

Conduction in Liquids and Gases

3

Chapter

3.1 Introduction

We are mostly familiar with many liquids and gases being used as dielectric materials in different applications in electrical engineering to ensure the required insulation level for proper functioning of different electrical appliances, devices and machines. But, in addition to the abovementioned, we also encounter many liquids and gases used for conduction purposes as well. In certain special circumstances, we are forced to use liquids and gases to ensure the conduction process takes place smoothly as demanded by the circumstance. There may be such special cases, where the same material is meant for dual purposes, firstly act as insulation and then during special conditions, act as conducting medium.

3.2 Ionic Conduction in Liquids

The practical aspect of ionic conduction in electrolytes can be visualized from the examples purposely done electrolysis electroplating electrolysis of water and naturally occurring phenomena.

In case of purposely done electrolysis or electroplating, what we do is to take aqueous media containing the metal to be plated in which two electrodes called cathode and anode are placed. When an external potential is applied, cation and anion are formed, and cation is attracted towards anode while the anion goes towards cathode.

In case of naturally occurring ionic conduction due to surrounding environmental conditions, aqueous media is formed. And, due to electron affinity of material, ionization takes place.

One of the examples of ionic conduction in liquids is electroplating in which following reaction takes place:

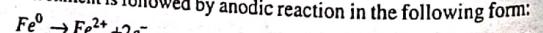


Cu^{++} goes to anode loses positive charge and sticks on to the anode;

SO_4^{-} goes to cathode which is made up of copper, so knocks copper to form CuSO_4 in solution making the concentration same as before.

Similar examples of electroplating of other metals and the electrolysis of water can also be recalled in which ionic conduction takes place.

In natural form of ionic conduction is exhibited by rusting of iron. Iron in free environment due to the humidity of the air, forms an aqueous environment is followed by anodic reaction in the following form:



All the above mentioned processes take place in the following manner. In ionic conduction the individual atoms transfer electrons between each other to form positively charged cation and negatively charged anions. The binding forces between the ions are electrostatic in nature and are thus very strong. The room temperature conductivity of such liquid materials are very low in comparison to conductivity of metals because only limited number of electrons from valence band can reach the conduction band and result in ionization.

The main conduction is caused due to movement of such positively or negatively ion in particular direction on application of electric field. The ionic conductivity is given by

$$\sigma_{\text{ionic}} = eN_{\text{ion}}\mu_{\text{ion}} \quad (3.4)$$

where N_{ion} is the number of ions, which are ionized, and moving under the influence of applied electric field. And anion is the mobility of these ions.

In order to move, these ions must possess sufficient kinetic energy and electric field. During the movement of these ions, there is collision and exchange of energy. Hence ionization may be increasing successively and directional in nature due to applied electric field.

From Einstein diffusion relation, we have

$$\mu_{\text{ion}} = \frac{eD}{kT} \quad (3.5)$$

where D is the diffusion coefficient for ions; k is Boltzmann's constant and T is the temperature of the liquid.

The diffusion coefficient varies with temperature as

$$D = D_0 \exp(-Q/kT) \quad (3.6)$$

where Q is the activation energy for the purpose under consideration, and D_0 is the pre-exponential factor which depends on the vibrational frequency of the atoms and other material properties.

Ionic conductivity can be expressed as

$$\begin{aligned} \sigma_{\text{ion}} &= eN_{\text{ion}}\mu_{\text{ion}} \\ &= eN_{\text{ion}} \frac{eD}{kT} \\ &= eN_{\text{ion}} \frac{e}{kT} D_0 \exp(-Q/kT) \\ &= \frac{e^2 N_{\text{ion}}}{kT} D_0 \exp\left(-\frac{Q}{kT}\right) \end{aligned} \quad (3.7)$$

i.e.

$$\sigma_{\text{ion}} = \sigma_0 \exp\left[-\frac{Q}{kT}\right] \quad (3.7)$$

$$\text{where } \sigma_0 = \frac{e^2 N_{\text{ion}}}{kT} D_0 \quad (3.8)$$

Taking natural logarithm on both the sides, we get

$$\ln \sigma_{\text{ion}} = \ln \sigma_0 - \frac{Q}{kT} = \ln \sigma_0 - \left(\frac{Q}{k}\right) \frac{1}{T} \quad (3.9)$$

The above relation gives the dependence of ionic conductivity on the inverse of temperature.

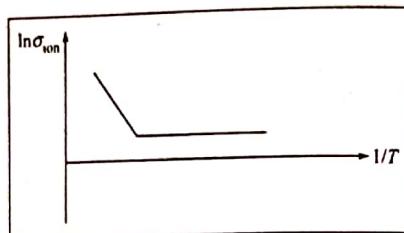


Fig. 3.1 Dependence of ionic conductivity on inverse of temperature.

In addition to the above-mentioned ionic conduction in electrolytes, conduction also takes place in liquid dielectric materials. When low electric fields of less than 1kV/cm are applied the conductivity of the liquid ranges from 10^{-18} to 10^{-20} Sm/cm due to the presence of impurities in the dielectric after the purification process. However, when the fields are very high ~100kV/cm, there is rapid rise in current with violent fluctuations. This conduction current shown in figure 3.2 is nearer to breakdown of dielectric.

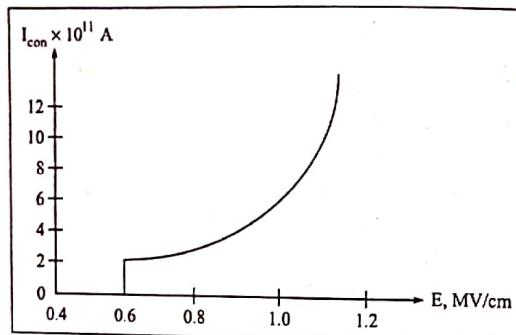


Fig. 3.2 Conduction current vs. electric field characteristics in hexane

However if this characteristics is redrawn from very small currents, we will see that at initial stages, the current is due to the dissociation of ions and with intermediate fields the current reaches saturation value and for high fields the current is due to field assisted emission. Although the exact mechanism of current growth is not known, however, it appears that the electrons generated from the cathode field emission get multiplied by a process similar to Townsend's primary and secondary ionization in gases.

Commercial liquid insulating materials such as hexane, benzene, transformer oil, silicone, liquid nitrogen, liquid oxygen, liquid argon, liquid helium, etc are not chemically pure. They normally consist impurities like air bubbles, suspended particles, etc. These impurities reduce the breakdown strength of these liquids considerably. When breakdown occurs in these commercial liquids, additional gases and bubbles are evolved and solid decomposition products are formed. Due to this, electrode surface becomes rough and breakdown in commercial liquids are classified as:

- (i) Suspended particle mechanism

- (ii) Cavitation and bubble mechanism
- (iii) Stressed oil volume mechanism

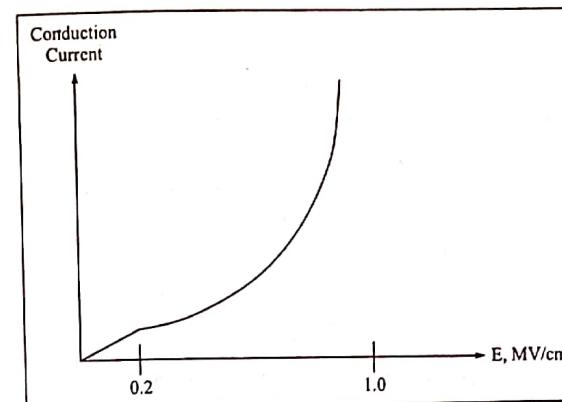


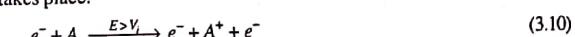
Fig. 3.3 Conduction current vs. electric field characteristic in hydrocarbon liquid.

3.3 Electrical Conduction in Gases

Gases are very easily available in the nature. Most of the gases are found everywhere in the nature, and so they are used widely in electrical engineering primarily for insulating purposes. When the applied voltage is low, a small current flows between the electrodes via gaseous medium separating these electrodes, and the gas retains its all insulating properties. But when the voltage applied is large enough, a sharp increase in currents between the electrodes through the gaseous medium is observed leading to electrical breakdown of the gaseous medium. A strongly conducting spark formed during breakdown produces a short circuit between the electrodes with heavy currents. The maximum voltage applied to the insulation (gas) at the moment of breakdown is called breakdown voltage.

A gas in its normal state is almost a perfect insulator. However, when a high voltage is applied between two electrodes immersed in a gaseous medium, the gas becomes a conductor and an electrical breakdown occurs. The processes primarily responsible for the breakdown of a gas are ionization by collision, photo-ionization and secondary emission processes.

In ionization process, an electron from a gas molecule is produced with simultaneous production of a positive ion. A free electron collides with a neutral gas molecule and gives rise to a new electron and a positive ion. In the figure shown below, if the energy E of the electron gained during travel between collisions exceeds the ionization potential V_i , which is the energy required to dislodge the electron from its atomic shell, then the ionization takes place.



where A is the atom, A^+ is the positive ion and e^- is the electron.

A few electrons produced by ultraviolet rays falling on the cathode, ionize neutral gas particles producing positive ions and additional electrons. These additional electrons make ionizing collisions and thus the process repeats itself. The number of electrons reaching the anode is greater than those liberated at the cathode. The positive ions also reach the cathode and give rise to secondary electrons.

In the figure 3.4, let n_0 be the number of electrons emitted from cathode and α (called Townsend's first ionization coefficient) be the average ionizing collisions made by electron per centimeter travel in the direction of the field, n_x be the number of electrons at a distance x from the cathode. At $x = 0$, $n_x = n_0$.

Similarly,

$$\frac{dn_x}{dx} = \alpha n_x$$

$$n_x = n_0 \exp(\alpha x) \quad (3.11)$$

Then the number of electron reaching the anode (i.e., $x = d$) will be

$$n_d = n_0 \exp(\alpha d) \quad (3.12)$$

And the current in the gap, which is equal to the number of electrons traveling per second is

$$I = I_0 \exp(\alpha d) \quad (3.13)$$

where I_0 is the initial current at cathode.

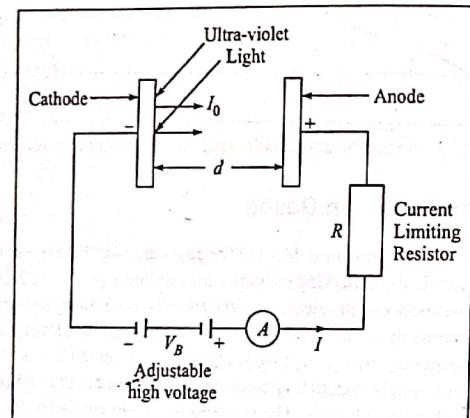


Fig. 3.4 Arrangement for Townsend discharge

3.4 Arc Discharge, Electric Breakdown in Gases

We mentioned how conduction in gases is the result of breakdown in gaseous medium due to various reasons. In the above-mentioned process, the probability of additional new electrons being liberated in the gap by other mechanisms increases and these new electrons further intensify the electron avalanche process. There are other mechanisms for liberation of electrons. The positive ions liberated may have sufficient energy to liberate electrons from the cathode when they impinge on it. The excited atoms or molecules in avalanche may emit photons, and this will lead to the emission of electrons due to photoemission. And, the metastable particles may diffuse back causing electron emission. The secondary electrons produced by these mechanisms are defined by secondary ionization coefficient. Let n'_0 be the number of secondary electrons produced due to secondary processes and n''_0 be the total number of electrons leaving the cathode, then

$$n''_0 = n_0 + n'_0 \quad (3.14)$$

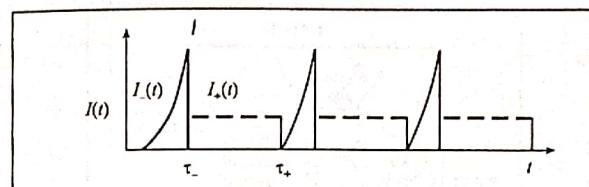


Fig. 3.5 Current as a function of time (Secondary electrons are produced at cathode by positive ions)

The total number of electrons reaching the anode now becomes

$$n = n''_0 \exp(\alpha d) = (n_0 + n'_0) \exp(\alpha d) \quad (3.15)$$

And

$$n'_0 = \gamma [n - (n_0 + n'_0)] \quad (3.16)$$

So,

$$n = \frac{n_0 \exp(\alpha d)}{1 - \gamma [\exp(\alpha d) - 1]} \quad (3.17)$$

$$I = \frac{I_0 \exp(\alpha d)}{1 - \gamma [\exp(\alpha d) - 1]} \quad (3.18)$$

The coefficients α and γ can be calculated by plotting V-I characteristics for different gap settings. The initial slope of $\log(I_0/I)$ plot versus gap distance gives the value of α . By using the value of α , γ can be found using equation (3.18) using points on the upcurving portion of the graph.

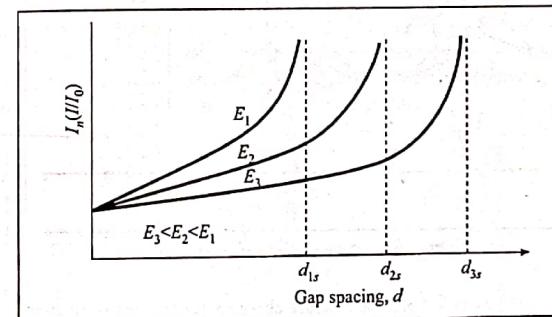


Fig. 3.6 Typical current growth curve in a Townsend discharge

In addition to Townsend mechanism of electric breakdown in gases, there is another theory known as streamer theory which is able to explain many nuances not explained by Townsend discharge mechanism. This theory predicts the development of spark discharge directly from a single electron avalanche in which the space charge developed by the avalanche itself is said to transform the avalanche into a plasma streamer. As shown in figure 3.7, a single electron starting at the cathode by ionization builds up an avalanche that crosses the gap. The electrons in the avalanche move very fast compared with the positive ions. So by the time electrons reach the anode, positive ions are at their initial position forming a positive

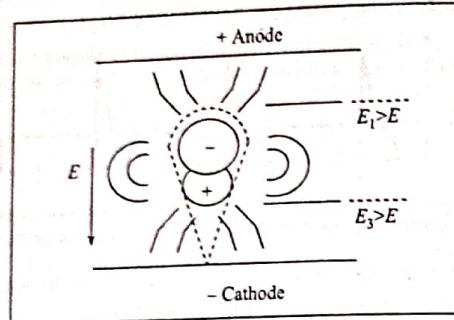


Fig. 3.7 Effect of space charge produced by an avalanche on the applied field

space charge layer at anode which enhances the field and secondary avalanche are formed due to presence of few electrons generated due to photo ionization in the space charge region. Although this occurs initially at the anode, due to the very high speed of the process spreads, space charge region will extend up to the cathode. This process creates a luminous track from anode to the cathode, which is called streamer. As soon as the streamer tip approaches the cathode, a cathode spot is formed and a stream of electrons rushes from the cathode to neutralize the positive charge in the streamer. And, the result is a spark, so spark breakdown has occurred.

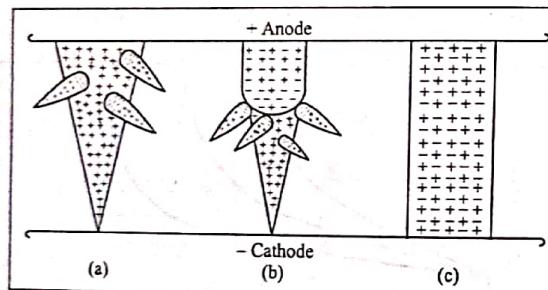


Fig. 3.8 Cathode directed streamer

According to Meek, the electric field due to space charge at the radius r is given by

$$E_r = 5.27 \times 10^{-7} \frac{\alpha \exp(\alpha x)}{(x/p)^{1/2}} \text{ V/cm} \quad (3.19)$$

where α is the Townsend's first ionization coefficient, p – gas pressure in torr, and x – distance to which the streamer has extended in the gap. According to Meek the minimum breakdown voltage is obtained when $E_r = E$ and $x = d$ in the above equation. The equation then simplifies into

$$\alpha d + \ln(\alpha/p) = 14.5 + \ln(E/p) + 0.5 \ln(d/p) \quad (3.20)$$

The equation is solved between α/p and E/p at which a given p and d satisfy the equation. The breakdown voltage is given by the corresponding product of E and d . Based on the experimental results,

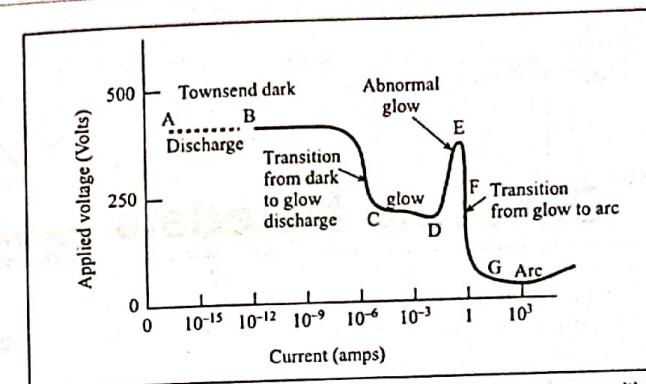


Fig. 3.9 DC voltage-current characteristics of an electrical discharge with electrodes having no sharp points or edges.

the breakdown potential of air is expressed as a function of product of pressure p and distance d as

$$V = 24.2 \times \frac{293pd}{760T} + 6.08 \times \left[\frac{293pd}{760T} \right]^{1/2} \quad (3.21)$$

where p is in torr, T in Kelvin and d in cm.

If the current in the gap is increased to about 1A or more, the voltage across the gap suddenly reduces to few volts ~ 20–25 V. The discharge becomes very luminous and noisy as shown in region E-G of figure 3.9. This phase is called arc discharge. The current density in the cathode region during the arc discharge increases to very high values ~ 10^3 to 10^7 A/cm^2 and the temperature also reaches to very high values ranging from 1000° C to several thousand degrees Celsius. The discharge contains very high electron density and positive ion density, which together are called arc plasma.

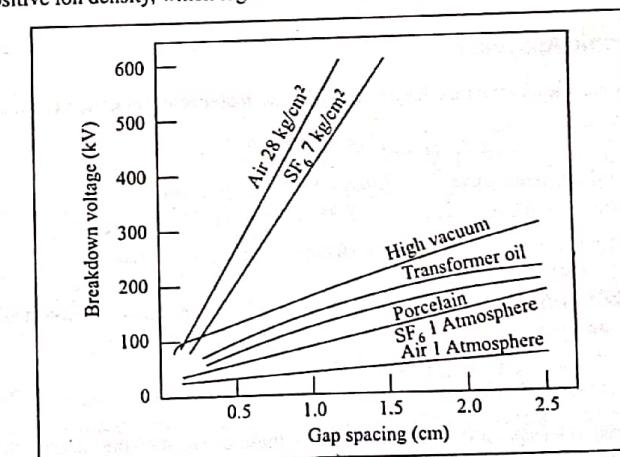


Fig. 3.10 DC breakdown strength of typical solid, liquid, gas and vacuum insulation in uniform fields.

Dielectric Materials

4
Chapter

4.1 Macroscopic Effects

4.1.1 Introduction

The importance of study of dielectric materials is justified by the fact that as the conductors are meant to conduct electricity with as low loss (low resistance) as possible, few also require these current carrying conductor at various high voltages to be insulated from any unwanted contact and this is achieved by using dielectric materials. As we know the dielectrics are the materials, which do not conduct electricity. There are very few free electrons in the conduction band at normal temperature and moderate electric field. The energy gap between valence band and conduction band is of the order of 5eV. The boundary between semiconductor and dielectric is assumed to be equal to 2.5eV in terms of energy gap. The failure of insulation is a big problem for various electric utilities as they use many generators, transformers and associated accessories whose normal functioning depends on the insulation used. The variation (increase) in the applied electric field can affect the normal functioning of insulating materials used for this or that application.

4.1.2 Macroscopic Approach

The relationship between the electric flux density at a point in a material and electric field strength is given by

$$D = \epsilon E \quad (4.1)$$

Where D = Electric Flux density, Cm^{-2} , E = Electric field strength, Vm^{-1}
 ϵ = Permittivity of the medium, $\epsilon = \epsilon_0 = 8.854 \times 10^{-12} \text{ Fm}^{-1}$ for free space

And $\epsilon = \epsilon_0 \epsilon_r$ for other material where ϵ_r is the relative permittivity of the material.

When a voltage V is applied across the parallel plate capacitor as shown in figure 4.1.a., charges $+Q_0$ and $-Q_0$ will be induced on the surfaces of these plates based on electrostatic induction. The capacitance of the parallel plate capacitor is then given by

$$C_0 = \frac{Q_0}{V_0} \quad (4.2)$$

When a dielectric material (medium) is inserted between these plates, then the charges on the surfaces of the plates get increased due to the appearance of the charge on the surface of the dielectric near to the

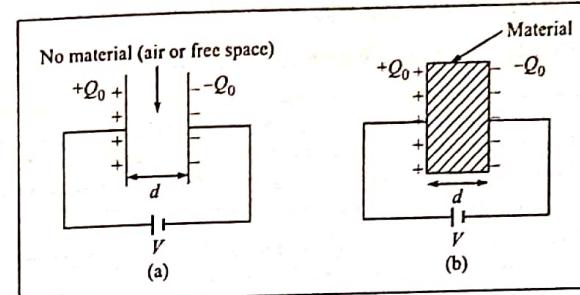


Fig. 4.1 Surface charges on metallic plates. Without material (a); with material (b).

In vacuum, the electric flux density is numerically equal to the surface charge, i.e., per unit area

$$\sigma_0 = Q_0 = \epsilon_0 \frac{V}{d} \quad (4.3)$$

Where d is the separation between the plates.

Similarly, when there is a dielectric medium inserted between the plates, the above relationship becomes

$$\sigma = Q = \epsilon_0 \epsilon_r \frac{V}{d} \quad (4.4)$$

The relative permittivity can be defined as increase in charge stored on the capacitor plates due to insertion of the medium compared to the charge without medium.

$$\epsilon_r = \frac{Q}{Q_0} = \frac{C}{C_0} \quad (4.5)$$

Increase in charge density when dielectric medium is inserted compared with that without dielectric is called polarization, i.e., $P = \sigma - \sigma_0 = Q - Q_0$

$$P = \epsilon_0 \epsilon_r \frac{V}{d} - \epsilon_0 \frac{V}{d}$$

$$\text{or, } P = \epsilon_0 \frac{V}{d} (\epsilon_r - 1) \quad (4.6)$$

$$\text{or, } P = \epsilon_0 (\epsilon_r - 1) E \quad X = \epsilon_r - 1 \quad \epsilon = \epsilon_0 \text{ no polarization}$$

$$\text{or, } P = \epsilon_0 \chi E \quad \text{where } \chi = \epsilon_r - 1 \quad \epsilon_r = \frac{\epsilon}{\epsilon_0} = 1$$

where $E = V/d$ electric field strength

$\chi = \epsilon_r - 1$ = dielectric susceptibility

P = Polarization vector of the material

Polarization P can also be defined as the induced charge within dielectric per unit area.

$$P = \frac{Q'}{A} \quad (4.9)$$

From the above relationship we can conclude that for the value of relative permittivity equal to unity, there will be no polarization ($P = 0$), hence, no additional surface charge on the plate.

4.2 Polarization, Dielectric Constant

4.2.1 Electric Dipole Moment, Electronic Polarization

To understand the polarization mechanism, we first need to know about electric dipole moment. Electric dipole moment is simply the separation between a positive charge and negative charge of equal magnitude. If Q is the magnitude of charge and a represents the vector from negative charge to the positive, then electric dipole moment is defined as the product of charge and the vector from negative to positive charge.

$$p = Qa \quad (4.10)$$

In a region containing a positive charge $+Q$ and negative charge $-Q$ will have zero net charge. Nonetheless, this region will have net dipole moment if their charge centers do not coincide.

In an atom when there is no applied field, the center of mass of atom and the charge center coincide with each other. So according to the definition of electric dipole moment, the dipole moment within the atom is zero. But when an external field is applied, the atomic center of mass does not change as the mass of an atom is mostly due to the contribution from the nucleus. In the other hand, the charge center for positive charge is unchanged; the negative charge center will be shifted in the direction opposite to the direction of applied field. The separation of charge centers means that there is induced electric dipole moment which is termed as polarization. Whenever an atom is said to be polarized, it possesses an effective dipole moment.

The induced dipole moment depends primarily on the electric field causing it. Induced dipole moment is related to applied electric field as

$$P_{ind} = \alpha E \quad (4.11)$$

where α = polarizability of the atom.

Since the polarization of a neutral atom involves the displacement of electrons, α is called electronic polarizability or coefficient of electronic polarization. Almost all atoms possess electronic polarizability as electrons are not rigidly fixed in atom.

Electronic polarizability can be calculated using the following expression (can be derived):

$$m_e \omega_0^2 r = \frac{Ze^2}{4\pi\epsilon_0 r^2} \quad (4.12)$$

$$\alpha_e = \frac{Ze^2}{m_e \omega_0^2} \quad (4.13)$$

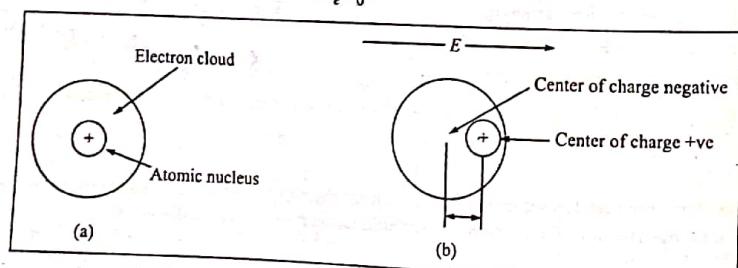


Fig. 4.3 A neutral atom $p = 0$ (a); induced dipole moment $p \neq 0$ (b)

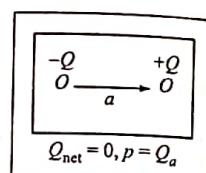


Fig. 4.2 Interpretation of electric dipole moment

Where Z = atomic number
 ω_0 = oscillation frequency of the center of mass of the electron cloud about the nucleus,
 m_e = mass of electron and, e = charge of electron

4.2.2 Polarization Vector and Local Electric Field

When a material is placed in an electric field, the atoms and the molecules of the material become polarized and the dipole moments are distributed in the material as shown in figure 4.4. If we consider only the bulk (fig 4.4.b), the dipoles are aligned head to tail. Every positive charge has a negative charge next to it and vice versa. There is no net charge within the material. But the negative charges of dipoles appearing on the left hand side of the material are not cancelled by positive charge of any dipoles at this face and in the same manner the positive charges of the dipole appearing on the right hand side are also not cancelled by any dipoles. The negative charge and positive charge on the far sides of the material are bound to the surface charge on the plate. So there are positive and negative surface charges on the right side and left side of the material represented here by $+Q_p$ and $-Q_p$ respectively. These charges shown in figure 4.4.c, which are bound, are called surface polarization charges.

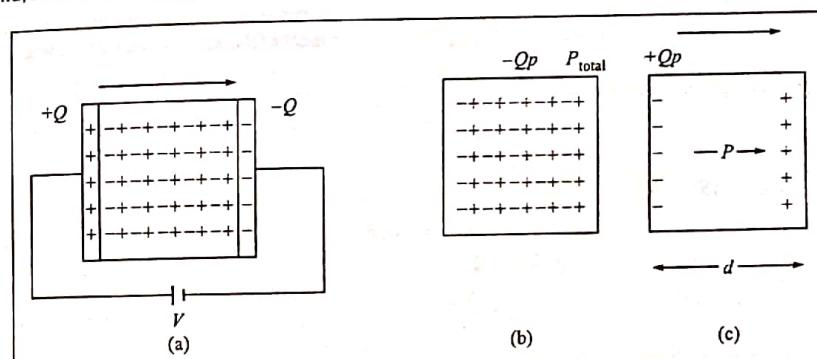


Fig. 4.4 Polarization of dielectric in the influence of applied field (a); Bound molecular polarization (b); Surface polarization (c).

Polarization of a dielectric medium is represented by polarization vector \mathbf{P} and defined as dipole moment per unit volume.

$$P = \frac{\sum_i^N p_i}{V} \quad (4.14)$$

where p_i = dipole moment of i^{th} molecule,
 N = number of molecules within the material,
 V = volume of the dielectric material.

If p_{av} is the average dipole moment per molecule in the dielectric, then an equivalent P is given by

$$P = Np_{av} \quad (4.15)$$

If $+Q_p$ and $-Q_p$ are the surface polarization charges and d the separation between them, then total dipole moment within the material is

$$P_{total} = Q_p d \quad (4.16)$$

Now the polarization is given by

$$P = \frac{P_{total}}{V} \quad (4.16)$$

$$P = \frac{Q_p d}{Ad} \quad (4.17)$$

$$P = \frac{Q_p}{A} \quad (4.18)$$

But as mentioned in equation (4.9), Q_p/A is also the surface polarization charge density. So

$$P = \sigma_p \quad (4.18)$$

Where σ_p = surface polarization charge density.

The charge per unit area appearing on the surface of a polarized medium is equal to the component of the polarization vector normal to this surface. If P_{normal} is the component of P normal to the surface where the polarization charge density is σ_p , then $P_{normal} = \sigma_p$.

Again recalling the expression (4.8)

$$P = \chi_e \epsilon_0 E$$

and

$$P_{ind} = \alpha_e E$$

So that from (4.15)

$$\begin{aligned} P &= Np_{ind} = N\alpha_e E \\ \chi_e \epsilon_0 E &= \chi_e \epsilon_0 E \end{aligned} \quad (4.19)$$

or,

$$\chi_e = \frac{N\alpha_e}{\epsilon_0} \quad (4.20)$$

The relationship (4.20) relates dielectric susceptibility to the electronic polarizability of the atom.

The relative permittivity is related to dielectric susceptibility as

$$\epsilon_r = 1 + \chi_e$$

$$\epsilon_r = 1 + \frac{N\alpha_e}{\epsilon_0} \quad (4.21)$$

The expression (4.21) relates macroscopic property of the material that is relative permittivity to the microscopic property that is electronic polarizability.

The actual field experienced by a molecule in dielectric is defined as local field E_{loc} which depends not only on the free charges on the plates but also on the arrangement of all polarized molecules around this point. The greater the polarization the greater is the local field. In a material with cubic structure or a liquid, the local field is given by

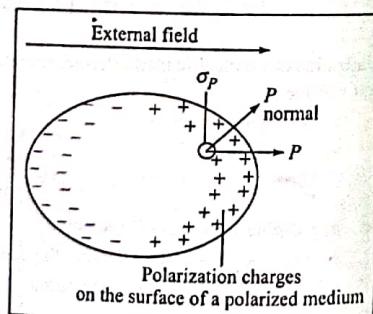


Fig. 4.5 Polarization charge density on the surface of a polarized medium.

$$E_{loc} = E + \frac{P}{3\epsilon_0} \quad (4.22)$$

Induced polarization of molecule depends on this local field rather than applied field E .

$$p_{ind} = \alpha_e E_{loc} \quad (4.23)$$

The fundamental definition of susceptibility given by equation (4.8) is unchanged.

$$P = Np_{ind} = N\alpha_e E_{loc} = N\alpha_e \left(E + \frac{P}{3\epsilon_0} \right)$$

$$\text{or, } P = N\alpha_e E + \frac{N\alpha_e P}{3\epsilon_0}$$

$$\text{or, } P = \frac{N\alpha_e E}{1 - \frac{N\alpha_e}{3\epsilon_0}}$$

But from (4.8), we have

$$P = \chi_e \epsilon_0 E.$$

$$\chi_e \epsilon_0 E = \frac{N\alpha_e E}{1 - \frac{N\alpha_e}{3\epsilon_0}}$$

$$N\alpha_e = \chi_e \epsilon_0 \left(1 - \frac{N\alpha_e}{3\epsilon_0} \right)$$

$$N\alpha_e = \chi_e \left(\frac{3\epsilon_0 - N\alpha_e}{3} \right) = \chi_e \epsilon_0 - \chi_e \frac{N\alpha_e}{3}$$

$$N\alpha_e + \chi_e \frac{N\alpha_e}{3} = \chi_e \epsilon$$

$$N\alpha_e \left(1 + \frac{\chi_e}{3} \right) = \chi_e \epsilon_0$$

$$N\alpha_e \left(\frac{3 + \chi_e}{3} \right) = \chi_e \epsilon_0$$

$$\frac{N\alpha_e}{3\epsilon_0} = \frac{\chi_e}{3 + \chi_e} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \quad (4.24)$$

And, at optical frequencies, ϵ_r is replaced by n^2 , where n is refractive index.

Equation (4.24) is known as Clausius-Mossotti equation, which relates relative permittivity with electronic polarizability more accurately. There are different polarization mechanisms in materials. The different polarization mechanisms are classified as:

- (i) Electronic polarization
- (ii) Ionic polarization

- (iii) Orientational (dipolar) polarization
- (iv) Interfacial polarization

Of the above-mentioned, electronic polarization has been discussed already. So we will be discussing other remaining three types of polarization mechanisms.

4.2.3 Ionic Polarization

Ionic crystals like NaCl, KCl, LiBr, etc witness ionic polarization. The ionic crystals have distinctly distinguishable ions located at well-defined lattice sites so that each pair of oppositely charged neighboring ions has a dipole moment. As shown in the figure 4.6.a, in the absence of applied field, there is no net polarization of the specimen because the dipole moments of equal magnitude are lined up head to head and tail to tail. The dipole moment p_+ in the +ve x direction has the same magnitude as dipole moment p_- in the -ve x direction giving zero net polarization. When external electric field is applied, Cl^- ions are pushed opposite to the direction of applied field and Na^+ ions in the direction of field about their equilibrium positions. Consequently, dipole moment p'_+ in the direction of $+x$ increases to p'_+ and that in the $-x$ direction decreases to p'_- and the net dipole moment is no longer zero. The average dipole moment per ion pair ($p'_+ - p'_-$) in the presence of electric field depends on the field. The ionic polarizability α_i is defined in terms of local field experienced by the ions as

$$P_{av} = \alpha_i E_{loc} \quad (4.25)$$

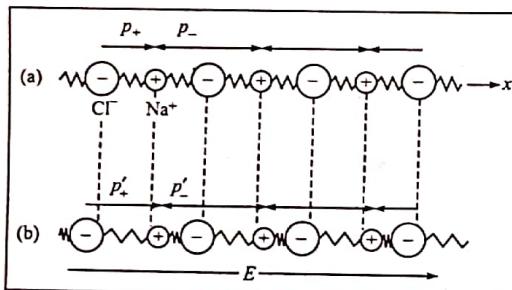


Fig. 4.6 NaCl chain with no applied field (a); in the presence of applied field (b).

Generally α_i is greater than α_e by a factor of 10 or more. Which leads to ionic solids having large dielectric constants. If N_i is the number of ions in the dielectric, then polarization is given by

$$P = N_i P_{av} = N_i \alpha_i E_{loc} \quad (4.26)$$

But

$$E_{loc} = E + \frac{P}{3\epsilon_0} \text{ and } P = \chi_e \epsilon_0 E$$

After some manipulations as in the previous case, we can obtain the familiar Clausius-Massotti equation for ionic polarization.

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{1}{3\epsilon_0} N_i \alpha_i \quad (4.27)$$

Electronic polarization also persists in these solids due to the displacement of negative charge center by the applied field. But in ionic solids their contribution is much smaller and can be neglected.

4.2.4 Orientational (Dipolar) Polarization

Certain molecules like HCl possess permanent dipole moments. However, in the absence of external electric field, these dipole moments are randomly oriented due to thermal agitation and the average dipole moment per molecule in the sample is zero. In HCl the dipole moment denoted by p_0 is directed from Cl^- ion to H^+ ion giving. When an external field E is applied, both the ions experience force in opposite directions. But the nearly rigid bond between these ions holds them together. The molecule thus experiences a torque τ about its center of mass, which tries to rotate dipole p_0 and align with the applied field. If all the molecules were to simply rotate and align with the field, the polarization of the solid would be

$$P = N p_0 \quad (4.28)$$

where N = number of molecules per unit volume,

P = Polarization vector for the specimen.

However, due to the thermal energy, the molecules collide with each other and with the container thereby destroying some of the dipole alignments which means the orientations of dipole moments are randomized. Nonetheless, there is net polarization of the sample which would have been greater had there not been the random orientation of the molecules. This type of polarization is termed as Orientational (dipolar) polarization.

When the dipole will be aligned by the external field, certain amount of work will be done. No work will be done if the dipole is already aligned with the field. Maximum work will be done when the torque has to rotate the field by an angle of 180° . The torque experienced by dipole is

$$\tau = (F \sin \theta)a = (EQ \sin \theta)a = p_0 E \sin \theta \quad (4.29)$$

If we take potential energy equal to zero for $\theta=0$, then its maximum would be for the angle $\theta=180^\circ$, so

$$PE_{max} = \int_0^{180^\circ} (p_0 E \sin \theta) d\theta = p_0 E \quad (4.30)$$

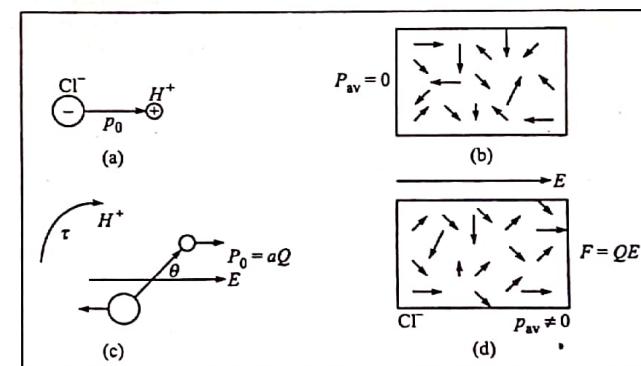


Fig. 4.7 HCl molecule possesses permanent dipole moment (a); Net average dipole moment is zero in the absence of external field (b); Permanent dipole experiencing a torque due to applied field (c); Presence of net average dipole moment per molecule due to the external applied field.

The average dipole potential energy is $PE_{\max}/2$. The average dipole energy along applied field is directly proportional to permanent dipole moment and the ratio of average dipole energy to the average thermal energy of the dipole gives the average dipole moment per molecule

$$p_{av} \propto p_0 \frac{P_0 E}{5kT/2} \quad (4.31)$$

where $5kT/2$ is the average thermal energy per molecule (for 5 degrees of freedom).

But by using Boltzmann statistics (see Appendix-2), we would find the average dipole moment per molecule as given by

$$p_{av} = \frac{p_0^2 E}{3kT} \quad (4.32)$$

As we already defined dipole moment as proportional to the applied field, so from equation (4.32), we can introduce the proportionality constant termed as dipolar or orientational polarizability and given by

$$\alpha_d = \frac{p_0^2}{3kT} \quad (4.33)$$

Dipolar (orientational) polarization is strongly temperature dependent and decreases with increase in temperature and so is the relative permittivity. Polar liquids such as water, alcohol and various electrolytes and polar gases such as gaseous HCl and steam exhibit this type of polarization.

4.2.5 Interfacial Polarization

Whenever there is accumulation of charge at an interface between two materials or two regions, there is interfacial polarization. The accumulation of charge near the dielectric in one of the electrodes takes place as shown in figure. There are always some defects and impurities within the dielectric material, which contribute, free charge carriers such as electron and holes. In a dielectric material like ceramic or glass, there are always equal numbers of positive and negative ions but positives ions like H^+ and Li^+ are far more mobile than the negative ions in the structure. Under the influence of the applied field, these positive ions migrate near to the negative electrode. These positive ions cannot leave the dielectric material and enter the crystal structure of metallic electrode. So, these positive ions simply pile up at the interface and give

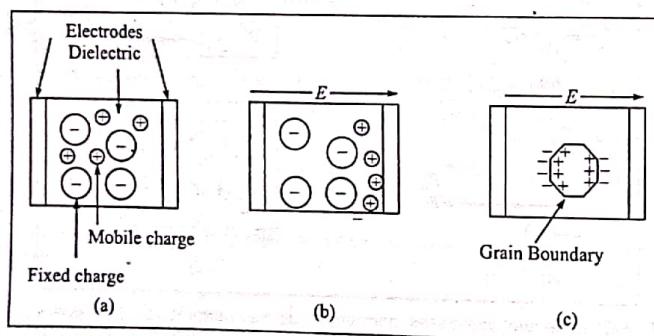


Fig. 4.8 No polarization (a); Net polarization with the field (b) and (c)

rise to a positive space charge near the negative electrode. These positive charges at the interface attract more electrons to the negative electrode and this additional charge on the electrode appears as an increase in the dielectric constant of the material. The positive charge near the interface with the electrode and negative charge that remain in the bulk together constitute dipole moments that appear in the polarization vector P and so the polarization due to this is termed as interfacial polarization.

Trapping of electrons or holes at the crystal defects near the interface between electrode and the dielectric is another example of interfacial polarization. The mobile charge carriers drifting under the influence of external field try to reach the electrodes but are trapped at these defects. Grain boundaries also lead to interfacial polarization, as the mobile charges cannot move beyond the grain boundaries, so give rise to the polarization.

The induced dipole moment per molecule in the presence of electronic, ionic and orientational polarizations is given by

$$p_{av} = \alpha_e E_{loc} + \alpha_i E_{loc} + \alpha_d E_{loc} \quad (4.34)$$

Each effect simply adds to the net dipole moment per molecule the fact that is verified by experiments. But the interfacial polarization cannot be simply added to the above because it occurs at the interfaces and cannot be put into an average dipole moment per molecule in the bulk. The local fields also cannot be simply approximated for Orientational polarization. So the dielectric constant under electronic and ionic polarization mechanisms can be found from

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N_e \alpha_e + N_i \alpha_i}{3\epsilon_0} \quad (4.35)$$

Table 4.1 Typical examples of polarization mechanisms

Polarization	Example	Static ϵ_r	Comments
Electronic	Ar gas	1.0005	
Electronic	Ar liquid ($T < 87.3$ K)	1.53	
Electronic	Si crystal	11.9	Due to valence electrons, bond polarization
Ionic	NaCl crystal	5.9	
Ionic	CsCl crystal	7.2	
Dipolar	Water	80	
Dipolar	PVC	7	Partly hindered in solid

4.3 Dielectric Losses, Frequency and Temperature Effect

The static dielectric constant is the effect of polarization under dc conditions. When the applied field is sinusoidal ac, polarization of material under these conditions leads to the ac dielectric constant, which is generally different from the dc. As an example we will consider the Orientational polarization in dipolar liquids where the varying field tries to change the alignment of dipoles in one and the other way. If the instantaneously induced dipole moment per molecule can readily follow the field variations, then at any instant the dipole is

$$p = \alpha_d E \quad (4.36)$$

where the orientational polarizability has its expected maximum value from the dc conditions, that is,

$$\alpha_d = p_0^2 / 3kT \quad (4.37)$$

There are two factors opposing the immediate alignment of dipoles with the applied field. First, the thermal agitation tries to randomize the dipole orientations. Second, the molecules rotate in the viscous medium by virtue of their interactions with neighbors, which is particularly strong in liquids and solids. So the dipoles cannot respond instantaneously to the changes in the field. At low frequencies the dipoles can respond with the change in applied field, so dipolar polarizability is maximum in this region. In high frequency range, the dipoles cannot respond to the change in applied field, consequently the high frequency field cannot polarize the material means it the orientational polarizability is zero.

In the figure above, the dielectric is applied by a dc field E_0 for long time before the field is reduced to E . $\alpha_d(0)$ corresponds to polarizability at zero frequency. The induced dipole moments are also reduced as shown in figure 4.9. The decrease in dipole moments is achieved by random collision between molecules. Assume τ as average relaxation time between molecular collisions. If p is the induced dipole moment, then $p - \alpha_d(0)E_0$ is the excess moment which must disappear as t tends to infinity. τ is the average time to eliminate the excess dipole moment. Rate of change of dipole moment is

$$\frac{dp}{dt} = -\frac{p - \alpha_d(0)E}{\tau} \quad (4.38)$$

The above equation can be used to describe dipolar polarizability in ac conditions. Electric field in ac condition is given by

$$E = E_0 \sin \omega t = E_0 \exp(j\omega t) \quad (4.39)$$

From equation (4.38) and (4.39), we have

$$\begin{aligned} \frac{dp}{dt} &= -\frac{p + \alpha_d(0)}{\tau} E_0 \exp(j\omega t) \\ p &= \alpha_d(0) E_0 \exp(j\omega t) \end{aligned} \quad (4.40)$$

where

$$\alpha_d(\omega) = \frac{\alpha_d(0)}{1 + j\omega\tau} \quad (4.41)$$

$\alpha_d(\omega)$ represents dipolar orientational polarizability under ac conditions. Equation (4.40) indicates that dipole moments and applied field are out of phase. So is the polarization vector $P = Np$ and E . At low frequencies, when $\omega\tau \ll 1$, $\alpha_d(\omega)$ is nearly equal to $\alpha_d(0)$, and dipole moment p is in phase with the applied field., the rate of relaxation, $1/\tau$, is much faster than the frequency of applied field or the rate at which the polarization is being changed, so p closely follows E_0 . At higher frequencies i.e. $\omega\tau \gg 1$, the rate of relaxation is very slow than the frequency of applied field and p can no longer follow E , so the polarization is zero.

From the complex dipolar polarizability, we can obtain the complex dielectric constant as polarizability is related to dielectric constant. So, the dielectric constant in ac is written as

$$\epsilon_r = \epsilon'_r - j\epsilon''_r \quad (4.42)$$

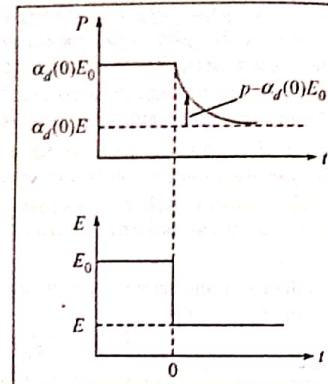


Fig. 4.9 When dc field is suddenly changed from E_0 to E at $t = 0$, the dipole moment p changes from $\alpha_d(0)E_0$ to $\alpha_d(0)E$

$$\text{where } \epsilon'_r = \epsilon_{rm} + \frac{\epsilon'_r(0) - \epsilon'_{rm}}{1 + \omega^2 \tau^2} \text{ and } \epsilon''_r = \frac{\epsilon''_r(0) - \epsilon''_{rm}}{1 + \omega^2 \tau^2}$$

where ϵ'_r is the real part and ϵ''_r is the imaginary part, and both are frequency dependent.

ϵ'_{rm} and $\epsilon'_r(0)$ is dielectric constant at high and low frequencies. As shown in the figure 4.10, the real part decreases from maximum value $\epsilon'_r(0)$ at dc corresponding to $\alpha_d(0)$, to unity at high frequencies when $\alpha_d(\omega) = 0$ as ω tends to infinity. The imaginary part ϵ''_r is zero at low and high frequencies and peaks when $\omega\tau = 1$ or $\omega = 1/\tau$. The real part of the capacitance will be used to calculate the capacitance whereas the imaginary part will be used to calculate the energy loss in the dielectric under ac conditions. The energy loss is due to the random collision of dipoles under varying fields.

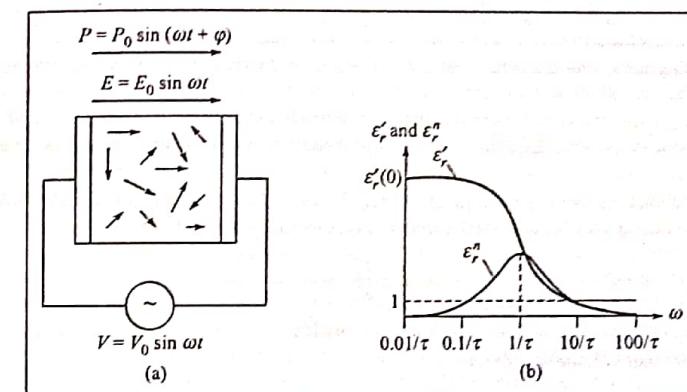


Fig. 4.10 AC field applied to dipolar medium (a). Relative permittivity in ac (b)

For a capacitor shown in figure 4.11.a, the admittance can be calculated as

$$Y = \frac{j\omega A \epsilon_0 \epsilon_r(\omega)}{d} = \frac{j\omega A \epsilon_0 \epsilon'_r(\omega)}{d} + \frac{\omega A \epsilon_0 \epsilon''_r(\omega)}{d} \quad (4.43)$$

or,

$$Y = j\omega C + G_p \quad (4.44)$$

where

$$C = \frac{A \epsilon_0 \epsilon'_r(\omega)}{d} \quad (4.45)$$

$$G_p = \frac{\omega A \epsilon_0 \epsilon''_r(\omega)}{d} \quad (4.46)$$

C represents the capacitance of the capacitor and depends on the real part of dielectric constant, and G_p is called conductance of the capacitor and represents the power loss in the capacitor under varying electric field. The capacitor can be represented by an ideal capacitor with capacitance C and a resistor with resistance $R = 1/G_p$ in parallel to represent the power loss in the dielectric medium.

Here there is no real power dissipation in C but there is real power dissipation in $G_p(R_p)$. The input power is given by

$$P_{in} = VI = V(VY) = j\omega CV^2 + V^2 G_p \quad (4.47)$$

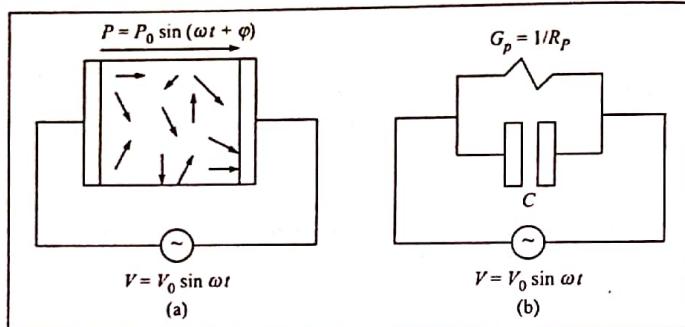


Fig. 4.11 Capacitor with dielectric medium (a), equivalent circuit diagram of real capacitor (b)

So the power dissipated in the dielectric medium depends only on the imaginary part of dielectric constant and is maximum when ϵ_r'' is maximum as G_p is directly proportional to ϵ_r'' as seen from expression (4.46).

For any capacitor, we should try to minimize ϵ_r'' for given ϵ_r' . The relative magnitude of ϵ_r'' with respect to ϵ_r' is defined through $\tan \delta$, called loss tangent or loss factor, given as

$$\tan \delta = \frac{\epsilon_r''}{\epsilon_r'} \quad (4.48)$$

which is frequency dependent and peaks just beyond $\omega=1/\tau$.

Power loss per unit volume is expressed as

$$P_{vol} = \frac{V^2 G_p}{V_{volume}} = V^2 \frac{\omega A \epsilon_0 \epsilon_r''}{d} \frac{1}{dA}$$

$$P_{vol} = \frac{V^2}{d^2} \omega \epsilon_0 \epsilon_r'' = E^2 \omega \epsilon_0 \epsilon_r'' = E^2 \omega \epsilon_0 \epsilon_r' \tan \delta \quad (4.49)$$

From equation (4.49), it is clear that power loss per unit volume is the function of frequency, applied field and loss tangent.

We have obtained the frequency dependence of dielectric loss for dipolar orientational polarization. But this can be extended to other forms of polarization as well. The frequency dependence of dielectric loss is shown in figure 4.12 below.

The dependence of dielectric constant on temperature can be seen from expressions relating polarizability to the temperature. In most of the cases, increased temperature indicates toward increased thermal agitation of molecules, which always hinders the orientation of dipoles within the material. So the dielectric constant is smaller at higher temperature compared to that at lower temperature. The dielectric strength will be lower at higher temperature than at lower one.

4.4 Dielectric Breakdown

A defining property of dielectric material is not only its ability to increase capacitance but also and equally important is its insulating behavior. Dielectric materials are widely used as insulating media between

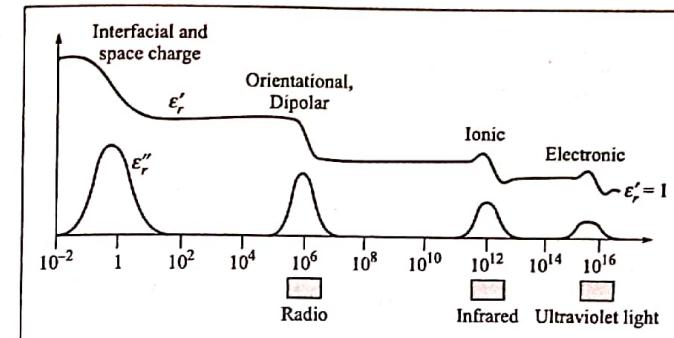


Fig. 4.12 Frequency dependence of real and imaginary parts of dielectric constant.

conductors at different voltages to prevent ionization of air and current flashovers between conductors. The voltage across conductors and hence within the dielectric cannot be increased without limit. A voltage will be eventually reached when substantial current starts flowing through the dielectric, which may lead to what is called **dielectric breakdown**. In gases and liquid, the breakdown is not permanent but in solids, there can be permanent damage due to the breakdown. The maximum electric field that can be applied to the insulating medium without causing dielectric breakdown is called **dielectric strength** of the material and is denoted by E_{br} . The dielectric strength of solid depends on molecular structure, impurities in the material, sample geometry, nature of the electrodes, temperature, ambient conditions, duration and frequency of applied field, and is different in ac and dc conditions. The ageing also affects the dielectric strength. Dielectric breakdown in solids, liquids and gases will be discussed separately.

4.4.1 Dielectric Breakdown in Gases

Due to cosmic radiation, there are always a few free electrons in a gas. If the field is sufficiently large, then one of these electrons can be accelerated to sufficiently large kinetic energies to impact ionize a neutral gas molecule and produce additional free electron and a positively charged gas ion. Both the first and liberated electrons are available to accelerate in the field and again impact-ionize more neutral gas molecules, and so on. The breakdown in gases depends upon pressure. Higher pressure means greater concentration of molecules and the reduced mean separation between molecules, hence the shorter mean free path of free electrons. This means free electrons should be accelerated by larger energies to impact ionize the neutral gas molecule. So, E_{br} is greater at higher pressure.

When a local region only is exhibiting discharge so that the discharge does not directly connect two electrodes, it is said that a partial discharge has occurred. For a high voltage cylindrical conductor above a grounded electrode, the electric field is highest near the surface of the conductor facing ground. This field initiated discharge locally because the field is sufficiently strong to continue the electron avalanche. But away from the conductor due to weak field, there will be no electron avalanche. Such local discharge is termed as corona discharge. Voids and cracks within the solid dielectric material and discontinuities at dielectric-electrode interface can also lead to partial discharge since the electric field in these voids and cracks are stronger than in dielectric material itself. Partial discharges in these voids and cracks erode the surrounding region of the dielectric both physically and chemically, which eventually can lead to major breakdown if not controlled in proper time.

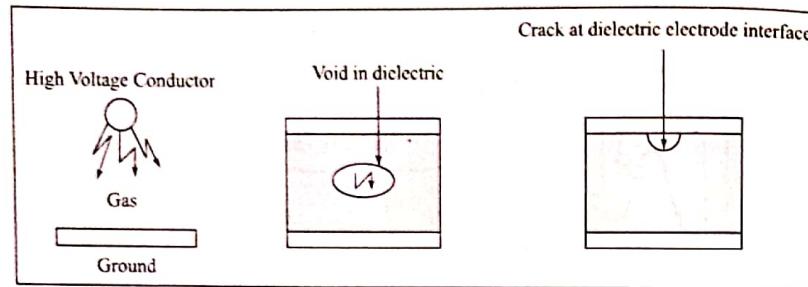


Fig. 4.13 Partial discharges in gases

4.4.2 Dielectric Breakdown in Liquids

The process that leads to the breakdown in liquid dielectric is not as clear as that in gases. In impure liquids with small particles in suspension, it is believed that these impurities coalesce end to end to form a conducting bridge between electrodes thereby give rise to breakdown. Sometimes discharge initiates as partial discharge in gas bubbles entrapped in the liquids. Moisture absorption and gas absorption deteriorates the dielectric strength of liquids. In oil in a long run, oxidation produces acidic products and some parts may become more conducting by this and can give rise to discharge. Electron emission from electrodes at high field also can initiate discharge in liquids.

4.4.3 Dielectric Breakdown in Solids

There are various mechanisms of dielectric breakdown in solids such as intrinsic breakdown, thermal breakdown, breakdown due to insulation aging, electromechanical breakdown, etc.

In intrinsic breakdown, which is also called electronic breakdown, an electron in the conduction band of dielectric can be accelerated to sufficiently large energy to impact ionize host atom. The collision of electron with energy more than the band gap energy with lattice vibrations is capable of exciting electron from valence band to conduction band, and, then further, both the primary and secondary electrons will impact ionize other host atoms thereby generating an electron avalanche effect. SiO_2 films with no microstructural defects in MOS capacitors exhibits intrinsic breakdown. Although the theoretical limit for $E_{br} \sim 1\text{MV cm}^{-1}$, due to micro-structural defects in reality there are lower values for breakdown.

Finite conductivity of the insulation means there is a Joule heat σE^2 being released within the solid. At high frequencies, dielectric loss $V^2 \omega \tan \delta$ is high. If the heat generated in the solid cannot be removed quickly by thermal conduction or other means, the temperature of dielectric increases and leads to higher electrical conductivity, which will cause more Joule heating. Certain local area can become hot spots and if there are many such hot spots in the sample there can be permanent conducting channel connecting two electrodes and hence a thermal breakdown of dielectric. At the same temperature the breakdown field can be drastically reduced if the duration is increased.

The basis of electromechanical breakdown is the force experienced by dielectric due to oppositely charged plates between which it is placed. The compressive force increases with the increase in voltage (hence the electric field). The thickness of the dielectric is reduced which will increase the charge on the electrode as given by $Q = CV = \epsilon_0 \epsilon_r AV/d$. This will lead to further increase in compressive force and decrease in thickness. This process will continue till the dielectric deforms plastically causing the insulation

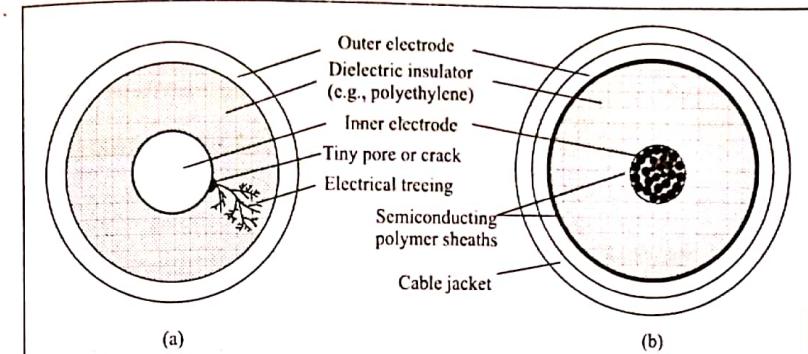


Fig. 4.14 Electrical treeing (a), high voltage coaxial cable with semiconductor sheath (b)

to breakdown by increase in field. The increase in E and decrease in d produces more Joule heat and dielectric loss heat causing more problems for mechanical stability of material. It is possible to reach thermal breakdown due to mechanical deformation. There is possibility of internal cracks being developed and these cracks experience shear forces, which combined with large electric field, can cause crack propagation and eventually mechanical and hence dielectric failure.

Internal discharges are the partial discharges that take place in micro-structural voids, cracks, or pores within dielectric. Initially the pores size can be small and the partial discharges insignificant, but with time partial discharges erode the internal surface of the void by locally melting the insulator. And, eventually an electrical tree type discharge is developed. The erosion of dielectric by partial discharge propagates like a branching tree, which are erosion channels in which gaseous discharge takes place and forms a conducting channel. In high voltage cables, to avoid this problem, the electrode is sheathed by an intermediate semiconducting material.

With the time of use the properties of insulating materials get degraded. And eventually dielectric breakdown takes place at fields below the values established by experiments. Aging determines the useful life of the insulating material. Although there are many factors which directly or indirectly affect the properties and performance of insulating material, we here are limited just to the fact that with time these materials loose their useful properties and the breakdown is eventual outcome.

4.5 Ferroelectricity and Piezoelectricity

4.5.1 Ferroelectricity

Certain crystals like Barium titanate (BaTiO_3) which have permanent polarization even in the absence of an applied field are called ferroelectric crystals and the phenomenon ferroelectricity. At above 130°C , the crystal structure of BaTiO_3 has cubic structure and center of mass of positive and negative charges coincide and the net polarization is zero. Below 130° , BaTiO_3 has tetragonal structure and center of mass of positive and negative charges do not coincide. So there is polarization vector P as there is separation between negative and positive charge centers. The temperature above which the ferroelectricity is lost is called Curie temperature T_c . Below this the whole specimen becomes spontaneously polarized and above T_c , disorderedness of dipole increases which is called pyroelectric state. By applying temporary field and

letting the crystal cool below T_c , we can induce spontaneous polarization to develop along the field direction. So, we can define the c axis shown in figure 4.15.b by imposing a temporary external field. The c axis is called ferroelectric axis. The dielectric strength of the material is different for the field applied along a-axis and c-axis. Ferroelectric crystals have large dielectric constants. Along a-axis the dielectric constant is ~4100 compared to that along c which is ~160.

Above T_c the dielectric constant usually follows a Curie-Weiss behavior which is given by

$$\epsilon_r = \frac{C}{T - T_0},$$

where C is Curie constant and T_0 is Curie-Weiss temperature.

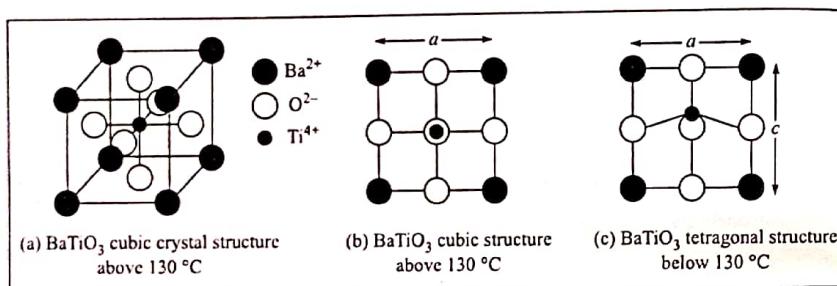


Fig. 4.15 BaTiO₃ at different temperatures giving different dielectric properties.

The variation of polarization with electric field is not linear for such crystals but forms a closed loop called the hysteresis loop which is similar to the ferromagnetic loop as discussed in chapter-5. The ferroelectric crystals may be classified into two main groups: the order-disorder and displacive group. In order-disorder group, the ferroelectric transition is associated with individual ordering of ions. These crystals contain hydrogen bond and in which the motion of protons is related to ferroelectric properties. Examples of such materials are KH₂PO₄, RbH₂PO₄ etc. The displacive group of ferroelectric materials is the one in which the ferroelectric transition is associated with displacement of a whole sub-lattice of ions of one type relative to a sub-lattice of another type. Few examples of such type of ferroelectric materials are BaTiO₄, PbTiO₃, KNbO₃, LiTiO₃, etc.

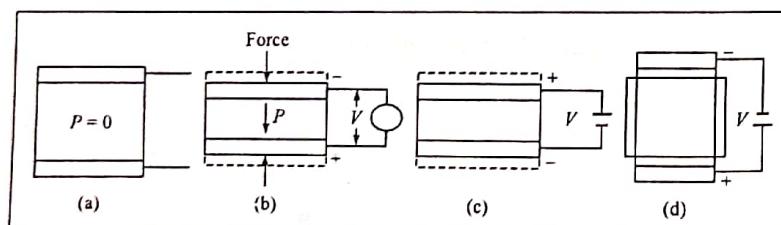


Fig. 4.16 Piezoelectric crystal with no applied field or stress (a), crystal strained by applied force (b), an applied field causes crystal to be strained (c), and direction of strain changes with change in applied field direction (d).

4.5.2 Piezoelectricity

Certain crystals become polarized under mechanical stress. Also, when an electric field is applied to these crystals, they will be mechanically deformed. These two effects are complementary and termed as piezoelectricity.

Only certain crystals exhibit piezoelectricity because the phenomenon requires a special crystal structure—that which has no center of symmetry. The NaCl unit cell has cubic symmetry and there is no net polarization when there is no force applied and the specimen is still not polarized after it has been applied by external force as shown in figure 4.17.

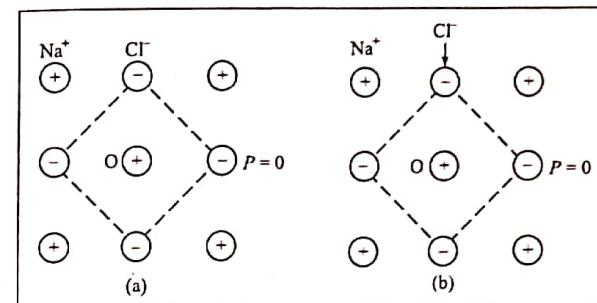


Fig. 4.17 Without external force (a) and with external force applied (b)

There are crystals that have no center of symmetry. The unit cell of such crystals is called noncentrosymmetric. In these crystals as shown in figure 4.18, in the absence of external force, the center of mass of both the positive and negative charges coincide. But when an external force is applied, the centers of masses of both the charges are shifted and there is now net polarization of the specimen. The direction of the induced polarization depends upon the direction of applied stress. If T_j is the applied mechanical stress along j direction and P_i is the induced polarization, then they are related as

$$P_i = d_{ij} T_j \quad (4.50)$$

where d_{ij} is called piezoelectric coefficient.

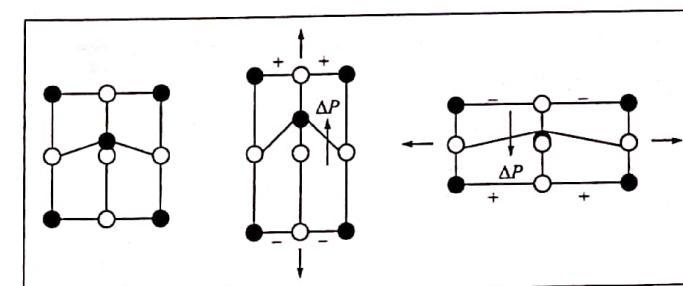


Fig. 4.18 Without applied force (a), Under force in y-direction P is along y (b), and under force in x-direction P is along x as well but in addition to x , P can be in y also.

The converse piezoelectric effect between an induced strain S_i along j -direction and applied electric field E_i along i -direction is given by

$$S_i = d_{ij} E_i \quad (4.51)$$

where the coefficient d_{ij} has the same meaning as before.

The electromechanical coupling between electrical and mechanical energies is denoted by coupling factor k and is defined in terms of k^2 as

$$k^2 = \frac{\text{Electrical energy converted to mechanical energy}}{\text{Electrical Input Energy}} \quad (4.52)$$

or conversely

$$k^2 = \frac{\text{Mechanical energy converted to electrical energy}}{\text{Mechanical Input Energy}} \quad (4.53)$$

All ferroelectric materials are piezoelectric but not all the piezoelectric materials are ferroelectric.

If C and C_0 be the mechanical and electrical capacitances, then

$$k^2 = \frac{\frac{1}{2}CV^2}{\frac{1}{2}CV^2 + \frac{1}{2}C_0V^2} = \frac{C}{C+C_0} = 1 - \frac{f_r^2}{f_a^2} \quad (4.54)$$

where f_r and f_a are resonance and anti-resonance frequencies and are measured when impedance is minimum and maximum respectively.

Table 4.2 Piezoelectric materials with their d and k values

Crystal	d, mV^{-1}	k	Application
Quartz (crystal SiO_2)	2.3×10^{-12}	0.1	Crystal oscillators, ultrasonic transducers, filters
Barium Titanate BaTiO_3	190×10^{-12}	0.49	Accelerometers
PZT (lead zirconate titanate)	480×10^{-12}	0.72	Earphones, microphones, spark generators, displacement transducers
Polyvinylidene fluoride (PVDF)	18×10^{-12}	-	Used in wide range, very cheap

4.6 Properties of Some Dielectric Materials

Table 4.3 Dielectric strength at room temperature for certain insulating material

Dielectric medium	Dielectric strength at 60Hz	Comments
Air at 1atm	31.7 kV cm^{-1}	Breakdown by electron avalanche by impact ionization
SF_6 gas	79.3 kV cm^{-1}	Used in high voltage circuit
Polybutene	$>138 \text{ kV cm}^{-1}$	Liquid dielectric used as oil filler
Transformer oil	128 kV cm^{-1}	Used for insulating medium in transformer, high voltage circuit breakers
Amorphous SiO_2	10 MV cm^{-1} dc	Very thin oxide films without defect. Intrinsic breakdown limit
Borosilicate glass	10 MV cm^{-1} duration $10\mu\text{s}$	Intrinsic and thermal breakdown
Polypylene	$295-314 \text{ kV cm}^{-1}$	Thermal breakdown or electrical treeing

Table 4.4 Experimentally determined permanent dipole moments ($\times 3.33 \times 10^{-30} \text{ C-m}$)

Molecule	p_0	Molecule	p_0	Molecule	p_0
NO	0.1	HBr	0.79	H_2O	1.84
CO	0.11	NO_2	0.4	H_2S	0.93
HCl	1.04	HI	0.38	CH_3Cl	1.15

The real part of relative dielectric constant and the loss tangent of various dielectrics at a number of frequencies are presented in table 4.5 below.

Table 4.5

Material	ϵ'	Frequency, Hz				
		10^2	10^4	10^6	10^8	3×10^9
Pyranol 1467	ϵ'	4.42	4.0	4.4	4.08	2.84
	$10^4 \times \tan \delta$	36	4	25	1300	1200
Cab-Oil 5314	ϵ'	2.25	2.25			2.22
	$10^4 \times \tan \delta$	3	0.4			18
Teflon	ϵ'	2.1	2.1	2.1	2.1	2.1
	$10^4 \times \tan \delta$	5	3	2	2	1.5
Polystyrene	ϵ'	2.59	2.56	2.56	2.55	2.55
	$10^4 \times \tan \delta$	0.5	0.5	0.7	1.0	3.3
Polyethylene	ϵ'	2.25	2.25	2.25	2.25	2.25
	$10^4 \times \tan \delta$	5	3	4		3
Nylon 66	ϵ'	3.88	3.6	3.33	3.16	3.03
	$10^4 \times \tan \delta$	144	233	257	210	128
Bakelite BM-120	ϵ'	4.87	4.62	4.36	3.95	3.7
	$10^4 \times \tan \delta$	300	200	280	380	438
Glass (Corning 0010)	ϵ'	6.68	6.57	6.43	6.33	6.1
	$10^4 \times \tan \delta$	77	35	16	23	360
Porcelain No. 4462	ϵ'	8.99	8.95	8.95	8.95	8.9
	$10^4 \times \tan \delta$	22	6	2	4	11

Table 4.6 Static and Optical Dielectric Constants for Alkali Halides

Solid	ϵ_{rs}	ϵ_{ro}	Solid	ϵ_{rs}	ϵ_{ro}
LiF	9.27	1.92	KF	6.05	1.85
LiCl	11.05	2.75	KCl	4.68	2.13
LiI	11.03	3.8	KI	4.94	2.69
NaCl	5.62	2.25	NaF	6.0	1.74
RbI	5.0	2.63	RbBr	5	2.33
KBr	4.78	2.33	RbF	5.91	1.93
RbCl	5.0	2.19	LiBr	12.1	3.16
NaBr	5.99	2.62	Nal	6.6	2.91

ϵ_{rs} - static dielectric constant

ϵ_{ro} - optical dielectric constant

4.7 Solved Numerical Examples

1. The electronic polarizability of Ar atom is $1.7 \times 10^{-40} \text{ Fm}^2$. Calculate the static dielectric constant of solid Ar given its density is 1.8 g cm^{-3} , $M_{\text{Ar}} = 39.95 \text{ g mol}^{-1}$ and $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$

Solution:

Number of Ar atoms per unit volume is

$$N = \frac{N_A \rho}{M_{\text{Ar}}} = \frac{(6.02 \times 10^{23} \text{ mol}^{-1})(1.8 \text{ g cm}^{-3})}{39.95 \text{ g mol}} = 2.71 \times 10^{22} \text{ cm}^{-3}$$

$$\text{Now, } \epsilon_r = 1 + \frac{N\alpha_e}{\epsilon_0} = 1 + \frac{(2.71 \times 10^{22} \text{ m}^{-3})(1.7 \times 10^{-40} \text{ Fm}^2)}{8.85 \times 10^{-12} \text{ Fm}^{-1}}$$

If we use Clausius-Massotti equation, then

$$\epsilon_r = \frac{1 + \frac{2N\alpha_e}{3\epsilon_0}}{1 - \frac{N\alpha_e}{3\epsilon_0}} = 1.63 \text{ which is higher than the value found without using Clausius-Massotti equation.}$$

To find the correct value we should use Clausius-Massotti equation.

2. The number of electrons per unit volume of Si is $5 \times 10^{28} \text{ m}^{-3}$ and its dielectric constant is 11.9. Calculate:

- a. Electronic polarizability due to valence electrons per Si atom (if one could portion the observed crystal polarization to individual atom);
- b. If the Si crystal sample is electroded on opposite faces, by how many times the local field is greater than the applied field?

Solution:

- a. Using Clausius-Massotti equation,

$$\alpha_e = \frac{3\epsilon_0(\epsilon_r - 1)}{N(\epsilon_r + 2)} = \frac{3(8.854 \times 10^{-12} \text{ Fm}^{-1})(11.9 - 1)}{5 \times 10^{28} \text{ m}^{-3}(11.9 + 2)} = 4.17 \times 10^{-40} \text{ Fm}^2$$

which is larger than electronic polarizability of an isolated Ar atom with larger number of electrons.

- b. The local field is $E_{\text{loc}} = E + \frac{P}{3\epsilon_0}$ And, $P = \chi_e \epsilon_0 E = (\epsilon_r - 1) \epsilon_0 E$

Substituting for P

$$E_{\text{loc}} = E + \frac{(\epsilon_r - 1)\epsilon_0 E}{3\epsilon_0} = E \left(1 + \frac{\epsilon_0(\epsilon_r - 1)}{3\epsilon_0}\right)$$

$$\frac{E_{\text{loc}}}{E} = 1 + \frac{11.9 - 1}{3} = 4.63$$

So, the local field is 4.63 times greater than the applied field.

3. CsCl crystal has one Cs^+ and one Cl^- pair of ions per unit cell and a lattice parameter $a = 0.412 \text{ nm}$. The electronic polarizability of Cs^+ and Cl^- ions are $3.35 \times 10^{-40} \text{ Fm}^2$ and $3.4 \times 10^{-40} \text{ Fm}^2$ respectively. The mean ionic polarizability per ion pair is $6 \times 10^{-40} \text{ Fm}^2$. Calculate the dielectric constant at low frequencies and what would its value at optical frequencies?

Solution:

Volume of the unit cell $V = a^3 = (0.412 \times 10^{-9} \text{ m})^3$

Since one unit cell contains one ion pair, the number of ions per unit volume can be calculated as

$$N_i = \frac{1}{V} = 1.43 \times 10^{28} \text{ m}^{-3}$$

N_i is the concentration of cation and anion separately. So, using Clausius-Massotti equation,

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{1}{3\epsilon_0} (N_i \alpha_e (\text{Cs}^+) + N_i \alpha_e (\text{Cl}^-) + N_i \alpha_i)$$

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{(1.43 \times 10^{28} \text{ m}^{-3})(3.35 \times 10^{-40} + 3.4 \times 10^{-40} + 6 \times 10^{-40}) \text{ Fm}^2}{3 \times 8.854 \times 10^{-12} \text{ Fm}^{-1}}$$

And solving for dielectric constant we get $\epsilon_r = 7.56$

At optical frequencies, the ionic polarization is too sluggish to allow polarization to contribute to ϵ_r . Thus the dielectric constant will be determined only by electronic polarization and given by

$$\frac{\epsilon_{ro} - 1}{\epsilon_{ro} + 2} = \frac{1}{3\epsilon_0} (N_i \alpha_e (\text{Cs}^+) + N_i \alpha_e (\text{Cl}^-))$$

$$\frac{\epsilon_{ro} - 1}{\epsilon_{ro} + 2} = \frac{(1.43 \times 10^{28} \text{ m}^{-3})(3.35 \times 10^{-40} + 3.4 \times 10^{-40}) \text{ Fm}^2}{3 \times 8.854 \times 10^{-12} \text{ Fm}^{-1}}$$

And solving for ϵ_{ro} , we will get $\epsilon_{ro} = 2.71$.

The experimental values of dielectric constants at low and optical frequencies are 7.2 and 2.62 respectively.

4. Calculate the heat generated per second due to dielectric loss per cubic centimeter of XLPE and alumina Al_2O_3 at 60 Hz and 1MHz at a field of 100 kV cm^{-1} . ϵ_r' and $\tan \delta$ for XLPE are 2.3 and 3×10^{-4} and for alumina – 8.5 and 10^{-3} respectively at 60 Hz. Similarly at 1 MHz, these values are 2.3 and 4×10^{-4} and 8.5 and 10^{-3} respectively.

Solution:

Power dissipated per unit volume is given by

$$P_{\text{vol}} = \omega E^2 \epsilon_0 \epsilon_r' \tan \delta$$

For XLPE at 60Hz, $P_{\text{vol}} = 230.3 \text{ Wm}^{-3}$ And, similarly for alumina at 60Hz,

$$P_{\text{vol}} = 5 \text{ Wm}^{-3}$$

At 1 MHz the corresponding power losses per unit volume are 2840 Wm^{-3} for XLPE and 330 Wm^{-3} for alumina.

5. The optical index of refraction and the dielectric constant for glass are 1.45 and 6.5 respectively. Calculate the percentage of ionic polarizability.

Solution:

We have from Clausius-Massotti equation

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N(\alpha_e + \alpha_i)}{3\epsilon_0}$$

At optical frequencies, we have

$$\frac{n^2 - 1}{n^2 + 2} = \frac{N\alpha_e}{3\epsilon_0}$$

Now, dividing the second relation by the first, we have

$$\frac{n^2 - 1}{n^2 + 2} \times \frac{\epsilon_r + 2}{\epsilon_r - 1} = \frac{\alpha_e}{\alpha_0 + \alpha_i}$$

So, percentage of ionic polarizability is

$$\begin{aligned}\frac{\alpha_i}{\alpha_e + \alpha_i} \times 100 &= \left[1 - \frac{\alpha_e}{\alpha_e + \alpha_i} \right] \times 100 \\ &= \left[1 - \left(\frac{1.1025}{4.1025} \right) \left(\frac{8.5}{5.5} \right) \right] \times 100 = 57.98\%\end{aligned}$$

Exercise

- A glass dielectric has dielectric constant 2.6 and loss tangent 7×10^{-5} at 1MHz. Calculate the loss of power per unit volume if the signal applied to the glass has peak amplitude of 0.71V/m. [5.05 $\times 10^{-9}$ W/m³]
- Calculate dielectric constant of a lead ceramic zirconate capacitor at 215°C if its capacitance is 2×10^{-9} F in pellet form, with radius 0.8cm and thickness 1.5mm. [1686.82]
- Prove that heat loss in a dielectric is half its maximum value at which imaginary part of the relative permittivity is maximum using single relaxation time of dipoles.
- Calculate the dielectric constant of Barium Strontium Titanate at 100°C if its Curie-Weiss temperature is 71°C and Curie constant 1.3×10^5 C. [4482.76]
- Determine the percentage of ionic polarizability of water for which refractive index and dielectric constant are 1.33 and 8.1 respectively. [71%]
- Determine real and imaginary parts of dielectric constant of a dielectric sample at 1kHz if the relaxation time for the material is 10^{-6} s. Given that the dielectric constant at low frequency and high frequency are 8 and 2 respectively. [7.99, 0.038]