new chapter # Two Port Network

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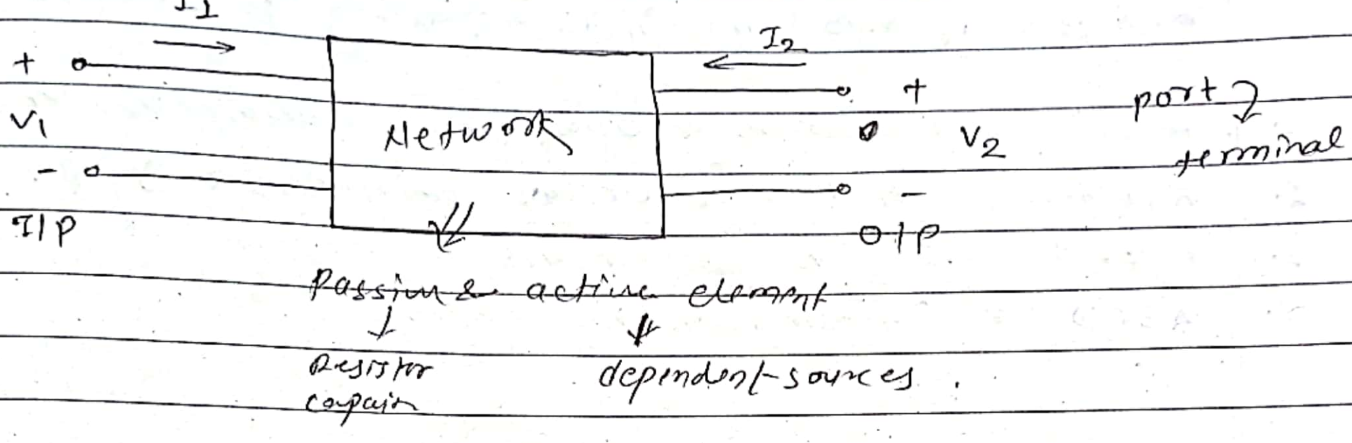
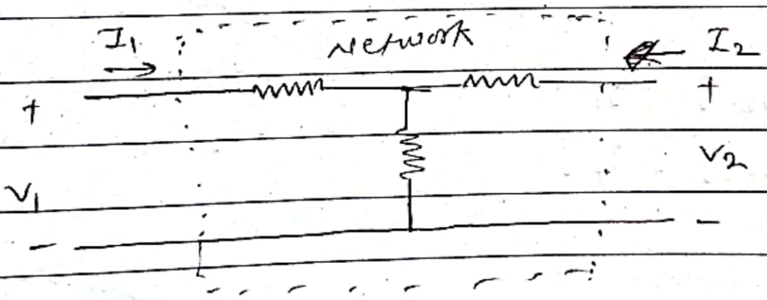
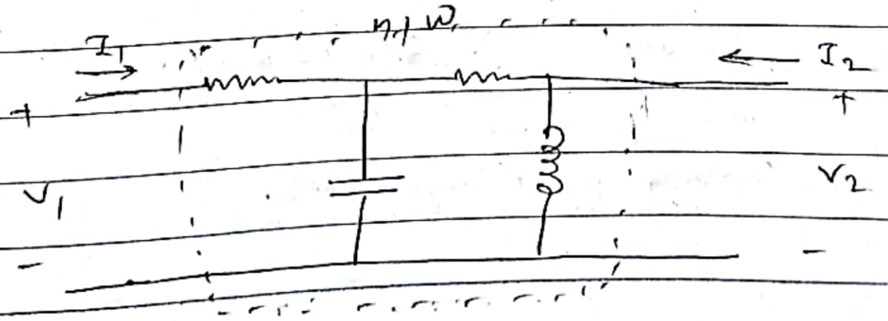


* Properties of two port network.

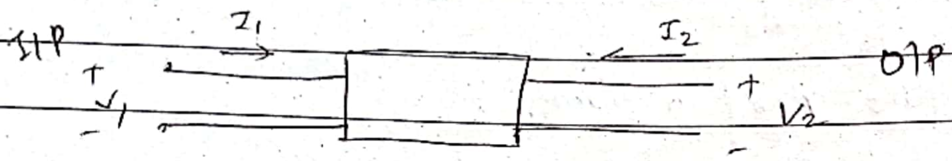
1. It consists of two pair of terminals
2. It has one I/P and one O/P side.
3. We can provide I/P and take O/P simultaneously.
4. The N/W consists of passive elements (resistor, inductor and capacitor) in relaxed state (no initial value) and active sources (dependent sources only).





* Analysis of two port network using parameters

1. Impedance parameters or open circuit parameters or Z-para
2. Admittance or short circuit parameters or Y-parameters
3. Hybrid or h-parameters
4. ABCD or Transmission parameters.



1. Z or Impedance or open circuit parameters.

Set of standard equations are,

$V_1 = Z_{11} I_1 + Z_{12} I_2$ —— ① voltage equation.

$V_2 = Z_{21} I_1 + Z_{22} I_2$ —— ②

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \underbrace{\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}}_{\text{Z- parameters.}} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

from eqn ①, $I_2 = 0$ (assume) ⇒ (O/P is opened)

$V_1 = Z_{11} I_1$

⇒ $Z_{11} = V_1 / I_1$ (Driving point impedance)

from eqn ②, $I_2 = 0$

$V_2 = Z_{21} I_1$

⇒ $Z_{21} = \dfrac{V_2}{I_1}$ (Forward impedance)

Assume $I_1 = 0$ (I/p is opened)

from ①

$$V_1 = Z_{11} \times 0 + Z_{12} I_2$$

$$\Rightarrow Z_{12} = \frac{V_1}{I_2} \quad \text{(Backward impedance)}$$

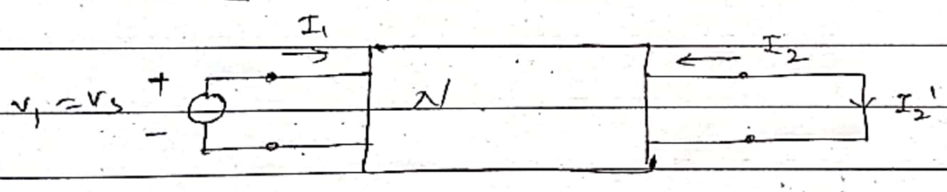
from ②

$$V_2 = 0 + Z_{22} I_2$$

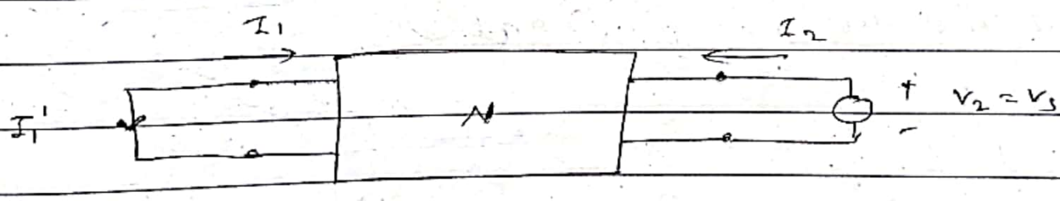
$$\Rightarrow Z_{22} = \frac{V_2}{I_2} \quad \text{(output impedance)}$$

* condition for reciprocity and symmetry.

Reciprocal :-



(a)



(b)

The network if produces the same current no matter exchanging the position of source is known as reciprocal.

In above ckt, the n/w is reciprocal if

$$I_1' = I_2'$$

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From fig. (a)

$V_1 = V_S, \quad I_2 = -I_2', \quad V_2 = 0$

Since,
$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{——— (1)}$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{——— (2)}$$

put above conditions in eqn (1) & (2)

$$V_S = Z_{11} I_1 \cancel{+} Z_{12} I_2' \quad \text{——— (3)}$$
$$0 = Z_{21} I_1 - Z_{22} I_2' \quad \text{——— (4)}$$

solving eqn (3) and (4) to get I_2'

$$I_2' = \frac{V_S Z_{21}}{Z_{11} Z_{22} - Z_{12} Z_{21}}$$

Again from fig (b),

$V_1 = 0, \quad I_1 = -I_1', \text{ and } V_2 = V_S.$

put above conditions in eqn (1) and (2)

$$0 = -Z_{11} I_1' + Z_{12} I_2 \quad \text{——— (5)}$$
$$V_S = -Z_{21} I_1' + Z_{22} I_2 \quad \text{——— (6)}$$

solving (5) and (6) to get I_1'

$$I_1' = \frac{V_S Z_{12}}{Z_{11} Z_{22} - Z_{12} Z_{21}}$$

To be reciprocal, $I_1' = I_2'$

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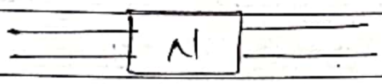
$$\frac{V_s Z_{21}}{Z_{11} Z_{22} - Z_{12} Z_{21}} = \frac{V_s Z_{21}}{Z_{11} Z_{22} - Z_{12} Z_{21}}$$

$\Rightarrow$ $\boxed{Z_{12} = Z_{21}}$

Symmetry :-



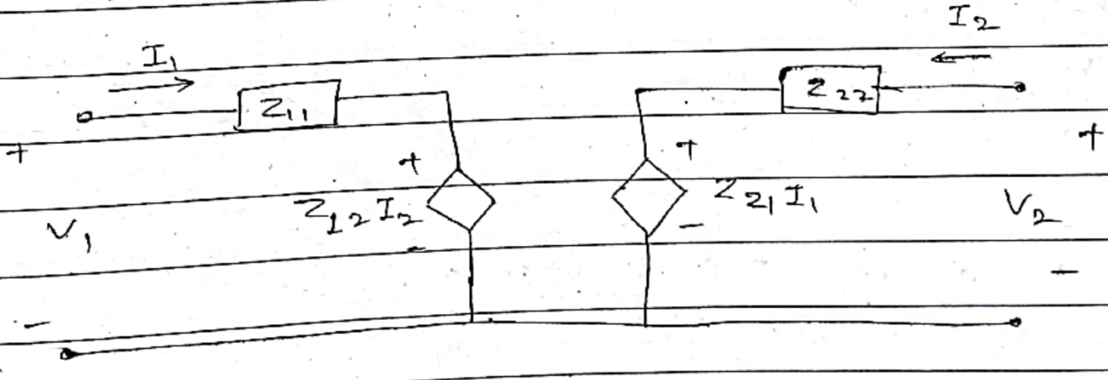
$$Z_{11} = \frac{V_1}{I_1} \qquad Z_{22} = \frac{V_2}{I_2}$$

$Z_{11} = Z_{22}$, To be symmetric.

* Equivalent circuit

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$



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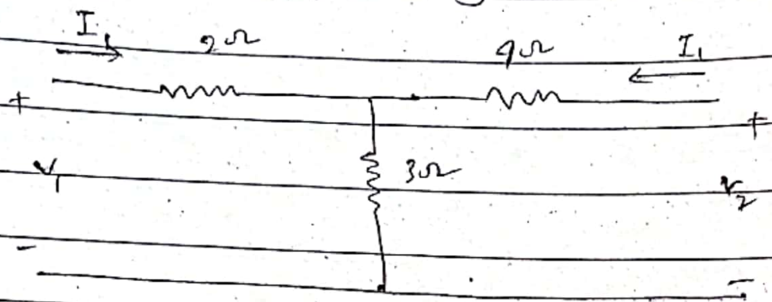
Q. Find the z-parameters of the circuit. Also check for reciprocity and symmetry. Draw the equivalent circuit.



Sol^n

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

let $I_2 = 0$, i.e. O/P is opening

in first mesh, using KVL

$-2I_1 - 3(I_1 + I_2) + V_1 = 0$

$+5I_1 = V_1$

$Z_{11} = \frac{V_1}{I_1} = 5\,\Omega$

and, $V_2 = 3 \times I_1$

$Z_{21} \; \frac{V_2}{I_1} = 3\,\Omega$

Again

let $I_1 = 0$ i.e. I/P is opening.

in 2nd mesh, applying KVL

$-4I_2 - 3(I_2 + I_1) + V_2 = 0$

$-4I_2 - 3I_2 + V_2 = 0$

$Z_{22} = \frac{V_2}{I_2} = 4\,\Omega.$

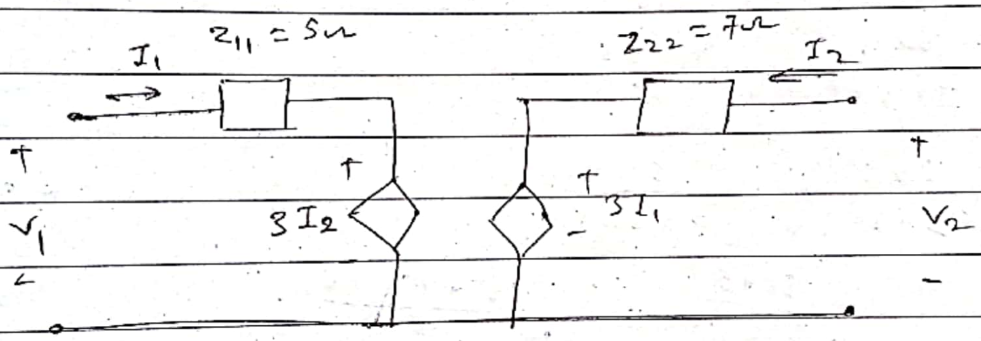
again, $V_1 = 3I_2$

$$Z_{12} = \frac{V_1}{I_2} = 3\Omega.$$

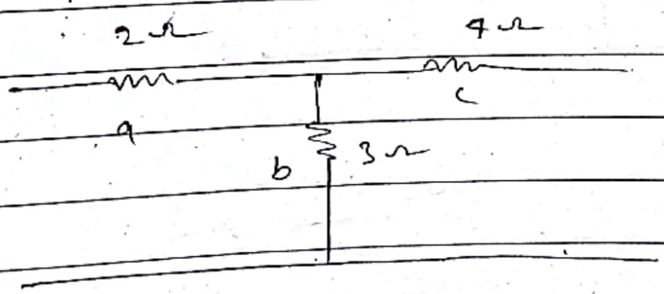
$$\begin{bmatrix} Z_{11} = 5\Omega & Z_{12} = 3\Omega \\ Z_{21} = 3\Omega & Z_{22} = 7\Omega \end{bmatrix}$$

since $Z_{12} = Z_{21} = 3\Omega$, the circuit is reciprocal.
since $Z_{11} \neq Z_{22}$, the circuit is not symmetrical.

Equivalent circuit

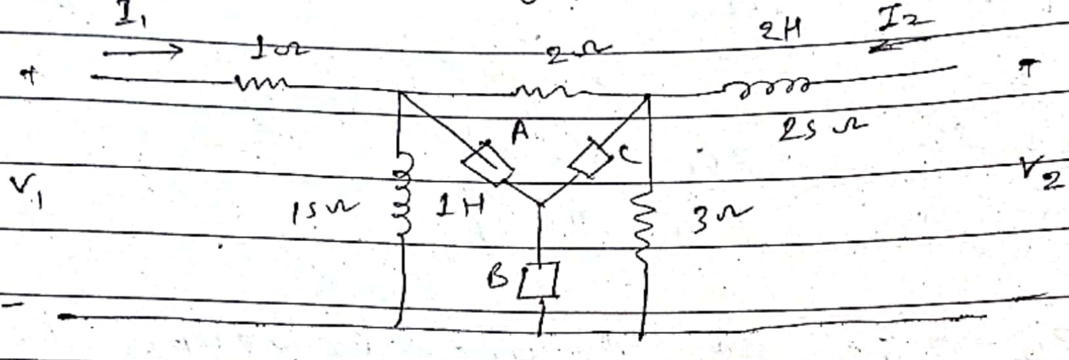


Note:
prototype n/w, T-n/w, star n/w.



$Z_{11} = a+b = 5\Omega$
$Z_{22} = b+c = 7\Omega$
$Z_{12} = Z_{21} = b = 3\Omega$

Q. Find the Z-parameters of the circuit. Also check for reciprocity and symmetry. Draw the equivalent circuit.



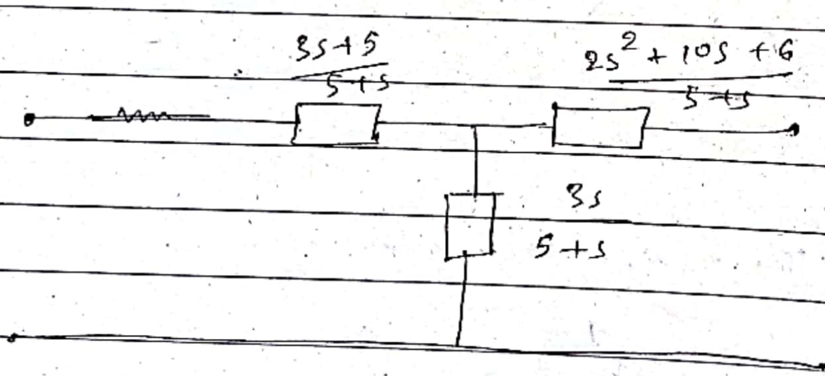
$$A = \frac{2 \times 1s}{2 + 1s + 3} = \frac{2s}{5+s}\ \Omega$$

$$A + 1 = \frac{2s}{5+s} + 1 = \frac{5+3s}{5+s}$$

$$B = \frac{3 \times 1s}{2 + 1s + 3} = \frac{3s}{5+s}\ \Omega$$

$$C = \frac{2 \times 3}{5+s} = \frac{6}{5+s}\ \Omega$$

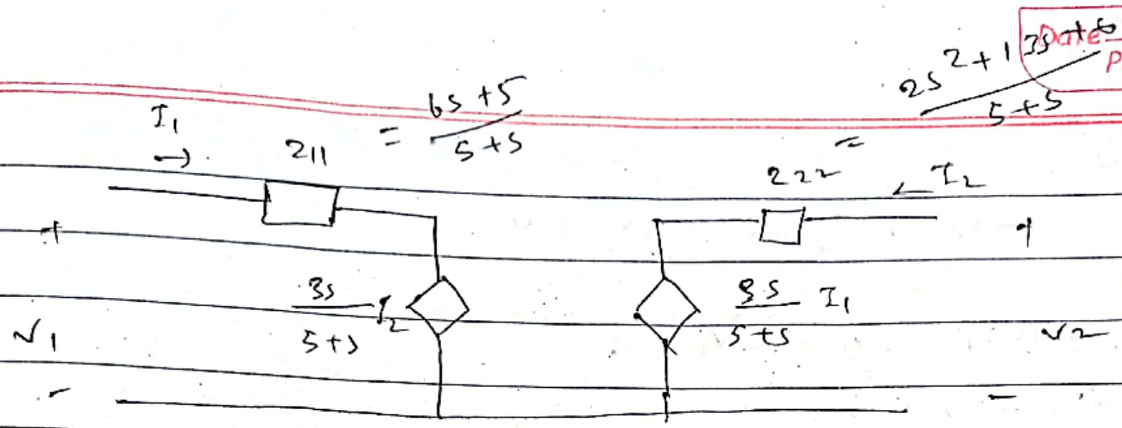
$$C + 2s = \frac{6}{5+s} + 2s = \frac{6 + 10s + 2s^2}{5+s}$$

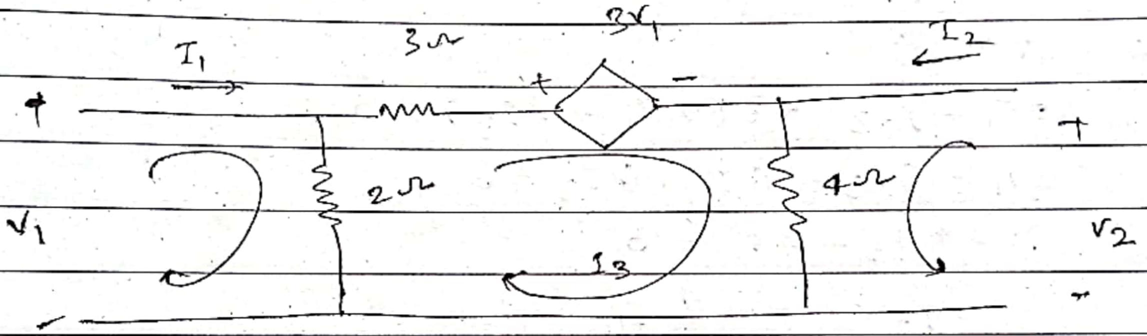


$$z_{11} = \frac{3s+5}{5+s} + \frac{3s}{5+s} = \frac{6s+5}{5+s}\ \Omega$$

$$z_{22} = \frac{2s^2+10s+6}{5+s} + \frac{3s}{5+s} = \frac{2s^2+13s+6}{5+s}\ \Omega.$$

$$z_{12} = z_{21} = \frac{3s}{5+s}\ \Omega.$$





Find the open circuit parameters. Also draw equivalent ckt.

Let, $I_2 = 0$, o/p ~~ckt~~ opening.

1st mesh using KVL

$$-2(I_1 - I_3) + V_1 = 0$$

$$V_1 = 2I_1 - 2I_3 \longrightarrow (1)$$

In 2nd mesh, KVL

$$-3I_3 - 3V_1 - 4(I_3 + I_2) - 2(I_3 - I_1) = 0$$

$$2I_1 - 9I_3 - 3V_1 = 0 \longrightarrow (2)$$

Again,

$$-4(I_3 + I_2) + V_2 = 0$$

$$V_2 = 4I_3 \longrightarrow (3)$$

$$I_3 = \frac{2I_1 - V_1}{2}$$

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from (2)

$$2I_1 - 9\frac{2I_1 - V_1}{2} - 3V_1 = 0$$

$$4I_1 - 18I_1 + 9V_1 - 6V_1 = 0$$

$$-14I_1 + 3V_1 = 0$$

$$3V_1 = 14I_1$$

$$\frac{V_1}{I_1} = \frac{14}{3} \Rightarrow z_{11} = \frac{14}{3}\,\Omega.$$

from (1) & (2)

$$2I_1 - 9I_3 - 3(2I_1 - 2I_3) = 0$$

$$2I_1 - 9I_3 - 6I_1 + 6I_3 = 0$$

$$-3I_3 - 4I_1 = 0$$

$$I_3 = -\frac{4}{3}I_1$$

from (3)

$$4I_3 = V_2$$

$$V_2 = 4 \times -\frac{4}{3}I_1$$

$$\frac{V_2}{I_1} = -\frac{16}{3} \Rightarrow z_{21} = -\frac{16}{3}\,\Omega.$$

let $I_1 = 0$, i/p is closing.

mesh (3).

$$V_2 - 4(I_2 + I_3) = 0$$

$$V_2 = 4I_2 + 4I_3 \quad \text{—— (4)}$$

mesh (2),

$$3I_3 - 3V_1 - 4(I_2 + I_3) - 2I_3 = 0$$

$$+3I_3 - 3V_1 - 4I_2 - 4I_3 - 2I_3 = 0$$

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$3V_1 + 4I_2 + 3I_3 = 0$ —— (5)

mesh(1).

$V_1 - 2(I_1 - I_3) = 0$

$V_1 + 2I_3 = 0$ —— (6)

$V_1 = -2I_3$

$-6I_3 + 4I_2 + 3I_3 = 0$

$4I_2 - 3I_3 = 0$

$I_3 = \frac{4I_2}{3}$

on, $V_1 = -2 \times \frac{4I_2}{3} = \frac{-8}{3} I_2$

$\frac{V_1}{I_2} = \frac{-8}{3}\Omega = Z_{12}$.

from (4)

$I_3 = \frac{V_2 - 4I_2}{4}$

from (1) & (4). $I_3 = \frac{4I_2}{3}$

$V_2 = 4I_2 + 4 \times \frac{4I_2}{3}$

$V_2 = 4I_2 + \frac{16I_2}{3}$

$V_2 = \frac{12I_2 + 16I_2}{3}$

$V_2 = \frac{28I_2}{3}$

$\frac{V_2}{I_2} = \frac{28}{3}\Omega = Z_{22}$.

in (5)

$3V_1 + 4I_2 + 3\frac{V_2 - 4I_2}{4} = 0$

$3V_1 + 4I_2 + \frac{3V_2 - 12I_2}{4} = 0$

$12V_1 + 16I_2 + 3V_2 - 12I_2 = 0$

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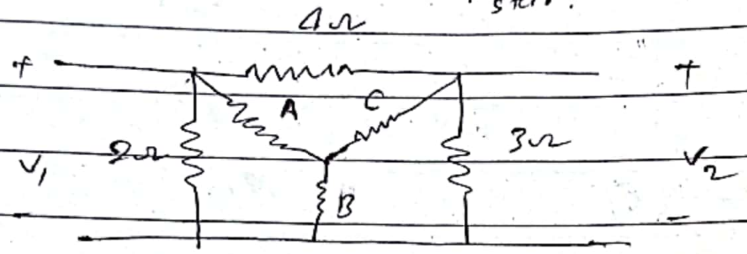
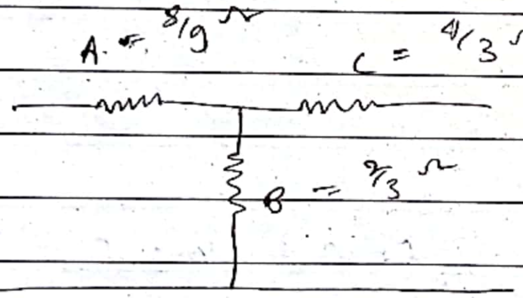
Q. Find the equivalent T-network of the given π-network.
π delta
T star.



$$A = \frac{2\times4}{2+4+3} = \frac{8}{9}\,\Omega$$

$$B = \frac{2\times3}{9} = \frac{6}{9} = \frac{2}{3}\,\Omega$$

$$C = \frac{4\times3}{9} = \frac{12}{9} = \frac{4}{3}\,\Omega$$

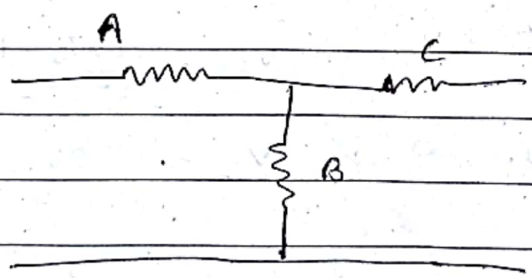


$z_{11} = A+B = \left(\frac{8}{9}+\frac{2}{3}\right)\Omega$

$z_{22} = B+C = \left(\frac{2}{3}+\frac{4}{3}\right)\Omega$

$z_{12} = z_{21} = B = \frac{2}{3}\,\Omega$

Q. If $z_{11} = 4\,\Omega$, $z_{22} = 6\,\Omega$, $z_{12} = z_{21} = 1\,\Omega$, then construct equivalent T-n/w.



$A+B = z_{11} = 4 \Rightarrow A = 4-B \Rightarrow A = 4-1 = 3\,\Omega$

$B+C = z_{22} = 6 \Rightarrow C = 6-B \Rightarrow 5\,\Omega.$

$z_{12} = z_{21} = B = 1$

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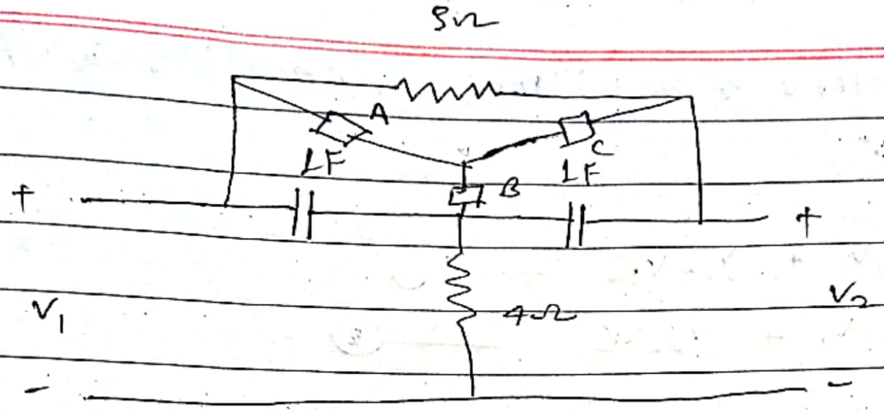
Q.



1F → $\frac{1}{s}$

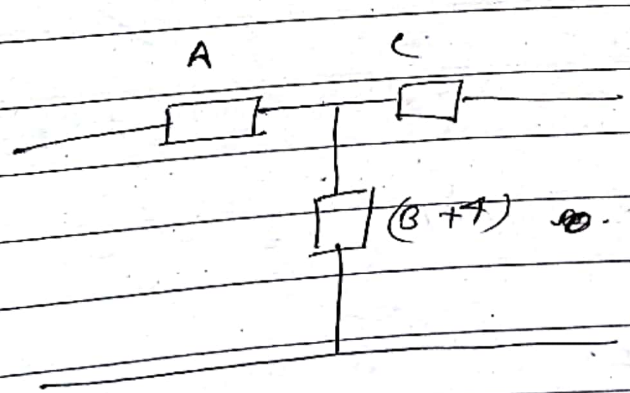
Find the equivalent T-network of the given π-network.

$$A = \frac{3 \times 1/s}{3 + \frac{1}{s} + 1/s} \Rightarrow \frac{3}{3s+2}$$

$$B = \frac{1/s \times 1/s}{3 + 1/s + 1/s} \Rightarrow \frac{1}{s(3s+2)}$$

$$C = \frac{3 \times 1/s}{3 + 1/s + 1/s} \Rightarrow \frac{3}{3s+2}$$

equivalent T-n/w,



Y - parameters or admittance parameters or short circuit parameters:

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{——— (1)}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{——— (2)}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \underbrace{\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}}_{\text{Y- parameters}} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

let, $V_2 = 0$ i.e. o/p is shorted

from (1)

$$I_1 = Y_{11}V_1 + Y_{12} \times 0$$

$$Y_{11} = \frac{I_1}{V_1} \; (℧) \quad (\text{mho})$$

from (2)

$$I_2 = Y_{21}V_1 + Y_{22} \times 0$$

$$Y_{21} = \frac{I_2}{V_1} \; (℧)$$

Let $V_1 = 0$ ie i/p is shorted

from (1)

$$I_1 = Y_{11} \times 0 + Y_{12} \times V_2$$

$$Y_{12} = \frac{I_1}{V_2} \; (℧)$$

from (2)

$$I_2 = Y_{21} \times 0 + Y_{22}V_2$$

$$Y_{22} = \frac{I_2}{V_2} \; (℧).$$

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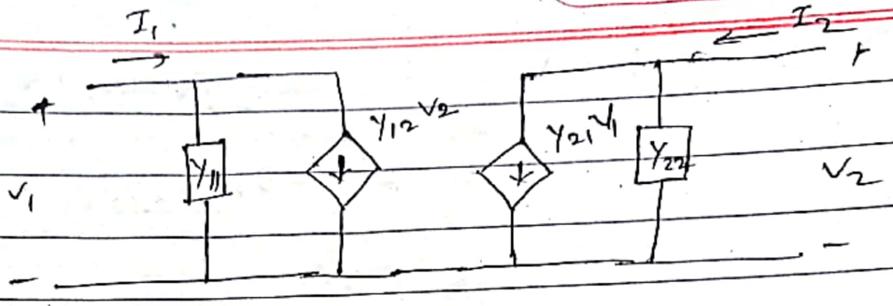
Reciprocal :-

$y_{12} = y_{21}$

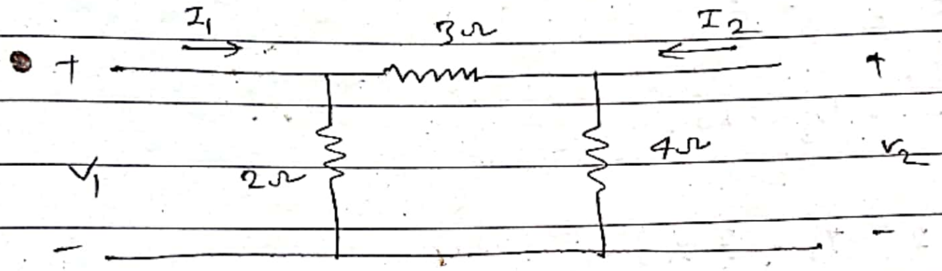
Symmetry :-

$y_{11} = y_{22}$



equivalent ckt of y - parameters.

Q.

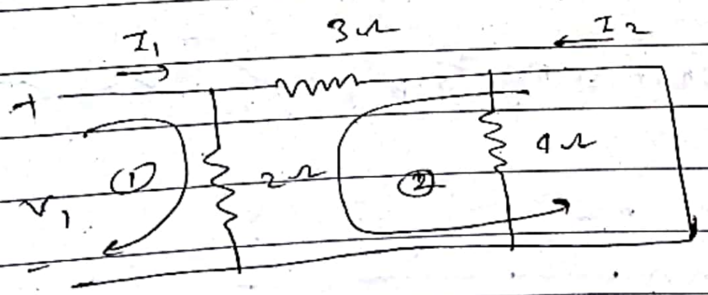


Find the y - parameters, Also check for reciprocity and symmetry. Draw equivalent circuit.

$I_1 = y_{11}V_1 + y_{12}V_2$ —— ①

$I_2 = y_{21}V_1 + y_{22}V_2$ —— ②

let, $V_2 = 0$ i.e. short ckting o/p port.



In mesh 1, KVL

$-2(I_1 + I_2) + V_1 = 0$ —— ①

$V_1 = 2(I_1 + I_2)$

In 2nd mesh,

$-3I_2 - 2(I_1 + I_2) = 0$

$-2I_1 - 5I_2 = 0$ —— ②

$-2I_1 = 5I_2$

$I_1 = -\frac{5}{2}I_2$

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frm (1) $V_1 = 2 \left(-\frac{5}{2} I_2 + I_2\right)$

$= \cancel{2} \frac{-5I_2 + 2I_2}{\cancel{2}}$

$= -3I_2$

~~$Z_{12} = \frac{V_1}{I_2} = -3$ mho~~

$\frac{I_2}{V_1} = -\frac{1}{3}$

$Y_{21} = -1/3$ mho.

$V_1 = 2\left(I_1 - \frac{2}{5} I_1\right)$

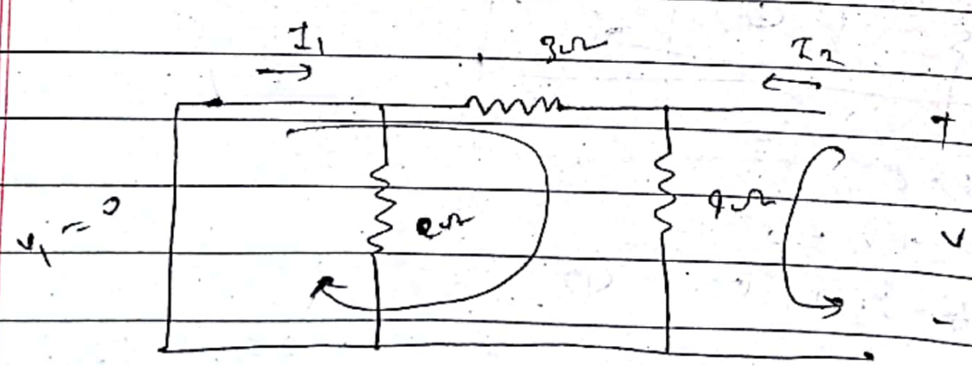
$= 2 \frac{(5-2) I_1}{5}$

$= \frac{6}{5} I_1$

$\frac{I_1}{V_1} = \frac{5}{6}$

$Y_{11} = \frac{5}{6}$ mho.

Let $V_1 = 0$, i.e, short sc-ting i/p port.



In first mesh, KVL

$$-3I_1 - 4(I_1 + I_2) = 0$$

$$-7I_1 - 4I_2 = 0 \longrightarrow (3)$$

In second mesh, KVL,

$$-4(I_2 + I_1) + V_2 = 0$$

$$V_2 = 4(I_2 + I_1) \longleftarrow (4)$$

$$V_2 = 4\left(I_2 - \frac{4}{7} I_2\right)$$

$$= 4 \cdot \frac{3 I_2}{7}$$

$$= \frac{12}{7} I_2$$

$$V_2 = 4\left(-\frac{7}{4} I_1 + I_1\right)$$

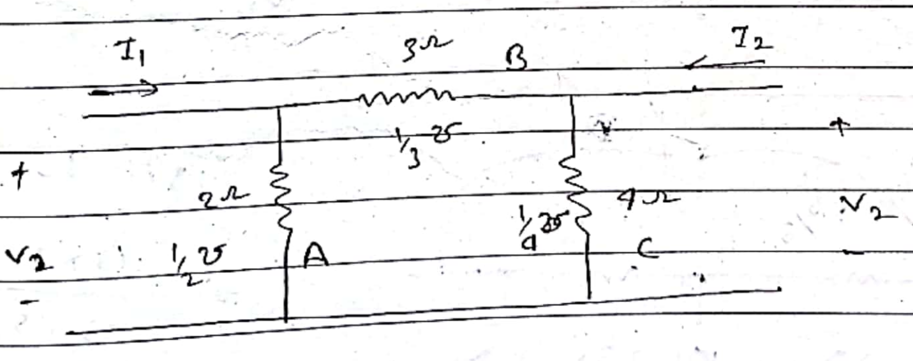
$$= 4\left(\frac{-7I_1 + 4I_1}{4}\right)$$

$$= -3I_1$$

$$Y_{22} = \frac{I_2}{V_2} = \frac{7}{12} \text{ mho.}$$

$$Y_{12} = \frac{I_1}{V_2} = -\frac{1}{3} \text{ mho.}$$

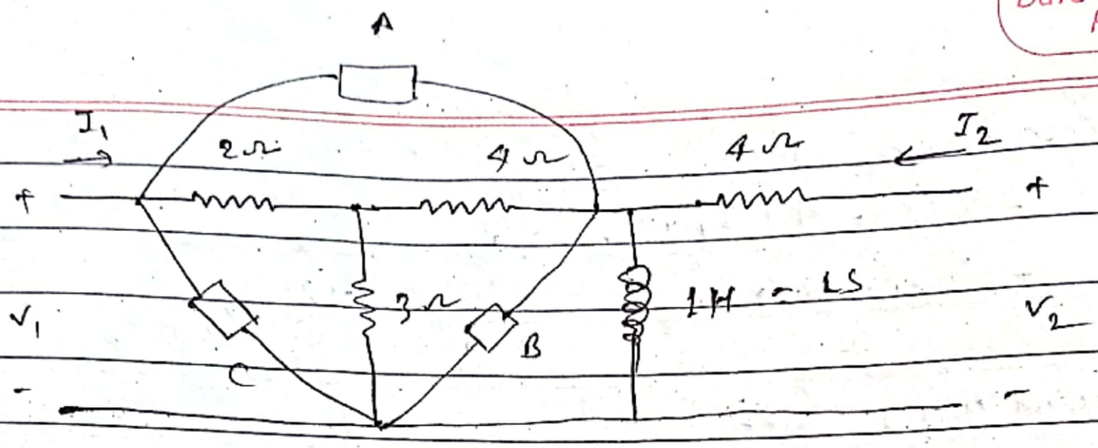
Z- parameters থেকে Y বের নির্ণয় দেখিঃ। হ গ।ড ইহা থেকে জানি admittance পাচ্ছি।



$$Y_{11} = A + B = \frac{1}{2} + \frac{1}{3} = \frac{5}{6} \text{ ℧}$$

$$Y_{22} = B + C = \frac{1}{3} + \frac{1}{4} = \frac{7}{12} \text{ ℧}$$

$$Y_{12} = Y_{21} = -B = -\frac{1}{3} \text{ ℧}$$

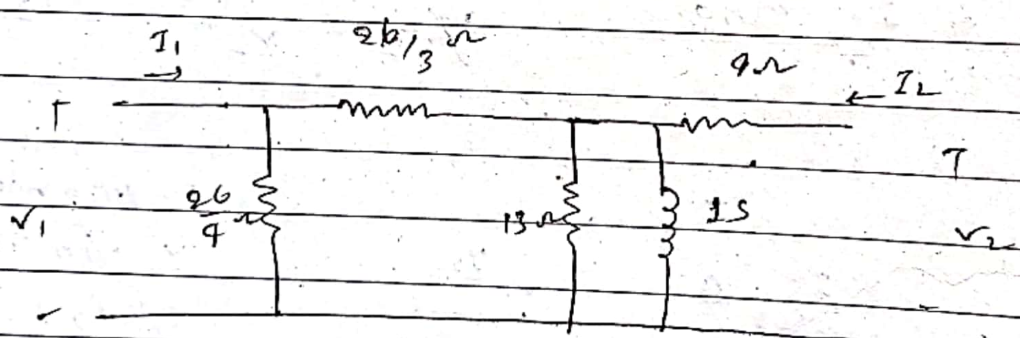


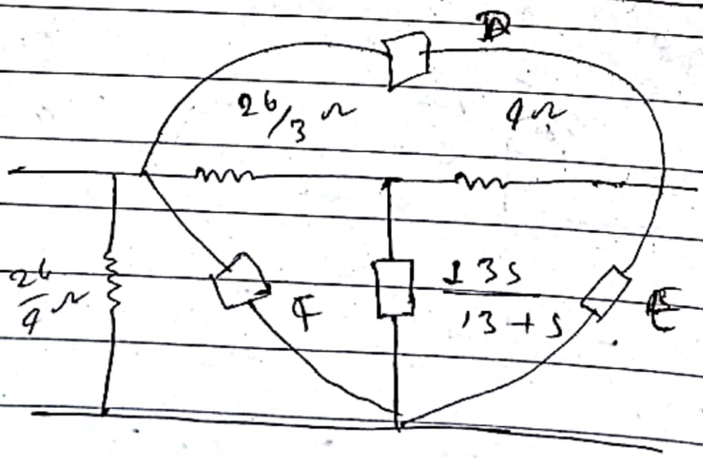
Find admittance parameters of the given network.

$$A = \frac{2\times4 + 4\times3 + 3\times2}{3} = \frac{8+12+6}{3} = \frac{26}{3}$$

$$B = \frac{26}{2}$$

$$C = \frac{26}{4}$$





$13 \,||\, 1s$

$$\Rightarrow \frac{13\times1s}{13+s} = \frac{13s}{13+s}$$

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$$A = \frac{\frac{26}{3}\times 4 + \frac{13s}{13+s}\times 4 + \frac{26}{3}\times\frac{13s}{13+s}}{\frac{13s}{13+s}}$$

$$= \frac{\frac{104}{3} + \frac{52s}{13+s} + \frac{338s}{3(13+s)}}{\frac{13s}{13+s}}$$

$$= \frac{104(13+s) + 3\times 52s + 338s}{3\times 13s}$$

$$= \frac{1352 + 598s}{39s}$$

$$E = \frac{(1352 + 598s) / (13+s)3}{\frac{26}{3}}$$

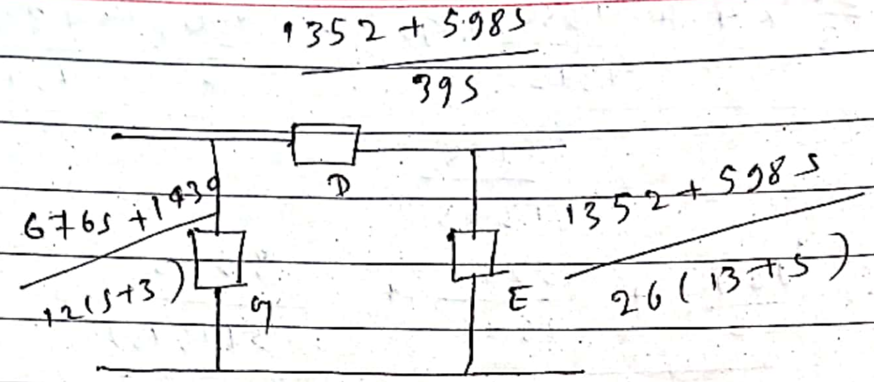
$$= \frac{1352 + 598s}{26(13+s)}$$

$$\eta = \frac{1352 + 598s}{12(s+13)}$$

$$\frac{\frac{26}{4}\times\frac{1352+598s}{12(s+13)}}{\frac{26}{4}+\frac{1352+598s}{12(s+13)}}$$

$$\frac{26\times 3(s+13) + 1352 + 598s}{12(s+3)} = \frac{676s + 2430}{12(s+3)}$$

=



$$y_{11} = G + D$$

$$= \frac{676s + 1430}{12(s+3)} + \frac{1352 + 598s}{39s}$$

$$y_{22} = E + D$$

$$= \frac{1352 + 598s}{26(13+s)} + \frac{1352 + 598s}{39s}$$

$$y_{12} = y_{21} = D = \frac{1352 + 598s}{39s}$$

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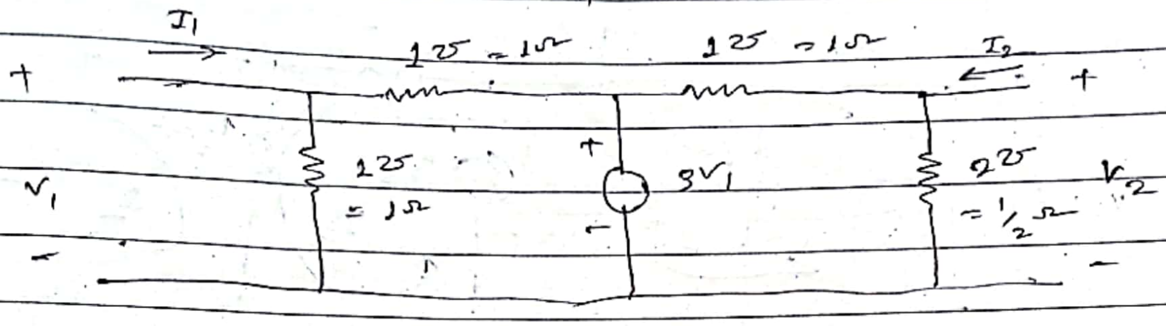
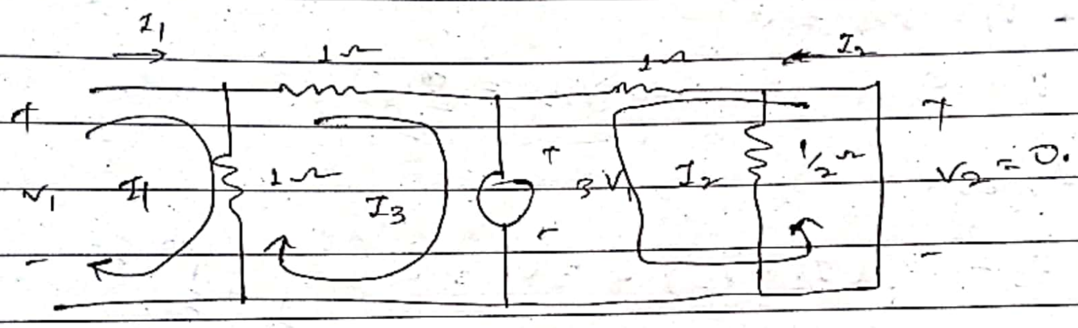
Determine the admitance parameters of the given network.



Sol^n

let $V_2 = 0$, i.e. short circuiting O/P port.



In 1st mesh,

$-(I_1 - I_3) + V_1 = 0$

$V_1 = I_1 - I_3$ ——— (1) $\Rightarrow I_3 = I_1 - V_1$

In 2nd mesh,

$-I_3 - 3V_1 - (I_3 - I_1) = 0$

$3V_1 = -2I_3 + I_1$ ——— (2)

$3V_1 = -2(I_1 - V_1) + I_1$

$3V_1 = -2I_1 + 2V_1 + I_1$

$3V_1 = -I_1 + 2V_1$

$V_1 = -I_1$

$\frac{I_1}{V_1} = -1$

$\Rightarrow Y_{11} = -1 \mho$.

In 3rd mesh,

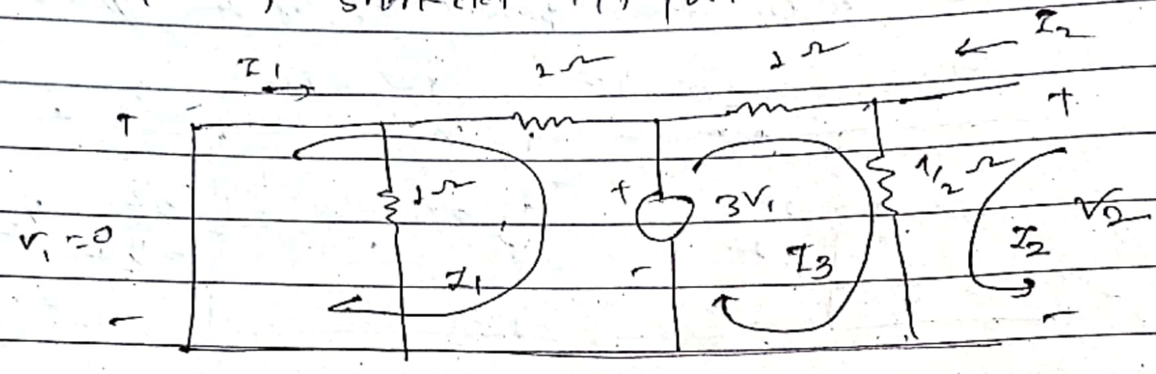
$-I_2 - 3V_1 = 0$

$-I_2 = 3V_1$

$\frac{I_2}{V_1} = -3$

$\Rightarrow Y_{21} = -3 \mho$

let $V_1 = 0$, short ckt I/P port.



in 1st mesh,

$$-I_1 - 3V_1 = 0$$

$$I_1 = 0 \quad \Rightarrow Y_{12} = \frac{I_1}{V_2} = 0$$

2nd mesh,

$$-I_3 - \frac{1}{2}(I_3 + I_2) + 3V_1 = 0$$

$$-I_3 - \frac{1}{2}I_3 - \frac{1}{2}I_2 = 0$$

$$-\frac{3}{2}I_3 - \frac{1}{2}I_2 = 0$$

$$3I_3 = -I_2$$

$-\frac{1}{6}$ $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{6}$ I_2 I_3

3rd mesh,

$$\frac{1}{2}(I_2 + I_3) - V_2 = 0$$

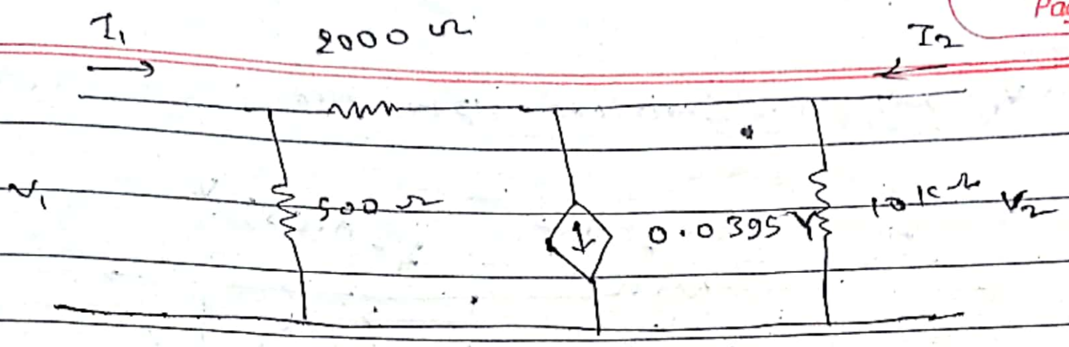
$$\frac{1}{2}I_2 + \frac{1}{2}I_3 - V_2 = 0$$

$$\frac{1}{2}I_2 - \frac{1}{2}\cdot\frac{I_2}{3} - V_2 = 0$$

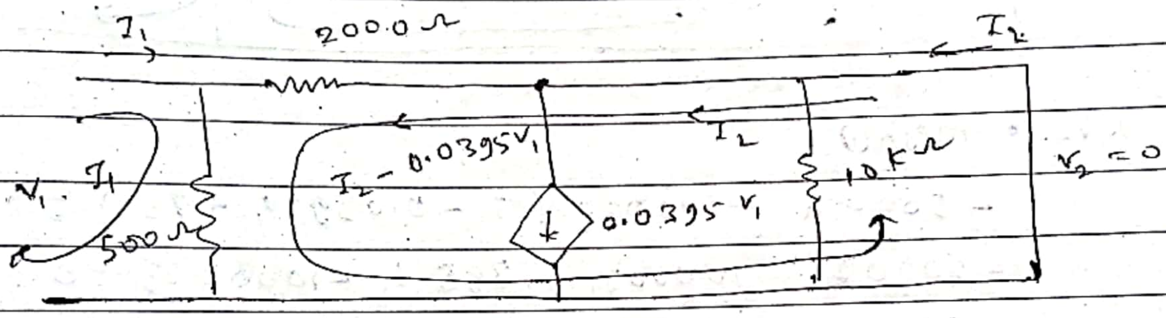
$$\frac{2I_2}{2\cdot 3} - V_2 = 0$$

$$\frac{I_2}{3} = V_2$$

$$\frac{I_2}{V_2} = 3 \quad \Rightarrow Y_{22} = 3 \ \mho$$



let, $V_2 = 0$ i.e. short ckt o/p loop.



KVL @ mesh 1,

$-500(I_1 + I_2 - 0.0395V_1) + V_1 = 0$

or $-500 I_1 - 500 I_2 + 19.75 V_1 + V_1 = 0$

$20.75 V_1 = 500 I_1 + 500 I_2$ —— (1)

KVL @ mesh 2,

$-2000(I_2 - 0.0395 V_1) - 500(I_2 - 0.0395 V_1 + I_1) = 0$

$-2000 I_2 + 79 V_1 - 500 I_2 + 19.75 V_1 - 500 I_1 = 0$

$98.75 V_1 = 500 I_1 + 2500 I_2$ —— (2)

From (1) & (2)

$98.75 V_1 = 20.75 V_1 - 500 I_2 + 2500 I_2$

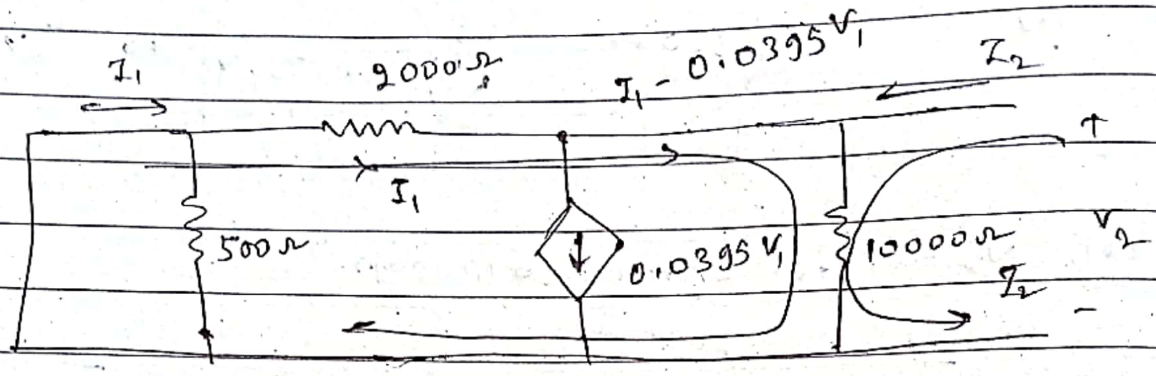
$2000 I_2 = 78 V_1$

$\frac{I_2}{V_1} = Y_{21} = \frac{2000}{78} = 25.64$ ℧.

~~$98.75 V_1 = 500 I_1 + 2500 \left(\frac{20.75 V_1 - 500 I_1}{500} \right)$~~

$98.75 V_1 = 500 I_1 + 103.75 V_1 - 2500 I_1 \Rightarrow \frac{I_1}{V_1} = 0.0025$ ℧. (Y_{11})

let, $v_1 = 0$ i.e. short ckt i/p two port.



KVL @ mesh 1

$$-2000 I_1 - 10000(I_1 - 0.0395 V_1 + I_2) = 0$$

$$-2000 I_1 - 10000 I_1 + 395 V_1 - 10000 I_2 = 0$$

$$-12000 I_1 - 10000 I_2 + 395 \cancel{V_1} = 0 \quad \text{——— (3)}$$

0

KVL @ mesh 2

$$-10000(I_2 + I_1 - 0.0395 V_1) + V_2 = 0$$

$$-10000 I_2 - 10000 I_1 + 395 \cancel{V_1} + V_2 = 0 \quad \text{——— (4)}$$

fr (3) & (4)

$$-1000\cancel{0} I_2 - 10000 I_1 + 10000 I_2 + 12000 I_1 + V_2 = 0$$

$$2000 I_1 + V_2 = 0$$

$$Y_{12} = \frac{I_1}{V_2} = \frac{-1}{2000} \mho = -0.0005 \mho$$

$$-10000 I_2 - 10000\left(-\frac{10000}{12000} I_2\right) + V_2 = 0$$

$$\Rightarrow -10000 I_2 + 100000 I_2 + V_2 = 0$$

$$90000 I_2 = -V_2$$

$$Y_{22} \Rightarrow \frac{I_2}{V_2} = \frac{-1}{90000} \mho$$

Hybrid parameters or h-parameters.

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \underbrace{\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}}_{\text{h-parameters}} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

condition of reciprocity

$$h_{12} = h_{21}$$

symmetry $\rightarrow$ $h_{11} h_{22} - h_{12} h_{21} = 1$.

Let, $V_2 = 0$ ie. O/P short ckt

from eqn ①

$$V_1 = h_{11} I_1 + h_{12} \times \cancel{V_2}$$

$$\Rightarrow h_{11} = \frac{V_1}{I_1} \quad \Omega$$

from eqn ② $I_2 = h_{21} I_1 + h_{22} \times 0$

$$\Rightarrow h_{21} = \frac{I_2}{I_1}$$

Again, let $I_1 = 0$ ie. opening I/P port.

from ①

$$V_1 = h_{11} \times 0 + h_{12} V_2$$

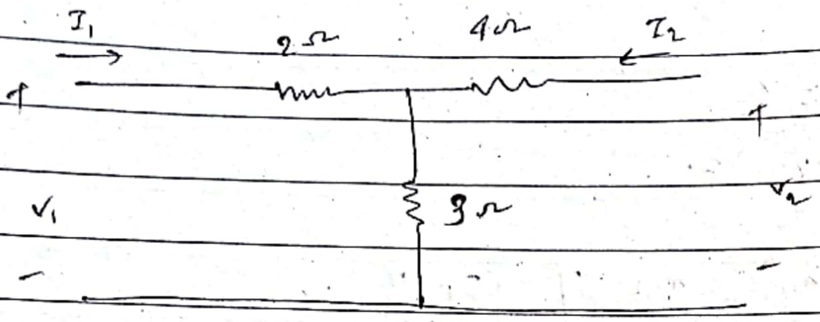
$$\Rightarrow h_{12} = V_1 / V_2$$

from ②

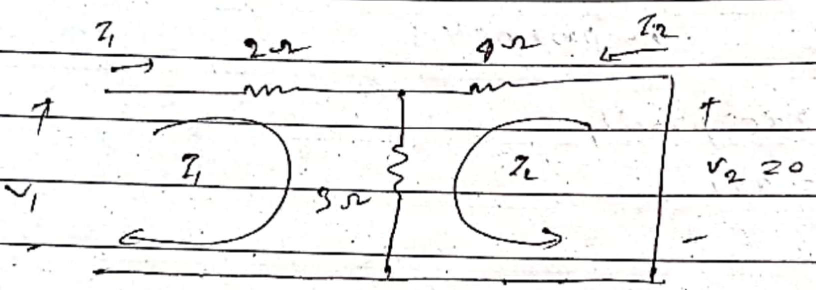
$$I_2 = h_{21} \times 0 + h_{22} V_2$$

$$\Rightarrow h_{22} = I_2 / V_2$$

Find h-parameters,



let $V_2 = 0$, i.e. short ckt o/p port.



1st mesh,

$-2I_1 - 3(I_1 + I_2) + V_1 = 0$

$V_1 = 5I_1 + 3I_2$ —— (1)

2nd mesh,

$-4I_2 - 3(I_2 + I_1) = 0$

$-7I_2 - 3I_1 = 0$

$-7I_2 = 3I_1$ —— (2)

from (2)

$h_{21} = \frac{I_2}{I_1} = -\frac{3}{7}$

from (1)

$V_1 = 5I_1 + 3 \times -\frac{3}{7} I_1$

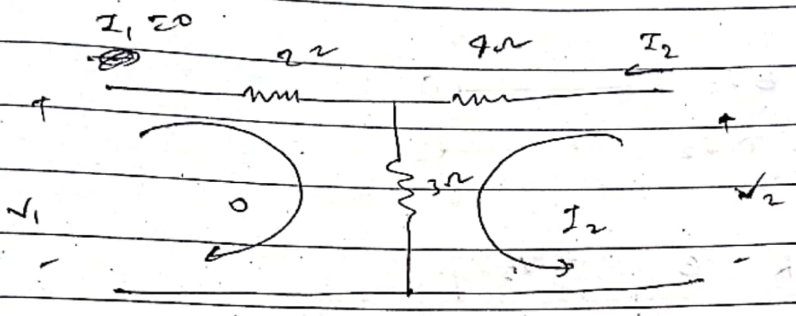
$= 5I_1 - \frac{9}{7} I_1$

$= \frac{35I_1 - 9I_1}{7}$

$= \frac{26}{7} I_1$

$h_{11} = \frac{V_1}{I_1} = \frac{26}{7}\ \Omega.$

let, $I_1 = 0$ i.e. open i/p port.



$-4I_2 - 3(I_1 + I_2) + V_2 = 0$ (mesh 2)

$-4I_2 - 3I_2 + V_2 = 0$

$V_2 = 7I_2$

$h_{22} = \frac{I_2}{V_2} = \frac{1}{7}$ ℧.

$V_2 = 7I_2$

$= 7 \cdot \frac{V_1}{3}$

KVL mesh (1)

$-3(0 + I_2) + V_1 = 0$

$h_{12} = \frac{V_1}{V_2} = \frac{3}{7}$

$-3I_2 + V_1 = 0$

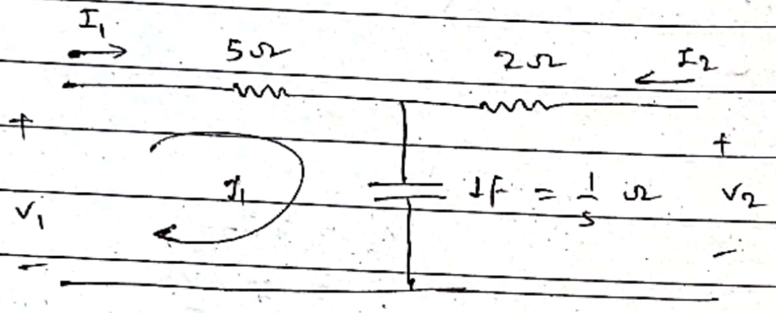
$V_1 = 3I_2$

2018/09/18

ABCD (Transmission) parameters:

$$V_1 = AV_2 - BI_2 \quad \text{——— (1)}$$
$$I_1 = CV_2 - DI_2 \quad \text{——— (2)}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$



obtain ABCD parameters.

Let $I_2 = 0$ i.e. opening o/p port

In first mesh, KVL

$$-5I_1 - \frac{1}{s}(I_1 + \cancel{I_2}^{\,0}) + V_1 = 0$$
$$V_1 = 5I_1 + \frac{1}{s}I_1$$
$$V_1 = \left(5 + \frac{1}{s}\right)I_1 \quad \text{——— (1)}$$

Again, $V_2 = \frac{1}{s}I_1$ ——— (ii)

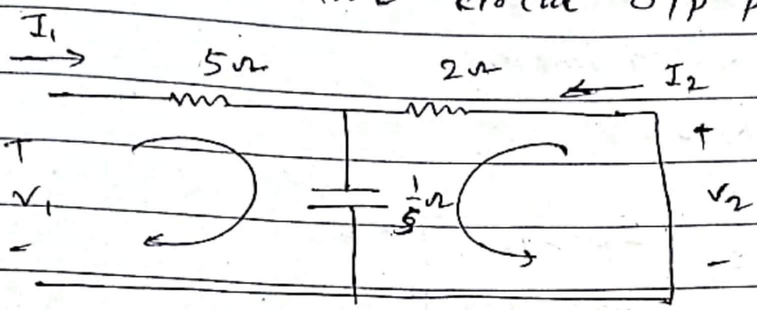
$$\Rightarrow C = \frac{I_1}{V_2} = s$$

Also, $V_1 = \left(5 + \frac{1}{s}\right) \cdot V_2 s$

$$= \frac{(5s+1)}{s} V_2 s$$

$$\Rightarrow A = \frac{V_1}{V_2} = 5s+1$$

Let $V_2 = 0$ i.e. short circuit o/p port



In first mesh, KVL

$$-5I_1 - \frac{1}{s}(I_1 + I_2) + V_1 = 0$$

$$V_1 = 5I_1 + \frac{1}{s}I_1 + \frac{1}{s}I_2$$

$$= \left(5 + \frac{1}{s}\right) I_1 + \frac{1}{s} I_2 \longrightarrow (3)$$

In second mesh, KVL

((ठूलो n/w तोडियो भने Δ-Y convert गरेर सानो n/w बनाएर मात्र solve गर्नो

$$-2I_2 - \frac{1}{s}(I_1 + I_2) = 0$$

$$2I_2 + \frac{1}{s}I_1 + \frac{1}{s}I_2 = 0$$

$$\left(2 + \frac{1}{s}\right) I_2 = -\frac{1}{s} I_1$$

$$\Rightarrow D = -\frac{I_1}{I_2} = \left(2 + \frac{1}{s}\right)s = (2s+1).$$

put I_1 in (3) we get.

$$V_1 = \frac{5s+1}{s} \times \frac{-(2s+1)}{s} \times sI_2 + \frac{1}{s} I_2$$

$$= \frac{-(5s+1)(2s+1)+1}{s} I_2$$

$$\Rightarrow B = -\frac{V_1}{I_2} = \frac{(5s+1)(2s+1)-1}{s}$$

$$= \frac{10s^2 + 7s}{s}$$

$$= (10s + 7)$$

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* Conversion of parameters.

1. z-parameters to y-parameters

z-parameters are known.

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1}$$

$$= \frac{1}{\Delta Z} \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$$

$$Y_{11} = \frac{Z_{22}}{\Delta Z} \qquad Y_{12} = \frac{-Z_{12}}{\Delta Z}$$

$$Y_{21} = \frac{-Z_{21}}{\Delta Z} \qquad Y_{22} = \frac{Z_{11}}{\Delta Z}$$

$\Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21}$ (determinant of z-parameters)

2. y-parameters to z-parameters

y-parameters are known

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1} = \frac{1}{\Delta Y} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

$$Z_{11} = \frac{Y_{22}}{\Delta Y} \qquad Z_{12} = \frac{-Y_{12}}{\Delta Y}$$

$$Z_{21} = \frac{-Y_{21}}{\Delta Y} \qquad Z_{22} = \frac{Y_{11}}{\Delta Y}$$

where,

$$\Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}$$

3. Z-parameters to ABCD (Transmission) parameters :

Z-parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{--- (1)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{--- (2)}$$

पहिलेको लाई जस्तै rearrange गरेर निकालेको पर्नो सँग compare गर्ने ।

ABCD - parameters.

$$V_1 = A V_2 - B I_2 \quad \text{--- (3)}$$

$$I_1 = C V_2 - D I_2 \quad \text{--- (4)}$$

from (2)

$$Z_{21} I_1 = V_2 - Z_{22} I_2$$

$$I_1 = \frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2 \quad \text{--- (5)}$$

put I_1 in (1)

$$V_1 = Z_{11}\left(\frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2\right) + Z_{12} I_2$$

$$= \frac{Z_{11}}{Z_{21}} V_2 - \frac{Z_{11} Z_{22}}{Z_{21}} I_2 + Z_{12} I_2$$

$$= \frac{Z_{11}}{Z_{21}} V_2 - \left(\frac{Z_{11} Z_{22} + Z_{12} Z_{21}}{Z_{21}}\right) I_2 \quad \text{--- (6)}$$

Comparing eq^n (3) & (6)

$$A = \frac{Z_{11}}{Z_{21}}$$

$$B = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}}$$

Comparing (5) & (4)

$$C = \frac{1}{Z_{21}}$$

$$D = \frac{Z_{22}}{Z_{21}}$$

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2. Z-parameters to h-parameters

Z-parameters,

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{--- (1)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{--- (2)}$$

h-parameters,

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{--- (3)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{--- (4)}$$

from (2)

$$I_2 Z_{22} = V_2 - Z_{21} I_1$$

$$I_2 = \frac{1}{Z_{22}} V_2 - \frac{Z_{21}}{Z_{22}} I_1 \quad \text{--- (5)}$$

from (1)

$$V_1 = Z_{11} I_1 + Z_{12} \left(\frac{1}{Z_{22}} V_2 - \frac{Z_{21}}{Z_{22}} I_1 \right)$$

$$= \left(Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22}} \right) I_1 + \frac{Z_{12}}{Z_{22}} V_2$$

$$= \left(\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{22}} \right) I_1 + \frac{Z_{12}}{Z_{22}} V_2 \quad \text{--- (6)}$$

Comparing (3) & (6)

$$h_{11} = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{22}}$$

$$h_{12} = \frac{Z_{12}}{Z_{22}}$$

Comparing (4) & (5)

$$h_{21} = -\frac{Z_{21}}{Z_{22}}$$

$$h_{22} = \frac{1}{Z_{22}}$$

* PU online exam.

If $Z_{11} = 30\Omega$, $Z_{22} = 20\Omega$, $Z_{12} = Z_{21} = 10\Omega$. obtain Transmission parameters.

Sol^n

→ we have, Transmission parameters A, B, C, D are

$$A = \frac{Z_{11}}{Z_{21}} = \frac{30}{10} = 3\ \Omega.$$

$$B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}} = \frac{30 \times 20 - 10 \times 10}{10} = 50\ \Omega.$$

$$C = \frac{1}{Z_{21}} = \frac{1}{10}\ \Omega$$

$$D = \frac{Z_{22}}{Z_{21}} = \frac{20}{10} = 2\ \Omega.$$

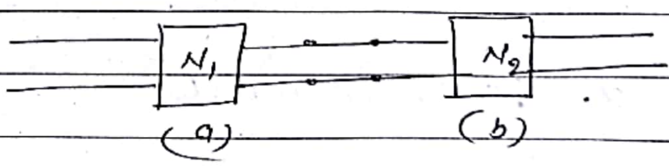
Interconnection of networks.

When two or more networks are connected together to form bigger network, it is said that they are interconnected.

Types:

1. Cascade.

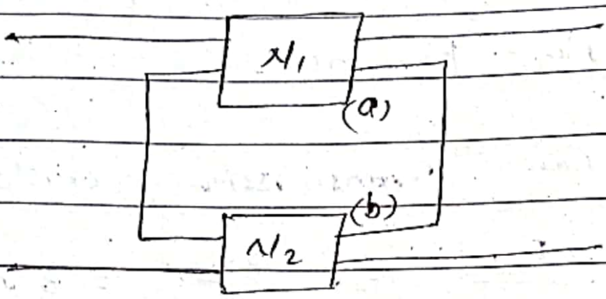
In cascade n/w, we find ABCD parameters of each n/w and multiply each to find total ABCD parameters.



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_a \times \begin{bmatrix} A & B \\ C & D \end{bmatrix}_b$$

2. Series - series.

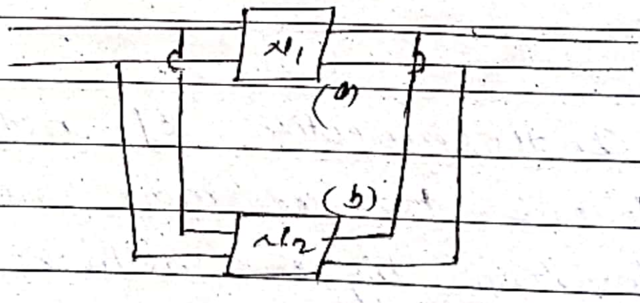
In series n/w, we find z-parameters of each n/w and add each to find overall z-parameters.



$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}_a + \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}_b$$

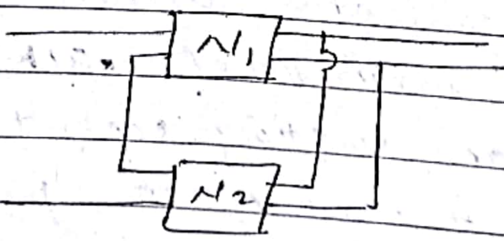
3. Parallel - parallel

In this connection, we find y-parameters of whole circuit by adding y-parameters of each network.



$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}_a + \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}_b$$

4. Series - parallel.



$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}_a + \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}_b$$

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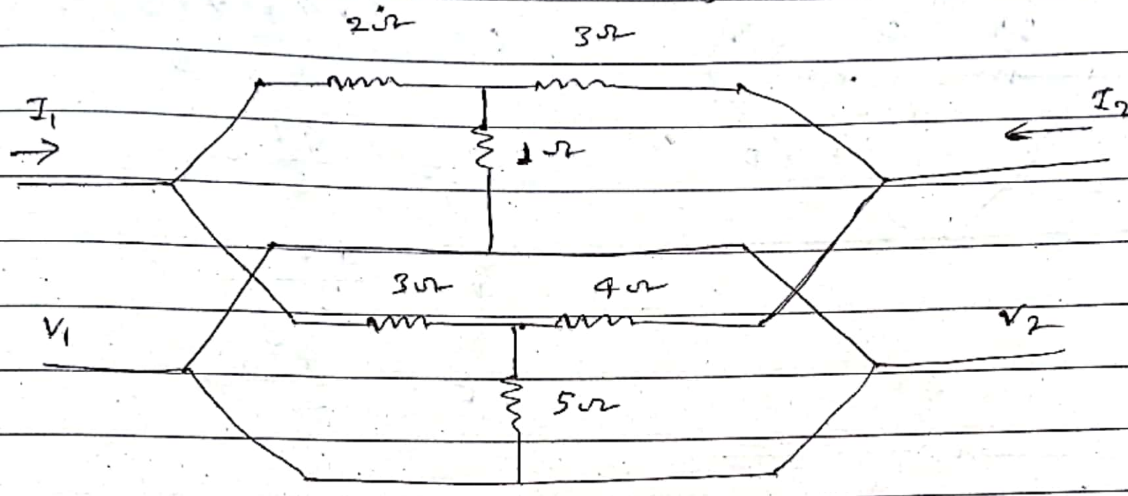
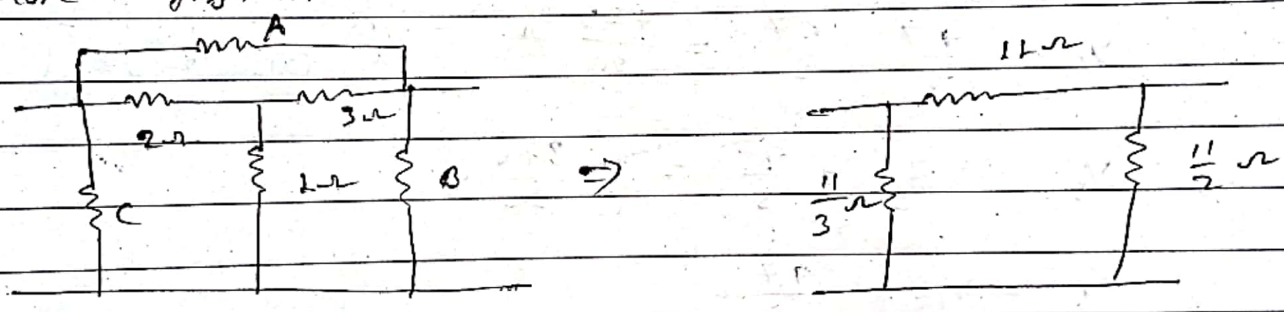
Q. Find y-parameters of the given network.



Here two n/w are connected in parallel connection so we get y-parameters.

consider first n/w.



$$A = \frac{2\times 3 + 3\times 1 + 1\times 2}{1} = 11\,\Omega$$

$$B = \frac{11}{2}\,\Omega \qquad Y_{11} = \frac{11}{3} + 11 = \frac{44}{3}\,\Omega$$

$$C = \frac{11}{3}\,\Omega \qquad Y_{22} = \frac{11}{2} + 11 = \frac{33}{2}\,\Omega$$

$$Y_{21} = Y_{12} = 11\,\Omega$$

$$\therefore \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}_a = \begin{bmatrix} 44/3 & 11 \\ 11 & 33/2 \end{bmatrix} \longrightarrow (1)$$

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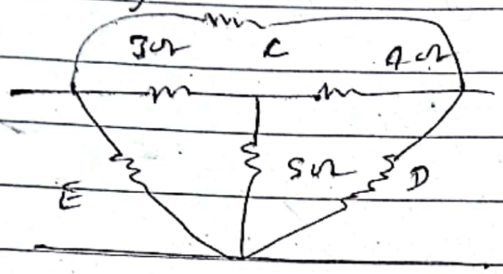
Considering second n/w.

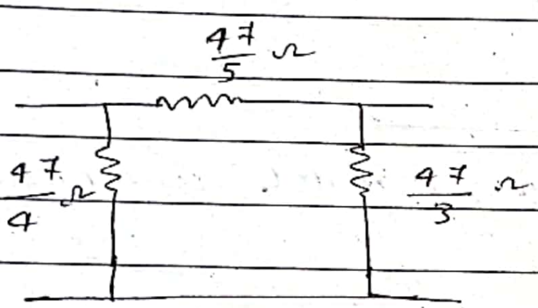


$$C = \frac{3\times4 + 4\times5 + 5\times3}{5} = \frac{47}{5}\,\Omega$$

$$D = \frac{47}{3}\,\Omega$$

$$E = \frac{47}{4}\,\Omega$$

⇓



$$Y_{11} = \frac{47}{4} + \frac{47}{5} = \frac{423}{20}\,\Omega$$

$$Y_{22} = \frac{47}{3} + \frac{47}{5} = \frac{376}{20}\,\Omega$$

$$Y_{21} = Y_{12} = \frac{47}{5}\,\Omega$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}_b = \begin{bmatrix} \frac{423}{20} & \frac{47}{5} \\ \frac{47}{5} & \frac{376}{20} \end{bmatrix} \quad (2)$$

Hence, from (1) & (2)

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{44}{3} & 11 \\ 11 & \frac{33}{2} \end{bmatrix} + \begin{bmatrix} \frac{423}{20} & \frac{47}{5} \\ \frac{47}{5} & \frac{376}{20} \end{bmatrix}$$

$$= \begin{bmatrix} \left(\frac{44}{3} + \frac{423}{20}\right) & \left(11 + \frac{47}{5}\right) \\ \left(11 + \frac{47}{5}\right) & \left(\frac{33}{2} + \frac{376}{20}\right) \end{bmatrix}$$