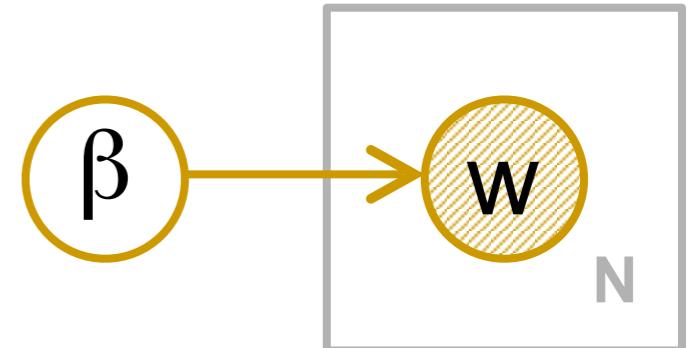
In Plate notation, the things we observe are shaded

The N says how many things are generated

$$p(\beta) \cdot p(w_1 | \beta) \cdot p(w_2 | \beta) \cdot p(w_3 | \beta) \dots$$

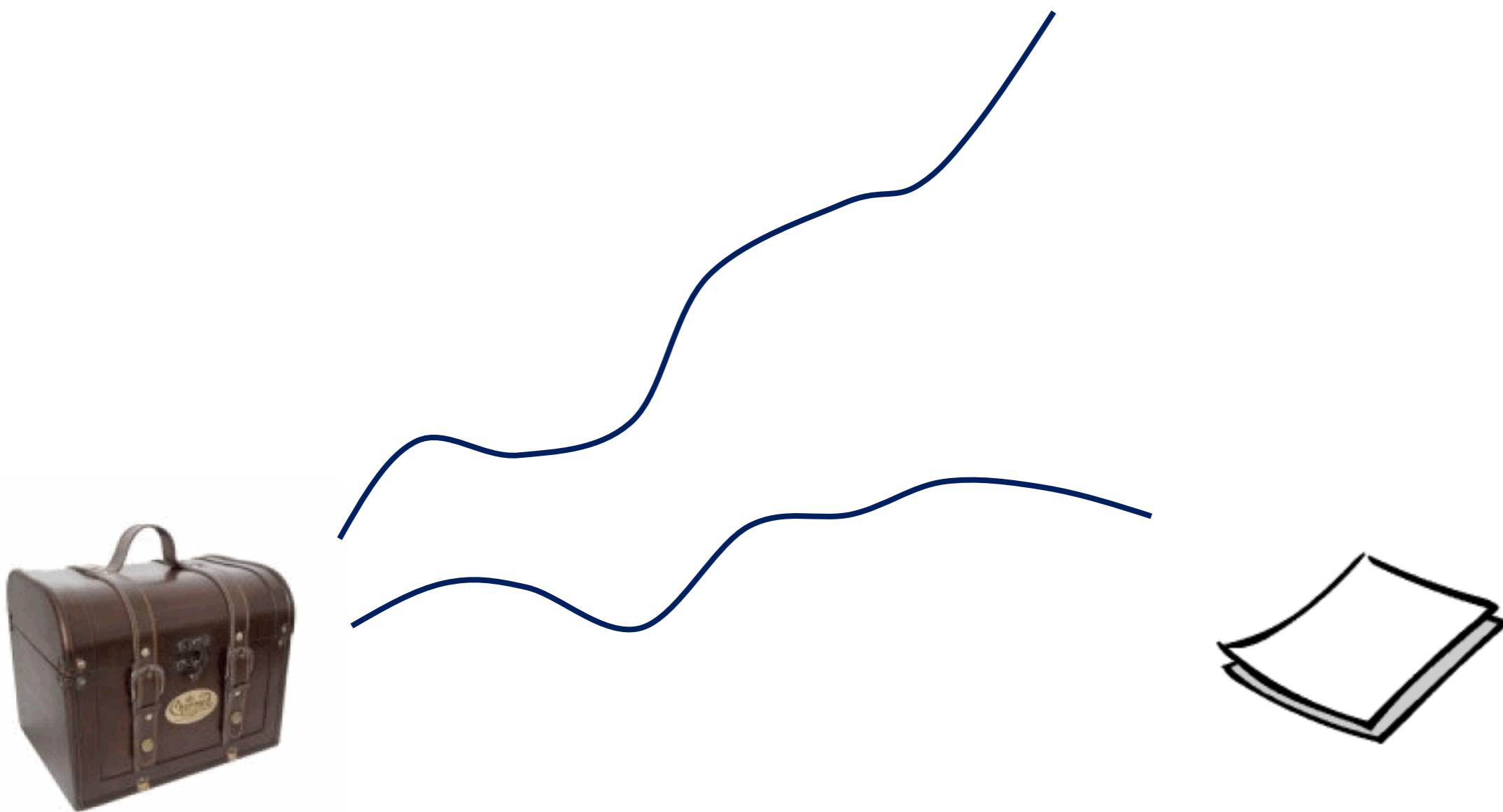
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Extending this model to topics

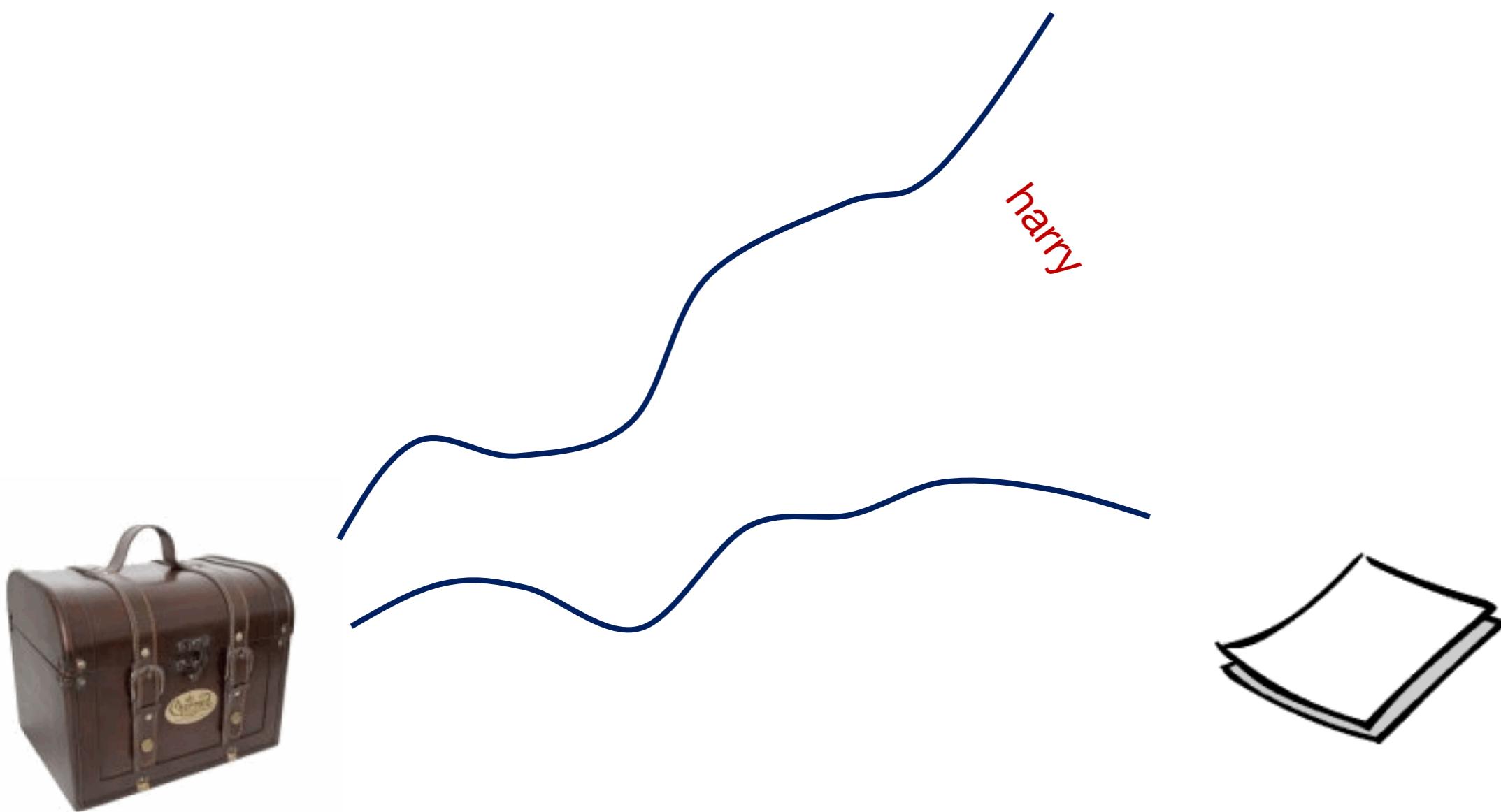


- β represents a probability distribution for generating words in all documents
- But many documents are on different topics
- What if we wanted wanted to have different β for different topics?

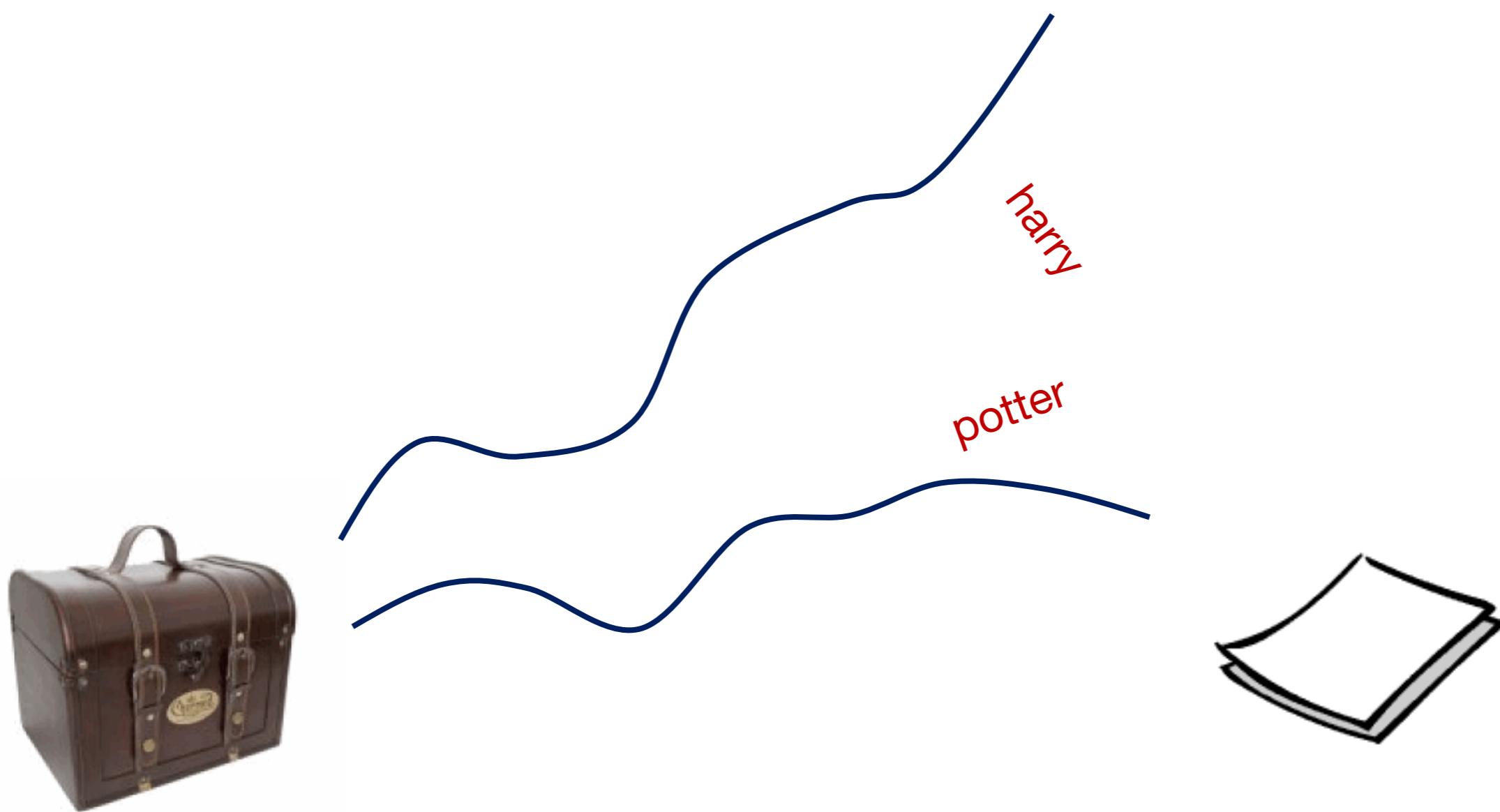
Generative Model of Text



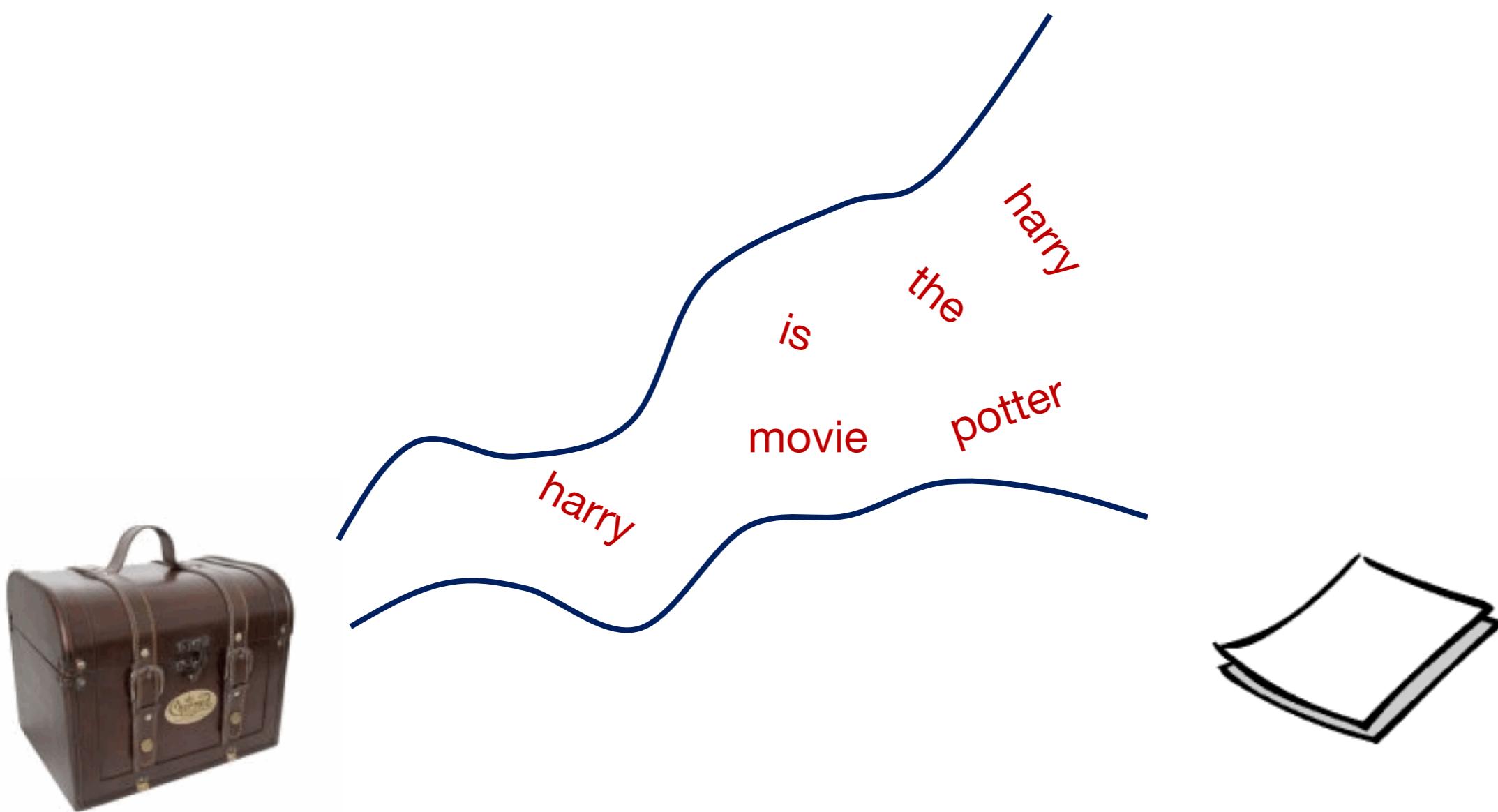
Generative Model of Text



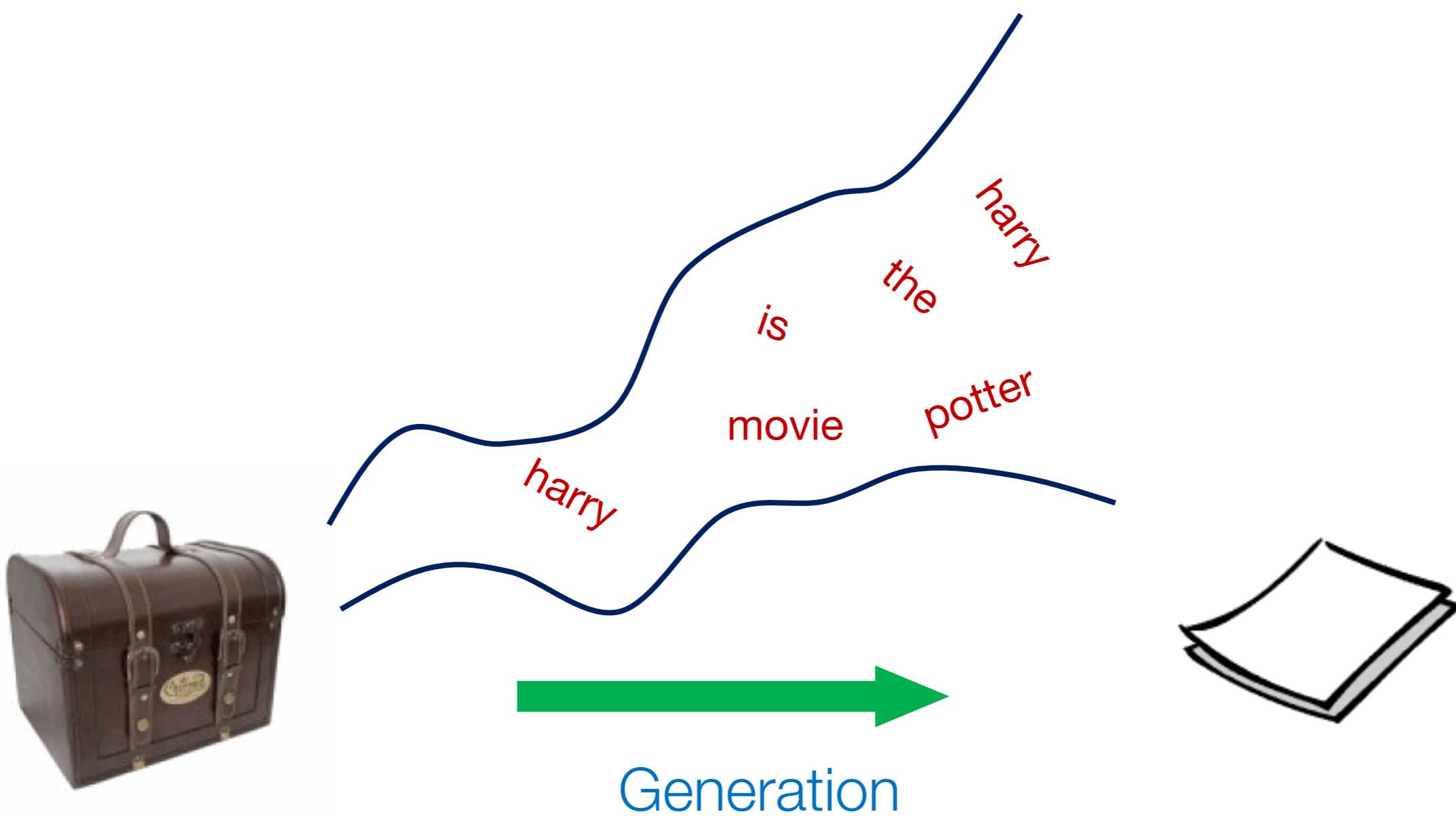
Generative Model of Text



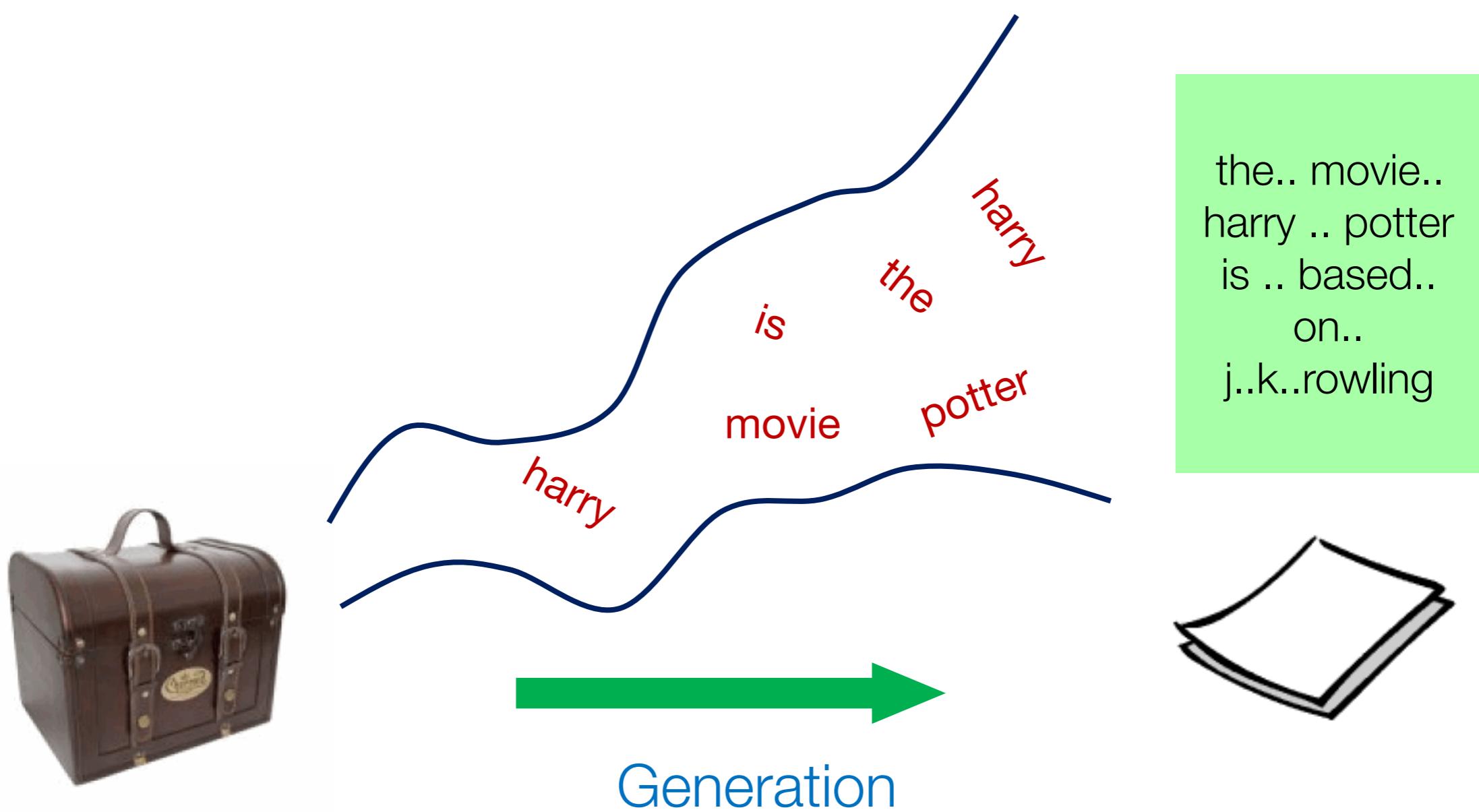
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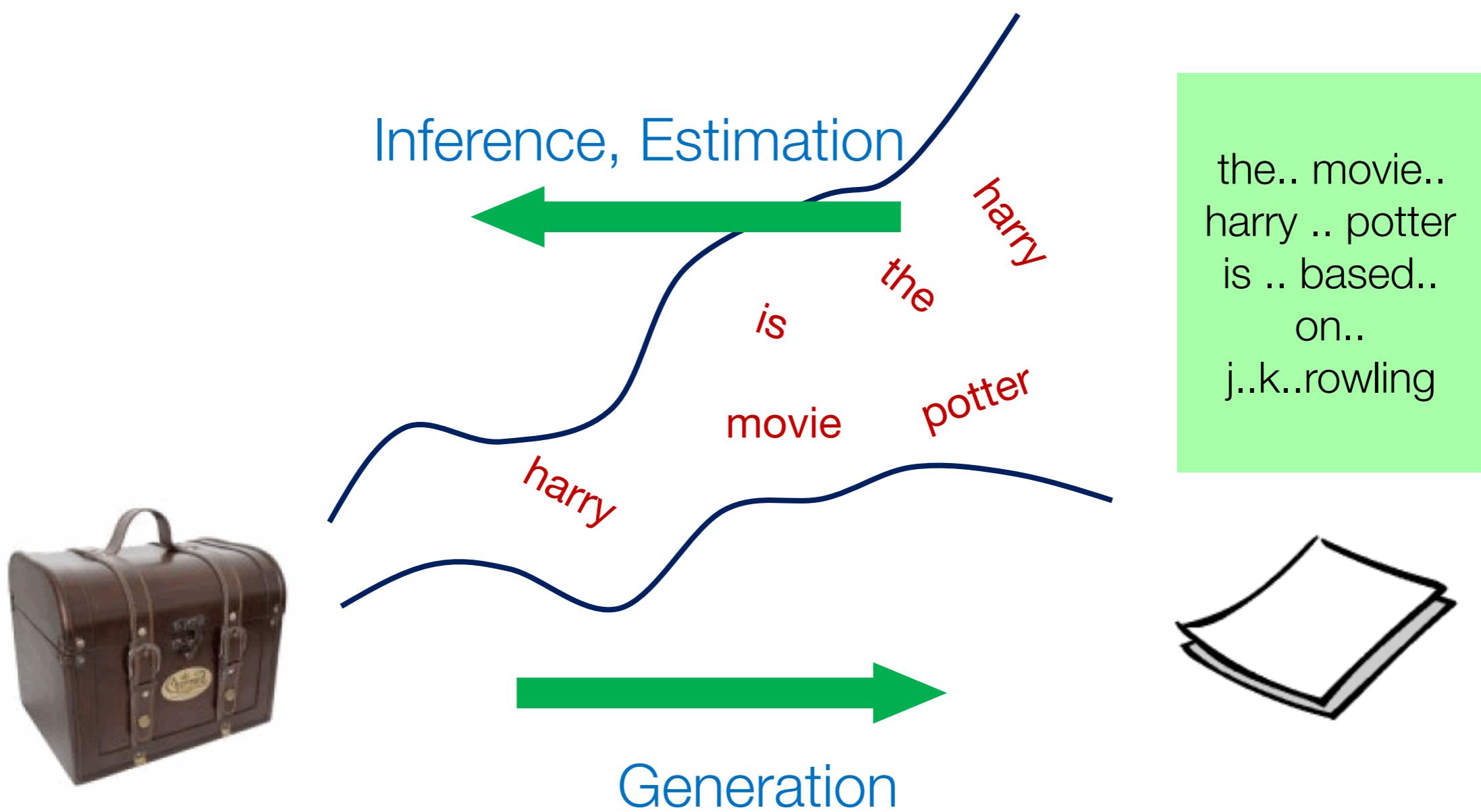
Generative Model of Text



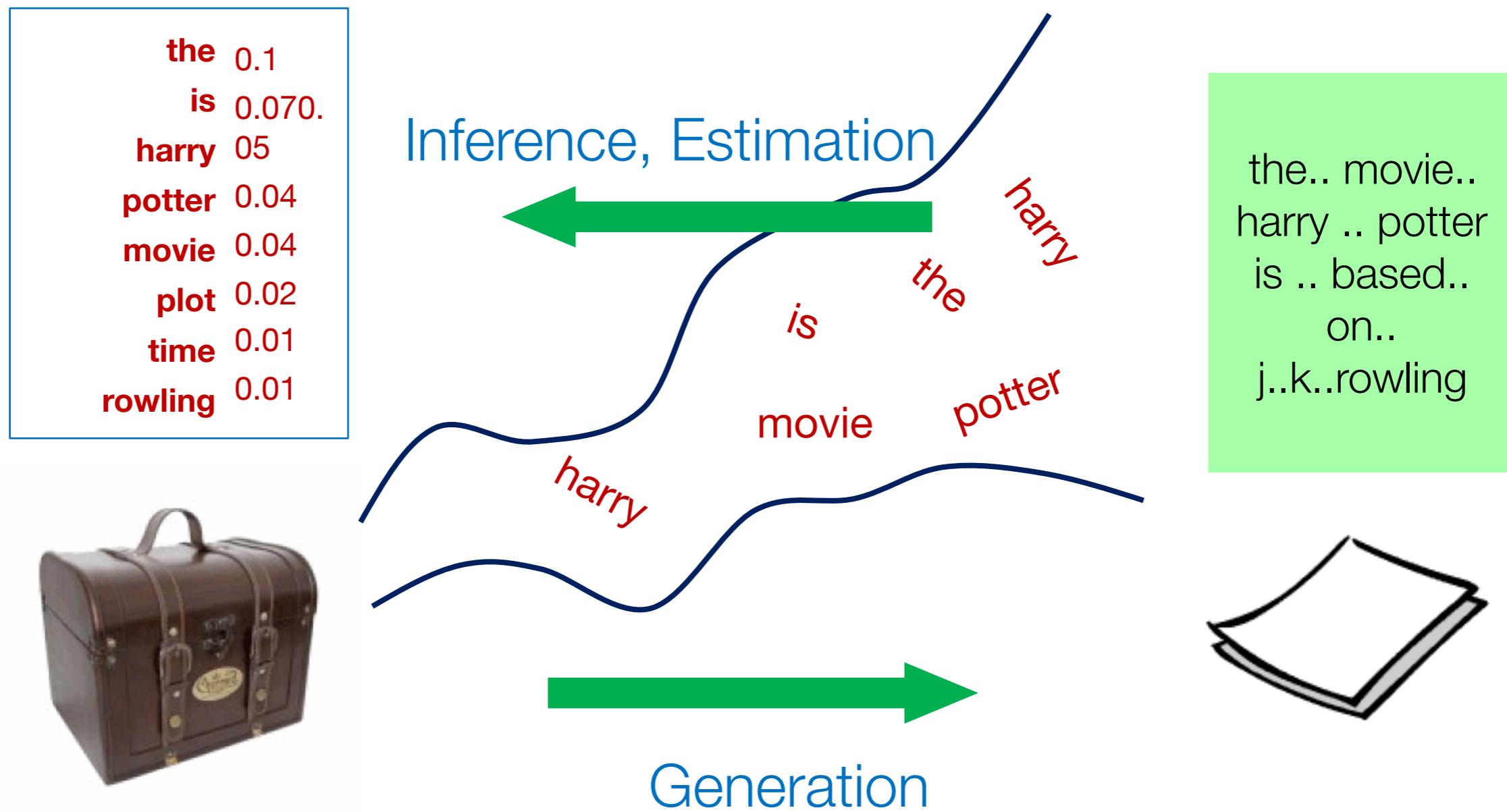
Generative Model of Text



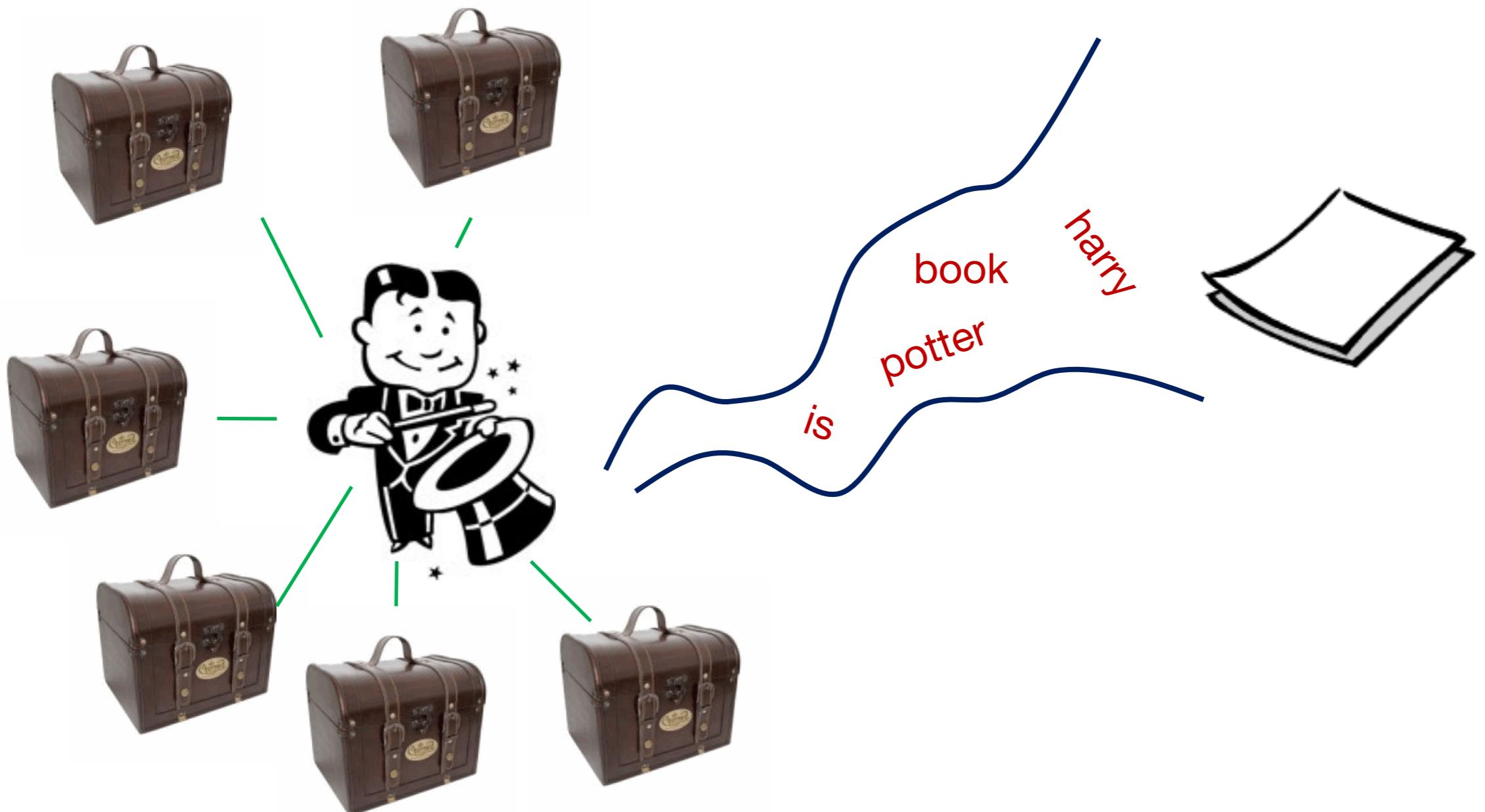
Generative Model of Text



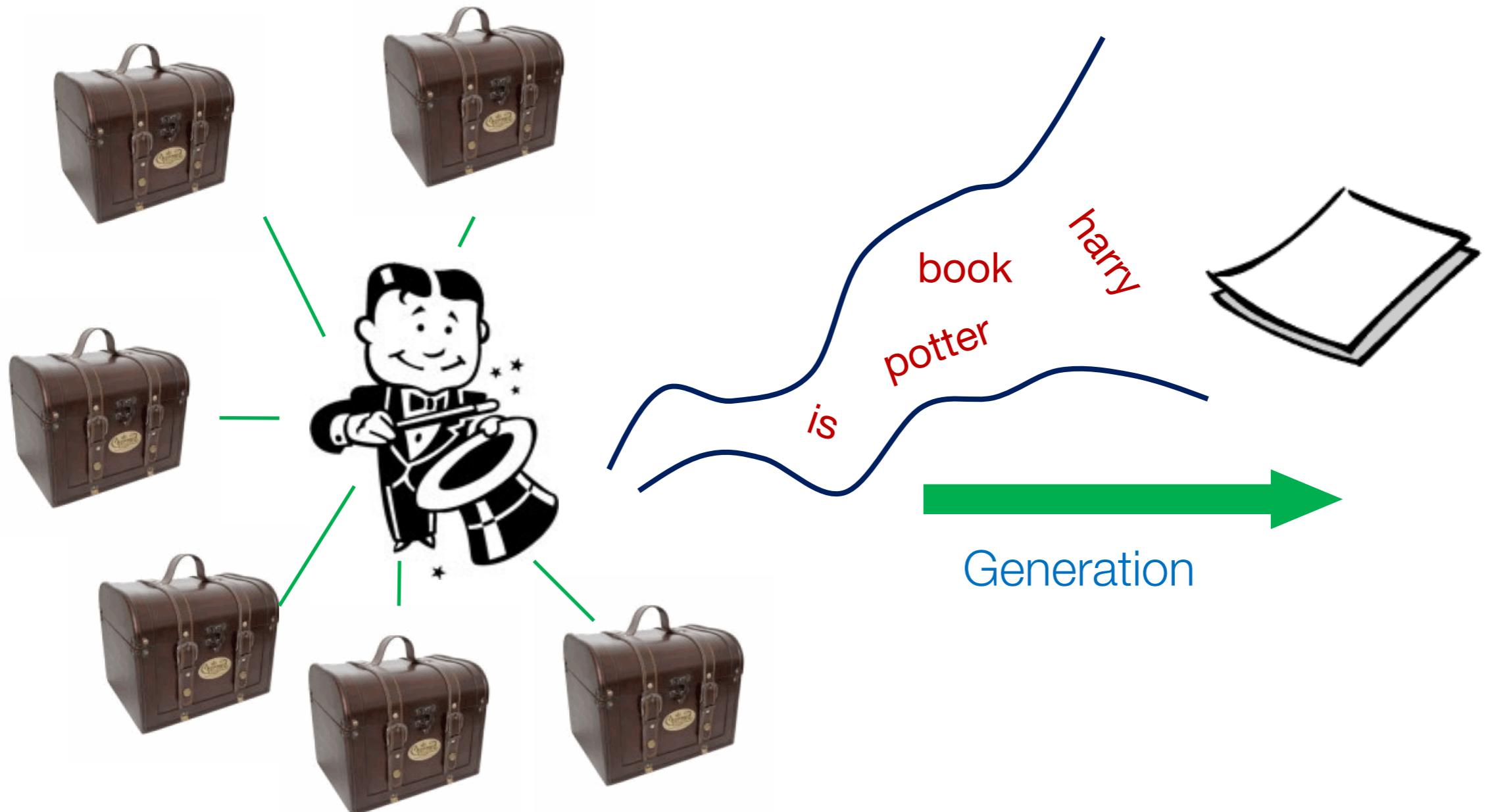
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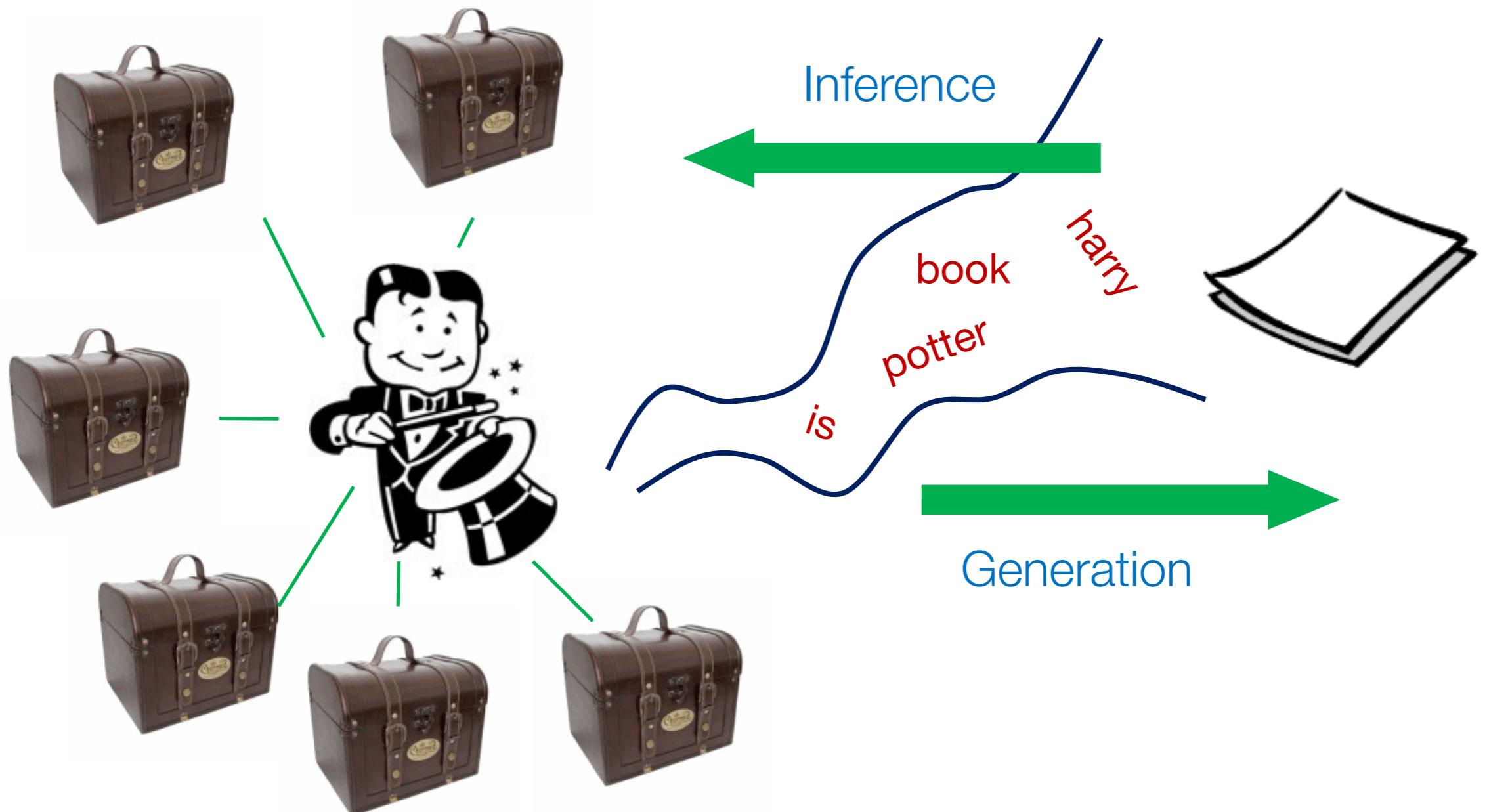
A Generative Model can be Much More Complicated (e.g., a Mixture Model)



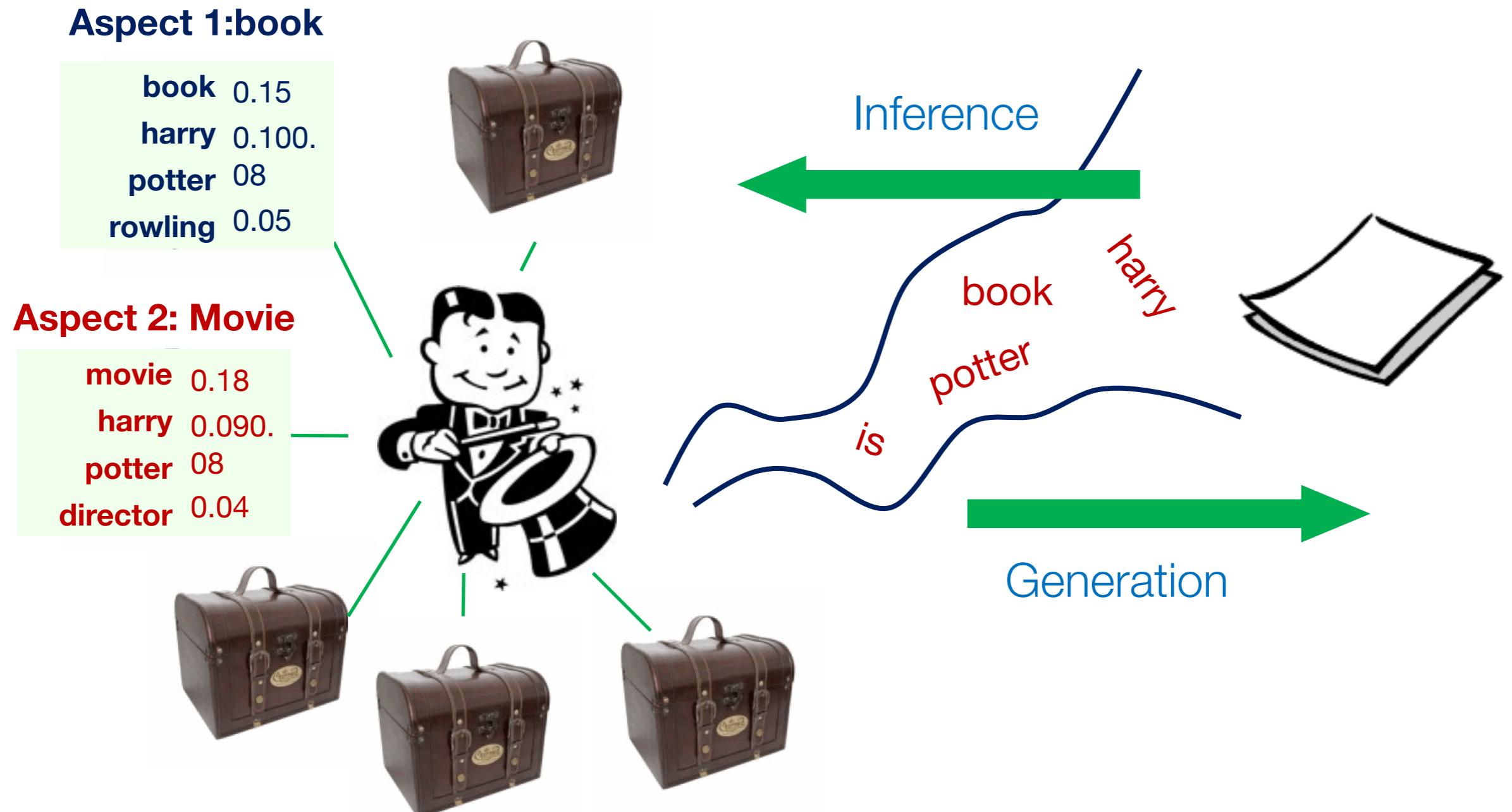
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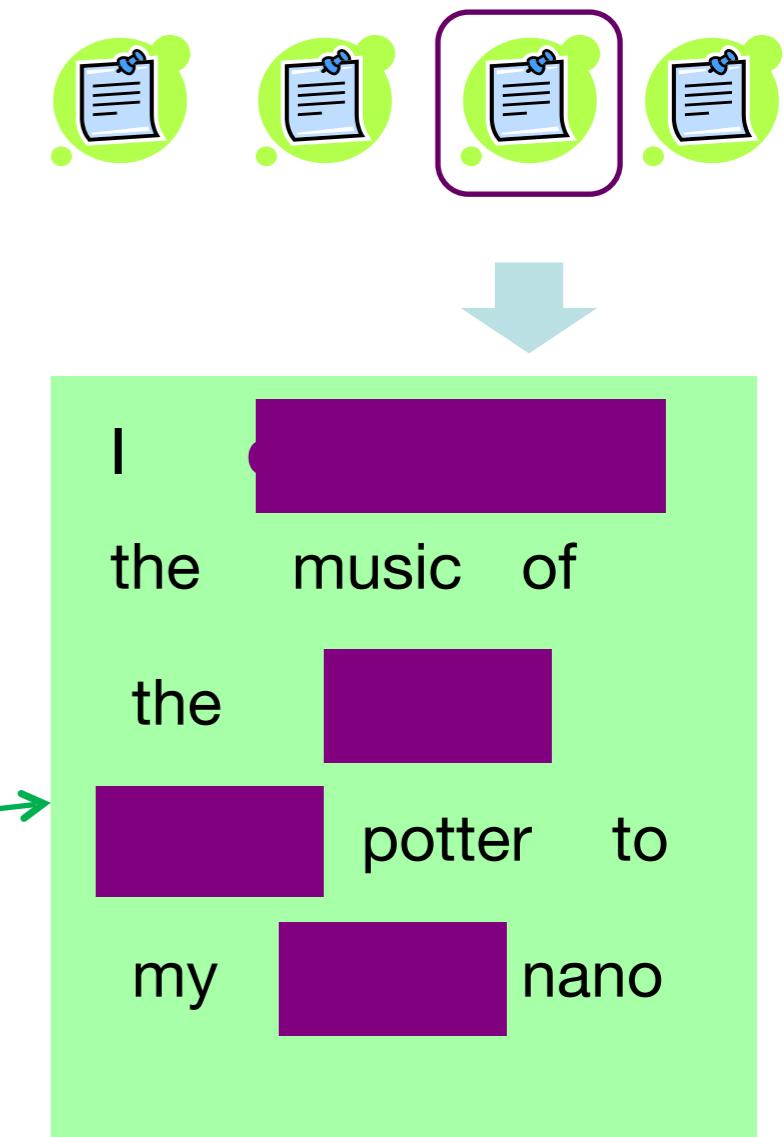
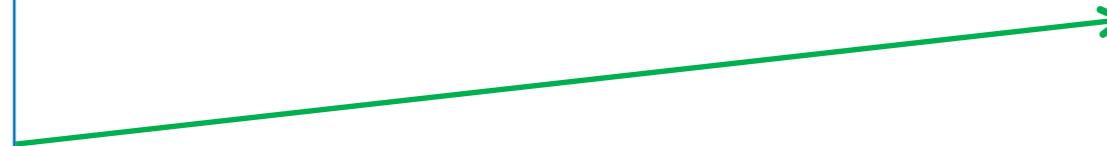
The Simplest Generative Model: Unigram Model

- Only one topic in the entire corpus, or one topic in each document
- Generative process:
- For each document d :
 - For each word token w_n in d :
 - Choose a word w according to the multinomial distribution $P(w)$.

Simple Unigram – Generative Process

$$P(d) = \prod_{w \in d} P(w)$$

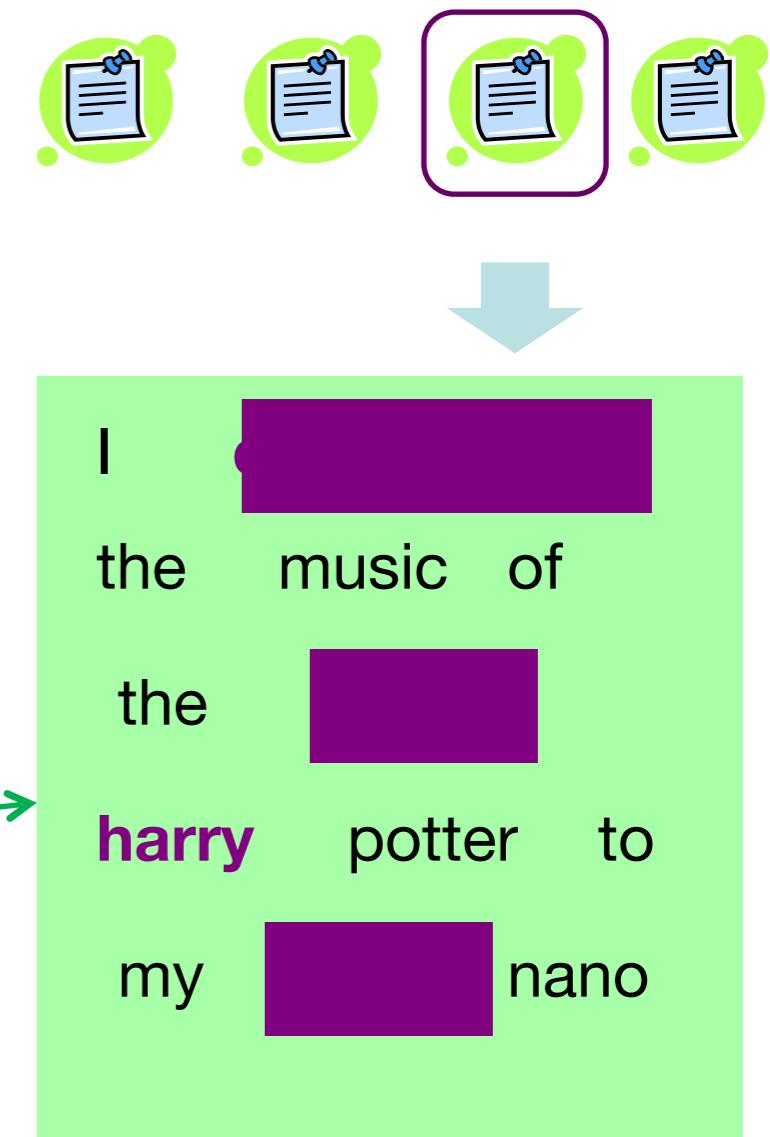
<i>movie</i>	0.100
<i>harry</i>	.090.
<i>potter</i>	05
<i>ipod</i>	0.01
<i>music</i>	0.02



Simple Unigram – Generative Process

$$P(d) = \prod_{w \in d} P(w)$$

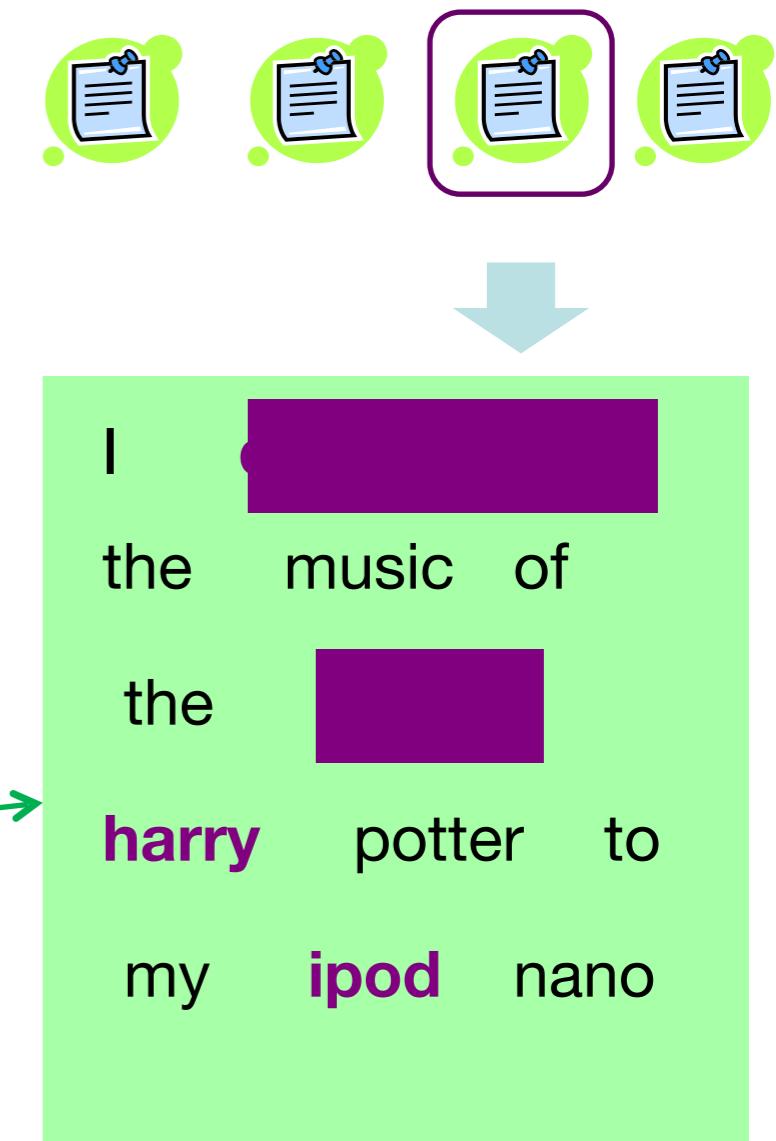
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<i>movie</i>	0.100
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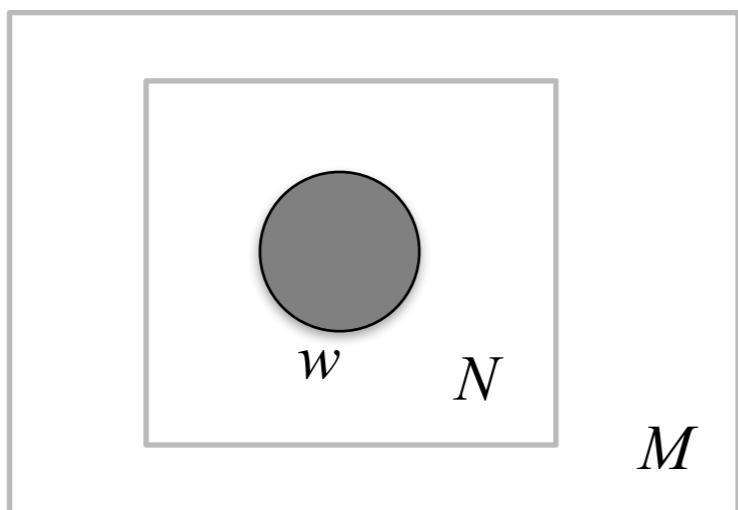
<i>movie</i>	0.100
<i>harry</i>	.090.
<i>potter</i>	05
<i>ipod</i>	0.01
<i>music</i>	0.02



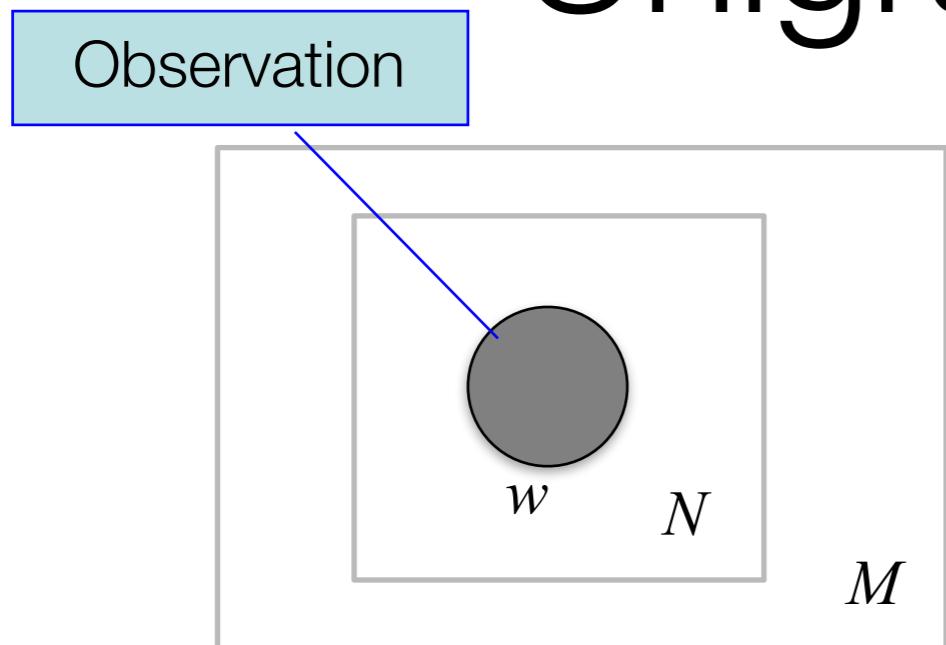
I **downloaded**
the music of
the **movie**
harry potter to
my **ipod** nano

Graphical Structure of Unigram Models

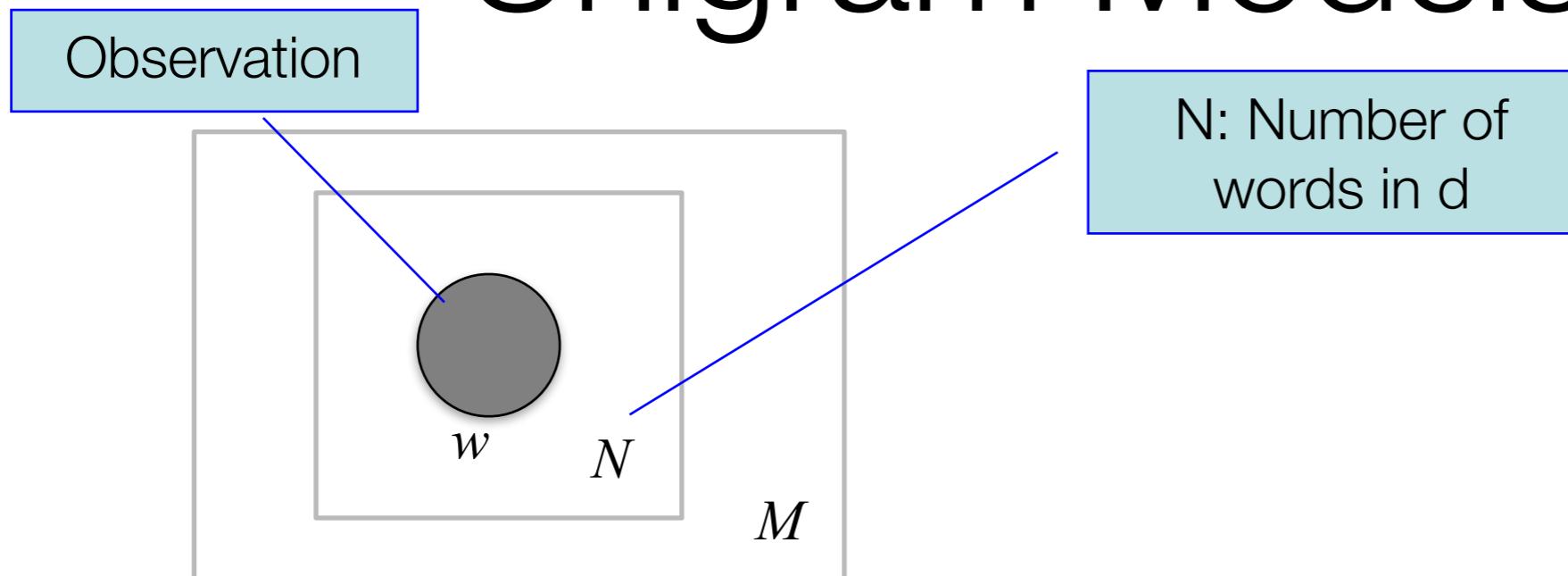
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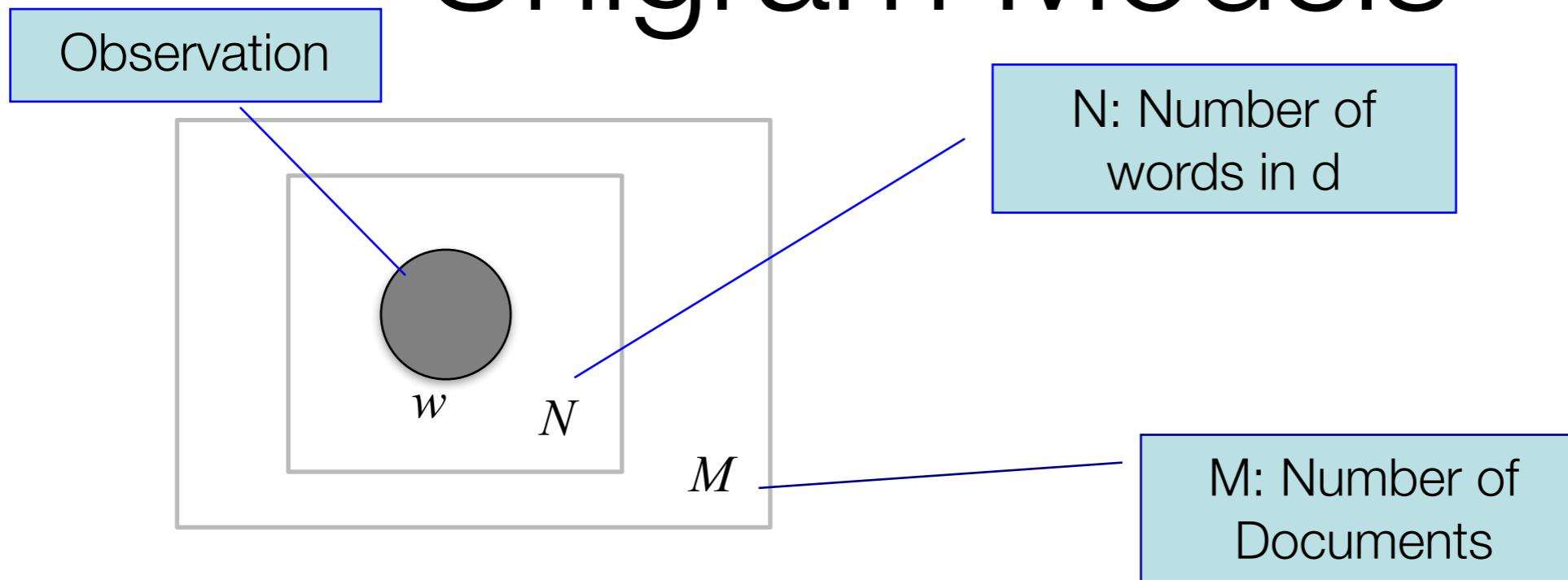
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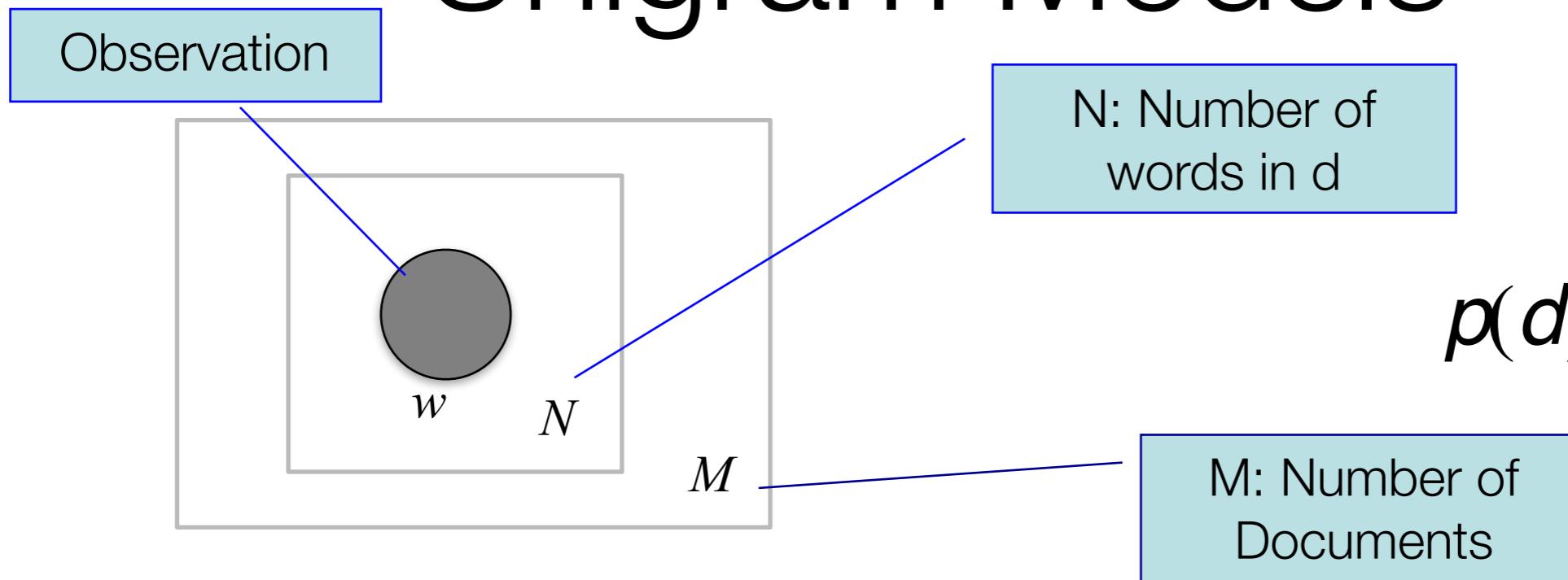
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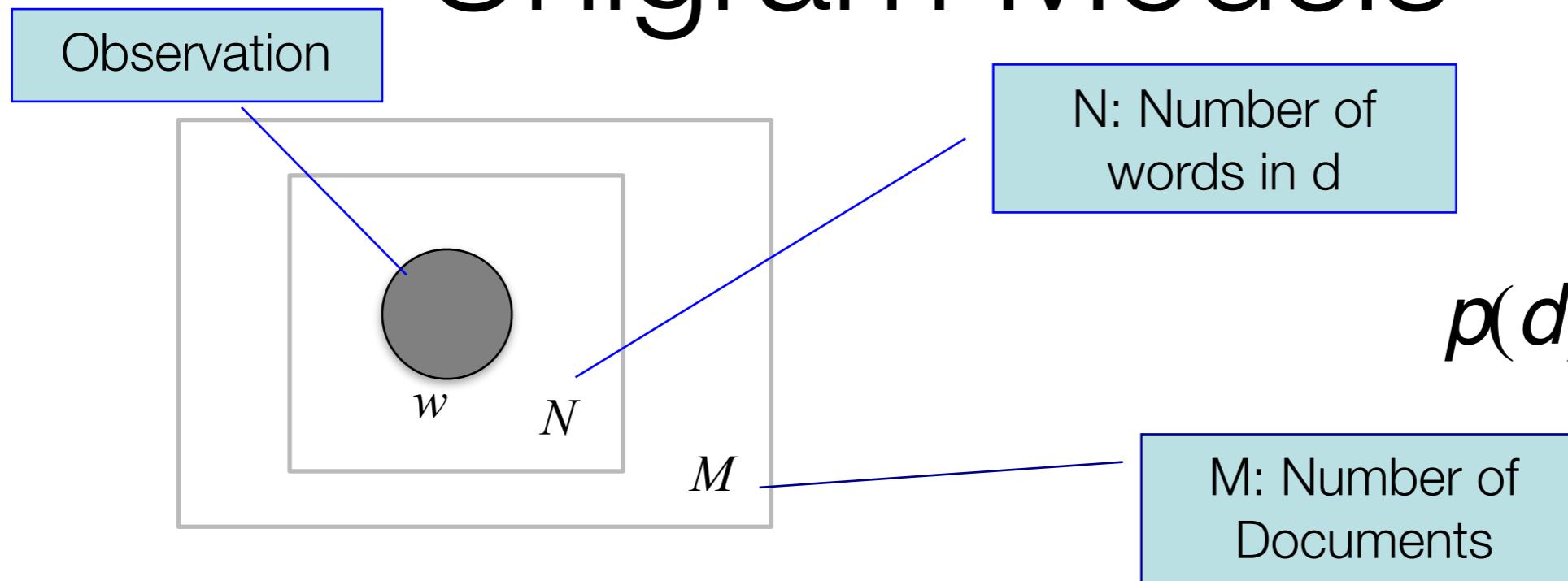


Graphical Structure of Unigram Models



$$p(d) = \prod_{n=1}^N p(w_n)$$

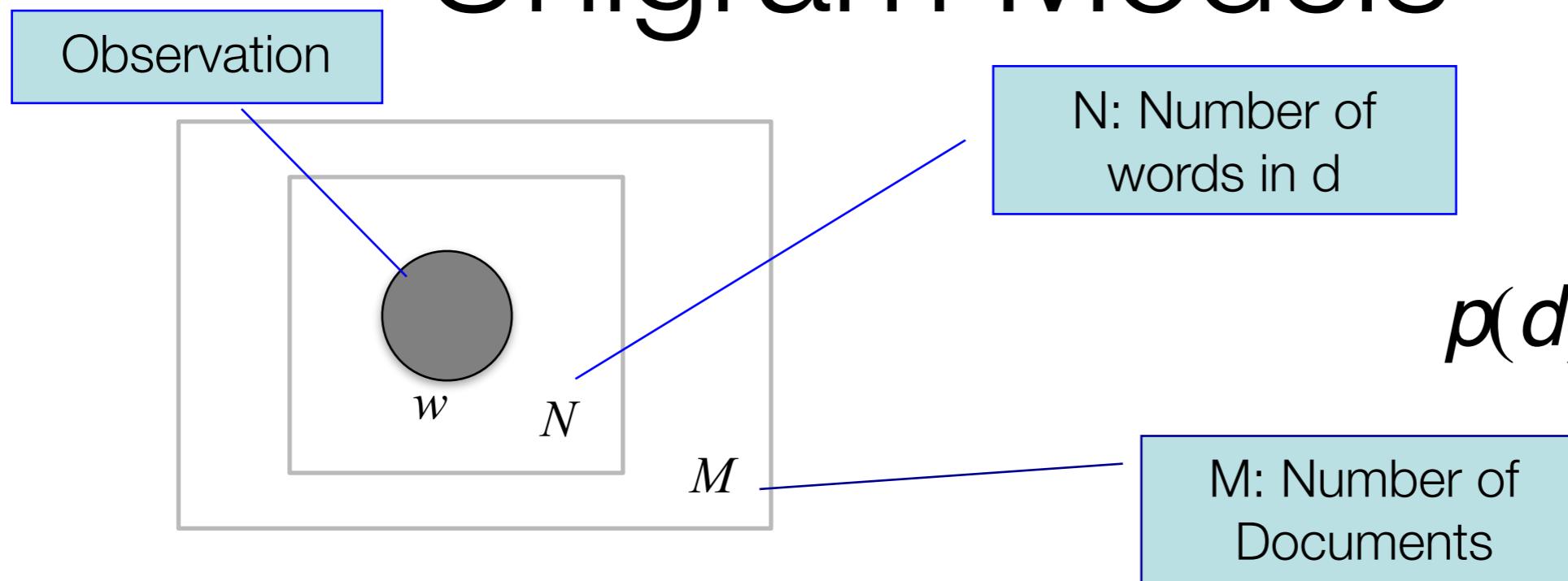
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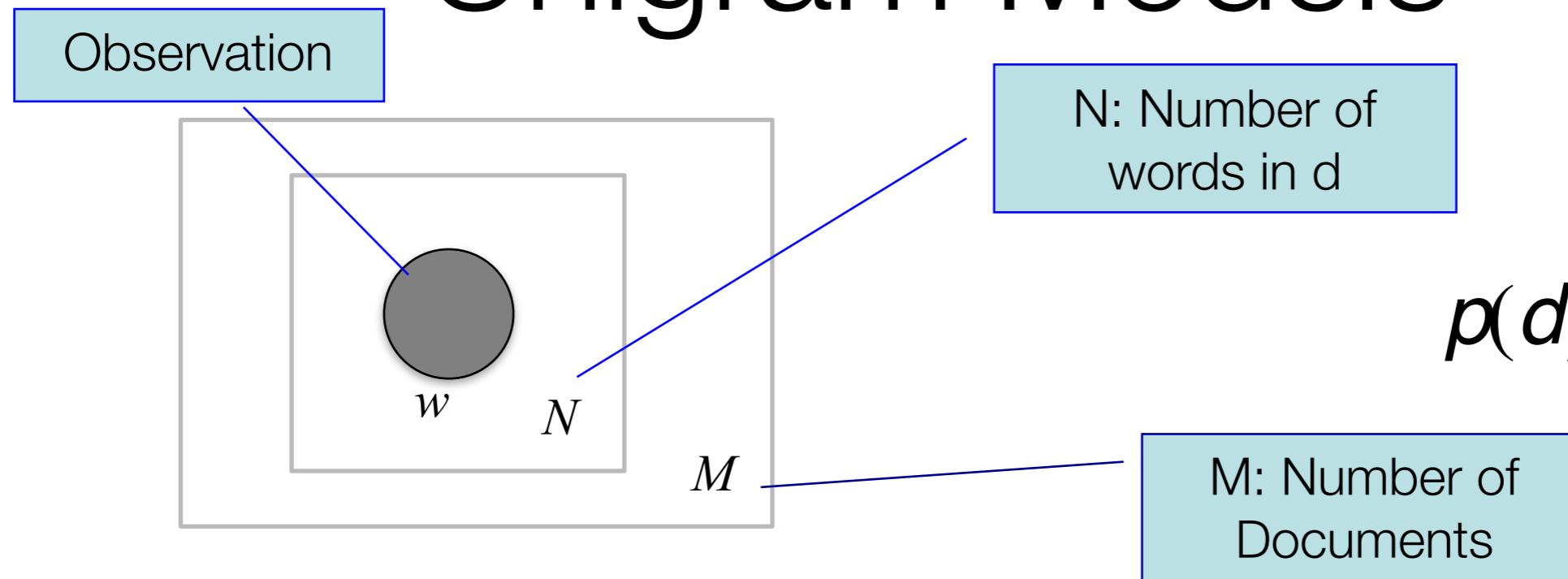
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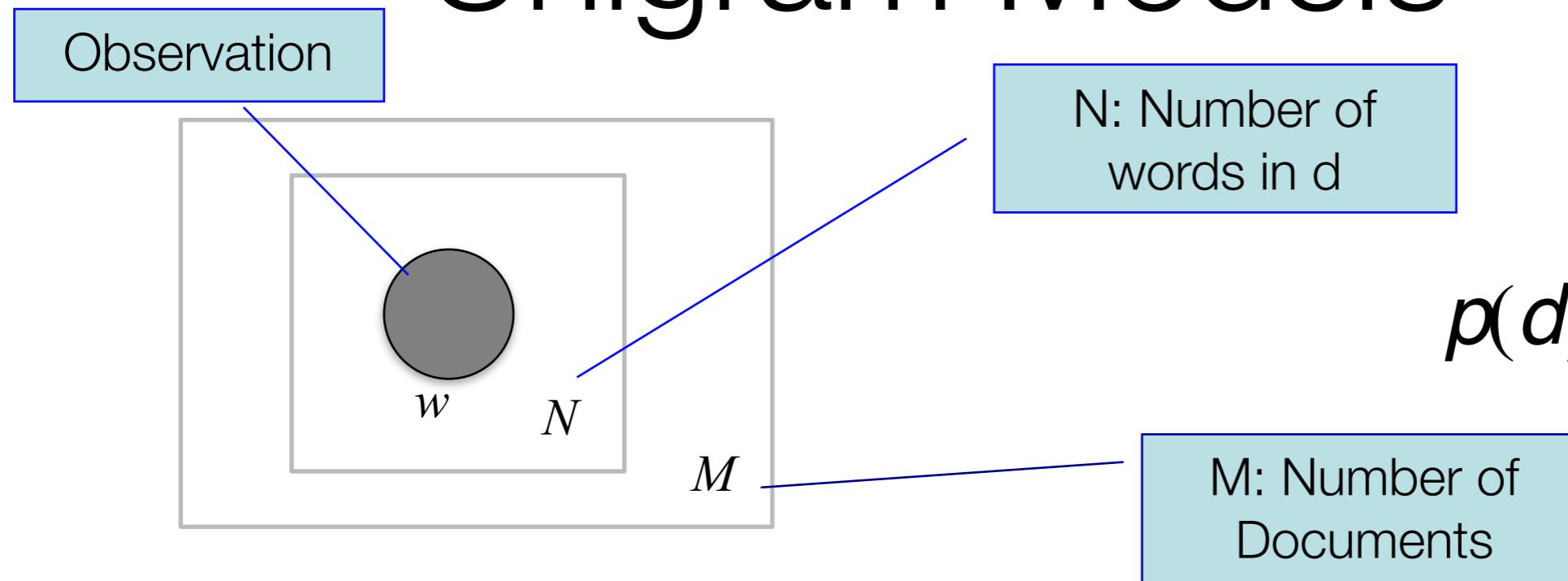
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- Applications:
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Mixture of Unigrams

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- The generative model of text becomes a mixture model
- Generative process:
 - For every document d
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 - For every word token in d
 - Choose a word w_n according to word distribution $P(w|z)$

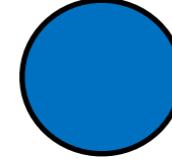
Mixture of Unigrams – Generative Process

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iPod	0.15
nano	0.080
music	.05
download	0.02
apple	0.01

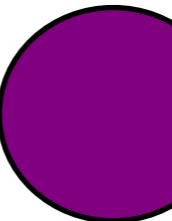
movie	0.100
harry	.090.
potter	05
iPod	0.01
music	0.02

Topic 1



Apple iPod

Topic 2



Harry Potter



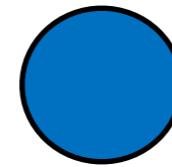
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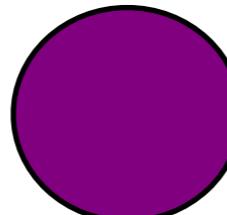
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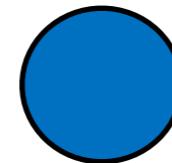
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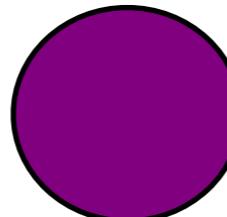
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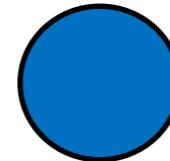
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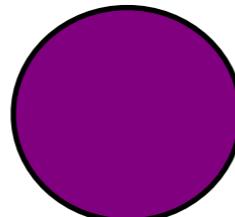
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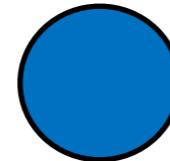
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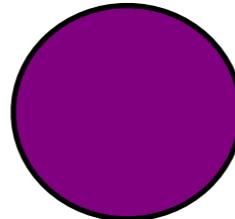
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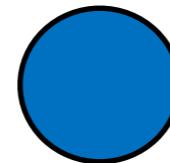
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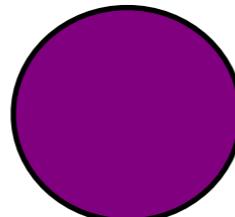
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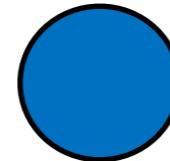
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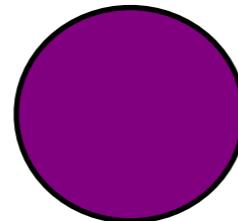
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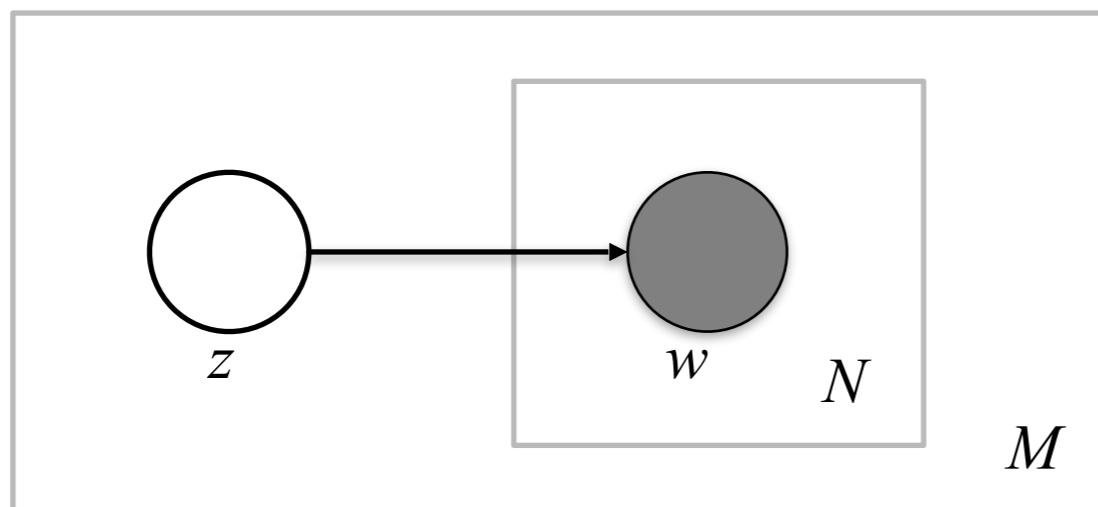
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- There are k topics in the collection, but each document only cover one topic

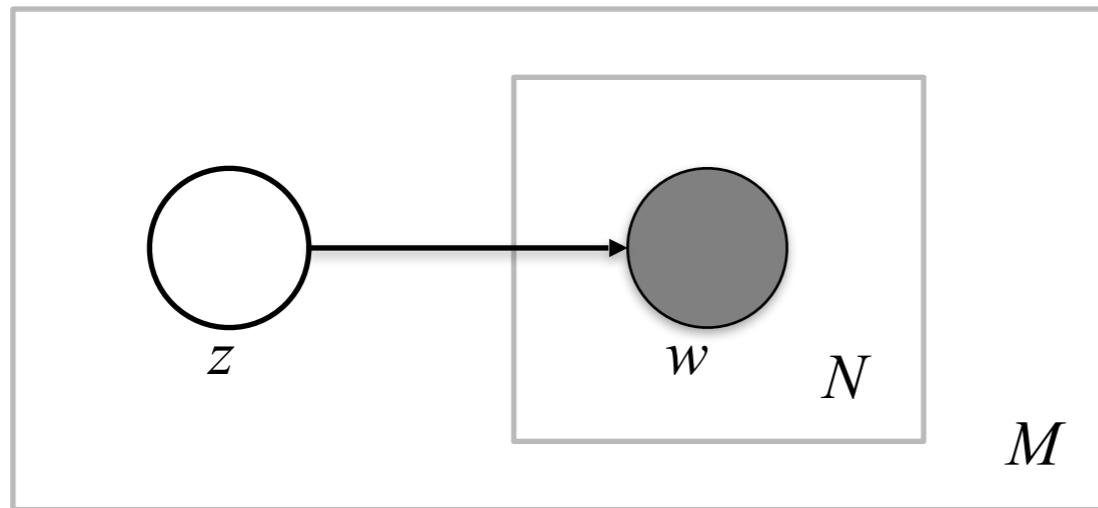
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$$p(d) = \sum_z p(z) \prod_{n=1}^N p(w_n | z)$$

- Estimation: MLE and EM Algorithm
- Application: simple document clustering

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- Cons:
 - Usually different parts of a document present different topics
 - Desirable: words in a document to take different identities

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(Hofmann, 1999)

- Also known as probabilistic latent semantic indexing (PLSI)
- Generative process:

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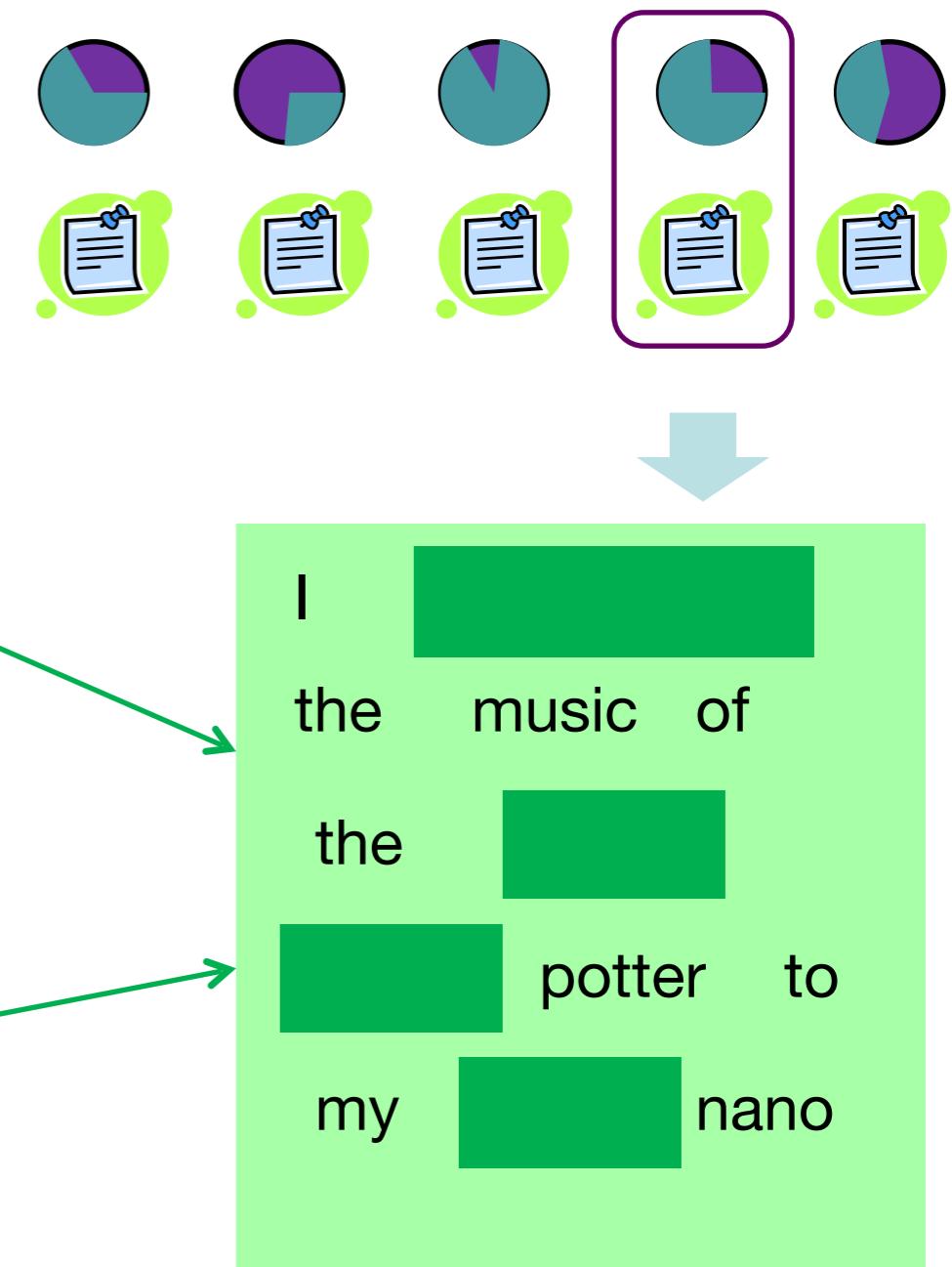
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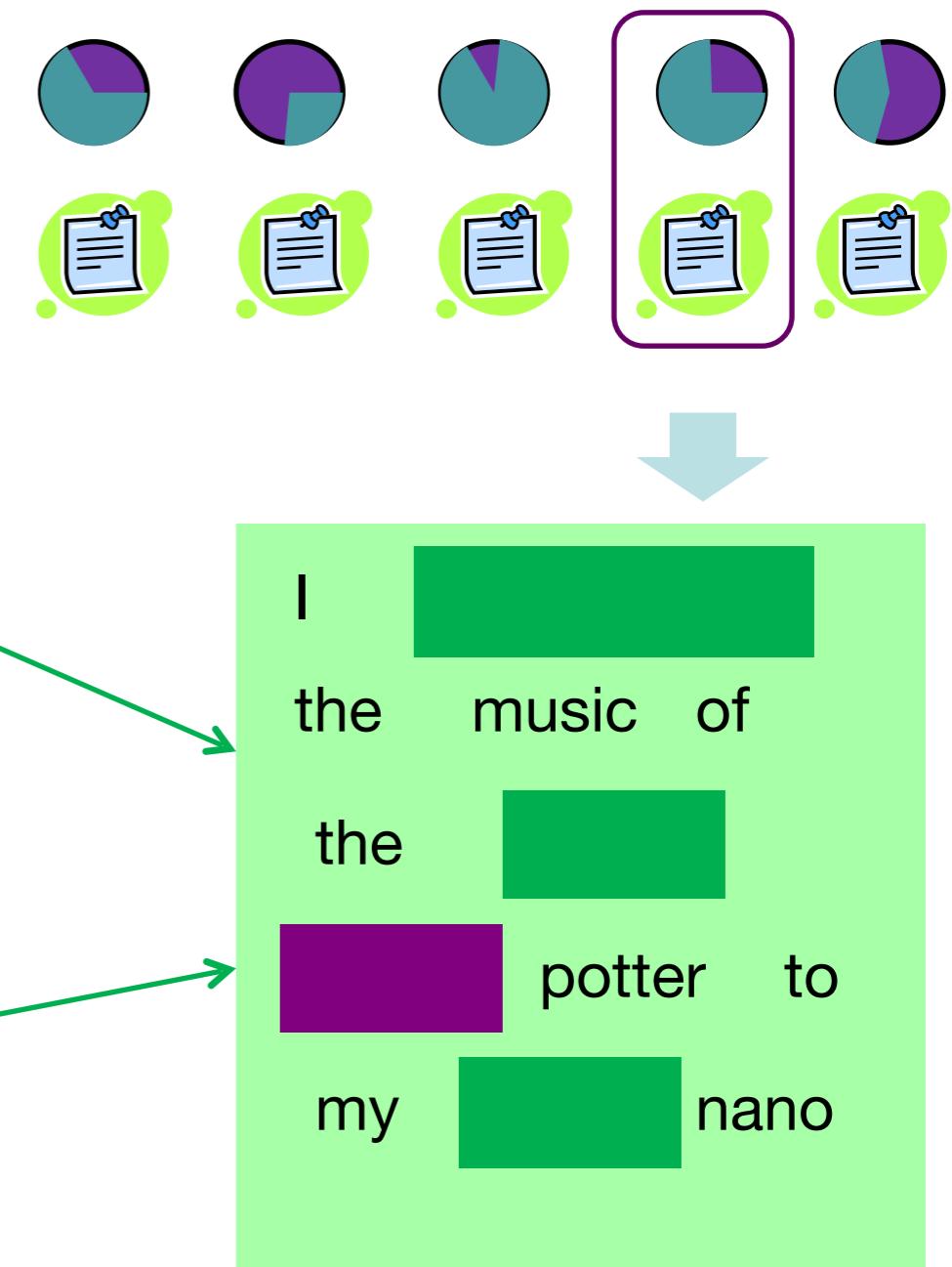


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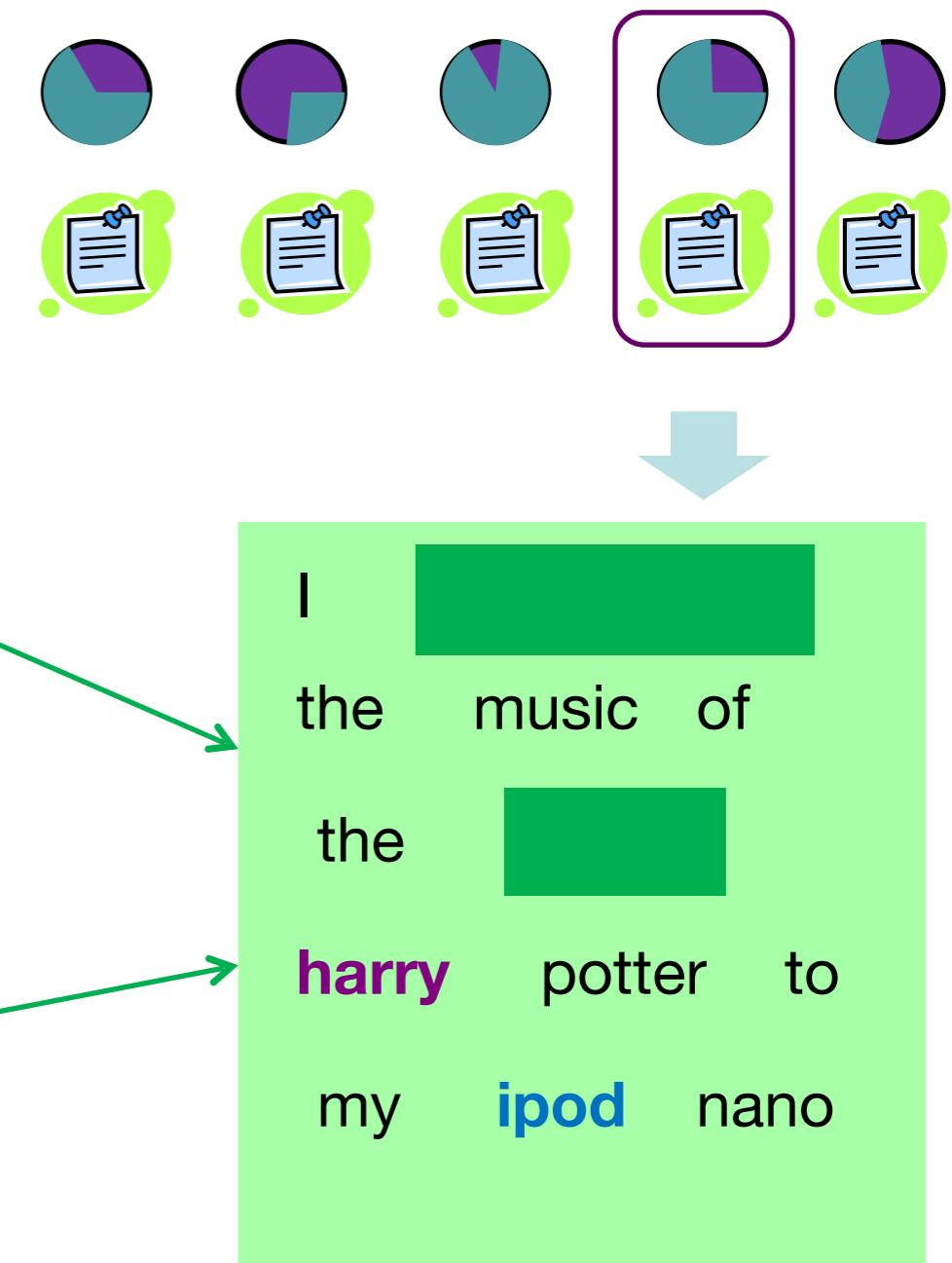
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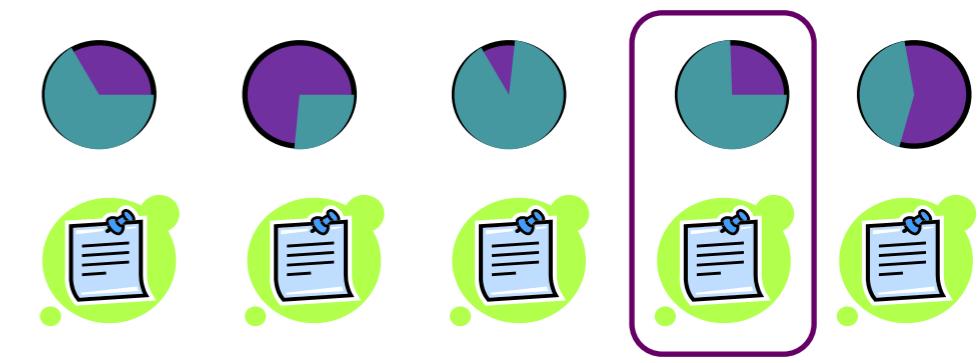
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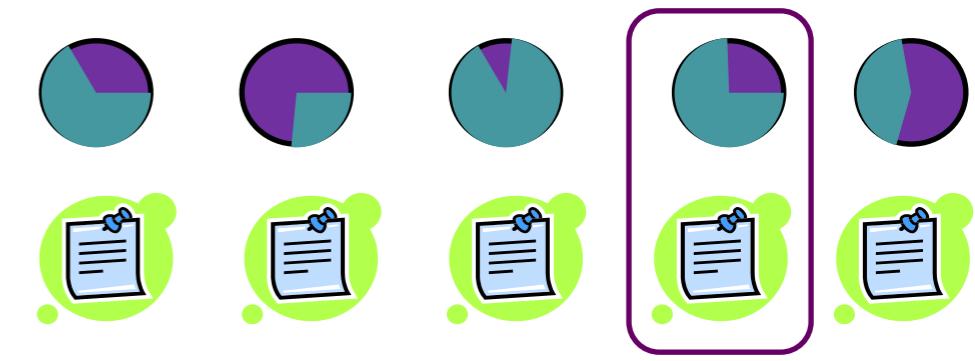
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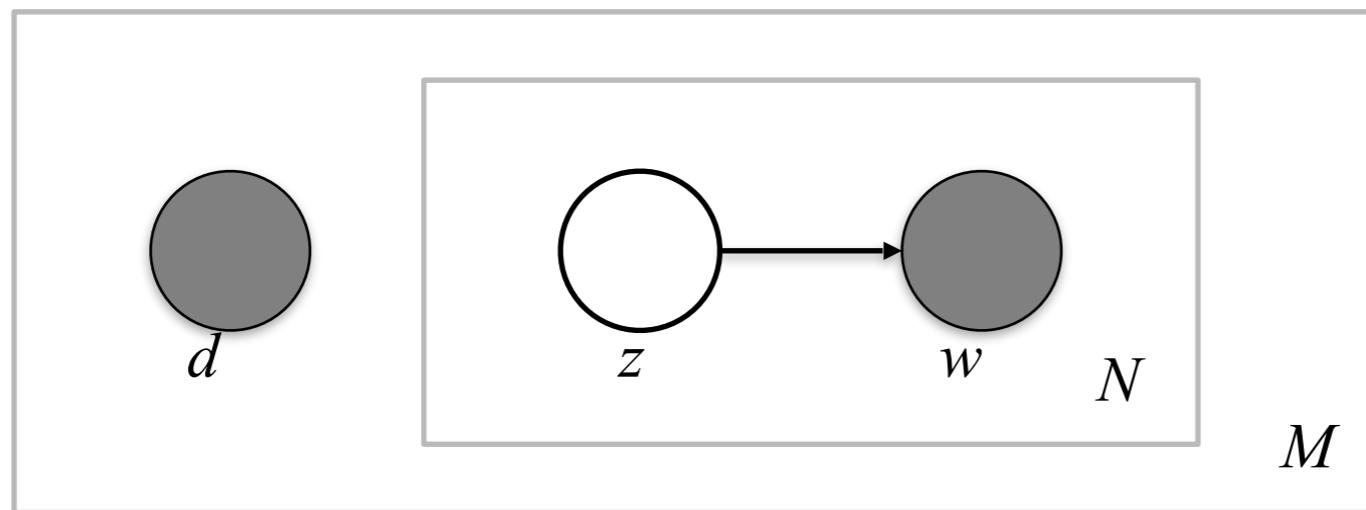
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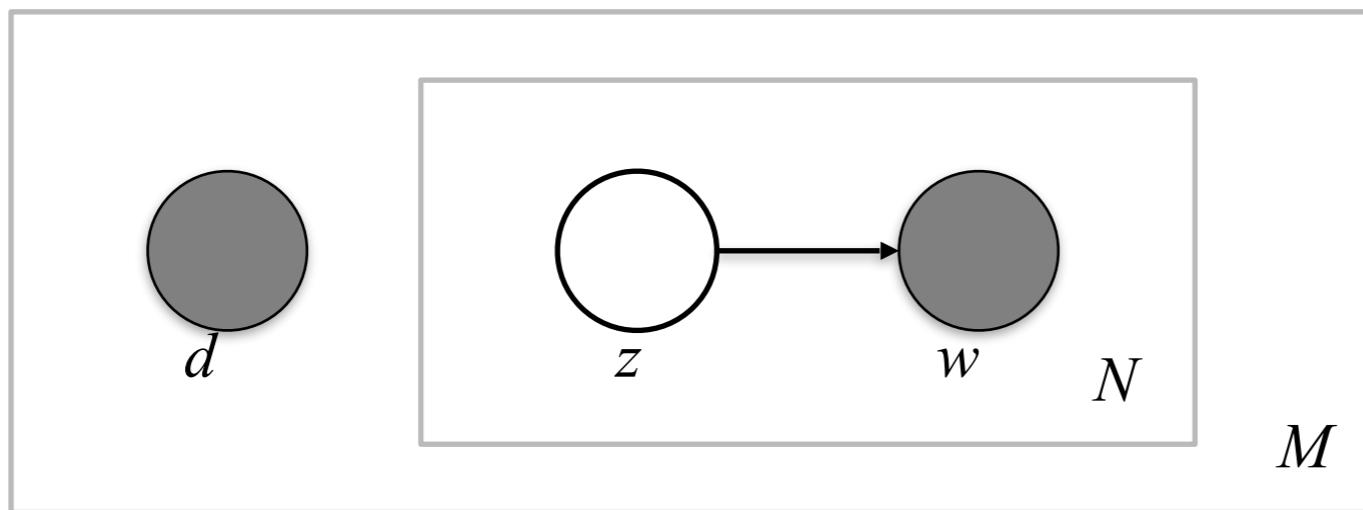
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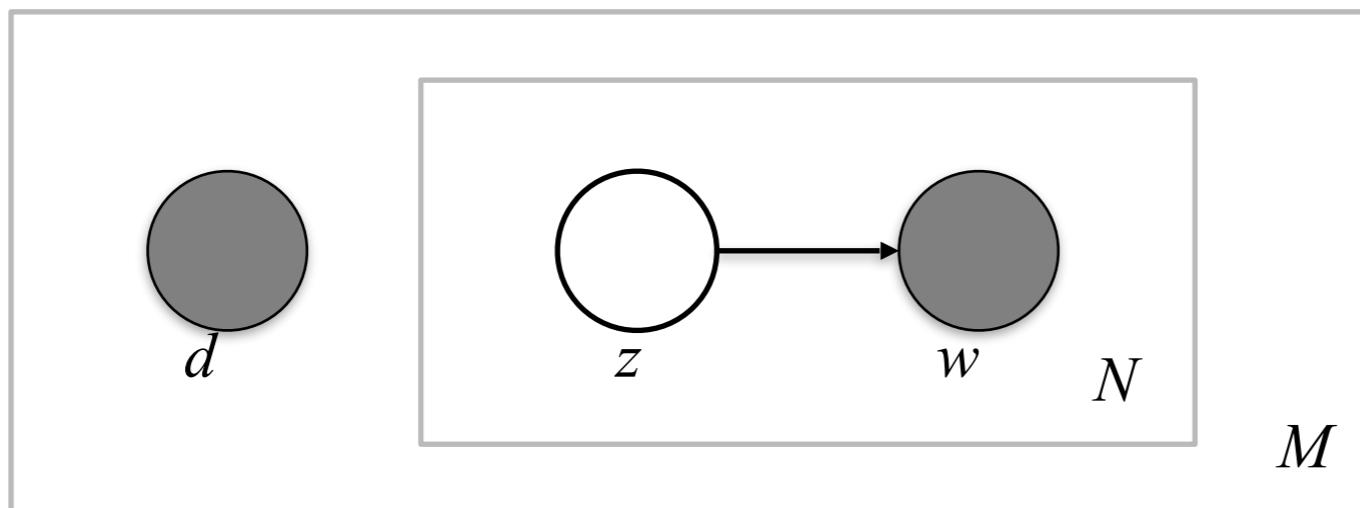


Graphical Model of PLSA



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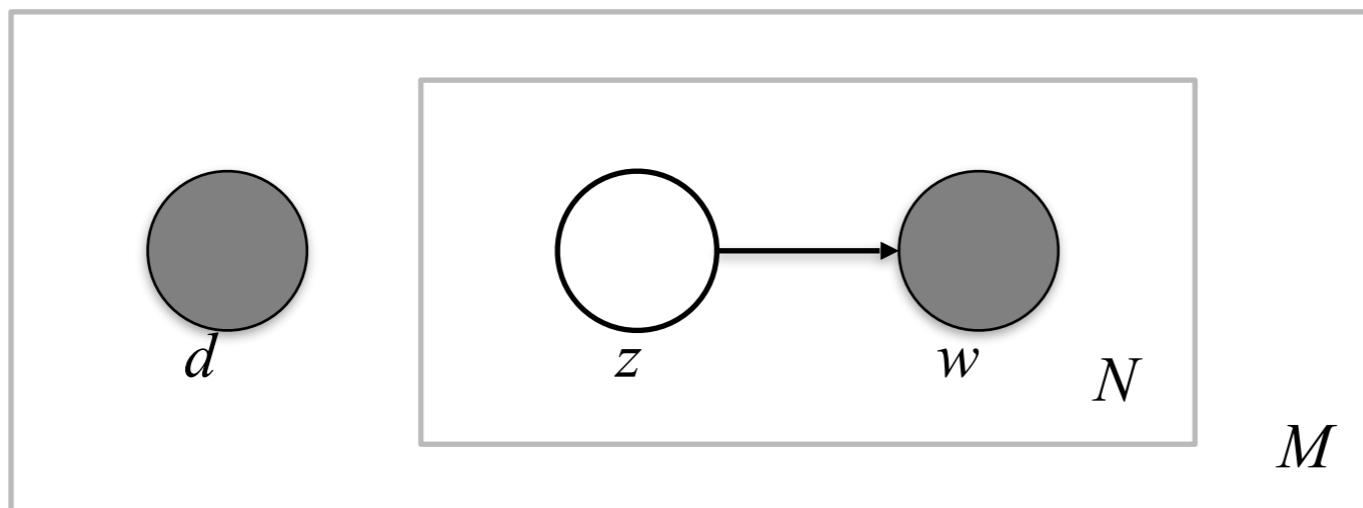
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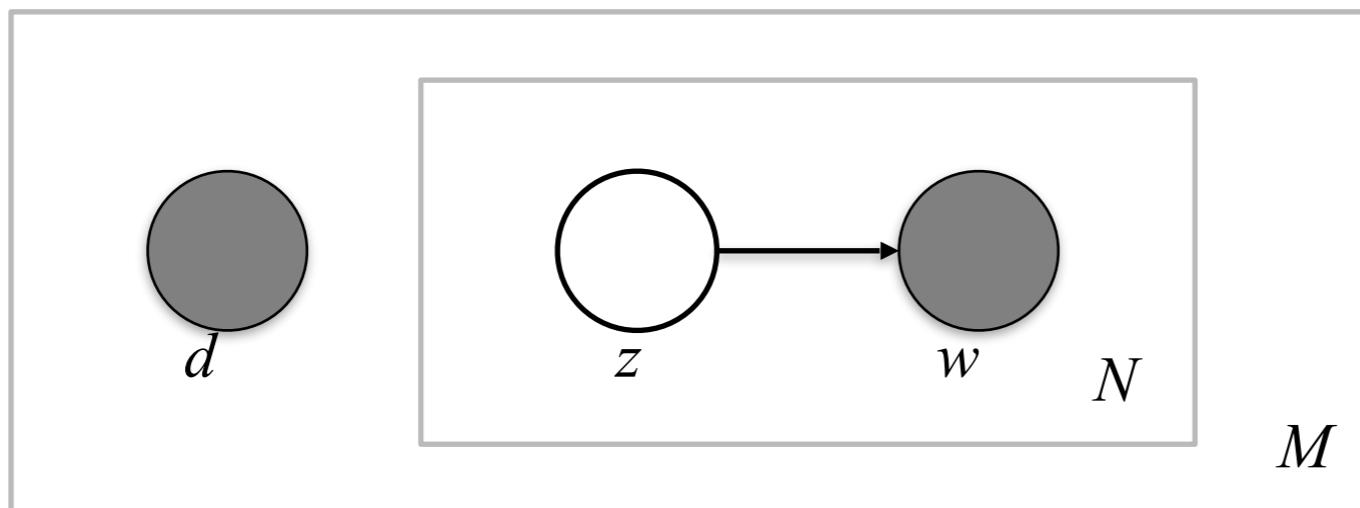
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- A “soft” version of K-means
- Cons:
 - Not a complete Bayesian model: hard to interpret unseen documents
 - Overfits the data (because number of parameters grows linearly with the number of documents).

Latent Dirichlet Allocation

(Blei&Ng&Jordan, 2003)

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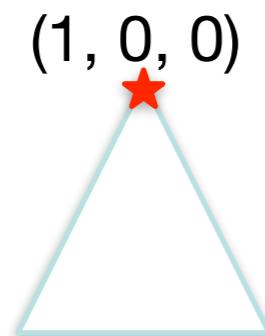
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- For a fuller Bayesian approach, can place a Dirichlet prior to these word multinomial distributions to smooth the probabilities.

Multinomial Distributions

- Distribution over discrete outcomes
- Represented by a non-negative vector that sums to one
- Picture repression of multinomial distributions over three discrete outcomes

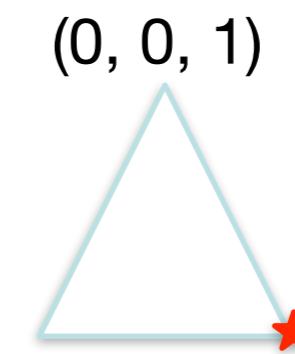
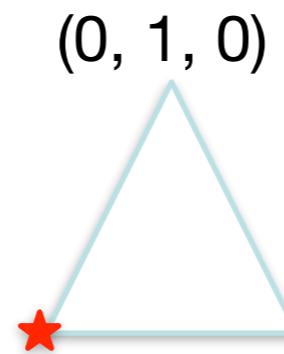
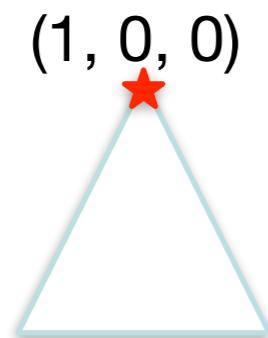
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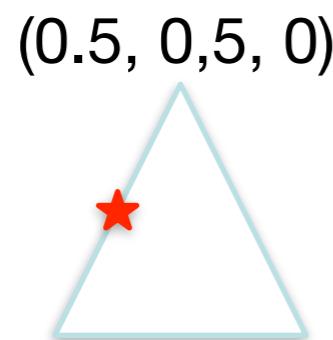
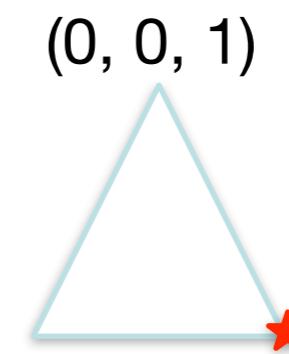
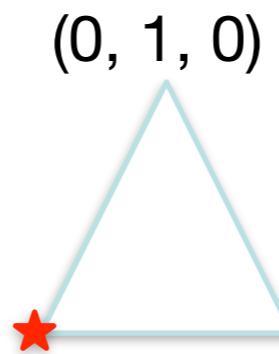
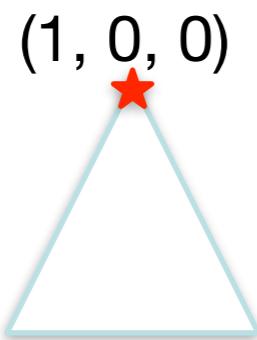
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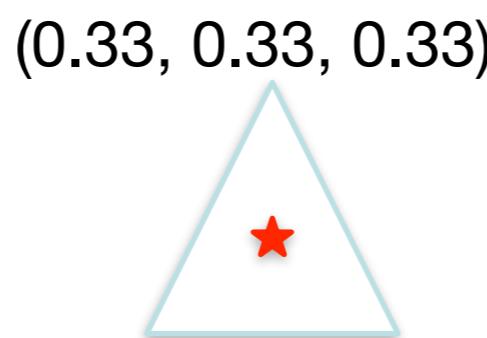
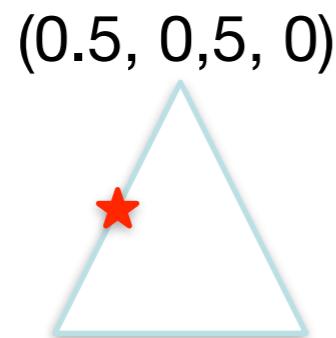
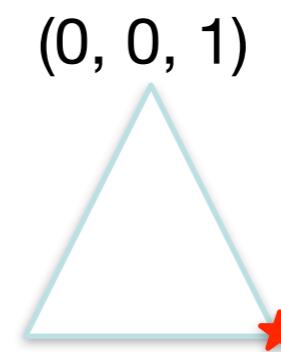
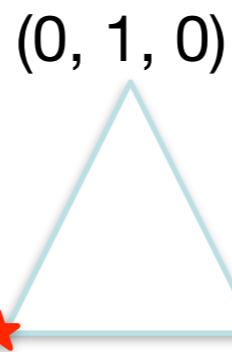
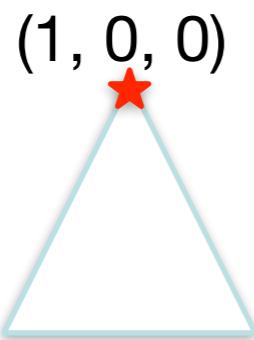
Multinomial Distributions

- Distribution over discrete outcomes
- Represented by a non-negative vector that sums to one
- Picture repression of multinomial distributions over three discrete outcomes



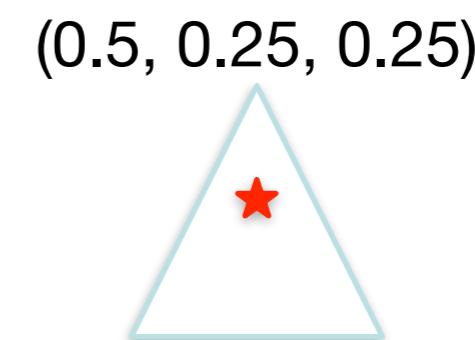
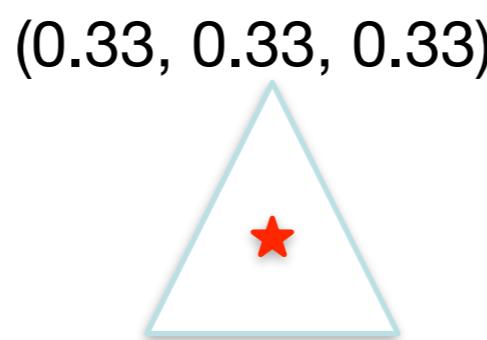
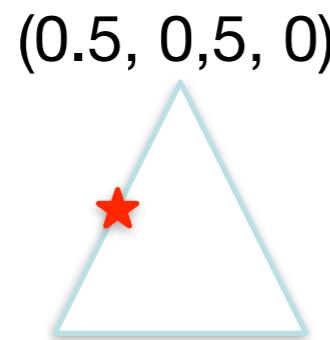
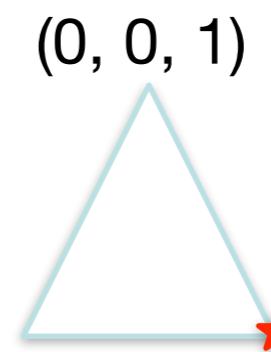
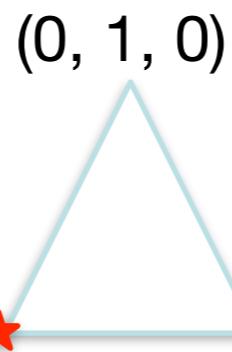
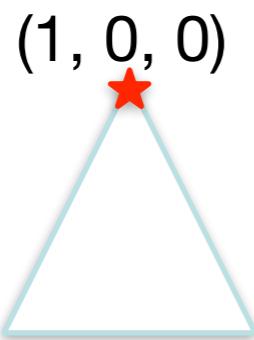
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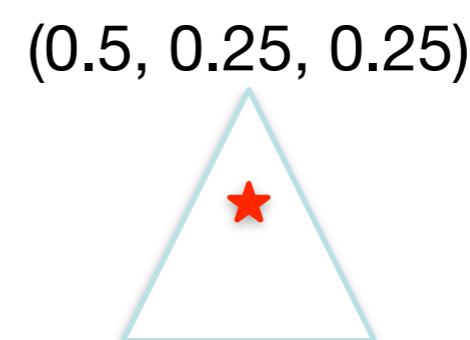
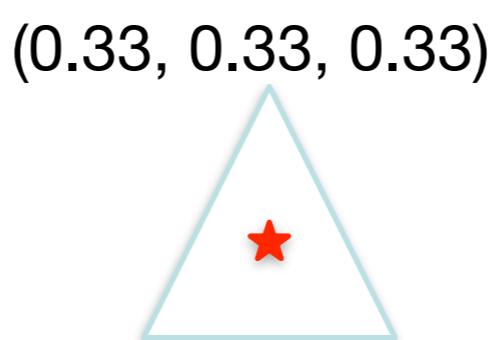
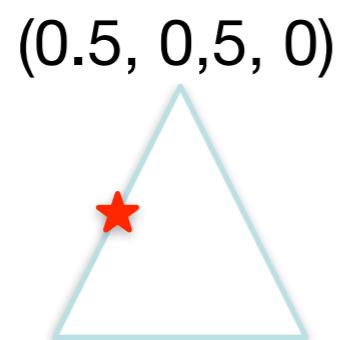
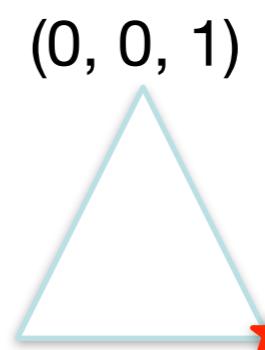
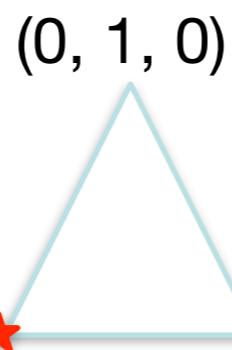
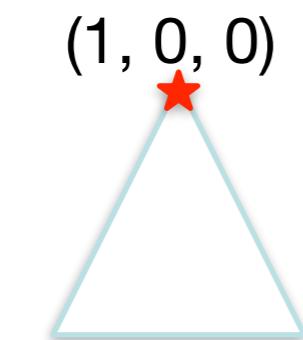
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Multinomial Distributions

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The Dirichlet distribution will give us a distribution over where a multinomial can be placed

Dirichlet Distribution as Priors over Multinomials

Think of α as a scale and m as a center

$$P(p|\alpha m) = \frac{\Gamma(\sum_k \alpha m_k)}{\prod_k \gamma(\alpha m_k)} \prod \rho_k^{\alpha m_k - 1}$$

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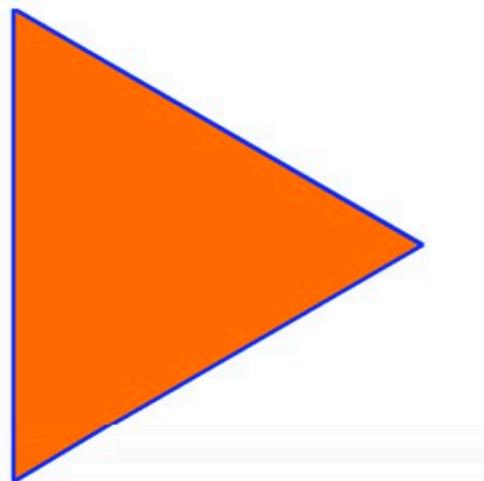
$$\alpha=3, \mathbf{m}=(0.3, 0.3, 0.3)$$

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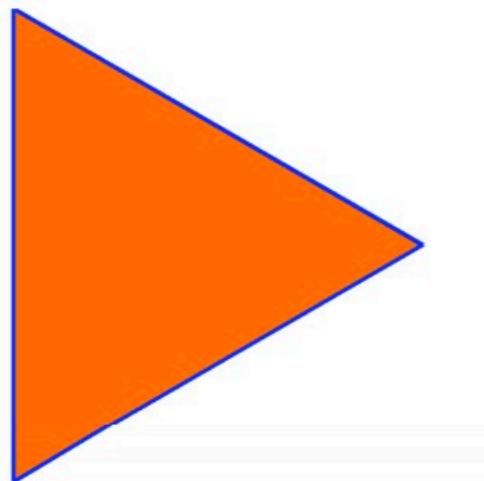


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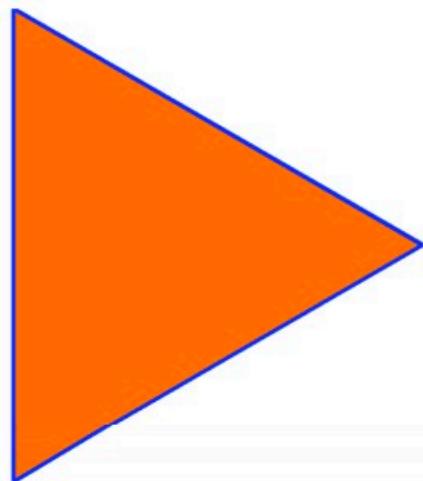


Dirichlet Distribution as Priors over Multinomials

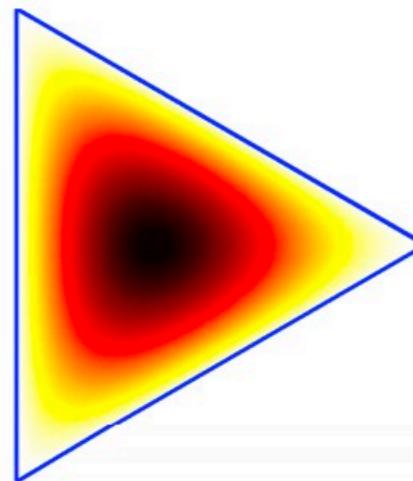
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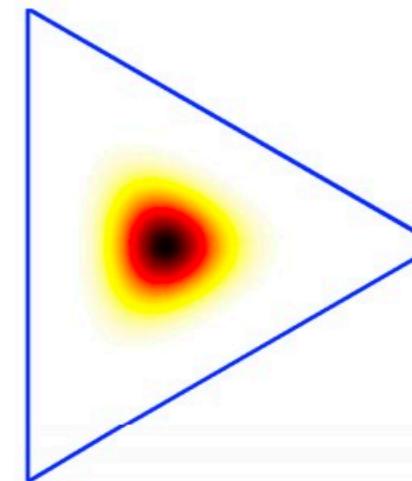
$\alpha=3, \mathbf{m}=(0.3,0.3,0.3)$



$\alpha=6, \mathbf{m}=(0.3,0.3,0.3)$



$\alpha=30, \mathbf{m}=(0.3,0.3,0.3)$

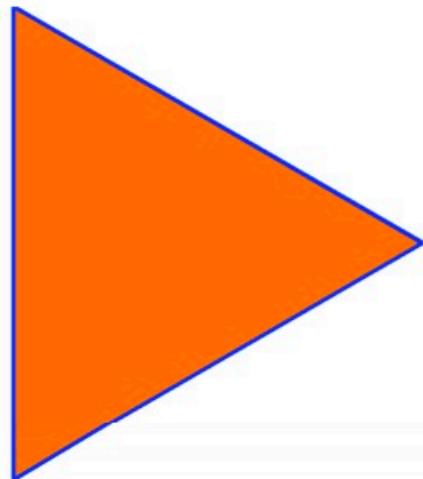


Dirichlet Distribution as Priors over Multinomials

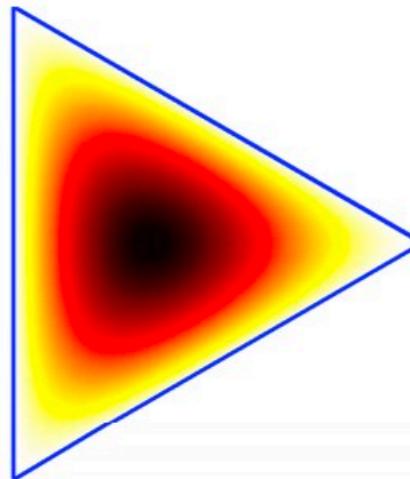
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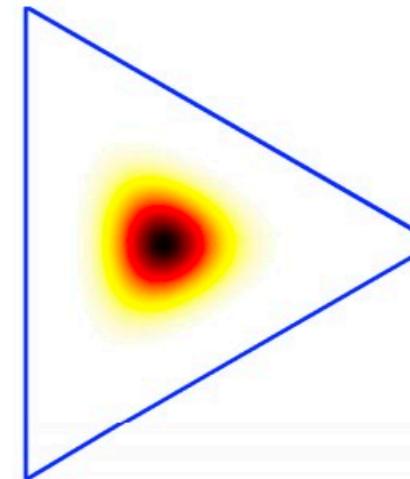
$\alpha=3, \mathbf{m}=(0.3,0.3,0.3)$



$\alpha=6, \mathbf{m}=(0.3,0.3,0.3)$



$\alpha=30, \mathbf{m}=(0.3,0.3,0.3)$



$\alpha=14, \mathbf{m}=(1/7, 5/7, 1/7)$

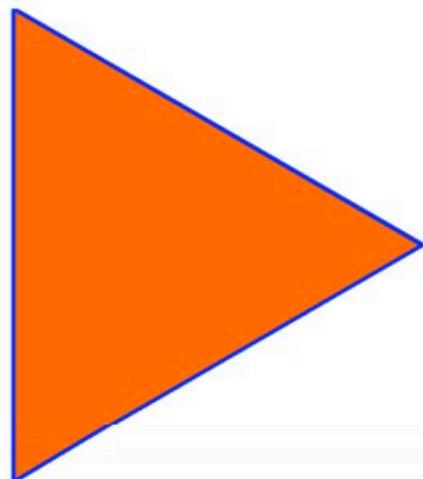
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Dirichlet Distribution as Priors over Multinomials

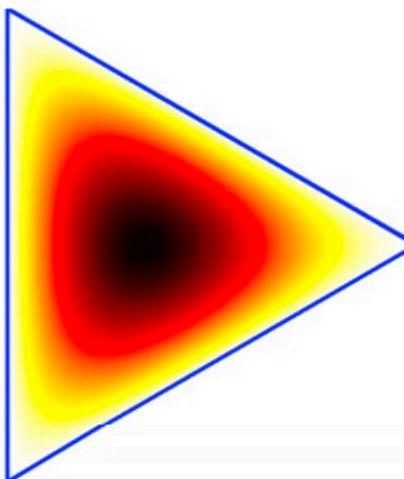
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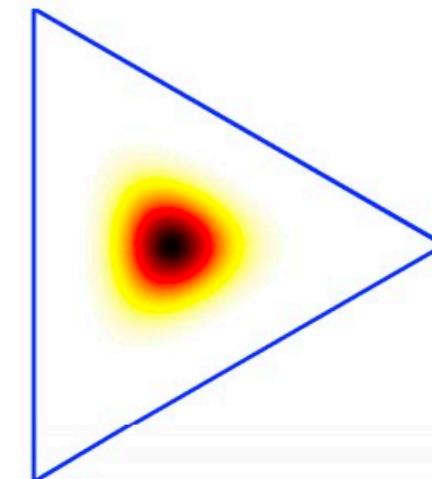
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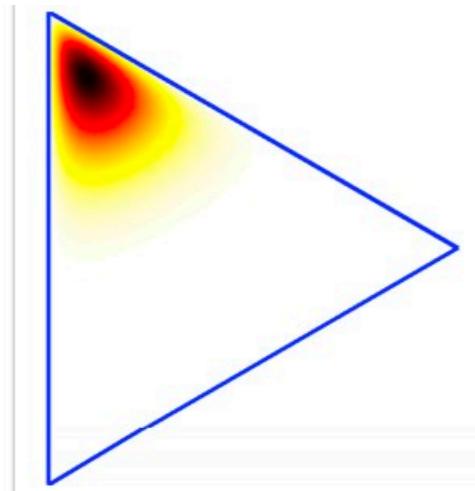
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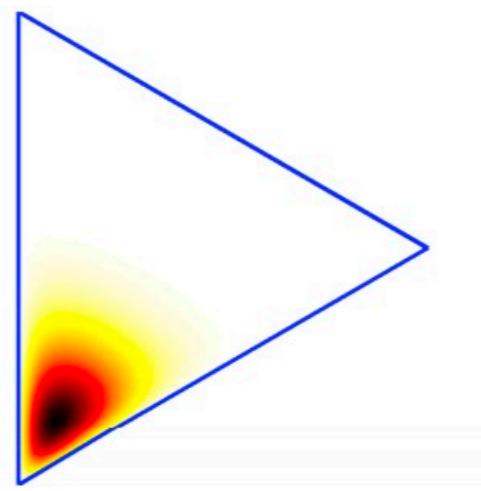
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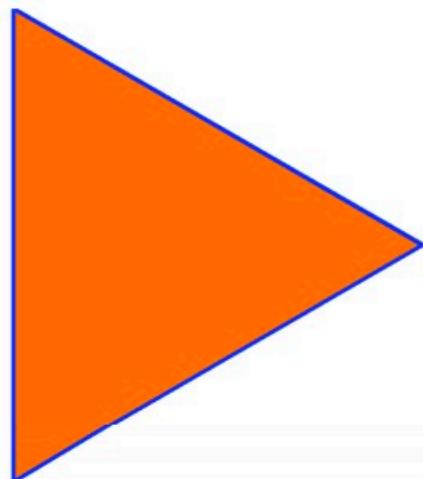


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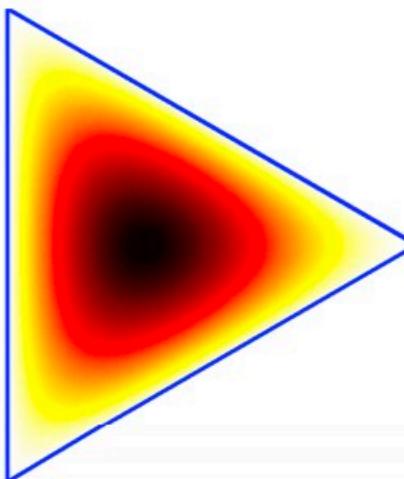
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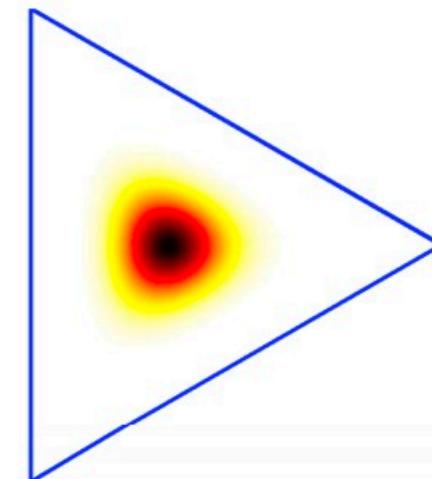
$\alpha=3, \mathbf{m}=(0.3,0.3,0.3)$



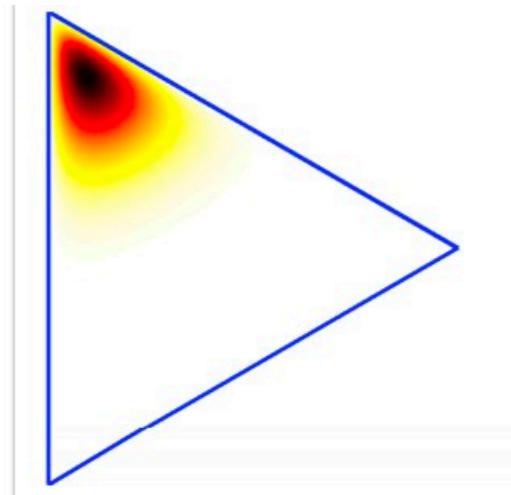
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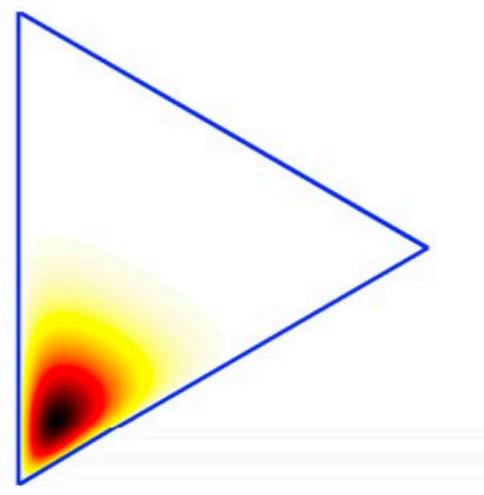
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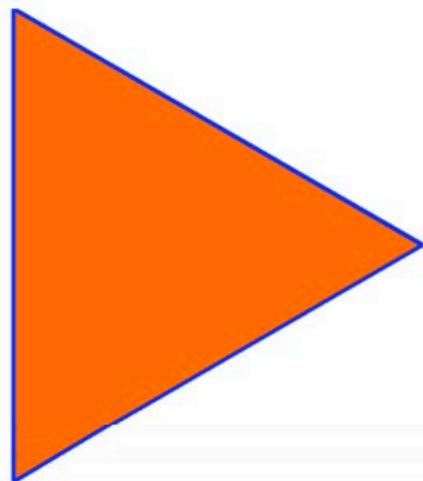


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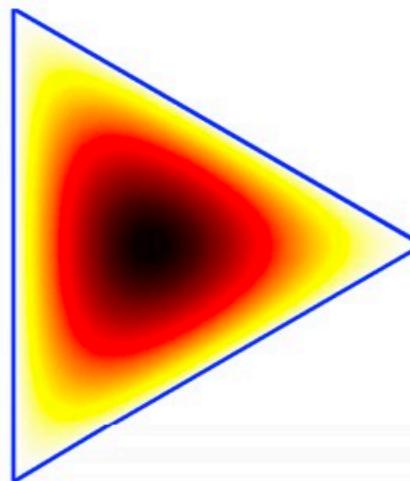
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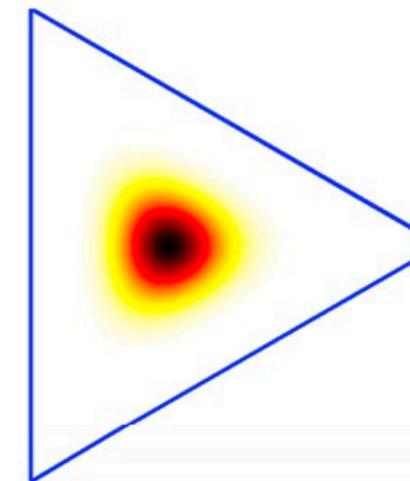
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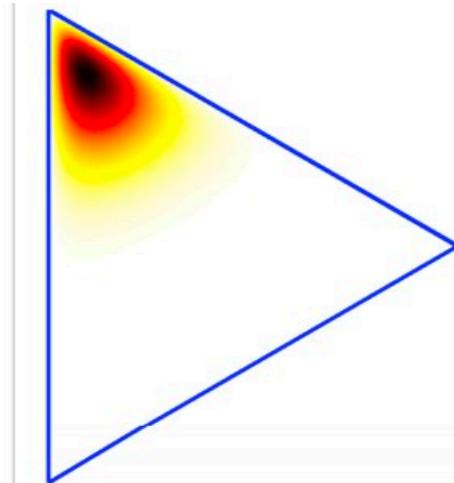
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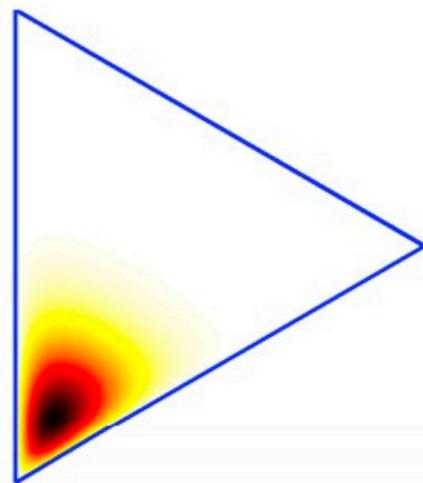
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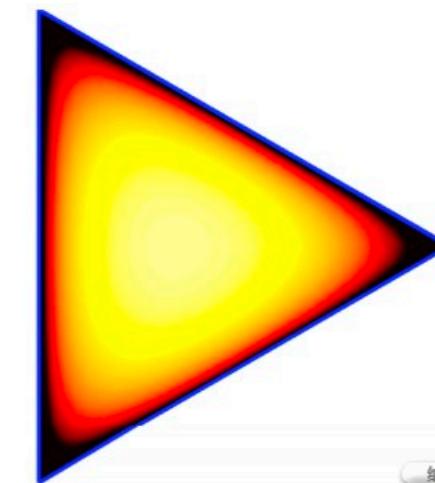
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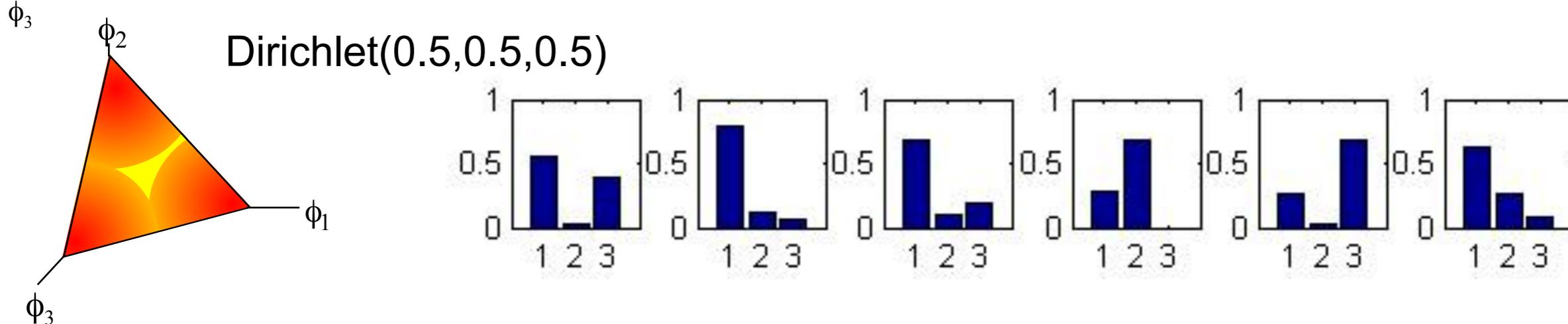
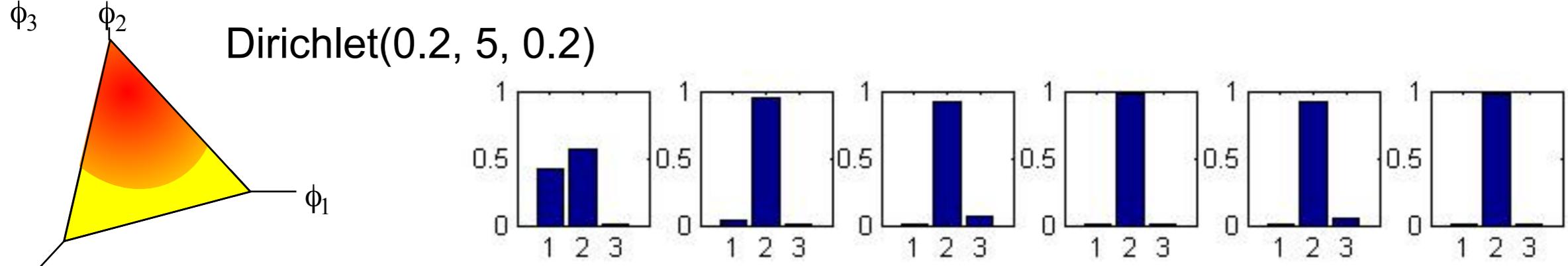
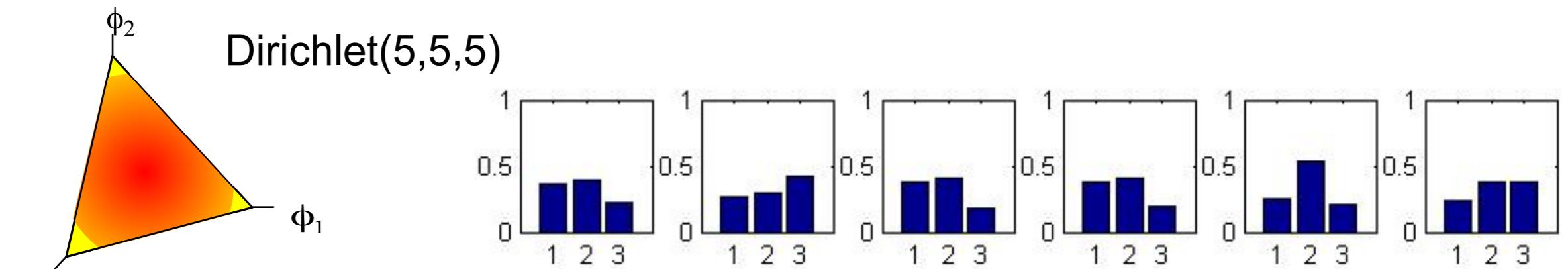


$\alpha=.2, \mathbf{m}=(0.3,0.3,0.3)$



Dirichlet Distribution

- Example draws from a Dirichlet Distribution over the 3-simplex:



LDA – Generative Process

- For each document d :

LDA – Generative Process

- For each document d :
 - Choose document length

LDA – Generative Process

- For each document d :
 - Choose document length $N \sim Poisson(\xi)$

LDA – Generative Process

- For each document d :
 - Choose document length $N \sim Poisson(\xi)$
 - Choose a topic mixture distribution

LDA – Generative Process

- For each document d :
 - Choose document length $N \sim Poisson(\xi)$
 - Choose a topic mixture distribution $\theta \sim Dir(\alpha)$

LDA – Generative Process

- For each document d :
 - Choose document length $N \sim Poisson(\xi)$
 - Choose a topic mixture distribution $\theta \sim Dir(\alpha)$
 - For each word token in d :
 - Choose a topic

LDA – Generative Process

- For each document d :
 - Choose document length $N \sim Poisson(\xi)$
 - Choose a topic mixture distribution $\theta \sim Dir(\alpha)$
 - For each word token in d :
 - Choose a topic $Z_n \sim multinomial(\theta)$

LDA – Generative Process

- For each document d :
 - Choose document length $N \sim Poisson(\xi)$
 - Choose a topic mixture distribution $\theta \sim Dir(\alpha)$
 - For each word token in d :
 - Choose a topic $z_n \sim multinomial(\theta)$
 - Choose a word w_n from a multinomial distribution conditioned on the topic z_n .

LDA – Generative Process

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$$[\beta]_{k \times V} \quad \beta_{ij} = p(w_j | z=i)$$

LDA – Generative Process

$$P(d) = \prod_{w \in d} \sum_{i=1..K} P(z=i | d) P(w | Topic_i)$$

iPod 0.15
nano 0.080
music .05
download 0.02
apple 0.01

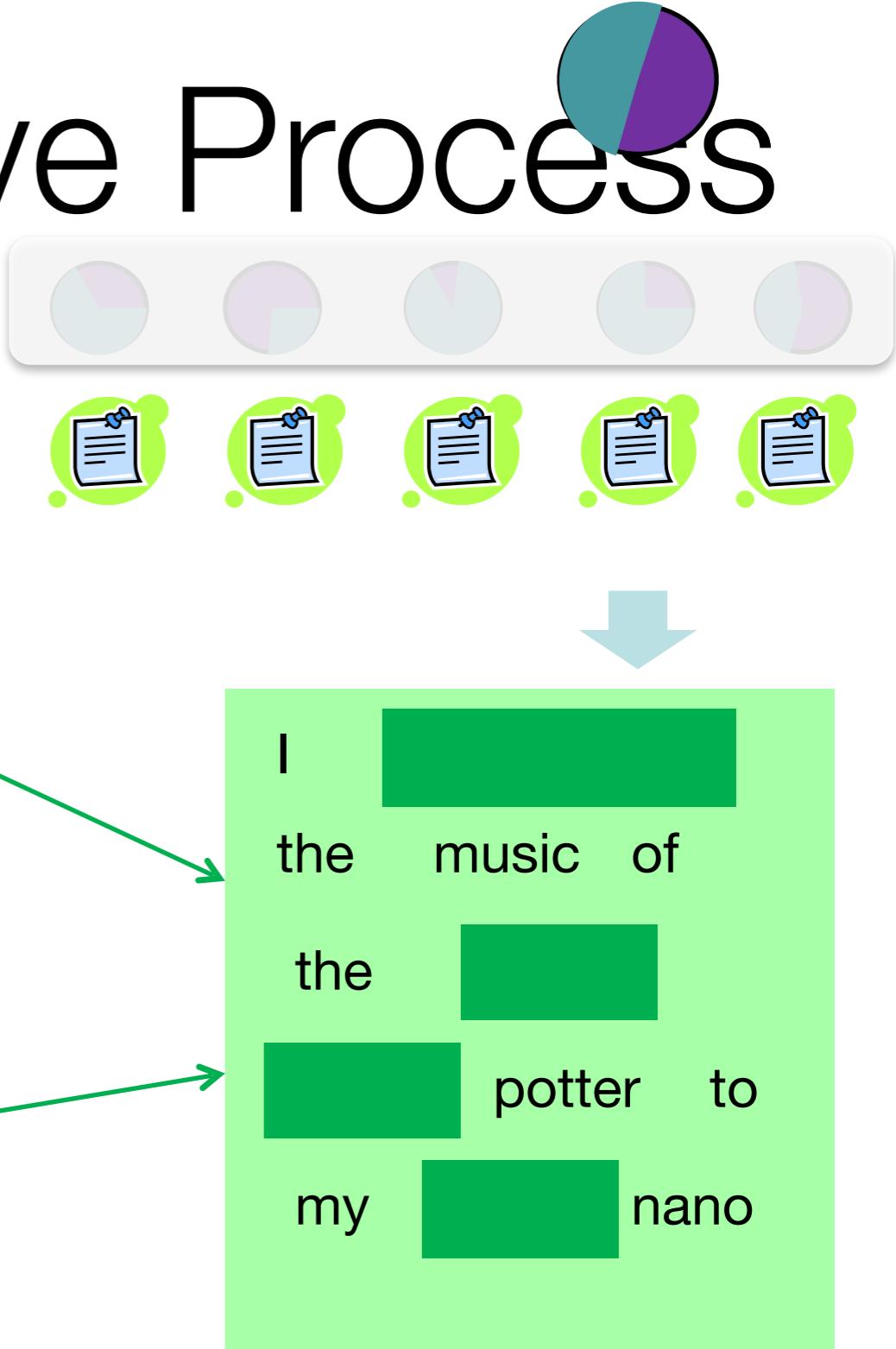
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harry .090.
potter 05
ipod 0.01
music 0.02

Topic 1

Apple iPod

Topic 2

Harry Potter



LDA – Generative Process

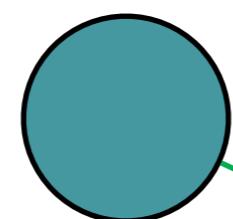
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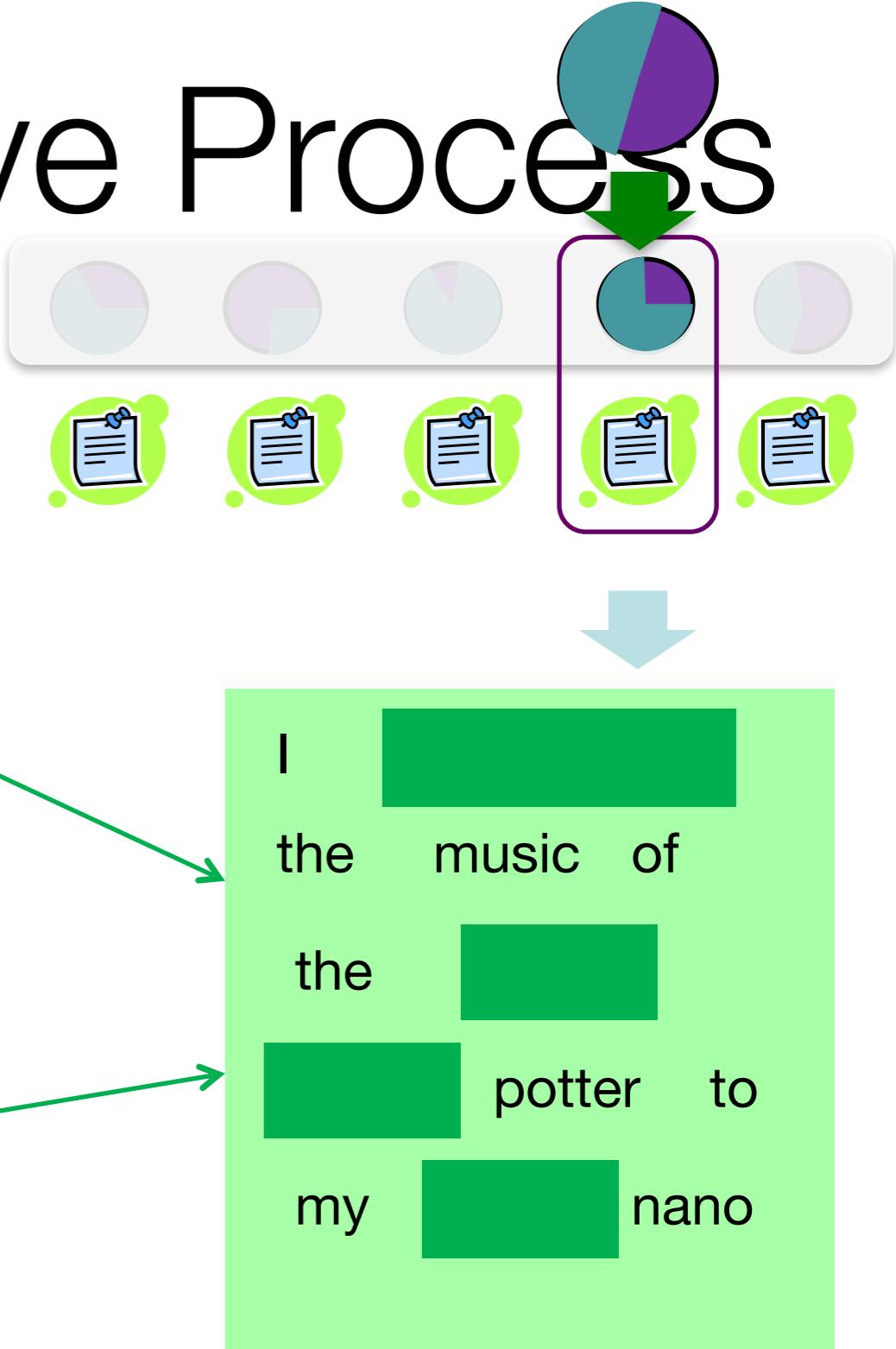
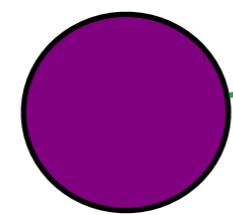
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Apple iPod



Topic 2

Harry Potter

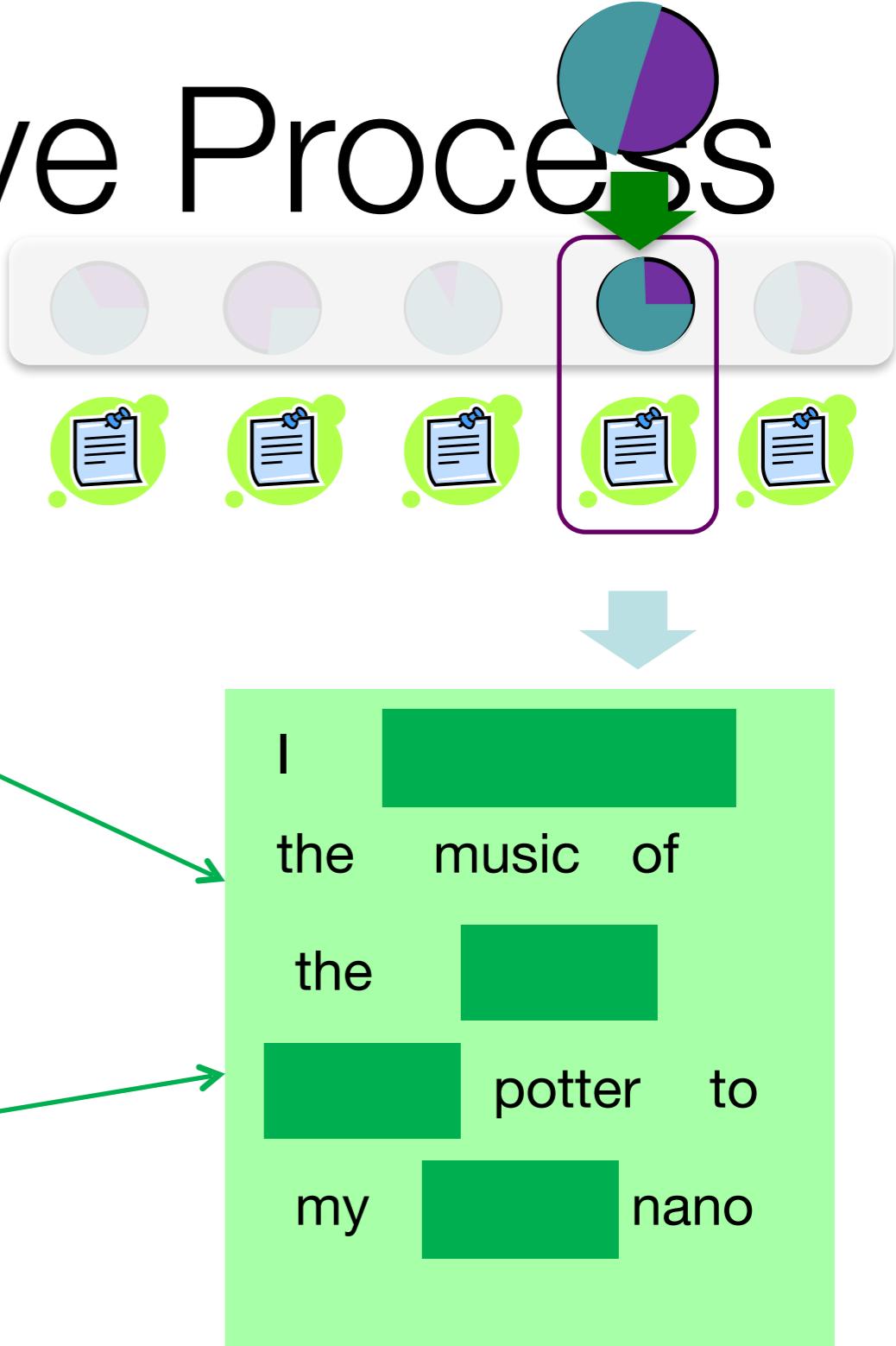


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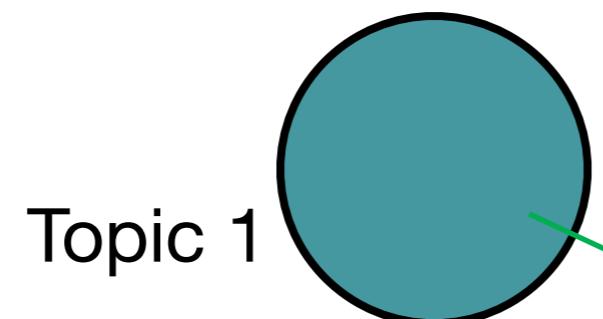
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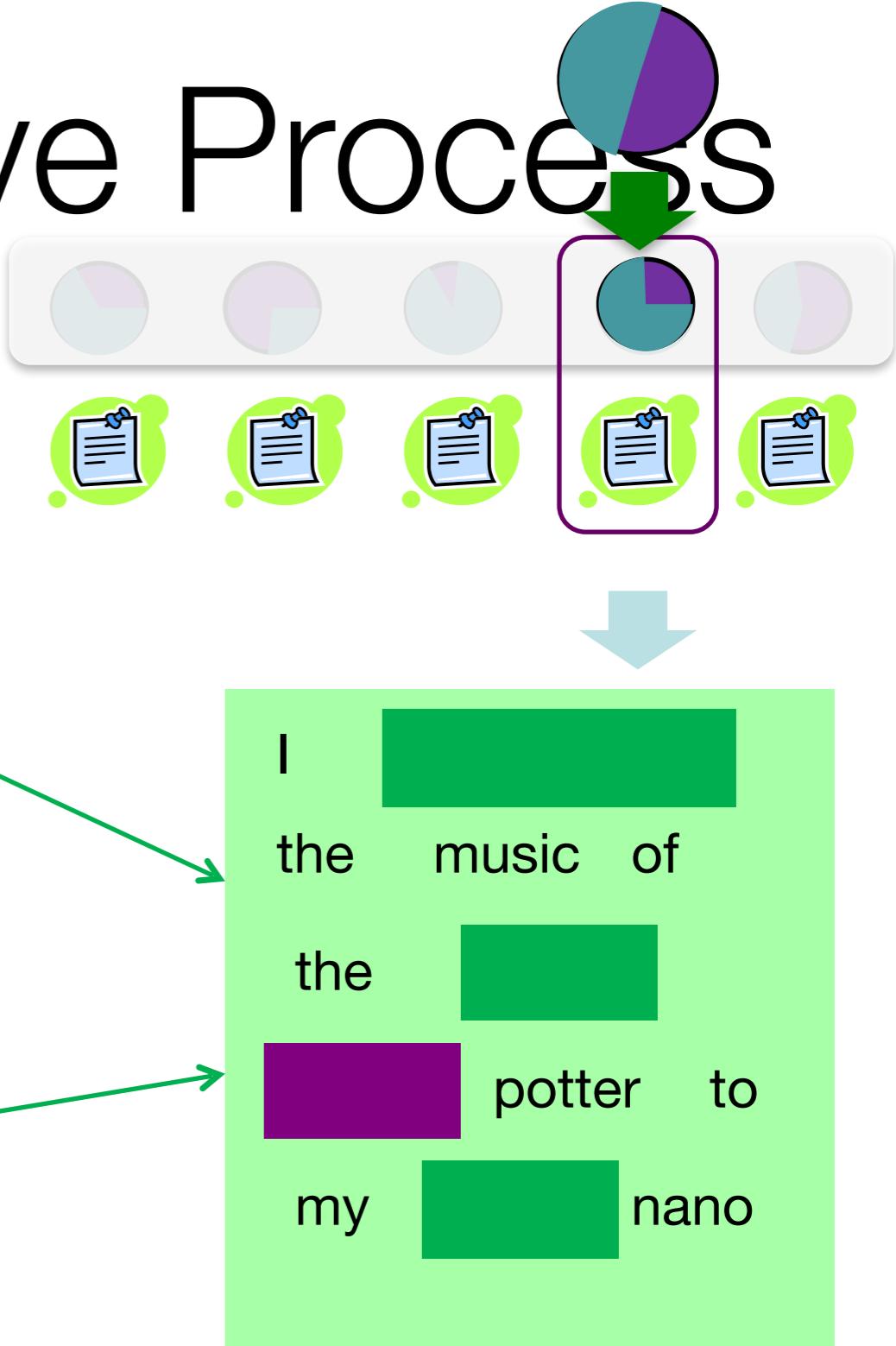
Apple iPod

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potter	05
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Topic 2



Harry Potter



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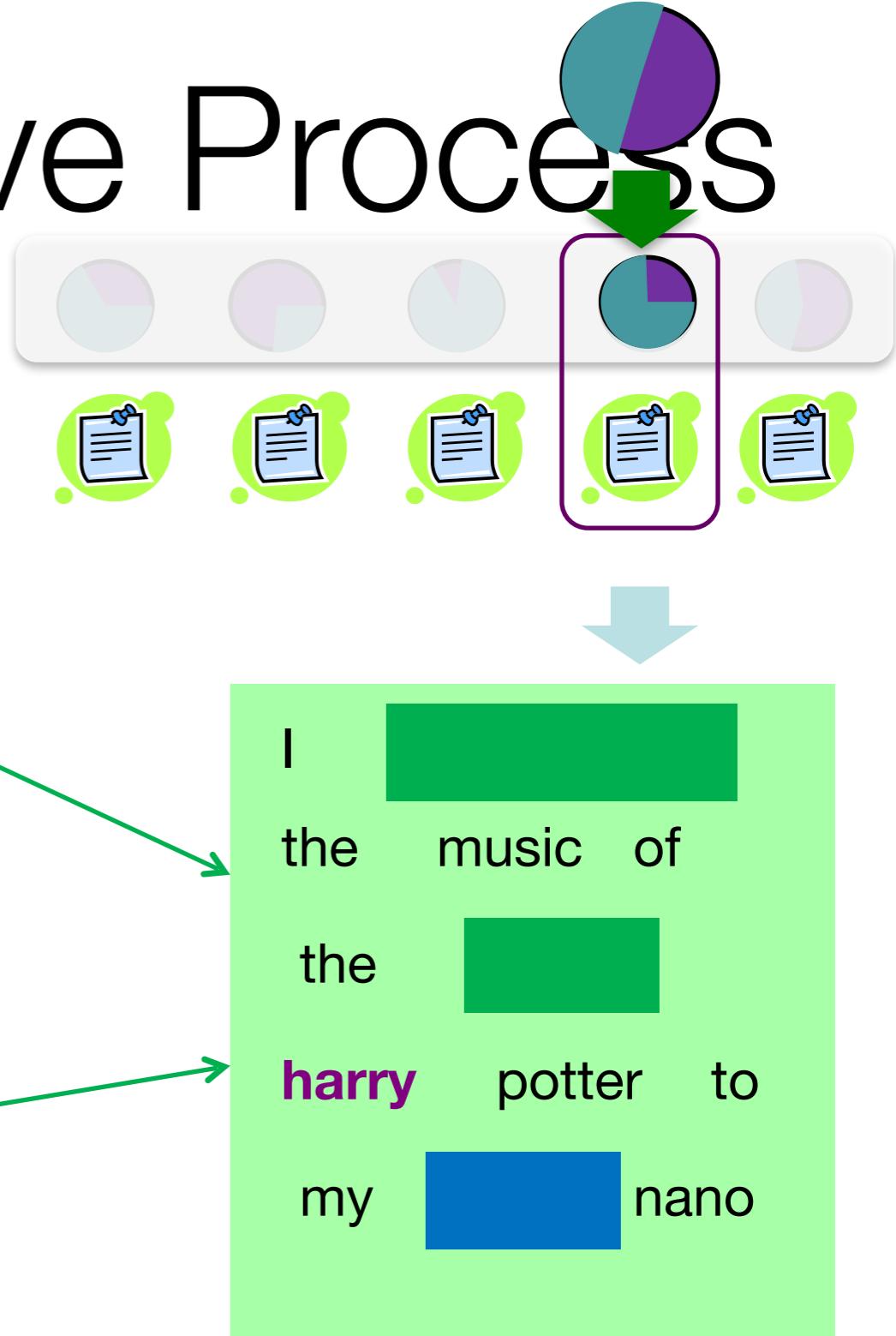
Topic 1

Apple iPod

movie	0.100
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music	0.02

Topic 2

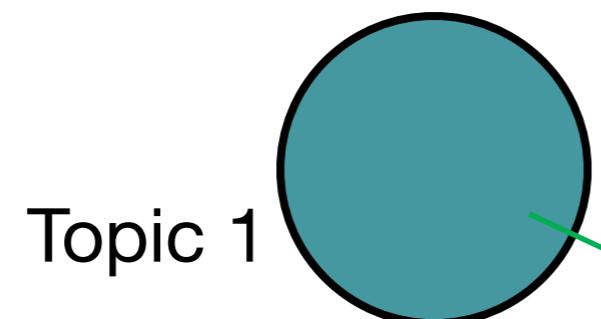
Harry Potter



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music	.05
download	0.02
apple	0.01



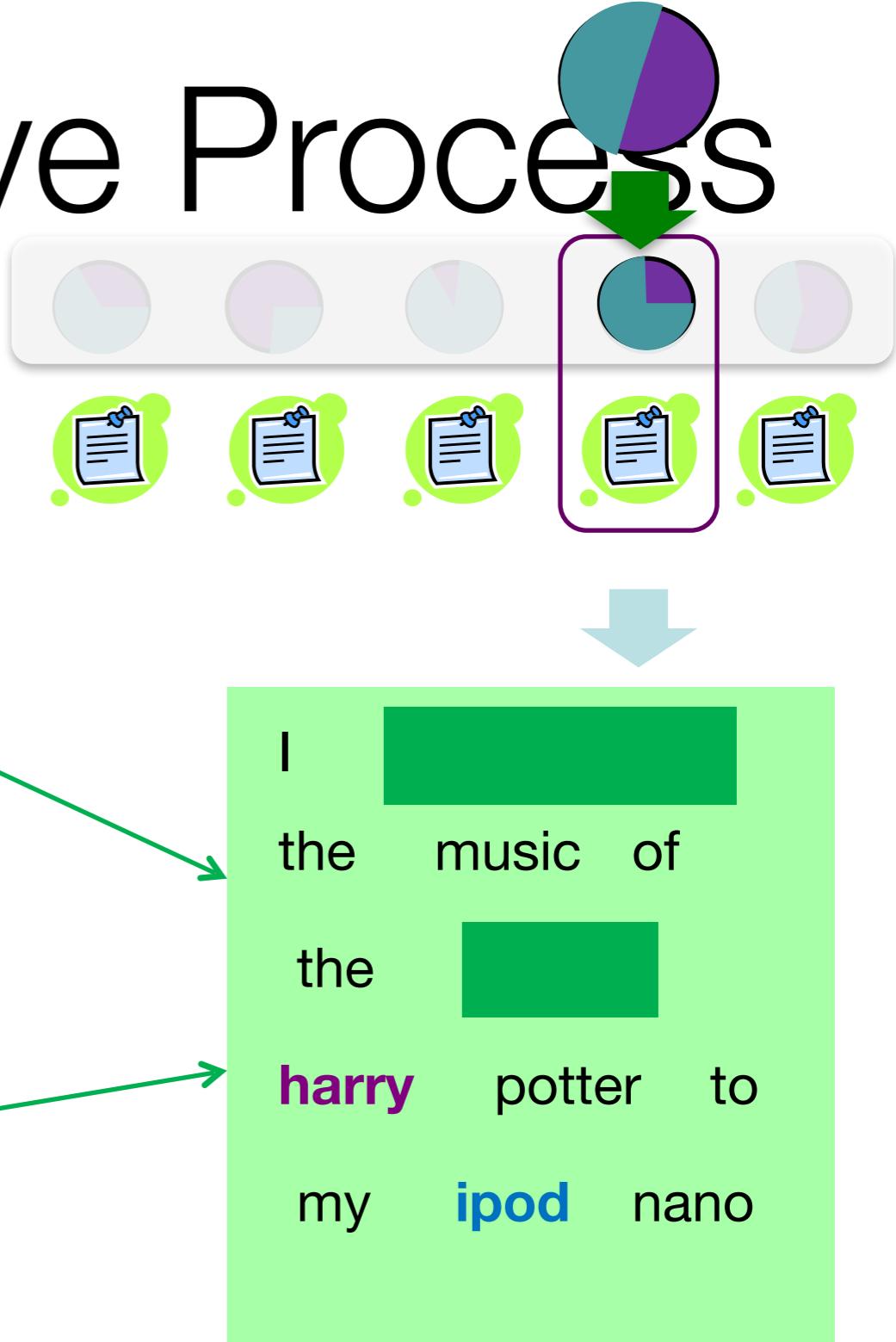
Apple iPod

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Topic 2



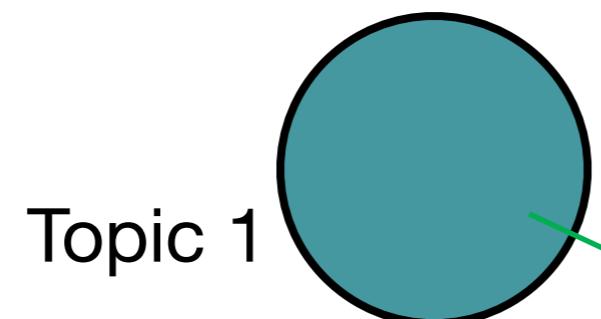
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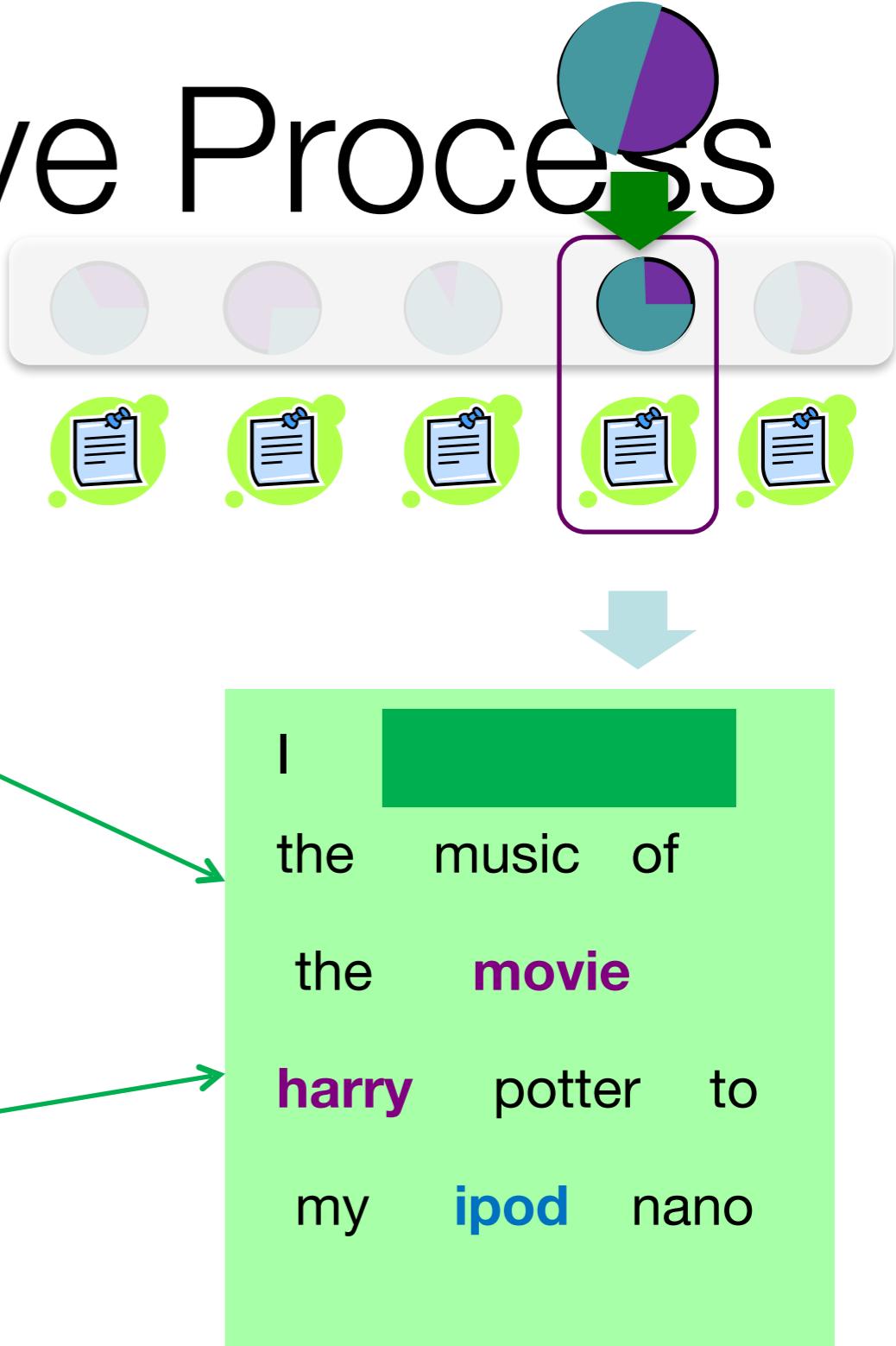
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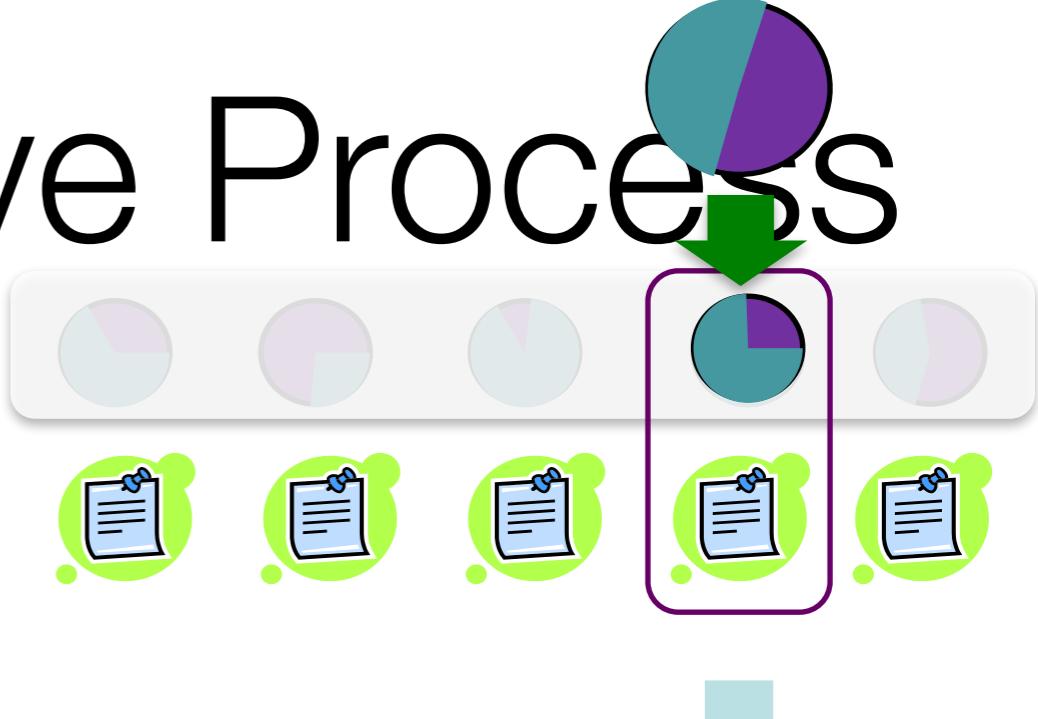
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music	0.02

Topic 2

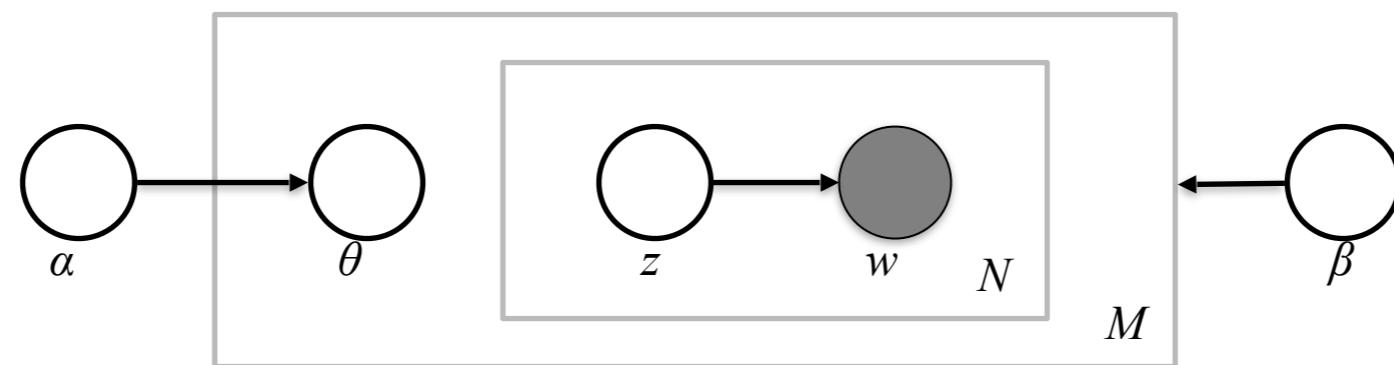
Harry Potter



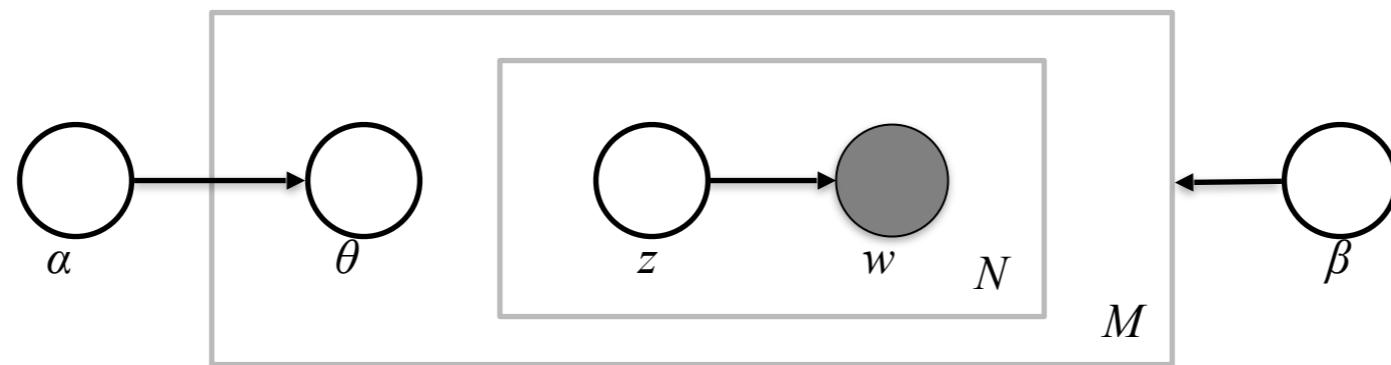
I **downloaded**
the music of
the **movie**
harry potter to
my **ipod** nano

Graphical Model of LDA

Graphical Model of LDA

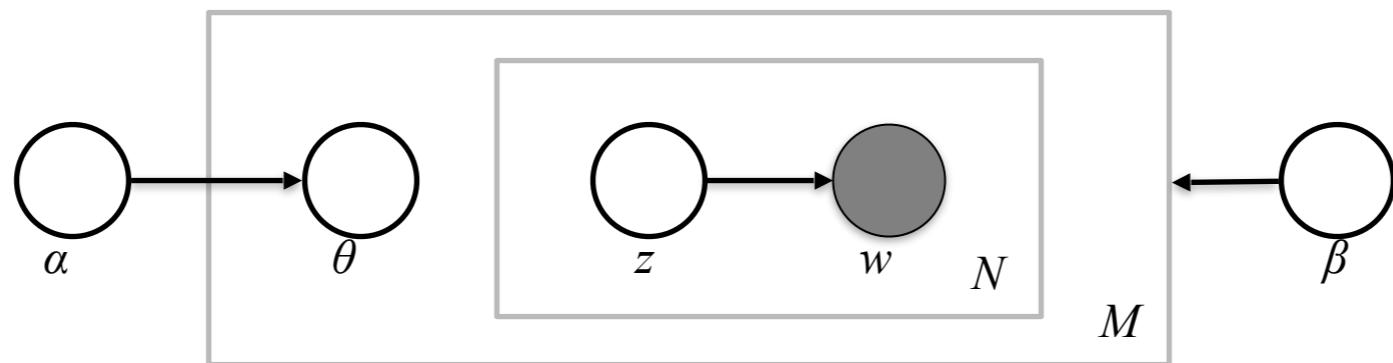


Graphical Model of LDA



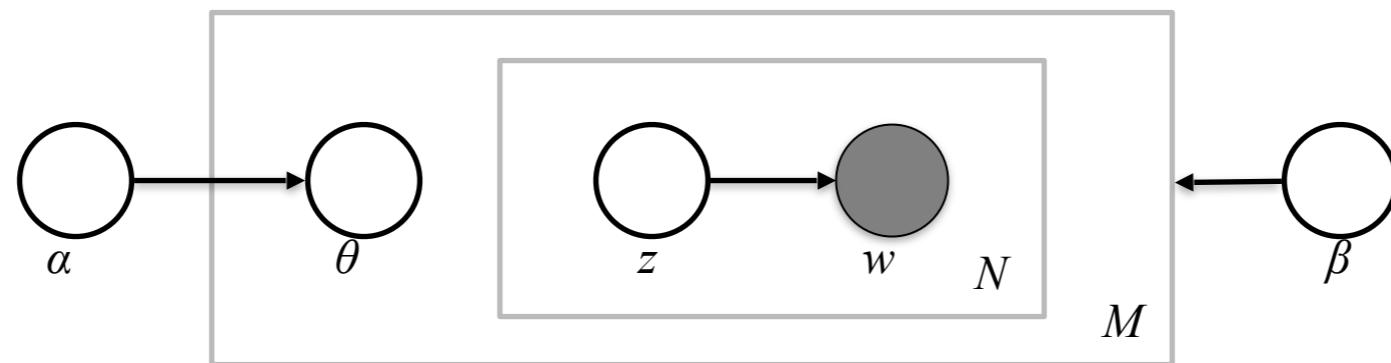
Complete likelihood:
(if we observe all latent variables)

Graphical Model of LDA



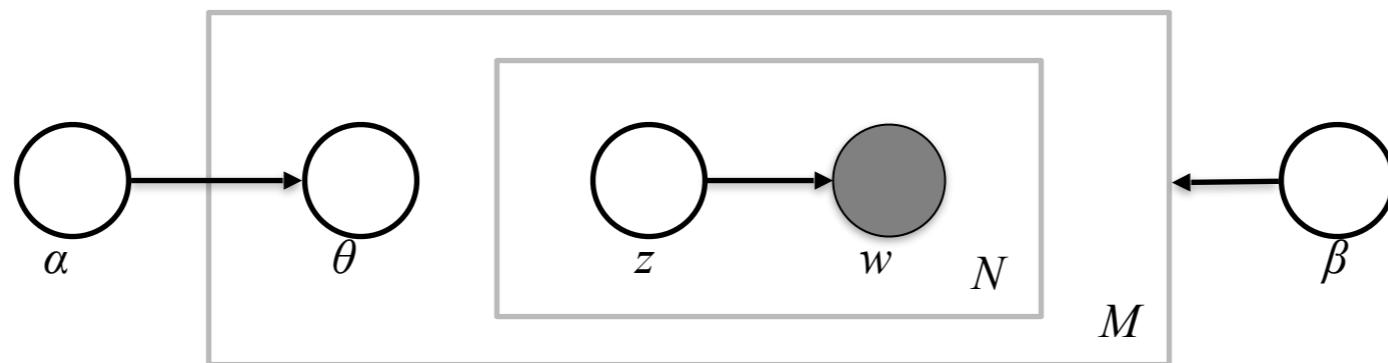
Complete likelihood: $p(\theta, \mathbf{z}, \mathbf{w} | \alpha, \beta) =$
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Graphical Model of LDA



Complete likelihood: $p(\theta, \mathbf{z}, \mathbf{w} | \alpha, \beta) = p(\theta | \alpha) \prod_{n=1}^N p(z_n | \theta) p(w_n | z_n, \beta)$
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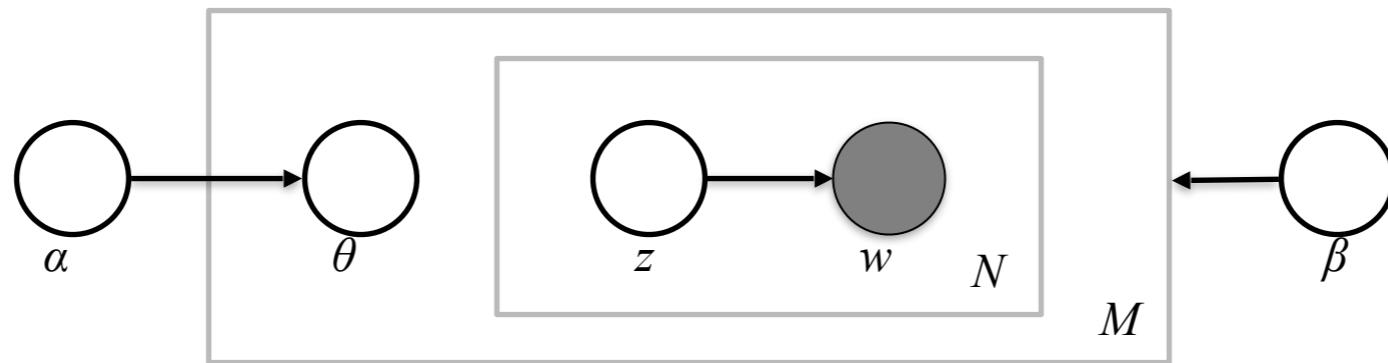
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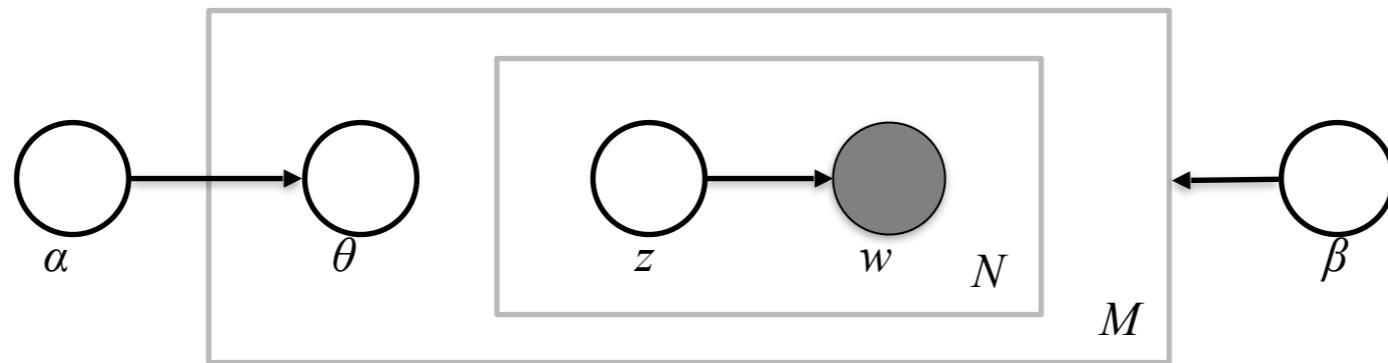


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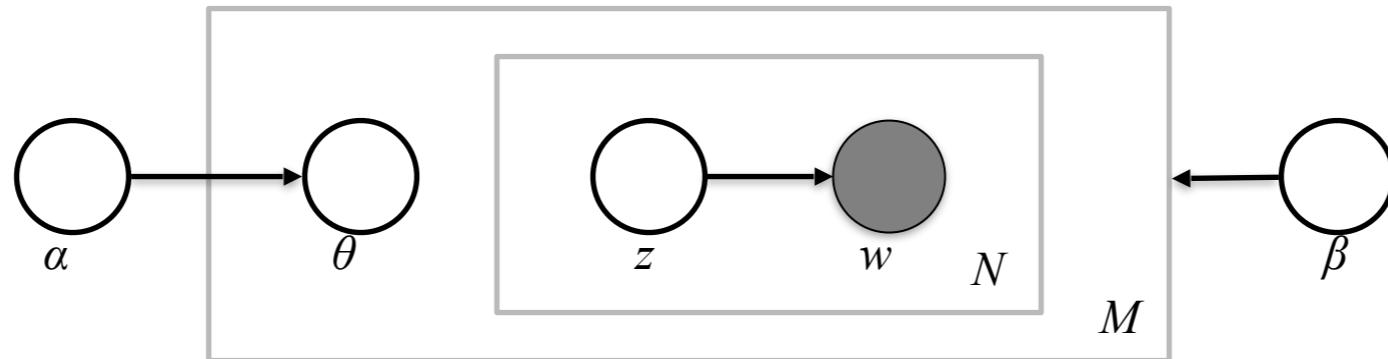
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Data likelihood:

- $P(z|d)$ are no longer fixed parameters to be estimated...
- But inference becomes much more complicated

Putting all the pieces together

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- Latent Dirichlet Allocation (LDA) is a fully Bayesian version of ...

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- Latent Semantic Indexing (LSI) is a dimensional reduction using the Singular Value Decomposition (SVD)
- PLSA is also proved to be equivalent to non-negative matrix factorization (with a specific loss function)

Application to corpus data

- TASA corpus: text from first grade to college
- 26414 word types, over 37000 documents, used approximately 6 million word tokens
- Run Gibbs for models with $T = 300, 500, \dots, 1700$ topics

A selection from 500 topics $[P(w|z = j)]$

THEORY	SPACE	ART	STUDENTS	BRAIN	CURRENT
SCIENTISTS	EARTH	PAINT	TEACHER	NERVE	ELECTRICITY
EXPERIMENT	MOON	ARTIST	STUDENT	SENSE	ELECTRIC
OBSERVATIONS	PLANET	PAINTING	TEACHERS	SENSES	CIRCUIT
SCIENTIFIC	ROCKET	PAINTED	TEACHING	ARE	IS
EXPERIMENTS	MARS	ARTISTS	CLASS	NERVOUS	ELECTRICAL
HYPOTHESIS	ORBIT	MUSEUM	CLASSROOM	NERVES	VOLTAGE
EXPLAIN	ASTRONAUTS	WORK	SCHOOL	BODY	FLOW
SCIENTIST	FIRST	PAINTINGS	LEARNING	SMELL	BATTERY
OBSERVED	SPACECRAFT	STYLE	PUPILS	TASTE	WIRE
EXPLANATION	JUPITER	PICTURES	CONTENT	TOUCH	WIRES
BASED	SATELLITE	WORKS	INSTRUCTION	MESSAGES	SWITCH
OBSERVATION	SATELLITES	OWN	TAUGHT	IMPULSES	CONNECTED
IDEA	ATMOSPHERE	SCULPTURE	GROUP	CORD	ELECTRONS
EVIDENCE	SPACESHIP	PAINTER	GRADE	ORGANS	RESISTANCE
THEORIES	SURFACE	ARTS	SHOULD	SPINAL	POWER
BELIEVED	SCIENTISTS	BEAUTIFUL	GRADES	FIBERS	CONDUCTORS
DISCOVERED	ASTRONAUT	DESIGNS	CLASSES	SENSORY	CIRCUITS
OBSERVE	SATURN	PORTRAIT	PUPIL	PAIN	TUBE
FACTS	MILES	PAINTERS	GIVEN	IS	NEGATIVE

A selection from 500 topics $[P(w|z = j)]$

MIND	STORY	FIELD	SCIENCE	BALL	JOB
WORLD	STORIES	MAGNETIC	STUDY	GAME	WORK
DREAM	TELL	MAGNET	SCIENTISTS	TEAM	JOBS
DREAMS	CHARACTER	WIRE	SCIENTIFIC	FOOTBALL	CAREER
THOUGHT	CHARACTERS	NEEDLE	KNOWLEDGE	BASEBALL	EXPERIENCE
IMAGINATION	AUTHOR	CURRENT	WORK	PLAYERS	EMPLOYMENT
MOMENT	READ	COIL	RESEARCH	PLAY	OPPORTUNITIES
THOUGHTS	TOLD	POLES	CHEMISTRY	FIELD	WORKING
OWN	SETTING	IRON	TECHNOLOGY	PLAYER	TRAINING
REAL	TALES	COMPASS	MANY	BASKETBALL	SKILLS
LIFE	PLOT	LINES	MATHEMATICS	COACH	CAREERS
IMAGINE	TELLING	CORE	BIOLOGY	PLAYED	POSITIONS
SENSE	SHORT	ELECTRIC	FIELD	PLAYING	FIND
CONSCIOUSNESS	FICTION	DIRECTION	PHYSICS	HIT	POSITION
STRANGE	ACTION	FORCE	LABORATORY	TENNIS	FIELD
FEELING	TRUE	MAGNETS	STUDIES	TEAMS	OCCUPATIONS
WHOLE	EVENTS	BE	WORLD	GAMES	REQUIRE
BEING	TELLS	MAGNETISM	SCIENTIST	SPORTS	OPPORTUNITY
MIGHT	TALE	POLE	STUDYING	BAT	EARN
HOPE	NOVEL	INDUCED	SCIENCES	TERRY	ABLE

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- [Relational topic model](#): Documents on the same topic are generated separately but tend to link to one another. (Why is this useful?)
- [Dynamic topic model](#): We also observe a year for each document. The k topics β used in 2011 have evolved slightly from their counterparts in 2010.



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How to train your LDA

How to train your LDA model

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- We're interested in the posterior distribution:

$$p(Z|X, \Theta)$$

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topics

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- Here, the latent variables are the topic assignments z and the topics are Theta. X is the words (divided into documents) and Θ are hyperparameters to the Dirichlet distributions: α for topic proportions and λ for topics

$$p(z, \beta, \theta | \mathbf{w}, \alpha, \lambda)$$

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topic $p(z, \beta, \theta | \mathbf{w}, \alpha, \lambda)$ Each document's
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$$p(\mathbf{w}, \mathbf{z}, \theta, \beta | \alpha, \lambda) = \prod_k p(\beta_k | \lambda) \prod_d p(\theta_d | \alpha) \prod_n p(z_{d,n} | \theta_d) p(w_{d,n} | \beta_{z_{d,n}})$$

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Quick Detour: Monte Carlo Methods

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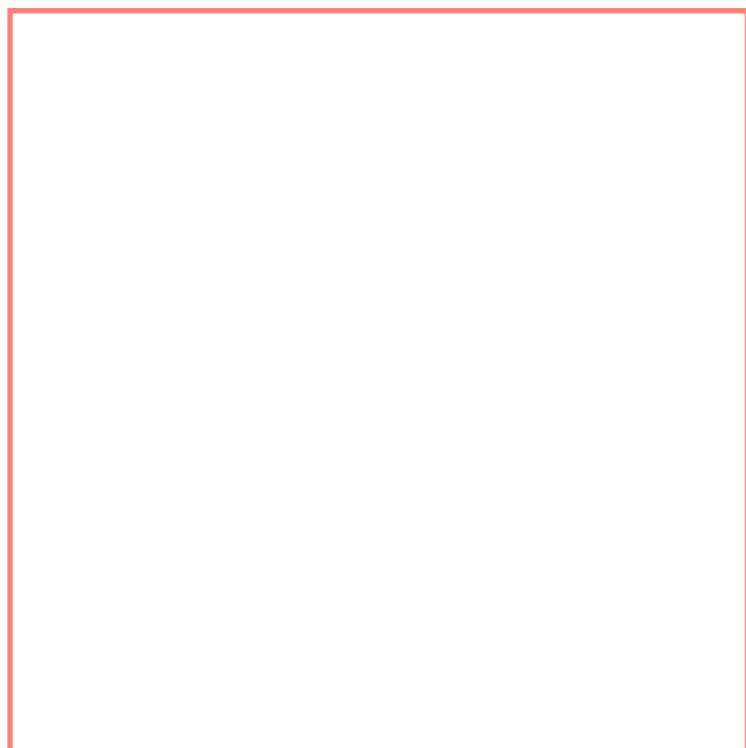
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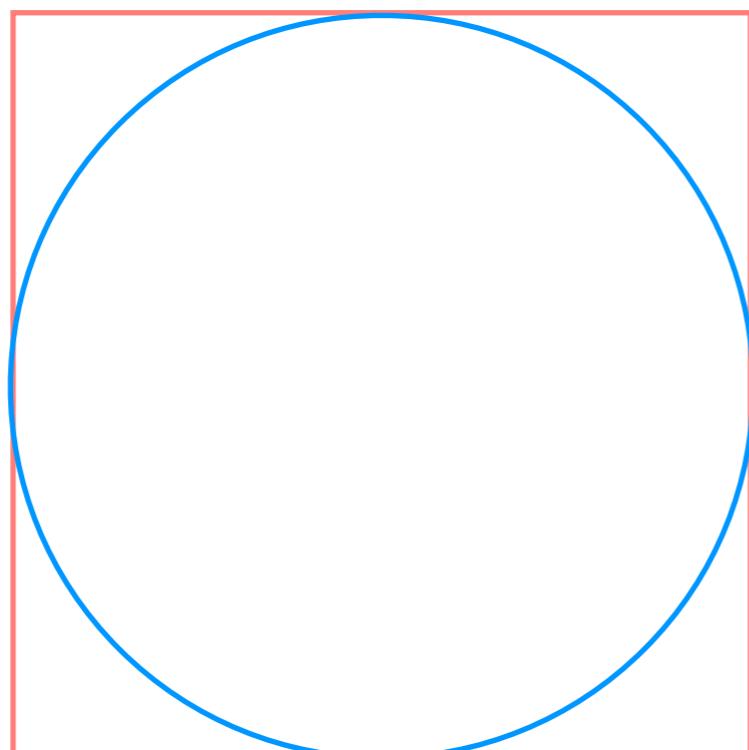
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- $\pi \approx C / S$

Gibbs Sampling

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Pretend we never saw the topic assignment for the n^{th} word in the d^{th} document

What Gibbs Sampling is doing at an abstract level

Topic 1

wizard
magic
wand
spell

Topic 2

movie
book
novel
media

Topic 3

Harry
Hermoine
Ron
Dumbledore

Topic 4

school
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(In the real version, all words are assigned a topic!)

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(In the real version, all words are used to infer the word's new topic!)

Gibbs Sampling Equation

$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i}$$

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How much this document
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How much this document
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How much this topic likes word
 $w_{d,n}$

Gibbs Sampling Equation

Number of times document d
uses topic k

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How much this document likes topic k

Number of times topic k uses word time $w_{d,n}$

How much this topic likes word $w_{d,n}$

Gibbs Sampling Equation

Dirichlet parameter for the document to topic distribution

Number of times document d uses topic k

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How much this document likes topic k

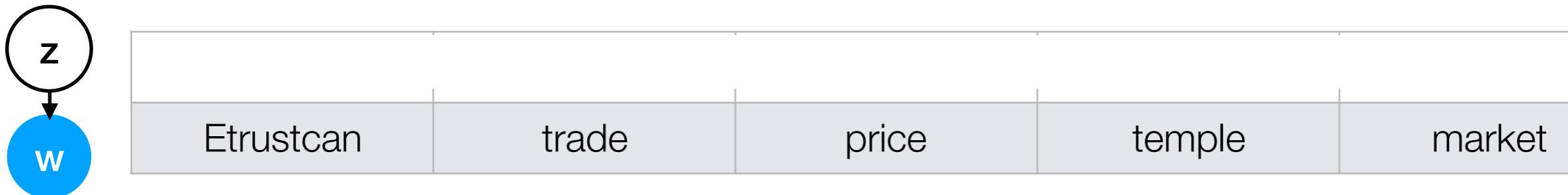
Dirichlet parameter for the topic to word distribution

Number of times topic k uses word time $w_{d,n}$

$$\frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i}$$

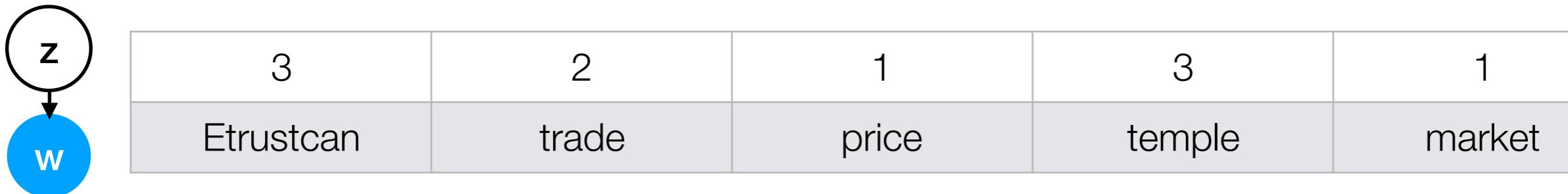
How much this topic likes word $w_{d,n}$

Let's do an example!



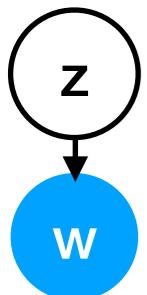
Start by assigning all words topics at random

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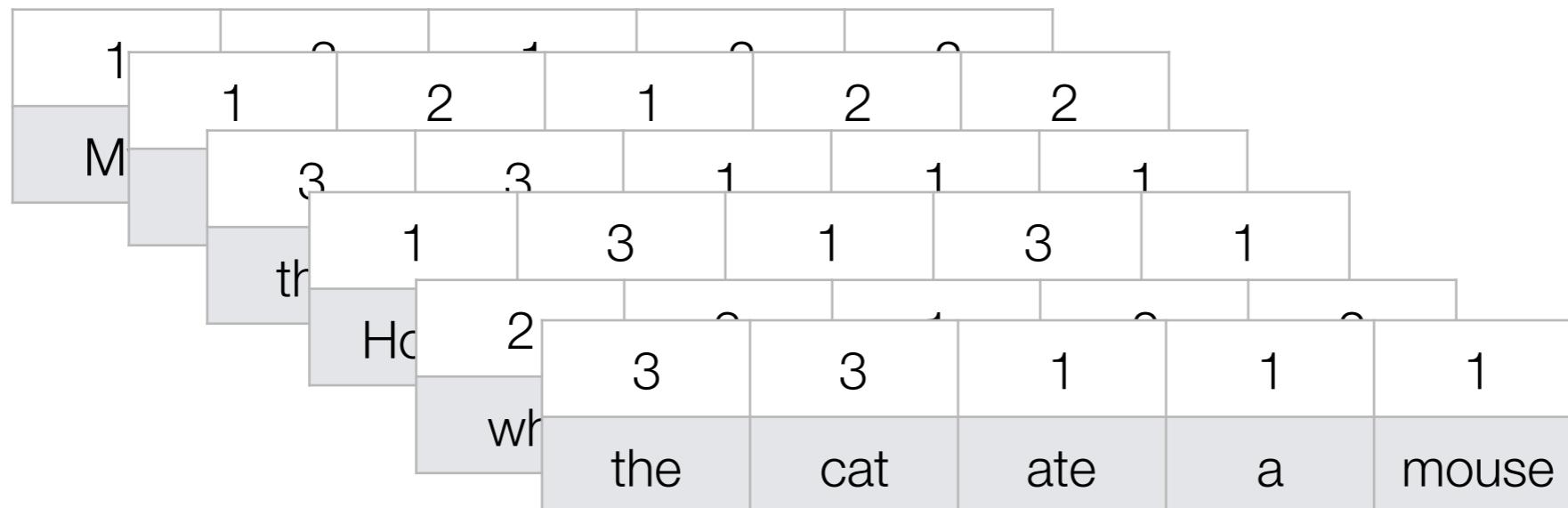
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3	2	1	3	1
Etrustcan	trade	price	temple	market

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Total Topic Counts

3	2	1	3	1
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Total counts from
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We want to sample this word

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Step 1: Unassign it and decrement its count

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Step 2: Recompute the conditional distribution —
How much does this document like each topic?

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Topic 1



Topic 2



Topic 3



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Topic 2



Topic 3



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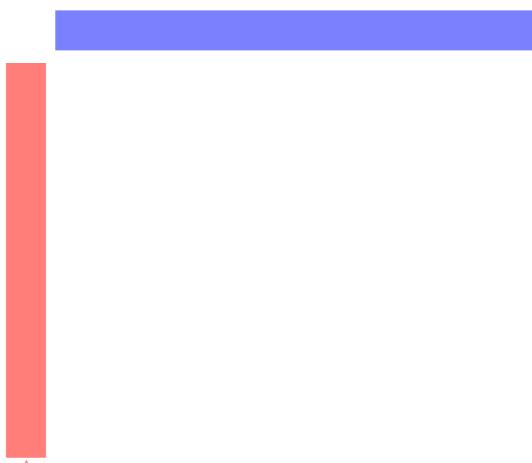


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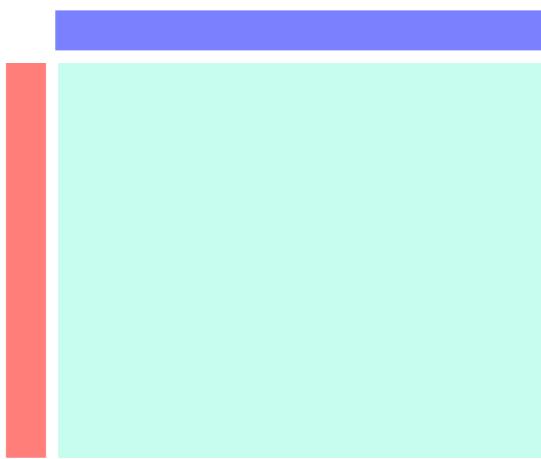


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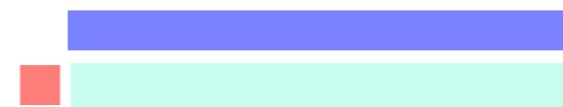
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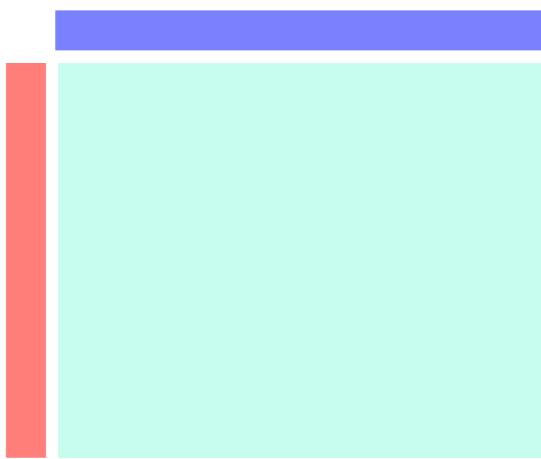


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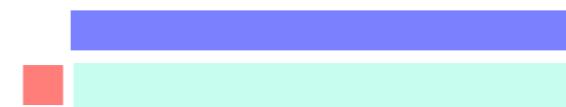
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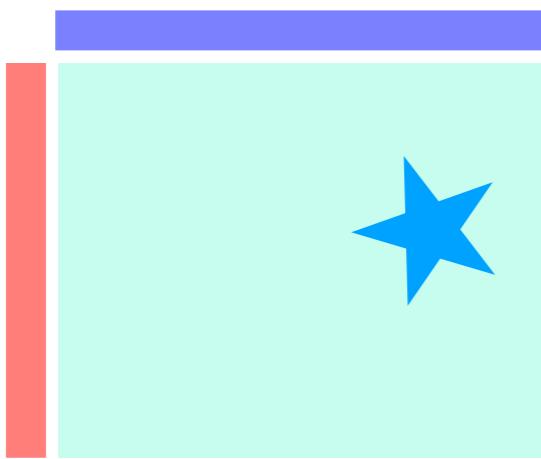


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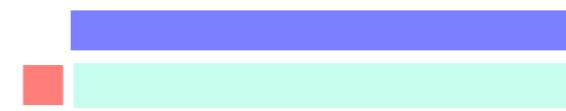
Topic 1



Topic 2



Topic 3



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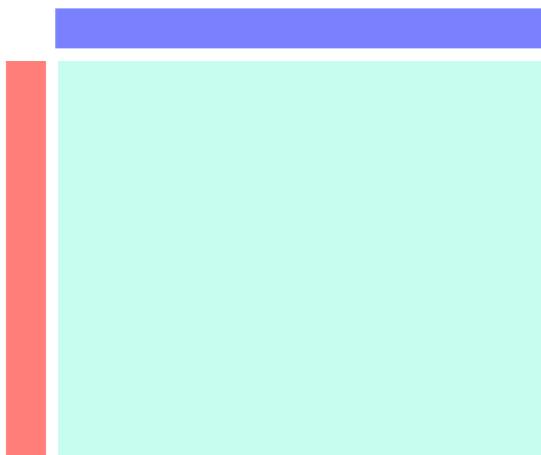
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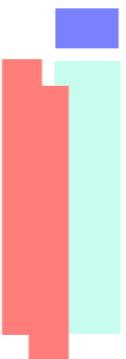
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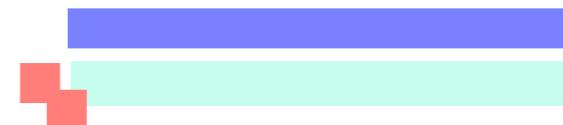
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Topic 2



Topic 3

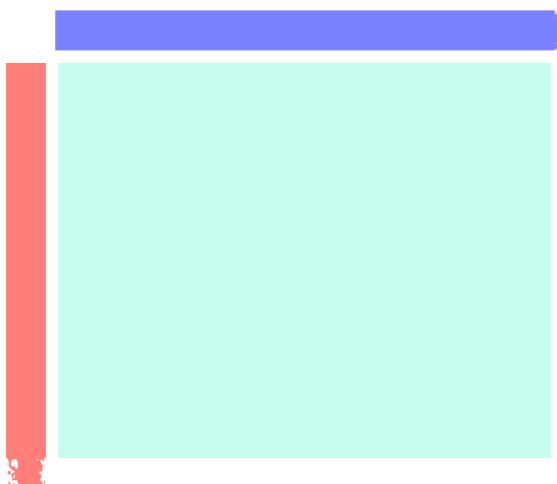


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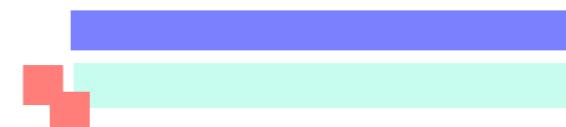
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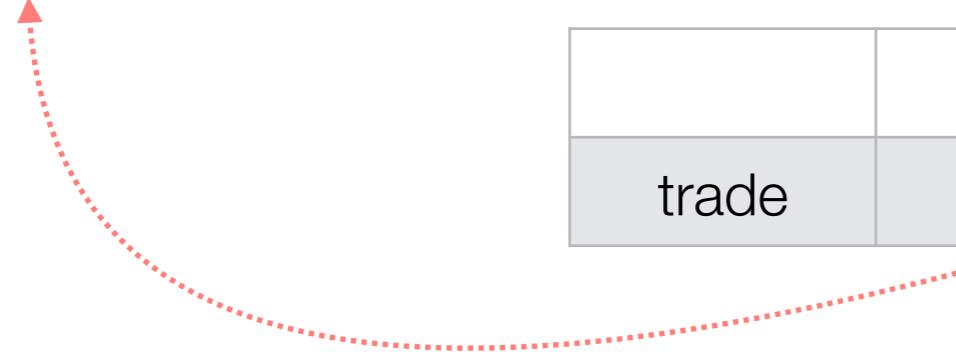
Topic 2



Topic 3



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Gibbs Sampling Algorithm

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 - Increment $n_{d,z_{new}}$ and $v_{z_{new},w_{d,n}}$

Homework 4

- You'll be implementing the Gibbs Sampling updates part of LDA!
- We've given you a bunch of skeleton code; your part is ~10 lines, very similar to what we covered today
- 90% thinking; 10% coding

Gibbs Sampling for the Uninitiated

Philip Resnik

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Institute for Advanced Computer Studies
University of Maryland
College Park, MD 20742 USA
`resnik AT umd.edu`

Eric Hardisty

Department of Computer Science and
Institute for Advanced Computer Studies
University of Maryland
College Park, MD 20742 USA
`hardisty AT cs.umd.edu`

Abstract

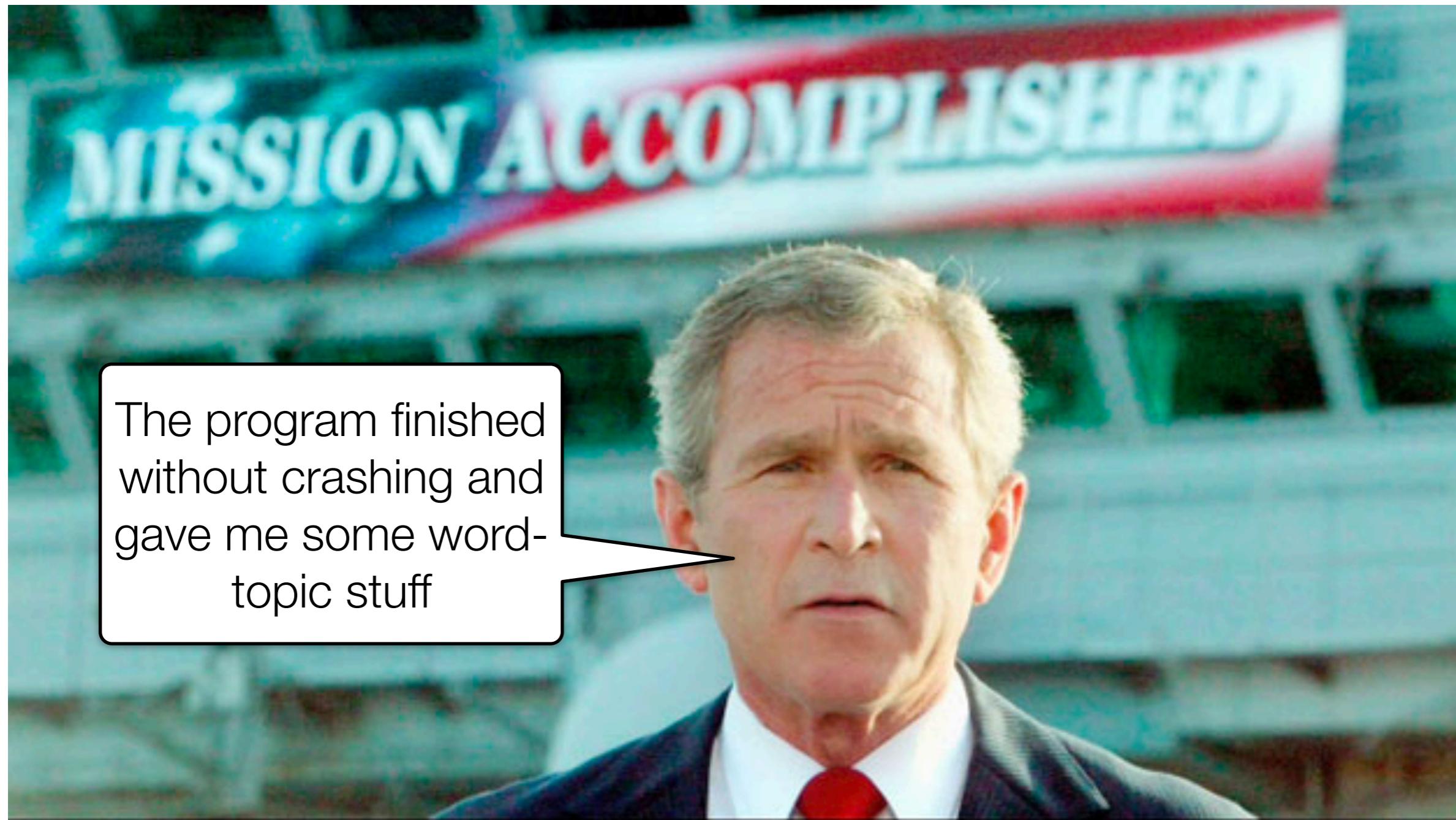
This document is intended for computer scientists who would like to try out a Markov Chain Monte Carlo (MCMC) technique, particularly in order to do inference with Bayesian models on problems related to text processing. We try to keep theory to the absolute minimum needed, though we work through the details much more explicitly than you usually see even in “introductory” explanations. That means we’ve attempted to be ridiculously explicit in our exposition and notation.

After providing the reasons and reasoning behind Gibbs sampling (and at least nodding our heads in the direction of theory), we work through an example application in detail—the derivation of a Gibbs sampler for a Naïve Bayes model. Along with the example, we discuss some practical implementation issues, including the integrating out of continuous parameters when possible. We conclude with some pointers to literature that we’ve found to be somewhat more friendly to uninitiated readers.

1 Introduction

Markov Chain Monte Carlo (MCMC) techniques like Gibbs sampling provide a principled way to approximate the value of an integral.

How do we know if the topics we learned are good?



The program finished
without crashing and
gave me some word-
topic stuff

Option 1: Test on held out data

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- Good:
 - Wallach (2009) show how to efficiently approximate this
- Bad:
 - Chang (2009) show that likelihood is not correlated with human judgements of goodness

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- Intrusion Detection (Blei et al., 2009)

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- Bad: Can't be done automatically

Many other metrics

- Qualitative
 - Expert judgements
 - Word Clouds
 - Visualizing word-topic assignments
- Quantitative
 - Topic Coherence: Better models have words that are highly related according to some external source (Newman, 2010; Mimno et al., 2011)

Fancier metrics are also possible

8. CONCLUSION

We proposed a unifying framework that span the space of all known coherence measures and allows to combine all main ideas in the context of coherence quantification. We evaluated 237 912 coherence measures on six different benchmarks for topic coherence—to the best of our knowledge, this is the largest number of benchmarks used for topic coherences so far. We introduced coherence measures from

Röder et al. (2015)

Summarizing what we learned about topic models



Topic Modeling in Practice

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- Number of topics have to be predetermined
 - Finding the right number of topics can be hard
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- Topics need to be interpreted
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 - Methods to generate labels for topics (Mei et al., 2007)
- Evaluation is subjective
 - Likelihood of held-out data, quality of topics, quality of document clusters, human judgments, ...

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- But can provide “deep” features for those tasks
- Works well when few training examples are available
- Extremely powerful when contextual variables are observed and concerned with
 - Very easy to extend, by adding variables into the Bayesian network (graphical structure)

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- Do not apply LDA directly if
 - You have too few documents
 - Your documents are too short
 - Your goal is to find smallish, tight clusters
 - The co-occurrence assumption doesn't hold in your data
- Try to leverage meta-data or context information to improve the performance of topic modeling
 - Authorship
 - Time, geographic location
 - Networks of documents and words
 - User-guidance, ...

Resources of Topic Modeling

- Topic Modeling Bibliography
 - Collected by David Mimno: <https://mimno.infosci.cornell.edu/topics.html>
- Toolkits:
 - LDA-C: command-line version from David Blei
 - LDA++: Easy-to-use C++ implementation
 - MALLET: Java package, with good support of text mining in general
 - Gensim: Python package for on-line topic modeling (*not so good*) and wraps MALLET in a python interface (good!)
 - Mahout: Machine learning toolkit adaptable on Hadoop
- Topic modeling in digital humanities
 - <http://mith.umd.edu/topic-modeling-in-the-humanities-an-overview/>
 - <http://journalofdigitalhumanities.org/2-1/words-alone-by-benjamin-m-schmidt/>



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- Brown clustering can help you learn categories of words — and the brown vectors are useful representations
- Topic modeling is ~~magic~~ a powerful technique for learning broad themes in text
 - But is difficult to evaluate—don't trust the output without looking at it and using it for something!