CHP = 4

Simplification of Boolean Function

Karnaugh Map (K-map)

$SOP = A.B = 11$ or $\bar{A}\bar{B} = 00$

$POS = A+B = 00$ or $\bar{A}+\bar{B} = 11$

→ It is extremely useful tool for simplifying boolean function involving 2, 3, 4, 5 or even more variable.

→ The map is a diagram made up of squares, each square is called cell. The no of cell in a map depends on the no of variables. If there are 'n' variables then the map contain 2^n cell.

→ An important property of K-Map is that one and only one variable changes when we move one cell to next cell along any row & column.

→

Two variables K-Map

A \ B	0	1
0	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01
1	$A\bar{B}$ 10	AB 11

$f(x,y) = \Sigma(1,2,3)$

x \ y	0	1
0	0	1: 1
1	2: 1	3: 1

$F = x + y$ (using KMap)

Without KMap

$$F = m_1 + m_2 + m_3$$
$$= x'y + xy' + xy$$
$$= x'y + x$$
$$= (x+x').(x+y)$$
$$= (x+y)$$

$F = \Sigma(0,3)$

$x \backslash y$	0	1
0	(0) 1	(1)
1	(2)	(3) 1

$F = x'y' + xy$

$F = \Sigma(0,1,2)$

$x \backslash y$	0	1
0	(0) 1	(1) 1
1	(2) 1	(3)

$F = \overline{x} + \overline{y}$

$F = \Sigma(0,1,2,3)$

$x \backslash y$	0	1
0	(0) 1	(1) 1
1	(2) 1	(3) 1

$F = 1$

<u>Without K-map</u>

$$
\begin{aligned}
F &= \Sigma(0,1,2) \\
&= m_0 + m_1 + m_2 \\
&= \overline{x}\,\overline{y} + \overline{x}y + x\overline{y} \\
&= \overline{x}(\overline{y} + y) + x\overline{y} \\
&= \overline{x} + x\overline{y} \\
&= (\overline{x} + x) \cdot \overline{x} + \overline{y} \\
&= (\overline{x} + \overline{y})
\end{aligned}
$$

$$
\begin{aligned}
F &= \Sigma(0,1,2,3) \\
&= m_0 + m_1 + m_2 + m_3 \\
&= \overline{x}y' + x'y + xy' + xy \\
&= x'(y + y') + x(y + y') \\
&= x' + x \\
&= 1
\end{aligned}
$$

Three-Variables K-Map:

No. of minterms $= 2^3 = 8$

x \ yz	00	01	11	10
0	0 m_0	1 m_1	3 m_3	2 m_2
1	4 m_4	5 m_5	7 m_7	6 m_6

or

xy \ z	0	1
00	0	1
01	2	3
11	6	7
10	4	5

Rules:

1) Combining of 8 adjacent squares gives the function value equals to 1.
2) Combination of '4' adjacent squares gives the term with single literal.
3) Combination of 2 adjacent squares gives the term in 2-literals.
4) An uncombined square gives the term with '3' literals.

→ An uncombined Square or single Square give the term with '4' literals.

Qu Simplify the Boolean function:

1) $f(w, x, y, z) = \Sigma(0, 1, 2, 3, 4, 5, 6, 8, 9, 12, 13, 14)$

wx \ yz	00	01	11	10
00	0: 1	1: 1	3: D	2: 1 I_2
01	4: I_2 1	5: I_1 1	7	6: 1
11	12: I_3 1	13: 1	15	14: 1 I_3
10	8: 1	9: 1	11	12: D

$F = \overline{y} + \overline{\omega}\,\overline{z} + x\overline{z}$

2) $F(A, B, C, D) = \Sigma(4, 6, 7, 15)$

AB \ CD	00	01	11	10
00	0	1	3	2
01	4: 1	5	7: 1	6: 1
11	12	13	15: 1	14
10	8	9	11	10

$F = B \bullet CD + \overline{A} B \overline{D}$

3) $F(w,x,y,z) = \Sigma(2,3,12,13,14,15)$

wx \ yz	00	01	11	10
00	0	1	3 1	2 1
01	4	5	7	6
11	12 1	13 1	15 1	14 1
10	8	9	11	10

$F = \overline{w}\,\overline{x}\,y + wx$

4) $f(A,B,C,D) = \Sigma(3,7,11,13,14)$

AB \ CD	00	01	11	10
00	0	1	3 1	2
01	4	5	7 1	6
11	12	13 1	15	14 1
10	8	9	11 1	10

$F = AB\overline{C}D + ABC\overline{D} + \overline{A}CD + \overline{B}CD$

5) $f(w,x,y,z) = \Sigma(1,4,5,6,12,14,15)$

wx \ yz	00	01	11	10
00	0	1 1	3	2
01	4 1 I_1	5 1	7	6 1 I_1
11	12 1	13	15 1	14 1
10	8	9	11	10

$F = x\overline{z} + \overline{w}\,\overline{y}z + wxy$

(6) $F = (A, B, C, D) = (0, 1, 2, 4, 5, 7, 15)$

AB \ CD	00	01	11	10
00	0 1	1 1	3	2 1
01	4 1	5 1	7 1	6
11	12	13	15 1	14
10	8	9	11 1	10

$F = \overline{A}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{D} + BCD + ACD$

7) $f(w, x, y, z) = \Sigma(2, 3, 10, 11, 12, 13, 14, 15)$

wx \ yz	00	01	11	10
00	0	1	3 1	2 1
01	4	5	7	6
11	12 1	13 1	15 1	14 1
10	8	9	11 1	10 1

$F = wx + \overline{x}y$

$f(A, B, C, D) = \Sigma(0, 2, 4, 5, 6, 7, 8, 10, 13, 15)$

AB \ CD	00	01	11	10
00	0: 1	1:	3:	2: 1
01	4: 1	5: 1	7: 1	6: 1
11	12:	13: 1	15: 1	14:
10	8: 1	9:	11:	10: 1

$F = \bar{A}B + \bar{B}\bar{D} + BD$

Simplify Boolean Function in;
SoP (b) POS.

To simplify any Boolean function in SoP form combine the 1's in the map.
To simplify any Boolean function in POS form combine 0's in the map.

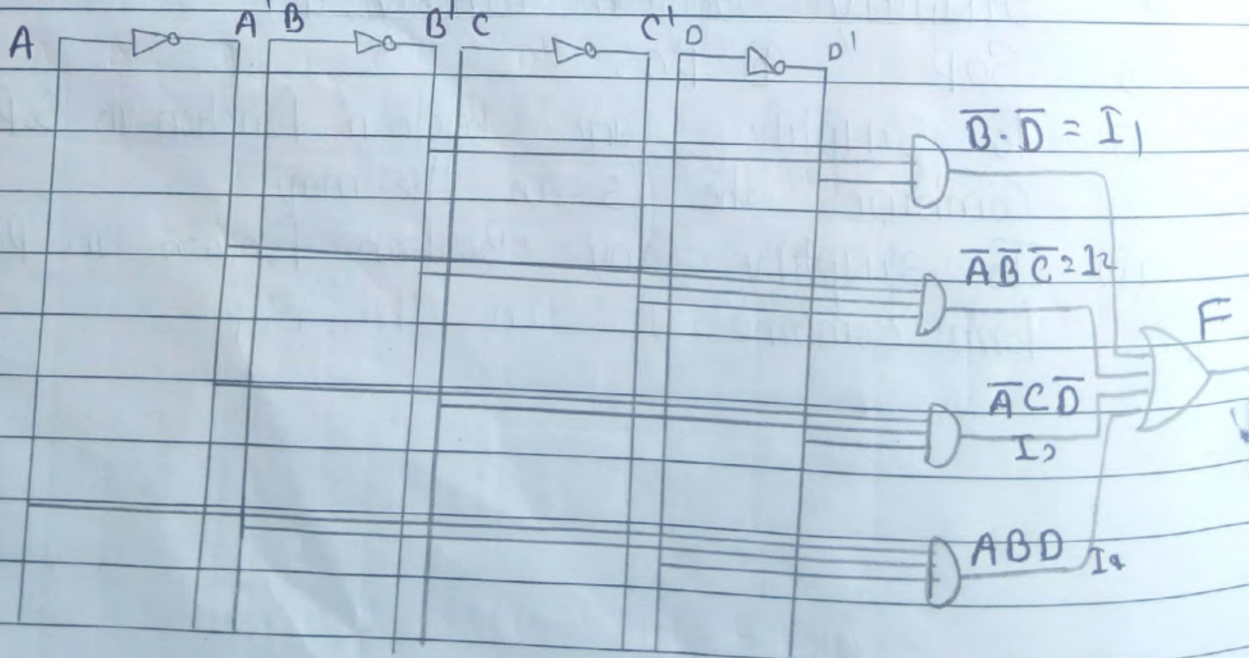
Some questions;

$F(ABCD) = \Sigma(1, 2, 6, 8, 10, 13, 15, 0)$

AB \ CD	00	01	11	10
00	0: 1	1: 1	3	2: 1
01	4	5	7	6: 1
11	12	13: 1	15: 1	14
10	8: 1	9	11	10: 1

$$F = \bar{B}\bar{D} + \bar{A}\bar{B}\bar{C} + \bar{A}C\bar{D} + ABD$$

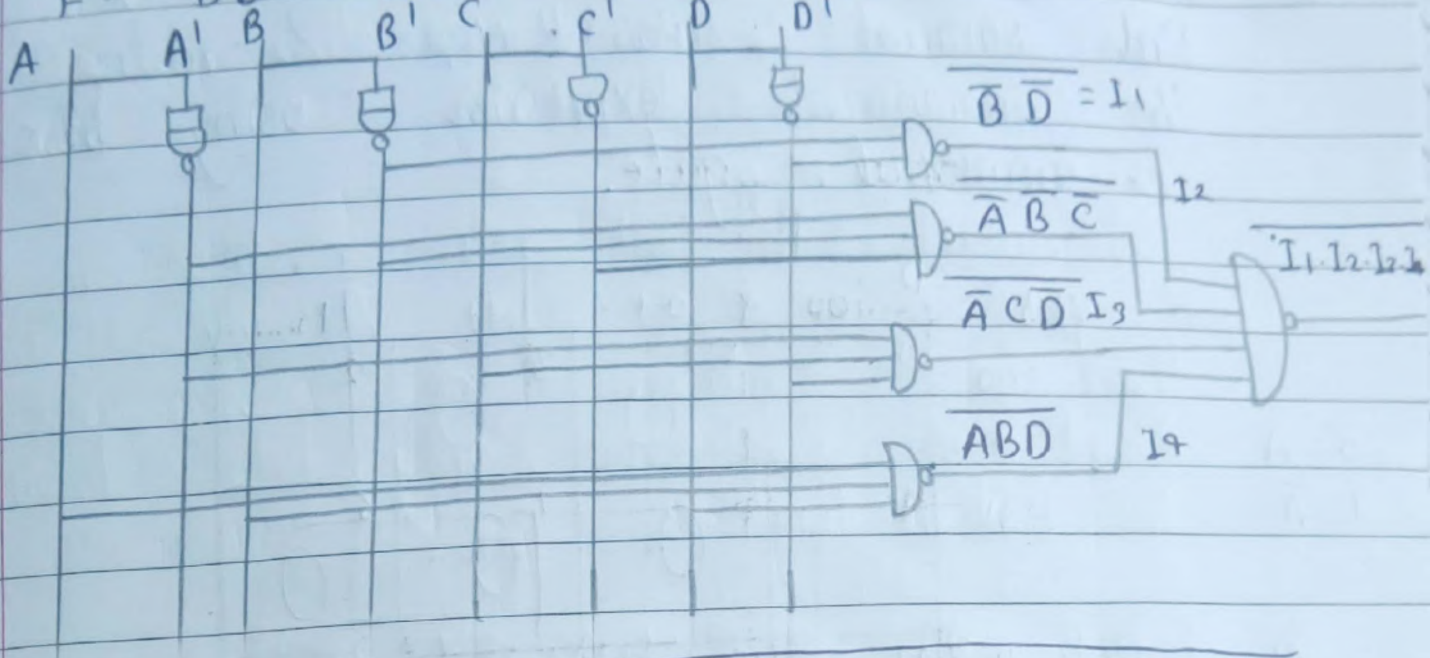
Realization using basic gate



$F = I_1 + I_2 + I_3 + I_4$

Realization using NAND gate :

$F = \bar{B}\bar{D} + \bar{A}\bar{B}\bar{C} + \bar{A}C\bar{D} + ABD$



Using deMorgan's law $= \overline{\overline{\bar{B}\bar{D}} \cdot \overline{\bar{A}\bar{B}\bar{C}} \cdot \overline{\bar{A}C\bar{D}} \cdot \overline{ABD}}$

$= \bar{B}\bar{D} + \bar{A}\bar{B}\bar{C} + \bar{A}CD + ABD$

$= F$

$F(A,B,C,D) = \prod(5,7,13,15,3,1,6,9,11)$

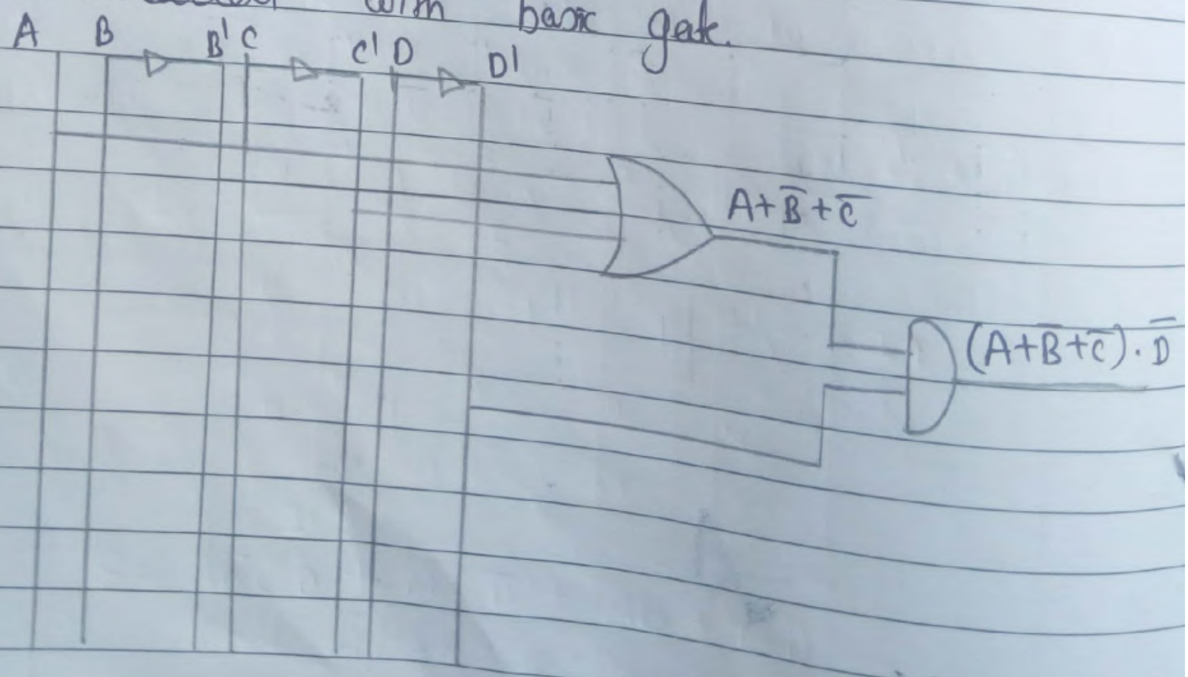
Find Minimal POS K-Map & realize the minimized expression using basic & universal gate.

$0 = A$
$1 = \bar{A}$

AB \ CD	00	01	11	10
00	0:	1: 0	3: 0	2:
01	4:	5: 0	7: 0	6: 0
11	12:	13: 0	15: 0	14:
10	8:	9: 0	11: 0	10:

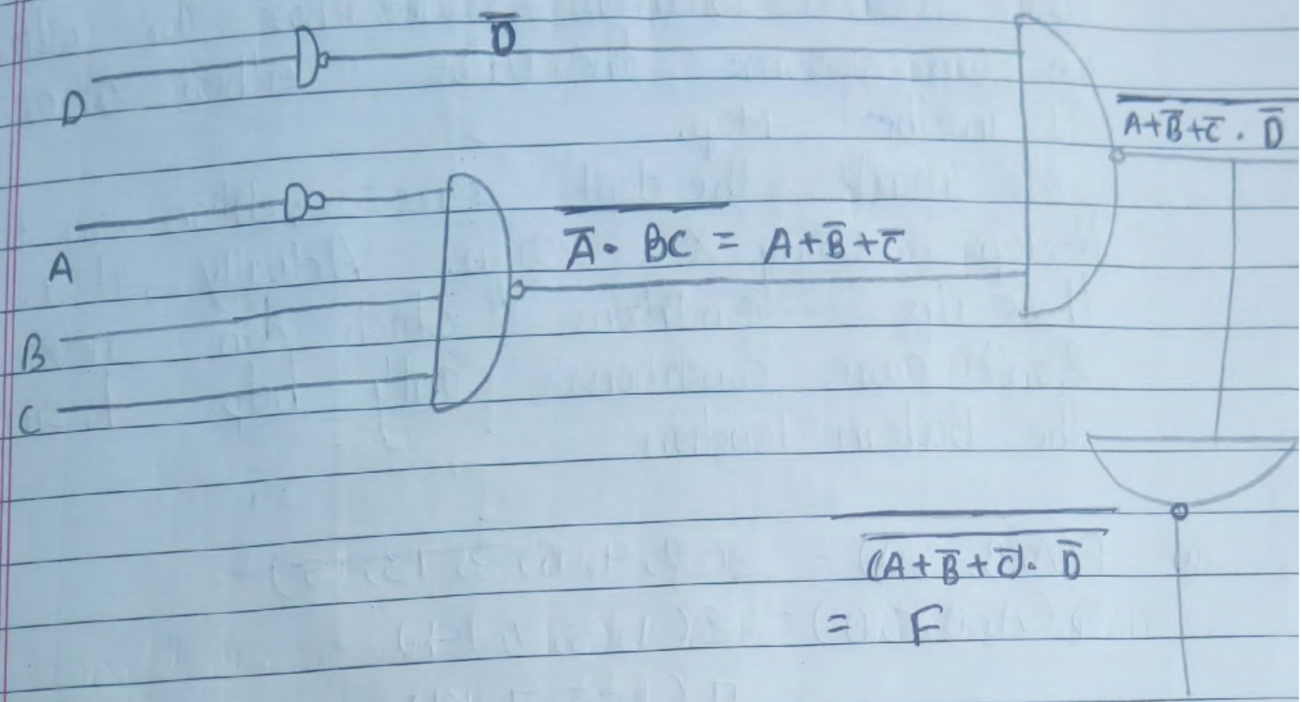
$F = \bar{D}\cdot(\bar{A}+\bar{B}+\bar{C})$

Realization with basic gate.

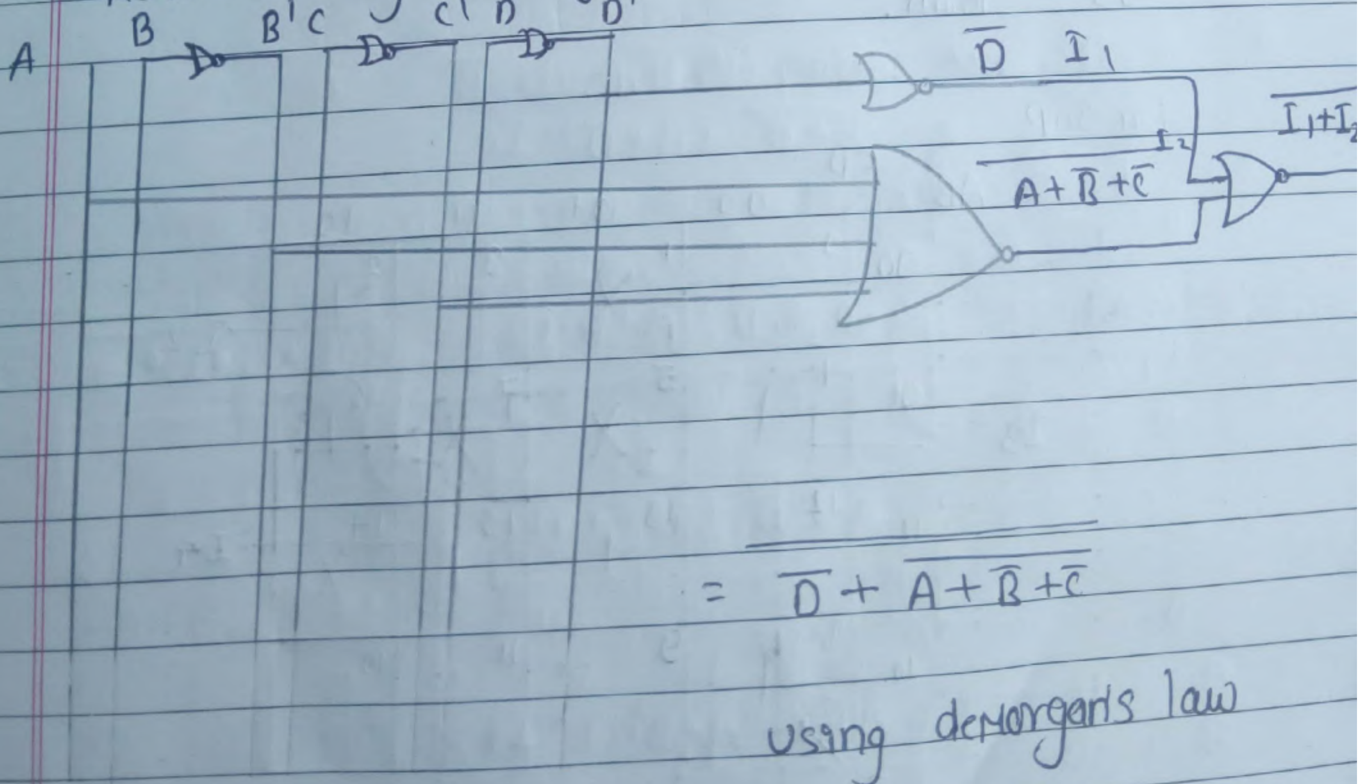


Realize using NAND.

→ $F = \overline{D}\cdot(A+\overline{B}+\overline{C})$



→ Realize using Nor gate:



$= \overline{\overline{D}+\overline{A+\overline{B}+\overline{C}}}$

using demorgan's law

$= \overline{\overline{D}}\cdot\overline{\overline{A+\overline{B}+\overline{C}}}$

$= D\cdot(A+\overline{B}+\overline{C})$

Don't care conditions:

Don't care conditions are those for which we can assume the value either '0' or '1' in the Map.

We mark the don't care condition in the K-Map using 'X' sign. Actually, it is those i/p conditions which can never occur. Don't care conditions only help to simplify the Boolean function.

Qu $F(A,B,C,D) = \Sigma(2,4,6,8,13,15)$

$Q(A,B,C,D) = \Sigma(1,5,7,14)$

$\Pi(1,5,7,14)$

Simplify the boolean expression in SoP & PoS form.

For SoP

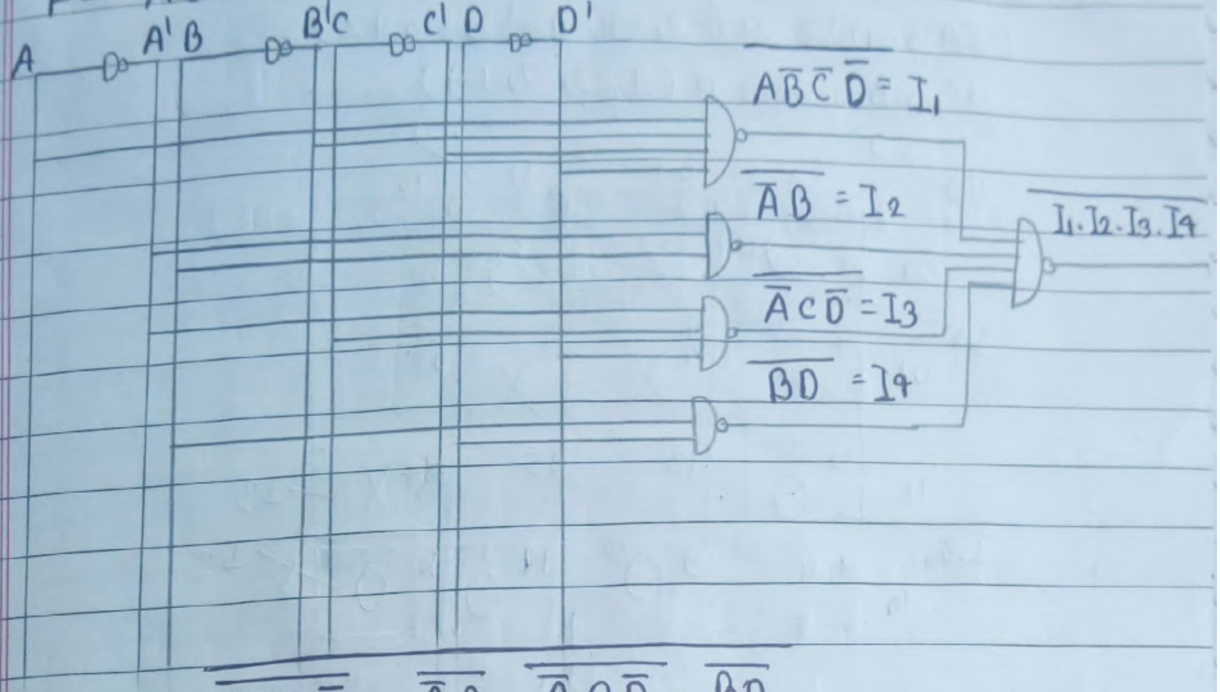
AB \ CD	00	01	11	10
00	0	1 X	3	2 1
01	4 1	5 X	7 X	6 1
11	12	13 1	15 1	14 X
10	8 1	9	11	10

I1 I2 I3 I4

$F = A\bar{B}\bar{C}\bar{D} + \bar{A}C\bar{D} + \bar{A}B + BD$

Realization using NAND gate:

$F = A\bar{B}\bar{C}\bar{D} + \bar{A}B + \bar{A}C\bar{D} + BD.$



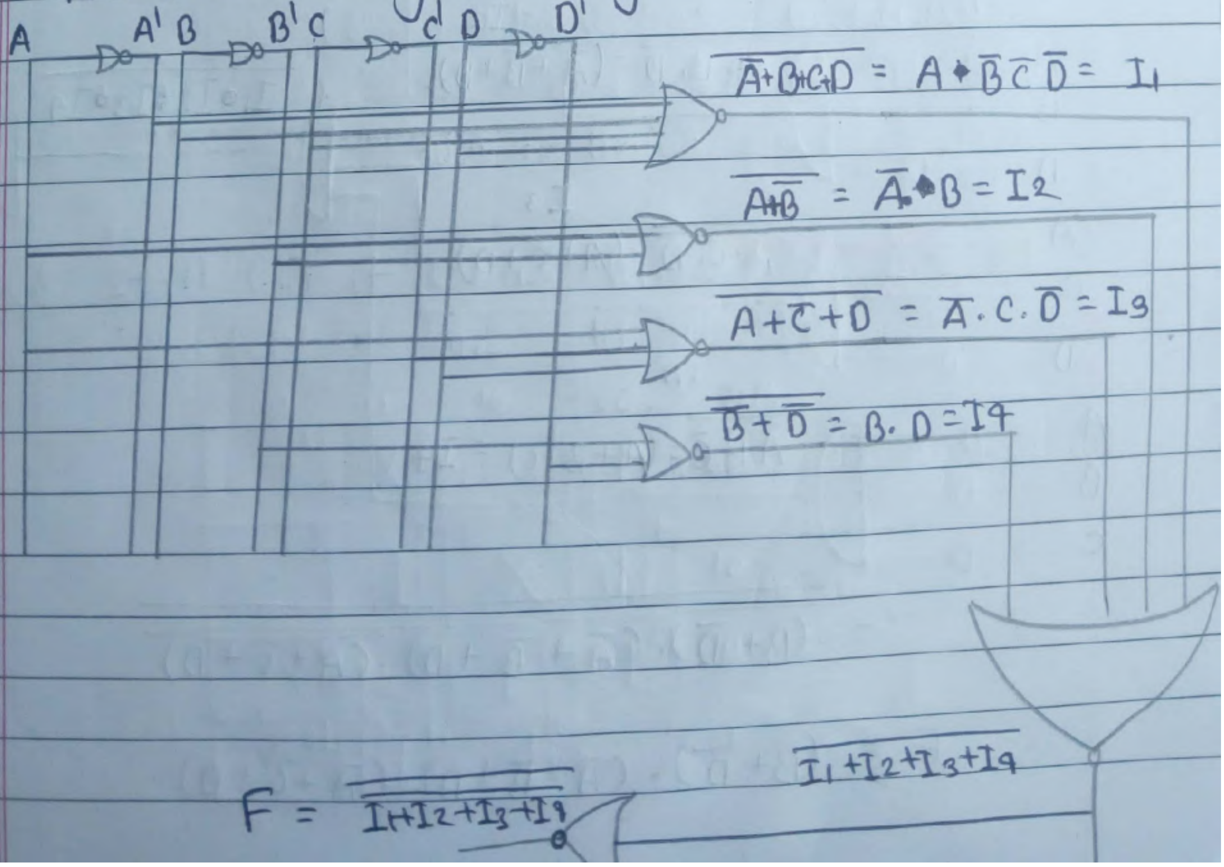
Here, $\overline{\overline{A\bar{B}\bar{C}\bar{D}} \cdot \overline{\bar{A}B} \cdot \overline{\bar{A}C\bar{D}} \cdot \overline{BD}}$

$= \overline{\overline{A\bar{B}\bar{C}\bar{D}}} + \overline{\overline{\bar{A}B}} + \overline{\overline{\bar{A}C\bar{D}}} + \overline{\overline{BD}}$

$= A\bar{B}\bar{C}\bar{D} + \bar{A}B + \bar{A}C\bar{D} + BD$

$= F$

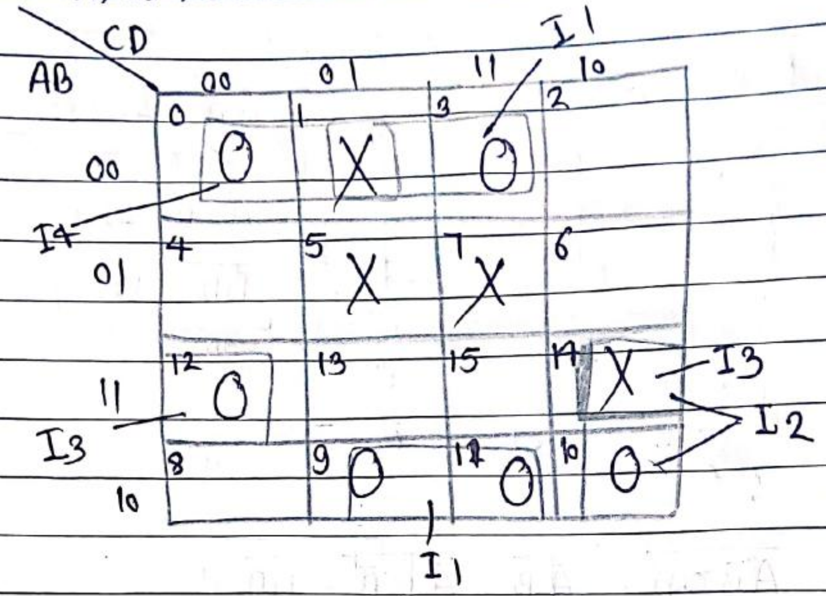
Relization using NOR gate;



For POS;

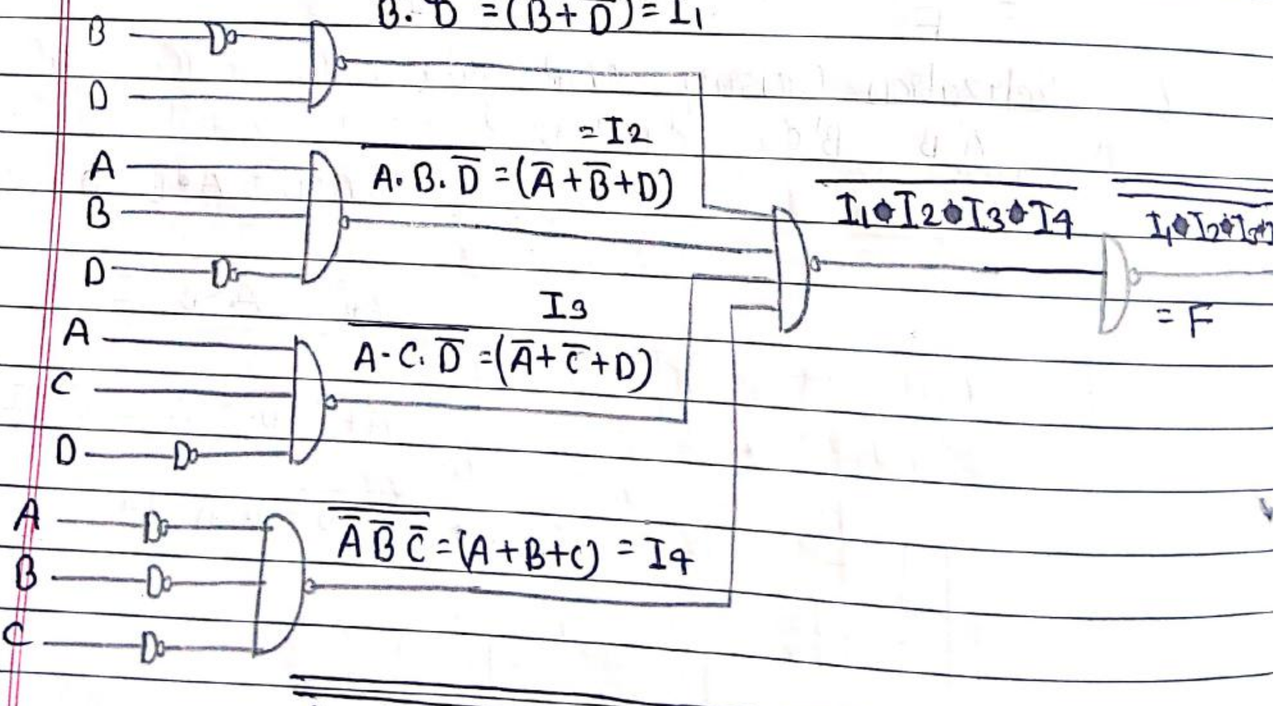
$F(ABCD) = \pi(0, 3, 9, 10, 11, 12)$

$d(A,B,C,D) = \pi(1, 5, 7, 14)$



$F = (B+\overline{D}) \cdot (\overline{A}+\overline{B}+D) \cdot (\overline{A}+\overline{C}+D)\ (A+B+C)$

Realization using NAND.

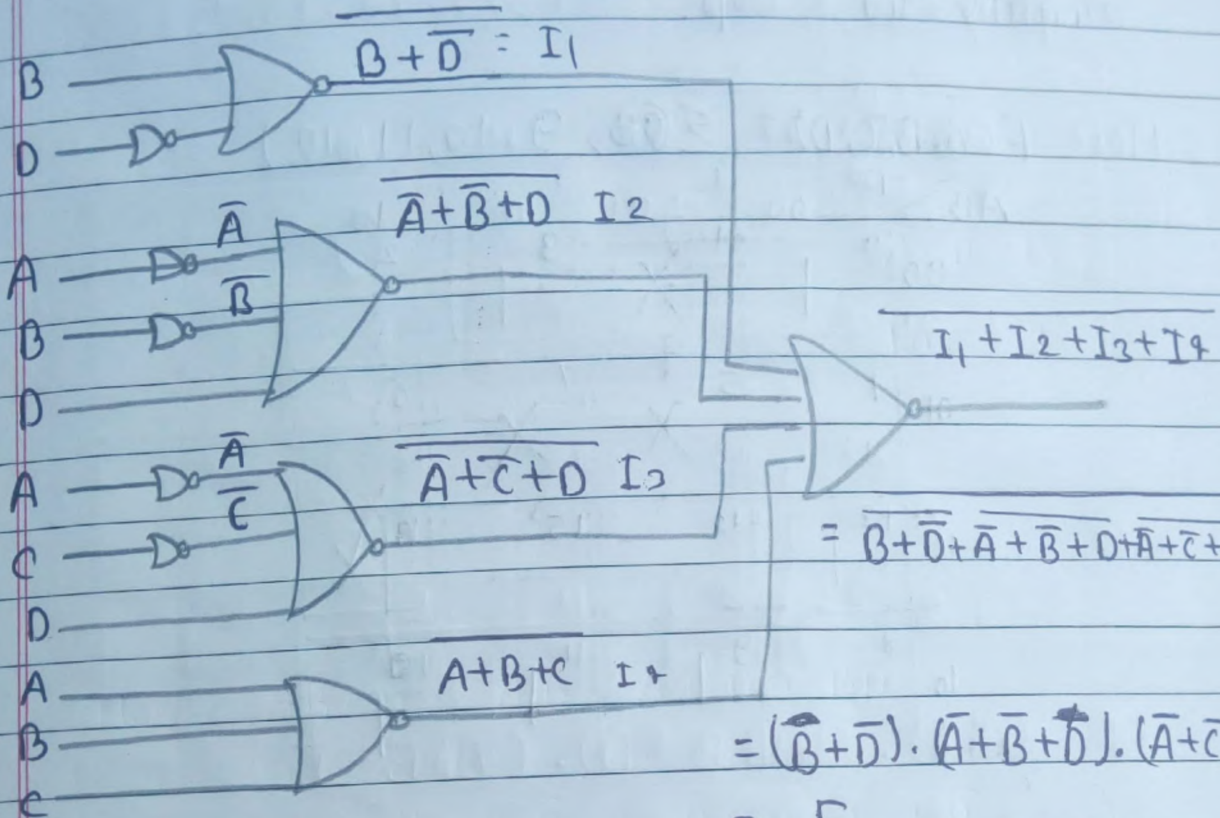


$= \overline{\overline{(B+\overline{D}) \cdot (\overline{A}+\overline{B}+D) \cdot (\overline{A}+\overline{C}+D)}}$

$F = (B+\overline{D}) \cdot (\overline{A}+\overline{B}+D) \cdot (\overline{A}+\overline{C}+D)$

Realization using nor gate;

$F = (B+\bar{D})\cdot(\bar{A}+\bar{B}+D)\cdot(\bar{A}+\bar{C}+D)\cdot(A+B+C)$



$= \overline{\overline{B+\bar{D}}+\overline{\bar{A}+\bar{B}+D}+\overline{\bar{A}+\bar{C}+D}+\overline{A+B+C}}$

$= (B+\bar{D})\cdot(\bar{A}+\bar{B}+D)\cdot(\bar{A}+\bar{C}+D)\cdot(A+B+C)$

$= F$

Q) $F(A,B,C,D) = \Pi(2,4,6,8,13,15)$
$\phi(A,B,C,D) = \Pi(1,5,7,14)$
Simplify in SOP.

Here, $F(A,B,C,D) = \Sigma(0,3,9,10,11,12)$

AB \ CD	00	01	11	10
00	(0) 1	(1) X	(3) 1	(2)
01	(4)	(5) X	(7) X	(6)
11	(12) 1	(13)	(15)	(14) X
10	(8)	(9) 1	(11) 1	(10) 1

$$F = \overline{B}D + AC\overline{D} + AB\overline{D} + \overline{A}\,\overline{B}\,\overline{C}$$

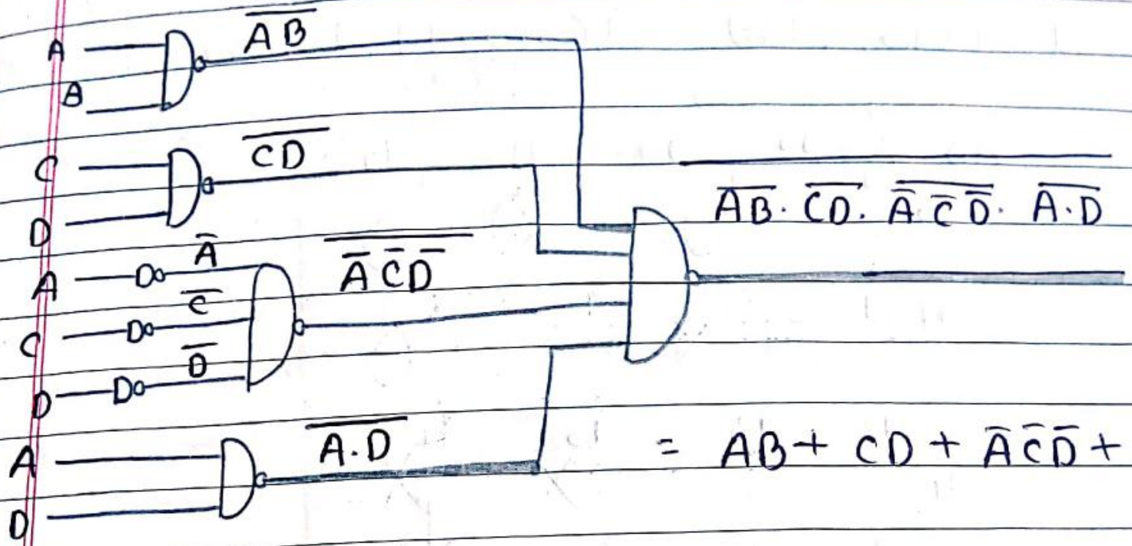
Q7) $F(A,B,C,D) = \Pi m(m_1, m_2, m_6, m_8)$
$d(A,B,C,D) = \Sigma m(3,5,14,15,10)$
Find the minimal K-Map for SOP & POS and realize using NAND or NOR.

For SOP: $F(A,B,C,D) = \Sigma m(0,4,7,9,11,12,13)$
$d(A,B,C,D) = \Sigma m(3,5,14,15,10)$

AB \ CD	00	01	11	10
00	(0) 1	(1)	(3) X	(2)
01	(4) 1	(5) X	(7) 1	(6)
11	(12) 1	(13) 1	(15) X	(14) X
10	(8)	(9) 1	(11) 1	(10) X

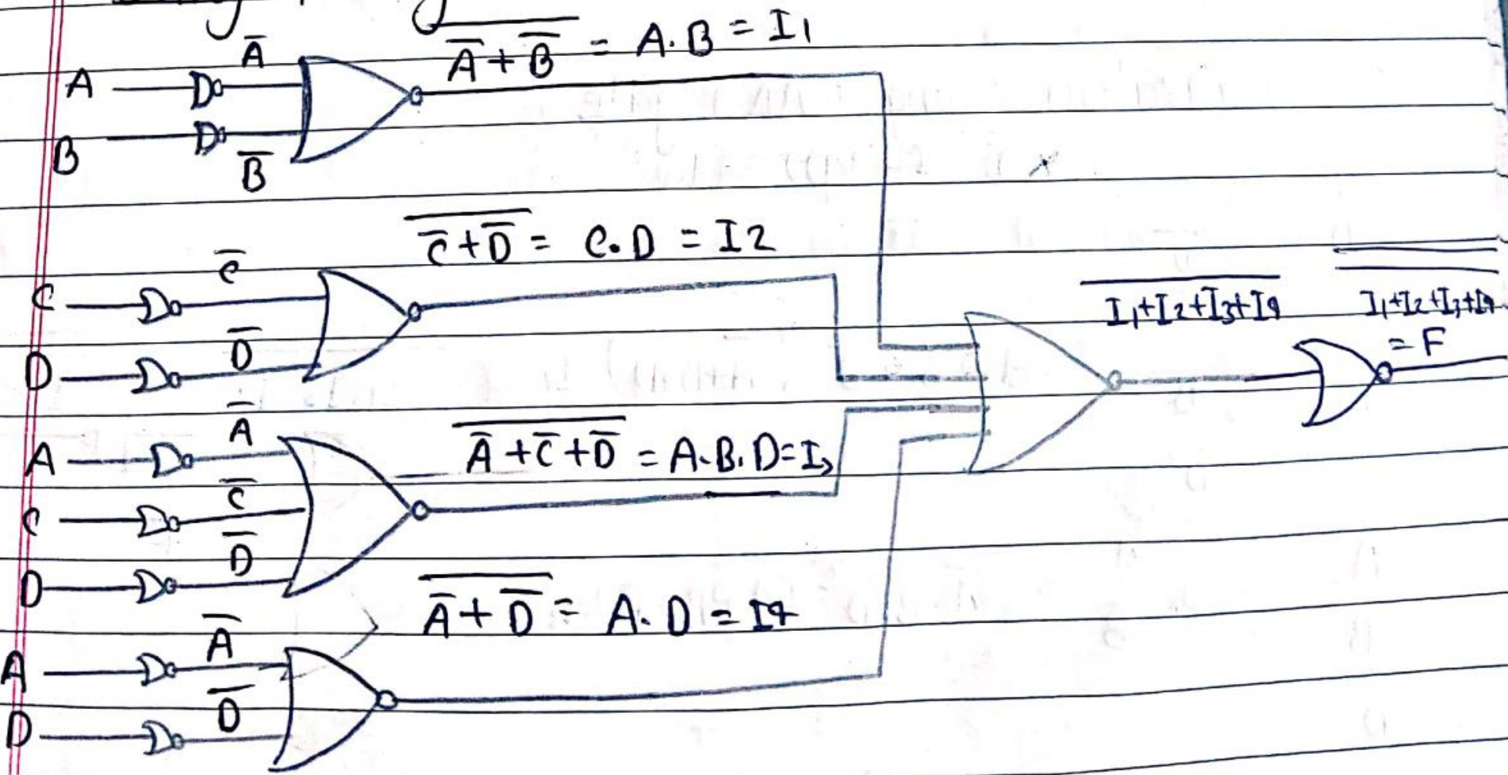
$F = CD + AB + \bar{A}\bar{C}\bar{D} + AD$

Realization using NAND gate;



$= AB + CD + \bar{A}\bar{C}\bar{D} + AD = F$

Using Nor gate



For POS

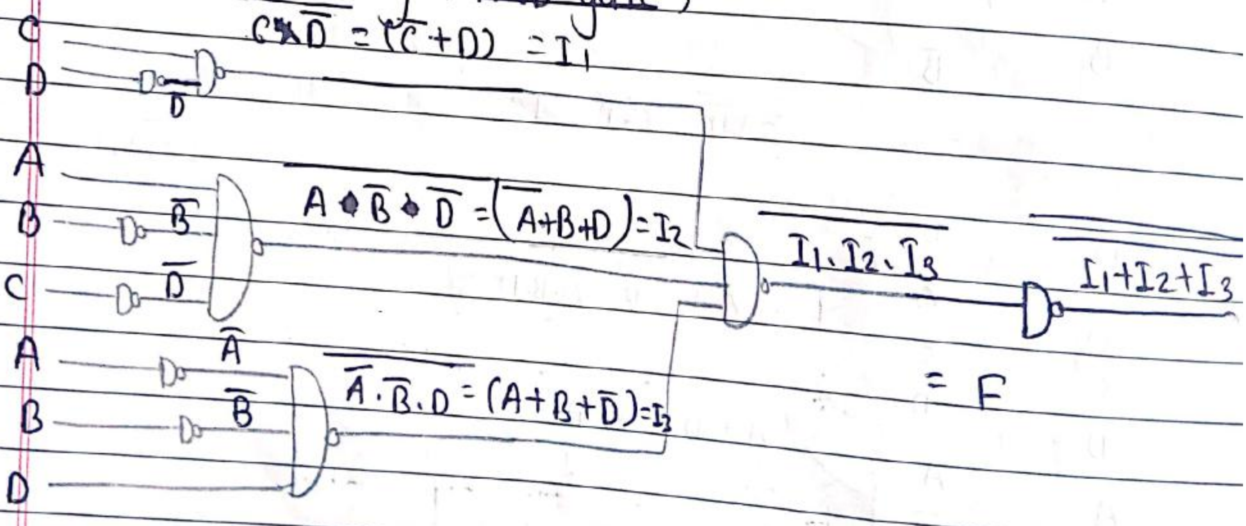
$F = \cancel{\Pi(1,2,6,8)} \quad \Pi(1,2,6,8)$

$d = \cancel{\Pi(1,5,7,14)} \quad \Pi(3,5,14,15,10)$

AB \ CD	00	01	11	10
00	0	1: 0	3: X	2: 0
01	4	5: X	7	6: 0
11	12	13	15: X	14: X
10	8: 0	9	11	10: X

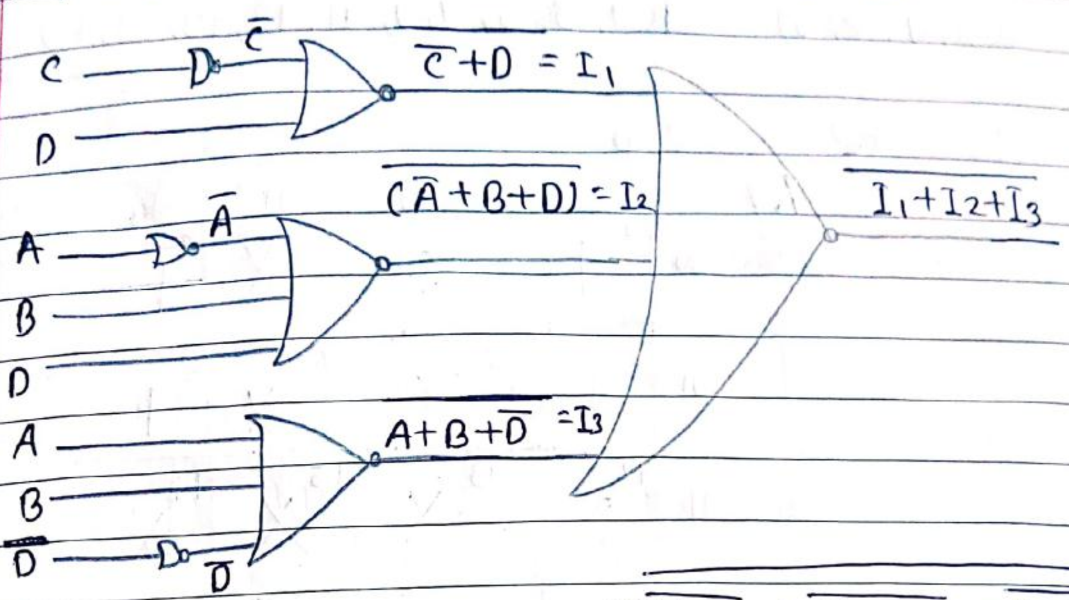
$F = (\overline{C}+D) \cdot (\overline{A}+B+D) \cdot (A+B+\overline{D})$

Realization using NAND gate;



Relrazation using NOR gate:

$F = (\bar{C}+D) \cdot (\bar{A}+B+D) \cdot (A+B+\bar{D})$



$= \overline{\overline{\bar{C}+D} + \overline{\bar{A}+B+D} + \overline{A+B+\bar{D}}}$

$= (\bar{C}+D) \cdot (\bar{A}+B+D) \cdot (A+B+\bar{D})$

$= F$

For SOP

$f(A,B,C,D) = \Sigma\,(0,4,7,9,11,12,13)$

$d(A,B,C,D) = \Sigma(3,5,10,14,15)$

AB \ CD	00	01	11	10
00	0: 1	1:	3: X	2:
01	4: 1	5: X	7: 1	6:
11	12: 1	13: 1	15: X	14: X
10	8:	9: 1	11: 1	10: X

$F = C\bar{D} + AB + AD + \bar{A}\bar{C}\bar{D}$

\# $F(x,y,z,\phi) = \Sigma(0,2,4,6,8,10)$
$d(x,y,z,\phi) = d(1,3,5,7,9,11,12,13,14,15)$

For SOP

XY \ Zφ	00	01	11	10
00	[0] 1	[1] X	[3] X	[2] 1
01	[4] 1	[5] X	[7] X	[6] 1
11	[12] X	[13] X	[15] X	[14] X
10	[8] 1	[9] X	[11] X	[10] 1

F = 1
For POS = 0.

\# Five Variable Map $(A,B,C,D,E) = \Sigma(0,1,3,5,8,9,19,23,28,31,27)$
No of Minterms $= 2^5 = 32$.

BC \ DE	00	01	11	10
00	[0] 1	[1] 1	[3] 1	[2]
01	[4]	[5] 1	[7]	[6]
11	[12]	[13]	[15]	[14]
10	[8] 1	[9] 1	[11]	[10]

A=0

BC \ DE	00	01	11	10
00	[16]	[17]	[19] 1	[18]
01	[20]	[21]	[23] 1	[22]
11	[28] 1	[29]	[31] 1	[30]
10	[24]	[25]	[27] 1	[26]

A=1

$F = ADE + \overline{B}\,\overline{C}DE + \overline{A}\,\overline{C}\,\overline{E} + \overline{B}\,\overline{D}E\overline{A}$
$+ ABC\overline{D}\,\overline{E}$.

- Combination of 32 adjacent squares gives the function value equals to 1.
- Combination of 16 adjacent squares gives the product term with single literal.
- Combination of 8 adjacent squares give the product term with two literals.
- Combination of '2' adjacent squares gives the product term with four literal.
- Uncombined square gives the product term with '5' literals