

5

Chapter

Magnetic Materials

5.1 Ferromagnetism, Ferrimagnetism and Paramagnetism

Many electrical engineering devices such as inductors, transformers, rotating machines, etc are based on the utilization of magnetic properties of materials. To understand magnetic materials and magnetism properly we need to know the following quantities first.

5.1.1 Magnetic Dipole Moment

A current carrying loop with A as enclosed area has a current I circulating through it as shown in figure 5.1. u_n is unit vector coming out from the area A . The direction of u_n is such that looking about it, the current circulates clockwise. Thus the magnetic dipole moment is defined by

$$\mu_m = IA u_n \quad (5.1)$$

In case of a coil having N turns, the magnitude of dipole moment

$$\mu_m = NIA \quad (5.2)$$

When a magnetic dipole is placed in a magnetic field, it experiences a torque that tries to rotate the magnetic moment to align its axis with the magnetic field. A magnetic dipole moment creates magnetic field just like a bar magnet.

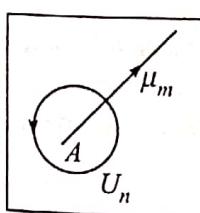


Fig. 5.1 A current loop

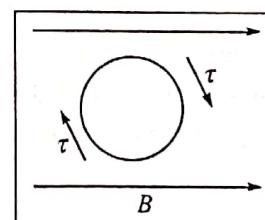


Fig. 5.2 A dipole moment in a magnetic field

5.1.2 Atomic Magnetic Moments

An orbiting electron in an atom behaves much like a current loop and has a magnetic dipole moment associated with it called orbital magnetic moment μ_{orb} as shown in figure 5.3. If ω is the angular frequency of the electron, then the current I due to the orbiting electron is

$$I = \text{Charge flowing per unit time} = \frac{-e}{\text{period}} = \frac{-e\omega}{2\pi} \quad (5.3)$$

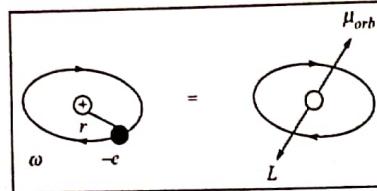


Fig. 5.3 An orbiting electron is equivalent to a magnetic dipole moment μ_{orb}

If r is the radius of the orbit, then the magnetic dipole moment is

$$\mu_{\text{orb}} = I(\pi r^2) = \frac{-e\omega r^2}{2} \quad (5.4)$$

Angular momentum of the electron is

$$L = (m_e v) r = m_e (\omega r) r = m_e \omega r^2 \quad (5.5)$$

which is quantized and is represented as

$$L = l\hbar$$

where l is the orbital quantum number.

So orbital magnetic moment is

$$\mu_{\text{orb}} = \frac{-e}{2m_e} L \quad (5.6)$$

where $e/2m_e$ is called gyromagnetic ratio, which is the ratio of magnetic moment to orbital magnetic moment and $\frac{e\hbar}{2m_e}$ is called Bohr magneton which is equal to $9.273 \times 10^{-24} \text{ A.m}^2$ or J.T^{-1} . The negative sign indicates that orbital magnetic moment is in opposite direction to angular momentum.

The intrinsic angular momentum of electron denoted by S gives rise to spin magnetic moment μ_{spin} .

$$\mu_{\text{spin}} = -\frac{e}{m_e} S = -s\hbar \frac{e}{2m_e} \quad (5.7)$$

The overall magnetic moment of the electron consists of μ_{orb} and μ_{spin} , appropriately added. They are both vector quantities, so have to be added vectorially. Only the electrons in unfilled sub-shells contribute to the overall atomic magnetic moment as for each electron in closed sub-shells there another electron with opposite spin.

5.1.3 Magnetization Vector M

The magnetic field inside the solenoid with free space inside it B_0 is given by

$$B_0 = \mu_0 nI = \mu_0 I \quad (5.8)$$

where n = number of turns per unit length,

I = current through the solenoid, A

μ_0 = absolute permeability of free space = $4\pi \times 10^{-7} \text{ H/m}$

I' = current per unit length = nI

If a cylindrical material is placed inside the solenoid, the magnetic field will be changed from B_0 to B . Each atom of the material responds to the applied field B_0 and develops a net magnetic moment μ_m along the applied field. The medium is thus magnetized. Magnetic vector M describes the extent of magnetization of the medium. So, magnetization vector M is defined as magnetic dipole moment per unit volume. If there are N atoms in a small volume of V and each atom, i , has a magnetic moment μ_{mi} , then M is defined as

$$M = \frac{1}{V} \sum_{i=1}^N \mu_{mi} = n_{\text{atomic}} \mu_{av} \quad (5.9)$$

where n_{atomic} = number of atoms per unit volume

μ_{av} = average magnetic dipole moment per atom

It can be assumed that each atom acquires a magnetic moment μ_{av} along B_0 which can be viewed as elementary current loops at the atomic scale. The elementary current loops are due to electronic currents within the atom and arise both from the orbital and spin motion. The current in the bulk cancel out each other, so there is no net current within the bulk. Only the surface current will be there. The surface currents are induced by the magnetization of the medium by the applied magnetic field and depend on the magnetization vector M of the specimen. Total magnetic moment of the cylindrical specimen is

$$\text{Total Magnetic Moment} = MV = MAL \quad (5.10)$$

If I_m is the magnetization current on the surface per unit length, then the total circulating current is $I_m L$ and total magnetic moment is by definition

$$\begin{aligned} \text{Total Magnetic Moment} &= \text{Total Current} \times \text{Cross Sectional area} = I_m AL \\ MAL &= I_m AL \\ M &= I_m \end{aligned} \quad (5.11)$$

Magnetizing current I_m is not due to the flow of free charge carriers as in a current carrying conductor. This is due to the localized electronic currents within the atoms of the solid at the surface. For a cylindrical specimen, I_m is perpendicular to M .

5.1.4 Magnetizing Field or Magnetic Field Intensity

When a medium is inserted within the solenoid as shown in figure 5.4, the magnetic field inside the solenoid changes from B_0 to B , and B is given by

$$B = \mu_0 (I' + I_m) = B_0 + \mu_0 I_m = B_0 + \mu_0 M \quad (5.12)$$

where B_0 , B and M all are vector quantities.

The field inside the magnetized material is the sum of the applied field B_0 and a contribution from the magnetization M at the material. The magnetizing field is defined as

$$\begin{aligned} H &= \frac{1}{\mu_0} B - M \\ H &= \frac{1}{\mu_0} B_0 \end{aligned} \quad (5.13)$$

H is simply related to the external conduction current of free charge carriers.

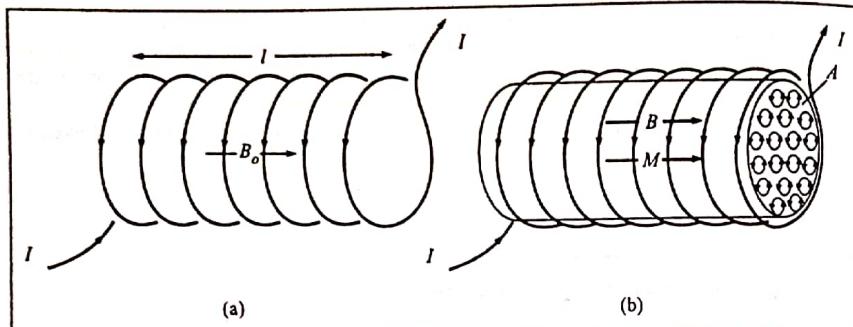


Fig. 5.4 Magnetic field of a solenoid with and without medium.

For a solenoid

$$\begin{aligned} B_0 &= n\mu_0 I \\ H &= nI \end{aligned} \quad (5.14)$$

Magnetizing field H is the cause and magnetic field B is its effect. H solely depends upon external conduction currents only whereas the magnetic field B depends on the magnetization of the material.

5.1.5 Magnetic Permeability and Susceptibility

The magnetic permeability of the medium at any point is defined as the magnetic field per unit magnetizing field

$$\mu = \frac{B}{H} \quad (5.15)$$

μ represents to what extent a medium is permeable by a magnetic field.

Relative permeability of the medium is defined as the fractional increase in the field with respect to the field in free space when a material medium is introduced.

$$\mu_r = \frac{B}{B_0} = \frac{B}{\mu_0 H} \quad (5.16)$$

Relating expression (5.16) with (5.15) we will get

$$\mu = \mu_0 \mu_r \quad (5.17)$$

where μ = absolute permeability of the medium

μ_r = relative permeability of the medium

μ_0 = absolute permeability of the free space = $4\pi \times 10^{-7} \text{ H.m}^{-1}$

The magnetization M produced in the material depends on the net magnetic field B . It should be natural to relate magnetization M to magnetic field B . But, for historic reasons, magnetization M is related to magnetizing field H . M is related to H as

$$M = \chi_m H \quad (5.18)$$

where χ_m = Magnetic susceptibility

The relationship given in (5.18) is obeyed by all materials.

Rewriting equation (5.12)

$$\begin{aligned} B &= B_0 + \mu_0 M \\ B &= \mu_0 H + \mu_0 M \\ B &= \mu_0 H + \mu_0 \chi_m H \\ B &= (1 + \chi_m) \mu_0 H \\ B &= \mu_0 \mu_r H \end{aligned} \quad (5.19)$$

where $\mu_r = 1 + \chi_m$

Expression (5.20) relates the relative permeability with magnetic susceptibility of the material.

5.1.6 Ferromagnetism

Ferromagnetic materials can possess large permanent magnetization even in the absence of an applied field. The magnetic susceptibility χ_m is positive and very large and depends on the applied field intensity. The relationship between magnetization M and $\mu_0 H$ is highly non-linear. At sufficiently high fields, magnetization M saturates. The origin of the ferromagnetism is the quantum mechanical exchange interaction between the constituent atoms that results in regions of material possessing permanent magnetization. The ferromagnetic crystal has magnetic moments of all crystal aligned in an orderly manner so as to give rise to the net magnetization vector M . Ferromagnetism occurs below a critical temperature T_c called Curie temperature. At temperatures above this ferromagnetism is lost, i.e., paramagnetism is obtained obeying

Curie-Weiss Law $\chi = \frac{C}{T - T_0}$. We will discuss about ferromagnetism in detail later on.

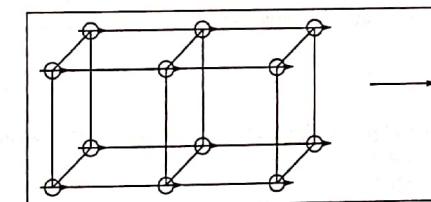


Fig. 5.5 Magnetized region of Ferromagnetic material

5.1.7 Antiferromagnetism

Antiferromagnetic materials have small positive susceptibility. In the absence of external field, there is no net magnetization of the material. They possess magnetic ordering in which the magnetic moments on alternating atoms in the crystals align in opposite directions. The opposite alignments of magnetic moments are due to quantum mechanical exchange interaction. Antiferromagnetism occurs below a critical temperature called Néel temperature. Above this they are paramagnetic.

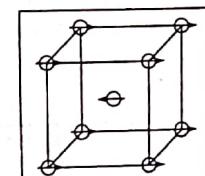


Fig. 5.6 Antiferromagnetic crystal (Cr)

Antiferromagnetic substances are feebly magnetized when subjected to a strong magnetic field. Examples of Antiferromagnetic substances are MnO, FeO, CoO, NiO, $\text{FeCl}_3\text{MnO}_4$, Cr, Mn, MnS, etc.

5.1.8 Ferrimagnetism

Ferrimagnetic materials exhibit magnetic behavior similar to ferromagnetism below a critical temperature called Curie temperature T_C . As shown in figure 5.7 above all A atoms has their spins aligned in one direction and all B atoms has their spins aligned in opposite direction. Magnetic moment of atom A is greater than that of atom B. Due to this there will be net magnetization M in the crystal even in the absence of external applied field. These materials are non-conducting, so they do not suffer from eddy current losses. This is the reason for these materials being used for high frequency electronic application. Ferrimagnetic materials have larger net magnetization as compared to antiferromagnetic materials. These materials are generally known as ferrites and represented by $X\text{Fe}_3\text{O}_4$ where $X = \text{Mg, Cu, Mn, Ni, Zn, Cd, etc.}$

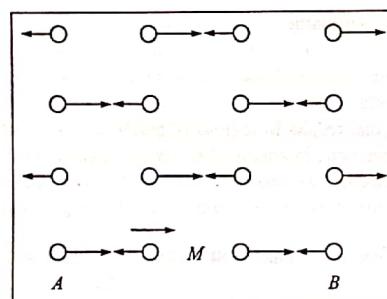


Fig. 5.7 Magnetic ordering in ferrimagnetic crystal

5.1.9 Paramagnetism

Paramagnetic materials have small positive magnetic susceptibility. In the absence of external magnetic field, molecular moments (atomic moments) are randomly oriented due to random collision of molecules. So the average dipole moment and the net magnetization both are zero. When an external field is applied, μ_{av} non-zero and depends upon the applied field $\mu_0 H$ and hence the magnetization is also non-zero and is equal to $\chi_m H$. Magnetization M increases with $\mu_0 H$ but decreases with increase in temperature. At higher

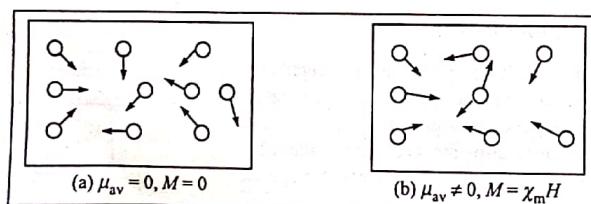


Fig. 5.8 Without external field (a), with external field (b)

temperatures, the molecular collisions destroy the alignment of molecular magnetic moments with applied field, i.e., Curie law ($\chi_m = C/T$) is obeyed. When a paramagnetic substance is placed in a non-uniform magnetic field, the induced magnetization M is along the direction of B and there is a net force toward greater fields. Many gases and metals are paramagnetic in nature. The typical value of magnetic susceptibility is $\chi_m = 1.2 \times 10^{-5}$ for metals and $\chi_m = 2.1 \times 10^{-6}$ for oxygen. Other examples are Pt, Al, Cr, Mn and dilute solutions of ferromagnetic materials.

5.1.10 Diamagnetism

Materials with negative magnetic susceptibility are termed as diamagnetic substances. Their relative permeability is slightly less than unity. When diamagnetic materials are placed in magnetic field, the magnetization vector M is in opposite direction to the applied field. This causes the magnetic field within the material to be less than the applied field. The negative susceptibility can be interpreted as the diamagnetic substance trying to expel the applied field from within the material. Covalent crystals and many ionic crystals are diamagnetic because the constituent atoms have no unfilled sub-shell. Superconductors are perfect diamagnet with $\chi_m = -1$ and totally expel the applied field. Other examples of diamagnetic substances are Bi, Sb, Au, Cu, Hg, water, air, alkalihalides, organic materials, many polymers, covalent solids like Si, Ge, diamond, etc. The average value of magnetic susceptibility of diamagnetic materials is in the range of -10^{-6} .

5.2 Domain Structure

5.2.1 Introduction

A single piece of iron does not necessarily possess a net permanent magnetization in the absence of an applied field. The reason for the absence of net magnetization is due to formation of magnetic domains that cancel each other. A magnetic domain is a region of the crystal in which all the spin magnetic moments are aligned to produce a magnet in one direction only.

In figure 5.9(a), a single crystal iron has a permanent magnetization as a result of aligning of all atomic spins. The potential energy called magnetostatic energy of the magnet can be reduced by dividing the crystal into two regions (domains) with opposite magnetization. The external magnetic field lines are reduced and there is now less potential energy stored in the magnetic field. There are only field lines at the end. The boundary between two domains is called domain wall or Bloch wall where the magnetization changes from one direction to the opposite. The domain wall is not simply one atomic spacing but has a finite thickness, which for iron is of the order of 0.1 micron, i.e., several hundred atom spacing.

The magnetostatic energy associated with the field lines at the ends (figure 5.9.b) can be further reduced by closing the ends with sideway domains with magnetization at 90°. These sideway domains at 90° with the main domains are called end domains. Although the magnetostatic energy has decreased, the potential energy in the walls has increased due to the addition of walls. The creation of magnetic domain continues spontaneously until the potential energy reduction in creating an additional domain is same as the increase in addition wall. The specimen then possesses minimum potential energy and is in equilibrium with no net magnetization. The sizes, shapes and distribution of domains depend on a number of factors including the size and shape of the whole specimen. For iron particles of dimension less than 0.01 micron, the increase in potential energy in creating a domain wall is too costly and these particles are single domains and hence always magnetized.

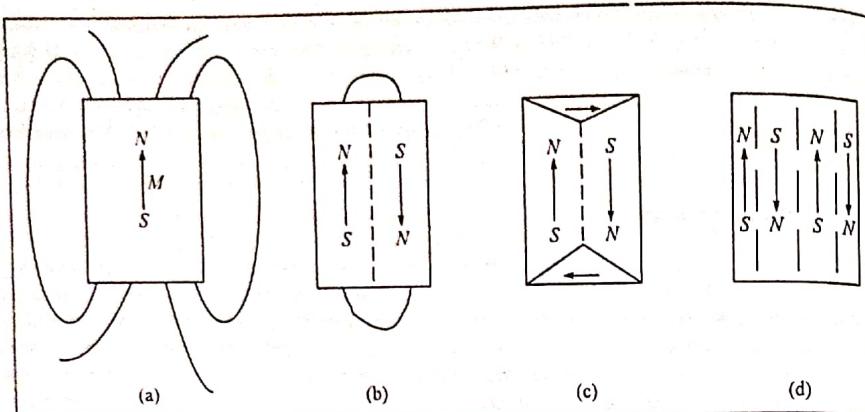


Fig. 5.9 Domain structure in ferromagnetic material. Magnetized bar of ferromagnet (a). Formation of two domains with opposite magnetization (b). Domains of closure fitting eliminating the external field lines(c). Specimen with several domains and closure domains (d).

The magnetization of the domains is normally along the one of the preferred directions in which the atomic spin alignment is easiest. For iron, the magnetization is easiest along one of the six $<100>$ directions (cube edges). The domains have magnetizations along these directions called easy directions. The magnetization of the crystal along an applied field occurs by growth of the domains with magnetizations along the applied field.

For simplicity magnetization is taken along an easy direction. The Bloch wall between domains A and B migrates toward right enlarging domain A and shrinking domain B. The crystal has net magnetization M along applied field H . The migration of Bloch wall is caused by the spins in the wall and also spins in the region B adjacent to the wall being gradually rotated by the field. The magnetization process involves the motions of Bloch walls in the crystal.

5.2.2 Domain Walls

The spin magnetic moments rotate across a domain wall which is several hundreds atom spacing thick. It is seen that the neighboring spin moments in the wall rotate gradually, and over several hundred atomic spacing the magnetic moments reach a rotation of 180° . Exchange forces between neighboring atomic spins favor very little relative rotation. Had it been left to the exchange forces alone, relative rotation of atomic spins would be so minute that the wall would have to be very thick (infinitely thick) to achieve a rotation of 180° . However, magnetic moments that are oriented away from the easy direction possess excess energy called the anisotropy energy. If the wall is thick, then it will contain many magnetic moments

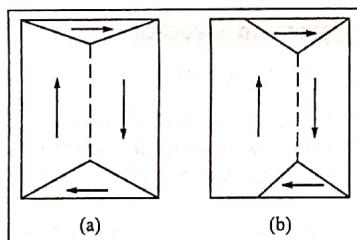


Fig. 5.10 Domain structure in the absence of applied field (a), in the presence of field (b)

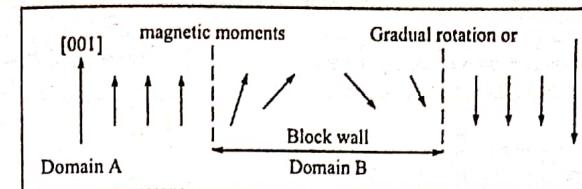


Fig. 5.11 Domains and domain wall

rotated away from the easy directions and there would be substantial anisotropy energy in the wall. Minimum anisotropy energy in the wall is obtained when the magnetic moment changes direction by 180° from the easy direction along $+z$ to another easy direction $-z$ without any intermediate rotations away from z . This requires a wall of one atomic spacing. In reality, the domain wall thickness is a compromise between the exchange energy requiring thick wall and anisotropy energy requiring a thin wall > the equilibrium wall thickness is that which minimizes the total potential energy which is the sum of exchange energy and anisotropy energy within the wall. For iron the thickness of the domain wall is of the order of 0.1 micron and for cobalt it is smaller due to higher anisotropy energy.

5.2.3 Magnetostriction

We can create strain in a ferromagnetic crystal by applying suitable stress along a certain direction thereby changing the interatomic spacing not only along this direction but also in other directions causing the change in exchange interaction between atomic spins. This will lead to change in magnetization properties of crystal. In the converse effect, magnetization of the crystal generates strains or changes in the physical dimensions of the crystal. When an iron crystal is magnetized along $[111]$ direction by a strong field (this being hard direction only strong field can magnetize along this), the atomic spins within the domains are rotated from their easy directions toward the hard direction. These electron spin rotations involve changes in the electron charge distributions around the atoms and therefore affect the interatomic bonding and hence the interatomic spacing. The longitudinal strain $\Delta L/L$ along the direction of magnetization is called the magnetostrictive constant λ . This constant depends on the crystal direction and may be positive (extension) or negative (contraction). The crystal lattice strain energy associated with magnetostriction is called the magnetostrictive energy, which is typically less than the anisotropy energy.

Magnetostriction is responsible for humming noise in transformers. As the core of a transformer is magnetized in one way and then in the opposite by an alternating current, the alternating changes in longitudinal strain vibrate the surrounding environment, air, oil, and so forth, and generate an acoustic noise at twice the main frequency and its harmonics. The magnetostriction constant can be controlled by alloying metals. The saturation strain for Ni along an easy direction is negative whereas that of iron is positive. So, by alloying 85% Ni with 15% Fe the resulting magnetostrictive constant is zero.

5.3 Hysteresis loop, Eddy Current Losses

The majority of magnetic materials used are polycrystalline having microstructure consisting of many grains of various sizes and orientations depending upon the preparation and thermal history of the specimen.

In unmagnetized polycrystalline sample, each crystal grain will possess domains, which will depend on the shape and size of the grain and to some extent, on the magnetization of the neighboring grains. The net

magnetization of the sample is zero in the absence of external field. It can be assumed that the component was heated to a temperature above Curie point and then allowed to cool to room temperature without applied field.

When a small field is applied in some direction (say $+x$), the domain walls within various grains begin to move small distances, and domain with net magnetization M along $+x$ grows a little larger at the expense of those pointing away from the field.

At the start there is very small net magnetization along H as indicated by region $a-a$. As we increase the field, the domain motions extend to larger distances and the walls encounter various obstacles such as crystal imperfections, impurities and so on, which try to attract the wall and thereby hinder the motion. A domain wall that is stuck or pinned at an imperfection at a given field cannot move until the field increases sufficiently to provide the necessary force to snap the wall free, which then suddenly surges to the next obstacle. Sudden snapping of the wall causes lattice distortion creates lattice waves that carry off some of the energy. Sudden changes in magnetization induce eddy currents that dissipate energy via joule heating (I^2R). The domain walls have finite electrical resistance. These processes involve energy conversion to heat and are irreversible. Sudden jerks in wall motion leads to a small jump in magnetization of the specimen as the magnetizing field is increased. This phenomenon is termed as *Barkhausen effect*.

As we continue to increase the field, magnetization vector M continues to increase by jerky domain wall motions that enlarge domain with favorably oriented magnetizations and shrink away those with magnetization pointing away from the applied field. Eventually domain wall motions leave each crystal grain with a single domain and magnetization in one of the easy directions.

If we decrease and remove the field, the magnetization in each grain will rotate to align parallel with the nearest easy direction in that grain. In some grain additional small domains may develop that reduce the magnetization within the grain (point e). This process from point d to e whereby the magnetizing field is reduced to zero leaves the specimen with the net permanent magnetization called *remnant* or *residual magnetization* M_r . If now a magnetizing field is applied in the reverse direction, the magnetization of the specimen would decrease and eventually at a sufficiently large applied field, magnetization M becomes zero. This means the specimen has been totally demagnetized. The magnetizing field required to totally demagnetize the sample is called *coercive field* and is denoted by H_c . Coercive field represents the opposition of the sample to demagnetization. The demagnetization process involves the generation of new domains, where nucleation at various crystal imperfections cancels out overall magnetization.

If the magnetization is further increased in reverse direction, the process from point e will be similar to that when magnetizing the sample. Point g will represent saturation magnetization $-M_{sat}$ in reverse direction. Decreasing the magnetizing field from $-H_{sat}$ to zero again leaves residual magnetization in the specimen and to eliminate this we need to apply magnetizing field in reverse direction (now in positive x direction). The complete M versus H curve obtained by changing magnetizing field from positive x to negative x for corresponding values of magnetization is a closed loop and called *hysteresis loop*. In both the positive and negative directions there are saturation magnetization M_{sat} and saturation magnetizing field H_{sat} . The initial magnetization curve for a previously unmagnetized specimen $o-a-b-c-d$ is called initial magnetization curve.

Under very small fields (a); boundary motions occur in sudden jerks (b); nearly all grains are single domains (c); magnetization in individual grains has to be rotated to align with H (d); after the external field

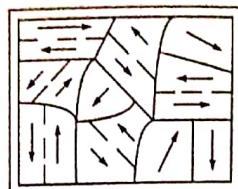


Fig. 5.12 Domains in the grains of an unmagnetized polycrystalline sample

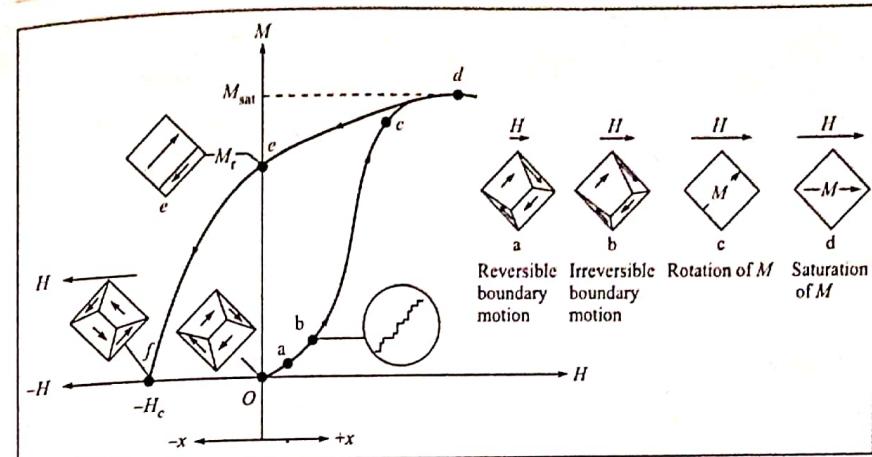


Fig. 5.13 M vs. H behavior of a previously unmagnetized polycrystalline iron specimen

has been removed (e); the remaining magnetization M_r has been removed by applying magnetizing field H_c in reverse direction (f).

We know the following relationship $B = \mu_0 M + \mu_0 H$ which leads to hysteresis loop in the magnetic field versus magnetizing field behavior. The very slight increase in B with H when M is in saturation is due to the permeability of free space. The area enclosed within B versus H hysteresis loop shown by hatched line in figure 5.14 (b) is the measure of energy dissipated per unit volume per cycle of applied field variation. If the magnetic field in the material does not reach B_{sat} but instead reaches some maximum value B_m for a magnetizing field H_m , still there will be a hysteresis loop formed since the magnetization and demagnetization are nonreversible and do not trace each other. Only the area of the loop will be smaller than that for $B_{sat}(M_{sat})$ versus H_{sat} . So the shape of hysteresis loop depends in addition to shape and size of the specimen on the magnitude of the applied field.

The energy dissipated during magnetization and demagnetization can be written as

$$dE_{vol} = H dB$$

When the magnetic field changes from B_1 to B_2

$$E_{vol} = \int_{B_1}^{B_2} H dB \quad (5.21)$$

The hysteresis power loss for steel cores can be empirically expressed as

$$P_h = K f B_m^n \quad (5.22)$$

where P_h = hysteresis power loss, W
 f = frequency of applied voltage, Hz
 B_m = maximum magnetic field (flux density), T

$n = 1.6$

K = constant which depends upon core material (typically $K = 150.7$)

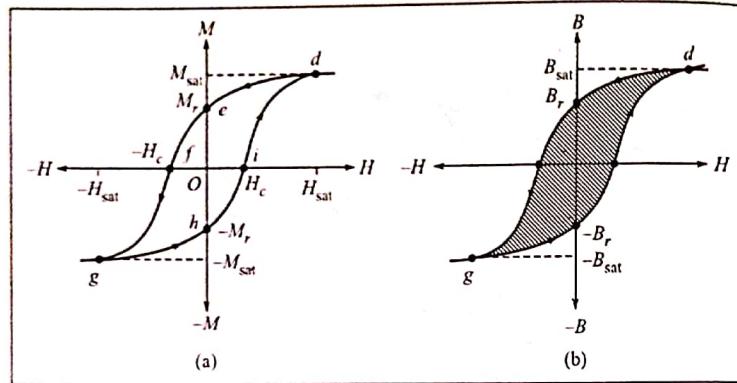


Fig. 5.14 A typical M vs. H hysteresis curve (a), and the corresponding B vs. H hysteresis curve (b).

5.4 Soft Magnetic Materials

When a magnetic material is magnetized and then demagnetized, energy loss takes place that is given by the area of the hysteresis loop. The size of the loop is different for different materials. There are materials, which can be easily magnetized and then demagnetized. And, other materials are hard both to magnetize and demagnetize. So, according to the B - H behavior, magnetic materials are classified into soft magnetic materials and hard magnetic materials.

Soft magnetic materials are easy to magnetize and demagnetize, and hence require relatively low magnetic field intensities. In other words, their B - H loops have small area, so the power loss per unit cycle is small. They are most suited for applications where repeated cycles of magnetization and demagnetization are involved. Electric motors, transformers, inductors have magnetic fields varying cyclically. So they require magnetic materials with low loss (small hysteresis loop). Electromagnetic relays which have to be switched on and off frequently for their assigned operation also require the iron core to be magnetized and demagnetized correspondingly. So, they require soft magnetic materials as well.

An ideal soft magnetic material should have zero coercivity (coercive field), a very large saturation magnetization M_{sat} (correspondingly B_{sat}), zero remanent magnetization M_r (and of course B_r), zero hysteresis loss and high initial and relative permeability μ_i and maximum relative permeability μ_{max} .

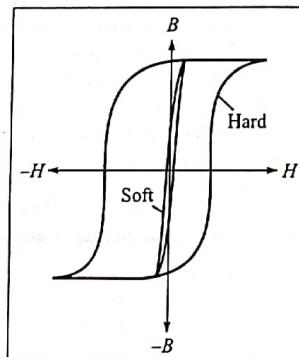


Fig. 5.15 B - H curve for typical soft and hard magnetic materials

Table 5.1 Selected soft magnetic materials (E_h is hysteresis loss, i.e., energy dissipated per unit volume per cycle, J/m^3 at $B_m = 1 T$)

Magnetic Material	$\mu_0 H_c, T$	B_{sat}, T	B_r, T	μ_i	μ_{max}	E_h, Jm^{-3}	Applications
Ideal soft iron	0	Large	0	Large	Large	0	Transformer cores, inductors, magnetic recording heads, relays, electromagnet cores, machines
Iron (commercial grade, 0.2% impurities)	<10 ⁻⁴	2.2	<0.1	150	10 ⁴	250	Due to large eddy current losses used only in some electromagnets and relays
Silicon Iron (2-4% Si)	<10 ⁻⁴	2	0.5-1	1000	10000 - 400000	30 - 100	Wide range of ac machinery
Supermalloy (79%Ni, 15%Fe, 5%Mo, 0.5%Mn)	2 × 10 ⁻⁷	0.7-0.8	<0.1	10 ⁵	10 ⁶	<0.5	High permeability low loss electrical devices, e.g. specialty transformers, magnetic amplifier
Permalloy (78.5%Ni, 21.5%Fe)	5 × 10 ⁻⁶	0.86	<0.1	8000	10 ⁵	<0.1	Low loss electrical devices, audio transformers, HF transformers, recording heads, filters
Glassy metals (Fe - Si - B)	2 × 10 ⁻⁶	1.6	<10 ⁻⁶	-	10 ⁵	20	Low loss transformer cores
Ferrites Mn-Zn ferrite	10 ⁻⁵	0.4	<0.01	2000	5000	<0.01	HF low loss applications. HF transformers, recording heads, inductors (E and U cores)

5.5 Fe-Ni Alloys

Pure iron despite being soft magnetic material is not normally used as magnetic material because its good conductivity allows large eddy currents to be induced under varying fields. So, other materials are added to obtain required properties of the magnetic material. By adding (2-4)% of silicon to iron, silicon steel is obtained. Such material obtained is still soft magnetic material with increased resistivity but reduced eddy current loss in the machinery using this material. Silicon steel is most widely used in manufacture of power transformers and other electrical machinery for alternating current applications.

Nickel-iron alloys with composition around 77% Ni and 23% Fe constitute a soft magnetic material with low coercivity, low hysteresis loss and high initial and maximum relative permeabilities. High initial relative permeability makes the material useful in low magnetic field applications that are typically found in high frequency works in electronics e.g. audio and wide band transformers. They have found many engineering applications in the form of sensitive relays, pulse and wide band transformers, current transformers, magnetic recording heads, magnetic shielding, etc. Alloying iron with nickel increases resistivity and hence reduces the eddy current losses in the machinery. The domain wall motions are easy in nickel composition resulting smaller hysteresis losses in the material. There are number of commercial Ni-Fe alloys whose application depends on exact composition (which may also have a few percentages of Mo, Cr, Cu, etc) and method of preparation (e.g. mechanical rolling).

Amorphous magnetic materials have no crystalline structure they only have short-range orders. So they do not possess crystal imperfections such as grain boundaries and dislocations. They are prepared by rapid solidification of melt by using special techniques. Typically they are thin ribbons by virtue of their preparation method. Since there is no crystalline structure, all the directions are easy. The absence of crystalline defects, which normally impedes domain wall motions, leads to low coercivity and hence soft magnetic properties. The coercivity is not zero due to the directional nature of strains frozen in metal during rapid solidification. By virtue of their disordered structure, amorphous materials have high resistivity, and hence smaller eddy current losses. Although they are ideally suited for various transformers and electric machinery, their limited sizes and shapes prevent their use in power applications.

5.6 Ferrites for High Frequency Transformers

Ferrites are ferromagnetic materials, which are typically oxides of mixed transition metals, one of which is iron. Manganese ferrite is $MnFe_2O_4$, Manganese zinc ferrite is $Mn_{1-x}Zn_xFe_2O_4$ where x is less than unity. Ferrites are normally insulators, so they do not suffer from eddy current losses (very high resistance of the material allows only very small current and consequently very small or no losses). This is the reason for these being suitable for high frequency applications where eddy current losses given by $K_e f^2 B_m^2$ would prevent the use of material with reasonable conductivity.

Although ferrites can have high initial relative permeabilities and low losses, they do not possess as large saturation magnetization as ferromagnets. Their useful temperature range determined by Curie temperature is also lower. There are many types of ferrites available depending on the application, tolerable losses and required upper frequency of operation. Manganese zinc ferrites have high initial relative permeability of 2000 but are only useful up to a frequency of 1MHz, whereas Nickel zinc ferrites have lower initial relative permeability of 200 but they can be used up to a frequency of 200 MHz. Generally, the initial relative permeability in the high frequency region decreases with increase in frequency.

Garnets are ferrimagnetic materials that are used at highest frequencies covering the microwave range (1-300GHz). The yttrium iron garnet YIG ($Y_3Fe_5O_{12}$) is one of the simplest garnets with very low hysteresis loss at microwave frequencies. They have excellent dielectric properties with high resistivities and consequently low losses. The main disadvantage of garnets is the low saturation magnetization (0.18T for YIG) and low Curie temperature (280°C for YIG). The compositions of garnets depend upon the properties required for the particular microwave application. $Y_{21}Gd_{0.98}Fe_5O_{12}$ is a garnet that is used in 8-12GHz three-port circulators handling high microwave powers.

5.7 Hard Magnetic Materials

As we already mentioned they are hard to magnetize and demagnetize, and hence require large magnetizing field for both magnetization and demagnetization. The $B-H$ curves of hard magnetic materials are broad and almost rectangular in shape. These materials possess relatively larger coercivities, which means they require large field to demagnetize them. The coercive field for hard magnetic materials can be millions of times greater than those for soft magnetic materials. Their characteristics make the hard magnetic materials suitable for making permanent magnets. It is clear that magnetization can be switched from one very persistent direction to other very persistent direction by a suitably large magnetizing field intensity. As the coercivity is strong, both the states $+B_r$ and $-B_r$ persist until a suitable large magnetic field intensity switches the field from one direction to the other. Hard magnetic materials can be used in magnetic storage of digital data where the states $+B_r$ and $-B_r$ can be made to represent 1 or 0 and vice versa.

An ideal hard magnetic material has a very large coercivity and remanent magnetic field. Since hard magnetic materials are used as permanent magnets, the energy stored per unit volume in external magnetic field should be as large as possible since this is the energy available to do work. This energy density in Joules per metre cube (Jm^{-3}) in external field depends upon the maximum value of the product of Band H in the second quadrant of $B-H$ characteristics and is denoted by $(BH)_{max}$. This corresponds to the largest rectangular area that fits the $B-H$ curve in the second quadrant.

When the size of a ferromagnetic sample falls below a certain critical dimension (0.1 micron for Co), the whole sample becomes a single domain because the energy required for generating a domain wall is too high compared to the reduction in external magnetostatic energy. These small particle-like pieces of magnetic materials are called single domain fine particles. Their magnetic properties depend both on the crystal structure and shape of the particle. For a spherical iron particle, the magnetization M will be in an easy direction e.g. along [001] in $+z$ direction. To reverse the magnetization from $+z$ to $-z$ by an external field, the magnetic spins have to be rotated around past the hard direction, since we can generate reverse domains. The rotation of magnetization involves substantial work due to magnetocrystalline anisotropy energy. So the sample has high coercivity. The higher the magnetocrystalline anisotropy energy, the greater is the coercivity. The energy involved in creating a domain wall increases with the magnetocrystalline anisotropy energy. The critical size below, which a particle becomes a single domain increases with magnetocrystalline anisotropy. Barium ferrite crystals have high degree of magnetocrystalline anisotropy due to their hexagonal shape. Critical size for single domain barium ferrite particles is about 1-1.5micron, and the coercivity about 0.3T compared with the coercivity of 0.02-0.1T for multi-domain barium ferrites.

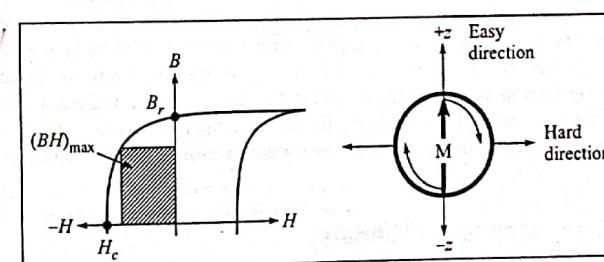


Fig. 5.16 $(BH)_{max}$ in hard magnetic material (a). A single domain fine particle (b)

Particles that are not spherical may even have higher coercivity as a result of shape anisotropy. For the single domain elongated particle, the potential energy in the external field is less for magnetization along long axis than for magnetization along short axis. So to rotate the magnetization from long axis to short, work has to be done. The magnetization of single domain fine particles along its elongated axis is termed as shape anisotropy. The elongation of the particle increases the coercivity and vice versa. Small spherical Fe-Cr-Co particles have a coercivity of 0.02T normally, but the elongated particles can have a coercivity as high as 0.075T due to shape anisotropy.

High coercivity magnets are fabricated by having elongated fine particles dispersed by precipitation in a structure. Alnico is one of the widely used permanent magnet material which is the alloy of aluminum, nickel, cobalt and iron. Fe-Co rich particles are dispersed in a matrix that is Al-Ni rich. The structure is obtained by appropriate heat treatment to allow Fe-Co particles to precipitate out from a solid solution of the alloy. Fe-Co particles possess strong magnetic field, whereas Al-Ni particles possess weak magnetic field. When heat treatment is carried out in the presence of strong magnetic field, the Fe-Co rich particles have their magnetization along elongated side. To demagnetize the elongated particles, high coercive field is required due to shape anisotropy. For manufacture of permanent magnets in permanent magnet machines, Alnico is widely used. The main disadvantage of this alloy is that it is mechanically hard and brittle. So, it cannot be shaped except by casting or sintering before heat treatment.

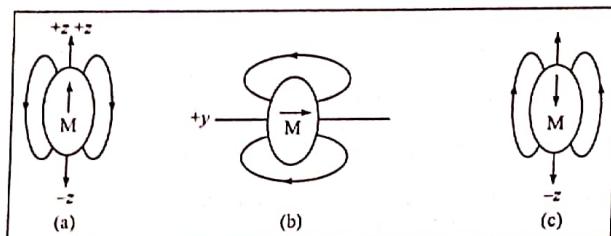


Fig. 5.17 Single domain elongated particle. Magnetization being changed from *a* to *b* to *c* by external field.

A variety of permanent magnets are made by compacting high coercivity particles by using powder metallurgy. The particles are magnetically hard because they are sufficiently small for each particle to be a single domain. Ceramic magnet, which is made by compacting barium ferrite, $\text{BaFe}_{12}\text{O}_9$ is very popular as permanent magnet. The barium ferrite has hexagonal crystal structure with large magnetocrystalline anisotropy giving high coercivity. These magnets are formed by wet pressing ferrite powder in the presence of strong magnetizing field to allow the single domain fine particles to be aligned in one of the easy directions, and then drying and carefully sintering the ceramic. Another example of ceramic magnet is strontium ferrite $\text{SrFe}_{12}\text{O}_9$. The ceramic magnets are mainly used to manufacture permanent magnets in low cost applications.

5.8 Square Loop Magnetic Materials

Rare earth cobalt permanent magnets based on samarium-cobalt (Sm-Co) alloys, which may possess other transition metals as well, have very high $(BH)_{max}$ values and are used in many applications such as stepper motors, servo motors, traveling wave tubes, gyroscopes, etc. SmCo_5 has a hexagonal crystal structure with high large magnetocrystalline anisotropy and hence high coercivity. SmCo_5 powder is pressed in the

presence of strong magnetic field and then by carefully sintering solid powder magnet is formed. The SmCo_{15} magnets are more recent and have $(BH)_{max}$ values of around 240kJm^{-3} .

The more recent neodymium-iron-boron, NdFeB, powdered solid magnets can have very large $(BH)_{max}$ values up to 275kJm^{-3} . The tetragonal crystal structure has the easy direction along long axis and possesses high magnetocrystalline anisotropy. Substantial amount of work has to be done to rotate magnetization around through the hard direction. These materials with very high coercivities are also termed as square loop magnetic materials because of the $B-H$ loop imitating the square shape. To store any information in the form of digitalized data, these materials are most suited. The only drawback of these materials are lower Curie temperature compared to other magnetic materials such as Alnico with Curie temperature of 700°C compare to 300°C for NdFeB.

Table 5.2 Occurrence of permanent atomic magnetic dipoles and interaction between neighboring dipoles

Type of material	Permanent dipoles	Interaction between neighboring dipoles
Paramagnetic	Yes	Negligible
Ferromagnetic	Yes	Parallel orientation
Antiferromagnetic	Yes	Antiparallel orientation of equal moments
Ferrimagnetic	Yes	Antiparallel orientation of unequal moments

Table 5.3 Susceptibilities of some paramagnetic materials at 300K

Substance	$\chi = \mu_r - 1, \times 10^{-3}$	Substance	$\chi = \mu_r - 1, \times 10^{-3}$
CrCl_2	1.5	Fe_2O_3	1.4
Cr_2O_3	1.7	$\text{Fe}_2(\text{SO}_4)_3$	2.2
CoO	5.8	FeCl_2	3.7
CoSO_4	2.0	FeSO_4	2.8
MnSO_4	3.6	NiSO_4	1.2

Table 5.4 Néel temperature (T_N) and paramagnetic Curie temperature (θ_c) for some antiferromagnetic materials.

Material	T_N, K	θ_c, K
MnF_2	72	113
MnO_2	84	316
MnO	122	610
MnS	165	528
FeO	198	570
NiF_2	73	116
CoO	292	280

Table 5.5 Susceptibility of some diamagnetic materials

Material	$\chi = \mu_r - 1, \times 10^{-5}$	Material	$\chi = \mu_r - 1, \times 10^{-5}$
Al_2O_3	-0.5	Cu	-0.9
BaCl_2	-2.0	Au	-3.6
NaCl	-1.2	Ge	-0.8
Diamond	-2.1	Si	-0.3
Graphite	-12	Se	-1.7

5.9 Solved Numerical Examples

1. Calculate the permeability and susceptibility of an iron bar of cross-sectional area 0.2cm^2 when a magnetizing field of 1200A.m^{-1} produces magnetic field of 24 micro Weber.

Solution:

We have,

$$\begin{aligned}\phi &= 2.4 \times 10^{-5} \text{Wb} \\ A &= 0.2\text{cm}^2 = 0.2 \times 10^{-4} \text{m}^2 \\ H &= 1200\text{A.m}^{-1}\end{aligned}$$

Using the relation,

$$B = \frac{\phi}{A} = \frac{2.4 \times 10^{-5}}{0.2 \times 10^{-4}} = 1.2\text{T}$$

Now, the permeability is given by

$$\mu = \frac{B}{H} = \frac{1.2}{1200} = 10^{-3} \text{H/m}$$

The susceptibility is then given by

$$\chi_m = \mu_r - 1 = \frac{\mu}{\mu_0} - 1 = \frac{10^{-3}}{4\pi \times 10^{-7}} - 1 = 737$$

2. Calculate the magnetization and flux density in a diamagnetic sample having susceptibility -0.3×10^{-5} and magnetic field strength 1000A/m .

Solution:

Given,

$$\begin{aligned}\chi &= -0.3 \times 10^{-5} \\ H &= 1000\text{A/m}\end{aligned}$$

We know that

$$M = \chi H = -0.3 \times 10^{-5} \times 1000 = -310^{-3} \text{A/m}$$

Now, the magnetic flux density is given by

$$B = \mu_0 H = 4\pi \times 10^{-7} \times 1000 = 1.257 \times 10^{-3} \text{T}$$

3. Calculate the magnetic moment of a bar of iron having length 0.5m and cross-sectional area 2mm^2 when placed in a long solenoid of 25 turns/cm carrying a current of 2A . Take the relative permeability of the material 400 .

Solution:

Given, Length of the iron bar, $l = 0.5\text{m}$

Cross-sectional area, $A = 2\text{mm}^2 = 2 \times 10^{-6} \text{m}^2$

No. of turns/length of the solenoid, $n = 25/\text{cm} = 25/10^{-2}\text{m} = 2500/\text{m}$

Current through the solenoid, $I = 2\text{A}$

We know that the magnetic field on the iron bar due to a solenoid is

$$B = \mu_0 \mu_r n I = 4\pi \times 10^{-7} \times 400 \times 2500 \times 2 = 2.51 \text{T}$$

Now,

$$M = \frac{B}{\mu_0} = \frac{2.51}{4\pi \times 10^{-7}} = 1.998 \times 10^6 \text{A.m}^{-1}$$

Hence the magnetic moment is equal to

$$\begin{aligned}MV &= MAl \\ &= 1.998 \times 10^6 \times 2 \times 10^6 \times 0.5 \\ &= 1.998 \text{Am}^2\end{aligned}$$

4. The saturation induction of Nickel is 0.65Wb.m^{-2} . If the density is 8906Kg.m^{-3} and its atomic weight is 58.7 ; calculate the magnetic moment of Nickel atom in Bohr magneton.

Solution:

We have, Saturation magnetic field, $B_s = 0.65\text{Wb.m}^{-2}$

Density of the Nickel, $\rho = 8906\text{Kg.m}^{-3}$

Atomic weight of Nickel, $A = 58.7\text{g}$

Magnetic moment for ferromagnetic materials is given by

$$\mu = \frac{\text{Magnetization}}{\text{No. of atoms/volume}} = \frac{M}{n}$$

where

$$n = \frac{\rho N_A}{M} = \frac{8906 \times 6.02 \times 10^{23}}{58.7 \times 10^{-3}} = 9.13 \times 10^{28} \text{m}^{-3}$$

$$\begin{aligned}\mu &= \frac{B_s}{\mu_0 n} \text{ where } M = \frac{B_s}{\mu_0} \\ &= \frac{0.65}{4\pi \times 10^{-7} \times 9.13 \times 10^{28}} \\ &= 5.66 \times 10^{-24} \text{A.m}^2 \\ &= 0.61\mu_B\end{aligned}$$

where $\mu_B = 9.273 \times 10^{-24} \text{A.m}^2$ is Bohr magneton.

Exercise

- Determine the permeability and relative permeability of a diamagnetic substance having susceptibility -9.5×10^{-9} [$1.25 \times 10^{-6} \text{H/m}$; 0.99]
- Calculate the relative permeability of a paramagnetic material at -73°C and 227°C if the susceptibility of the paramagnetic material at 27°C is 3.7×10^{-3} . [5.56×10^{-3} and 2.22×10^{-3}]

3. If a piece of ferric oxide has susceptibility 1.5×10^{-3} and a magnetic field intensity 10^6 A/m , then what are the magnetic flux density and magnetization of the material? [1.2T and 1500 A.m^{-1}]
4. If an atom of a magnetic material consists of 10 electrons distributed uniformly and revolving round a circular path of radius 10^{-11} m , then calculate orbital dipole moment of the atom in Bohr magneton, [$1.74 \mu_B$]
5. The magnetic field at northern magnetic pole of earth is vertical and is $6.2 \times 10^{-5} \text{ T}$. Assuming the earth's magnetic field to be a dipole field and mean radius of the earth $6.4 \times 10^6 \text{ m}$, calculate (i) the earth's magnetic moment and (ii) the amount of current flowing round the single turn of wire round the equator to produce the dipole moment. [(i). $8.1 \times 10^{22} \text{ A.m}^2$ (ii) $6.3 \times 10^8 \text{ A}$] [3.98 $\times 10^2 \text{ A.m}^{-1}$ and 10^6 A.m^{-1}]
6. Calculate the field intensity of a magnetic field and the intensity of magnetization if 0.2A current is passed through a winding of 20 turns/cm over an iron anchor ring having magnetic field 1.26T. [3.98 $\times 10^2 \text{ A.m}^{-1}$ and 10^6 A.m^{-1}]
7. The dipole moment associated with an atom of iron in an iron bar is $2.1 \times 10^{-23} \text{ J/T}$. Assume that all the atoms in the bar, which is 5cm long and has a cross-sectional area of 10^{-4} m^2 , have their dipole moments aligned. Calculate the dipole moment of the bar. [8.9Am²]
8. A copper wire has $10^{29} \text{ electrons/m}^3$, cross-sectional area of 2 mm^2 and carries current of 5A. Calculate the force acting on each electron if the wire is placed in a magnetic flux density 0.15 T which is perpendicular to the wire. [3.75 $\times 10^{-24} \text{ N}$]
9. Calculate the current necessary to produce magnetic flux of $5 \times 10^{-4} \text{ Wb}$ in a soft iron ring of cross-sectional diameter 8cm, mean circumference 200cm with 400 turns of wire and relative permeability 1800. [0.22A]