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K068

Probability & Statistics Lab-4

```
1.library(moments)
```

```
x = c(0,1,2)
```

```
p = c(1/3,1/3,1/3)
```

```
m0 = 1
```

```
m1 = sum(x*p)
```

```
m2 = sum(x*x*p)
```

```
m = c(m0,m1,m2)
```

```
raw2central(m)
```

```
print(m)
```

```
> library(moments)
> x = c(0,1,2)
> p = c(1/3,1/3,1/3)
> m0 = 1
> m1 = sum(x*p)
> m2 = sum(x*x*p)
> m = c(m0,m1,m2)
> raw2central(m)
[1] 1.0000000 0.0000000 0.6666667
> print(m)
[1] 1.000000 1.000000 1.666667
> |
```

$$1. \text{MGF} = e^{t_0} \cdot \frac{1}{3} + e^t \cdot \frac{1}{3} + e^{2t} \cdot \frac{1}{3} \\ = \frac{1}{3} (1 + e^t + e^{2t})$$

$$\text{Raw moments } \mu_1' = E(x^1) = \sum x P_x \\ = 0 \left(\frac{1}{3}\right) + 1 \left(\frac{1}{3}\right) + 2 \left(\frac{1}{3}\right) \\ = 1$$

$$\mu_2' = E(x^2) = \sum x^2 P_x \\ = 0 + \frac{1}{3} + \frac{4}{3} \\ = \frac{5}{3} \\ = 1.667$$

$$\mu_3' = E(x^3) = \sum x^3 P_x \\ = \frac{1}{3} + \frac{8}{3} \\ = \frac{9}{3} \\ = 3$$

$$\mu_4' = E(x^4) = \sum x^4 P_x \\ = \frac{1}{3} + \frac{16}{3} \\ = \frac{17}{3} \\ = 5.66$$

$$\text{Central moments } \mu_1 = 0 \\ \mu_2 = \mu_2' - (\mu_1')^2 \\ = \frac{5}{3} - (1)^2 \\ = \frac{2}{3}$$

$$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2\mu_1'^3 \\ = 3 - 3 \left(\frac{5}{3}\right) + 2(1)^3 \\ = 3 - 5 + 2 \\ = 0$$

$$\mu_4 = \frac{2}{3}$$

2.library(moments)

m = c(1,2,10,-30)

a = raw2central(m)

print(a)

mean = 5

print(mean)

variance=a[3]

print(variance)

skewness=a[4]

print(skewness)

Console Terminal x Jobs x

R 4.1.0 · ~/

```
> library(moments)
> m = c(1,2,10,-30)
> a = raw2central(m)
> print(a)
[1] 1 0 6 -74
> mean = 5
> print(mean)
[1] 5
> variance=a[3]
> print(variance)
[1] 6
> skewness=a[4]
> print(skewness)
[1] -74
>
```

$$\begin{aligned} \mu_1' &= E((x-p)^1) = E((x-3)^1) \\ &= E(x) - E(3) = 5 - 3 = 2 \end{aligned}$$

$$\mu_1' = E(x-3)$$

$$2 = E(x) - E(3) = (x)3 = 10$$

$$2 = E(x) - 3 \implies E(x) = 2 + 3 = 5$$

$$E(x) = 2 + 3 = 5$$

$$\mu_2' = \mu_2' - (\mu_1')^2$$

$$= 10 - (2)^2$$

$$= 10 - 4$$

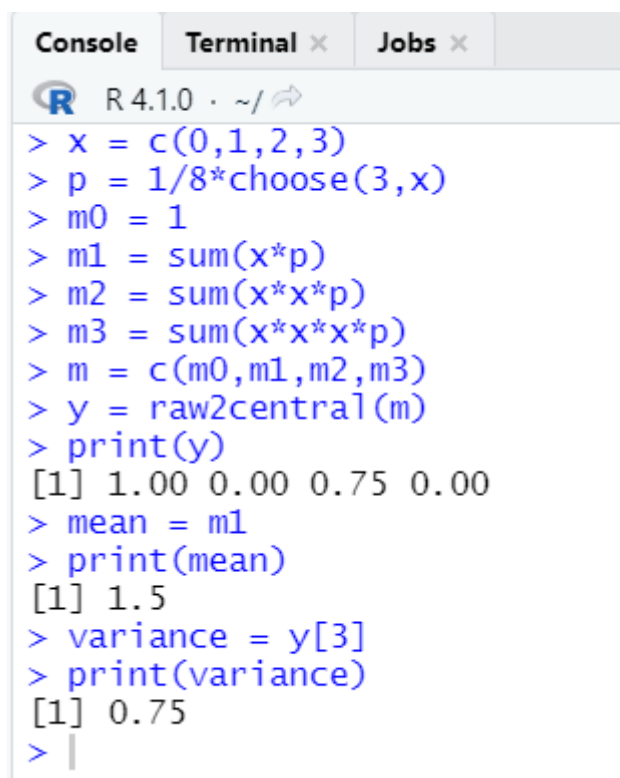
$$= 6 \implies \text{Variance}$$

$$\mu_3' = \mu_3' - 3\mu_1'\mu_2' + 2\mu_1'^3$$


$$= (-30) - 3(2)(10) + 2(2)^3$$

$$= -74 \implies \text{skewness}$$

```
3.x = c(0,1,2,3)
p = 1/8*choose(3,x)
m0 = 1
m1 = sum(x*p)
m2 = sum(x*x*p)
m3 = sum(x*x*x*p)
m = c(m0,m1,m2,m3)
y = raw2central(m)
print(y)
mean = m1
print(mean)
variance = y[3]
print(variance)
```



The screenshot shows an R console window with the following content:

```
R 4.1.0 · ~/ 
> x = c(0,1,2,3)
> p = 1/8*choose(3,x)
> m0 = 1
> m1 = sum(x*p)
> m2 = sum(x*x*p)
> m3 = sum(x*x*x*p)
> m = c(m0,m1,m2,m3)
> y = raw2central(m)
> print(y)
[1] 1.00 0.00 0.75 0.00
> mean = m1
> print(mean)
[1] 1.5
> variance = y[3]
> print(variance)
[1] 0.75
> |
```


$$\begin{aligned}
 3. \mu_x(t) &= E(e^{tx}) \\
 &= \sum e^{tx} p_x \\
 &= e^{t \cdot 0} (1/8) + e^{t \cdot 1} (3/8) + e^{t \cdot 2} (3/8) + e^{t \cdot 3} (1/8) \\
 \mu_x(t) &= \frac{1}{8} + e^t \frac{3}{8} + e^{2t} \frac{3}{8} + e^{3t} \frac{1}{8} \\
 &= \frac{1}{8} (1 + 3e^t + 3e^{2t} + e^{3t}) \\
 \mu'_1 &= \frac{d}{dt} \mu_x(t) \Big|_{t=0} \\
 \mu'_1 &= \frac{d}{dt} \mu_x(1) \Big|_{t=0} \\
 \mu'_1 &= \frac{d}{dt} \frac{1}{8} (1 + 3e^t + 3e^{2t} + e^{3t}) \\
 &= \frac{1}{8} (0 + 3e^t + 6e^{2t} + 3e^{3t}) \Big|_{t=0} \\
 &= \frac{3}{8} e^t + \frac{6}{8} e^{2t} + \frac{3}{8} e^{3t} \\
 &= \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8} \\
 &= \frac{3}{2} = 1.5 \Rightarrow \text{Mean} \\
 \mu'_2 &= \frac{d^2}{dt^2} \frac{1}{8} (1 + 3e^t + 3e^{2t} + e^{3t}) \\
 &= \frac{1}{8} \frac{d}{dt} (0 + 3e^t + 6e^{2t} + 3e^{3t}) \\
 &= \frac{1}{8} (3e^t + 12e^{2t} + 9e^{3t}) \Big|_{t=0} \\
 &= \frac{1}{8} (3 + 12 + 9) \\
 &= \frac{24}{8} = 3 \\
 \mu_2 &= \mu'_2 - (\mu'_1)^2 \\
 &= 3 - (1.5)^2 \\
 &= 3/4 \\
 &= 0.75 \rightarrow \text{variance}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{8} (3 + 12 + 9) \\
 &= \frac{24}{8} = 3 \\
 \mu_2 &= \mu'_2 - (\mu'_1)^2 \\
 &= 3 - (1.5)^2 \\
 &= 3/4 \\
 &= 0.75 \rightarrow \text{variance}
 \end{aligned}$$

```

4.x = c(-2,3,1)
p = c(1/3,1/2,1/6)
m0 = 1
m1 = sum(x*p)
m2 = sum(x*x*p)
m3 = sum(x*x*x*p)
m4 = sum(x*x*x*x*p)
m = c(m0,m1,m2,m3,m4)
y = raw2central(m)
print(y)
mean=m1
print(mean)
c_of_skewness = (y[4]/y[3]^(3/2))
print(c_of_skewness)
c_of_kurtosis = (y[5]/y[3]^(2))
print(c_of_kurtosis)

```

Console	Terminal x	Jobs x
 R 4.1.0 · ~/		
<pre> > m1 = sum(x*p) > m2 = sum(x*x*p) > m3 = sum(x*x*x*p) > m4 = sum(x*x*x*x*p) > m = c(m0,m1,m2,m3,m4) > y = raw2central(m) > print(y) [1] 1 0 5 -5 35 > mean=m1 > print(mean) [1] 1 > c_of_skewness = (y[4]/y[3]^(3/2)) > print(c_of_skewness) [1] -0.4472136 > c_of_kurtosis = (y[5]/y[3]^(2)) > print(c_of_kurtosis) [1] 1.4 > </pre>		

4. Raw moments: $\mu_1 = E(x^1) = \sum x^1 p_x$

$$\mu_0' = 1$$

$$\mu_1' = \sum x p_x$$

$$= (-2)(1/3) + 3(1/2) + 1(1/6)$$

$$= 1$$

$$\mu_2' = \sum x^2 p_x$$

$$= (4)(1/3) + 9(1/2) + 1(1/6)$$

$$= 6$$

$$\mu_3' = \sum x^3 p_x$$

$$= (-8)(1/3) + 27(1/2) + 1(1/6)$$

$$= 11$$

$$\mu_4' = \sum x^4 p_x$$

$$= 16(1/3) + 81(1/2) + 1(1/6)$$

$$= 46$$

$$\mu_1 = 0$$

$$\mu_2 = 5$$

$$\mu_3 = -5$$

$$\mu_4 = 35$$

$$\text{coeff. of skewness} = \frac{-5}{5^{3/2}} = -0.4472$$

$$\text{coeff. of kurtosis} = \frac{-5}{(5)^2} = \frac{5}{25}$$

$$= \frac{1}{5}$$

$$= 1.4$$