Chapter 1 | Basic Probability Concepts

2079/07/16
Wednesday

Basic Terms

1) Experiment → any task or action that is performed to get some results or outcomes

Deterministic Experiment
↓
those whose possible outcomes can be priorly predicated

~~Non~~ Probabilistic / Random experiment
↓
those whose possible outcomes can ~~be~~ not be perfectly predicated
eg. Tossing a fair coin
Tossing a coin twice

2) Sample Space → It is the set of all possible outcomes.

→ It is denoted by S or Ω

eg. S = {Head, Tail}

S = {HH, HT, TH, TT}

Sample space for rolling a die (S) = {1, 2, 3, 4, 5, 6}

Sample space for rolling a dice (i.e. two dies)

(S) = {11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66}

3) Trial and Event

→ Trial is the individual performance of random experiment.

→ Result or outcome of trial or random experiment is called as Event

<u>Example of events</u> :-

let, E_1 = Getting 2 heads

E_2 = getting 1 head

E_3 = Getting no head

E_4 = Getting at least 1 head

E_5 = Getting at most 1 head

here, E_1, E_2, E_3, E_4 & E_5 are events.

Events

Simple Event
↓
event having single result or single outcome

Compound Event
↓
event having more than one result / outcome.

<u>Example</u>

For rolling a die,

$S = \{1, 2, 3, 4, 5, 6\}$.

Let E_1 = Getting 1 (Simple event)

E_2 = Getting odd number (compound event)

E_3 = Getting Even number (compound event).

For rolling two dice simultaneously,

$S = \{11, 12, 13, \ldots\ldots, 66\}$

Let, E_1 = Getting both odd numbers (compound event)

E_2 = Getting both even numbers (compound event)

E_3 = Getting both faces same (compound event)

E_4 = Getting different faces (compound event)

E_5 = Getting greater number in first die than second (compound event)

E_6 = Getting sum of both faces of dice greater than 5 (compound event)

⋮

(many more ~~pre~~ events)

4) Equally likely events

→ Those events or possible outcomes ~~whose occu~~ which have equal probability of occurance are called as equally likely events.

eg. Tossing a coin

$S = \{H, T\}$; both Head and Tail have equal probability of occurance (i.e. $\frac{1}{2}$)

~~Toss~~ Rolling a die

$S = \{1, 2, 3, 4, 5, 6\}$; all the numbers have equal probability of occurance (i.e. $\frac{1}{6}$)

5) Mutually Exclusive Events

→ Two or more events are said to be mutually exclusive events if both the events ~~do not~~ cannot occur simultaneously at the same time.

6) Exhaustive cases or events

→ The total number of all possible outcomes of a random experiment is called as exhaustive cases/events.

→ It is denoted by n.

eg: $S = \{1, 2, 3, 4, 5, 6\}$

$\therefore \quad n = 6$

7) Favourable cases or events

→ It is the total number of possible outcome of a particular event.

→ denoted by m

eg. $S = \{1, 2, 3, 4, 5, 6\}$

Let,

E_1 = Getting even number

$\therefore \quad m = 3$

8) Independent Events

→ Two events are said to be independent events if the occurance of one event does not affect the occurance of another event.

9) Dependent Events

→ Two events are said to be dependent events if the occurance of one event affects the occurence of another event.

2079/07/18
Friday

Various Approaches to Probability

1. Classical or Mathematical Approach

→ This concept or approach is based on Theoretical or Ideal conception.

→ Let there are 'n' number of mutually exclusive and equally likely exhaustive cases or events of a random experiment.

Out of n, 'm' number of cases fabour to an event A (say).

Then the probability of happening of an event A, denoted by P(A) is given by :-

$$\boxed{P(A) = \frac{\text{Favourable cases}}{\text{Exhaustive Cases}} = \frac{m}{n}}$$

Similarly, from the def.^{n},

The prob. of non-happening of an event A, denoted by $P(\bar{A})$ or $P(A^c)$ is denoted as :-

$$P(\bar{A}) = \frac{n-m}{n} = 1 - \frac{m}{n}$$

$$\therefore P(\bar{A}) = 1 - P(A)$$

Remarks:

1. $P(A) \geq 0$ and $P(\bar{A}) \geq 0$

   such that $\boxed{P(A) + P(\bar{A}) = 1}$

   (i.e. probability lies always between 0 and 1)

   or, $\boxed{p + q = 1}$

2. $0 \leq P(A) \leq 1$, $m \leq n$

3. $P(S) =$ Total probability $= 1$

4. $P(\phi) =$ probability of impossible event $= 0$

5. If A and B are two mutually exclusive events, then the probability of getting at least one event (i.e. Either A or B) is given by,

   $$\boxed{P(A \text{ or } B) \text{ or } P(A \cup B) = P(A) + P(B)}$$

   Similarly, for three mutually exclusive events A, B and C,

   $$\boxed{P(A \cup B \cup C) = P(A) + P(B) + P(C)}$$

Example 1

Q. If you toss a fair coin twice, then find the probability of getting

(i) at least one head

(ii) 2 heads

(iii) 1 head

(iv) no head

(v) at most one head

Solun

Sample space $(S) = \{HH, HT, TH, TT\}$

$\therefore$ Exhaustive cases $(n) = 4$

(i) soln Let E_1 = getting at least 1 head

$\therefore$ favourable cases $(m) = 3$

So, probability of getting at least 1 head is,

$$P(C_1) = \frac{m}{n} = \frac{3}{4} \text{ ans}$$

(ii) soln Let C_2 = getting two heads

$\therefore m = 1$

So, $P(C_2) = \frac{m}{n} = \frac{1}{4}$ ans

(iii) soln Let C_3 = getting 1 head

$\therefore m = 2$

So, $P(C_3) = \frac{m}{n} = \frac{2}{4} = \frac{1}{2}$ ans

(iv) Soln Let C_4 = getting no head

$\therefore m = 1$

So, $P(C_4) = \frac{m}{n} = \frac{1}{4}$ ans

(v) Soln

Let C_5 = getting at most one head

$\therefore m = 3$

So, $P(C_5) = \frac{m}{n} = \frac{3}{4}$ ans

At most 1 head meaning बढिमा एउटा Head अथवा एउटै पनि Head नआए पनि हुन्छ । दुवै situation लाई count गर्नु पर्यो ।

Example 2

Q. If you roll two dice simultaneausly, then find the ~~probabily~~ probability of getting

(i) sum greater than 8

(ii) sum of 8 or 9

(iii) same faces

(iv) Different faces

$\underline{\text{Sol}^n}$

$S = \{11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66\}$

$\therefore$ Exhaustive cases $(n) = 36$

i) sum greater than 8

$\underline{\text{Sol}^n}$

Let D_1 = getting sum greater than 8

$\therefore m = 10$

So, $P(D_1) = \frac{m}{n} = \frac{10}{36}$ ans

(ii) sum of 8 or 9

$\underline{\text{Sol}^n}$ Let D_2 = getting sum of 8 or 9

$\therefore m = 9$

So, $P(D_2) = \frac{m}{n} = \frac{\cancel{9}^{1}}{\cancel{36}_{4}} = \frac{1}{4}$

(iii) $\underline{\text{Sol}^n}$ Let D_3 = getting same faces

$\therefore m = 6$

So, $P(D_3) = \frac{m}{n} = \frac{\cancel{6}^{1}}{\cancel{36}_{6}} = \frac{1}{6}$ ans

(iv) $\underline{\text{Sol}^n}$ Let D_4 = getting different faces

~~$\therefore m =$~~

$\therefore P(D_4) = 1 - P(D_3) = 1 - \frac{1}{6} = \frac{5}{6}$ ans

2. Statistical or Empirical Approach

→ This approach is based on experiments and observations.

→ Let an event A occurs 'm' times in 'n' repetations of a random experiment.

In the limiting case when 'n' becomes sufficiently large, then the probability of happening of an event A is finite and unique. That is,

$$\boxed{P(A) = \lim_{n \to \infty} \frac{m}{n}}$$

3. Subjective Approach

→ This approach is based on personal views or expeations or knowledge or experience.

4. Axiomatic or Modern Approach

→ This approach is the mixed concept of classical and statistical approach.

→ Let S be a sample space of a random experiment and A be any event which is defined on sample space (S). Then P(A) is a probability function satisfying the following axioms.

(i) $P(A) \geq 0$; Axiom of Non-negativity

(ii) $P(S) = 1$; Axiom of certainity
or,
Axiom of Totality

(iii) Let $A_1, A_2, ---, A_n$ be a finite or infinite sequence of mutually exclusive events. Then, by axiom of additivity we have,

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$$

Similarly,

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$

Counting Rules

→ It has 2 Fundamental Rules.

Addition Rule　　　　Multiplication Rule

Addition Rule

→ When event A occurs in n_1 ways and event B occurs in n_2 ways and when they are mutually exclusive, then event A or B (at least one event) can occur in $(n_1 + n_2)$ ways.

→ Example: A bag contains 5 Red and 6 Black balls, then the total number of cases for either Red or Black ball is $5+6=11$.

Multiplication Rule

→ When event A occurs in n_1 ways and event B occurs in n_2 ways, then both events A and B can occur simultaneously in $(n_1 \cdot n_2)$ ways.

→ Example: On rolling two dice simultaneously, first die has 6 possible outcomes and second die also has 6 possible outcomes. Then the total possible outcome is $6 \cdot 6 = 36$

Basic Methods of Counting Rule

2 Methods

Permutation

Combination

Permutation

→ Permutation means arrangement.

→ In permutation, the position or order of objects play the role.

$n \rightarrow$ total number of different objects

$r \rightarrow$ objects that are to be arranged among n

Then,

$$\boxed{{}^{n}P_{r} = \frac{n!}{(n-r)!}}$$

→ For repeated cases, the permutation of n objects in which p, q and r times objects are repeated is given by,

$$\boxed{\text{Permutation} = \frac{n!}{p! \cdot q! \cdot r!}}$$

Example 1 (Assignment 1)

Q. In how many ways the word 'STATISTICS' can be arranged?

Soln

$S \rightarrow 3$

$T \rightarrow 3$

$A \rightarrow 0$

$I \rightarrow 2$

$C \rightarrow 0$

$n = 10$

$$\frac{n!}{p!\,q!\,r!}$$

$$= \frac{10!}{3!\,3!\,2!}$$

$= 50400$ ways

Therefore, the word can be arranged in 50400 ways.

2079/07/20

Sunday

Combination (= Selection)

$n \rightarrow$ total objects

$r \rightarrow$ number of objects to be selected

→ In combination, order or position of objects does not play role.

→ <u>notation:</u> ^nC_r or $\binom{n}{r}$ or $C(n,r)$

$$\boxed{\therefore\ ^nC_r = \frac{n!}{(n-r)!\,r!} \quad \text{for } r \le n}$$

<u>Example 1</u>: A box contains 4 white and 6 Red balls. Two balls are drawn at random manner from the box. Then, find out the probability that both balls are red?

Soln given,

number of white balls = 4

number of red balls = 6

Total balls = 4 + 6 = 10

Out of 10 balls, 2 balls can be drawn in $^{10}C_2$ ways.

so, exhaustive cases $(n) = {}^{10}C_2$

$= 45$

Let, E = getting both balls red

Out of 6 red balls, 2 can be drawn in ${}^6C_2$ ways.

$\therefore$ favourable cases $(m) = {}^6C_2 = 15$

Hence, the probability of getting both balls red

$$P(E) = \frac{m}{n} = \frac{15}{45} = \frac{1}{3} \text{ ans}$$

Example 2:

A bag contains 15 IC chips with 5 defectives. If a random sample of 3 chips is drawn from the bag, then what is the probability that :-

(i) all are defectives
(ii) two are defectives
(iii) one defective
(iv) no defective

Soln given, Total number of IC chips = 15
number of defective chips = 5
number of non-defective chips = 15-5 = 10

Out of 15 chips, 3 can be drawn in ${}^{15}C_3$ ways.

So, exhaustive cases $(n) = {}^{15}C_3 = 455$

(i) Soln Let E_1 = getting all are defectives

Out of 5 defective chips, 3 can be drawn in ${}^5C_3$ ways

so, favourable cases $(m) = {}^5C_3 = 10$

Hence, the probability of getting all defective chips $P(E_1) = \frac{m}{n} = \frac{10}{455}$ ans

(ii) Soln let E_2 = getting two defectives
(i.e. getting two defectives and 1 non-defective)

$\therefore P(E_2) = P$ (getting two defectives) $\times P$ (getting 1 non-defective)

again,
out of 5 defective chips, 2 can be drawn in $5C_2$ ways.
so, $m = 5C_2 = 10$

$\therefore P$ (getting two defectives) $= \frac{m}{n} = \frac{10}{455}$

similarly,
out of 10 non-defective chips, 1 can be drawn in $10C_1$ ways.
so, $m = 10C_1 = 10$

$\therefore P$ (getting 1 non-defective) $= \frac{m}{n} = \frac{10}{455}$

Now,
$P(E_2) = \frac{10}{455} \times \frac{10}{455} = \frac{100}{207025}$ ans

(iii) Soln let E_3 = getting 1 defective
(i.e. getting 1 defective and getting 2 non-defective)

$\therefore P(E_3) = P$(getting 1 defective) $\times P$ (getting 2 non-defective)

Sol
out of 5 defective chips, 1 can be drawn in $^5C_1$ ways.

$\therefore m = {^5C_1} = 5$

$\therefore P(\text{getting 1 defective}) = \frac{m}{n} = \frac{5}{455}$

Similarly,
out of 10 non-defective chips, 2 can be drawn in $^{10}C_2$ ways.

So, $m = {^{10}C_2} = 45$

$\therefore P(\text{getting 2 non-defective}) = \frac{m}{n} = \frac{45}{455}$

Now, $P(E_3) = \frac{5}{455} \times \frac{45}{455} = \frac{9}{8281}$ ans

v) Soln let, E_4 = getting no defectives
(i.e. getting all non-defective)

So, out of 10 non-defective, 3 can be drawn in $^{10}C_3$ ways.

$\therefore m = {^{10}C_3} = 120$

$\therefore P(E_4) = \frac{m}{n} = \frac{120}{455}$ ans

Example 3 : A lot of IC chips of 10 good, 4 with minor defects and 2 with major defects.
2 chips are selected randomly from the lot.
what is the probability that :
(i) at least 1 chip is good ?
(ii) no chips are good

Soln given,
number of good chips = 10
number of bad chips = 4 + 2 = 6
Total number of chips = 10 + 6 = 16

Out of 16 IC chips, 2 chips can be selected in $^{16}C_2$ ways.
i.e. exhaustive cases $(n) = {}^{16}C_2 = 120$

(i) Soln
Let E_1 = getting at least 1 ~~chip~~ good chip.
(i.e. getting 1 good chip and 1 bad chip or 2 good chips)

So favourable cases $(m) = {}^{10}C_1 \cdot {}^{6}C_1 + {}^{10}C_2 = 105$

$\therefore P(E_1) = \frac{m}{n} = \frac{105}{120}$ ans

(ii) Soln let E_2 = getting no good chips (i.e. getting all bad chips)

so $m = {}^{6}C_2 = 15$

$\therefore P(E_2) = \frac{15}{120}$ ans

Example 4 :

If a box contains 75 good IC chips and 25 defective chips, and 12 chips are selected at random. What is the probability that at least one chip is defective ?

Soln given,

number of good IC chips = 75

number of defective chips = 25

Total number of chips = 75 + 25 = 100

Out of 100 IC chips, 12 chips can be selected in $^{100}C_{12}$ ways.

So, exhaustive cases $(n) = {}^{100}C_{12}$

Let E_1 = getting no defective chips (i.e. all chips good)

$\therefore m = {}^{75}C_{12}$

So, $P(E_1) = \frac{m}{n} = \frac{^{75}C_{12}}{^{100}C_{12}} = 0.0248$

Now, let $\bar{E_2}$ = getting at least one chip defective

$\therefore P(\bar{E_2}) = 1 - P(E_1) = 1 - 0.0248 = 0.975$ ans

Assignment 2:

Q-1. From a group of 15 chess players, 8 are selected to represent a group at a convention. What is the probability that the selected group include 3 out of 4 best players in the group?

Soln
given,

total number of chess players = 15
number of best players = 4
number of ordinary players = 15 − 4 = 11

Out of 15 chess players, 8 players can be selected in ${}^{15}C_8$ ways.

So, exhaustive cases $(n) = {}^{15}C_8 = 6435$

Let, E = Getting 3 best players
(i.e. getting 3 best players and 5 ordinary players)

So, no. of favourable cases $(m) = {}^{4}C_3 \cdot {}^{11}C_5 = 1848$

Hence, the probability that the selected group include 3 out of 4 best players $P(E) = \frac{m}{n}$

$$= \frac{1848}{6435}$$

$$= 0.287$$ ans

2079 / 07 / 23

wednesday

Laws of Probability

(i) Addition Law

(ii) Multiplication Law

Addition Law

(i) Let A and B are two mutually exclusive events. Then the probability of getting either A or B (i.e. at least one event) is given by,

$$\boxed{P(A \text{ or } B) \text{ or } P(A \cup B) = P(A) + P(B)}$$

Similarly,

for three mutually exclusive events A, B and C,

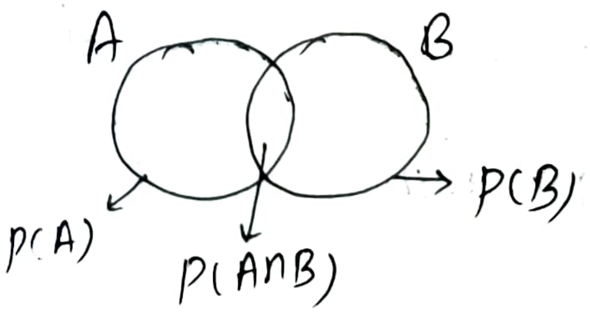
$$\boxed{P(A \cup B \cup C) = P(A) + P(B) + P(C)}$$

(ii) Let A and B are non-mutually exclusive events. Then,

the probability of getting at least one event (either A or B or both) is given by,

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

↳ occurence of both events



Similarly,

for three non-mutually exclusive events A, B and C,

$$\boxed{P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)}$$

Multiplication law

(i) Let A and B are two independent events. Then the probability of getting both events A and B is given by,

$$P(A \cap B) = P(A) \cdot P(B)$$

(ii) Let A and B are two dependent events. Then the probability of getting both events A and B is given by,

$$P(A \cap B) = P(A) \cdot P(B/A)$$

also, $P(A \cap B) = P(B) \cdot P(A/B)$

where, $P(A|B) \rightarrow$ probability of getting event A given that event B has already occured.

$P(B|A) \rightarrow$ probability of getting B given that event A has already occured.

This is called Conditional probability.

Example 1: The probability that a new airport will get an award for its design, award for its efficient use of materials and award for both is 0.16, 0.24 and 0.11 respectively. Then find the probability that the new airport get at least one award. Also what is the probability that it will get only one of two awards?

Soln given,

Let,

E_1 = getting award for design

E_2 = getting award for efficient use of materials

Then,

we also have,

$P(E_1) = 0.16$

$P(E_2) = 0.24$

$P(E_1 \cap E_2) = 0.11$

Now, the probability of getting at least one award

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= 0.16 + 0.24 - 0.11 = 0.29$ ans

Again, the probability of getting only one of two awards

$P(\text{only one of two award}) = P(E_1 \text{ only or } E_2 \text{ only})$

$= P(E_1 \text{ only}) + P(E_2 \text{ only})$

$= P(E_1) - P(E_1 \cap E_2) + P(E_2) - P(E_1 \cap E_2)$

$= 0.16 - 0.11 + 0.24 - 0.11$

$= 0.18$ ans

Example 2: The probability that a contractor will get a plumbing contract is $\frac{2}{3}$ and the probability that he will not get an electric contract is $\frac{5}{9}$. If the probability of getting at least one contract is $\frac{4}{5}$, then what is the probability of getting both contracts?

Soln given,

let, E_1 = getting a plumbing contract

E_2 = getting an electric contract

Then, we also have,

$P(E_1) = \frac{2}{3}$

$P(E_2) = 1 - \frac{5}{9} = \frac{9-5}{9} = \frac{4}{9}$

$P(E_1 \cup E_2) = 4/5$

now,

probability of getting both contracts $P(E_1 \cap E_2)$ is given by,

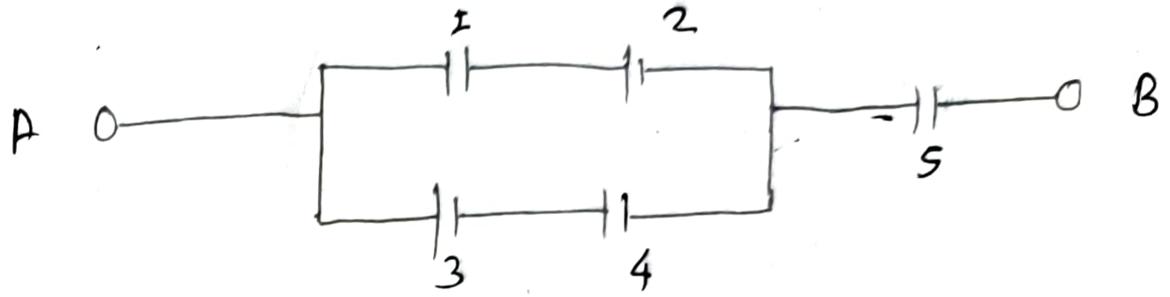
$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

or, $\frac{4}{5} = \frac{2}{3} + \frac{4}{9} - P(E_1 \cap E_2)$

or, $P(E_1 \cap E_2) = \frac{2}{3} + \frac{4}{9} - 4/5$

$\therefore P(E_1 \cap E_2) = \frac{14}{45} = 0.312$ ans

Example 3 : For the circuit given below, the probability of closing each relay of the circuit is 0.7. Assume that the relays act independently. What is the probability that a current will exist between the terminals A and B.



Soln
Given,
the relays are 1, 2, 3, 4 and 5
Let,
E_1 = closing of relay 1, 2 and 5
E_2 = closing of relay 3, 4 and 5

Since the relays are acting independently.
so,
$P(E_1) = P(1 \text{ and } 2 \text{ and } 5) = P_1 . P_2 . P_5$
we are also given that, the probability of closing each relay of the circuit is 0.7.
i.e. $P_1 = P_2 = P_3 = P_4 = P_5 = 0.7$

$\therefore P(E_1) = 0.7 \times 0.7 \times 0.7 = 0.343$

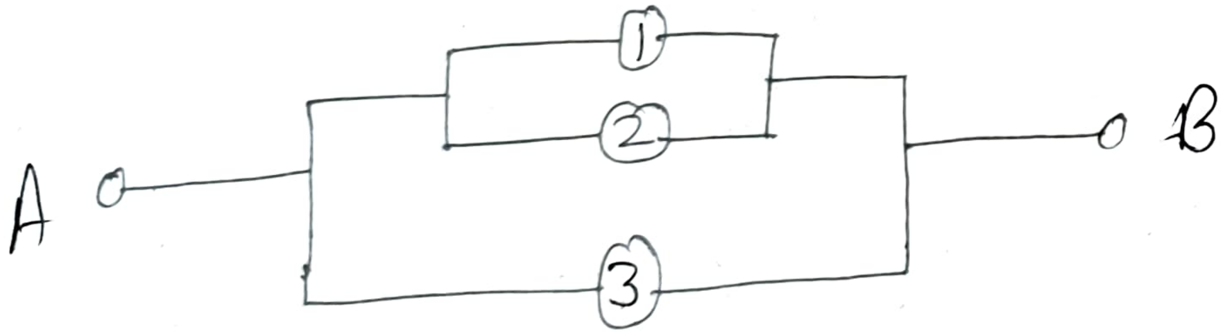
again,
$P(E_2) = P_3 . P_4 . P_5 = 0.7 \times 0.7 \times 0.7 = 0.343$

So $P(E_1 \cap E_2) = P(\text{closing } 1 \text{ and } 2 \text{ and } 3 \text{ and } 4 \text{ and } 5)$
$= P_1 . P_2 . P_3 . P_4 . P_5$
$= 0.7 \times 0.7 \times 0.7 \times 0.7 \times 0.7$
$= \cancel{0.168}\ \cancel{0.240}\ 0.168$

Now,
the probability of current flowing from A to B,
p(current flow) = p(closing of at least one path)

$$= P(E_1 \cup E_2)$$
$$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$$
$$= 0.322 \text{ ans}$$

Example 4: Consider the following portion of an electric circuit with 3 relays. Current will flow from point A to B if there is at least one closed path when the relays are activated. The relays may malfunction and not closed when activated. Suppose that the relays act independently of one another and closed properly when activated with probability of 0.90. What is the probability that the current will flow when the relays are activated.



Sol^n given,
let,
E_1 = closing of relay 1
E_2 = closing of relay 2
E_3 = closing of relay 3
Then, we also have, probability of closing each relay is 0.90.

i.e. $p(E_1) = 0.90$
$p(E_2) = 0.90$
and, $p(E_3) = 0.90$

So, probability of closing relay 1 and relay 2 is,
$$P(E_1 \cap E_2) = P(E_1).P(E_2) = 0.90 \times 0.90 = 0.81.$$

Similarly,
probability of closing relay 1 and relay 3 is,
$$P(E_1 \cap E_3) = P(E_1).P(E_3) = 0.9 \times 0.9 = 0.81$$

Also, probability of closing relay 2 and relay 3 is,
$$P(E_2 \cap E_3) = P(E_2).P(E_3) = 0.9 \times 0.9 = 0.81$$

So, probability of closing all three relays is,
$$P(E_1 \cap E_2 \cap E_3) = P(E_1).P(E_2).P(E_3) = 0.9 \times 0.9 \times 0.9 = 0.729$$

Now, the probability of current flow when the relays are activated (i.e., probability of closing at least one relay) is,
$$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_1 \cap E_3) - P(E_2 \cap E_3) + P(E_1 \cap E_2 \cap E_3)$$
$$= 0.9 + 0.9 + 0.9 - 0.81 - 0.81 - 0.81 + 0.729$$
$$= 0.999$$ ans

another method

given that,
probability of closing each relay of the circuit is 0.9.

i.e., $p_1 = p_2 = p_3 = 0.9$

Then,
the probability of opening each relay 1, 2 and 3 is,

$q_1 = q_2 = q_3 = 1 - 0.9 = 0.1$

Now,
the probability of opening all relay, (i.e. opening relay 1 and 2 and 3)

$P(\text{all open}) = q_1 \cdot q_2 \cdot q_3 = 0.1 \times 0.1 \times 0.1 = 0.001$

∴ The probability of at least one relay closed is,

$P(\text{current flow}) = P(\text{at least one relay closed})$

$= 1 - P(\text{all open})$

$= 1 - 0.001 = 0.999$ ans

Assignment 3

Q. There are three switches in a college network namely A, B and C working independently. These switches are configured in series so that all switches should be ON to have successful transmission of data. The individual probability of being switch ON for these are $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{1}{4}$ respectively. Find the probability that :-

(i) there will be a successful data transfer

(ii) there will be not successful data transfer.

Soln given,

let,

E_1 = switch A being ON

E_2 = switch B being ON

E_3 = switch C being ON

we also have that,

probability of switch A being ON is given by,

$P(E_1) = \frac{1}{2}$

Similarly,

probability of switch B being ON is given by,

$P(E_2) = 3/4$

And, probability of switch C being ON is given by,

$P(E_3) = 1/4$

(i) Soln let, S_1 = getting a successful data transfer (i.e. all switches should be ON)

Thus, the probability of getting a successful data transfer is given by,

$P(S_1) = P(E_1 \cap E_2 \cap E_3) = P(E_1) \cdot P(E_2) \cdot P(3)$

$= 1/2 \times 3/4 \times 1/4 = \underline{0.09375}$

(ii) Soln let $\bar{S_1}$ = not getting a successful data transfer

$\therefore P(\bar{S_1}) = 1 - P(S_1) = 1 - 0.09375$

$= \underline{0.90625}$ ans

2079/07/25

Friday

Example 1:

The odds in favour of A solving a mathematical problem are 3 to 4 and the odds against B solving the problem are 5 to 7.

Find the probability that the problem will be solved if they try independently?

Soln given,

Let, A = solving problem by A

and, B = solving problem by B

Then we have,

$P(A) = 3/7$

$P(B) = 7/12$

now,

probability that the problem will be solved is,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= P(A) + P(B) - P(A) \cdot P(B)$$

$$= \frac{3}{7} + \frac{7}{12} - \frac{\cancel{3}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{12}\,4}$$

$$= \frac{3}{7} + \frac{7}{12} - \frac{1}{4}$$

$$= \frac{16}{21} = 0.762 \text{ ans}$$

Example 2:

A problem in statics is given to 3 students A, B and C whose chances of solving it are $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{1}{4}$ respectively.

What is the probability that the problem will be solved if all of them try independently?

Soln Let,

E_1 = solving the problem by A

E_2 = solving the problem by B

E_3 = solving the problem by C

Then we have;

$P(E_1) = 1/2$, $P(E_2) = \frac{3}{4}$ and $P(E_3) = 1/4$

now,

the probability that the problem will be solved is,

$$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_1 \cap E_3) - P(E_2 \cap E_3) + P(E_1 \cap E_2 \cap E_3)$$

so,

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$$
$$= 1/2 \cdot 3/4 = \frac{3}{8}$$

$$P(E_1 \cap E_3) = 1/2 \cdot 1/4 = 1/8$$

$$P(E_2 \cap E_3) = 3/4 \cdot 1/4 = 3/16$$

$$P(E_1 \cap E_2 \cap E_3) = 1/2 \cdot 3/4 \cdot 1/4 = \frac{3}{32}$$

$$\therefore P(E_1 \cup E_2 \cup E_3) = 1/2 + \frac{3}{4} + 1/4 - 3/8 - \frac{1}{8} - \frac{3}{16} + \frac{3}{32}$$
$$= 0.90625$$ ans

Conditional Probability (Imp. for short notes)

→ ~~Proba~~

→ Conditional probability is the probability of an event occuring given that another event has already occured.

→ In other words, conditional probability means the probability of occuring an event based on the occurence of previous event or outcome.

→ For example:

- Let us consider that a fair die has been rolled and we are asked to give the probability that it was a five. In this case, there are six equally likely outcomes, so our answer will be $\frac{1}{6}$.

- But, suppose ~~if we~~ ~~suppose~~ we are given the condition that the number rolled was odd. Since, there are only three odd numbers that are possible, one of which is five. In this case, our answer will not be $\frac{1}{6}$ rather it will by $\frac{1}{3}$.

→ The conditional probability of getting event A given that event B has already occured is denoted by P(A|B) and is given by,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \; ; \; P(B) \neq 0$$

→ Similarly, the conditional probability of getting event B given that event A has already occured is denoted by P(B/A) and is given by,

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \;; \text{ such that } P(A) \neq 0$$

Example 1: A manufacturer of airplane parts knows that the probability is 0.8 that an order will be ready for shipment on time and it is 0.7 that an order will be ready for shipment on time and the order will be delivered on time. Then, what is the probability that such an order will be delivered on time given that it was also ready for shipment on time? ↳ condition

Soln given,

let, E_1 = order will be ready for shipment on time

E_2 = order will be delivered on time.

Then we have,

$P(E_1) = 0.8$

$P(E_1 \cap E_2) = 0.7$

Now, the probability that such an order will be delivered on time given that it was also ready for shipment on time is given by,

$$P(E_2/E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{0.7}{0.8} = 0.875 \text{ ans}$$

Example 2: A box contains 4 bad and 6 good tubes. 2 are drawn out from the box at a time. One of them is tested and found to be good. What is the probability that the other one is also good?

Soln given,

number of bad tubes = 4

" " good " = 6

total number of tubes = 4+6 = 10, ~~(i.e. n = 10)~~

Let,

E_1 = First tube is good

E_2 = Second tube is good

Then we have,

$$P(E_1) = \frac{6}{10}$$

also,

Out of 6 good tubes, 2 can be selected in ${}^6C_2$ ways

$\therefore$ favourable cases $(m) = {}^6C_2 = 15$

~~$\therefore P(E_1 \cap E_2) = \frac{15}{10}$~~

Similarly,

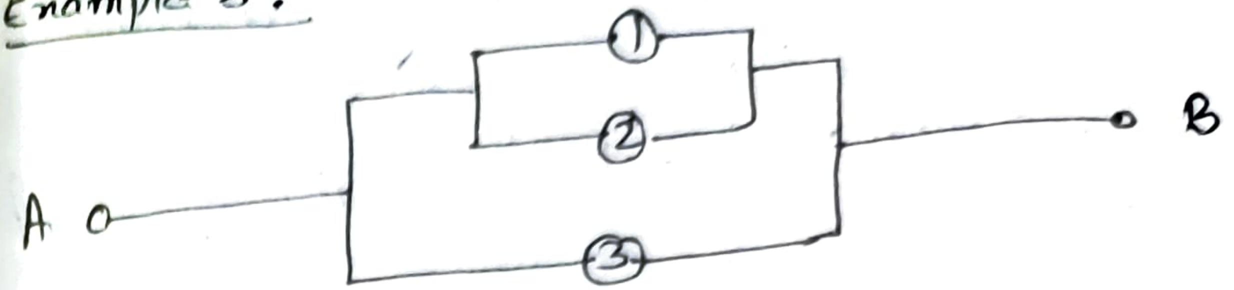
Out of 10, 2 can be selected in ${}^{10}C_2$ ways,

$\therefore n = {}^{10}C_2 = 45$

$\therefore P(E_1 \cap E_2) = 15/45$

$$\therefore P(E_2/E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{15/45}{6/10} = 0.556 \text{ ans}$$

Example 3 :



Given that current flows when the relays were activated, what is the prob. that relay 1 functioned?

Soln Let,

A = current flows (i.e. at least one relay is closed)

B = relay 1 functioned

Then,
$$P(A) = 0.999$$

Now,
$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{P(B)}{P(A)} \quad \left(\text{since, } B \text{ is the subset of } A\right)$$

$$= \frac{0.9}{0.999} = 0.9009 \text{ ans}$$

Assignment 4: A, B and C toss a fair coin in an order. The first one to throw a head wins. If A starts the game, what is the probability of winning the game by A?

Sol^n given,

A → winning the game by A

→ (i) → winning the game by A in 1st toss.

so, $P(i) = 1/2$ (i.e. A_1)

→ (ii) → winning the game by A in 2nd toss.

i.e. $\bar{A}_1 \bar{B}_1 \bar{C}_1 A_2$

so, $P(ii) = 1/2 \cdot 1/2 \cdot 1/2 \cdot 1/2 = (1/2)^4$

→ (iii) → winning the game by A in 3rd toss

i.e. $\bar{A}_1 \bar{B}_1 \bar{C}_1 \bar{A}_2 \bar{B}_2 \bar{C}_2 A_3$

so, $P(iii) = (1/2)^7$

⋮

and so on.

Now, the probability of winning game by A is,

$$P(A) = P(i) + P(ii) + P(iii) + \cdots$$

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^4 + \left(1/2\right)^7 + \cdots$$

$$= \frac{1}{2}\left(1 + \frac{1}{8} + \left(\frac{1}{8}\right)^2 + \left(\frac{1}{8}\right)^3 + \cdots\right)$$

Formula:

$S_n = \dfrac{a}{r-1}$; if $r > 1$; where r is common ratio and a is the first term

$S_n = \dfrac{a}{1-r}$; if $r < 1$

$$= \frac{1}{2}\left(\frac{1}{1-1/8}\right)$$

$$= \frac{1}{2}\left(1/7/8\right) = 1/2 \times 8/7 = 4/7 = 0.572 \text{ ans}$$

Assignment 5: A and B alternately throw a pair of die. A will win if he throws 6 before B throws 7 and B will win if he throws 7 before A throws 6. If A begins, what is the chance of winning the game by A?

Soln

the possible outcomes after throwing a pair of die alternately are given by,

Sample space (S) = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

∴ exhaustive cases (n) = 36

given that, A wins if he throws 6 before B throws 7.

So, the favourable cases for A is, (m) = 5

~~Hence, the probability~~

Similarly,

B wins if he throws 7 before A throws 6.

so, the favourable cases for (m) = 6

B is, $\therefore P(B) = \frac{6}{36} = 1/6$ (probability of winning the game by B)

Let,

$E_1 \rightarrow$ wining the game by A

i → wining the game by A in first throw

$\therefore p(i) = \frac{5}{36}$ (i.e. A_1)

ii → winning the game by A in 2nd throw

i.e. $\bar{A}_1 \bar{B}_1 A_2$

$\therefore p(ii) = \left(1-\frac{5}{36}\right)\left(1-\frac{1}{6}\right)\frac{5}{36}$

$= \frac{5}{36}\left(\frac{31}{36}\right)\left(\frac{5}{6}\right) = \frac{5}{36}\left(\frac{155}{216}\right)$

iii. → winning the game by A in 3rd throw

i.e. $\bar{A}_1 \bar{B}_1 \bar{A}_2 \bar{B}_2 A_3$

$\therefore p(iii) = \left(1-\frac{5}{36}\right)\left(1-\frac{1}{6}\right)\left(1-\frac{5}{36}\right)\left(1-\frac{1}{6}\right)\frac{5}{36}$

$= \frac{5}{36}\left(\frac{31}{36}\right)^2\left(\frac{5}{6}\right)^2 = \frac{5}{36}\left(\frac{155}{216}\right)^2$

iv → winning the game by A in 4th throw

i.e. $\bar{A}_1 \bar{B}_1 \bar{A}_2 \bar{B}_2 \bar{A}_3 \bar{B}_3 A_4$

$\therefore P(iv) = (1-5/36)(1-1/6)(1-5/36)(1-1/6)(1-5/36)$
$(1-1/6)\ 5/36$

$= \frac{5}{36}\left(\frac{31}{36}\right)^3\left(\frac{5}{6}\right)^3 = \frac{5}{36}\left(\frac{155}{216}\right)^3$

⋮

and so on.

now, the probability of winning the game by A is,

$P(G) = P(i) + P(ii) + P(iii) + P(iv) + \cdots$

$= \frac{5}{36} + \frac{5}{36}\left(\frac{155}{216}\right) + \frac{5}{36}\left(\frac{155}{216}\right)^2 + \frac{5}{36}\left(\frac{155}{216}\right)^3 + \cdots$

$= \frac{5}{36}\left[1 + \left(\frac{155}{216}\right) + \left(\frac{155}{216}\right)^2 + \left(\frac{155}{216}\right)^3 + \cdots\right]$

here, $a = 1$ (first term)

common ratio $(r) = \frac{\frac{155}{216}}{1} = \frac{155}{216} < 1$

$= \frac{5}{36}\left[\frac{a}{1-r}\right]$

$= \frac{5}{36}\left[\frac{1}{1-\frac{155}{216}}\right]$

$= \frac{5}{36} \times \frac{216}{61}$

$= \frac{30}{61}$

$= 0.492$ ans

2079/07/27

Sunday

Bayes' Theorem (Inverse Probability)

Statement:

Let $E_1, E_2, ---, E_n$ be 'n' mutually exclusive events which is defined on sample space (S) with $P(E_i) \neq 0$; where $i = 1, 2, 3, 4, ---, n$.

For any arbitary event A (say) which is the subset of sample space such that $P(A) > 0$, then the probability of getting E_i given that event A has already occured is given by:

$$\boxed{P(E_i/A) = \frac{P(E_i) \cdot P(A/E_i)}{\sum_{i=1}^{n} P(E_i) \cdot P(A/E_i)}}$$

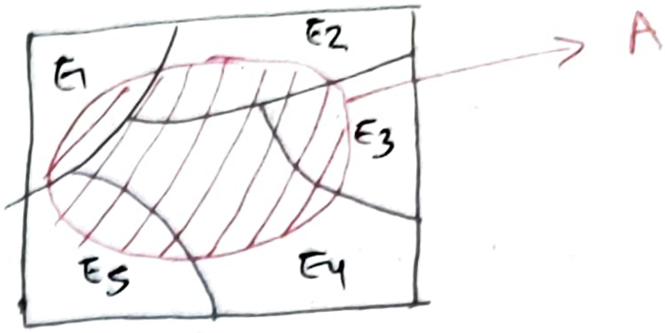
where,

$P(E_i)$ = prior probability

$P(A/E_i)$ = conditional probability

$P(E_i/A)$ = posterior probability (updated)

Proof:



Since, A is a subset of sample space (S) and sample space (S) is given by,

$$S = E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n$$

Then,
we can write as,

$$A = A \cap S$$

$$= A \cap [E_1 \cup E_2 \cup \ldots \cup E_n]$$

now using Distributive Law,

$$A = (A \cap E_1) \cup (A \cap E_2) \cup \ldots \cup (A \cap E_n)$$

Since $E_1, E_2, \ldots, E_n$ are mutually exclusive events, then $(A \cap E_1), (A \cap E_2), \ldots, (A \cap E_n)$ are also mutually exclusive events.

now, taking probability on both sides,

$$P(A) = P[(A \cap E_1) \cup (A \cap E_2) \cup \ldots \cup (A \cap E_n)]$$

$$= P(A \cap E_1) + P(A \cap E_2) + \ldots + P(A \cap E_n)$$

$$\therefore P(A) = \sum_{i=1}^{n} P(A \cap E_i) \quad \text{(Theorem of Totality)}$$

or,

$$P(A) = \sum_{i=1}^{n} P(E_i) . P(A | E_i) \quad \text{—— (i)}$$

now, we know that,

from the def^n of conditional probability;

$$P(E_i/A) = \frac{P(A \cap E_i)}{P(A)} = \frac{P(E_i) \cdot P(A/E_i)}{P(A)} \quad \text{——— (ii).}$$

now, from eq^n (1) and (ii) we get,

$$P(E_i/A) = \frac{P(E_i) \cdot P(A/E_i)}{\sum_{i=1}^{n} P(E_i) \cdot P(A/E_i)}$$

proved.

IMP **Example 1:** In a bolt factory, machine A, B and C manufactures 25%, 35% and 40% of the total respectively. Out of their outputs, 5%, 4% and 2% are defective bolts respectively. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by:

(i) machine A

(ii) machine B

(iii) machine C

$\underline{\underline{\text{soln}}}$ let,

$E_1 \rightarrow$ bolts manufactured by machine A

$E_2 \rightarrow$ bolts manufactured by machine B

$E_3 \rightarrow$ bolts manufactured by machine C

~~There~~ we have,

$$P(E_1) = \frac{25}{100} = 0.25$$

$$P(E_2) = \frac{35}{100} = 0.35$$

$$P(E_3) = \frac{40}{100} = 0.40$$

Also,
let, $D \rightarrow$ produced bolt is defective

then we have,

$$P(D|E_1) = \frac{5}{100} = 0.05$$

$$P(D|E_2) = \frac{4}{100} = 0.04$$

$$P(D|E_3) = \frac{2}{100} = 0.02$$

(i) $\underline{\underline{\text{soln}}}$
The probability that the defective bolt was manufactured by A is,

$$P(E_1|D) = \frac{P(E_1).P(D|E_1)}{P(E_1).P(D|E_1) + P(E_2).P(D|E_2) + P(E_3).P(D|E_3)}$$

$$= \frac{0.25 \times 0.05}{(0.25 \times 0.05) + (0.35 \times 0.04) + (0.40 \times 0.02)}$$

$$= \frac{0.0125}{0.0345} = 0.363$$ $\underline{\underline{\text{ans}}}$

(ii) Soln

The probability that the defective bolt was manufactured by machine B is,

$$P(E_2/D) = \frac{P(E_2).P(D|E_2)}{0.0345}$$

$$= \frac{0.35 \times 0.04}{0.0345} = 0.406 \text{ ans}$$

(iii) Soln

— — — — — — — — — — —

— — by machine C is,

$$P(E_3/D) = \frac{P(E_3).P(D|E_3)}{0.0345}$$

$$= \frac{0.40 \times 0.02}{0.0345}$$

$$= 0.232 \text{ ans}$$

Example 2: A manufacturing company produces steel pipes in three plants with daily production of 500, 1000 and 2000 units respectively. According to the past experience, it is known that the fraction of defective outputs produced by the three plants are 0.005, 0.008 and 0.01 respectively. If a pipe is selected from the total daily production and found to be defective, from which plant the defective pipe is expected to have been produced?

Sol^n let,

$E_1 \to$ pipe produced by the 1st plant

$E_2 \to$ pipe produced by the 2nd plant

$E_3 \to$ pipe produced by the 3rd plant

From the question,

Total no. of pipes = 500 + 1000 + 2000 (daily production)

= 3500 units

so we have,

$$P(E_1) = \frac{500}{3500} = 1/7 = 0.143$$

$$P(E_2) = \frac{1000}{3500} = 2/7 = 0.286$$

$$P(E_3) = \frac{2000}{3500} = 4/7 = 0.572$$

Also,

let, $D \to$ defective pipe produced

Then from the question, we have,

$$P(D|E_1) = 0.005$$

$$P(D|E_2) = 0.008$$

$$P(D|E_3) = 0.01$$

Now, probability of defective pipe being produced by 1st plant is,

$$P(E_1|D) = \frac{P(E_1) \cdot P(D|E_1)}{P(E_1) \cdot P(D|E_1) + P(E_2) \cdot P(D|E_2) + P(E_3) \cdot P(D|E_3)}$$

$$= \frac{0.05 \times 0.143}{(0.143 \times 0.005) + (0.286 \times 0.008) + (0.572 \times 0.01)}$$

$$= \frac{7.15 \times 10^{-3}}{\cancel{8.579}\ 8.723 \times 10^{-3}} = \cancel{2}\ 0.082$$

Similarly,

$$P(E_2 | D) = \frac{P(E_2) \cdot P(D | E_2)}{8.723 \times 10^{-3}}$$

$$= \frac{0.286 \times 0.008}{8.723 \times 10^{-3}} = 0.263$$

Again,

$$P(E_3 | D) = \frac{P(E_3) \cdot P(D | E_3)}{8.723 \times 10^{-3}}$$

$$= \frac{0.572 \times 0.01}{8.723 \times 10^{-3}}$$

$$= 0.656$$

here, $P(E_3|D) > P(E_1|D)$ and $P(E_3|D) > P(E_2|D)$.
Hence, the defective pipe is expected to have been produced from ~~pipe~~ 3rd plant. ans

Example 3

The contents of Urns I, II and III are as follows:

1 white, 2 black and 3 red balls
2 white, 1 black and 1 red balls
4 white, 5 black and 3 red balls

One urn is chosen at random and two balls are drawn. They are found to be white and Red. What is the probability that they come from Urn 1.

Soln

defining the events:
Let,
$E_1 \rightarrow$ choosing Urn I
$E_2 \rightarrow$ choosing Urn II
$E_3 \rightarrow$ choosing Urn III

Then we have,
$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$
$= 0.334$

Also, Let, $A \rightarrow$ choosing 1 white and 1 red ball

Then,
~~the probability~~

$$P(A|E_1) = \frac{{}^1C_1 \cdot {}^3C_1}{{}^6C_2} = 0.2$$

$$P(A|E_2) = \frac{{}^2C_1 \cdot {}^1C_1}{{}^4C_2} = 0.334$$

$$P(A|E_3) = \frac{{}^4C_1 \cdot {}^3C_1}{{}^{12}C_2} = 0.182$$

Now,

$$P(E_1|A) = \frac{P(E_1) \cdot P(A|E_1)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2) + P(E_3) \cdot P(A|E_3)}$$

$$= \frac{0.334 \times 0.2}{(0.334 \times 0.2) + (0.334 \times 0.334) + (0.334 \times 0.182)} = \frac{0.0668}{0.2392} = 0.273$$

ans

Example 4 : A given lot of chips [illegible] defective. Each chips is tested before delivery. The tester itself is not totally reliable. Probability of tester says the chip is good when it is really good is 0.95 and the chip is defective when it is actually defective is 0.94. If a tested device is indicated to be defective, what is the probability that it is actually defective?

Solⁿ defining the events:

Let, $E_1 \rightarrow$ chips are actually defective

$E_2 \rightarrow$ chips are actually good

then we have,

~~$P(E_1) = 2\% = \frac{2}{100} = 0.02$~~

actually defective chips = 2%.

So, actually good chips = (100 − 2)% = 98%.

Therefore,

$$P(E_1) = \frac{2}{100} = 0.02$$

$$P(E_2) = \frac{98}{100} = 0.98$$

Again,

Let $A \rightarrow$ Tester says chip is defective

Then we have,

$$P(A|E_1) = 0.94$$

$$P(A|E_2) = 1 - 0.95 = 0.05$$

Now, the probability that the tested device (chip) is defective ~~the~~ given that the tester indicates the chip to be defective is given by,

$$P(E_1|A) = \frac{P(E_1) \cdot P(A|E_1)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)}$$

$$= \frac{0.02 \times 0.94}{(0.02 \times 0.94) + (0.98 \times 0.05)}$$

$$= \frac{0.0188}{0.0678}$$

$$= 0.277 \text{ ans}$$

2079/08/07
Wednesday

Example 5: A Binary communication channel carries data as one of two types denoted by 0 and 1. Due to noise, sometimes transmitted 0 is received as 1 and sometimes transmitted 1 is received as 0.

For a given channel, assume that the probability of 0.94 that a transmitted 0 is correctly received as 0 and probability of 0.91 that a transmitted 1 is correctly received as 1. Further assume that the probability of 0.45 of transmitting 0. If a signal is sent then find the probability that :-

(i) a 1 is received

(ii) a 0 is received

(iii) a 1 is transmitted given that 1 is received

(iv) a 0 is transmitted given that 0 is received

(v) Error can occur

Soln defining the events :

$T_0 \rightarrow$ Transmitted 0

$T_1 \rightarrow$ Transmitted 1

$R_0 \rightarrow$ Received 0

$R_1 \rightarrow$ Received 1

From the question, we have;

$P(T_0) = 0.45$

$P(T_1) = 1 - 0.45 = 0.55$

$P(R_0 | T_0) = 0.94$

$P(R_1 | T_0) = 1 - 0.94 = 0.06$

And, $P(R_1 | T_1) = 0.91$

$P(R_0 | T_1) = 1 - 0.91 = 0.09$

i) soln

$P(R_1) = ?$

we have,

$P(R_1) = P(T_0 \cap R_1)$ or $P(T_1 \cap R_1)$

i.e. $P(R_1) = P(T_0 \cap R_1) + P(T_1 \cap R_1)$

$= P(T_0) . P(R_1 | T_0) + P(T_1) . P(R_1 | T_1)$

$= (0.45 \times 0.06) + (0.55 \times 0.91)$

$= 0.5275$

ii) soln $P(R_0) = ?$

we have,

$P(R_0) = P(T_0 \cap R_0)$ or $P(T_1 \cap R_0)$

$= P(T_0) . P(R_0 | T_0) + P(T_1) . P(R_0 | T_1)$

$= (0.45 \times 0.94) + (0.55 \times 0.09)$

$= 0.4725$

(iii) soln $P(T_1 | R_1) = \dfrac{P(T_1) . P(R_1 / T_1)}{P(T_1) . P(R_1 / T_1) + P(T_0) . P(R_1 / T_0)}$

$= \dfrac{P(T_1) . P(R_1 | T_1)}{P(R_1)} = \dfrac{0.55 \times 0.91}{0.5275}$

$= 0.9489$ ans

(iv) soln $P(T_0|R_0) = ?$

$$P(T_0|R_0) = \frac{P(T_0) \cdot P(R_0|T_0)}{P(R_0)}$$

$$= \frac{0.45 \times 0.94}{0.4725} = 0.895 \quad \text{ans}$$

(v) soln

$$P(\text{Error}) = P(T_0 \cap R_1) \text{ or } P(T_1 \cap R_0)$$

$$= P(T_0) \cdot P(R_1|T_0) + P(T_1) \cdot P(R_0|T_1)$$

$$= (0.45 \times 0.06) + (0.55 \times 0.09)$$

$$= 0.0765 \quad \text{ans}$$

Example 6: State Bayes' Theorem. In a population of workers, suppose 40% are grade school graduates, 50% are high school graduates and 10% are college graduates.

Among the grade school graduates, 10% are unemployed, among the high school graduates 5% are unemployed and among the college graduates 2% are unemployed.

If a worker is chosen at random and found to be unemployed, what is the probability that he is a college graduate?

Soln

defining the events :

$E_1 \rightarrow$ being grade school graduates

$E_2 \rightarrow$ being high school graduates

$E_3 \rightarrow$ being college graduates

Then we have,

$$P(E_1) = \frac{40}{100} = 0.4$$

$$P(E_2) = \frac{50}{100} = 0.5$$

$$P(E_3) = \frac{10}{100} = 0.1$$

Let, $U \rightarrow$ being unemployed

$$P(U|E_1) = \frac{10}{100} = 0.1$$

$$P(U|E_2) = \frac{5}{100} = 0.05$$

$$P(U|E_3) = \frac{2}{100} = 0.02$$

Then, $P(E_3|U) = ?$

we have,

$$P(E_3|U) = \frac{P(E_3) \cdot P(U|E_3)}{P(E_1) \cdot P(U|E_1) + P(E_2) \cdot P(U|E_2) + P(E_3) \cdot P(U|E_3)}$$

$$= \frac{(0.1 \times 0.02)}{(0.4 \times 0.1) + (0.5 \times 0.05) + (0.1 \times 0.02)}$$

$$= \frac{2 \times 10^{-3}}{0.067} = 0.0298 \quad \underline{\underline{\text{ans}}}$$

Chapter 2 | Random Variable

- Random variable is a real valued function which is associated with each outcomes of a random experiment.
- In simple words, R.V. (Random Variable) is defined as the outcomes associated with a random experiment.
- It is denoted by Capital Letters. Generally, we use $X, Y, Z, \ldots$ etc.
- The value taken by a random variable is denoted by small letters like $x, y, z, \ldots$ etc.
- There are two types of random variable.
  - → Discrete Random Variable
    - that takes discrete (i.e. integer or countable) values
  - → Continuous Random Variable