Chapter-1

Introduction To Digital Image Processing
(4 hours)

Introduction:

The pictorial representation of an object is called formation of an image

A **digital image** is a 2-D light intensity function $f(x, y)$ where x and y denotes spatial co-ordinates and value of function f at any point (x, y) is proportional to the brightness (gray level) of an image at that point (x, y).

Digital Image Processing refers to the processing of digital image by using digital computer on the basis of digitization (sampling and quantization).

Digital Image Representation:

A digital image is an image $f(x, y)$ that has been discretized both in spatial co-ordinates and in brightness

It is represented by 2-D integer array. The digitized brightness value is called gray level values.

Digital image can be represented as a matrix as shown in figure below:

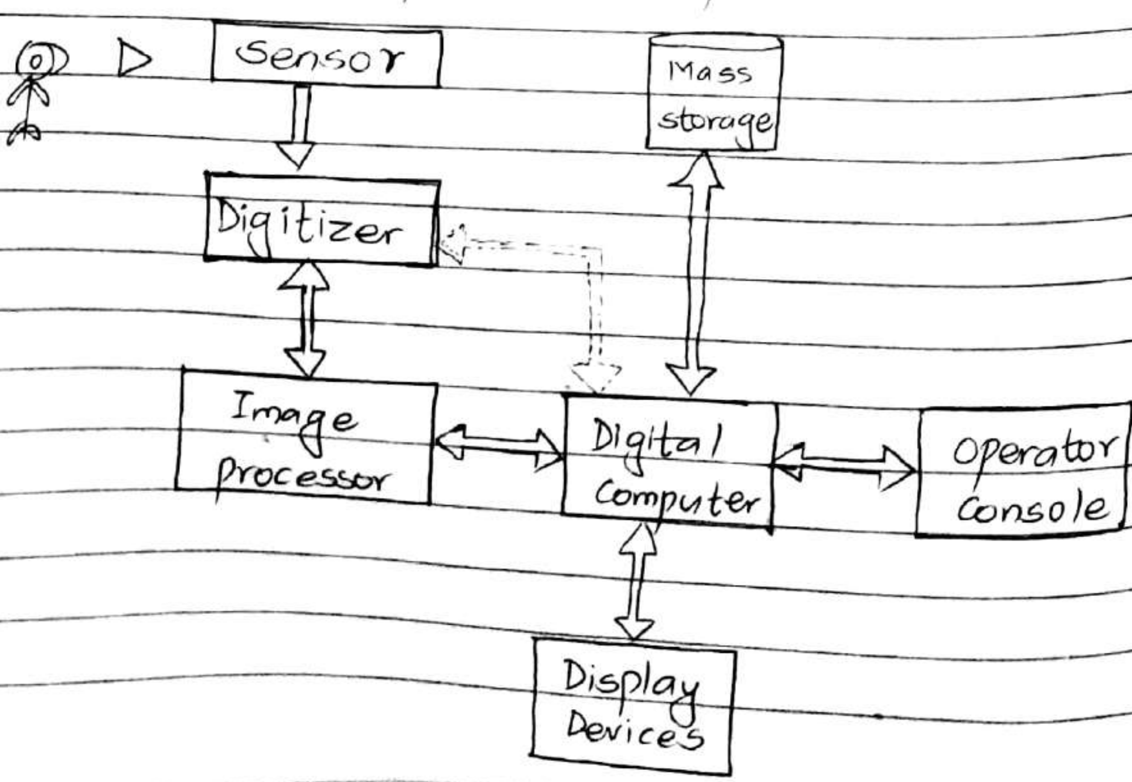
$$f(x,y) = \begin{bmatrix} f(0,0), & f(1,0) & \cdots \\ & & \\ \cdots & & f(x_{max}, y_{max}) \end{bmatrix}$$

(fig: Representation of Digital Image As Matrix)

Here,
the row and column identify the point in the image and the corresponding matrix elements values identify the gray level at that point.

The element in digital image is called Pixel or pels or Picture Points.

Elements (Components) of DIP:



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1. **Sensor:**

It intercepts the energy propagation from the scene and transform it to produce an intensity image.

Camera and scanner are the examples of photo sensor.

2. **Digitizer:**

The digitizer produce an image by the digitization technique.

Normally, a digitizer converts an intensity image into numerical representation which is suitable for import into a digital computer.

3. **Image Processor / Pre-Processor:**

It interface with digital computer to provide easy of programming. For this purpose, We can use micro-computer or mainframe computer.

Specially, it does functions like image acquistion, low level processing, scaling and rotating.

4. **Digital Computer:**

It always digitize the required image by the help of image pre-processor.

5. **Mass storage:**

It is used to store digital image line by line. The storage devices may be magnetic tape, hard disk etc.

⑥ Display Devices:
It produce and shows a visual of numerical values stored in a computer.

The example of display devices are: CRT monitor, printer etc.

⑦ Operator Console:
It controls the full mechanism of image processing system.

Applications of Digital Image Processing:

① Office Automation:

BIO MECI

Ex: OCR (optical character Reader) & document processing etc.

② Industrial Automation:

Ex: Robotics, Automatic assembly etc.

③ Bio Medical Automation:

Ex: ECG, EEG, X-Ray etc.

④ ~~Bi~~ Information Technology (IT):

Ex: Video phone, fax, video conferencing, etc.

(5) Entertainment:

Ex: Video game, visual Animations, etc.

(6) Military Application:

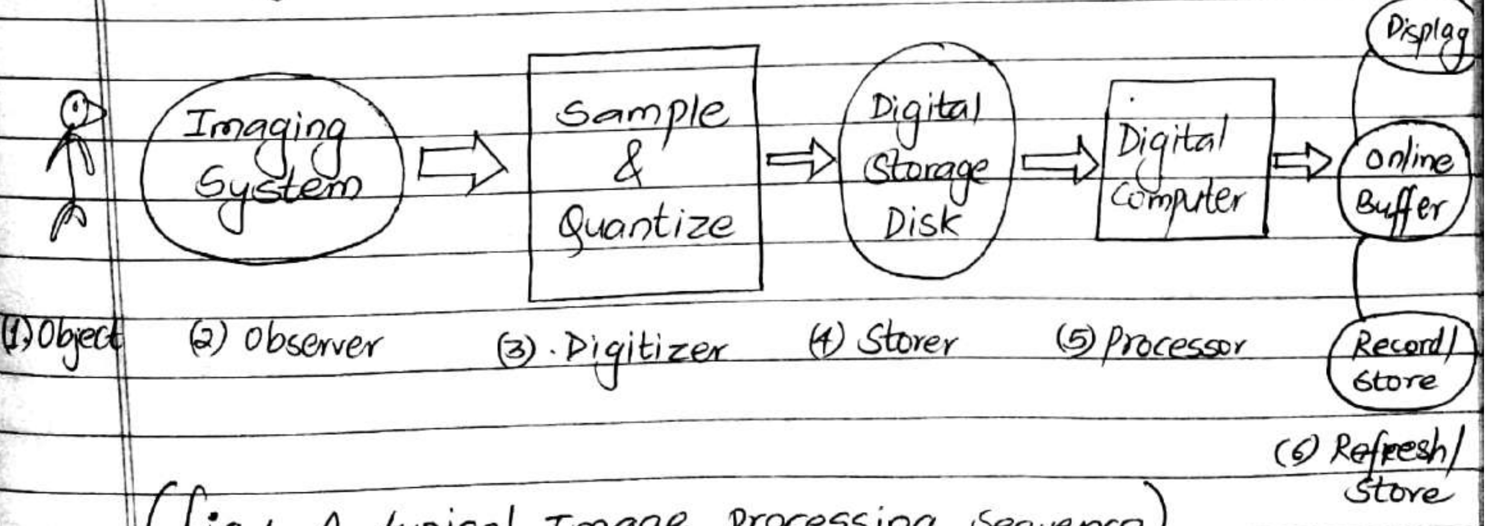
Ex: Target Indentification and missile guidance and detection.

(7) Criminology:

Ex: Bio-metric detection such as finger print detection, face detection, retina detection, etc.

(2010 spring)

Imp # A typical Image Processing Sequence:



(fig: A typical Image processing sequence)

In typical image processing sequence involves various components.

The image of an object travels along the different blocks.

Firstly, the object is ~~cha~~ sensed by the Imaging system and image is formed. This image is digitized by the process of sampling and quantization.

Now, image is in a digital form. A digital image is stored in digital storage system. The digital computer is connected to digital storage disk where all the manipulation of digital image is done.

Here, also online buffer mechanism is available where the information are stored as memory.

The image is displayed through the display block which may be color monitor, color TV etc. Finally, the image is recorded on a recording block.

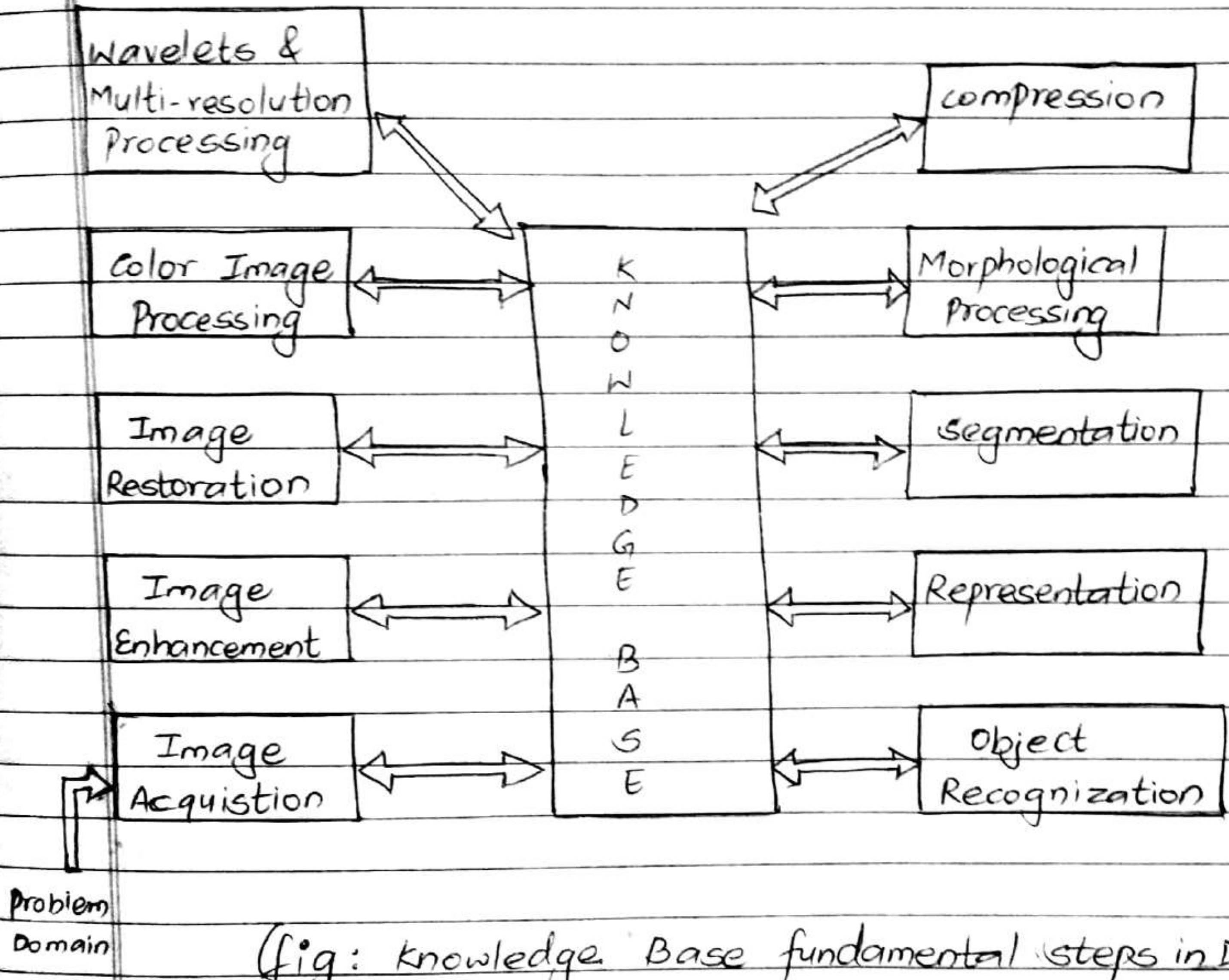
But sometimes some problem may occur when the image is processing sequentially into a digital computer.

AER / Cp wp / JT / SRO

(2015 fall)

Imp

Fundamental steps in DIP:



(fig: knowledge Base fundamental steps in DIP)

1) Image Acquistion:

It is the first process in which pre-processing activities like scaling and rotating are done.

2) Image Enhancement:

In this stage, the original image is

processed in such a manner that the resultant image will be more suitable with compared to original image.

③ Image Restoration:

It is an area that also deals with improving apparance of an image. Basically,
image enhancement is subjective process but image restoration is objective process.

④ Color Image Processing:

With the help of color, object identification and abstraction from the scene is processed.

⑤ Wavelets & Multi-Resolution Processing:

Foundation for the representing image in various degree of resolution deals to the wavelets and multi-resolution.

Sometimes, it is also used for image compression.

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(6) Compression:

It deals with the technique for reducing the storage required to save an image to transmit it.

(7) Morphological Processing:

It is the combination of structure. so, it deals with the tools for extraction of an image component that are useful in representation and description of shape.

(8) Segmentation:

It deals with the partation of an image into different parts and objectives.

(9) Representation:

After an image has been segmented into sub-regions then the resultant segmented pixels is represent and described in a suitable form for further analysis.

(10) Object Recognation:

It process that, the level to an object based on its descriptors.

(2011 fall)

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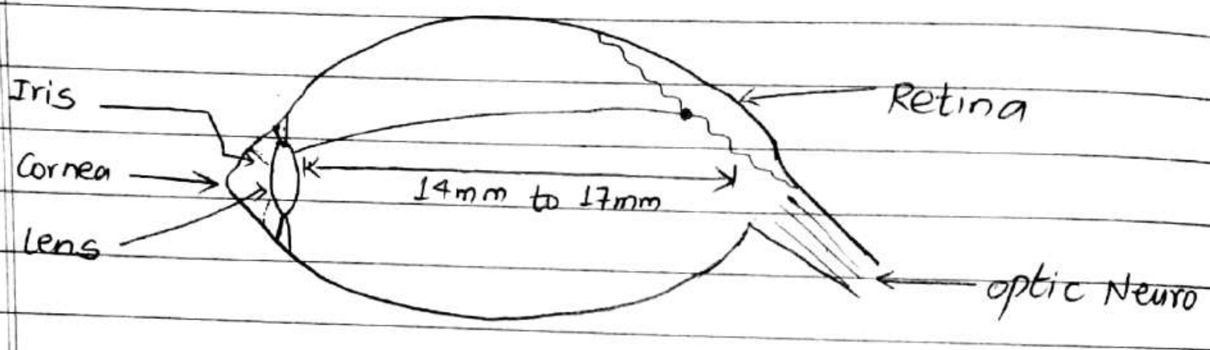
Elements of Visual Perception:

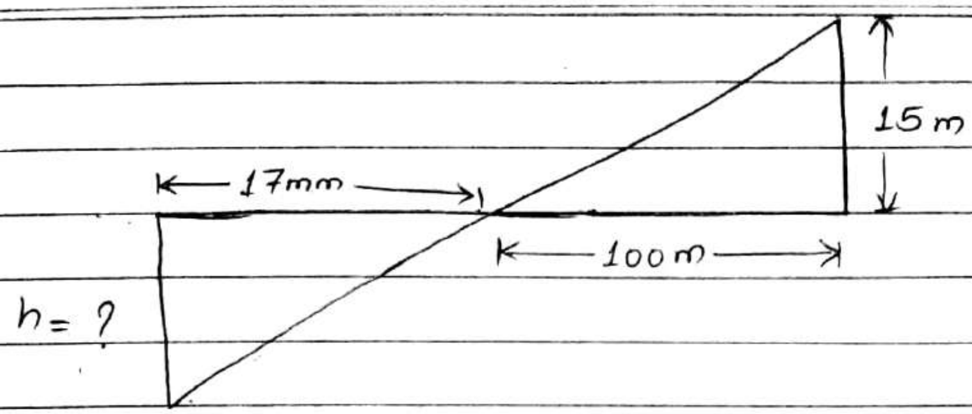


(fig: Cross Section of Human Eye)

Image analysis plays central role in the image processing. Therefore, the study of human visual perception is important which is represented as cross-section of human eye as shown in figure above.

The human eye nearly a sphere with an average diameter of 14mm to 17mm. The lens of the eye is flexible than the ordinary optical lens.

let, the observer is looking at a tree 15m height at the distance of 100m. Find the height of retrival image.



Here,

$$\frac{h}{17\,mm} = \frac{15\,m}{100\,m}$$

$$\Rightarrow h = \frac{15 \times 17}{100} = 2.55\,mm$$

$\therefore$ Height of Retrival Image(h) = 2.55 mm

V.Imp

Sampling & Quantization:

A physical image is changed into digital image by performing sampling and quantization. This process is also known as Digitization:

Sampling:

(It is the process where measurement are taken at regular space intervals.) Digitizing the spatial co-ordinates value (x, y) is called sampling.

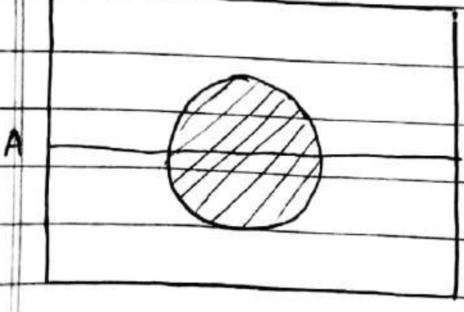
It is the process by which image is formed over a patch (domain) in a continous domain is mapped into a discrete point with integer co-ordinates.

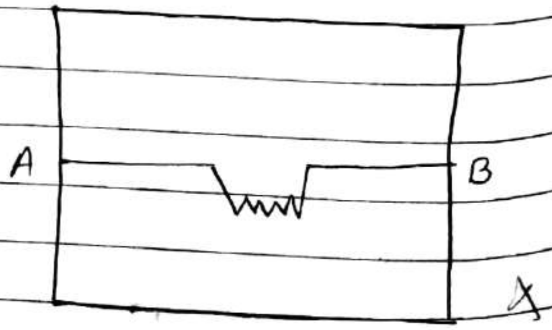
Hence, Sampling can be defined as a $\boxed{\text{selection of discrete location}}$ in the continous 2-D space.

Quantization:

It is the process of mapping the measured intensity to one of the finite number of discrete levels.

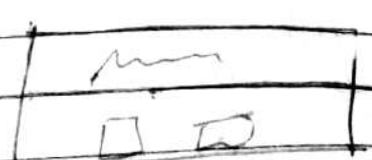
Hence, digitizing the amplitude values is called quantization or gray level quantization.

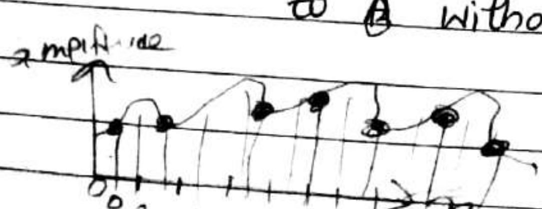


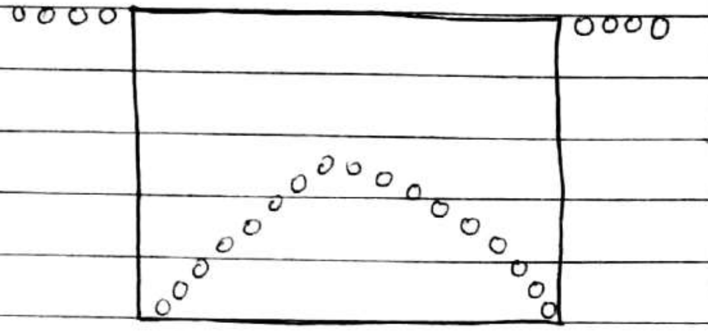


(1) Continuous Type

(2) Ideal Scan line A to B without noise



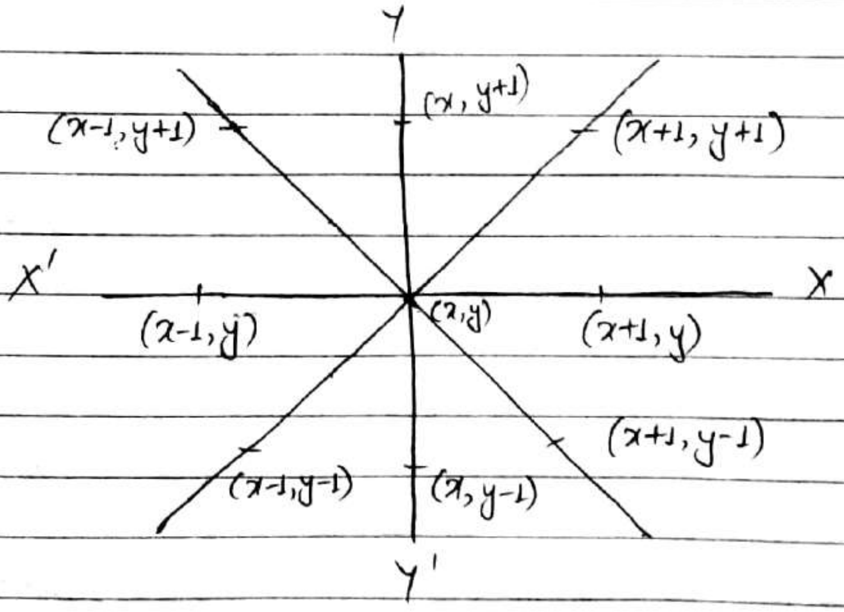




(3) Scan line A to B in image with noise

(4) Sampling & quantization

Neighbours of pixels:
(Arrangement of pixel in Image)



(fig: Neighbours of pixel)

A pixel at co-ordinate (x, y) has a four horizontal and four vertical neighbours whose co-ordinates are given by $(x-1, y+1)$, $(x, y+1)$, $(x+1, y+1)$, $(x+1, y)$, $(x+1, y-1)$, $(x, y-1)$, $(x-1, y-1)$, $(x-1, y)$.

These set of pixel is called four neighbours of horizontal and four neighbours of vertical.

Each pixel is at unit distance from (x,y) and called as $N_8(P)$.

To create a digital image, we need to convert the continous sensed data into digital form.

This involves two processes : Sampling & Quantization

An image may be continous with respect to x- and y-coordinates, and also in amplitude.

To convert it to digital form, we have to sample the function in both co-ordinates & in amplitude.

Digitizing the coordinate value is called Sampling.

And, Digitizing the amplitude value is called Quantization.

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Chapter-2

Image Enhancement In Spatial Domain
[7 hours]

Introduction:

Image enhancement refers to the sharpening or to improve the quality of an image by enhancement of pixel or point in a spatial Domain.

Some basic gray level transformation is needed to enhance the image in a spatial domain.

Some basic gray level transformation is listed as below:

(1) Point Operation
(2) Contrast Stretching
(3) Thresholding
(4) Digital Negative
(5) Intensity level slicing (Gray level slicing)
(6) Bit Plane slicing

[P.T.O]

Point Operation:

(pre-university)

(Defn)

Point operation is defined as a function that are performed on each pixel of an image.

Point operation is a uninary point operation if single image is modified.

The operation can be binary which means that two images combined in the same manner.

The operation can be ternary if three images are used in same manner at operation.

Unary Operation can be defined as,

$$\boxed{T_{xy} = function\ (P(x,y))}$$

Where,

T_{xy} = Transformed Pixel value

function = Numeric Transform

$P(x,y)$ = Input Pixel value

This operation can be used in many situation and for different purpose such as contrast stretching, noise clipping, Window slicing and histogram modeling.

So, in point operation a given gray level $u \in [0, L-1]$ is mapped into a gray level $v \in [0, L-1]$ according to a transformation

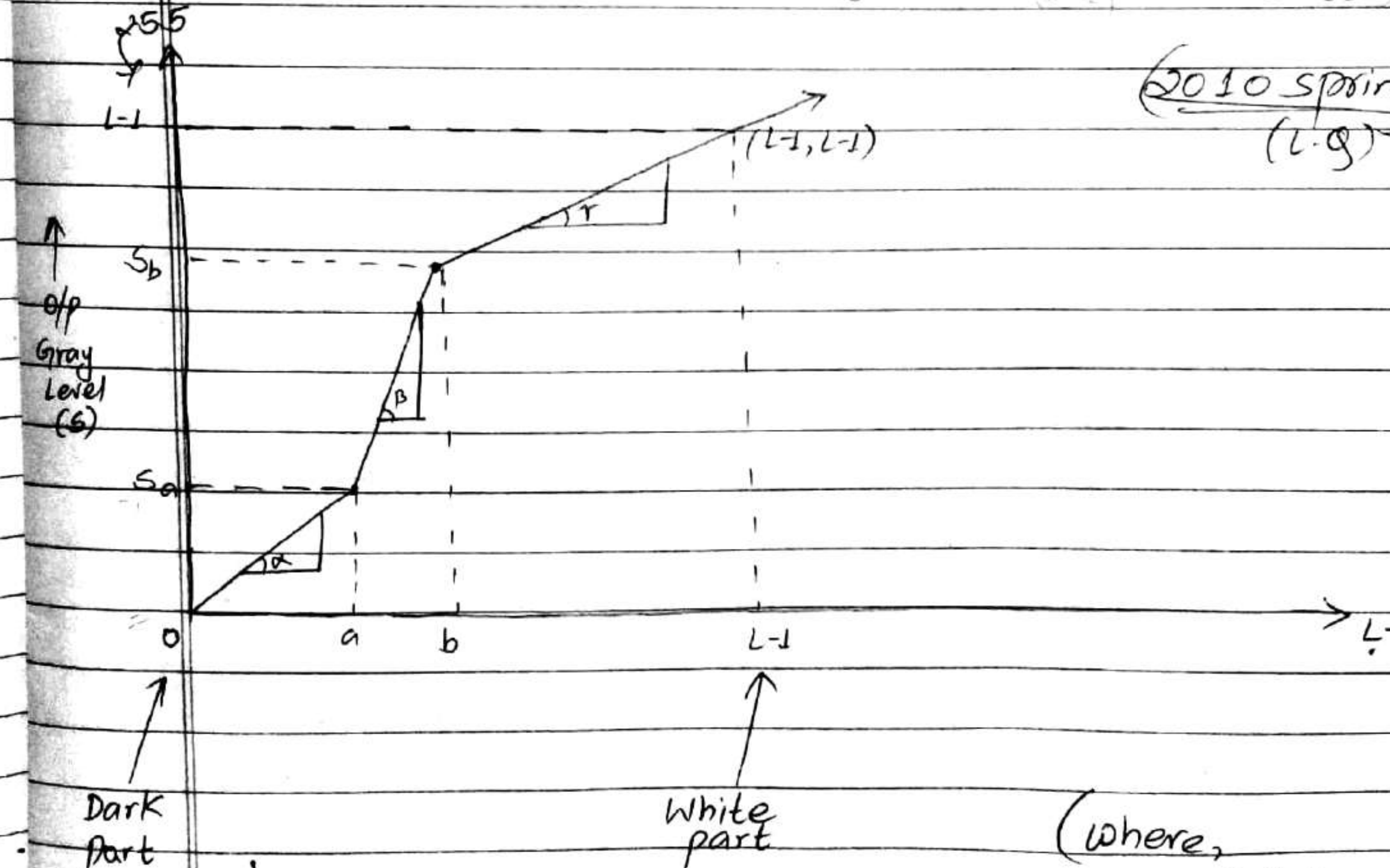
i.e $\boxed{v = f(u)}$ ($L-1 = 255$, $L \rightarrow$ Max. Gray level)

(Imp)

Contrast Stretching:

(Def^n → 2012)

(2010 spring) (L.Q)



(where, α, β & γ are the slopes)

(fig: Contrast Stretching Transformation)

In many times, we obtain low contrast image due to poor illumination or due to the wrong setting of the lens aparture.

The idea behind the contrast stretching is to increase the dynamic range of gray level in the image.

(Defn)

The main idea is to increase the contrast of the image by making the dark portion darker and bright portion brighter.

To obtain the contrast stretching we should have some formulations as below:

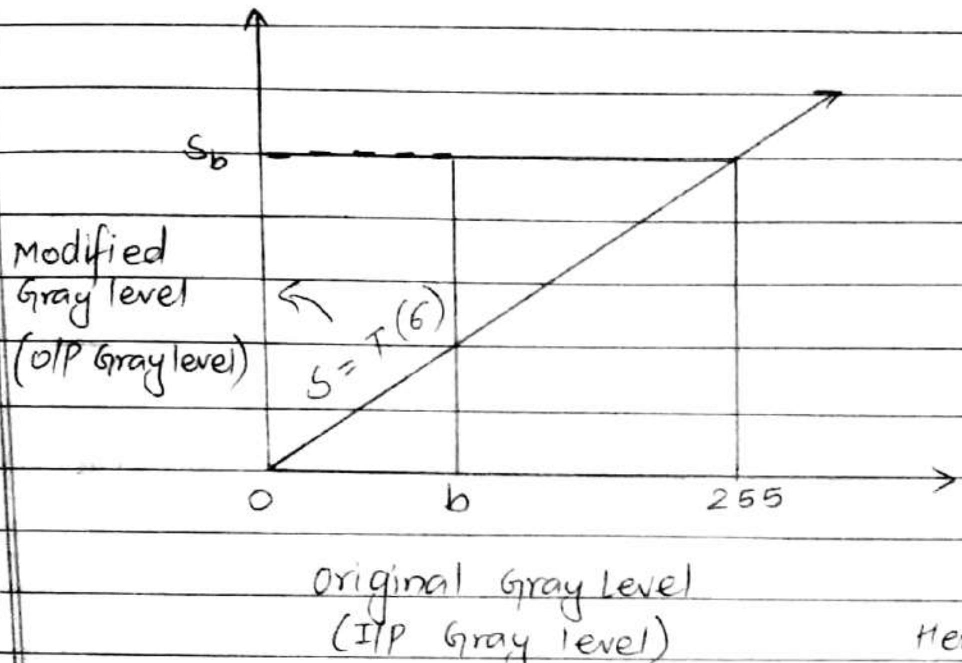
$$s = \begin{cases} \alpha r & , \quad 0 \le r < a \\ \beta(r-a) + s_a & , \quad a \le r < b \\ \gamma(r-b) + s_b & , \quad b \le r < k-1 \end{cases}$$

In the figure, 'r' represents the horizontal axis (i.e I/p gray level) and 's' represents the vertical axis (i.e o/p gray level.)

Here,

darker gray level is obtained by making a slope <1 and brighter is obtained by making a slope >1.

Thresholding (Binary Image)



Here,
$b = 100$ (Threshold value)

(fig: Thresholding)

Thresholding is the simplest method of the image processing. It can be used to create binary image.

If we observe the contrast stretching diagram, we notice that the first and last slopes are made to zero.

If $r_1 = r_2$ & $s_1 = 0$, $s_2 = L-1$

Now, The thresholding function is given below:

$S = 0$, if $r < b \Rightarrow 0$
$S = L-1$, if $r \geq b \Rightarrow 1$

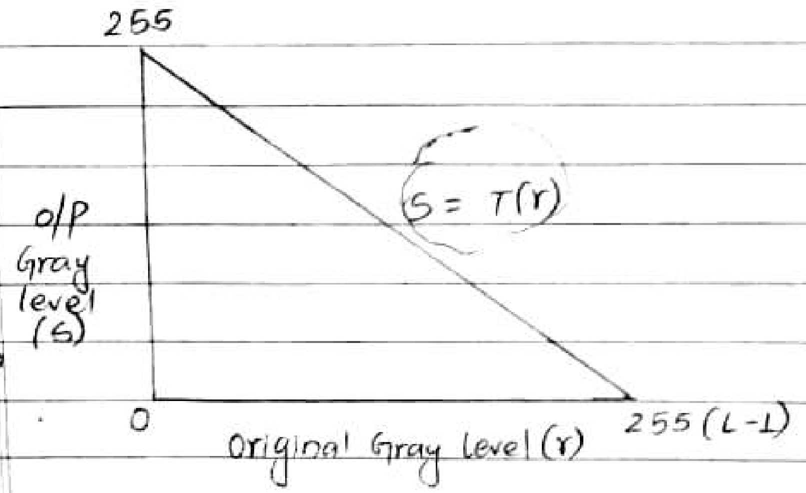
Where,

b = Thresholding value

L = Maximum Gray level

Important notice is that, threshold image has maximum contrast on it having black and white gray level only.

(Imp) # Digital Negative (x-ray): (2011 fall)



(fig: Digital Negative)

It is useful in a lots of application. The common example of digital negative is X-ray image.

As the name suggest, negative means simply inverting the gray level i.e black into white and vice-versa.

So, the negative of digital image

with the gray level in the range from 0 to L-1 is obtained by using negative transformation.

i.e

$$\boxed{s = T(r) = [(L-1) - r]}$$

Where,

r = Input pixel value

L = No. of Gray level or Maximum Gray level

$T(r)$ = Transformation Function

s = output Gray level.

Hence,

$r = 0$, $s = 255$ (White)

$r = 255$, $s = 0$ (Black)

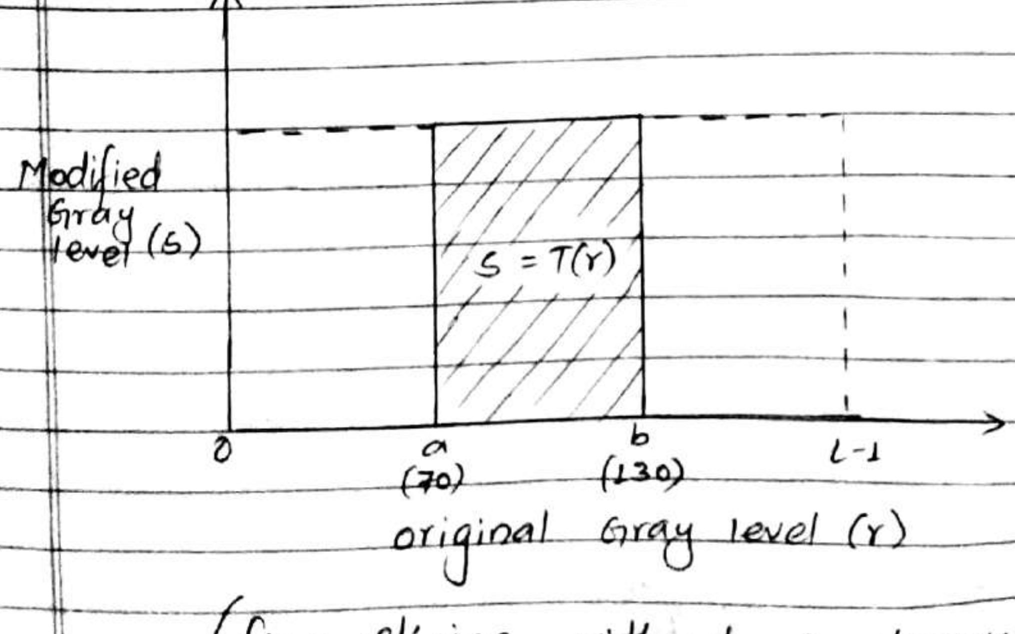
(IMP) # Intensity Level Slicing:

[Gray Level Slicing]

~~Linear~~

~~Logarithmic~~

~~power law~~



(fig: Slicing without Background)

5	2	6
7	8	3
9	1	2

perform intensity level slicing for $r_1 - r_2$ (3-5)

(I) with background

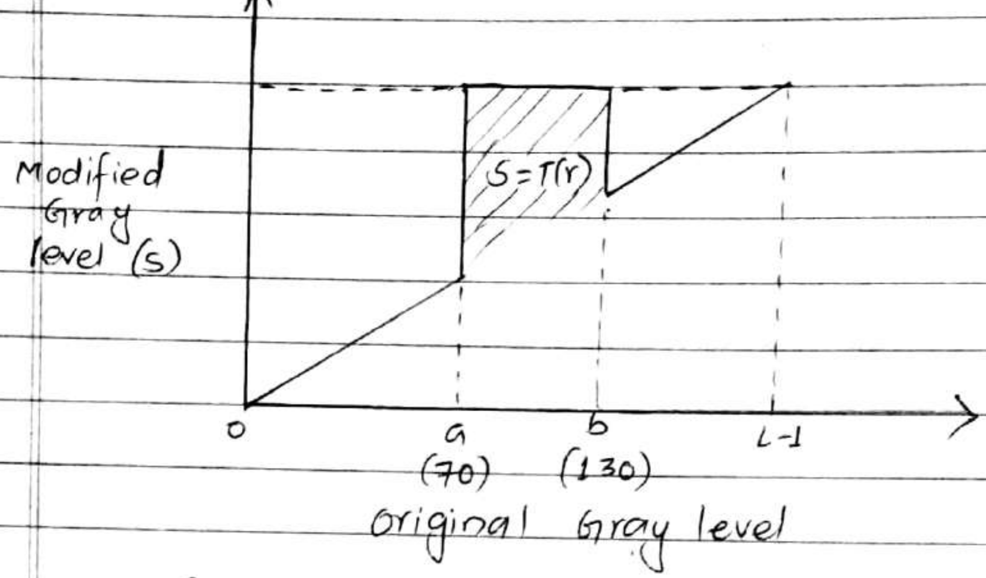
(II) without background

9	2	1
7	8	9
9	1	2

with background

9	0	0
0	0	9
0	0	0

without background



(fig: Slicing With Background)

Thresholding splits the gray level into two parts. But many times we need highlight a specific range of gray level.

In such a situation, use a transformation function on a gray level called as Gray Level Slicing.

There are two types of slicing:

(1) Slicing without background:

Here,
We have completely lost the background and this can be implemented by using the following function:

$s = L-1 \quad , \quad \text{if } a \leq r \leq b$
$s = 0 \quad , \quad \text{otherwise}$

(2) Slicing With Background:

Here,
in some applications we donot only need of enhancement to the band of gray level but also need to retain the background.

This can be implemented by using the following formation:

$$s = L-1 \quad , \quad \text{if} \quad a \leq r \leq b$$
$$s = r \quad , \quad \text{otherwise}$$

(2012 fall)

Bit Plane Slicing (clipping):

* heighlights interesting aspect

* isaaury particular bit

It is used to enhance features with an image. We must provide a threshold ~~labe~~ level to determine how the clipping occurs.

The threshold level or value is a reference value where other values set changed.

In general, it is performed for maximum and minimum operators. An expression that contain an image with threshold level.

For example:

If we have image variable and want to slice it to $\geq$ or = to 125, then the resulting image stated as:

Clipped Image = (Image (original)) > 125

Here,
it means if the element value is less than 125 then it is set as equal to 125. Otherwise, it is unchanged.

(V. Imp) # Introduction to Histogram:

Histogram of an image provides a global description of the apparance of an image.

The information obtained from histogram is innermost and hence histogram modeling is important in digital image processing (DIP).

By the defination,
the histogram of an image represents the relative frequency of occurance of various gray level in an image.

Histogram of an image can be plotted as:

(i) The X-axis has the gray level and Y-axis has the number of pixel in each gray level.

(ii) The X-axis represent the gray level and Y-axis represents the probability of occurance of that gray level.

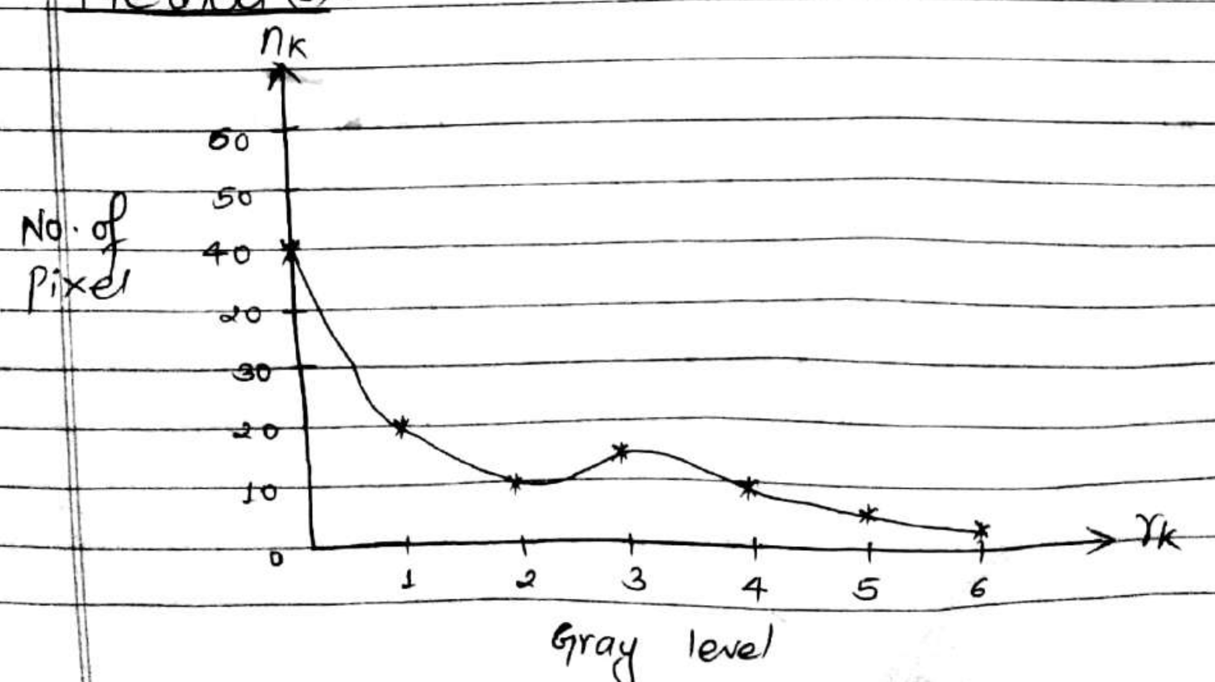
The method of histogram is shown as below:

Step(1): Original Image

Gray level (r_K)	No. of pixel (n_K)
0	40
1	20
2	10
3	15
4	10
5	3
6	2

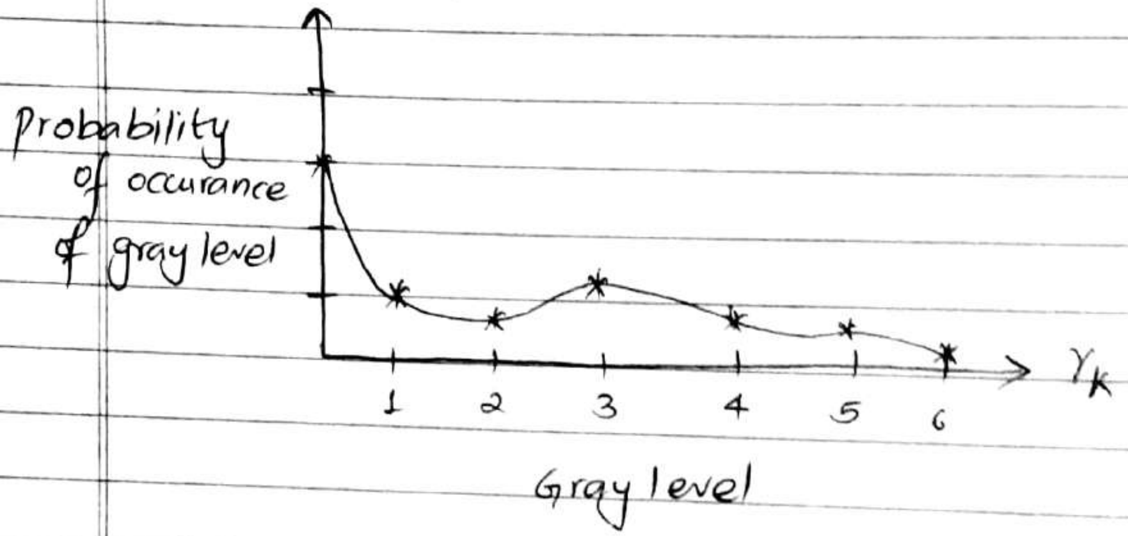
Step(2): Histogram Modeling

Method (1):



Method (2):

$p(r_k) = n_k / n$



(fig: Histogram Representation by 2-methods)

Here,

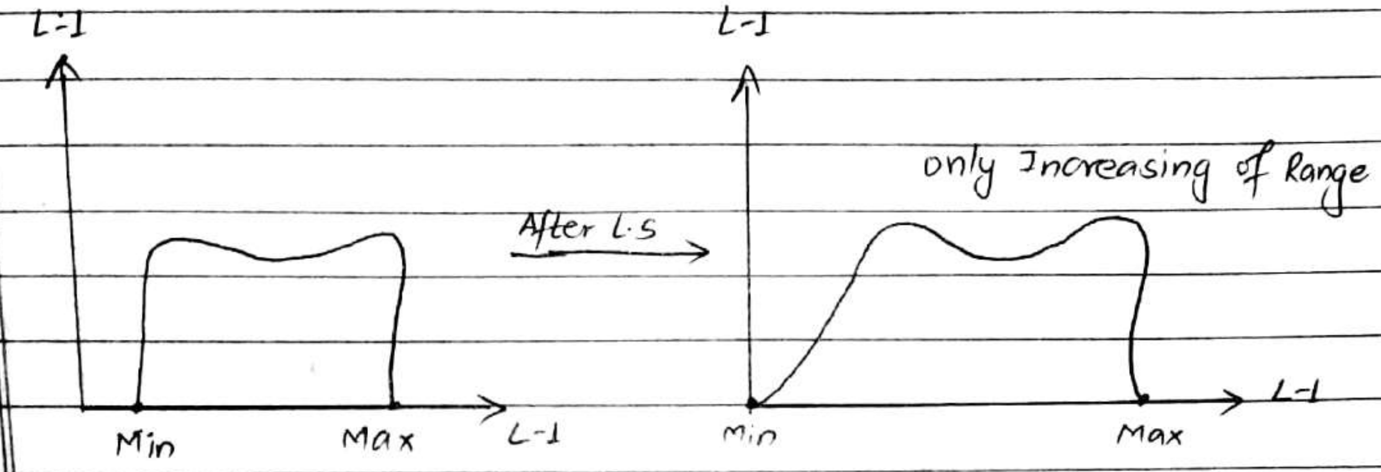
r_k = k^{th} Gray level
n_k = No. of pixel in k^{th} Gray level
n = Total no. of pixel

Histogram Techniques:

(1) Histogram stretching | Linear stretching
(2) Histogram Equalization | Histogram linearization
(3) Histogram specification | Histogram Matching

(pre-university)

(1) Histogram stretching :



(fig: Linear strectching)

To increase the dynamic range by using a technique is known as Histogram strecthing.

In this method, We donot change basic shape of the histogram but spread it over the entire dynamic range.

For this, we use this technique on the basis of slope. It can be given as:

$$\text{slope}(m) = \frac{S_{max} - S_{min}}{r_{max} - r_{min}} \quad \text{——— (i)}$$

For transformation of histogram stretching then, transformation can be defined as:

$$S = T(r) = \frac{S_{max} - S_{min}}{r_{max} - r_{min}}(r - r_{min}) + S_{min} \quad \text{——— (ii)}$$

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$$\boxed{S = m\,(r - r_{min}) + S_{min}}$$

Where,

S_{max} = Max^m gray level in o/p image

S_{min} = Min^m gray level in o/p image

r_{max} = Max^m gray level in i/p image

r_{min} = Min^m gray level in i/p image

This transformation function stretches or increase the dynamic range of given image.

Example: stretch the contrast of histogram over the entire range. [2011 fall]

(r) Gray Level	0	1	2	3	4	5	6	7
No. of pixel	0	0	50	60	50	20	10	0

Now, perform histogram stretching. show that the new image has a dynamic range of entire range ie [0, 7]

Soln: Given,

$r_{min} = 2$ $\quad$ $S_{min} = 0$

$r_{max} = 6$ $\quad$ $S_{max} = 7$

Now, We perform transformation function on histogram stretching as:

$$S = T(r) = \frac{S_{max} - S_{min}}{r_{max} - r_{min}}\,(r - r_{min}) + S_{min}$$

(S) -

Note:

Now,

When $r=2$: $s = \frac{7-0}{6-2}(2-2)+0 = 0$

$\therefore \boxed{r=2,\ s=0}$

When $r=3$: $s = \frac{7-0}{6-2}(3-2)+0 = 1.75 \simeq 2$

$\therefore \boxed{r=3,\ s=2}$

When $r=4$: $s = \frac{7-0}{6-2}(4-2)+0 = 3.5 \simeq 4$

$\therefore \boxed{r=4,\ s=4}$

When $r=5$: $s = \frac{7-0}{6-2}(5-2)+0 = 5.2 \simeq 5$

$\therefore \boxed{r=5,\ s=5}$

When $r=6$: $s = \frac{7-0}{6-2}(6-2)+0 = 7$

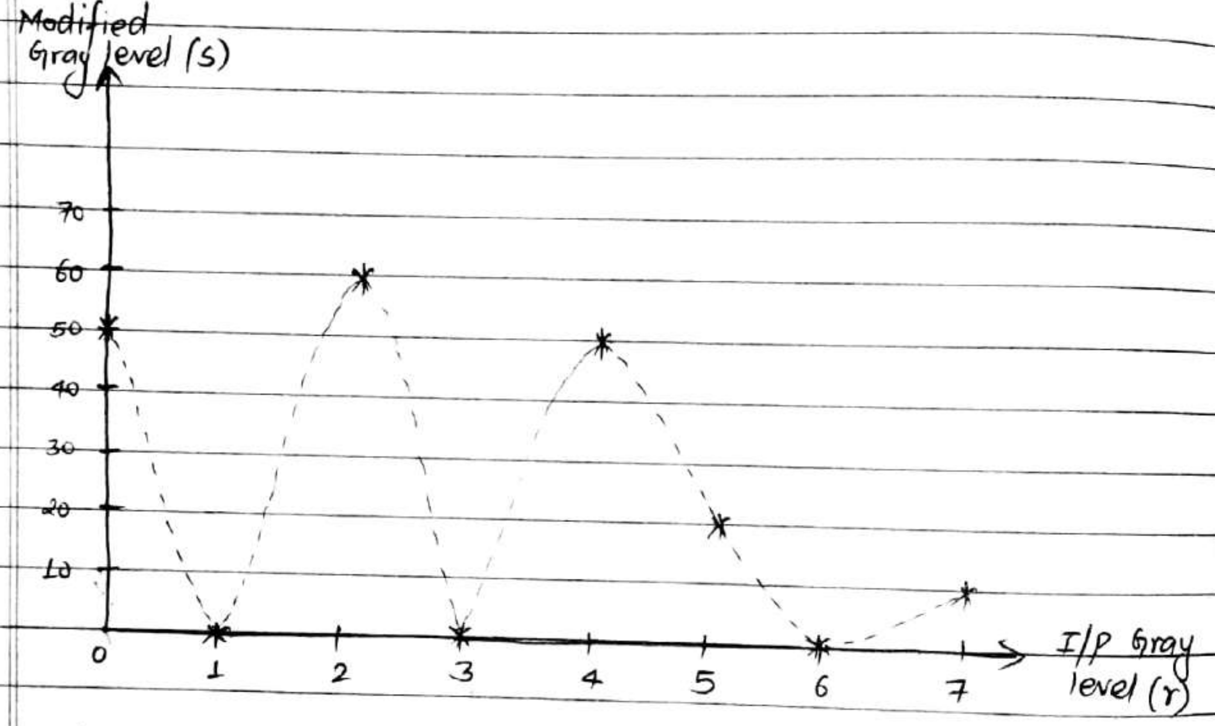
$\therefore \boxed{r=6,\ s=7}$

Hence,
Modified histogram or stretched range can be illustrated as:

(S) → Gray Level	0	1	2	3	4	5	6	7
No. of pixel	50	0	60	0	50	20	0	10

Note: $\boxed{s=0}$ ⇒ old Table मा $\boxed{r=2}$ को value हेर्ने त्यो value New Table $s=0$ मा राख्ने and so on

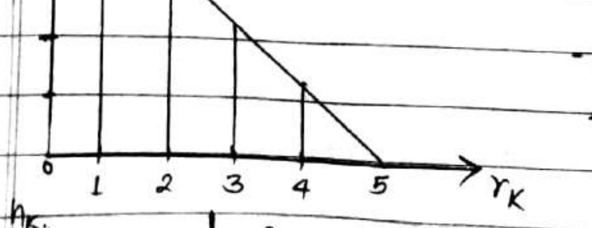
Now, The output histogram image is shown as below:

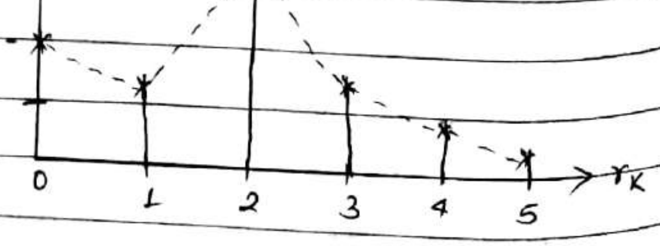


(fig: stretched / Modified / o/P Histogram Image)

(Imp) ② Histogram Equalization:

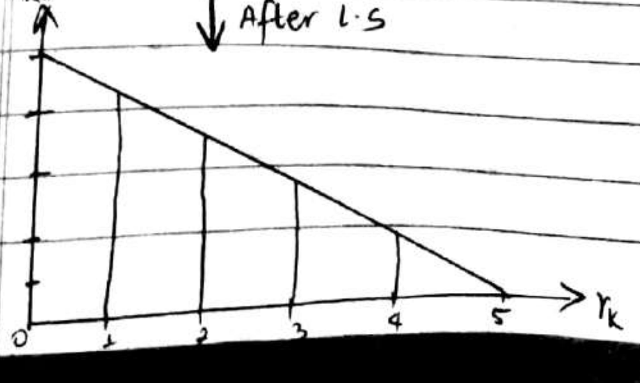
(2012 fall)





[redacted] Increase Dynamic Range But spread Equal no. of pixel in an Image

↓ After L.S



(fig: Histogram Equalization)

(3) The gray level for continous variable can be characterized by proability density function (Pdf) ie $P_r(r)$ and $P_s(s)$

We know that, we need to find the transformation which could be the flat histogram.

(1) linear stretching is a good technique but shape remain same. There are many applications where we need a flat histogram so ~~that~~ this cannot be achieved by histogram stretching.

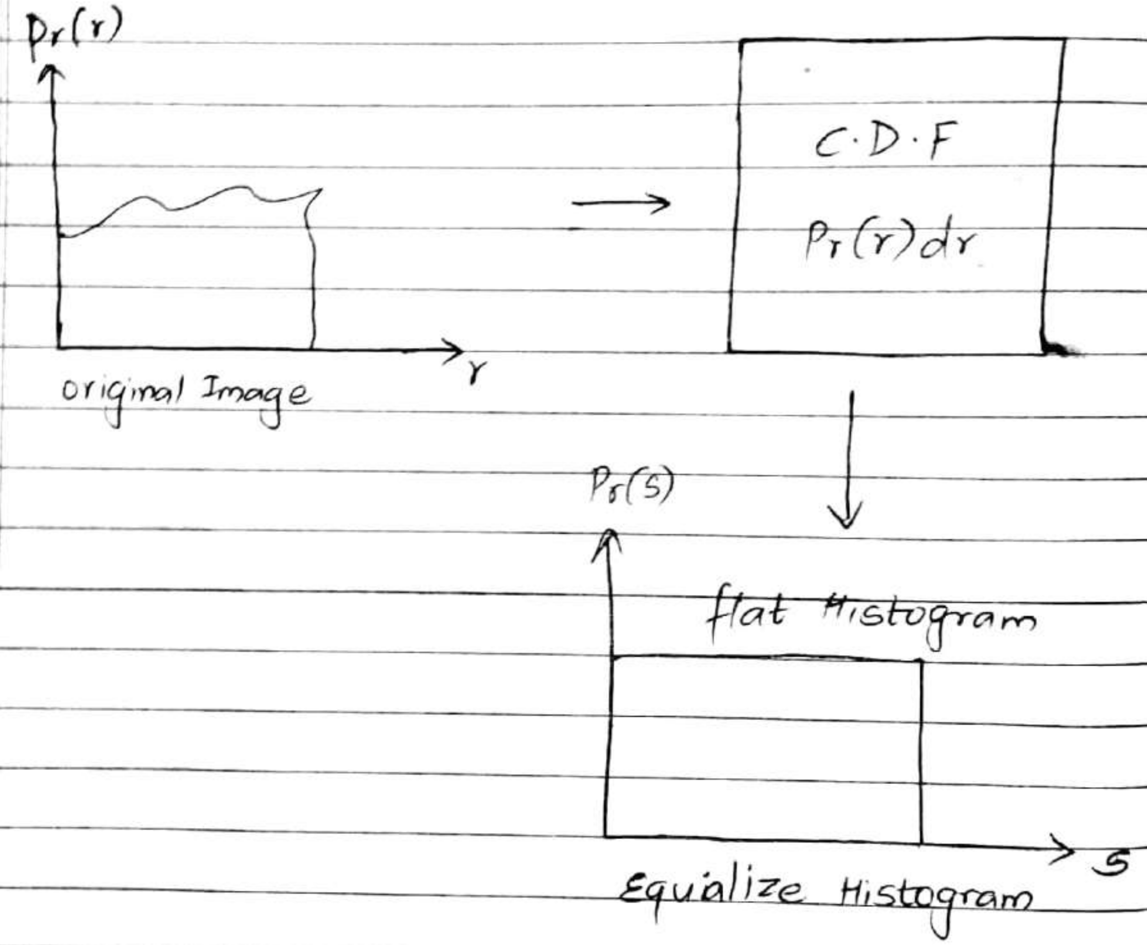
(2) Hence, a technique that can be used to obtain a uniform histogram is known as Histogram Equalization or Histogram Linearization.

A perfect image is one which has equal number of pixels in all gray level. Therefore, In this technique,
→ image is not only spread over the dynamic range but also to have equal number of pixel in all gray level.

We know that,

$$s = T(r) = \int_0^r P_r(r)\, dr , \quad 0 \leq r \leq 1$$

Above equation shows the transformation function of cumulative pdf (c.d.f) and it can be shown as:



Example: (V. Imp)

Equalize the given histogram:

Gray level	0	1	2	3	4	5	6	7
No. of pixel	790	1023	850	656	329	245	122	81

Soln: Here,

Step(1):

Gray level	n_K	P.D.F $P_r(r_K) = n_K/n$	C.D.F $S_K = \sum P_r(r_K)$	$7 * S_K$	Rounding off
0	790	0.19	0.19	1.33	1
1	1023	0.25	0.44	3.08	3
2	850	0.21	0.65	4.55	5
3	656	0.16	0.81	5.67	6
4	329	0.08	0.89	6.23	6
5	245	0.06	0.95	6.65	7
6	122	0.03	0.98	6.86	7
7	81	0.02	1	7	7
	n = 4096				

Step (2): Representing New Gray Level

Old Gray Level	No. of pixel	New Gray Level
0	790	1
1	1023	3
2	850	5
3	656	6
4	329	6
5	245	7
6	122	7
7	81	7

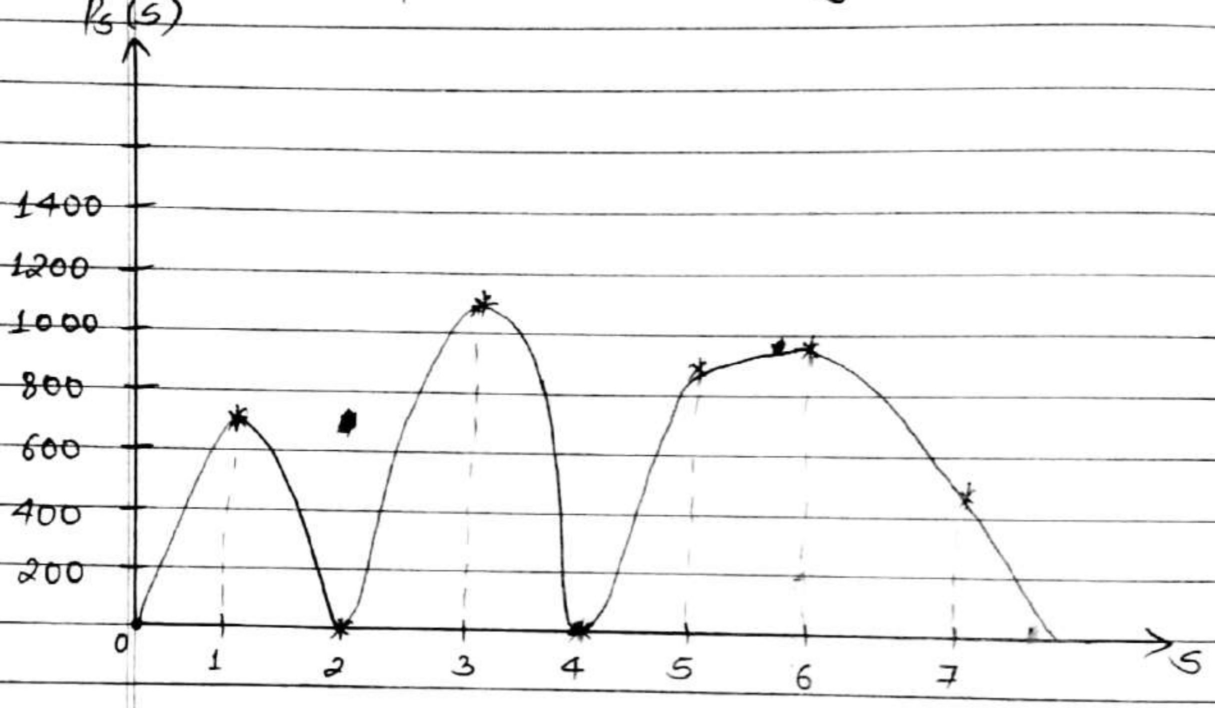
Step (3): Modified Histogram

Gray level	0	1	2	3	4	5	6	7
No. of pixel	0	790	0	1023	0	850	985	448

(656 + 329)

(245 + 122 + 81)

Hence, Equalized Histogram:



(fig: Equalized Histogram)

③ Histogram specification:

(Th + Nm)

Histogram specification is not interactive because it always gives one result i.e an approximation to an uniform histogram.

It is at time desirable to have an interactive methods in which certain gray levels are highlighted.

let, us suppose that $P_r(r)$ is the original pdf (Probability density function) and $P_z(z)$ is the ~~original~~ desired pdf.

Suppose that, histogram equalization is first applied on the original image.

i.e

$$S = T(r) = \int_0^r P_r(r)\, dr$$

Histogram equalization of desired image 'z' is that:

$$v = G(z) = \int_0^z P_z(r)\, dz$$

Now,

The inverse process $z = G^{-1}(s)$ which have the desired pdf that provide histogram specification.

i.e $z = G^{-1}(s) = G^{-1}(T(r)) = G^{-1}\int_0^r P_r(r)\, dr$

Example:

Matching the given histogram:

Histogram for image (a):

Gray level	0	1	2	3	4	5	6	7
No. of pixel	790	1023	850	656	329	245	122	81

Histogram for image (b):

Gray level	0	1	2	3	4	5	6	7
No. of pixel	0	0	0	614	819	1230	819	614

soln: Here,

step(1):

Equalized Histogram of Image (a):

Gray level	0	1	2	3	4	5	6	7
No. of pixel	0	790	0	1023	0	850	985	448

Now,

Equalized Histogram for Image (b) is:

Gray level	n_k	P.D.F $P_r(r_k) = n_k/n$	C.D.F $S_k = \Sigma P_r(r_k)$	$S_k * 7$	Rounding off
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	614	0.149	0.149	1.05	1
4	819	0.20	0.35	2.45	2
5	1230	0.30	0.65	4.55	5
6	819	0.2	0.85	5.77	6
7	614	0.15	1	7	7
	n = 4096				

image(a) 790

Equalized Histogram of Image (b) is:

Gray level	0	1	2	3	4	5	6	7
No. of pixel	0	614	819	0	0	1230	819	614

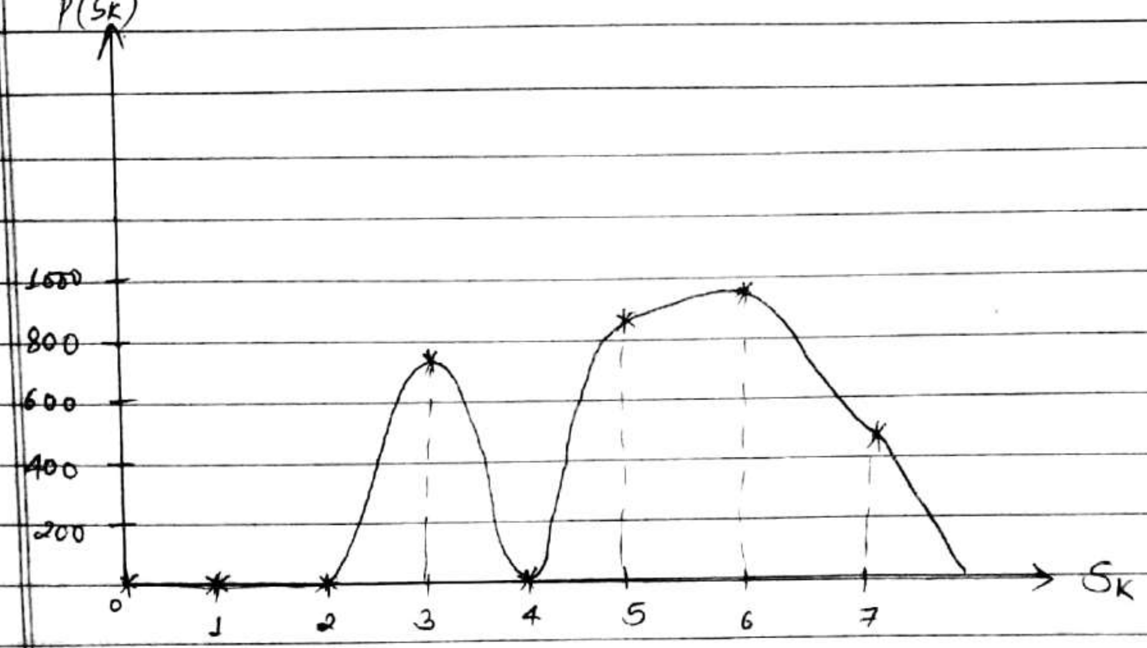
Step(2):

To obtain histogram specification, we apply inverse transformation comparing both equalized histogram.

Image (b) → Gray level → Rounding off no.
Then, Image (a) → Same Gray level
Then, No. of pixel of Image (a)

Gray level	0	1	2	3	4	5	6	7
No. of pixel	0	0	0	790	0	850	985	448

Hence,
The matched or resultant histogram is shown as below:



(fig: Specified Image)

(S.N) # Spatial Operations:

Many image enhancement techniques are based in spatial operations. Most of the spatial operations are performed on the basis of local neighbourhood of the input pixel.

Some of the example of spatial operation for image enhancement are as below:

(1) Neighbourhood processing
(2) Spatial Averaging
(3) Smoothing spatial Filter
(4) Zooming
(5) low Pass, High Pass and Band Pass

(1) Neighbourhood Processing:

k \ l	-1	0	1
1	$g(x-1, y+1)$	$g(x, y+1)$	$g(x+1, y+1)$
0	$g(x-1, y)$	$g(x, y)$	$g(x+1, y)$
-1	$g(x-1, y-1)$	$g(x, y-1)$	$g(x+1, y-1)$

(fig: 3 x 3 neighbourhood pixel)

We change the value of pixels based on the values of its 8 neighbourhood as shown in figure above.

Instead of 3 x 3 neighbourhood, we can also use 5 x 5, 7 x 7 etc.

Neighbourhood processing deals to the point operation. The above neighbourhood pixel can be masked into window as shown in the figure below:

$$
\begin{array}{c|ccc}
 & & l & \\
 & -1 & 0 & 1 \\
\hline
k \quad -1 & w_1 & w_2 & w_3 \\
0 & w_4 & w_5 & w_6 \\
1 & w_7 & w_8 & w_9
\end{array}
$$

(fig : 3×3 window)

The above figure is a one type of tablet which is also called as mask or window.

To achieve the neighbourhood processing, we place this 3×3 mask on the image corresponding to each of the component and get some information at the centre.

If g is the original image and f is the modified image then, f is derived as:

$$
\begin{aligned}
\therefore f(x,y) = {} & g(x-1, y+1) * w_1 + g(x, y+1) * w_2 + \\
& g(x+1, y+1) * w_3 + g(x-1, y) * w_4 + \\
& g(x, y) * w_5 + g(x+1, y) * w_6 + \\
& g(x-1, y-1) * w_7 + g(x, y-1) * w_8 + \\
& g(x+1, y-1) * w_9
\end{aligned}
$$

Hence,
the sum of products of the mask coefficients with corresponding pixel takes place.

(2) Spatial Averaging:
[Averaging Filtering]

(2010 spring)

In spatial averaging, each pixel is replaced by the weighted average of its neighbourhood pixel i.e

$$\boxed{v(m,n) = \sum\sum_{(k,l)\in W} a(k,l)\, y(m-k, n-l)}$$

where,

$y(m-k, n-l)$ = i/p image
$v(m,n)$ = o/p image
w = Choosen window
$a(k,l)$ = Filter Weight

A common class of spatial average filter has all equal weight i.e

$$\boxed{v(m,n) = \frac{1}{N_w} \sum\sum_{(k,l)\in w} y(m-k, n-l)}$$

where,

$a(k,l) = \frac{1}{N_w}$; N_w is the no. of pixel in window.

Now,
Each pixel is replaced by its

average with the average of its nearest pixel value.

Spatial average is used for noise smoothing, low pass filtering and sub-sampling of the image.

Example of spatial averaging can be represented as:

L

K	-1	0	1
-1	1/9	1/9	1/9
0	1/9	1/9	1/9
1	1/9	1/9	1/9

(fig: 3×3 window)

l=2

k=2		
	1/4	1/4
	1/4	1/4

(fig: 2×2 window)

(3) Smoothing Spatial Filter:
[Median & Mean Filter]

Median Filter:

The median filter is also a spatial filter but it replaces the central value in the window with median of all the pixel values in the window.

The kernel or template or mask is usually square but can be initaited in different window.

An example of median filtering is

i.e

unfiltered values in order like
0, 1, 2, 3, 4, 6, 10, 15, 19, 97

Now,

$$\text{o/p value} \text{ OR } \text{Median value} = \left(\frac{n+1}{2}\right)^{th}$$

$$= \left(\frac{9+1}{2}\right)^{th}$$

$$= 5^{th}$$

$$= 6$$

Hence,

6	2	0
3	97	4
19	15	10

Before filtering

$\xrightarrow[\text{filtering}]{\text{Median}}$

6	2	0
3	6	4.
19	15	10

After filtering

The center value 97 is replaced by the median value of all nine values i.e 6.

Formula for median filtering is :

$$\boxed{\vartheta(m,n) = \text{Median}\{y(m-k, n-l)\ (k,l) \in W\}}$$

Mean Filter:

Mean filter is a spatial filter that replaces the center value in the window

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with the average of all the pixel values in the window.

Here, the window can be of any shape but can be initaited as a square mask. Therefore, average of smoothing filter shows the properties of lowpass filter. So, it reduces the noise from an image.

An example of mean filter is:
Unfiltered values in order like:
5, 3, 6, 2, 1, 9, 8, 4, 7

Now,

$$\text{O/P value} \text{ OR } \text{Mean value} = \frac{\Sigma x}{N} = 5$$

Hence,

5	3	6
2	1	9
8	4	7

Before filtering

$\xrightarrow[\text{filtering}]{\text{Mean}}$

5	3	6
2	5	9
8	4	7

After filtering

The center value '1' is replaced by the mean value '5'

(P.T.O)

V. Imp

(4) Zooming :

(2012) [Magnification & Interpolation]

Another important application is image enhancement where spatial domain neighbourhood operation is used.

Such type of image enhancement technique is known as zooming.

Actually, it is the process of making an image larger. It involves two steps:

(1) Creating new pixel location.
(2) Assigning gray level to those new location.

Different types of zooming technique can be used to zoom. They are:

(i) Replication
(ii) linear Interpolation

(i) Replication:

In replication, we simply replicate each pixel and then replicate row.

let, us consider an image as:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 8 & 6 & 7 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

As ~~we~~ we start from the first row, first we replicate each pixel and then replicate each row.

Now,
The first row looks as:

$$\begin{bmatrix} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \end{bmatrix}$$

We will now replicate this row then it looks as:

$$\begin{bmatrix} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \\ 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \end{bmatrix}$$

Now,
Performing replication operation on an entire image then we obtain output image as:

$$\begin{bmatrix} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \\ 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \\ 5 & 5 & 6 & 6 & 7 & 7 & 8 & 8 \\ 5 & 5 & 6 & 6 & 7 & 7 & 8 & 8 \\ 9 & 9 & 8 & 8 & 6 & 6 & 7 & 7 \\ 9 & 9 & 8 & 8 & 6 & 6 & 7 & 7 \\ 0 & 0 & 1 & 1 & 2 & 2 & 3 & 3 \\ 0 & 0 & 1 & 1 & 2 & 2 & 3 & 3 \end{bmatrix}$$

Here,
4x4 image is zoomed to 8X8 image.
This method can be repeated to get larger image but no new data is added.

Zooming By zero-Interlace Technique:

step(1): Adding zero ~~to every~~ after every pixel of the first row then we obtain as:

$$\begin{bmatrix} 1 & 0 & 2 & 0 & 3 & 0 & 4 & 0 \end{bmatrix}$$

step(2): Adding zero at every column.
Now,
Insert row of zero and then this is known as zero-Interlacing.

$$\begin{bmatrix} 1 & 0 & 2 & 0 & 3 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

step(3): Now,
Performing zero-Interlace technique on an entire image then we obtain output image as:

$$\begin{bmatrix} 1 & 0 & 2 & 0 & 3 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 6 & 0 & 7 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 0 & 8 & 0 & 6 & 0 & 7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(fig: 8 × 8 final Pseudo Image)

Hence, this image is known as zero-Interlace Image.

(ii) Linear Interpolation:

In this method, instead of replicating each pixel, average of the two adjacent pixel along the row is taken and placed between the two pixels.

Accordingly, same operation is then performed along the column.

linear Interpolation is also derived from the zero-interlace technique.

step(1): zero- Interlace

$$\begin{bmatrix} 1 & 0 & 2 & 0 & 3 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 6 & 0 & 7 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 0 & 8 & 0 & 6 & 0 & 7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

step(2): Interpolate row

$$\begin{bmatrix} 1 & 1.5 & 2 & 2.5 & 3 & 3.5 & 4 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 5.5 & 6 & 6.5 & 7 & 7.5 & 8 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 8.5 & 8 & 7 & 6 & 6.5 & 7 & 3.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 1 & 1.5 & 2 & 2.5 & 3 & 3.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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Step(3): Interpolate column

$$\begin{bmatrix} 1 & 1.5 & 2 & 2.5 & 3 & 3.5 & 4 & 2 \\ 3 & 3.5 & 4 & 4.5 & 5 & 5.5 & 6 & 3 \\ 5 & 5.3 & 6 & 6.5 & 7 & 7.5 & 8 & 4 \\ 7 & 7 & 7 & 6.25 & 6.5 & 7 & 7.5 & 3.25 \\ 9 & 8.5 & 8 & 7 & 6 & 6.5 & 7 & 3.5 \\ 4.5 & 4.5 & 4.5 & 4.5 & 4 & 4.5 & 5 & 2.5 \\ 0 & 0.5 & 1 & 1.5 & 2 & 2.5 & 3 & 1.5 \\ 0 & 0.25 & 0.5 & 0.75 & 1 & 1.25 & 1.3 & 0.7 \end{bmatrix}$$

(5) Filtering:

(i) Low Pass Filtering:

It is employed to remove high frequency (unnecessary noise) from a digital image.

This type of filtering is usually used to attenuate the image noise.

Here, an image is $\boxed{\text{smoothed}}$ by decreasing the disparity between the pixel value by average value.

Example of mask or kernel or window of low pass filter can be shown as below:

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

(fig: 3 x 3 mask)

1/16	1/16	1/16	1/16
1/16	1/16	1/16	1/16
1/16	1/16	1/16	1/16
1/16	1/16	1/16	1/16

(fig: 4 X 4 mask)

(ii) High Pass Filtering: (Sharpening filters)

A high pass filtering is the basis for most sharpening methods. Sharpening is the opposite of smoothing.

The sharpening can make it possible to highlight the boarders between homogenous region.

This is used to return high frequency information.

The kernel of the high pass filter is designed to increase the brightness of the center pixel.

Example of mask or kernel or window of high pass filter is as shown below:

~~1/9~~	~~1/9~~	~~1/9~~
~~1/9~~	~~-1/9~~	~~1/9~~
~~1/9~~	~~1/9~~	~~1/9~~

$$\frac{1}{9}\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

(fig: 3X3 mask high pass filter)

(iii) Band Pass Filtering :

This filter is used to remove selected frequency region between low and high frequency. It is also used for image restoration. It passes a limited range of frequency.

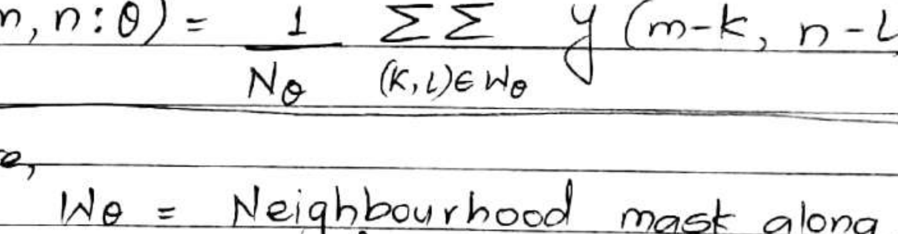
Calculating spatial averaging for band pass filtering is :

$$\boxed{\vartheta(m, n : \theta) = \frac{1}{N_\theta} \sum\sum_{(k,l)\in W_\theta} y(m-k, n-l)}$$

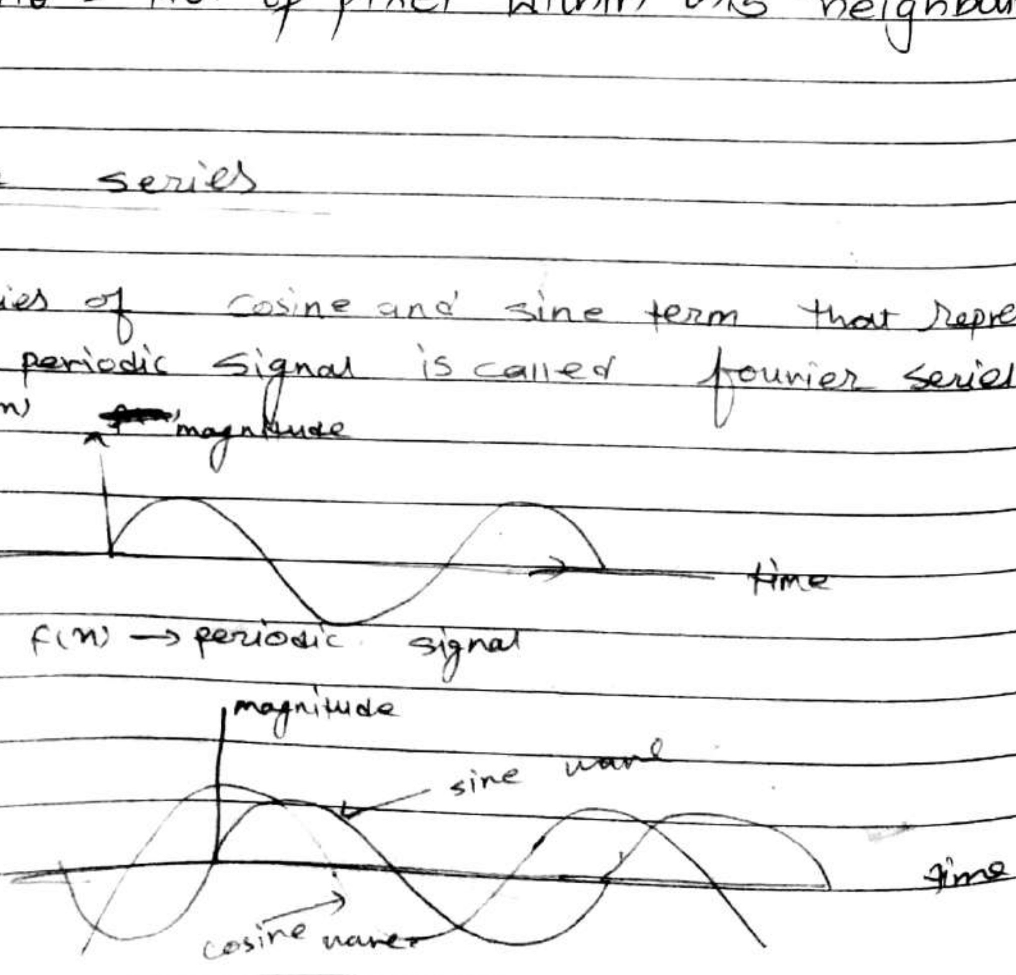
Where,
W_θ = Neighbourhood mask along direction θ
N_θ = No. of pixel within this neighbourhood

fourier series

* A series of cosine and sine term that represent the periodic signal is called fourier series.



f(n) → periodic signal



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Chapter-3

Image Enhancement In Frequency Domain

[6 hours]

Fourier Transform :

Fourier Transform is a mathematical tool to analyze and design linear system. It is used to reduce the number of calculation to a fraction.

It also helps to quantify the effects of digitizing system.

The application of Fourier transform is:

- Image Enhancement
- X-Ray
- Enhancement of TV transmission
- and video signal.

(A) 1-D Fourier Transform:

Let, $f(x)$ be a continous real variable of x.

Then,

Fourier Transform $= F[f(x)] = F(u)$

$$\therefore \boxed{F(u) = \int_{-\infty}^{\infty} f(x) \cdot e^{-j2\pi ux} \, dx}$$

P

And,

Inverse Fourier Transform is:

$$F^{-1}[F(u)] = f(x)$$

$$\boxed{\therefore f(x) = \int_{-\infty}^{\infty} F(u) \cdot e^{j2\pi ux} \, dx}$$

(B) 2-D Fourier Transform:

A 2-D function $f(x)$ has 2-D transform and,

$$\boxed{F[f(x,y)] = F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) \cdot e^{-j2\pi(ux+vy)} \cdot dxdy}$$

Now,

Inverse of fourier transform is:

$$\boxed{F^{-1}[F(u,v)] = f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v) \cdot e^{j2\pi(ux+vy)} \, dxdy}$$

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Properties Of Fourier Transform:

(1) Addition Theorem:

$$F[f(x,y) + g(x,y)] = F(u,v) + G(u,v)$$

(2) Shift Theorem:

$$F[f(x-a, y-b)] = F(u,v)\, e^{-j2\pi(U_a, V_b)}$$

(3) Similarity Theorem:

$$F[f(ax, by)] = \frac{1}{|ab|} F(U/a, V/b)$$

(4) Convolution Theorem:

$$F[f(x,y) * g(x,y)] = F(U,V) * G(U,V)$$

(5) Rayleigh's Theorem:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |f(x,y)|^2 \, dx.dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |F(U,V)|^2 \, dU.dV$$

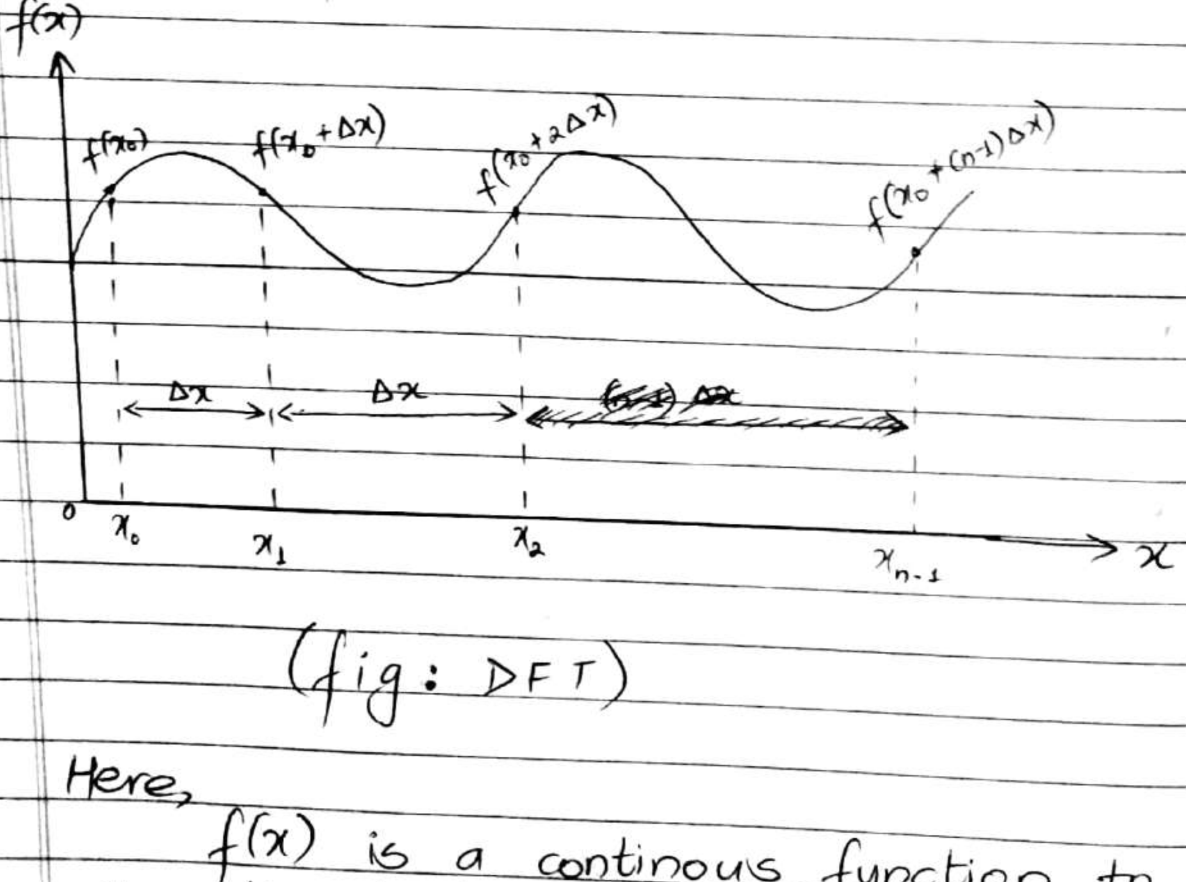
(6) Evenness & oddness:

$$f_e(x) \Rightarrow f(x) = f(-x) \text{ and}$$
$$f_o(x) \Rightarrow f(-x) = -f(x)$$

(7) Power Theorem:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) * g^*(x,y)\, dx\, dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v) * G^*(u,v)\, du\, dv$$

Discrete Fourier Transform (DFT) and It's Importance:



(fig: DFT)

Here,

$f(x)$ is a continous function to be discretized by taking 'n' sample i.e

$$\{f(x_0),\ f(x_0+\Delta x),\ f(x_0+2\Delta x),\ ---\ f(x_0+(n-1)\Delta x)\}$$

It is defined as:

$$\boxed{F(u) = \frac{1}{n}\sum_{x=0}^{n-1} f(x)\cdot e^{-j2\pi \frac{ux}{n}}}$$

Inverse Discrete Fourier Transform is defined as :

$$\boxed{f(x) = \sum_{U=0}^{N-1} F(U) \cdot e^{j2\pi \frac{Ux}{n}}}$$

These two given equation are discrete fourier transform and it is used to calculate the following :

(1) Fourier spectrum:

$$|F(U)| = \sqrt{R^2(U) + I^2(U)}$$

(2) Power spectrum:

$$|P(U)| = |F(U)|^2 = R^2(U) + I^2(U).$$

(3) Phase Angle (spectrum):

$$\phi = \tan^{-1}\left[\frac{I(U)}{R(U)}\right]$$

Where,
'U' is the transform frequency variable.
'x' is sapital or Image Variable.

Discrete Fourier Transform is most widely used orthogonal transform in the field of image processing and it can diagonilized circuilant matrix.

However, in this transform, it takes n^2 multiplication and $n(n-1)$ addition to calculate 1-D discrete fourier transform of 'n' data points.

Discrete Cosine Transform And It's properties: (DCT)

(2010 spring)

Discrete cosine transform is a Fourier related transform similar to DFT but it is only concerned with real number only.

It can be defined as:

$$DCT \Rightarrow \boxed{C(U) = \alpha(U) \sum_{x=0}^{N-1} f(x) \cdot \cos\left[\frac{(2x+1)U\pi}{2N}\right]}$$

Where,

$U = 0, 1, \ldots N-1$

$$\alpha(U) = \sqrt{\frac{1}{N}} = \frac{1}{\sqrt{N}} \text{ for } U = 0$$

$$= \sqrt{\frac{2}{N}} \text{ for } U = 1, \ldots N-1$$

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∴ Inverse DCT is defined as:

$$IDCT \Rightarrow f(x) = \sum_{u=0}^{N-1} \alpha(u) \cdot \cos\left[\frac{(2x+1)u\pi}{2N}\right]$$

Where,

$$u = 0, 1, \ldots N-1$$

<u>For 2-D</u>:

$$DCT \Rightarrow C(u,v) = \alpha(u,v) \cdot \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y) \cdot \cos\left[\frac{(2x+1)u\pi}{2N}\right] \cos\left[\frac{(2y+1)u\pi}{2N}\right]$$

Where,

$$\alpha(u,v) = \frac{1}{\sqrt{N}} \quad \text{for } u, v = 0$$

$$= \sqrt{\frac{2}{N}} \quad \text{for } u, v = 1, \ldots N-1$$

$$IDCT \Rightarrow f(x,y) = \sum_{u=0}^{N-1}\sum_{v=0}^{N-1} \alpha(u,v) \cos\left[\frac{(2x+1)u\pi}{2N}\right] \cos\left[\frac{(2y+1)u\pi}{2N}\right]$$

where,

$$x = y = 0, \ldots N-1$$

(P.T.O)

Arthogonal: $A^T A = I$
symmetric: $A^T = A$

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Properties Of DCT:

(1) The cosine Transform is real and orthogonal.

i.e

$$C = C^* \Rightarrow C^{-1} = C^T$$

Where,

C^* = conjugate of C
C^T = Transform of C

(2) The Casine Transform is not real part of Unitary DFT.

i.e

$$T^{-1} = T^{*T}$$

(3) It is a fast transform. The cosine transform of 'N' elements can be calculated in $O(N\log_2 N)$ operation through N point FFT (fast fourier Transform).

(4) It has excellent energy compaction for highly co-related data.

Uses Of DCT:

DCT is used in image processing and signal processing especially for loose data compression. ~~dense~~

Since it has strong anergy compaction features.

It is also used for JPEG image compression, MPEG video compression and MJPEG video compression.

Imp # Hadamard Transform: (2015 fall) (2012 fall)

Hadamard Transform is a fast transform which is real, symmetric and orthogonal.

1-D Hadamard Transform can be implemented in (Big oh) $O(N \log_2 N)$ addition and subtraction. It has only binary values i.e 1 or -1 in its kernel matrix.

It is defined as:

$$H(U) = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} f(x) \cdot \underbrace{(-1)^{\sum_{x=0}^{n-1} b_i(x) \cdot b_i(U)}}_{\text{Kernel}}$$

For N = 2:

$$N = 2^n \Rightarrow 2 = 2^n \Rightarrow n = 1$$

Where, $U = 0, \ldots N-1$

Now,

Hadamard Matrix can be defined as:

$$H_n = \frac{1}{\sqrt{N}} \begin{bmatrix} H_{n-1} & H_{n-1} \\ H_{n-1} & -(H_{n-1}) \end{bmatrix}$$

where,

$$\frac{1}{\sqrt{N}} = \text{sample value}$$

$$\begin{bmatrix} H_{n-1} & H_{n-1} \\ H_{n-1} & -(H_{n-1}) \end{bmatrix} = \text{Indicate kernel}$$

$$H_0 = 1$$

<u>Hadamard Matrix at 1</u> : (H_1) i.e $n=1$

$$H_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} H_{1-1} & H_{1-1} \\ H_{1-1} & -(H_{1-1}) \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} H_0 & H_0 \\ H_0 & -H_0 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Also,

<u>For N = 4</u>, $N = 2^n \Rightarrow 4 = 2^n \Rightarrow n = 2$

$$\therefore H_2 = \frac{1}{\sqrt{4}} \begin{bmatrix} H_{2-1} & H_{2-1} \\ H_{2-1} & -(H_{2-1}) \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} H_1 & H_1 \\ H_1 & -H_1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

(2012 Fall) (pre-university)

Properties of Hadamard Transform:

(1) It is real, symetric and orthogonal i.e

$$H = H^* \Rightarrow H^{-1} = H^T$$

(2) It is a fast transform and 1-D Hadamard Transform can be implemented in $O(N\log_2 N)$ addition and subtraction.

(3) It has only binary values i.e 1 or -1 in its kernel matrix. No multiplications are required in the transform.

(4) It is used for digital image processing and digital signal processing.

(5) It has good energy compaction for highly co-related image.

(6) It is also used in digital hardware implementation of image processing algorithm.

2010 spring
(S.N) 2012 fall

(S.N) # Haar Transform:

Haar Transform is the orthogonal transform which is derived from the Haar Matrix.

It is also a fast transform and can be implemented in $O(N\log_2 N)$ operation.

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The main advantage of this transform is that it samples the input data sequence into fine resolution and takes the difference between the adjacent pairs.

The resolution increases by a power of 2 and as a result, in the transform domain, differential energy is more and more localized.

It only exist for $N = 2^n$, where n is an integer.

Haar Transform is defined as:

$$\boxed{H_T = H_k(x)}$$

Where, $k = 0, ---- N-1$

k is given as:

$$\boxed{k = 2^p + q - 1}$$

And,

$$0 \leq p \leq (n-1)$$
$$1 \leq q \leq 2^p, \text{ for } p \neq 0$$
$$q = 0, 1, \text{ for } p = 0$$

(pre-university)

Properties Of Haar Transform:

(1) It is symmetric, seperable unitary transform that uses haar function for its basis.

(2) It is orthogonal and real.
ie

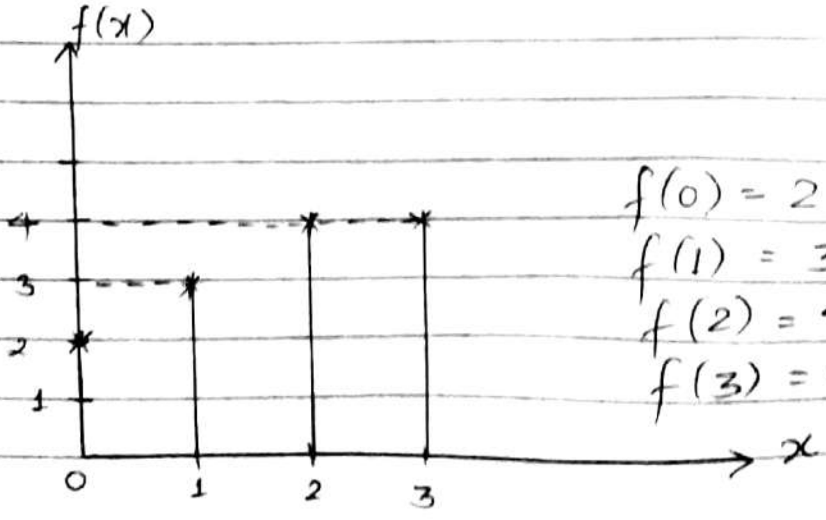
$$T^{-1} = T^{T} \Rightarrow H_r = H_r^{*}$$

(3) It is a fast transform and can be implemented in O(N) operation. Where, N is a number of samples.

(4) It exist for $N = 2^n$, where 'n' is an integer.

(5) It has poor energy compaction property.

Q. Find the Fourier spectrum of the function at point 0 and 1 as shown in figure below:



$f(0) = 2$
$f(1) = 3$
$f(2) = 4$
$f(3) = 4$

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Soln: Here,

Given that:

$f(0) = 2$, $f(1) = 3$, $f(2) = 4$, $f(3) = 4$

We know that:

DFT ⇒ $$\boxed{F(U) = \frac{1}{N} \sum_{x=0}^{N-1} f(x) \cdot e^{\frac{-j2\pi Ux}{N}}}$$

Where,

$N = 4$ $(0, 1, 2, 3)$

Now,

For $U=0$:

$$F(0) = \frac{1}{4} \sum_{x=0}^{3} f(x) \cdot e^{-j2\pi \frac{0 \cdot x}{4}}$$

$$= \frac{1}{4} \sum_{x=0}^{3} f(x)$$

$$= \frac{1}{4} \left[f(0) + f(1) + f(2) + f(3) \right]$$

$$= \frac{1}{4} \left[2 + 3 + 4 + 4 \right]$$

$$= 13/4$$

$\therefore$ Fourier spectrum at point 0 is:

$$|F(0)| = \sqrt{R^2(0) + I^2(0)}$$

$$= \sqrt{\left(\frac{13}{4}\right)^2 + 0}$$

$$= \frac{13}{4} \quad \text{Ans}$$

Note:

Power spectrum at point 0 is:

$$|P(0)| = |F(0)|^2 = R^2(u) + I^2(u)$$

$$= \left(\frac{13}{4}\right)^2$$

$$= \frac{169}{16}$$

Now,

<u>For $u=1$:</u>

$$F(1) = \frac{1}{4} \sum_{x=0}^{3} f(x) \cdot e^{-j2\pi \frac{1 \cdot x}{4}}$$

$$= \frac{1}{4} \sum_{x=0}^{3} f(x) \cdot e^{\frac{-j\pi x}{2}}$$

$$= \frac{1}{4}\left[f(0) \cdot e^{\frac{-j\pi \cdot 0}{2}} + f(1) \cdot e^{\frac{-j\pi \cdot 1}{2}} + f(2) \cdot e^{\frac{-j\pi \cdot 2}{2}} + f(3) \cdot e^{\frac{-j\pi \cdot 3}{2}} \right]$$

$$= \frac{1}{4}\left[f(0)\cdot e^{0} + f(1)\cdot e^{\frac{-j\pi}{2}} + f(2)\cdot e^{-j\pi} + f(3)\cdot e^{\frac{-j\pi 3}{2}}\right]$$

$$= \frac{1}{4}\left[2\times 1 + 3\times\left(\cos\frac{\pi}{2} - j\sin\frac{\pi}{2}\right) + 4\times\left(\cos\pi - j\sin\pi\right) + 4\times\left(\cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2}\right)\right]$$

$$= \frac{1}{4}\left[2 + 3\times(0 - j\cdot 1) + 4\times((-1) - j0) + 4\times(0 - j\cdot(-1))\right]$$

$$= \frac{1}{4}\left[2 - 3j - 4 + 4j\right]$$

$$= \frac{1}{4}\left[j - 2\right]$$

$$= \frac{1}{4}j - \frac{1}{2}$$

$$\therefore\ F(1) = \frac{j}{4} - \frac{1}{2}$$

Now,

Fourier spectrum at point 1 is:

$$|F(1)| = \sqrt{R^2(1) + I^2(1)}$$

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$$= \sqrt{\left(\frac{-1}{2}\right)^2 + \left(\frac{1}{4}\right)^2}$$

$$= \sqrt{\frac{1}{4} + \frac{1}{16}}$$

$$= \sqrt{\frac{5}{16}}$$

$$= \frac{\sqrt{5}}{4}$$

$\therefore \; |F(1)| = \sqrt{5}/4$ Ans

FFT (Fast Fourier Transform)

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Chapter-4
Image Restoration
[4 hours]

Introduction:

The objective of image restoration is to improve a given image in some pre-defined sense.

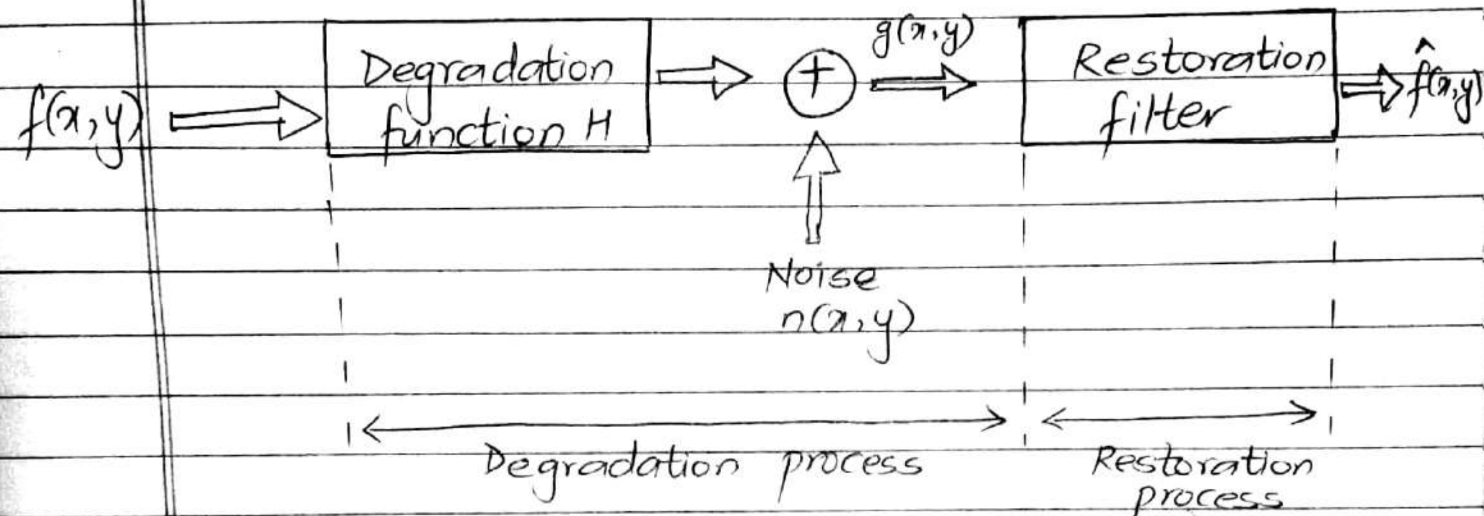
Image Enhancement is a subjective process whereas Image Restoration is an objective process.

Image Restoration is to re-construct or recover an image that has been degraded by using a prior knowledge of the degradation phenomena. So, this technique is a process of degradation and applying the inverse process in order to recover the original image.

If there is a presence of noise and image bluring we use the concept of image restoration.

Degraded Image = Original Image + Noise

A model of the Image Degradation/ Restoration Process :



(fig: A model of Image Degradation/ Restoration process)

The Degraded image $g(x,y)$ can be produced as :

$$g(x,y) = H[f(x,y)] + n(x,y) \quad ------(i)$$

Where,

$f(x,y)$ = Input Image
$n(x,y)$ = Additive Noise term
H = Degradation function

Now,

The degraded image can be defined in spatial Domain as :

$$g(x,y) = h(x,y) * f(x,y) + n(x,y) \quad ---(ii)$$

in function analysis convolution is a mathematical operation of two function (f & g) it produce the 3rd function

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Where,

$h(x,y)$ = spatial Representation of the degradation function

Then,

the degraded image in spatial domain is equivalent to the convoluted frequency domain represented as:

$$G(u,v) = H(u,v) * F(u,v) + N(u,v) \quad \text{.....(iii)}$$

∴ The degradation process is also referred to as convolving the image with a point spread function (PSF).

Similarly,

Restoration process is also referred to as De-convolution.

Hence,

Restoration Process can be obtained as,

$\hat{f}(x,y) = R[g(x,y)]$ in linear spatial Invariant Form.

$\hat{f}(u,v) = R[G(u,v)]$ in Deconvolution Form.

(2015 fall) (in) Image spatial Domain)

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"Noise Models Restoration" In the Presence of Noise only Spatial Filtering:

The method of choice for reducing the noise is spatial filtering. After filtering the noise, there should have a restoration process to have or obtain original image.

For noise reduction, following summarize and implement two types of spatial filters:

(1) Spatial Noise Filters
(2) Adaptive spatial filters

(1) Spatial Noise Filters:

In spatial noise filters, image is independently off how image characteristics vary from one location to another location.

In this condition, the spatial noise filters can be used and explained as:

(i) Arithmetic Mean, $A = \boxed{\frac{1}{n} * \sum_{i=1}^{n} x_i}$ ← value of individual item; n ← number of terms

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s,t)$$

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(ii) Geometric Mean,

$$\hat{f}(x,y) = \left[\prod_{(s,t) \in S_{xy}} g(s,t) \right]^{1/mn}$$

(iii) Harmonic Mean,

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t) \in S_{xy}} 1/g(s,t)}$$

(iv) Median,

$$\hat{f}(x,y) = \underset{(s,t) \in S_{xy}}{\text{Median}} \{g(s,t)\}$$

(v) Maximum,

$$\hat{f}(x,y) = \underset{(s,t) \in S_{xy}}{\text{Max}} \{g(s,t)\}$$

(vi) Minimum,

$$\hat{f}(x,y) = \underset{(s,t) \in S_{xy}}{\text{Min}} \{g(s,t)\}$$

Where,

$m \& n$ = Variables denotes image rows & columns spanned by filter respectively

S_{xy} = Denotes an $m \times n$ subImage of the input noisy image g.

$\hat{f}(x, y)$ = filter response at (x, y) co-ordinate

(x, y) = co-ordinates of sub-image.

(2) Adaptive Spatial Filter:

In some application, results can be ~~implement~~ improved by using filters (capable of adapting behaviour) based on the characteristics of the image in the region being filtered.

The adaptive median filtering algorithm uses two processing levels for adaptive spatial filter that can be denoted as:

Level A : $\left(\begin{array}{l} A_1 = Z_{med} - Z_{min} \\ A_2 = Z_{med} - Z_{max} \end{array} \right)$

Algorithm

→ If $A1 > 0$ AND $A2 < 0$, go to level B

* If $Z_{min} < Z_{med} < Z_{max}$ go to level B
* Else Increase the window size

→ If window size $\leq S_{max}$, repeat level A

→ Else output Z_{med}

$$\begin{pmatrix} B_1 = Z_{xy} - Z_{min} \\ B_2 = Z_{xy} - Z_{max} \end{pmatrix}$$

algorithm **Level B :** (If $B_1 > 0$ AND $B_2 < 0$, output Z_{xy})

If $Z_{min} < Z_{xy} < Z_{max}$, output Z_{xy}

Else ~~out~~ output Z_{med}

Where,

Z_{min} = Min^m intensity (→ gray level) value in S_{xy}

Z_{max} = Max^m intensity value in S_{xy}

Z_{med} = Median of the intensity value in S_{xy}

Z_{xy} = Intensity value at co-ordinates (x,y)

S_{xy} = Maxm allowed size of the adaptive filter window

(~~Frequency Domain~~)

Periodic Noise Reduction Using Frequency Domain Filtering :

Periodic Noise produces impluse like burst that often are visible in the Fourier Spectrum.

The principle approach for filtering these components is to use Notch reject filtering.

The general expression for Notch Reject Filtering having "Q" notch pair can be defined as :

$$H_{NR}(U,V) = \prod_{K=1}^{Q} H_K(U,V)\ H_{-K}(U,V)$$

$\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

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Where,

$H_K(U, V)$ & $H_{-K}(U, V)$ = High pass filter with centers at (U_K, V_K) & $(-U_K, -V_K)$ respectively.

These translated centers are specified with respect to the center of the frequency rectangle $(M/2, N/2)$.

Therefore, the distance computation for the filters are given by following expression:

$$D_K(U, V) = \left[\left(U - M/2 - U_K\right)^2 + \left(V - N/2 - V_K\right)^2 \right]^{1/2}$$

&

$$D_{-K}(U, V) = \left[\left(U - M/2 + U_K\right)^2 + \left(V - N/2 + V_K\right)^2 \right]^{1/2}$$

→ In a video stream, periodic noise is typically caused by the presence of electrical or electromagnetical interference during vedio acquisition or Transmission.

→ periodic noise can be ~~d~~ reduced with frequency domain filtering.

∴ frequency domain filtering isolate the frequency occupied by the noise and suppresses them using ~~band~~ Notch - Reject filter.