

M.1812



A Reference Course

In

Thermal Science

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Preface

While teaching the subject at Nepal Engineering College, one of Nepal's premier institutes, I have come across three kinds of problems that the students face with this subject.

On the one hand, there are several good books on this subject written by reputed authors. Invariably, these books are voluminous, containing more information than required by beginners who wish to learn just the basics. On the other hand, there are books that, though concise, oversimplify the subject under the pretext that the information contained in them is sufficient to score well in the examination. And thirdly, there is hardly any book that deals with thermodynamics along with heat transfer.

The second category of books does a disservice to the students as it escapes basics of the subject matter making it more difficult to understand. What students really need are the elaborate explanations with the correlation of their previous knowledge.

The fact that there is a need for a book that bridges the gap between advanced but voluminous books and simple but inadequate books is what prompted me to write this book. This is reflected in the adequate detail that has been given while keeping in mind the fact the students need a reference book as an introductory course to thermal science.

Teaching thermal science to first – year students has given me a good understanding of how the subject matter should be presented so that a beginner will find the material useful and beneficial. I have attempted to explain the topics in such a manner that it will be easy for the students even when reading it for the first time.

Chapter 1-6 focus primarily on the basics of the subject such as the concepts of system and thermodynamic property, properties of pure substance, heat and work interaction, first law and second law of thermodynamics with its corollaries, and entropy. Chapter 7 and 8 present the essential concepts of thermodynamic cycles and preliminaries of heat transfer.

In preparing the book, I have referred to several good books written on the subject. I have listed these books at the end. I wish to acknowledge my indebtedness to Prof. Dr. Deepak Bhattarai, the principal of Nepal Engineering College; Prof. Dr. Hari Krishna Shrestha, the vice-principal, NEC; and Mr. Dhrub Banjade, chief, student affairs, who have been a source of inspiration to me.

Last but not the least, I wish to express my deep gratitude to Mr. Dekha Bahadur Pandit, Mr. Prassana Muni, Mr. Ramesh Khamal, Mr. Jahir Ahmed Zibran, and Mr. Roshan Gyawali—my colleagues at civil engineering department, NEC – who have given me so much of themselves that this book is far better than it could possibly have been without them.

I sincerely believe that this book, *A reference Course in Thermal Science*, will be of immense help to the students especially of Pokhara University and that the book will enjoy wide acceptance.

Constructive criticism and suggestion for the improvement from the users of this book are welcome.

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Basic Concepts of Thermodynamics

1.1 Thermodynamics

It is the branch of science which deals with energy transfer and its effect on the state condition of a system. Essentially, thermodynamics pertains to the study of:

- Interaction of system and surrounding; it relates the changes which the system undergoes to the influences to which it is put.
- Energy and its transformation; energy inter-conversion in the form of heat and work
- Relationship between heat, work and physical properties such as pressure, volume and temperature of working substance employed to obtain energy conversion.

1.1.1 Applications of thermodynamics

- Power producing devices:- internal combustion engine, gas turbines, steam and nuclear power plants
- Power consuming devices:- fans, blowers and compressors, refrigeration and air conditioning plants.
- Chemical processing plants.

1.2 Macroscopic and Microscopic Approach Classical and Statistical Thermodynamics

The above two approaches are used to investigate the behavior of a system. In macroscopic approach, the following aspects are taken into consideration:

- Structure of the matter is not considered; no attention is focused on the behavior of individual particles.
- The volumes considered are very large compared to molecular dimensions.
- Study is made of overall effect of several molecules; the behavior and activities of the molecules are space-averaged i.e., their effect integrated. Such space-averaged quantity describes the state or condition of matter.
- Only a few variables are used to describe the state of the matter under consideration.

In microscopic approach, the matter is considered to be composed of a large number of particles called molecules. At a particular instant, each particle has a definite position, velocity and energy and these characteristics change very frequently due to collision between particles. The overall behavior of matter is then predicted by statistical time-average of the behavior of individual particle. This approach is complex, cumbersome and time consuming. The salient aspects of this approach may be summed up as :

- A large number of variables are required for a complete specification of the state of matter under consideration.
- The variables used to describe the state of matter can not be measured easily and precisely.
- Knowledge of the structure of matter under consideration is essential.

1.3 Thermodynamic System, Surrounding and Boundary

System: The thermodynamic system is defined as a quantity of matter or a region in space upon which attention is concentrated to analyze a problem; to study the changes in its properties due to exchange of energy in the form of heat and work.

Surrounding: Everything external to the system is called surrounding.

Boundary: The system and surrounding are separated by an envelope called boundary. It represents the limit of the system and may be either real or imaginary and may change shape, volume, position and/or orientation relative to the observer.

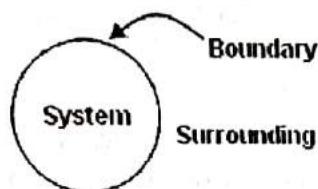


Fig 1.1 Thermodynamic System

Adiabatic boundary/wall: The boundary/wall which does not allow the heat transfer to take place across them is called adiabatic boundary/wall.

Diathermic boundary/wall: The boundary/wall which does allow heat interaction across them is called diathermic boundary/wall.

1.3.1 Types of thermodynamic system

(i) *Closed System:*

The system in which there is exchange of energy in the form of heat and work with its environment but there is no mass transfer across the system boundary is called closed system. The mass within the system remains the same and constant though its volume can change against a flexible boundary.

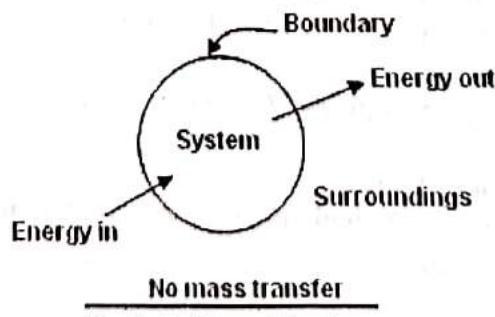


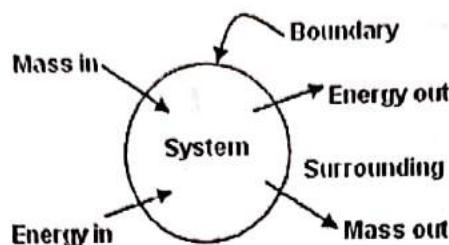
Fig 1.2 Closed System

Examples:

- Cylinder fitted with a movable piston
- Bomb Calorimeter.

(ii) *Open System*:

An open system has mass exchange with the surrounding along with transfer of energy in the form of heat and work. The mass within the system does not necessarily remain constant; it may change depending upon mass inflow and mass outflow.

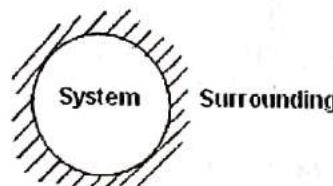


Both mass and energy transfer

Fig 1.3 Open System

(iii) *Isolated System*:

An isolated system is of fixed mass and energy; it exchanges neither mass nor energy with another system or with the surrounding. Such system has no interaction with the surrounding.



No mass or energy transfer

Fig 1.4 An Isolated System

Example:

- A system and its surrounding taken together
- The universe
- Thermosflask

iv) *Adiabatic System*:

The system having no heat interaction with its surrounding is called adiabatic system. A system whose boundary is covered by an insulating material tends to be an adiabatic system.

System on the basis of phase

i) *Homogeneous System*:

If a system consists of single phase, it is called homogeneous system.

Example

- Mixture of water vapour and air
- Solution of ammonia in water
- Mixture water and nitric acid

ii) *Heterogeneous System*:

If a system consists of more than one phase, it is called heterogeneous system.

Example

- Mixture of ice and water
- Mixture of water and oil
- Mixture of water and steam.

1.4 Thermodynamic Property

Dyn P

The characteristics which can be used to describe the condition or state of a system e.g., temperature, pressure, volume, energy etc are called properties of a system.

Salient features of property:

- It is a measurable quantity.
- It has a definite unique value when the system is in a particular state.
- It is dependant only on the state of the system.
- Its differential is exact.

If P is a thermodynamic property, then the change in the property in going from the initial state 1 to final state 2 is given by:

$$\int_1^2 dP = P_2 - P_1$$

If $P=f(x,y)$, then

$$dP = \left(\frac{\partial P}{\partial x} \right)_y dx + \left(\frac{\partial P}{\partial y} \right)_x dy$$

This may be written as

$$dP = M dx + N dy$$

Where

$$M = \left(\frac{\partial P}{\partial x} \right)_y \quad \text{and} \quad N = \left(\frac{\partial P}{\partial y} \right)_x$$

Then,

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y} \left(\frac{\partial P}{\partial x} \right)_y = \frac{\partial^2 P}{\partial y \partial x} \quad \text{and} \quad \frac{\partial N}{\partial x} = \frac{\partial}{\partial x} \left(\frac{\partial P}{\partial y} \right)_x = \frac{\partial^2 P}{\partial x \partial y}$$

Since the order of differentiation is immaterial, we have

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

In fact, this provides a test for exactness and the condition for a quantity to be a thermodynamic property.

1.4.1 Types of property

(a) *Intensive property*

It is independent of the extent or mass of the system. Its value remains same whether one considers the whole system or only part of it. Examples are: Pressure, temperature, density, composition, viscosity, thermal conductivity etc.,

(b) *Extensive property*

It depends upon mass or extent of the system; its value depends on how big a portion of the system is being considered. Examples are: energy, enthalpy, entropy. Volume etc.,

(c) *Specific property*

An extensive property expressed per unit mass of the system is known as specific property. Examples are: specific volume, specific energy, specific enthalpy, specific entropy etc.,

(d) *Molar Property*

An extensive property expressed per unit mole of the system is known as molar property. Examples are : molar volume, molar heat capacity etc.,

1.5 Equilibrium

When a system shows the absence of tendency for spontaneous change in the value of any microscopic property, it is said to exist in some equilibrium.

(a) *Mechanical Equilibrium*

In absence of any unbalanced force within the system itself and also between the system and the surrounding, the system is said to exist in a state of mechanical equilibrium.

(b) *Chemical Equilibrium*

If there is no chemical reaction or transfer of matter from one part of the system to another, such as diffusion, the system is said to exist in a state of chemical equilibrium.

(c) *Thermal Equilibrium*

A system is said to be in thermal equilibrium if there exists uniformity of temperature throughout the system or between the system and surrounding.

(d) *Thermodynamic Equilibrium*

A system which is simultaneously in a state of mechanical equilibrium, thermal equilibrium and chemical equilibrium is said to be in a state of thermodynamic equilibrium.

1.6 State, Path, Process and Cycle

State

When all the properties of a system have definite values, the system is said to exist at a definite state. State is thus the condition of the system identified by thermodynamic properties.

Change of state

Any operation in which one or more of the properties of a system changes is called change of state.

Path

The locus of a series of states through a system passes in going from one state to another state is known as path.

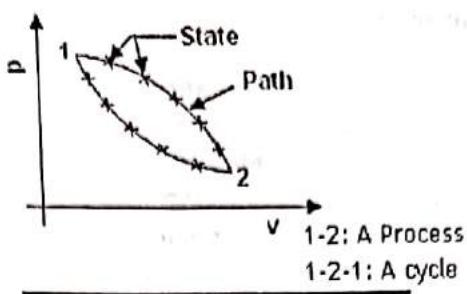


Fig 1.5 A Process and a Cycle

Process

When the path is completely specified, the change of state is called a process.

Cycle

A cycle is defined as a series of change of state such that the final state is identical with the initial state. The change in the value of any property of a system for a cyclic process is zero. Mathematically,

$$\oint dP = 0, \text{ Where } P \text{ is any property.}$$

1.7 Quasi-static (Quasi-equilibrium) Process

The process in which every state passed through by the system is an equilibrium state is known as quasi-static process. Infinite slowness is the characteristic feature of such process. It can be represented graphically as a continuous line on a state diagram.

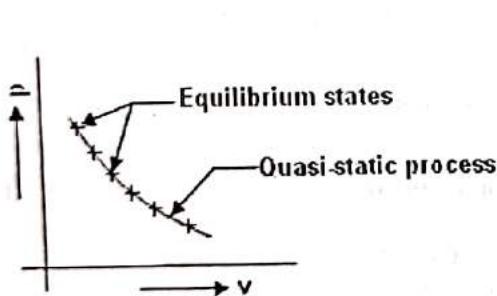


Fig 1.6 A Quasi-static process

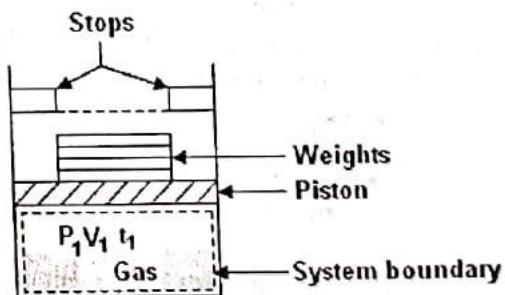


Fig 1.7 System undergoing quasi-static Process

Let us consider a system of gas contained in a cylinder as shown in figure. The system initially is in equilibrium state, represented by the properties (P_1, V_1, t_1) . The weight on the piston just balances the upward force exerted by the gas. If the total weight is removed, there will be an unbalanced force between the system and the surroundings and the piston will move up till it hits the stops. The system again comes to an equilibrium state being described by the properties (P_2, V_2, t_2) . But the intermediate states passed through by the system are non-equilibrium states which can't be described by the thermodynamic co-ordinates. Now if the single weight on the piston is made up of innumerable pieces of infinitesimally small weight and these weights are removed one by one very slowly from the top of the piston, the piston will move upwards very slowly. At any particular instant of piston travel, the unbalanced force would be infinitesimally small and the system would be almost close to the state of equilibrium. And as such every state passed by the system will be an equilibrium state. The locus of a series of such equilibrium states is called a quasi-equilibrium process and can be located on a state diagram by a continuous line.

1.8 Reversible process

A thermodynamic process is reversible if the system passes through a continuous series of equilibrium states.

Let the state of a system be represented by A as shown in figure and let the system be taken to state B by following the path A-B. If the system and also the surroundings are restored to their initial states and no change in the universe is produced, then the process A-B will be reversible. In the reverse process, the system has to be taken from state B to A by following the same path B-A.

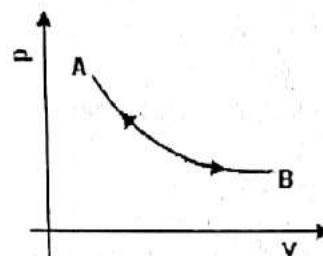


Fig 1.8 A reversible process

Characteristics of a reversible process

- A reversible process must be such that the system and surroundings can be made to traverse in the reverse order through the states it passes through during original process, without producing any changes in the rest of the universe.
- All energy interactions of the original process would be reversed in direction but unchanged in form or magnitude.
- The direction of a reversible process can be changed by making infinitesimal changes in the conditions that control it.
- The process must be quasi-equilibrium.
- A reversible process must involve no friction, unrestrained expansion, mixing, heat transfer across a finite temperature difference or inelastic deformation.

A reversible process must meet each of the conditions listed above. Thus, we can apply these conditions to test any process to be reversible.

Features

- A reversible process yields maximum efficiency in engines and requires minimum work in devices such as refrigerators, pumps and compressors.
- Work done during the reversible process is represented by area under the process curve. For example, $W = \int pdv$
- No real process can be reversible.

Example:

- Motion without friction
- Isothermal and reversible adiabatic process
- Elastic stretching of a solid
- Polarization and magnetization effects

1.9 Irreversible Process

A process is irreversible if the system passes through a sequence of non-equilibrium states. During such process, the properties of the system do not have a unique value and accordingly can not be located on any state diagram. When an irreversible process is made to proceed in the backward direction, the original state of the system is not restored.

Characteristics

- It occurs at finite rate i.e. the time allowed for a process to occur is finite.
- It can not be reversed without causing permanent change in the rest of the universe.
- The system is not in equilibrium state at any instant during irreversible process.

Features

- All spontaneous processes are irreversible.
- There is degradation of energy by such factors as friction, turbulence, diffusion, etc.

Factors causing irreversibility

- a. Heat transfer across a finite temperature difference
- b. Free expansion
- c. Mixing
- d. Friction
- e. Combustion
- f. Inelastic deformation
- g. Electric current flow through a resistance etc.,

Example:

- Viscous flow, fluid flow with friction
- Diffusion of gases
- Free expansion of gases
- Throttling process.

1.9.1 Internal and External Reversibility Internal and External Irreversibility

During a given process, the factors that make the process irreversible may exist either only in the system, or in the surroundings or in both the system and the surroundings.

- i) If the factors causing irreversibility exist only in the system and not in the surroundings, the process is said to be internally irreversible but externally reversible.
- ii) If the factors causing irreversibility exist only in the surroundings and not in the system, the process is said to be internally reversible but externally irreversible.
- iii) If the factors causing irreversibility exist in both the system and the surroundings, the process is said to be totally irreversible.

For example, let us consider a system undergoing a Carnot cycle explained above. During the isothermal expansion process, the system is not only at a steady temperature but also has heat interaction with a reservoir which is at the same temperature as its own. Similarly, during the isothermal compression process, the system is at a steady temperature and is transferring heat to the reservoir at a temperature equal to its own temperature. In other words, the heat transfers in a Carnot cycle occur practically with no temperature difference

between the system and the reservoirs. Hence, the Carnot heat engine or the Carnot cycle is both internally and eternally reversible.

Let us now consider another system. Consider a system undergoing a cycle consisting of two reversible isotherms and two reversible adiabatics. The system may be said to be undergoing a carnot cycle. During the isothermal expansion process, although the system temperature is steady and uniform at t_1 let the high temperature reservoir transferring heat to the system be at t_H where $t_H > t_1$. Similarly, during the isothermal compression process let the system be at a steady and uniform temperature of t_2 but let it transfer heat to the low temperature reservoir at t_L where $t_L < t_2$. In such a cycle, although there are no factors internally that make the cycle irreversible, there are such factors outside the system (heat transfer with finite temperature difference). Hence, although the cycle undergone by the system is internally reversible, it is externally irreversible.

1.10 Concept of Continuum

From microscopic viewpoint, even a very small volume of a system is assumed to contain a large number of molecules so that statistical averaging is meaningful and a property value can be assigned to it. Disregarding the behavior of individual molecules, matter is treated as continuous. Now, if the volume is continuously decreased to such a level that it occupies only significantly few molecules, the statistical average value of property substantially fluctuates with time as molecules pass into and out of the volume in random motion and so it is difficult to speak of a definite value. The smallest volume of matter which can be considered as continuous is regarded as continuum.

1.11 Specific Volume

The specific volume of a substance is defined as the volume per unit mass. From continuum consideration, the specific volume at a point is defined as

$$v = \lim_{\delta v \rightarrow \delta v'} \frac{\delta V}{\delta m}$$

Where, $\delta V'$ is the smallest volume for which the system can be considered as a continuum

1.12 Pressure

Pressure is the normal force exerted by a system against unit area of the bounding surface. If δA is a small area and $\delta A'$ is the smallest area from continuum point of view, and δF_n is the component of force normal to δA , the pressure p at a point on the wall is defined as,

$$p = \lim_{\delta A \rightarrow \delta A'} \frac{\delta F_n}{\delta A}$$

Unit: Pa, bar, atm

(i) Atmospheric Pressure

This is the pressure exerted by the envelope of air surrounding the earth's surface.

(ii) Absolute pressure

A zero pressure is considered to occur when there is perfect vacuum i.e. in absence of molecules or molecular motion. "Pressure measured relative to this state of vacuum or zero pressure is called absolute pressure."

(iii) Gauge Pressure

Pressure measured relative to the atmospheric pressure is called gauge pressure. When the pressure is more than atmospheric pressure, the relative pressure is known as gauge. Relative pressure for the pressure less than atm. Pressure is simply known as vacuum, rarefaction or negative pressure.

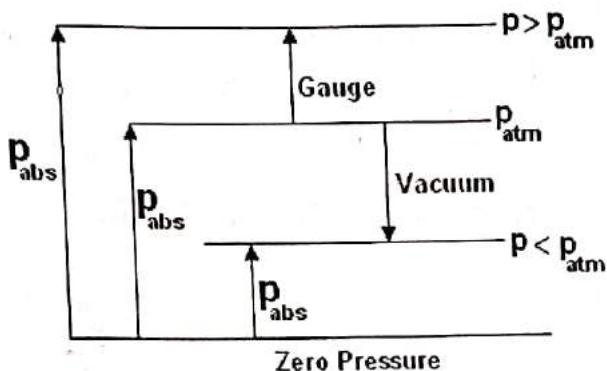


Fig 1.9 Relationship between pressures

$$P_{abs} = P_{atm} + P_g \text{ and } P_{abs} = P_{atm} - P_{vac}$$

Column Pressure

The pressure difference between two points in a fluid separated by a distance h is given by

$$\Delta p = \rho gh$$

1.13 Equality of temperature

Two systems are equal in temperature if no change in property occurs in either of them when they are brought into thermal communication.

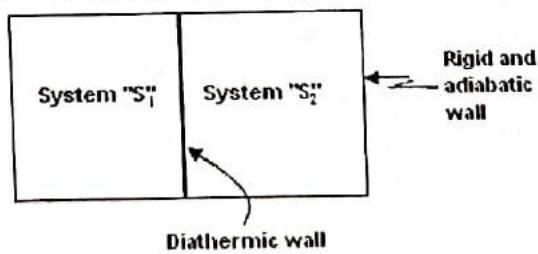


Fig 1.10 Systems in thermal equilibrium

Let us consider two systems S_1 and S_2 that are in contact with each other through a diathermic wall. Let the system S_1 be hot and the system S_2 be cold. Because of the interaction, the energies of both the system as well as their properties undergo a change. The hot body becomes cold and the cold body becomes hot. After some time, the states of either of the systems do not undergo any further change and the systems are said to be in thermal equilibrium. The properties of either of the two systems, though unique, may be different from each other except one- the temperature. Thus, temperature is the property which has the same value for all the systems in thermal equilibrium.

1.14 Zerоth law of Thermodynamics *Fwd*

When two systems are in thermal equilibrium with a third system separately, they in turn, will be in thermal equilibrium with each other.

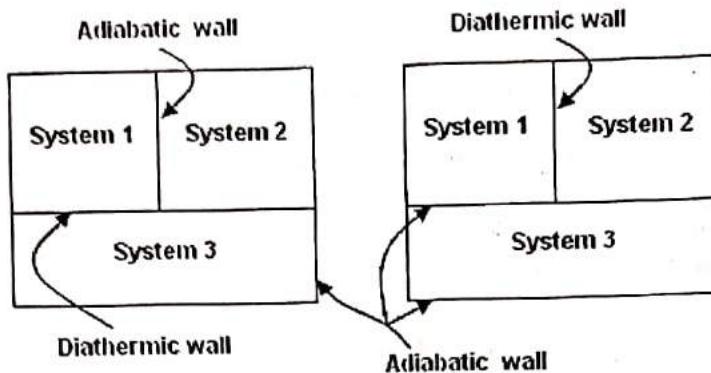


Fig 1.11 Demonstration of zeroth law of thermodynamics

Let us consider three systems 1,2 and 3 placed in an adiabatic enclosure as shown in 1st figure above. The system 1 and 2 do not interact with each other, and they are separated by an adiabatic wall. But systems 1 and 2 interact separately with system 3 through diathermic wall . Then system 1 is in thermal equilibrium with system 3 and system 2 is also in thermal equilibrium with the system 3. By intuition, we can say that though systems 1 and 2 are not interacting, they are in thermal equilibrium with each other. Now, suppose systems 1 and 2 are brought into contact with each other by replacing the adiabatic wall separating them by a diathermic wall as shown in 2nd figure above.. Further, they are isolated from the system 3 by replacing the diathermic wall, separating systems 1 and 2 from 3, by an adiabatic wall. Then , no change in the state of systems 1 and 2 is observed. This fact is known as zeroth law of thermodynamics and can be stated as p: when two systems are in thermal equilibrium with a third system separately, they in turn will be in thermal equilibrium with each other.

1.14.1 Significance of zeroth law

The significance of this law lies in the fact that it forms the basis for the temperature measurement. Let S_2 be a quantity of melting ice at standard atmospheric pressure, S_3 a thermometer and S_1 a system whose temperature is to be measured. The temperature of S_2 (ice point) is universally assigned a fixed value of 0°C . This is one of the system International fixed points. Let the thermometer S_3 be brought in communication with S_2 . The mercury level in the capillary of the thermometer keeps dropping. Once equilibrium between S_2 and S_3 is reached, there is no further change in the level of the mercury. Let this level be marked as 0°C , in accordance with the value assigned for the fixed point. Now, let S_1 be brought in communication with S_3 , the thermometer. If the mercury level were to stay put at the zero mark, then S_1 and S_3 , are in thermal equilibrium. According to the zeroth law, S_1 must then be in temperature equilibrium with S_2 . As S_2 is assigned the value of 0°C , S_1 must also be at 0°C .

Similarly, considering boiling water at standard atmospheric pressure as S_2 , which is given a fixed value of 100°C (steam point), the above exercise can be repeated again to get the 100°C mark on the thermometer and use it to measure the temperature S_1 .

To measure temperatures between the ice point and the steam point, the distance between the two marks are divided into 100 equal parts, and each graduations is marked as 1°C on the Celsius scale.

Chapter 2

Pure Substance

2.1 Introduction

A pure substance is one which is (i) homogeneous in composition (ii) homogeneous in chemical aggregation, and (iii) invariable in chemical aggregation irrespective of the phase or phases in which it exists. The pure substance is sometimes called one component system.

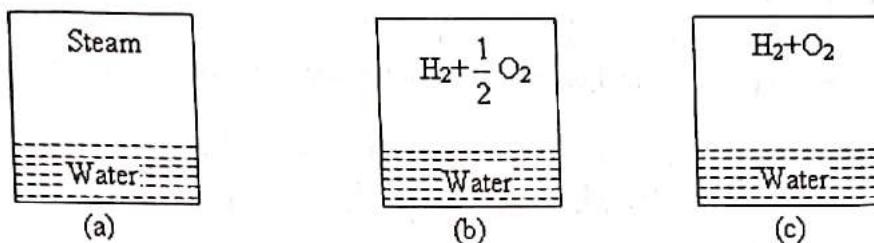


Fig 2.1 Pure substance or not?

- * “homogeneous in composition” means that the composition of each part of the system is the same as the composition of every other part. “Composition” means the relative proportions of the chemical elements into which the sample can be analyzed. It does not matter how these elements are combined.
For example, in fig (a), the system comprising steam and water is homogeneous in composition since chemical analysis would reveal that hydrogen and oxygen atoms are present in the ratio of 2:1 whether the sample be taken from the steam or from the water. The same is true to the system (b) containing uncombined hydrogen and oxygen gas in the atomic ratio 2:1 in the upper part and water in the lower part. System (c) , however , is not homogeneous in composition since hydrogen and oxygen are present in the ration 1:1 in the upper part but in the lower part , it is 2:1.
- “homogeneous in chemical aggregation” means that the chemical elements must be combined chemically in the same way in all parts of the system. In the above figure it is clear that the system (a) satisfies this condition as steam and water consist of identical molecules. System (b) on the other hand is not homogeneous in chemical aggregation since in the upper part of it, the hydrogen and oxygen are not combined chemically whereas in the lower part of it, the hydrogen and oxygen are combined to form water.
- “Invariable in chemical aggregation” means that the state of chemical combination of the system does not change with time . Condition (ii) refers to variation with position. Thus a mixture of hydrogen and oxygen which changed into steam during the time that the system was under consideration can not be a pure substance.

Other examples of pure substance:

- A mixture of different gases may also be treated as a pure substance provided that its composition is uniform. If one of the gases condenses, the mixture can no longer be treated as a pure substance since the condensed phase has a different composition than the original mixture. For example, air is a pure substance only as long as it is all gas or all liquid. A mixture of dry gaseous air and liquid air is not pure substance because the chemical composition of the liquid phase (liquid phase will be richer in N₂) is different from that of the vapour.
- Other examples of pure substance are the vapours of Fr-12, Fr-22 and NH₃ etc. which are used in refrigeration and air conditioning system for producing cooling effect.

2.1.1 Phase

- A phase is any homogeneous part of a system that is physically distinctive.
- It is essentially the homogeneous, chemical and physical state of aggregation of molecules comprising the substance.
- A pure substance may exist in different phases namely solid, liquid and vapour.
- Each phase of a system is separated by from other phases by interfaces called phase boundary. A system of an ice cube in liquid water has two phases viz, solid and liquid and the phase boundary is the surface of ice cube.
- The solid phase of a substance can have different forms known as allotropes. Examples are carbon(graphite) and carbon (diamond).
- The substance of mixed phase of gas and liquid is called vapour. Unlike a gas, vapour can be easily liquefied by applying pressure at constant temperature. The simple law of perfect gas does not apply to the vapours and hence variation between their properties are depicted in charts and tables.

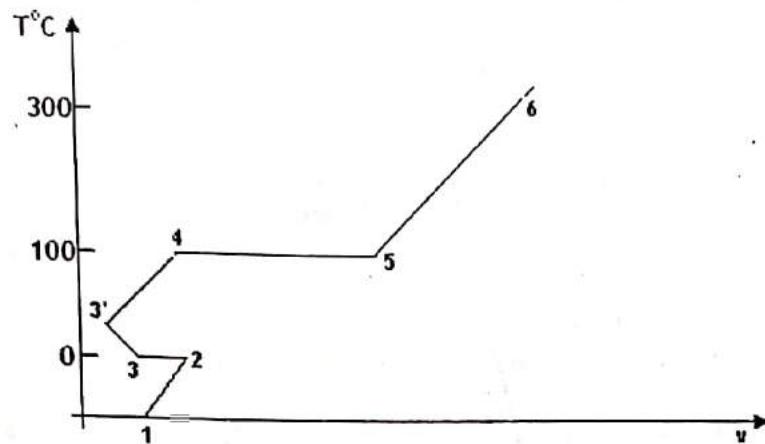


Fig 2.3 T-v diagram for phase change of ice into steam

1. When heat is supplied to ice at -10°C , 1 atm, its temperature rises to 0°C (1-2). During this process, the volume increases as solid expands on heating
2. On further heating, melting starts (2-3). During the process of melting:
 - there exists a mixed phase of ice and water.
 - The temperature and pressure of this mixed phase remains constant.
 - The volume decreases upto 3.
3. At point 3, the entire ice gets melted when there is only one phase i.e. water.
4. On further heating, the temperature continuously increases until boiling or vaporization starts as is clear in part (3-4) of figure A. But as far as volume is concerned, it decreases from 0°C to 4°C (3-3') and then increases continuously upto point 4 till boiling starts.
5. Again on further heating, water starts boiling i.e. phase change takes place from liquid to vapour(4-5). Durin this process:
 - There exists a two phase mixture of water and its vapour.
 - The temperature and pressure remains constant i.e. it is isobaric and isothermal process.
 - There is considerable increase increase in volume.
6. At state point 5, all the water has vaporized and state of steam at this point is called dry saturated steam.
7. Once the steam attains dry and saturated state, its temperature and volume start increasing with further supply of heat. The steam so obtained is called superheated steam and it behaves like a perfect gas.

2.3 T-v Diagram of water

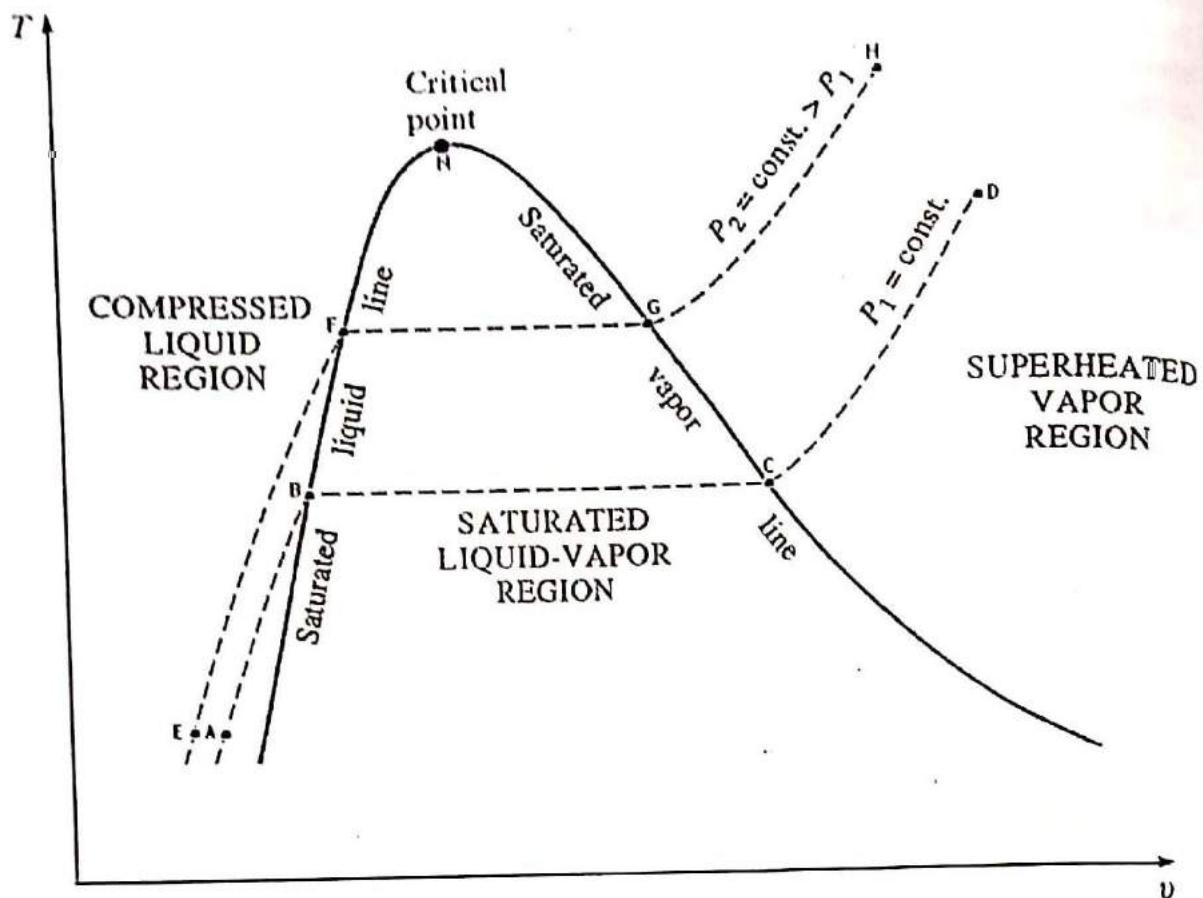


Fig 2.4 T-v diagram of water

1. Let us consider 1 kg of water contained in a vessel. Let it be heated at constant pressure. A number of isobars can be plotted on T-v diagram. Let us consider one such isobar ABCD at 1 atm, the various parts of which represent the followings:

a. AB: Heating of liquid to saturation temperature.

During the process AB, heating produces an increase in temperature and volume. If the liquid has temperature less than saturation temperature, it is said to be sub-cooled or compressed liquid.

b. BC: Change of phase from liquid to vapour at constant temperature.

At point B, the liquid starts changing the phase to vapour. The change of phase ends at C where all the water gets converted to vapour. The points B and C are known as saturated liquid state and saturated vapour state respectively.

c. CD: Superheating of vapour

On further heating after Point C is achieved, both temperature and volume increases. When the temperature of the vapour is higher than the saturation temperature at a given pressure, it is called superheated steam.

2. The locus of different saturation states like B and C for different pressures gives a curve known as saturation curve. Also the locus of points denoting saturated liquid is called saturated liquid line and the locus of the points denoting saturated vapour state is called saturated vapour line.
3. In the dome BFNGC, both the liquid and vapour phases are present while in the region to the left of the dome, only liquid is present. And in the region to the right of the dome, only superheated vapour is present.
4. The difference between the specific volumes of vapour and liquid ($v_g - v_l$) decreases with increasing pressure i.e. as we go upwards the dome and becomes zero at a point known as critical point.
5. Critical point is the point on the saturation curve beyond which liquid-vapour transformation is not possible. At critical state, $v_{fg}=0$. At critical point the liquid and vapour phases are indistinguishable and at which liquid and vapour phases co-exist together.
6. The temperature, pressure and volume at the critical point are called critical temperature t_c , critical pressure p_c and critical volume v_c and are given below:
- $$t_c = 374.15^\circ\text{C}, p_c = 221.2 \text{ bar and } v_c = 0.00310 \text{ m}^3/\text{kg}$$

2.4 P-v Diagram of water

Let us consider 1 kg of steam being compressed isothermally along the isotherm DEFG.

- If the vapour at D is compressed slowly and isothermally, the pressure rises until saturation state E comes.
- If compression is continued even at E, condensation takes place at constant pressure and temperature known as saturation pressure and temperature. It continues till all the steam is converted into water at F.
- Points E and F represent saturated vapour and saturated liquid. At any point between E and F, both liquid and vapour phase exist together.
- Since water is nearly incompressible, the volume does not change significantly with further compression after saturation liquid state at F. Hence, the part FG of the isotherm is almost vertical.
- If the above process is carried out at different temperatures, a number of isotherms like HIJK etc can be plotted. The locus of all saturated liquid states gives saturated liquid line FJC and the locus of all the saturated vapour states gives saturated vapour line CIE. And the locus of all saturated states whether it is liquid or vapour is called saturation curve.

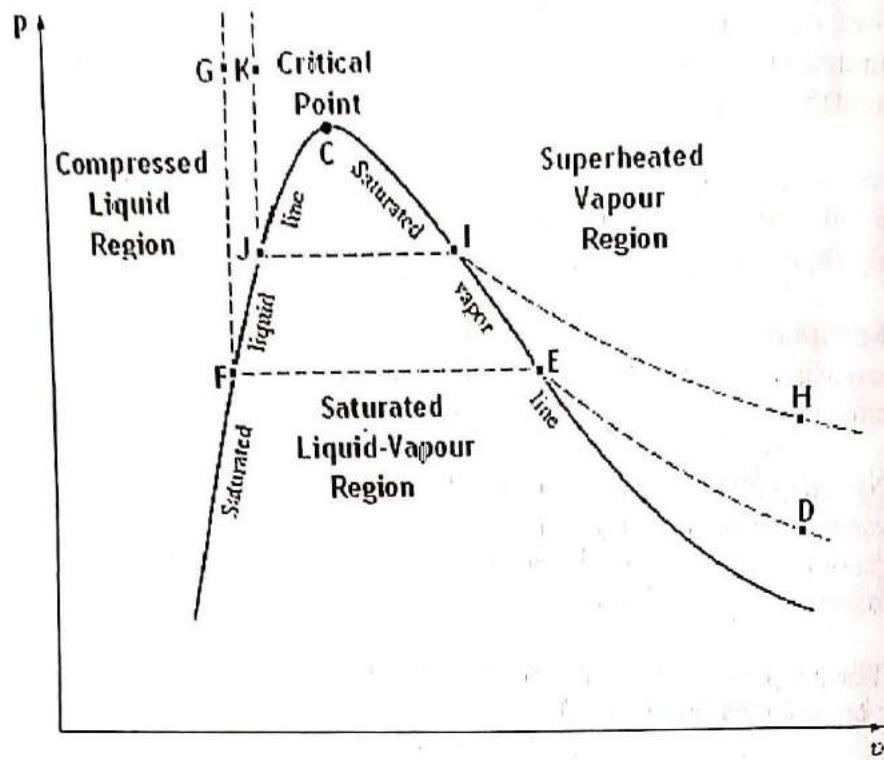


Fig 2.5 P-v diagram of water

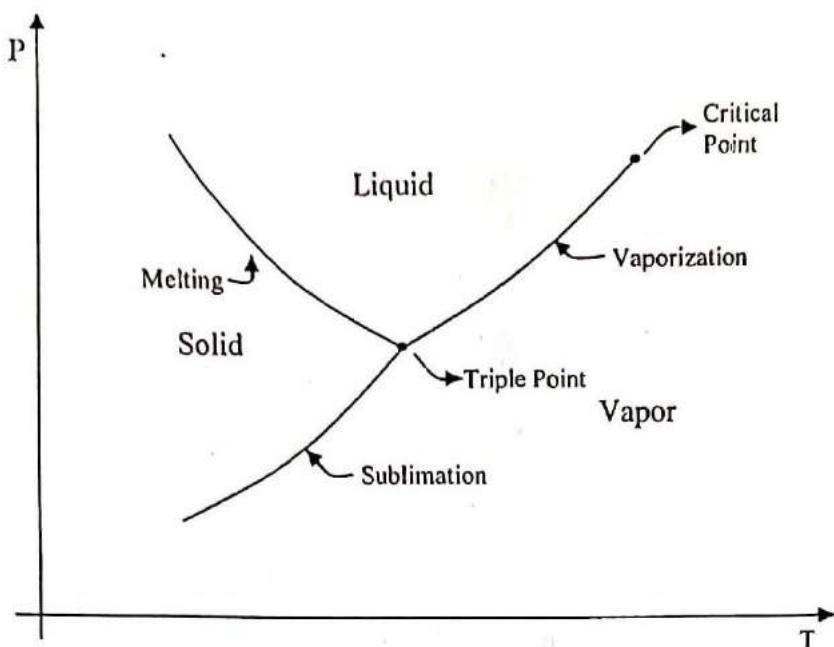
6. The transition zone say EF decreases with the increase in saturation temperature and pressure and it becomes zero at a point where the saturated liquid line and saturated vapour line meet together. Such a point is called critical point. At critical point, the phase change from liquid to vapour is indistinguishable.

7. The properties such as pressure, temperature and specific volume at critical point are called critical pressure, temperature and specific volume of water and are as follows:
 $t_c=374.15^\circ\text{C}$, $p_c=221.2 \text{ bar}$ and $v_c=0.00310 \text{ m}^3/\text{kg}$

8. In the dome FJCIE, both the liquid and vapour phases are present while in the region to the left of the dome, only compressed liquid is present. And in the region to the right of the dome, only superheated vapour is present.

2.5 P-T diagram of water

1. Phase change ($\text{solid} \rightarrow \text{liquid}$, $\text{liquid} \rightarrow \text{vapour}$, $\text{solid} \rightarrow \text{vapour}$) takes place at constant pressure and temperature known as saturation pressure and temperature. At such pressure and temperature, two phase exists together as a mixture. There can be a number of pairs of such pressure and temperature which when plotted on P-v diagram gives curve for different phase change. The solid and vapour phase co-exist along sublimation curve l-T, the solid and liquid phase co-exist along Fusion line (Melting line), and the liquid and vapour phase co-exist along vaporization line T-C.



2. The fusion line for water has a negative slope ($\partial p / \partial T < 0$) indicating that melting point of ice decreases with increasing pressure.
3. The vaporization line ends at critical point where there is no distinct change from liquid to vapour phase i.e. the liquid and vapour are indistinguishable.
4. The sublimation, fusion and vaporization lines meet at a point called triple point. The triple point may thus be defined as the point at which all the three phases co-exist in equilibrium. For water, triple point values are:
 $t_c = 0.01^\circ\text{C}$ & $p_c = 0.611 \text{ kPa}$
5. Sublimation refers to the heating process where solid(ice) gets directly transformed to saturated vapour phase without first melting into liquid(water)
6. The saturation curve gives the saturation pressure as a function of saturation temperature.

2.6 P-V-T surface of water

1. In this presentation, pressure, specific volume and temperature are plotted on mutually perpendicular co-ordinates and each possible equilibrium state is thus represented by a point on the surface.
2. Any point on the p-v-T surface is determined by any two independent properties such as p and T .
3. In this presentation, pressure, specific volume and temperature are plotted on mutually perpendicular co-ordinates and each possible equilibrium state is thus represented by a point on the surface.

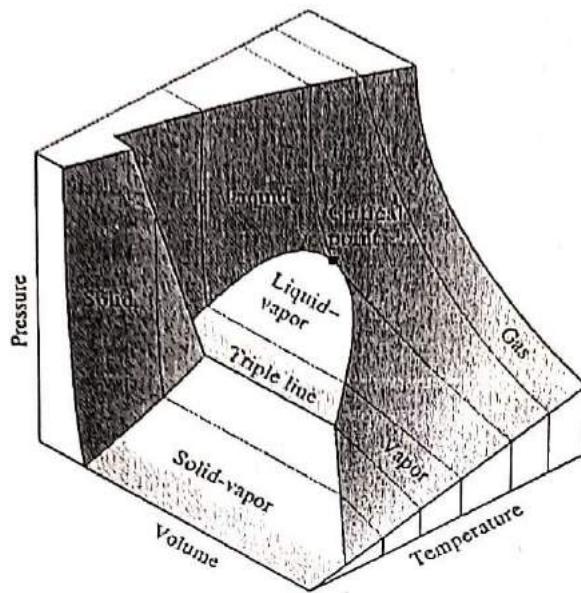


Fig 2.7 P-V-T surface diagram

4. In this presentation, pressure, specific volume and temperature are plotted on mutually perpendicular co-ordinates and each possible equilibrium state is thus represented by a point on the surface.
5. Any point on the p-v-T surface is determined by any two independent properties such as p and T .
6. The regions of the surface that represent a single phase namely the solid, liquid and vapour phase are labeled as curved.
7. The two phase regions namely the solid-liquid, solid - vapour and liquid-vapour are ruled surfaces and are made up of straight lines parallel to the specific volume axis.
8. In the two phase regions, lines of constant pressure are also the lines of constant temperature though the specific volume may change.
9. The triple point actually appears as the triple line as the pressure and temperature of the triple point are fixed but the specific volume may vary depending upon the proportion of each phase.

2.7 Wet Steam

Wet steam is defined as a mixture of dry steam and suspended water particles. Thus it is a two phase mixture. The relative amount of each phase present in the wet steam determines the quality or dryness fraction of the steam. *The quality or dryness fraction of steam is defined as the ratio of the mass of dry steam to the total mass of the wet steam (i.e. mixture).* It is represented by X and is given by:

$$X = \frac{\text{mass of dry vapour}}{\text{mass of mixture}}$$

If m_f = mass of suspended water of fluid particle

The superheated steam is usually treated as ideal gas. Applying the equation of state to the superheated steam along the constant pressure line on T-v diagram, we get

$$pV = mRT \Rightarrow p \frac{V}{m} = RT \Rightarrow pv = RT$$

Since p is constant, we have

$$\frac{v}{T} = \text{constant}$$

So, for superheated steam

$$\frac{v_{\text{sup}}}{T_{\text{sup}}} = \frac{v_g}{T_g}$$

$$\therefore v_{\text{sup}} = \frac{T_{\text{sup}}}{T_g} \times v_g$$

2.8.2 Specific enthalpy

We know that

$$\Delta h = \Delta q \quad \text{for constant pressure process}$$

So, for superheated steam along isobar on T - v diagram, we have

$$h_{\text{sup}} - h_g = 1 \cdot c_{p,s} (t_{\text{sup}} - t_g)$$

$$\therefore h_{\text{sup}} = h_g + c_{p,s} (t_{\text{sup}} - t_g)$$

2.8.3 Specific Entropy

We know that

$$ds = \frac{\delta q}{T} \quad \text{for reversible process}$$

For superheated steam along isobar on T - s diagram, we have

$$ds = \frac{c_{p,s} dT}{T}$$

On integration, it yields to

$$\int_{s_g}^{s_{\text{sup}}} ds = \int_{T_g}^{T_{\text{sup}}} \frac{c_{p,s} dT}{T} = c_{p,s} \int_{T_g}^{T_{\text{sup}}} \frac{dT}{T}$$

$$[s]_{T_g}^{T_{\text{sup}}} = c_{p,s} [\ln T]_{T_g}^{T_{\text{sup}}} = c_{p,s} [\ln T_{\text{sup}} - \ln T_g]$$

$$\text{or, } s_{\text{sup}} - s_g = c_{p,s} \ln \frac{T_{\text{sup}}}{T_g}$$

$$\therefore s_{\text{sup}} = s_g + c_{p,s} \ln \frac{T_{\text{sup}}}{T_g}$$

m_g = mass of dry saturated steam
 m = mass of the wet steam i.e. mixture

$$X = \frac{m_g}{m_f + m_g} = \frac{m_g}{m}$$

2.7.1 Specific volume for wet steam

Let us consider m kg of wet steam having volume V m^3 . The steam consists of m_f kg of suspended water and m_g kg of dry saturated vapour the respective volumes of which are V_f and V_g .

Now we can write

$$V = V_f + V_g$$

Dividing both the sides by m , we get

$$\frac{V}{m} = \frac{V_f}{m} + \frac{V_g}{m}$$

$$\text{or, } v = \frac{V_f}{m_f} \times \frac{m_f}{m} + \frac{V_g}{m_g} \times \frac{m_g}{m}$$

$$\text{or, } v = v_f \cdot \frac{m - m_g}{m} + v_g X$$

$$\text{or, } v = v_f \left(\frac{m}{m} - \frac{m_g}{m} \right) + v_g X$$

$$\text{or, } v = v_f (1 - X) + v_g X$$

$$\text{or, } v = v_f - X v_f + X v_g$$

$$\text{or, } v = v_f + X v_g - X v_f$$

$$\text{or, } v = v_f + X(v_g - v_f)$$

$$\therefore v = v_f + X v_{fg}$$

$$\text{Where } v_{fg} = v_g - v_f$$

2.8 Determination of properties of superheated steam

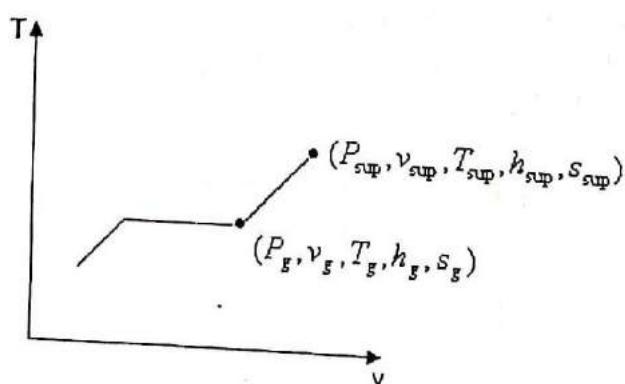


Fig 2.8 Co-relation between superheated and saturated states of steam

2.8.1 Specific Volume

Solved Examples

1. Determine the state of steam i.e. whether it is wet, dry or superheated in the following cases:
a) steam has a pressure of 10 bar and specific volume of $0.175 \text{ m}^3/\text{kg}$
b) steam has a pressure of 15 bar and a temperature of 220°C .

Solution:

Case a: steam has a pressure of 10 bar and specific volume of $0.175 \text{ m}^3/\text{kg}$ i.e.

$$P=10 \text{ bar}=1000 \text{ kPa}$$

$$v=0.175 \text{ m}^3/\text{kg}$$

At 1000 kPa

From steam table, we have

$$v_f = 0.001127 \text{ m}^3/\text{kg}$$

$$v_g = 0.19444 \text{ m}^3/\text{kg}$$

Obviously,

$$v_f < v < v_g$$

So, our steam is wet one.

Case b: steam has a pressure of 15 bar and a temperature of 220°C i.e.

$$P=15 \text{ bar}=1500 \text{ kPa}$$

$$t=220^\circ\text{C}$$

At 1500 kPa

From steam table, we have

$$\text{Saturation temperature } t_s = 198.32^\circ\text{C}$$

$$\text{But, given temperature of steam, } t = 220^\circ\text{C}$$

Since, $t > t_s$, our steam must be superheated.

2. A vessel of 0.3 m^3 capacity contains 1.5 kg mixture of water and steam in equilibrium at a pressure of 5 bar. Calculate:
a) the volume and mass of liquid
b) the volume and mass of steam.

Solution:

Given:

Mass of the mixture, $m=1.5 \text{ kg}$

Volume of the mixture, $V=0.3 \text{ m}^3$

Equilibrium pressure, $P=5 \text{ bar}=500 \text{ kPa}$

We know that,

$$\text{Specific volume of the mixture, } v = \frac{V}{m} = \frac{0.3}{1.5} = 0.2 \text{ m}^3/\text{kg}$$

At 500 kPa

From steam table, we have

$$v_f = 0.001093 \text{ m}^3/\text{kg}; \quad v_{fg} = 0.37380 \text{ m}^3/\text{kg}; \quad v_g = 0.37489 \text{ m}^3/\text{kg}$$

Since the mixture contains water and steam at equilibrium temperature, we have

$$v = v_f + X.v_{fx}$$

$$Or, 0.2 = 0.001093 + X(0.37380)$$

$$\therefore X = \frac{0.2 - 0.001093}{0.37380} = 0.532$$

Also we know that,

$$X = \frac{m_g}{m} \Rightarrow m_g = X \cdot m = 0.532 \times 1.5 = 0.798 \text{ kg}$$

And, $m_f = m - m_g = 1.5 - 0.798 = 0.702 \text{ kg}$, where m_f and m_g are mass of liquid and steam respectively and X is the dryness fraction.

Specific volume is defined as, $v = \frac{V}{m} \Rightarrow V = mv$

So that,

$$\text{Volume of steam, } V_g = m_g v_g = 0.798 \times 0.37489 = 0.299 \text{ m}^3$$

$$\text{Volume of the liquid, } V_f = m_f v_f = 0.702 \times 0.001093 = 7.67 \times 10^{-4} \text{ m}^3$$

3. A vessel contains one kg of steam which contains $\frac{1}{3}$ liquid and $\frac{2}{3}$ vapor by volume. The temperature of the steam is 151.86°C . Find the quality, specific volume and specific enthalpy of the mixture.

Solution:

Given:

Mass of the mixture, $m = 1 \text{ kg}$

$$\text{Volume of the liquid, } V_f = \frac{V}{3}$$

$$\text{Volume of the steam, } V_g = \frac{2V}{3}$$

Temperature of the steam $t = 151.86^{\circ} \text{C}$

Where, V is the total volume of the vessel

If m_f and m_g are the masses of the liquid and steam, then we have

$$m_f + m_g = m$$

Putting the values of V_f and V_g in (I), we get

$$\frac{V}{3v_f} + \frac{2V}{3v_g} = 1$$

$$Or, \frac{V}{3} \left(\frac{1}{v_f} + \frac{2}{v_g} \right) = 1$$

At 151.86⁰C

From steam table, we have

$$v_f = 0.001093 \text{ m}^3/\text{kg} \quad ; \quad v_{fg} = 0.37380 \text{ m}^3/\text{kg} \quad ; \quad v_g = 0.37489 \text{ m}^3/\text{kg}$$

On substitution, eqⁿ(II) yields to,

$$V = \frac{3}{\frac{1}{0.001093} + \frac{2}{0.37489}} = 3.26 \times 10^{-3} \text{ m}^3$$

And the specific volume, $v = \frac{V}{m} = \frac{3.26 \times 10^{-3}}{1} = 0.00326 \text{ m}^3 / \text{kg}$

But we know that,

$$v = v_f + X.v_{f_0}$$

$$Or, 0.00326 = 0.001093 + X(0.37380)$$

$$\therefore X = \frac{0.00326 - 0.001093}{0.37380} = 0.00579$$

At 151.86°C

From steam table , we have

$$h_f = 640.21 \text{ kJ/kg} \quad ; \quad h_{fg} = 2108.47 \text{ kJ/kg}$$

So, the specific enthalpy of the mixture is given by

$$h = h_f + X \cdot h_{fg} = 640.21 + (0.00579) \times (2108.47) = 652.42 \text{ kJ/kg}$$

4. A closed system containing dry saturated steam undergoes expansion according to the law $pv^n = c$ from an initial pressure of 10 bar to a final pressure of 2 bar. If the steam is finally wet with dryness fraction 0.85, evaluate the index of expansion and the work done.

Solution:

Initial condition

Pressure, $P_1 = 10 \text{ bar} = 10 \times 10^5 \text{ Pa}$

Condition: dry saturated

At $P_1 = 1000 \text{ kPa}$

From steam table, we have

$$v_g = 0.19444 \text{ m}^3 / \text{kg}$$

Since the steam at initial condition is dry saturated, its specific volume is given by

$$v_1 = v_g = 0.19444 \text{ m}^3 / \text{kg}$$

Final Condition

Pressure $P_2 = 2 \text{ bar} = 2 \times 10^5 \text{ Pa}$

Dryness fraction, $X_2=0.85$

But at $P_2=200\text{ kPa}$

From steam table, we have

$$v_f = 0.001061 \text{ m}^3/\text{kg} ; v_{fg} = 0.88467 \text{ m}^3/\text{kg}$$

$$\therefore v_2 = v_f + X_2 v_{fg} = 0.001061 + (0.85)(0.88467) = 0.753 \text{ m}^3/\text{kg}$$

Again, by question, we have

$$pv^n = c$$

$$\therefore p_1 v_1^n = p_2 v_2^n$$

Taking log on both sides, we get

$$\ln p_1 + n \ln v_1 = \ln p_2 + n \ln v_2$$

$$\text{Or, } n(\ln v_1 - \ln v_2) = \ln p_2 - \ln p_1$$

$$\text{Or, } n \ln \frac{v_1}{v_2} = \ln \frac{p_2}{p_1}$$

$$\therefore n = \frac{\ln(p_2/p_1)}{\ln(v_1/v_2)} = \frac{\ln(2/10)}{\ln(0.19444/0.753)} = 1.19$$

Hence,

$$\begin{aligned}\text{Work done, } W &= \frac{p_2 v_2 - p_1 v_1}{1-n} \\ &= \frac{2 \times 10^5 \times 0.753 - 10 \times 10^5 \times 0.19444}{1-1.19} \\ &= 230736.84 \text{ J/kg} \\ &= 230.74 \text{ kJ/kg}\end{aligned}$$

5. Tank A has a volume of 0.1 m^3 and contains steam at 200°C , 10% liquid and 90% vapour by mass, while tank B is evacuated. The valve is opened and the tanks eventually come to pressure of 4 bar and 200°C . What is the volume of the tank B?

Solution:

Given:

Volume of the steam in tank A, $V_A = 0.1 \text{ m}^3$

Temperature of steam in tank A, $t_1 = 200^\circ\text{C}$

Mass of liquid in tank A, $m_l = 0.1 \text{ kg}$

Mass of steam in tank A, $m_g = 0.1 \text{ kg}$, where m is the total mass inside the tank A

$$\therefore X = \frac{m_g}{m} = \frac{0.9m}{m} = 0.9$$

At 200°C

From steam table, we have

$$v_f = 0.001156 \text{ m}^3/\text{kg} ; v_{fg} = 0.12620 \text{ m}^3/\text{kg} ; v_g = 0.12736 \text{ m}^3/\text{kg}$$

Specific volume of steam in tank A is given by,

$$\begin{aligned}v_A &= v_f + X v_{fg} \\ &= 0.001156 + (0.9)(0.12620) \\ &= 0.114736 \text{ m}^3/\text{kg}\end{aligned}$$

$$\text{Mass of steam, } m = \frac{V_A}{v_A} = \frac{0.1}{0.114736} = 0.872 \text{ kg}$$

When the valve is opened

Pressure, $p_2 = 4 \text{ bar} = 400 \text{ kPa}$

Temperature, $t_2 = 200^\circ\text{C}$

At 400 kPa

From steam table, we have

Saturation temperature, $t_s = 143.63^\circ\text{C}$

Since $t_2 > t_s$, the steam must be superheated.

From superheated steam table at 400k Pa and 200°C, we have

$$v = 0.53422 \text{ m}^3/\text{kg}$$

So, specific volume of the steam inside tank (A+B) is given by

$$v_{A+B} = v = 0.53422 \text{ m}^3/\text{kg}$$

Since the mass remains same, we have

Volume of steam inside tank(A+B) is,

$$\begin{aligned} V_{A+B} &= \text{Mass} \times \text{Sp. Volume} \\ &= 0.872 \times 0.53422 \\ &= 0.466 \text{ m}^3 \end{aligned}$$

$$\text{Volume of tank B, } V_B = V_{A+B} - V_A = 0.466 \text{ m}^3 - 0.1 \text{ m}^3 = 0.366 \text{ m}^3$$

6. A cylinder contains 1 kg of water and steam at a pressure of 3.8 bar and 0.4 dry. Heat is supplied at constant volume until pressure is reached to 10 bar. The steam is then expanded according to the law $pv = \text{const.}$ until the pressure is 2 bar. Calculate the volume of the cylinder and the temperature of the steam after expansion.

Solution:

Initial condition

$$P_1 = 3.8 \text{ BAR} = 380 \text{ kPa}$$

$$X_1 = 0.4$$

The properties of steam can not be obtained directly from the steam table. So, it is to be determined with the help of interpolation as follows.

P(kPa)	$v_f (\text{m}^3/\text{kg})$	$v_{fg} (\text{m}^3/\text{kg})$
375	0.001081	0.49029
380	?	?
400	0.001084	0.46138

Interpolating for v_f , we get

$$v_f - 0.001081 = \frac{0.001084 - 0.001081}{400 - 375} (380 - 375)$$

$$\therefore v_f = 0.0010816 \text{ m}^3/\text{kg}$$

Similarly, interpolating for v_{fg} , we get

$$v_{fg} - 0.49029 = \frac{0.46138 - 0.49029}{400 - 375} (380 - 375)$$

$$\therefore v_{fg} = 0.484508 \text{ m}^3/\text{kg}$$

So, at $p_1 = 380 \text{ kPa}$, we have

$$v_f = 0.0010816 \text{ m}^3/\text{kg}$$

$$v_{fg} = 0.484508 \text{ m}^3/\text{kg}$$

$$\therefore v_1 = v_{f|p_1} + X_1 v_{fg|p_1} = 0.0010816 + 0.4 \times 0.484508 = 0.1949 \text{ m}^3/\text{kg}$$

2nd condition

$$p_2 = 10 \text{ bar} = 1000 \text{ kPa}$$

$$v_2 = v_1 = 0.1949 \text{ m}^3/\text{kg}$$

3rd condition

$$p_3 = 2 \text{ bar} = 200 \text{ kPa}$$

The process 2-3 is an expansion process complying $pv = \text{const.}$

$$\therefore p_3 v_3 = p_2 v_2 \Rightarrow v_3 = \frac{p_2}{p_3} \times v_2 = \frac{10}{2} \times 0.1949 = 0.9745 \text{ m}^3/\text{kg}$$

After expansion, the volume of the cylinder, V_3 , is given by

$$V_3 = m \times v_3 = 1 \times 0.9745 = 0.9745 \text{ m}^3$$

Now, at final condition i.e. at $p_3 = 200 \text{ kPa}$

We have from steam table

$$v_f = 0.001061 \text{ m}^3/\text{kg}$$

$$v_{fg} = 0.88467 \text{ m}^3/\text{kg}$$

$$v_g = 0.88573 \text{ m}^3/\text{kg}$$

Since $v_3 > v_g$, the steam at the final condition must be superheated.

Now, from steam table for superheated region, we have

$v (\text{m}^3/\text{kg})$	$t (^{\circ}\text{C})$
0.95964	150
0.9745	?
1.08034	200

Interpolating for v_{fg} from the above table, we get

$$t - 150 = \frac{200 - 150}{1.08034 - 0.95964} (0.9745 - 0.95964)$$

$$\therefore t = 156.15^{\circ}\text{C}$$

Problems

1. A vessel of 0.05 m^3 capacity contains a mixture of saturated steam and water at a temperature of 300°C ; the mass of liquid water being 10 kg. Determine the pressure, mass, enthalpy, specific volume, internal energy and entropy of contents present in the vessel.
2. Steam at 100 kPa and 300°C enters an engine and expands to 20 kPa. If the exhaust steam has a dryness fraction of 0.9, calculate for the drop in enthalpy and change in entropy.
3. A closed metallic boiler drum of 0.25 m^3 capacity holds steam at 10 bar pressure and 195°C . Determine the quantity of steam in the drum. At what pressure in the drum the steam would become dry saturated if the drum is cooled? What would be the pressure and condition of steam in the drum if the cooling continues till the temperature of the steam becomes 120.2°C ?
4. Steam at 10 bar pressure and 0.9 dryness fraction is cooled at constant volume at 160°C . What will be its final condition?
5. A closed vessel of 0.2 m^3 capacity contains steam at a pressure of 10 bar and a temperature of 250°C . The vessel is cooled till the pressure of steam in the vessel falls to 3.5 bar, Find: a) final temperature and dryness fraction (b) change in internal energy and entropy
6. During a steady flow process, 5 kg of steam at 20 bar and 300°C loses 4150 kJ of heat at constant pressure. Determine the final condition of steam.
7. Steam at 10 bar and 250°C expands polytropically to a pressure of 1 bar. If the exponent n for the polytropic process is 1.2, calculate (i) specific volume and temperature after expansion (ii) change in entropy.
8. Steam initially at 6 MPa and 450°C is allowed to cool to 200°C at constant volume. Find the final condition and heat transfer if the mass of steam is 0.75 kg.
9. 0.1 kg saturated steam expands reversibly from 10 to 1 bar in a piston-cylinder device according to $p v^{1.3} = \text{constant}$. Find the work and heat interactions during the expansion process.
10. A rigid vessel of volume 0.2 m^3 contains wet steam at a pressure of 1 bar. Given that the ratio of mass of the liquid to the vapour is $\frac{5}{95}$, find the mass of liquid and vapour each and the dryness fraction of the steam.
11. A sealed rigid vessel contains 0.1 m^3 of wet steam at 2 bar. By volume, 95% is in the vapour form. Find the dryness fraction of the steam. Also find the specific volume.
12. A rigid cylinder contains steam at 10 bar and a temperature of 300°C . The steam is cooled until its pressure decreases to 1 bar. Find the final state and the heat transfer per kg of steam.

13. 0.1 kg steam at 5 bar, 0.9 dry expands to a pressure of 1 bar in a resisted process that follows the path $pv^{1.2} = \text{constant}$. Find work transfer, heat transfer and change in entropy.
14. A vessel contains wet steam at a pressure of 2 bar, 12% of whose mass is in liquid. Find the following: (a) temperature (b) dryness fraction (c) specific volume (d) specific enthalpy and (e) specific internal energy
15. A vessel of one cubic meter contains wet steam at a pressure of 2 bar, 12% of whose volume is liquid. Find the following : (a) temperature (b) dryness fraction (c) specific volume (d) specific enthalpy.
16. It is found that wet steam at 150 kPa when heated in a rigid vessel of one cubic metre volume passes through the critical state. Find the mass of the steam and its initial dryness.
17. Ten kg of water at 45°C is heated at a constant pressure of 10 bar until it becomes superheated vapour at 300°C . Find the changes in volume, enthalpy, internal energy and entropy.
18. A spherical aluminium vessel has an inside diameter of 0.3 m and 0.62 cm thick wall. The vessel contains water at 25°C with a quality of 1%. The vessel is then heated until the water inside is saturated vapour. Calculate the mass of fluid and the mass of vessel. The density of aluminium is 2.7 g/cm^3 and its specific heat is 0.896 kJ/kg K .
19. A vessel having a volume of 0.4 m^3 contains 2 kg of a liquid water and water vapour mixture in equilibrium at a pressure of 600 kPa. Calculate:
 - the volume and mass of liquid
 - the volume and mass of vapour.
20. Steam at 9 bar and 0.93 dry is contained inside a vessel of 0.9 m^3 volume. The delivery valve is opened and the steam is blown off until the pressure reduces to 4 bar. The delivery valve is then closed and the vessel cooled until the pressure is 3 bar. Assume that enthalpy per kg of steam remains constant during blowing off period. Calculate:
 - the mass of steam blown off.
 - quality of steam in the vessel after cooling.
21. Saturated water vapour at 60°C has its pressure decreased to increase the volume by 10% keeping the temperature constant. To what pressure should it be expanded ?
22. Water vapour is heated in a closed rigid tank from saturated vapour at 160°C to a final temperature of 400°C . Determine the initial and final pressure, in bars, and sketch the process on T-v diagram.
23. A two-phase liquid –vapour mixture of H_2O is initially at a pressure of 10 bars. If , on heating at fixed volume, the critical point is attained, determine the quality at the initial state.

24. A closed metallic boiler drum of capacity 0.24 m^3 and contains steam at a pressure of 11 bar and temperature of 200°C . Calculate the quantity of steam in the vessel. At what pressure in the vessel will the steam be dry and saturated if the vessel is cooled? The cooling is further continued till the temperature of the steam in the vessel becomes 116.9°C . Calculate the pressure and the condition of the steam at the end of the process.
25. Steam at 3 bar and 0.9 dry expands in a cylinder till the volume is four times that at the commencement. The law of expansion is $pv^{1.1} = c$. Determine the quality of steam after expansion.

Heat & Work

3.1 Definition of work

Work as defined in Mechanics

Work is said to be done when the point of application of a force moves in the direction of force. It is given by the dot product of the force and the displacement i.e.

$$W = \vec{F} \cdot \vec{d}$$

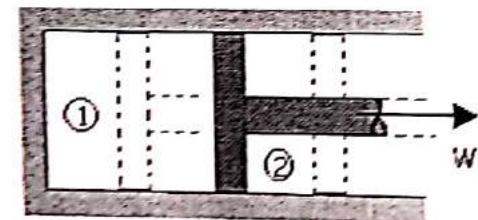
Work as defined in Thermodynamics

- Work refers to the energy transferred because of a property difference, other than temperature, that exists between the system and its surroundings.
- Work refers to the interaction between the system and surroundings whose sole effect external to the system, could be reduced to the lifting of mass against the gravitational force.

3.2 Displacement Work for Quasi-static process

It is the kind of work done by a system due to its boundary movement.

Let us consider a system formed by a certain amount of gas contained in a piston cylinder arrangement as shown. Let the piston move outward through a small distance dx ($dx \rightarrow 0$) during a small interval dt . Since the piston



State 1 : P_1, V_1
State 2 : P_2, V_2

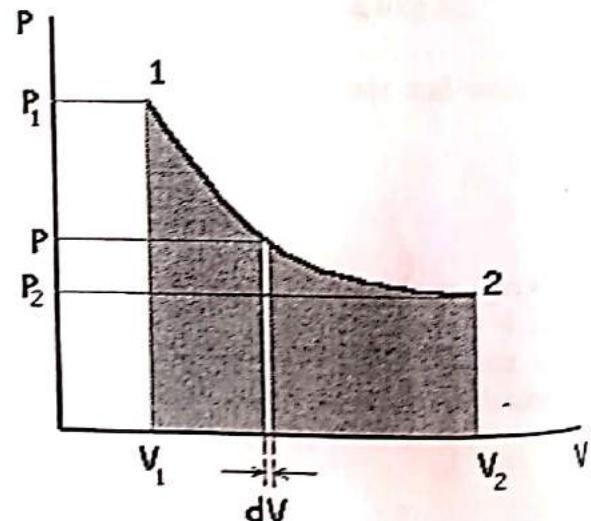


Fig 3.1 Work done during quasi-static process

Since the piston moves through an infinitesimally small distance, the pressure p acting on the face of the piston can be assumed to remain constant. In this case, the infinitesimal work done by the system,

$$\begin{aligned}\delta W &= F dx \\ &= p A dx \\ &= p dV\end{aligned}$$

Where, dV is the infinitesimal volume swept by the piston.

If the piston moves through a finite distance, the work done by the piston can be evaluated by integrating the above equation,

$$W = \int_1^2 p dV$$

The integral can be evaluated and hence computation of work made only if the relation between p and V or the path of the process is known. The area under p - V diagram gives the magnitude of work whatsoever the reversible process may be.

3.3 Salient aspects of work

- Work is energy in transit and can be identified only when the process is being executed. Work does not exist either before or after the interaction.
- Work is boundary phenomenon; it is an interaction across the boundary. It will be wrong to say that work is possessed or stored by the system. It is transferred energy that can be stored indefinitely by the system and not the work.
- Work is something which happens to a system but it is not a characteristic of the system.
- Being an inexact differential, work interaction is represented as δW instead of dW .

Further, $\delta W \neq W_2 - W_1$. Rather $\int_1^2 \delta W = W_{1-2}$ or $W_2 - W_1$

3.3.1 Sign convention for Work

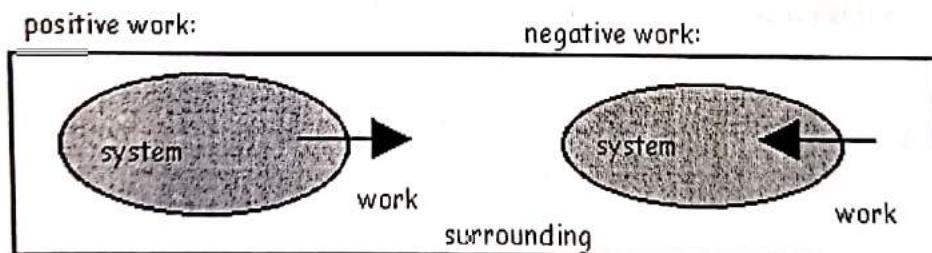


Fig 3.2 Sign convention for work

3.4 Heat

Heat is a form of energy that is transferred across the boundaries of a system because of temperature difference between the system and its surrounding and is always from a higher temperature to a lower temperature. No heat interaction is possible between the system and its surrounding if both are in thermal equilibrium.

3.4.1 Sign convention for heat

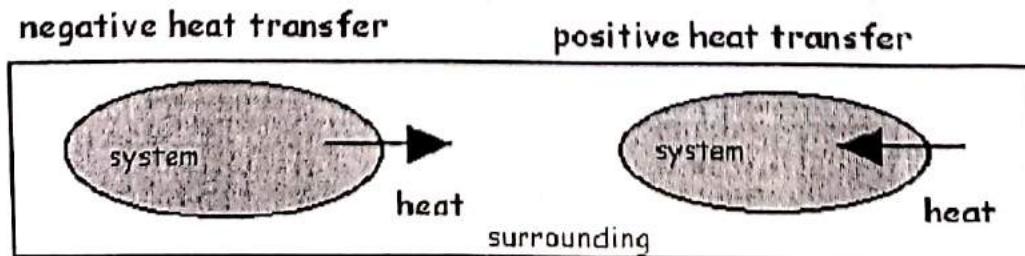


Fig 3.3 Sign convention for heat

3.5 Comparison between Heat and Work

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Similarities:

- Both heat and work are path functions; they are not properties of the system and the differentials are not exact.
- Both are boundary phenomena.
- Both represent transient phenomenon; these energy interactions occur only when system undergoes change of state.
- Concepts of heat and work are not associated with a 'store' but with a 'process'

Dissimilarities

- There can not be work transfer in a stable system. No such restriction exists for heat interaction.
- Heat is energy interaction due to temperature difference only; work is by reason other than temperature difference.
- Sole effect external to the system could be reduced to the raising of mass against gravitational force in case of work. However, during heat transfer other effects are also seen.
- Heat is low grade energy but work is high grade energy.

3.6 Displacement work in various processes

1. Isobaric or Isopiestic process

In this process, $p = \text{constant}$.

$$W_2 = \int_{V_1}^{V_2} pdV = p \int_{V_1}^{V_2} dV = p[V]_{V_1}^{V_2} = p(V_2 - V_1)$$

2. Isobaric Process

In this process, volume remains constant.

$$W_2 = \int_{V_1}^{V_2} pdV = 0, \text{ since } dV = 0 \text{ for } V = \text{constant}$$

3. Isothermal or Hyperbolic process

In this process, temperature remains constant i.e. $T = \text{const}$. So, by equation of state, $PV = RT = C$ (constant.) i.e. $PV = P_1V_1 = P_2V_2 = C \Rightarrow P = \frac{C}{V}$

$$W_2 = \int_{V_1}^{V_2} pdV = \int_{V_1}^{V_2} \left(\frac{C}{V} \right) dV = C \int_{V_1}^{V_2} \frac{dV}{V} = C[\ln V]_{V_1}^{V_2} = C(\ln V_2 - \ln V_1) = C \ln \frac{V_2}{V_1}$$

Therefore,

$$W_2 = P_1V_1 \ln \frac{V_2}{V_1} = P_2V_2 \ln \frac{V_2}{V_1}$$

$$\text{Also, as } P_2V_2 = P_1V_1 \Rightarrow \frac{V_2}{V_1} = \frac{P_1}{P_2}$$

$$\text{Hence, } W_2 = P_1V_1 \ln \frac{P_1}{P_2} = P_2V_2 \ln \frac{P_1}{P_2}$$

4. Polytropic process

In this process, $pV^n = p_1V_1^n = p_2V_2^n = C$ (constant) $\Rightarrow p = \frac{C}{V^n}$

Hence,

$$\begin{aligned} W_2 &= \int_{V_1}^{V_2} pdV = \int_{V_1}^{V_2} \frac{C}{V^n} dV = C \int_{V_1}^{V_2} \frac{1}{V^n} dV = C \int_{V_1}^{V_2} V^{-n} dV = C \left[\frac{V^{1-n}}{1-n} \right]_{V_1}^{V_2} = \frac{CV_2^{1-n} - CV_1^{1-n}}{1-n} \\ &= \frac{p_2V_2^nV_2^{1-n} - p_1V_1^nV_1^{1-n}}{1-n} = \frac{p_2V_2^{n+1-n} - p_1V_1^{n+1-n}}{1-n} = \frac{p_2V_2 - p_1V_1}{1-n} \end{aligned}$$

3.7 Equation of state for simple compressible substance

Based on experimental observations, the p-v-T behaviour of gases at low density is closely co-related by the following equation of state:

$$p\bar{v} = \bar{R}T \quad \dots \dots \dots \text{(i)}$$

where $\bar{R} = 8.3145 \text{ kJ/k mol K}$ is called Universal gas constant and \bar{v} is volume per unit mole of the gas. Dividing both sides of the above equation by M, we get

$$\frac{p\bar{v}}{M} = \frac{\bar{R}T}{M}$$

or, $pv = RT$

where

$R = \frac{\bar{R}}{M}$ is a constant for a particular gas known as Gas constant. v is volume per unit mass.

Equation (i) can be re-written as $p \frac{V}{n} = \bar{R}T \Rightarrow pV = n\bar{R}T$

Solved Examples

- 0.2 m³ of an ideal gas at a pressure of 2 MPa and 600K is expanded isothermally to 5 times the initial volume. It is then cooled to 300K at constant volume and then compressed back polytropically to its initial state. Determine the total work done during the cycle.

Solution:

Given,

Referring to the adjoining figure, we have

$$V_1 = 0.2 \text{ m}^3$$

$$V_2 = 2 \text{ MPa} = 2 \times 10^6 \text{ Pa}$$

$$T_1 = 600 \text{ K}$$

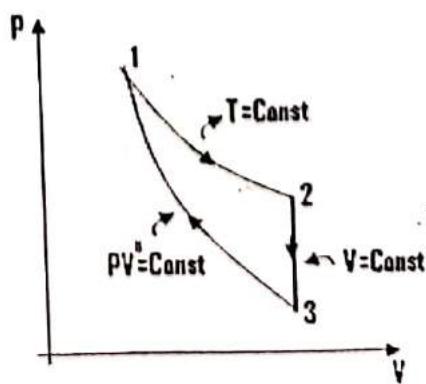
$$V_2 = 5V_1 = 5 \times 0.2 = 1 \text{ m}^3$$

$$T_2 = T_1 = 600 \text{ K}$$

$$T_3 = 300 \text{ K}$$

$$V_3 = V_2 = 1 \text{ m}^3$$

Now,



$$W_2 = P_1 V_1 \ln \frac{V_2}{V_1} = 2 \times 10^6 \times 0.2 \times \ln \frac{1}{0.2} = 643775.165 \text{ J} = 643.78 \text{ kJ}$$

$W_3 = 0$ (Since the process 2-3 is isochoric one)

Now, Process 1-2 is isothermal one

$$\therefore PV = k$$

$$\text{So, } P_2 V_2 = P_1 V_1$$

$$\text{Or, } P_2 = P_1 \times \frac{V_1}{V_2} = 2 \times 10^6 \times \frac{0.2}{1} = 0.4 \times 10^6 \text{ Pa}$$

We know that,

$$PV = mRT$$

When $V = \text{Const}$, we have

$$\frac{P}{T} = \text{Const.}$$

Applying this to the process 2-3, we get

$$\frac{P_3}{T_3} = \frac{P_2}{T_2} \Rightarrow P_3 = P_2 \times \frac{T_3}{T_2} = 0.4 \times 10^6 \times \frac{300}{600} = 0.2 \times 10^6 \text{ Pa}$$

Now applying $PV^n = k$ to the polytropic process 3-1, we get

$$P_3 V_3^n = P_1 V_1^n$$

$$\text{Or, } \left(\frac{V_3}{V_1} \right)^n = \frac{P_1}{P_3}$$

Taking log on both the sides, we get

$$\ln \left(\frac{V_3}{V_1} \right)^n = \ln \frac{P_1}{P_3}$$

$$\text{Or, } n \ln \left(\frac{V_3}{V_1} \right) = \ln \frac{P_1}{P_3}$$

$$\therefore n = \frac{\ln \frac{P_1}{P_3}}{\ln \frac{V_3}{V_1}} = \frac{\ln \frac{2 \times 10^6}{0.2 \times 10^6}}{\ln \frac{1}{0.2}} = 1.431$$

So,

$$W_1 = \frac{P_1 V_1 - P_3 V_3}{1-n} = \frac{2 \times 10^6 \times 0.2 - 0.2 \times 10^6 \times 1}{1-1.431} = -464037.123 \text{ J} = -464.04 \text{ kJ}$$

Thus,

Total work done in the cycle is given by,

$$\begin{aligned} W_{\text{net}} &= W_2 + W_3 + W_1 \\ &= 643.78 + 0 - 464.04 \\ &= 179.74 \text{ kJ} \end{aligned}$$

2. A cylinder fitted with a movable piston contains 0.04m^3 of air at 10bar pressure and 400K temperature. The air expands according to the law $p = \frac{A}{V^2} - \frac{B}{V}$ to a final pressure of 1 bar and volume of 0.2m^3 . Determine the work done.

Solution:

Given,

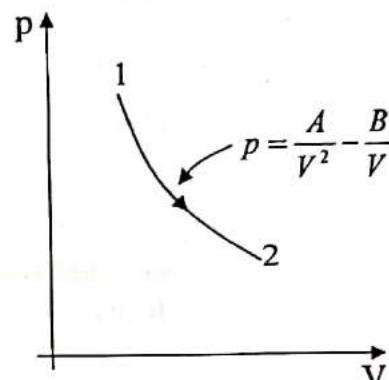
Referring to the adjoining figure, we have

$$V_L = 0.04 \text{ m}^3$$

$P_1 = 10 \text{ bar}$

$T_1=400\text{ K}$

$P_2 = 1 \text{ bar}$



By the question, we have

$$P = \frac{A}{V^2} - \frac{B}{V}$$

When $p_1=10$ bar & $V_1=0.04 \text{ m}^3$, we get

$$10 = \frac{A}{(0.04)^2} - \frac{B}{0.04}$$

Multiplying both the sides by $(0.04)^2$, we get

$$10(0.04)^2 = \frac{A}{(0.04)^2} \times (0.04)^2 - \frac{B}{(0.04)} \times (0.04)^2$$

Similarly, when $p_2=1$ bar and $V_2=0.2 \text{ m}^3$, we have

$$1 = \frac{A}{(0.2)^2} - \frac{B}{0.2}$$

Multiplying both the sides by $(0.2)^2$, we get

$$1(0.2)^2 = \frac{A}{(0.2)^2} \times (0.2)^2 - \frac{B}{(0.2)} \times (0.2)^2$$

Subtracting eqⁿ (II) from eqⁿ (I), we get

$$A - (0.04)B - A + (0.2)B = 10(0.04)^2 - (0.2)^2$$

$$\text{Or}, 0.16B = -0.024$$

$$\therefore B = -\frac{0.024}{0.16} = -0.15$$

Putting the value of B in eqⁿ(II), we get

$$A - (0.2)(-0.15) = (0.2)^2$$

$$\therefore A = 0.01$$

Thus,

$$p = \frac{0.01}{V^2} - \frac{-0.15}{V} \text{ bar}$$

$$= \left[\frac{0.01}{V^2} + \frac{0.15}{V} \right] \times 10^5 Pa$$

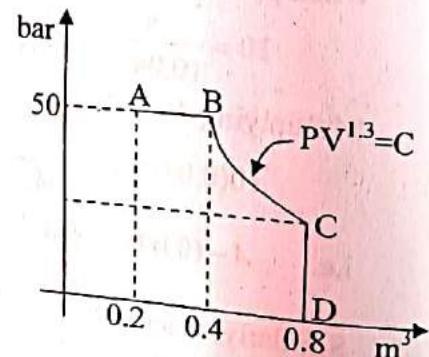
$$\begin{aligned}
 W_2 &= \int_1^2 pdV = \int_1^2 \left(\frac{0.01}{V^2} + \frac{0.15}{V} \right) \times 10^5 dV \\
 &= 10^5 \left[\int_1^2 \frac{0.01}{V^2} dV + \int_1^2 \frac{0.15}{V} dV \right] \\
 &= 10^5 \left[0.01 \int_1^2 \frac{1}{V^2} dV + 0.15 \int_1^2 \frac{dV}{V} \right] \\
 &= 10^5 \left[0.01 \left[-\frac{1}{V} \right]_{0.04}^{0.2} + 0.15 [\ln V]_{0.04}^{0.2} \right] \\
 &= 10^5 \left[0.01 \left(-\frac{1}{0.2} + \frac{1}{0.04} \right) + 0.15 (\ln 0.2 + \ln 0.04) \right] \\
 &= 44141.57 J \\
 &= 44.14 kJ
 \end{aligned}$$

3. Determine the total work done by a gas system following an expansion process as shown in the figure:

Solution:

From the figure, we have

$$\begin{aligned}
 P_A &= P_B = 50 \text{ bar} \approx 50 \times 10^5 \text{ Pa} \\
 V_A &= 0.2 \text{ m}^3 \\
 V_B &= 0.4 \text{ m}^3 \\
 V_C &= 0.8 \text{ m}^3
 \end{aligned}$$



Now, process A-B is constant pressure process

$$\begin{aligned}
 \therefore {}_A W_B &\approx P_B V_B - P_A V_A \\
 &= 50 \times 10^5 \times 0.4 - 50 \times 10^5 \times 0.2 \\
 &= 1000000 \text{ J} = 1 \text{ MJ}
 \end{aligned}$$

Applying $PV^{1.3}=C$ to the process B-C, we get

$$\begin{aligned}
 P_C V_C^{1.3} &= P_B V_B^{1.3} \\
 \therefore P_C &= P_B \left(\frac{V_B}{V_C} \right)^{1.3} = 50 \times 10^5 \times \left(\frac{0.4}{0.8} \right)^{1.3} = 20.31 \times 10^5 \text{ Pa}
 \end{aligned}$$

$$\therefore {}_B W_C = \frac{P_C V_C - P_B V_B}{1-n} = \frac{20.31 \times 10^5 \times 0.8 - 50 \times 10^5 \times 0.4}{1-1.3} \\ = 1250666.667 \text{ J} = 1.251 \text{ MJ}$$

And in the process C-D, Volume remains constant

$$\therefore c^W_D = 0$$

$$\begin{aligned}\text{Total Work, } W_{\text{net}} &= {}_A W_B + {}_B W_C + {}_C W_D \\&= 1 + 1.251 + 0 \\&= 2.251 \text{ MJ}\end{aligned}$$

4. a Cylinder fitted with a piston contains propane gas at 100 kPa, 300K with a volume of 0.2m³. The gas is now slowly compressed according to the relation $PV^{1.1}=\text{Const.}$ to a final temperature of 340K.

a) What is the final pressure ?

b) How much work is done during the process?

Solution:

Given,

Referring to the adjoining figure, we have

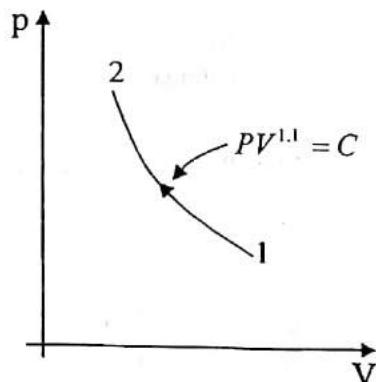
$$P_1 = 100 \text{ kPa}$$

$$T_c = 300K$$

$$V_1 = 0.2m^3$$

$$T_2 = 340\text{ K}$$

We know that



$$\frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1}$$

$$\Rightarrow P_2 V_2 = \frac{P_1 V_1}{T_1} \times T_2 = \frac{100 \times 10^3 \times 0.2}{300} \times 340$$

$$\therefore P_2 V_2 = 22666.67 \dots \dots \dots (I)$$

Also, by question, we have

$$PV^{1,1} = C$$

$$\therefore P_2 V_2^{1.1} = P_1 V_1^{1.1} = 100 \times 10^3 \times (0.2)^{1.1} = 17026.79$$

Dividing (II) by (I), we get

$$\frac{P_2 V_2^{1.1}}{P_2 V_2} = \frac{17026.79}{22666.67}$$

$$Or, V_2^{0.1} = \frac{17026.79}{22666.67}$$

$$\therefore V_2 = \left(\frac{17026.79}{22666.67} \right)^{\frac{1}{0.1}} = 0.0572 \text{ m}^3$$

Now, putting the value of V_2 in eqⁿ (I), we get

$$P_2(0,0572) = 22666.67$$

$$\therefore P_2 = \frac{22666.67}{0.0572} = 396270.45 \text{ Pa} = 0.396 \text{ MPa}$$

Thus,

$$\begin{aligned} \text{Work done, } W_2 &= \frac{P_2 V_2 - P_1 V_1}{1-n} \\ &= \frac{22666.67 - 100 \times 10^3 \times 0.2}{1-1.1} \\ &= -26666.7 \text{ J} \\ &= -26.67 \text{ kJ} \end{aligned}$$

5. A cylinder with a frictionless piston contains 0.1m^3 of gas at 200kPa pressure. The piston is connected to a coil spring which exerts a force proportional to the displacement from its equilibrium position. The gas is heated until the volume is doubled at which pressure is 500kPa . Determine the work done by the gas. Take atmospheric pressure equal to 100kPa .

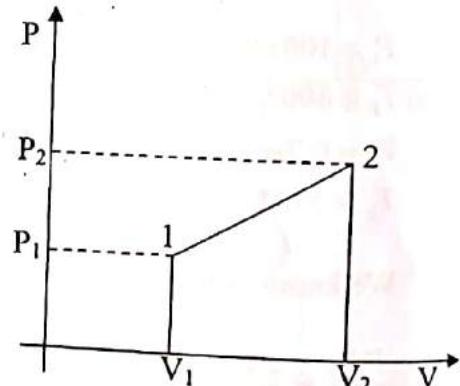
Solution:

Force balance at any position of the piston gives:

$$PA = P_a A + Kx, \quad \text{where } x \text{ is displacement} = \frac{\text{Volume}}{\text{Area}}$$

$$= P_s A + \frac{KV}{A}$$

$$Or, P = P_a + \frac{K}{A^2} V$$



We know that

$$\begin{aligned}W_2 &= \int_1^2 P dV \\&= \int_1^2 \left(P_a + \frac{K}{A^2} V \right) dV \\&= \int_1^2 P_a dV + \int_1^2 \frac{K}{A^2} V dV \\&= P_a \int_1^2 dV + \frac{K}{A^2} \int_1^2 V dV\end{aligned}$$

$$\begin{aligned}
&= P_a \left| V \right|_{V_1}^{V_2} + \frac{K}{A^2} \left| \frac{V^2}{2} \right|_{V_1}^{V_2} \\
&= P_a (V_2 - V_1) + \frac{K}{A^2} \left(\frac{V_2^2}{2} - \frac{V_1^2}{2} \right) \\
&= P_a (V_2 - V_1) + \frac{K}{2A^2} (V_2^2 - V_1^2) \\
&= P_a (V_2 - V_1) + \frac{K}{2A^2} (V_2 - V_1)(V_2 + V_1) \\
&= (V_2 - V_1) \left[P_a + \frac{K}{2A^2} (V_2 + V_1) \right]
\end{aligned}$$

But from (i), we have

$$V_2 = \frac{A^2}{K} (P_2 - P_a)$$

$$V_1 = \frac{A^2}{K} (P_1 - P_a)$$

So that,

$$V_2 + V_1 = \frac{A^2}{K} [(P_2 - P_a) + (P_1 - P_a)]$$

Hence,

$$\begin{aligned}
W_2 &= (V_2 - V_1) \left[P_a + \frac{K}{2A^2} (V_2 + V_1) \right] \\
&= (V_2 - V_1) \left[P_a + \frac{K}{2A^2} \times \frac{A^2}{K} [(P_2 - P_a) + (P_1 - P_a)] \right] \\
&= (V_2 - V_1) \left[P_a + \frac{1}{2} [(P_2 - P_a) + (P_1 - P_a)] \right] \\
&= (V_2 - V_1) \left[\frac{2P_a + P_2 - P_a + P_1 - P_a}{2} \right] \\
&= (V_2 - V_1) \left[\frac{P_2 + P_1}{2} \right]
\end{aligned}$$

Given: $P_1 = 200 \text{ kPa}$; $P_2 = 500 \text{ kPa}$
 $V_1 = 0.1 \text{ m}^3$; $V_2 = 2V_1 = 0.2 \text{ m}^3$

$$\therefore W_2 = (0.2 - 0.1) \left[\frac{200 + 500}{2} \right] = 35 \text{ kJ}$$

Problems

1. 3 kg of air kept at an absolute pressure of 100kPa and temperature of 300K is compressed polytropically until the pressure and temperature becomes 1500kPa and 500K respectively. Evaluate the index of compression and the work of compression. Take $R=287 \text{ J/kg K}$
2. Air initially at 60 kPa ,800K and occupying a volume of 0.1 m^3 is compressed isothermally until the volume is halved and subsequently it goes further compression at constant pressure till the volume is again halved. Calculate the total work done.
3. A mass of 1.5 kg air is compressed in a quasi-static process from 0.1 MPa to 0.7 MPa for which $pV=\text{Constant}$. The initial density of air is 1.6 kg/m^3 . Find the work done by the piston.
4. One kg of air at a pressure of bar and temperature of 360 K undergoes a reversible polytropic process which may be represented by $PV^{1.1}=\text{Const}$. If the final pressure is 1.4 bar, evaluate (a) final specific volume and temperature (b) Work done.
5. A system consisting of 1 kg of an ideal gas at 6 bar pressure and 0.01 m^3 volume executes a cyclic process comprising the following three distinct operations:
 - Reversible expansion to 0.05 m^3 volume and 2 bar pressure, assuming $p=a+bV$
 - Reversible cooling at constant pressure ,and
 - Reversible hyperbolic compression according to the law $pV=\text{Const}$. This brings the gas back to the initial conditions. Evaluate the net work.
6. A piston-cylinder arrangement is containing a fluid at 10 bar, the initial volume being 0.05 m^3 . Find the work done by the fluid when it expands reversibly to a final volume of 0.2 m^3 for the following cases:
 - at constant pressure
 - according to the linear law and a final pressure of bar
 - according to the law $PV=\text{Constant}$
 - according to the law $PV^3=\text{Constant}$
 - according to the law $p = \frac{A}{V^3} - \frac{B}{V}$ to a final pressure of 1 bar.
7. Air at 200 kPa, 30°C is contained in a cylinder-piston arrangement with initial volume 0.1 m^3 . The inside pressure balances ambient pressure of 100 kPa plus an externally imposed force that is proportional to $V^{0.5}$. Now heat is transferred to the system to a final temperature of 200°C . Find the final pressure and the work done in the process.
8. A gas undergoes a reversible non-flow process according to the relation $p = (-3V + 15)$ where V is the volume in m^3 and p is the pressure in bar. Determine the work done when the volume changes from 3 to 6 m^3 .
9. A non-flow reversible process occurs for which pressure and volume are correlated by the expression $p = \left(V^2 + \frac{6}{V}\right)$ where p is in bar and V is in m^3 . What amount of work will be done when volume changes from 2 to 4 m^3 ?

10. Consider a gas enclosed in a piston-cylinder arrangement. The gas is initially at 150 kPa and occupies a volume of 0.03 m^3 . The gas is now heated until the volume of the gas increases to 0.1 m^3 . Calculate the work done by the gas if volume of the gas is inversely proportional to the pressure.

11. The pressure-volume correlation for a non-flow reversible (quasi-static) process is given by, where V is in m^3 . If 150 kJ of work is supplied to the system, determine the final pressure and volume of the system. Take initial volume = 0.6 m^3 .

12. A fluid at a pressure of 3 bar and with specific volume of $0.18 \text{ m}^3/\text{kg}$ is contained in a cylinder behind a piston. The fluid expands reversibly to a pressure of 0.6 bar according to the law $p = \frac{C}{V^2}$ where C is a constant. Calculate the work done by the fluid on the piston.

13. A system of volume V contains a mass m of gas at pressure p and temperature T . The macroscopic properties of the system obey the following relationship:

$$\left(p + \frac{a}{V^2} \right) (V - b) = mRT$$

Where a , b and R are constants.

Obtain an expression for the displacement work done by the system during a constant-temperature expansion from volume V_1 to volume V_2 . Calculate the work done by a system which contains 10 kg of this gas expanding from 1 m^3 to 10 m^3 at a temperature of 293 K. Use the values $a = 157 \text{ Nm}^4$, $b = 0.0107 \text{ m}^3$ and $R = 0.287 \text{ kJ/kgK}$.

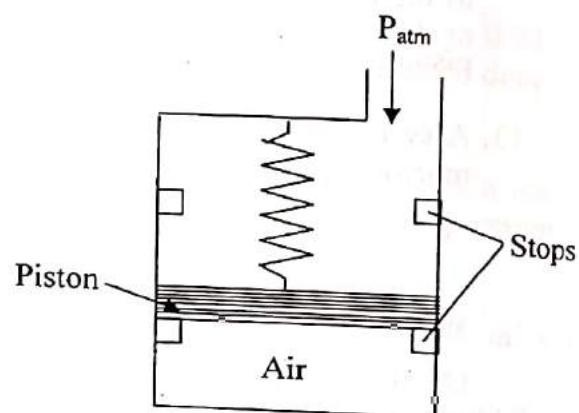
14. A mass of gas is compressed in a quasi-static process from 80 kPa, 0.1 m^3 to 0.4 MPa, 0.03 m^3 . Assuming that the pressure and volume are related by $pv^n = \text{constant}$, find the work done by the gas system.

15. A spherical balloon has a diameter of 25 cm and contains air at a pressure of 1.5 bar. The diameter of the balloon increases to 30 cm in a certain process during which the pressure in the balloon is observed to be proportional to the diameter. (a) Calculate the magnitude and direction of work for air inside the balloon. (b) Given that the atmospheric pressure is 1 bar, find the magnitude and direction of work for the atmosphere and the balloon.

16. (a) A certain mass of air is compressed from 1 bar, 0.1 m^3 to 6 bar in a piston-cylinder device according to $Pv^{1.4} = \text{constant}$. Find the work of compression for air. (b) Had the compression been carried out hyperbolically between the same initial state and the same final pressure as in (a), what would be the work done on air?

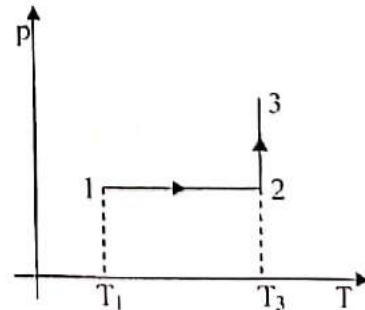
17. A gaseous system undergoes three quasi-static processes in sequence. The gas initially at 5 bar, 0.01 m^3 is expanded at constant pressure. It is then further expanded according to the relation $PV^{1.4} = \text{constant}$ to 2 bar, 0.025 m^3 . The gas is then returned to its initial state during which process $PV = \text{constant}$. Calculate the work interaction in each of three processes and the net work for the system.

18. Consider the system consisting of air in frictionless vertical piston-cylinder mechanism as shown in figure below. The cylinder's diameter is 20 cm. Initially, as shown in figure, when the spring just touches the top surface of the piston but does not exert any force on it, the pressure and volume of the system are found to be 90 kPa and 0.1 m^3 . The mass of the piston is 16 kg and the atmospheric pressure is 100 kPa. In a certain process, during which the force exerted by the spring on the piston is proportional to the square of the displacement of the piston from its initial position, the piston rises slowly but steadily. When the piston is half way between the two stops (when the volume of air is 0.15 m^3), the system pressure is found to be 200 kPa. Finally, the piston rests against the top stop, and the system pressure is 550 kPa. (a) Show the process undergone by the system on p-V diagram. (b) Calculate the net work interaction for (i) the air, (ii) the piston, (iii) the spring and (iv) the atmosphere.



19. In a frictionless, horizontal piston-cylinder mechanism, the piston is forced against the gas by a spring which exerts a force directly proportional to the volume of the gas. In addition to the spring force, the atmospheric pressure of 1 bar acts upon the outer side of the piston. (a) Considering the gas as the system, calculate the work when from an initial state of 0.03 m^3 , 2 bar the gas volume increases to 0.09 m^3 . (b) Considering the spring as the system, find the work interaction.
20. A gas in a piston-cylinder assembly undergoes an expansion process for which the relationship between pressure and volume is given by $pV^n = \text{constant}$. The initial pressure is 3 bars, the initial volume is 0.1 m^3 . Determine the work for the process, in kJ, if (a) $n=1.5$ (b) $n=1.0$, and (c) $n=0$.
21. A gas expands from an initial state where $p_1=500 \text{ kPa}$ and $V_1=0.1 \text{ m}^3$ to a final state where $p_2=100 \text{ kPa}$. The relationship between pressure and volume during the process is $PV=\text{constant}$. Sketch the process on p-V diagram and determine the work, in kJ.
22. Air undergoes two processes in series:
- Process 1-2: polytropic compression, with $n=1.3$, from $p_1=100 \text{ kPa}$, $v_1=0.04 \text{ m}^3/\text{kg}$ to $v_2=0.02 \text{ m}^3/\text{kg}$*
- Process 2-3: constant-pressure process to $v_3=v_1$*
- Sketch the process on p-v diagram and determine the total work per unit mass of air, in kJ/kg

23. A gas undergoes a thermodynamic cycle consisting of three processes:
- Process 1-2: compression with $pV=$ constant, from $p_1=1$ bar, $V_1=1.6 \text{ m}^3$ to $V_2=0.2 \text{ m}^3$; $U_2-U_1=0$*
- Process 2-3: constant pressure to $V_3=V_1$*
- Process 3-1: constant volume, $U_1-U_3=-3549 \text{ KJ}$*
- There are no significant changes in kinetic energy or potential energy. Determine the net work for the cycle.
24. A piston-cylinder arrangement contains carbon dioxide at $300\text{kPa}, 100^\circ\text{C}$ with a volume of 0.2 m^3 . Weights are added to the piston such that the gas compresses according to the law $PV^{1.2}=\text{constant}$ to a final temperature of 200°C . Determine the work done during the process.
25. A cylinder fitted with a movable piston contains 9 m^3 of air at 1 bar and 400 K . The air expands according to the law $V=Ap^4+Bp^3$ to a final pressure of 2 bar when the volume is 112 m^3 . Calculate the work done during the expansion.
26. Determine the rate of change of pressure with respect to volume in a reversible isothermal process for an ideal gas system when its pressure is 4 bar and volume is 2 m^3 .
27. Starting from an initial state of 10 bar, 0.01m^3 , a gaseous system undergoes a resisted expansion process according to $pV^{1.4}=\text{constant}$. At the end of the process, it was calculated that the work output was 6050J. Starting from the same initial state, had the system undergone a resisted expansion process according to $pV=\text{constant}$, to the same final volume as in the previous case, what would be the work output?
28. Monoatomic ideal gas at 12 bar is contained in a tank of capacity 0.028 m^3 . The tank is placed in a room where the pressure and temperature are 1 bar and 20°C respectively. Find the amount of work done in each of the following cases:
- Leakage occurs which may be considered to be sufficiently slow so that the tank and the contents remain at a constant temperature.,
 - The slow leakage process may be considered reversible adiabatic assuming the tank thermally insulated.
- Assume $C_p=20 \text{ kJ/kgK}$ and $C_v=12 \text{ kJ/kgK}$
29. A piston encloses a gas within a cylinder and is restrained by a linear spring. The initial pressure and volumes are 150kPa and 0.001m^3 respectively. The spring touches the piston but exerts no force at the initial position. The gas is heated until the volume is tripled and the pressure is 1000 kPa . Calculate the work done by the gas. What is the work done against the piston and spring?
30. Show that work done by an ideal gas system executing the processes shown by the path in the figure is
- $$w_3 = RT_3 \left(1 - \frac{T_1}{T_3} + \ln \frac{P_1}{P_3} \right); \text{ where } P \text{ and } T \text{ are known at states 1 and 3.}$$



First Law Of Thermodynamics

4.1 First law of thermodynamics for a cyclic Process

"When a system executes a cyclic process, the net heat transfer during the process is always equivalent to the net work transfer." Mathematically, the 1st law of thermodynamics can be stated as :

$$\oint \delta Q = \oint \delta W$$

4.1.1 Verification by Joule's Wheel- Paddle Experiment

The experiment conducted by Joule is shown in the figure given below. In this experiment, work was done on the fluid by means of a paddle wheel. This work caused a rise in temperature of a fluid.

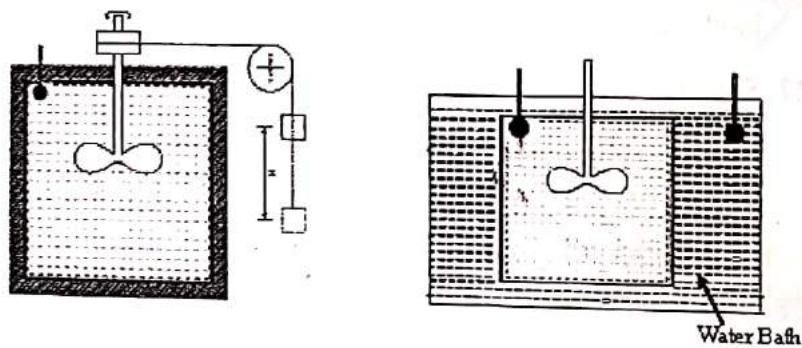


Fig 4.1 Joule's Wheel-paddle experiment

The amount of work done was measured by the change in potential energy(mgz) as the weight W fell through a height 'z'. Then the system was placed in a water bath so that the heat was transferred from the liquid to the water until the original state of the fluid was regained which is indicated by the pressure and temperature of the fluid. In this way the system was taken through a cycle. The amount of heat rejected by the fluid is equal to the rise in energy of the water bath and may be determined by the measuring the rise in the temperature of the water bath.

A number of such experiments were conducted for different work interactions. Let the magnitudes of work and heat interactions in each such experiment be (W_1, Q_1) , (W_2, Q_2) , (W_3, Q_3) etc. The experimental result confirmed that the ratio $W:Q$ in each experiment turned out to be same and equals to 4.189 Nm/cal. i.e. $\frac{W_1}{Q_1} = \frac{W_2}{Q_2} = \frac{W_3}{Q_3} = 4.189$ Thus, for a closed system undergoing a cyclic process,

$$\oint \delta Q \propto \oint \delta W$$

If heat and work are expressed in the same unit, then

$$\oint \delta Q = \oint \delta W$$

4.2 First law for a closed system undergoing process

Let us consider a system undergoing a cycle, in which it changes from state 1 to state 2 through a process A and returns from state 2 to state 1 through process path B or path C as shown below.

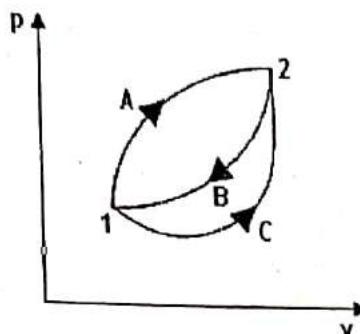


Fig 4.2 Stored Energy- A property of a system

Now for a cycle, we know that $\oint \delta Q = \oint \delta W$

For the cycle made up of two process A and B i.e. 1-A-2-B-1, we have

$$\int_1^2 \delta Q_A + \int_2^1 \delta Q_B = \int_1^2 \delta W_A + \int_2^1 \delta W_B \quad \dots \dots \dots \text{(I)}$$

Similarly, for the cycle made up of the processes A and C i.e. 1-C-2-B-1, we have

$$\int_1^2 \delta Q_C + \int_2^1 \delta Q_B = \int_1^2 \delta W_C + \int_2^1 \delta W_B \quad \dots \dots \dots \text{(II)}$$

Now subtracting the second from the first equation, we get

$$\int_1^2 \delta Q_A - \int_1^2 \delta Q_C = \int_1^2 \delta W_A - \int_1^2 \delta W_C$$

On rearranging,

$$\int_1^2 (\delta Q - \delta W)_A = \int_1^2 (\delta Q - \delta W)_C$$

Thus, we see that

- i) the quantity $\int_1^2 (\delta Q - \delta W)$ is same irrespective of the path i.e. whether the system undergoes change through path A or C .
- ii) $\int_1^2 (\delta Q - \delta W)$ depends solely upon the initial and final states, and not on the path followed,

So, $(\delta Q - \delta W)$ is an exact differential and its integral is a property or a point function. This property represents the total energy stored within a system and is usually denoted by E.
Thus, stored energy within a system is a property or a point function

$$\therefore dE = (\delta Q - \delta W)$$

$$\text{or, } \delta Q = dE + \delta W$$

On integration we get,

$$_1 Q_2 = E_2 - E_1 + _1 W_2$$

This is the general expression of first law of thermodynamics for a closed system undergoing a process.

Note:

The property E , the stored or total energy, represents all the energy (Potential, Kinetic, Internal, Chemical, Electrical, Magnetic, Surface or Strain Energy) contained by the system at given state. So, $E = E_p + E_k + U + E_{\text{Che}} + E_{\text{Elec}} + E_{\text{Mag}} + E_{\text{Sur}}$. For a system showing negligible or no chemical, electrical, magnetic and surface effects, we have.

$$E = E_p + E_k + U$$

In differential form, it can be written as

$$dE = dE_p + dE_k + dU$$

For reversible process, $dV \approx 0 \Rightarrow dE_k = 0$ and $dh \approx 0 \Rightarrow dE_p = 0$

So, for a closed system undergoing reversible process and showing negligible or no chemical, electrical, magnetic and surface effect (like system having pure substance and ideal gas), we have

$$dE = dU$$

and 1st law of thermodynamics for such system yields to

$$\delta Q = dU + \delta W$$

4.2.1 First Law as Rate Equation

Let us consider a time interval during δt which an amount of heat δQ crosses the control surface, an amount of work δW is done by the control mass, the stored energy change is ΔE . From 1st law of thermodynamics, we have

$$\delta Q = \delta E + \delta W$$

Dividing both sides by δt , we get

$$\frac{\delta Q}{\delta t} = \frac{\delta E}{\delta t} + \frac{\delta W}{\delta t}$$

Taking limit for each of these quantities as δt approaches zero, we have

$$\lim_{\delta t \rightarrow 0} \frac{\delta Q}{\delta t} = \lim_{\delta t \rightarrow 0} \frac{\delta E}{\delta t} + \lim_{\delta t \rightarrow 0} \frac{\delta W}{\delta t}$$

$$\frac{dQ}{dt} = \frac{dE}{dt} + \frac{dW}{dt}$$

$$\therefore \dot{Q} = \frac{dE}{dt} = \dot{W}$$

This is the rate equation form of the first law.

4.3 Joule's Law

"Internal energy of an ideal gas is a function of absolute temperature only and is independent of any changes in pressure and volume"

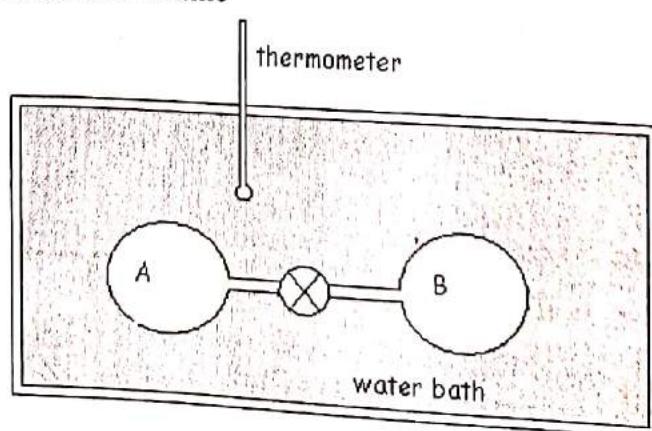


Fig 4.3 Joule's Experiment

The experimental set-up consists of a chamber A containing gas in thermodynamic equilibrium and an evacuated chamber B. Both of them are connected by a stopcock. The two chambers constitute a closed system and are immersed in a water bath which is insulated from outside. Let the system be in thermal equilibrium with the environment at temperature t_1 . When the stopcock is opened, the gas in A expands freely into B. Initially the temperature of A decreases while that of B increases. But the experiment shows that there is no change in the reading of the thermometer inserted inside water bath. This implies that chamber A loses as much energy as chamber B gains and there is no net heat interaction between the system and surrounding.

Therefore,

$$\delta Q = 0, \text{ and } \delta W = 0$$

Now, from 1st law of thermodynamics, we have

$$\begin{aligned} dU + \delta W &= \delta Q \\ \text{or, } dU + 0 &= 0 \\ \therefore dU &= 0 \Rightarrow U = \text{Constant} \end{aligned}$$

Thus, we see that internal energy does not change as the temperature does not change. So. It can be concluded that internal energy of an ideal gas is a function of absolute temperature only.

4.4 Specific Heat Capacity of gas

4.4.1 Specific heat capacity of gas at constant volume(c_v)

It is defined as the heat required to raise the temperature of unit mass of a substance by unit degree keeping the volume constant.

$$c_v = \frac{1}{m} \left. \frac{\delta Q}{\delta T} \right|_v \dots \dots \dots \quad (i)$$

But from 1st law of thermodynamics, we have

$$\delta Q = \partial U + P \partial V \dots \dots \dots \quad (a)$$

At constant volume i.e. $V = \text{constant}$, we have

$$\partial V = 0$$

So, relation (a) reduces to

$$\delta Q = \partial U$$

On substitution, relation (i) yields to

$$c_v = \frac{1}{m} \left. \frac{\partial U}{\partial T} \right|_v = \frac{1}{m} \left(\frac{\partial U}{\partial T} \right)_v$$

In case of ideal gas,

$$U=f(T)$$

$$\therefore \left(\frac{\partial U}{\partial T} \right)_V = \frac{dU}{dT}$$

Thus, for ideal gas, we get

$$c_v = \frac{1}{m} \frac{dU}{dT} = \frac{d}{dT} \left(\frac{U}{m} \right) = \frac{du}{dT}$$

$$i.e. \quad c_v = \frac{du}{dT}$$

4.4.2 Enthalpy

It is defined as the sum of the internal energy U plus the product of the pressure p and volume V . Using the symbol H for the enthalpy:

$$H = U + pV$$

$$\text{And, } \frac{H}{m} = \frac{U}{m} + p \frac{V}{m}$$

i.e. $h = u + p v$

Enthalpy is then a precisely measurable state variable, since it is defined in terms of three other precisely definable state variables(U, p, V).

4.4.3 Specific heat capacity of gas at constant pressure(c_p)

It is defined as the heat required to raise the temperature of unit mass of a substance by unit degree keeping the pressure constant.

$$c_p = \frac{1}{m} \left. \frac{\partial Q}{\partial T} \right|_V \quad \dots \dots \dots \text{(i)}$$

But from 1st law of thermodynamics we have

$$\delta Q = \partial U + P \partial V \quad (a)$$

At constant pressure i.e. n

$$\delta Q = \partial U \delta T + \partial V \delta P - \delta PV$$

But $w = 1$

But we know

$\vdash \neg p \vee q = H$

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Also,

$$\int \partial Q = \int \partial H$$

$$Q_2 = H_2 = H$$

Thus, we see that the net heat transfer during constant pressure process is equal to the change in enthalpy.

On substitution from (b), relation (i) yields to

$$c_p = \frac{1}{m} \left. \frac{\partial H}{\partial T} \right|_p = \frac{1}{m} \left(\frac{\partial H}{\partial T} \right)_p$$

In case of ideal gas,

$$\begin{aligned} H &= U + pV \\ &= f(T) + n\bar{R}T \\ &= \phi(T) \end{aligned}$$

$$\therefore \left(\frac{\partial H}{\partial T} \right)_p = \frac{dH}{dT}$$

Thus, for ideal gas, we get

$$\begin{aligned} c_p &= \frac{1}{m} \frac{dH}{dT} = \frac{d}{dT} \left(\frac{H}{m} \right) = \frac{dh}{dT} \\ \text{i.e. } c_p &= \frac{dh}{dT} \end{aligned}$$

4.5 First law of thermodynamics for a control volume

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Let us consider, as shown in figure, a control volume, CV, bound by the control surface C. In a small time interval δt , let the mass of the material entering the control volume through the inlet manifold at cross section 1-1 be δm_1 and the mass of the material leaving the control volume at cross section 2-2 in the exit manifold be δm_2 .

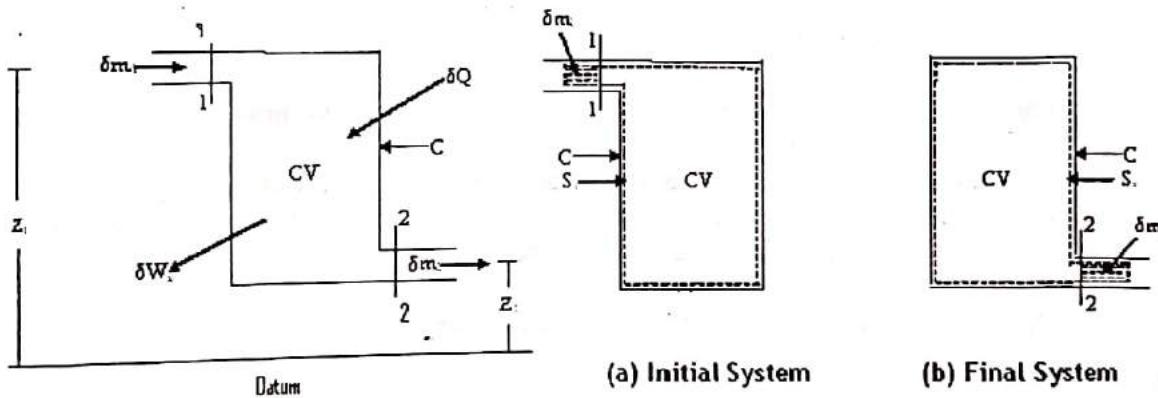


Fig 4.4 Schematic diagram for control volume analysis

At the same time, let δQ and δW_x be the heat and work interactions across the control volume. S_i and S_f are initial and final system boundaries that contain same mass of the flowing material inside them. The process undergone by the system is the displacement of the system boundary at the inlet, causing the material δm_1 to enter the CV and the displacement of the system boundary at the exit, causing material δm_2 to leave the CV. Let the properties of the fluid at the entry be p_1, v_1, e_1, h_1 and V_1 and the corresponding properties at the exit be p_2, v_2, e_2, h_2 and V_2 where p is pressure, v the specific volume, e the net specific energy, h the specific enthalpy and V the velocity. If δW is the net work interaction across the system boundary, we have

$$\delta W = \delta W_x + \text{Displacement Work at inlet and outlet}$$

But, we know that

$$\text{Displacement Work} = p.dV = p.v dm \quad [v = \frac{\delta V}{\delta m} \Rightarrow \delta V = v \cdot \delta m]$$

Therefore,

Displacement Work at the inlet = $- p_1 v_1 \delta m_1$

(-ve sign shows that the boundary is compressed due to the inflow of the material i.e. work is done on the system)

Displacement work at the outlet = $+ p_2 v_2 \delta m$,

(+ve sign shows that the boundary expands as the material leaves the outlet i.e. work is done by the system)

Hence,

Let E_{c1} and E_{c2} be the initial and final energies of the material in the control volume respectively and that E_1 and E_2 are the initial and final energies of the material enclosed in the system boundary.

Then,

$$E_1 = E_{c1} + e_1 \cdot \delta m_1 \quad \text{and} \quad E_2 = E_{c2} + e_2 \cdot \delta m_2$$

So,

$$\begin{aligned} E_2 - E_1 &= (E_{c2} + e_2 \delta m_2) - (E_{c1} + e_1 \delta m_1) \\ &= (E_{c2} - E_{c1}) + (e_2 \delta m_2 - e_1 \delta m_1) \\ &= \delta E_c + (e_2 \delta m_2 - e_1 \delta m_1) \end{aligned}$$

Where, δE_c is the change in energy contents of the control volume.

Since the system boundary is assumed to contain fixed mass of material, it can be treated as open system and thus, 1st law of thermodynamics can be applied to the mass enclosed in the system boundary and not to the control volume.

Thus, For the system enclosed by the system boundary, we have

$$\delta O \equiv E_{\text{c}} - E_{\text{i}} + \delta W$$

Substituting for δW and $(E_2 - E_1)$ from (i) and (ii), we get

But we know that.

$$e = \frac{1}{m}(U + KE + PE) = \frac{U}{m} + \frac{KE}{m} + \frac{PE}{m} = u + \frac{\frac{1}{2}mV^2}{m} + \frac{mgz}{m} = u + \frac{V^2}{2} + gz$$

So, on substitution for e_1 and e_2 , equation (iii) yields to

$$(u_1 + \frac{V_1^2}{2} + gz_1 + p_1 v_1) \delta m_1 + \delta Q = \delta E_c + (u_2 + \frac{V_2^2}{2} + gz_2 + p_2 v_2) \delta m_2 + \delta W_x$$

$$\text{or, } (u_1 + p_1 v_1 + \frac{V_1^2}{2} + gz_1) \delta m_1 + \delta Q = \delta E_c + (u_2 + p_2 v_2 + \frac{V_2^2}{2} + gz_2) \delta m_2 + \delta W_x$$

But, by definition of enthalpy ($u + pv = h$), we have

$$u_1 + p_1 v_1 = h_1 \quad \text{and} \quad u_2 + p_2 v_2 = h$$

So, equation (iv) reduces to

$$(h_1 + \frac{V_1^2}{2} + gz_1) \delta m_1 + \delta Q = \delta E_c + (h_2 + \frac{V_2^2}{2} + gz_2) \delta m_2 + \delta W_x$$

Dividing both sides by δt , we get

$$(h_1 + \frac{V_1^2}{2} + gz_1) \frac{\delta m_1}{\delta t} + \frac{\delta Q}{\delta t} = \frac{\delta E_c}{\delta t} + (h_2 + \frac{V_2^2}{2} + gz_2) \frac{\delta m_2}{\delta t} + \frac{\delta W_x}{\delta t}$$

Assuming $\delta t \rightarrow 0$, we have

$$(h_1 + \frac{V_1^2}{2} + gz_1) \frac{dm_1}{dt} + \frac{dQ}{dt} = \frac{dE_c}{dt} + (h_2 + \frac{V_2^2}{2} + gz_2) \frac{dm_2}{dt} + \frac{dW_x}{dt}$$

$$\text{i.e., } \dot{m}_1(h_1 + \frac{V_1^2}{2} + gz_1) + \dot{Q} = \frac{dE_c}{dt} + \dot{m}_2(h_2 + \frac{V_2^2}{2} + gz_2) + \dot{W}_x \dots \dots \dots \text{(v)}$$

Where, \dot{m}_1 and \dot{m}_2 represent the flowrate of mass at inlet and at outlet and \dot{Q}, \dot{W}_x represent flowrate of heat and work transfer.

If there are more than one inlet and outlet, we can have

$$\sum \dot{m}_i(h_i + \frac{V_i^2}{2} + gz_i) + \dot{Q} = \frac{dE_c}{dt} + \sum \dot{m}_o(h_o + \frac{V_o^2}{2} + gz_o) + \dot{W}_x \dots \dots \dots \text{(vi)}$$

Equations (v) and (vi) are known as general energy flow equation for the control volume.

4.5.1 Conservation of mass for control volume

Let us consider the system as shown in figure 4.4. For this, we have

Mass under system boundary at time t , $m = m_{cv}(t) + \delta m_1$

In time interval δt , the system boundary at the inlet is squeezed and it expands at the outlet as shown in figure 4.4 (b). For this, we have

Mass under system boundary at time at $t + \delta t$, $m = m_{cv}(t + \delta t) + \delta m_2$

Since the system boundary has been selected in such a way that it contains fixed mass.

$$\therefore m_{cv}(t + \delta t) + \delta m_2 = m_{cv}(t) + \delta m_1$$

$$m_{cv}(t + \delta t) - m_{cv}(t) = \delta m_1 - \delta m_2$$

Dividing both sides by δt , we get

$$\frac{m_{cv}(t + \delta t) - m_{cv}(t)}{\delta t} = \frac{\delta m_1 - \delta m_2}{\delta t} = \frac{\delta m_1}{\delta t} - \frac{\delta m_2}{\delta t}$$

Taking limit as $\delta t \rightarrow 0$, we get

$$\lim_{\delta t \rightarrow 0} \frac{m_{cv}(t + \delta t) - m_{cv}(t)}{\delta t} = \lim_{\delta t \rightarrow 0} \frac{\delta m_1}{\delta t} - \lim_{\delta t \rightarrow 0} \frac{\delta m_2}{\delta t}$$

$$\therefore \frac{dm_{cv}}{dt} = \dot{m}_1 - \dot{m}_2$$

4.6 Steady State Steady Flow Energy Equation (SFEE) (Pm)

Basic Assumptions for SFEE:

1. The control volume does not move relative to the co-ordinate frame
2. The properties of fluid at each point in the control volume do not vary with time i.e.

$$\frac{dE_c}{dt} = 0$$
3. The rate of flow of the fluid into and out of the control volume must be same i.e.

$$\sum \dot{m}_i = \sum \dot{m}_o$$
4. The rate of heat and work transfer across the system boundary remains constant.

Applying these assumptions to the general energy equation to the CV, we get

$$\sum \dot{m}_i (h_i + \frac{V_i^2}{2} + gz_i) + \dot{Q} = \frac{dE_c}{dt} + \sum \dot{m}_o (h_o + \frac{V_o^2}{2} + gz_o) + \dot{W}_x$$

For devices having only one inlet and outlet, the assumption '3' gives

$$\dot{m}_i = \dot{m}_o = \dot{m}$$

So that the above equation yields to

$$\dot{m}(h_1 + \frac{V_1^2}{2} + gz_1) + \dot{Q} = \dot{m}(h_2 + \frac{V_2^2}{2} + gz_2) + \dot{W}_x \dots \text{[Time based SFEE]}$$

Dividing both sides by \dot{m} , we get

$$h_1 + \frac{V_1^2}{2} + gz_1 + \frac{\dot{Q}}{\dot{m}} = h_2 + \frac{V_2^2}{2} + gz_2 + \frac{\dot{W}_x}{\dot{m}}$$

$$\text{Or, } h_1 + \frac{V_1^2}{2} + gz_1 + \frac{d\dot{Q}}{dm} = h_2 + \frac{V_2^2}{2} + gz_2 + \frac{d\dot{W}_x}{dm}$$

$$\text{Or, } h_1 + \frac{V_1^2}{2} + gz_1 + \frac{d\dot{Q}}{dm} = h_2 + \frac{V_2^2}{2} + gz_2 + \frac{d\dot{W}_x}{dm}$$

$$\therefore h_1 + \frac{V_1^2}{2} + gz_1 + q = h_2 + \frac{V_2^2}{2} + gz_2 + w \dots \text{[Mass Based SFEE]}$$

4.7 Applications of SFEE

4.7.1 Heat Exchanger

Heat exchangers are devices in which heat is transferred from one flowing fluid to another without physical mixing of the two. During the process of heat exchange, a hot fluid entering the heat exchanger transfers heat to another cold fluid.

Assumptions

- The heat exchanger is an ideal one i.e. it is adiabatic with energy interaction occurring only internally ($\dot{Q}=0$)
- There is no work interaction ($\dot{W}_x=0$)
- The change in kinetic and potential energies are negligible.

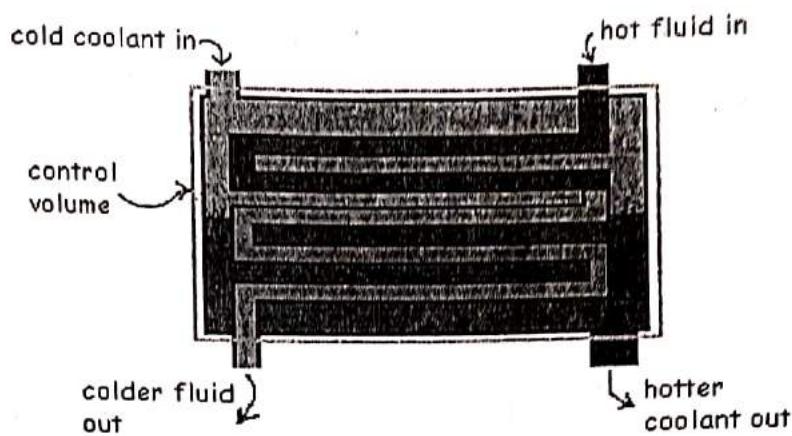


Fig 4.6 A simple heat exchanger

We know that,

$$\sum \dot{m}_i (h_i + \frac{V_i^2}{2} + gz_i) + \dot{Q} = \sum \dot{m}_o (h_o + \frac{V_o^2}{2} + gz_o) + \dot{W}_x$$

Since $\dot{Q} = 0$ and $\dot{W}_x = 0$, we have

$$\sum \dot{m}_i (h_i + \frac{V_i^2}{2} + gz_i) + 0 = \sum \dot{m}_o (h_o + \frac{V_o^2}{2} + gz_o) + 0$$

$$or, \sum \dot{m}_i h_i + \sum \dot{m}_i \frac{V_i^2}{2} + \sum \dot{m}_i gz_i = \sum \dot{m}_o h_o + \sum \dot{m}_o \frac{V_o^2}{2} + \sum \dot{m}_o gz_o$$

The terms containing K.E. and P.E. can be cancelled out from both sides as the change in kinetic and potential energies are negligible.

Thus, the above equation finally yields to,

$$\sum \dot{m}_i h_i = \sum \dot{m}_o h_o$$

4.7.2 Nozzle

A nozzle is a variable-area duct or passage. It is used to increase the velocity of a fluid flowing through it. The pressure of the fluid decreases continuously and the velocity increases. In the convergent nozzle shown below, the fluid enters the nozzle at section 1-1 and leaves at 2-2.

Assumptions

- The nozzle is an ideal one i.e. the flow through it is adiabatic ($\dot{Q}=0$)
- There is no work interaction ($\dot{W}_x=0$)
- The change potential energy is negligible.
- The velocity of approach is very small as compared to the velocity of exit

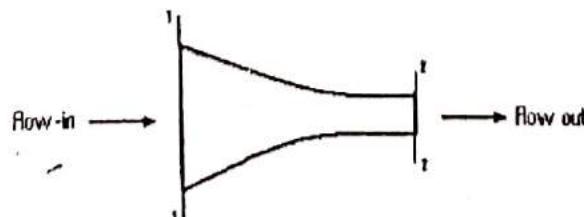


Fig 4.7 A simple nozzle

Applying SFEE to the inlet and outlet of the nozzle, we get

$$\dot{m}(h_1 + \frac{V_1^2}{2} + gz_1) + \dot{Q} = \dot{m}(h_2 + \frac{V_2^2}{2} + gz_2) + \dot{W}_x$$

Since $\dot{Q} = 0$ and $\dot{W}_x = 0$, we have

$$\dot{m}(h_1 + \frac{V_1^2}{2} + gz_1) + 0 = \dot{m}(h_2 + \frac{V_2^2}{2} + gz_2) + 0$$

$$\text{or, } \dot{m}(h_1 + \frac{V_1^2}{2} + gz_1) = \dot{m}(h_2 + \frac{V_2^2}{2} + gz_2)$$

$$\text{or, } h_1 + \frac{V_1^2}{2} + gz_1 = h_2 + \frac{V_2^2}{2} + gz_2$$

Neglecting the 3rd term as there is no change in PE, we have

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

Since $V_1 \ll V_2$, the 2nd term on LHS can be neglected

$$\text{Thus, } h_1 = h_2 + \frac{V_2^2}{2}$$

$$\text{or, } \frac{V_2^2}{2} = h_1 - h_2$$

$$\therefore V_2 = \sqrt{2(h_1 - h_2)}$$

4.7.3 Diffuser

A diffuser is a variable-area duct or passage. However, a diffuser does exactly opposite to that of a nozzle. In a diffuser, the pressure gradually but continuously increases while the flow velocity keeps on decreasing in the direction of flow.

Assumptions

- The diffuser is an ideal one i.e. the flow through it is adiabatic ($\dot{Q}=0$)
- There is no work interaction ($\dot{W}_x=0$)
- The change potential energy is negligible.
- The velocity of approach is very high as compared to the velocity of exit

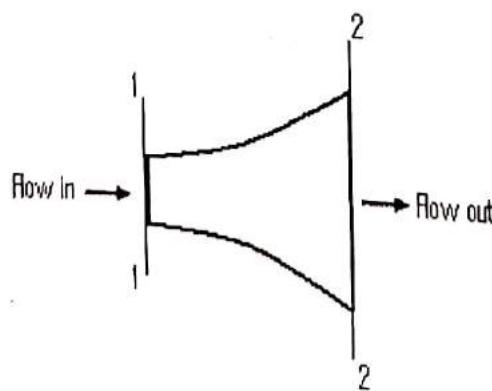


Fig 4.8 A simple diffuser

Applying SFEE to the inlet and outlet of the diffuser, we get

$$\dot{m}(h_1 + \frac{V_1^2}{2} + gz_1) + \dot{Q} = \dot{m}(h_2 + \frac{V_2^2}{2} + gz_2) + \dot{W}_x$$

Since $\dot{Q} = 0$ and $\dot{W}_x = 0$, we have

$$\dot{m}(h_1 + \frac{V_1^2}{2} + gz_1) + 0 = \dot{m}(h_2 + \frac{V_2^2}{2} + gz_2) + 0$$

$$\text{or, } \dot{m}(h_1 + \frac{V_1^2}{2} + gz_1) = \dot{m}(h_2 + \frac{V_2^2}{2} + gz_2)$$

$$\text{or, } h_1 + \frac{V_1^2}{2} + gz_1 = h_2 + \frac{V_2^2}{2} + gz_2$$

Neglecting the 3rd term as there is no change in PE, we have

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

Since $V_1 >> V_2$, the 2nd term on RHS can be neglected

$$\text{Thus, } h_1 + \frac{V_1^2}{2} = h_2$$

$$\text{or, } \frac{V_1^2}{2} = h_2 - h_1$$

$$\therefore V_2 = \sqrt{2(h_2 - h_1)}$$

4.7.4 Turbine

A turbine is a device that converts the heat energy of the fluid into useful work. The energy of level of the working fluid decreases continuously along the flow stream.

Assumptions

- The turbine is an ideal one i.e. the flow through it is adiabatic ($\dot{Q}=0$)
- The change in kinetic energy is negligible.
- The change potential energy is negligible.

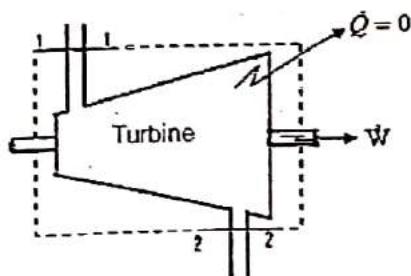


Fig 4.9 A simple turbine

Applying SFEE to the inlet and outlet of the turbine, we get

$$\dot{m}(h_1 + \frac{V_1^2}{2} + gz_1) + \dot{Q} = \dot{m}(h_2 + \frac{V_2^2}{2} + gz_2) + \dot{W}_x$$

Since $\dot{Q} = 0$, we have

$$\dot{m}(h_1 + \frac{V_1^2}{2} + gz_1) + 0 = \dot{m}(h_2 + \frac{V_2^2}{2} + gz_2) + \dot{W}_x$$

$$or, \dot{m}(h_1 + \frac{V_1^2}{2} + gz_1) = \dot{m}(h_2 + \frac{V_2^2}{2} + gz_2) + \dot{W}_x$$

$$or, \dot{m}h_1 + \frac{\dot{m}V_1^2}{2} + \dot{m}gz_1 = \dot{m}h_2 + \frac{\dot{m}V_2^2}{2} + \dot{m}gz_2 + \dot{W}_x$$

Neglecting the 2nd and 3rd terms as there is no change in KE & PE, we have

$$\dot{m}h_1 = \dot{m}h_2 + \dot{W}_x$$

$$\therefore \dot{W}_x = \dot{m}(h_1 - h_2)$$

4.7.5 Rotary Compressor(Turbo-compressor)[Examples: Centrifugal or Axial Compressor]

The function of rotary compressor is to compress the fluid with the help of a rotating element called rotor followed by a stator. This increases the energy level of the fluid in the direction of flow-stream. The pressure increases with the decrease in specific volume in this machine.

Assumptions

- The compressor is an ideal one i.e. the flow through it is adiabatic($\dot{Q}=0$)
- Shaft work is done on the system and the fluid is compressed; so \dot{W}_x is taken -ve
- The change in kinetic energy is negligible.
- The change potential energy is negligible.

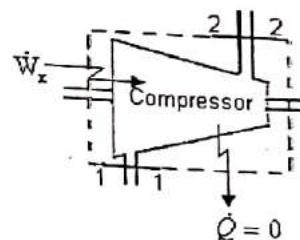


Fig 4.10 A simple compressor

Applying SFEE to the inlet and outlet of the compressor, we get

$$\dot{m}(h_1 + \frac{V_1^2}{2} + gz_1) + \dot{Q} = \dot{m}(h_2 + \frac{V_2^2}{2} + gz_2) - \dot{W}_x$$

Since $\dot{Q} = 0$, we have

$$\dot{m}(h_1 + \frac{V_1^2}{2} + gz_1) + 0 = \dot{m}(h_2 + \frac{V_2^2}{2} + gz_2) - \dot{W}_x$$

$$or, \dot{m}(h_1 + \frac{V_1^2}{2} + gz_1) = \dot{m}(h_2 + \frac{V_2^2}{2} + gz_2) - \dot{W}_x$$

$$or, \dot{m}h_1 + \frac{\dot{m}V_1^2}{2} + \dot{m}gz_1 = \dot{m}h_2 + \frac{\dot{m}V_2^2}{2} + \dot{m}gz_2 - \dot{W}_x$$

Neglecting the 2nd and 3rd terms as there is no change in KE & PE, we have

$$\dot{m}h_1 = \dot{m}h_2 - \dot{W}_x$$

$$\therefore \dot{W}_x = \dot{m}(h_2 - h_1)$$

4.7.6 Throttling Process

A throttling process is one in which a fluid flowing through a duct experiences a heavy resistance. Flow of a fluid across an almost closed valve, or through a pipe plugged with porous material such as glass wool or through a long capillary are examples of throttling processes. There is very high pressure drop during the process which serves to overcome the friction for the flow.

Assumptions

- The flow in this process is adiabatic ($\dot{Q}=0$)
- The change in kinetic energy is negligible.
- The change potential energy is negligible.
- There is no means of external work ($\dot{W}_x=0$)
- The process is extremely irreversible.

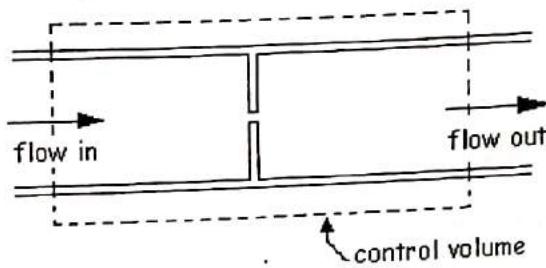


Fig 4.11 A simple throttling device

Applying SFEE to the inlet and outlet of the throttling device, we get

$$\dot{m}(h_1 + \frac{V_1^2}{2} + gz_1) + \dot{Q} = \dot{m}(h_2 + \frac{V_2^2}{2} + gz_2) + \dot{W}_x$$

Since $\dot{Q} = 0$ and $\dot{W}_x = 0$, we have

$$\dot{m}(h_1 + \frac{V_1^2}{2} + gz_1) + 0 = \dot{m}(h_2 + \frac{V_2^2}{2} + gz_2) + 0$$

$$\text{or, } \dot{m}(h_1 + \frac{V_1^2}{2} + gz_1) = \dot{m}(h_2 + \frac{V_2^2}{2} + gz_2)$$

$$\text{or, } h_1 + \frac{V_1^2}{2} + gz_1 = h_2 + \frac{V_2^2}{2} + gz_2$$

Neglecting the 2nd and 3rd term as there is no change in KE & PE, we have

Neglecting the 2nd and 3rd term as there is no change in KE & PE, we have
Thus, throttling process is an isenthalpic process.

4.7.7 Boiler

Boiler is an apparatus used for the generation of steam. Thermal energy released by the combustion of fuel is transferred to the water which vaporizes and gets converted into steam at the desired pressure and temperature.

Assumptions

- There is no external means of work transfer ($\dot{Q}=0$).
- The change in kinetic energy is negligible.
- The change potential energy is negligible.

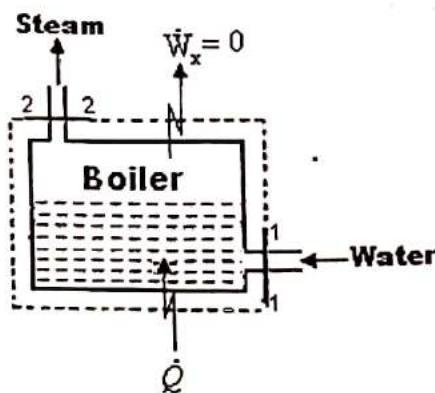


Fig 4.12 A simple boiler

Applying SFEE to the inlet and outlet of the boiler, we get

$$\dot{m}(h_1 + \frac{V_1^2}{2} + gz_1) + \dot{Q} = \dot{m}(h_2 + \frac{V_2^2}{2} + gz_2) + \dot{W}_x$$

Since $\dot{W}_x = 0$, we have

$$\dot{m}(h_1 + \frac{V_1^2}{2} + gz_1) + \dot{Q} = \dot{m}(h_2 + \frac{V_2^2}{2} + gz_2) + 0$$

$$\text{or, } \dot{m}(h_1 + \frac{V_1^2}{2} + gz_1) + \dot{Q} = \dot{m}(h_2 + \frac{V_2^2}{2} + gz_2)$$

$$\text{or, } \dot{m}h_1 + \frac{\dot{m}V_1^2}{2} + \dot{m}gz_1 + \dot{Q} = \dot{m}h_2 + \frac{\dot{m}V_2^2}{2} + \dot{m}gz_2$$

Neglecting the 2nd and 3rd term as there is no change in KE & PE, we have

$$\dot{m}h_1 + \dot{Q} = \dot{m}h_2$$

$$\therefore \dot{Q} = \dot{m}(h_2 - h_1)$$

4.8 Uniform State Uniform Flow Process

Assumptions

1. The control volume remains constant (stationary) relative to the coordinate frame.
2. The state of mass within the control volume may change with time, but at any instant of time the state is uniform throughout the entire control volume.
3. The state of the mass crossing each of the areas of the flow on the control surface is constant with time although the mass flow rates may be time varying.

At any instant of time during the process, the continuity equation is

$$\frac{dm_{c.v.}}{dt} + \sum \dot{m}_e - \sum \dot{m}_i = 0$$

where the summation is over all areas on the control surface through which the flow occurs.

Integrating over time t gives the change of mass in control volume during the overall process

$$\therefore \int_0^t \left(\frac{dm_{c.v.}}{dt} \right) dt = (m_2 - m_1)_{c.v.}$$

The total mass leaving the control volume during time t is

$$\int_0^t (\sum \dot{m}_e) dt = \sum m_e$$

And the total mass entering the control volume during time t is

$$\int_0^t (\sum \dot{m}_i) dt = \sum m_i$$

Therefore, for this period of time t, the continuity equation for this process becomes,

$$(m_2 - m_1)_{c.v.} + \sum m_e - \sum m_i = 0$$

Since at any instant of time the state within the control volume is uniform, the first law for the uniform state and uniform flow process becomes,

$$\dot{Q}_{c.v.} + \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) = \sum m_e \left(h_e + \frac{V_e^2}{2} + gz_e \right) + \frac{d}{dt} \left[m \left(u + \frac{V^2}{2} + gz \right) \right]_{c.v.} + \dot{W}_{c.v.}$$

Let us integrate this equation over time t, during which we have

$$\begin{aligned} \int_0^t \left[\dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) \right] dt &= \sum m_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) \\ \int_0^t \left[\dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right) \right] dt &= \sum m_e \left(h_e + \frac{V_e^2}{2} + gz_e \right) \\ \int_0^t \dot{W}_{cv} dt &= W_{cv} \\ \int_0^t \left[\frac{d}{dt} \left[m \left(u + \frac{V^2}{2} + gz \right) \right] \right]_{c.v.} dt &= \left[m_2 \left(u_2 + \frac{V_2^2}{2} + gz_2 \right) - m_1 \left(u_1 + \frac{V_1^2}{2} + gz_1 \right) \right]_{c.v.} + W_{cv}. \end{aligned}$$

So, 1st law for uniform state uniform flow process becomes,

$$\begin{aligned} \dot{Q}_{c.v.} + \sum m_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) &= \sum m_e \left(h_e + \frac{V_e^2}{2} + gz_e \right) + \\ &\quad \left[m_2 \left(u_2 + \frac{V_2^2}{2} + gz_2 \right) - m_1 \left(u_1 + \frac{V_1^2}{2} + gz_1 \right) \right]_{c.v.} + W_{cv}. \end{aligned}$$

Solved Examples

1. A piston and cylinder machine contains a fluid system which passes through a complete cycle of four processes. During the cycle, the sum of all heat transfers is -170 kJ. The system completes 100 cycles per min. Complete the following table showing the method for each item, and compute the net rate of work output in kW.

Process	<u>Q(kJ/min)</u>	<u>W(kJ/min)</u>	<u>$\Delta E(kJ/min)$</u>
a-b	0	2170	-
b-c	21000	0	-
c-d	-2100	-	-36600
d-a	-	-	-

Solution:

For process a-b:

$$Q = \Delta E + W$$

$$0 = \Delta E + 2170$$

$$\therefore \Delta E = -2170 \text{ kJ/min}$$

For process b-c

$$Q = \Delta E + W$$

$$21000 = \Delta E + 0$$

$$\therefore \Delta E = 21000 \text{ kJ/min}$$

For process c-d:

$$Q = \Delta E + W$$

$$-2100 = -36600 + W$$

$$\therefore W \approx 34500 \text{ kJ/min}$$

For process d-a

$$\sum_{\text{cycle}} Q = -170 \text{ kJ}$$

The system completes 100 cycles/min

$$\therefore Q_{ab} + Q_{bc} + Q_{cd} + Q_{da} = -17000 \text{ kJ/min}$$

$$\Rightarrow 0 + 21000 - 2100 + Q_{da} = -17000$$

$$\therefore Q_{da} = -35900 \text{ kJ/min}$$

Now, $\oint dE = 0$, since cyclic integral of any property is zero

$$\therefore \Delta E_{a-b} + \Delta E_{b-c} + \Delta E_{c-d} + \Delta E_{d-a} = 0$$

$$\text{Or, } -2170 + 21000 - 36600 + \Delta E_{d-a} = 0$$

$$\therefore \Delta E_{d-a} = 17770 \text{ kJ/min}$$

$$\therefore W_{d-a} = Q_{d-a} - \Delta E_{d-a} = -35900 - 17770 = -53670 \text{ kJ/min}$$

Now, the table becomes,

<u>Process</u>	<u>$Q(\text{kJ/min})$</u>	<u>$W(\text{kJ/min})$</u>	<u>$\Delta E(\text{kJ/min})$</u>
a-b	0	2170	-2170
b-c	21000	0	21000
c-d	-2100	34500	-36600
d-e	-35900	-53670	17770

Since $\sum Q_{\text{cycle}} = \sum W_{\text{cycle}}$, we have

Rate of work output = -17000 kJ/min = -283.3 kW

2. A fluid is confined in a cylinder by a spring loaded frictionless piston so that the pressure in the fluid is a linear function of the volume ($p=a+bV$). The internal energy of the fluid is given by the following relation

$$U = 34 + 3.15pV$$

Where U is in kJ/kg, p is in kPa, and V is in m^3 . If the fluid changes from an initial state of 170 kPa, 0.03 m^3 to a final state of 400 kPa, 0.06 m^3 , with no work other than that done on the piston, find the direction and magnitude of the work and heat transfer.

Solution:

The change in the internal energy of the fluid during the process.

$$\begin{aligned} U_2 - U_1 &= (34 + 3.15p_2V_2) - (34 + 3.15p_1V_1) \\ &= 3.15(p_2V_2 - p_1V_1) \\ &= 3.15(400 \times 0.06 - 170 \times 0.03) \\ &= 59.5 \text{ kPa} \end{aligned}$$

Now,

$$\begin{aligned} \because p &= a + bV \\ \therefore 170 &= a + b \times 0.03 \\ 400 &= a + b \times 0.06 \end{aligned}$$

On solving these two equations, we get

$$\begin{aligned} a &= -60 \text{ kN/m}^2 \\ b &= 7667 \text{ kN/m}^5 \end{aligned}$$

Work transfer involved during the process is given by

$$\begin{aligned} W_{1-2} &= \int_{V_1}^{V_2} pdV = \int_{V_1}^{V_2} (a + bV)dV \\ &= a(V_2 - V_1) + b \left(\frac{V_2^2 - V_1^2}{2} \right) \\ &= (V_2 - V_1) \left[a + \frac{b}{2}(V_1 + V_2) \right] \\ &= (0.06 - 0.03)m^3 \left[-60 \text{ kN/m}^2 + \frac{7667}{2} \frac{\text{kN}}{\text{m}^5} \times (0.03 + 0.06)\text{m}^2 \right] \\ &= 8.55 \text{ kJ.} \end{aligned}$$

Work done by the system is 8.55 kJ

∴ Heat transfer involved is given by

$$\begin{aligned} Q_{1-2} &= U_2 - U_1 + W_{1-2} \\ &= 59.5 + 8.55 \\ &= 68.05 \text{ kJ} \end{aligned}$$

68.05 kJ of heat flow into the system during the process.

3. The heat capacity at constant pressure of a certain system is a function of temperature only and may be expressed as

$$C_p = 2.093 + \frac{41.87}{t+100} \text{ J}/^\circ\text{C}$$

Where t is the temperature of the system in ${}^\circ\text{C}$. The system is heated while it is maintained at a pressure of 1 atmosphere until its volume increases from 2000 cm^3 to 2400 cm^3 and its temperature increases from 0°C to 100°C . Find (a) the magnitude of the heat interaction (b) change in internal energy (c) change in enthalpy.

Solution:

Work done during constant pressure non-flow process is

$$W_{1-2} = p(V_2 - V_1) = 1.013 \times 10^5 (0.0024 - 0.002) = 40.45 \text{ J}$$

Heat supplied at constant pressure for unit mass of gas is

$$\begin{aligned} Q_{1-2} &= \int_{T_1}^{T_2} C_p dT \\ &= \int_{273}^{373} \left(2.093 + \frac{41.87}{T - 273 + 100} \right) dT \quad [\because T = t + 273 \Rightarrow t = T - 273] \\ &= \int_{273}^{373} \left(2.093 + \frac{41.87}{T - 173} \right) dT \\ &= [2.093T + 41.87 \ln(T - 173)] \Big|_{273}^{373} \\ &= 2.093(373 - 273) + 41.87 [\ln(373 - 173) - \ln(273 - 173)] \\ &= 2.093 \times 100 + 41.87 \times \ln \frac{200}{100} \\ &= 238.32 \text{ J} \end{aligned}$$

From 1st law of thermodynamics, we have

$$\begin{aligned} Q_{1-2} &= U_2 - U_1 + W_{1-2} \\ \Rightarrow U_2 - U_1 &= Q_{1-2} - W_{1-2} = 238.32 - 40.45 = 197.87 \text{ J} \end{aligned}$$

Since the change in enthalpy in a constant pressure process is equal to the change in enthalpy, we have

$$H_2 - H_1 = Q_{1-2} = 197.87 \text{ kJ}$$

4. A steam turbine receives steam from the boiler at the rate of 5 kg/s and at 3.5 MPa and in dry saturated condition. The velocity of steam entering the turbine is 75 m/s and the elevation at the entry is 5 m. It leaves the turbine at a pressure of 7 kPa and with a dryness fraction of 0.8. The velocity of steam at the exit of the turbine is 160 m/s and elevation at the exit is 1 m. The heat loss from the turbine to the surrounding is 18 MJ/hr. Calculate the power developed by the turbine.

Solution:

<u>Inlet Condition</u>	<u>Outlet Condition</u>
$\dot{m} = 5 \text{ kg}$	$\dot{m} = 5 \text{ kg}$
$p_1 = 3.5 = 3.5 \text{ MPa} = 3500 \text{ kPa}$	$p_2 = 7 \text{ kPa}$
$V_1 = 75 \text{ m/s}$	$V_2 = 160 \text{ m/s}$
$Z_1 = 5 \text{ m}$	$Z_2 = 1 \text{ M}$
Dry saturated	$X_2 = 0.8$
$\dot{Q} = -18 \text{ MJ/h} = -\frac{18 \times 10^6}{3600} = -5000 \text{ J/s}$	

At $P_1 = 3500 \text{ kPa}$

From steam table, we have

$$h_g = 2803.43 \text{ kJ/kg}$$

Since the steam with this pressure at the inlet is dry saturated, we have

$$h_1 = h_g = 2803.43 \text{ kJ/kg}$$

The properties of steam can not be obtained directly from the steam table at 7 kPa. So, it has to be determined with the help of interpolation as follows:

<u>Pressure(kPa)</u>	<u>$h_f(\text{kJ/kg})$</u>	<u>$h_{fg}(\text{kJ/kg})$</u>
5	137.79	2423.66
7	?	?
7.5	168.77	2406.02

Interpolating for h_f , we get

$$h_f - 137.79 = \frac{168.77 - 137.79}{7.5 - 5} (7 - 5)$$

$$\therefore h_f = 162.574 \text{ kJ/kg}$$

Similarly interpolating for h_{fg} , we get

$$h_{fg} - 2423.66 = \frac{2406.02 - 2423.66}{7.5 - 5} (7 - 5)$$

$$\therefore h_{fg} = 2409.55 \text{ kJ/kg}$$

So, finally we have at 7 kPa,

$$h_{f|p_2} = 162.574 \text{ kJ/kg}$$

$$h_{fg|p_2} = 2409.55 \text{ kJ/kg}$$

$$\therefore h_2 = h_{f|p_2} + X_2 h_{fg|p_2} = 162.574 + (0.8)(2409.55) = 209021 \text{ kJ/kg}$$

Now, applying SFEE to the turbine, we get

$$\dot{m} \left(h_1 + \frac{V_1^2}{2} + gZ_1 \right) + \dot{Q} = \dot{m} \left(h_2 + \frac{V_2^2}{2} + gZ_2 \right) + \dot{W}_x$$

$$\text{Or. } 5 \left[2803.43 \times 10^3 + \frac{(75)^2}{2} + 9.81 \times 5 \right] - 5000 = 5 \left[2090.21 \times 10^3 + \frac{(160)^2}{2} + 9.81 \times 1 \right] + \dot{W}_x$$

$$\therefore \dot{W}_x = 3516358.7 \text{ J/s} = 3.52 \text{ MW}$$

5. Steam enters a steam turbine at a pressure of 1 MPa, a temperature of 300°C and a velocity of 50 m/s. The steam leaves the turbine at a pressure of 150 kPa and velocity of 200 m/s. Determine the work per kg of steam flowing through the turbine, assuming the process to be reversible and adiabatic.

Solution:

Given

<u>Inlet condition</u>	<u>Outlet condition</u>
$p_1 = 1 \text{ MPa} = 1000 \text{ kPa}$	$p_2 = 150 \text{ kPa}$
$t_1 = 300^\circ\text{C}$	$V_2 = 200 \text{ m/s}$
$V_1 = 50 \text{ m/s}$	

From SFEE based on unit mass, we know that:

$$h_1 + \frac{V_1^2}{2} + gz_1 + q = h_2 + \frac{V_2^2}{2} + gz_2 + w$$

Assuming no change in P.E. and the process to be adiabatic i.e. $q=0$, the above eqⁿ reduces to

To find out w , we have to determine the values of h at inlet and outlet i.e. h_1 and h_2 .

At $p_1 = 1000 \text{ kPa}$

From steam table, we have

$$t_s = 179.91^\circ\text{C}$$

But, $t_1 = 300^\circ\text{C}$

Since $t_1 > t_s$, the steam at the inlet must be superheated.

Now, from steam table A-4 we have

$$h_b = 3051.15 \text{ kJ/kg} \quad \text{and} \quad s_b = 7.1228 \text{ kJ/kg}$$

Since the process is reversible and adiabatic, it must be isentropic:

$$s_2 \equiv s_1 \equiv 7.1228 \text{ kJ/kgK}$$

So at the exit we have

$$p_c = 150 \text{ kPa}$$

$$s_2 = 7.1228 \text{ kJ/kgK}$$

But, at $P_2 = 150\text{ kPa}$

We have from the steam table

$$s_f = 1.4335 \text{ kJ/kgK} \quad : \quad h_f = 467.08 \text{ kJ/kg}$$

$$s_u = 7.2232 \text{ kJ/kgK} : h_u = 2693.54 \text{ kJ/kg}$$

$$s_f = 5.7897 \text{ kJ/kgK} : h_f = 2226.461 \text{ J}$$

Since $s_f < s_2 < s_g$, the steam at the outlet must be wet. Let X_w be its quality.

$$\therefore s_2 = s_{d+} + X_2 s_{-}.$$

$$Or.7.1228 = 14335 + Y \quad (5.7807)$$

$$\Rightarrow X_2 = \frac{7.1228 - 1.4335}{5.7897} = 0.9826$$

Similarly, we have

$$h_2 = h_{J|p_2} + X_2 h_{Jg|p_2} = 467.08 + 0.9826 \times 2226.46 = 2654.79 \text{ kJ/kg}$$

So from eqⁿ(i), we have

$$3051 \times 10^3 + \frac{50^2}{2} = 2654.79 \times 10^3 + \frac{200^2}{2} + w$$

$$\therefore w = (3051.15 - 2654.79) \times 10^3 + \frac{50^2}{2} - \frac{200^2}{2}$$

$$= 377610 W$$

$$= 377.61 \text{ kW.}$$

6. Air flows steadily through an air compressor, entering at 7 m/s velocity, 100 kPa pressure and 0.95 m³/kg volume and leaving at 5 m/s, 700 kPa and 0.19 m³/kg. The internal energy of the air leaving is 90 kJ/kg greater than that of the air entering. Cooling water in the compressor jackets absorb heat from the air at the rate of 58 kW.
- Compute the rate of shaft work input to the air in kW
 - Find the ratio of the inlet pipe diameter to the outlet pipe diameter.

Solution

Given

Inlet Condition	Outlet Condition
$V_1 = 7 \text{ m/s}$	$V_2 = 5 \text{ m/s}$
$P_1 = 100 \text{ kPa}$	$P_2 = 700 \text{ kPa}$
$v_1 = 0.95 \text{ m}^3/\text{kg}$	$v_2 = 0.19 \text{ m}^3/\text{kg}$
$\dot{m} = 0.5 \text{ kg/s}$	$\dot{Q} = -58 \text{ kW}$
	$u_2 = (u_1 + 90) \text{ kJ/kg}$

(a)

From SSEE, we have

$$\dot{m} \left(h_1 + \frac{V_1^2}{2} + gZ_1 \right) + \dot{Q} = \dot{m} \left(h_2 + \frac{V_2^2}{2} + gZ_2 \right) + \dot{W}_x$$

Neglecting the change in P.E., we have

$$\begin{aligned} \dot{m} \left(h_1 + \frac{V_1^2}{2} \right) + \dot{Q} &= \dot{m} \left(h_2 + \frac{V_2^2}{2} \right) + \dot{W}_x \\ \therefore \dot{W}_x &= \dot{m} \left[(h_1 - h_2) + \frac{V_1^2}{2} - \frac{V_2^2}{2} \right] + \dot{Q} \\ &= \dot{m} \left[(u_1 + p_1 v_1) - (u_2 + p_2 v_2) + \frac{V_1^2}{2} - \frac{V_2^2}{2} \right] + \dot{Q} \\ &= \dot{m} \left[(u_1 - u_2) + (p_1 v_1 - p_2 v_2) + \frac{V_1^2}{2} - \frac{V_2^2}{2} \right] + \dot{Q} \\ &= 0.5 \left[-90 \times 10^3 + (100 \times 10^3 \times 0.95 - 700 \times 10^3 \times 0.19) + \frac{7^2}{2} - \frac{5^2}{2} \right] - 58 \times 10^3 \\ &= -121994 \text{ J/s} = -121.994 \text{ kJ/s} = -121.994 \text{ kW} \end{aligned}$$

-ve sign shows that work is done on the compressor.

(b)

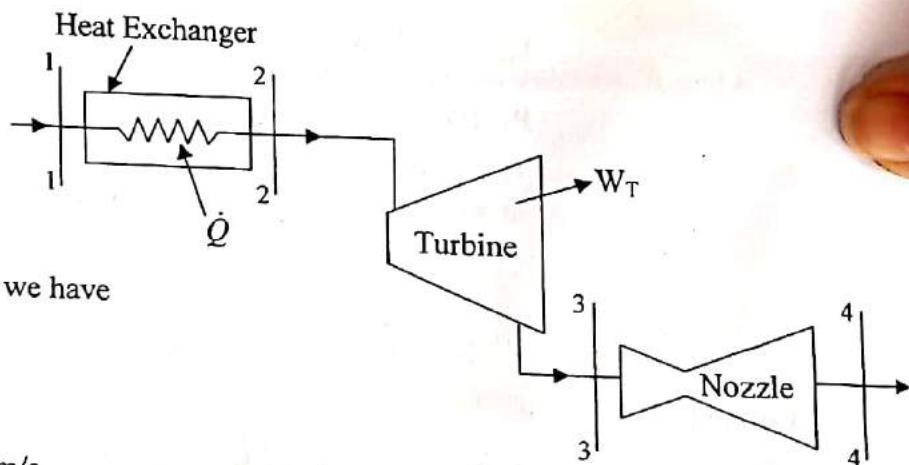
From mass balance, we have

$$\frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2} \Rightarrow \frac{A_1}{A_2} = \frac{v_1}{v_2} \times \frac{V_2}{V_1} = \frac{0.95}{0.19} \times \frac{5}{7} = 3.57$$

$$\therefore \frac{\pi d_1^2}{\pi d_2^2} = 3.57 \Rightarrow \left(\frac{d_1}{d_2} \right)^2 = 3.57 \Rightarrow \frac{d_1}{d_2} = \sqrt{3.57} = 1.89$$

7. Air at a temperature of 15°C passes through a heat exchanger at a velocity of 30 m/s where its temperature is raised to 800°C . It then enters a turbine with the same velocity of 30 m/s and expands until the temperature falls to 650°C . On leaving the turbine, the air is taken at a velocity of 60 m/s to a nozzle where it expands until the temperature has fallen to 500°C . If the air flow rate is 2 kg/s, calculate: (a) the rate of heat transfer to the air in the heat exchanger (b) The power output from the turbine, assuming no heat loss (c) the velocity at the exit from the nozzle, assuming no heat loss.

Solution:



Given

Referring to the figure, we have

$$t_1 = 15^\circ\text{C}$$

$$t_2 = 800^\circ\text{C}$$

$$V_1 = 30 \text{ m/s} ; V_2 = 30 \text{ m/s}$$

$$t_3 = 650^\circ\text{C} ; V_3 = 60 \text{ m/s}$$

$$t_4 = 500^\circ\text{C} ; V_4 = ?$$

Writing SFEE for heat exchanger, we have

$$\dot{m} \left(h_1 + \frac{V_1^2}{2} + gZ_1 \right) + _1 \dot{Q}_2 = \dot{m} \left(h_2 + \frac{V_2^2}{2} + gZ_2 \right) + _1 \dot{W}_2$$

Assuming no change in K.E. and P.E. and no means of external work, we have

$$\dot{m} h_1 + _1 \dot{Q}_2 = \dot{m} h_2$$

$$\begin{aligned} \therefore _1 \dot{Q}_2 &= \dot{m} (h_2 - h_1) \\ &= \dot{m} c_p (t_2 - t_1) \\ &= 2 \times 1.005 \times (800 - 15) \\ &= 1580 \text{ kJ/s} \end{aligned}$$

Similarly, SFEE for turbine is

$$\dot{m} \left(h_2 + \frac{V_2^2}{2} + gZ_2 \right) + _2 \dot{Q}_3 = \dot{m} \left(h_3 + \frac{V_3^2}{2} + gZ_3 \right) + _2 \dot{W}_3$$

Assuming no change in P.E. and no heat exchange (${}_{_2}\dot{Q}_3 = 0$), we get

$$\begin{aligned} \dot{m} \left(h_2 + \frac{V_2^2}{2} + gZ_2 \right) &= \dot{m} \left(h_3 + \frac{V_3^2}{2} + gZ_3 \right) + {}_{_2}\dot{W}_3 \\ \therefore {}_{_2}\dot{W}_3 &= \dot{m} \left[(h_2 - h_3) + \frac{V_2^2 - V_3^2}{2} \right] \\ &= \dot{m} \left[c_p(t_2 - t_3) + \frac{V_2^2 - V_3^2}{2} \right] \\ &= 2 \times \left[1.005 \times 10^3 (800 - 650) + \frac{30^2 - 60^2}{2} \right] \\ &= 298800 \text{ J/s} \\ &= 298.8 \text{ kW.} \end{aligned}$$

i.e. $\dot{W}_T = 298.8 \text{ kW}$

Again applying SFEE to the nozzle, we have

$$\dot{m} \left(h_3 + \frac{V_3^2}{2} + gZ_3 \right) + {}_3\dot{Q}_4 = \dot{m} \left(h_4 + \frac{V_4^2}{2} + gZ_4 \right) + {}_3\dot{W}_4$$

Assuming no change in P.E., no heat exchange and no work exchange, we have

$$\begin{aligned} \dot{m} \left(h_3 + \frac{V_3^2}{2} \right) &= \dot{m} \left(h_4 + \frac{V_4^2}{2} \right) \\ \text{Or, } \frac{V_4^2}{2} &= h_3 - h_4 + \frac{V_3^2}{2} \\ \text{Or, } V_4^2 &= 2 \left[c_p(t_3 - t_4) + \frac{V_3^2}{2} \right] \\ \text{Or, } V_4^2 &= 2 \left[1.005 \times 10^3 (650 - 500) + \frac{60^2}{2} \right] \\ \text{Or, } V_4^2 &= 305100 \\ \therefore V_4 &= \sqrt{305100} = 552.36 \text{ m/s} \end{aligned}$$

Problems

1. The internal energy of a certain substance is given by the following equation:

$$u = 3.56pv + 84$$

where u is given in kJ/kg, p is kPa, and v is in m^3/kg .

A system composed of kg of this substance expands from an initial pressure of 500 kPa and a volume of 0.22 m^3 to a final pressure 100 kPa in a process in which pressure and volume are related by $pv^{1.2} = \text{const}$.

- (a) If the expansion is quasi-static, find Q , ΔU , and W for the process
- (b) In another process the same system expands according to the same pressure-volume relationship as in part (a), and from the same initial state to the same final state as in part (a), but the heat transfer in this case is 30 kJ. Find the work transfer for this process.
- (c) Explain the difference in work transfer in parts (a) and (b).

- 2.. A stationary fluid system goes through a cycle shown in figure comprising the following processes:

- (i) Process 1-2; isochoric heat addition of 235 kJ/kg
- (ii) Process 2-3; adiabatic expansion to its original pressure with loss of 70 kJ/kg in internal energy.
- (iii) Process 3-1; isobaric compression to its original volume with heat rejection of 200 kJ/kg.

Prepare a balance sheet of energy quantities and find the overall changes during the cycle.

3. The properties of certain fluid are related as follows:

$$u = 196 + 0.718t \text{ and } pv = 0.287(t + 273)$$

where u is the specific internal energy (kJ/kg), t is in $^{\circ}\text{C}$, p is pressure (kN/m^2), and v is specific volume (m^3/kg). For this fluid, find c_v and c_p .

4. A slow chemical reaction takes place in a fluid at the constant pressure of 0.1 MPa. The fluid is surrounded by a perfect heat insulator during the reaction which begins at state 1 and ends at state 2. The insulation is then removed and 105 Kj of heat flow to the surroundings as the fluid goes to state 3. The following data are observed for the fluid at states 1, 2 and 3:

State	$V(\text{m}^3)$	$t(^{\circ}\text{C})$
1	0.003	20
2	0.3	370
3	0.06	20

For the fluid system, calculate E_2 and E_3 , if $E_1=0$.

5. A gas of mass 1.5 kg undergoes a quasi-static expansion which follows a relationship $p = a + bV$, where a and b are constants. The initial and final pressures are 1000 kPa and 200 kPa respectively and corresponding volumes are 0.20 m^3 and 1.20 m^3 . The specific internal energy of the gas is given by the relation

$$u = 1.5pv - 85 \text{ kJ/kg}$$

where p is in kPa and v is in m^3/kg . Calculate the net heat transfer and the maximum internal energy of the gas attained during expansion.

6. A gas undergoes a thermodynamic cycle consisting of the following processes:

- (i) process 1-2: Constant pressure $p = 1.4 \text{ bar}$, $V_1 = 0.028 \text{ m}^3$, $W_{12} = 10.5 \text{ kJ}$
- (ii) process 2-3: Compression with $pV = \text{constant}$, $U_3 = U_2$,
- (iii) process 3-1: Constant volume, $U_1 - U_3 = -26.4 \text{ kJ}$.

There are no significant changes in KE and PE (a) calculate the net work for the cycle in kJ. (b) calculate the heat transfer for the process 1-2 (c) show that $\sum_{\text{cycle}} Q = \sum_{\text{cycle}} W$

7. A gas undergoes a thermodynamic cycle consisting of three processes beginning at an initial state where $p_1 = 1 \text{ bar}$, $V_1 = 1.5 \text{ m}^3$ and $U_1 = 5.2 \text{ kJ}$. The processes are as follows:

- (i) process 1-2 : compression with $pV = \text{Const.}$ to $p_2 = 2 \text{ bar}$, $U_2 = 690 \text{ kJ}$
- (ii) process 2-3 : $W_{23} = 0$, $Q_{23} = -150 \text{ kJ}$, and
- (iii) process 3-1: $W_{31} = +50 \text{ kJ}$.

Neglecting KE and PE changes, determine the heat interactions Q_{12} and Q_{31} .

8. A perfect gas undergoes the following three separate and distinct processes to execute a cycle:

- (i) constant volume process during which 80 kJ of heat is supplied to the gas,
- (ii) constant pressure process during which 85 kJ of heat is lost to the surroundings and 20 kJ of work is done on it.

- (iii) adiabatic process which restores the gas back to its initial state.

Evaluate the work done during adiabatic process and the value of internal energy at all the state points if initially its value is 95 kJ.

9. The property of a system, during a reversible constant pressure non-flow process at $P=1.6$ bars, changes from $V_1=0.3 \text{ m}^3/\text{kg}$, $T_1=20^\circ\text{C}$ to $V_2=0.55 \text{ m}^3/\text{kg}$, $T_2=260^\circ\text{C}$. The specific heat capacity of the fluid is given by

$$c_p = 1.5 + \frac{75}{T+45} \text{ kJ/kg}^\circ\text{C} \text{ where } T \text{ is in } ^\circ\text{C.}$$

Determine: (i) Heat added per kg (ii) Change in internal energy and in the enthalpy per kg.

10. For an adiabatic process show that $T^{a_2} e^{\left(\frac{bT}{2} + \frac{c}{2}T^2\right)} V^{(a_1 - a_2)} = \text{const}$, where $c_p = a_1 + bT + cT^2$
and $c_v = a_2 + bT + cT^2$

11. Prove that the formula $T^b v^{a-b} e^{KT} = \text{const.}$ for the adiabatic expansion of the gas if $c_p = a + KT$ and $c_v = b + KT$, where a , b and K are constants and T is in Kelvin. Calculate the work done if 2 kg of the gas originally occupying 0.2 m^3 at 40 bar expands adiabatically until the temperature is 260°C , given that $a=0.948$, $b=0.656$ and $K=0.0001$.

12. Steam enters a nozzle operating at steady state with $p_1=40$ bar, $T_1=400^\circ\text{C}$, and a velocity of 10 m/s. The steam flows through the nozzle with negligible heat transfer and no significant change in P.E. At the exit, $p_2=15$ bars, and the velocity is 665 m/s. The mass flow rate is 2 kg/s. Determine the exit area of the nozzle, in m^2 .

13. Steam enters a turbine operating at steady state with a mass flow rate of 4600 kg/hr. The turbine develops a power output of 1000 kW. At the inlet, the pressure is 60 bar, the temperature is 400°C and the velocity is 10 m/s. At the exit the pressure is 0.1 bar, the quality is 0.9, and the velocity is 50 m/s. Calculate the rate of heat transfer between the turbine and surroundings, in kW.

14. Steam flows through a small turbine at the rate of 5000 kh/hr entering at 15 bar, 300°C and leaving at 0.1 bar with 4% moisture. The steam enters at 80 m/s at a point 2 m above the discharge and leaves at 40 m/s. Compute the shaft power assuming that the device is adiabatic but considering K.E. and P.E. changes. How much error would be made if these terms were neglected? Calculate the diameters of the inlet and discharge tubes.

15. Steam expands isentropically in a nozzle from 1 MPa, 250°C to 10kPa. The steam flow rate is 1 kg/s. Find the velocity of steam at the exit from the nozzle, and the exit area of the nozzle. Neglect the velocity of steam at the inlet to the nozzle. The exhaust steam from the nozzle flows into a condenser and flows out as saturated water. The cooling water enters the condenser at 25°C and leaves at 35°C . Determine the mass flow rate of cooling water.

16. Steam at 10 bar, 250°C flowing with negligible velocity at a rate of 3 kg/min mixes adiabatically with steam at 10 bar, 0.75 quality, flowing also with negligible velocity at the rate of 5 kg/min. The combined stream of steam is throttled to 5 bar and expanded isentropically in a nozzle to 2 bar. Determine (a) the state of steam after mixing, (b) the state of steam after throttling (c) the increase in entropy due to throttling, (d) the velocity of steam at the exit from the nozzle, and (e) the exit area of the nozzle. Neglect the K.E. of steam at the inlet to the nozzle.
17. Steam at 0.6 MPa, 200°C enters an insulated nozzle with a velocity of 50 m/s. It leaves at a pressure of 0.15 MPa and a velocity of 60 m/s. Determine the final temperature if the steam is superheated.
18. Steam is allowed to pass through a throttling valve from an initial state of 1.4 MPa and 0.95 quality. Its pressure is reduced to 0.45 MPa. Find, with the help of steam table, the final condition of steam and change of entropy for the process.
19. Steam initially at 1.5 MPa, 300°C expands reversibly and adiabatically in a steam turbine to 40°C . Determine the ideal work output of the turbine per kg of steam.
20. Steam at 0.8 MPa, 250°C and flowing at the rate of 1 kg/s passes into a pipe carrying wet steam at 0.8 MPa, 0.95 dry. After adiabatic mixing, the flow rate is 2.3 kg/s. Determine the condition of steam after mixing.
The mixer is now expanded in a frictionless nozzle isentropically to a pressure of 0.4 MPa. Determine the velocity of the steam leaving the nozzle. Neglect the velocity of steam in the pipeline.
21. Steam enters a turbine at a pressure of 100 bar and a temperature of 400°C . At the exit of the turbine the pressure is 1 bar and the entropy is 0.6 J/gK greater than that at inlet. The process is adiabatic and the changes in K.E. and P.E. may be neglected. Find the work done by the steam in J/g. What is the mass flow rate of steam required to produce a power output of 1 kW?
22. Steam expands isentropically in a nozzle from 1 MPa, 250°C to 10 kPa. The steam flow rate is 1 kg/s. Neglecting the K.E. of steam at inlet to the nozzle, find the velocity of steam at exit from nozzle and the exit area of the nozzle.
23. Steam at 800 kPa, 300°C is throttled to 200 kPa. Changes in K.E. are negligible for this process. Determine the final temperature of the steam
24. The mass rate of flow into a steam turbine is 1.5 kg/s, and the heat transfer from the turbine is 8.5 Kw. The following data are known for the steam entering and leaving the turbine:

Pressure	Inlet Conditions	Exit conditions
Temperature	2.0 MPa	0.1 MPa
Quality	350°C	-
Velocity	50 m/s	100%
Elevation above Reference Plane $g = 9.8066 \text{ m/s}^2$	6 m	100 m/s 3 m

Determine the power output of the turbine.

25. Steam at 3 MPa, 400°C enters a turbine with a volume flow rate of $5 \text{ m}^3/\text{s}$. An extraction of 15% of the inlet mass flow rate exits at 600 kPa, 200°C . The rest exits the turbine at 20 kPa with a quality of 90%, a velocity of 20 m/s. Determine the volume flow rate of the extraction flow and the diameter of the final exit pipe.
26. In a steady flow apparatus, 135 kJ of work is done by each kg of fluid. The specific volume of the fluid, pressure, and the velocity at the inlet are $0.37 \text{ m}^3/\text{kg}$, 600 kPa, and 16 m/s respectively. The inlet is 32 m above the floor, and the discharge pipe is at floor level. The discharge conditions are $0.62 \text{ m}^3/\text{kg}$, 100 kPa and 270 m/s. The total heat loss between the inlet and discharge is 9 kJ/kg of fluid. In flowing through this apparatus, does the specific internal energy increase or decrease, and by how much?
27. A certain water heater operates under steady flow conditions receiving 4.2 kg/s of water at 75°C temperature, enthalpy 313.93 kJ/kg . The water is heated by mixing with steam which is supplied to the heater at temperature 100.2°C and enthalpy 2676 kJ/kg . The mixer leaves the heater as liquid water at temperature 100°C and enthalpy 419 kJ/kg . How much steam must be supplied to the heater per hour?
28. In a gas turbine the gas enters at the rate of 5 kg/s with a velocity of 50 m/s and enthalpy of 900 kJ/kg and leaves the turbine with a velocity of 150 m/s and enthalpy 400 kJ/kg . The loss of heat from the gases to the surrounding is 25 kJ/kg . Assume for gas $R=0.285 \text{ kJ/kgK}$ and $c_p = 1.004 \text{ kJ/kgK}$ and the inlet conditions to be at 100 kPa and 27°C . Determine the power output of the turbine and the diameter of the inlet pipe.
29. A blower handles 1 kg/s air at 20°C and consumes a power of 15 kW. The inlet and outlet velocities of air are 100 m/s and 150 m/s respectively. Find the exit air temperature, assuming adiabatic conditions. Take $c_p = 1.005 \text{ kJ/kgK}$ for air.
30. Nitrogen is heated while it flows steadily through a constant area tube. At the inlet the nitrogen is 200 kPa, 100°C , and has a velocity of 175 m/s. It is heated to 300°C . At the outlet pressure is 115 kPa. At the inlet, $u_1 = 227 \text{ kJ/kg}$ and $h_1 = 387 \text{ kJ/kg}$; at the exit $u_2 = 428 \text{ kJ/kg}$ and $h_2 = 598 \text{ kJ/kg}$. Determine the heat transfer per kg of nitrogen.
31. In an oil cooler, oil flows steadily through a bundle of metal tubes submerged 90°C and leaves at 30°C , while the water enters at 25°C and leaves at 70°C . The enthalpy of oil at $t^{\circ}\text{C}$ is given by $h = 1.68t + 10.5 \times 10^{-4}t^2 \text{ kJ/kg}$. What is the cooling water flow required for cooling 2.78 kg/s of oil?
32. A turbo compressor delivers $2.33 \text{ m}^3/\text{s}$ at 0.276 MPa, 43°C which is heated at this pressure to 430°C and finally expanded in a turbine which delivers 1860 kW. During the expansion, there is a heat transfer of 0.09 MJ/s to the surroundings. Calculate the turbine exhaust temperature if the changes in K.E. and P.E. are negligible.
33. A reciprocating air compressor takes in $2 \text{ m}^3/\text{min}$ at 0.11 MPa, 20°C which it delivers at 1.5 MPa, 111°C to an after cooler where the air is cooled at constant pressure to 25°C . The power absorbed by the compressor is 4.15 kW. Determine the heat transfer in (a) the compressor, and (b) the cooler
34. Air at 101.325 kPa, 20°C is taken into a gas turbine power plant at a velocity of 140 m/s through opening of 0.15 m^2 cross-sectional area. The air is compressed, heated, expanded through a turbine, and exhausted at 0.18 MPa, 150°C through an opening of

0.10 m² cross-sectional area. The power output is 375 kW. Calculate the net amount of heat added to the air in kJ/kg. Assume that air obeys the law $pv = 0.287(t + 273)$, where p is the pressure in kPa, v is the specific volume in m³/kg, and t is the temperature in °C. Take $c_p = 1.005$ kJ/kgK.

35. A gas flows steadily through a rotary compressor. The gas enters the compressor at a temperature of 16°C, a pressure of 100 kPa, and an enthalpy of 391.2 kJ/kg. The gas leaves the compressor at a temperature of 245°C, a pressure of 0.6 MPa, and an enthalpy of 534 kJ/kg. There is no heat transfer to or from the gas as it flows through the compressor. (a) evaluate the external work done per kg mass of gas, assuming the gas velocity at entry and exit to be negligible. (b) evaluate the external work done per kg of gas when the gas velocity at entry is 80 m/s and that at the exit is 160 m/s.
36. Two air streams $A(m_A = 0.8 \text{ kg/s}, p_A = 15 \text{ bar} \text{ and } t_A = 250^\circ\text{C})$ and $B(m_B = 0.5 \text{ kg/s}, p_B = 15 \text{ bar} \text{ and } t_B = 200^\circ\text{C})$ are mixed in a large chamber and the chamber supplies the mixed air, at 10 bar, to a turbine. The turbine exhausts air at 30°C to the atmosphere. Assume steady adiabatic flow all through. Neglect K.E. and P.E. Assume that enthalpy and internal energy of air are functions of temperature only and that $c_p = 1 \text{ kJ/kgK}$ and $c_v = 0.718 \text{ kJ/kgK}$. Determine (a) the temperature of air at inlet to the turbine and (b) the power developed by the turbine.
37. A perfect gas flows through a nozzle where it expands in a reversible adiabatic manner. The inlet conditions are 22 bar, 500°C, 38 m/s. At the exit the pressure is 2 bar. Determine the exit velocity and exit area if the flow rate is 4 kg/s. Take $R=190 \text{ J/kgK}$ and $\gamma=1.35$.
38. In a steady flow process, the fluid flows through a machine at the rate of 15 kg/min. Between the entrance and exit of the machine, the relevant data regarding the working fluids is:

Parameter	Inlet	Outlet
Velocity	5 m/s	8 m/s
Pressure	100 kPa	700 kPa
Specific volume	0.45 m ³ /kg	0.125 m ³ /kg

The working fluid leaves the machine with internal energy 160 kJ/kg greater than at entrance and during the process 7200 kJ/min of heat lost to the surroundings. Assuming entrance and exit pipes to be the same level, calculate the shaft work and the ratio of inlet pipe diameter to the outlet pipe diameter.

39. Air at 1 bar, 290 K flows steadily at the rate of 120 m³/hr into a compressor where its pressure and temperature are respectively raised to 15 bar and 390 K. During the compression process, the heat transfer from the compressor is 10 % of the work transfer from the machine. Neglecting changes in K.E. and P.E., evaluate the work and heat interactions. Presume that air behaves as a perfect gas.
40. Air at 290K temperature passes through a heat exchanger at 30 m/s velocity and its temperature gets raised to 1100 K. Subsequently, the heated air enters a turbine with the same velocity and its expansion continues till the temperature drops to 900 K. After the exit from the turbine at 45 m/s, further expansion occurs in a nozzle and the temperature falls to 790 K. If the mass flow rate of air is 2 kg/s, determine: (a) rate of heat transfer to the air in the heat exchanger, (b) power output from the turbine, and (c) velocity at the exit from the nozzle.

Chapter 5

Second Law of Thermodynamics

5.1 Limitations of 1st Law of Thermodynamics

1. Although it gives the relationship between heat and work, it does not depict anything about the actual conversion of heat to work or work to heat.
2. It does not impose any restriction as to whether any particular process under consideration is possible or not i.e., it does not depict the direction of flow of work or heat.

Examples

- A thermodynamic cycle in which heat transferred to the system is completely converted to an equal amount of work does not violate the 1st law of thermodynamics but such a cycle is not possible in practice.
- Let us consider two heat reservoirs, one at a higher temperature T_1 and the other at lower temperature T_2 . There is a flow of heat from higher temperature reservoir to lower temperature reservoir. When they are brought in contact with each other, the amount of energy lost by the higher temperature reservoir will be equal to the amount of energy gained by the lower temperature reservoir. The flow of heat from lower temperature to higher temperature reservoir does not violate the 1st law of thermodynamics if heat coming out from one is equal to heat entering into the other. But this is not possible in actual practice.
- Let us consider a rotating fly-wheel brought to rest by friction in its bearing. As a result, the temperature of the wheel and bearings rises and the increase in their internal energy is equal to the original kinetic energy of the fly-wheel. The 1st law is valid here. Now, by cooling down of the wheel and bearing to get the kinetic energy of the fly-wheel also satisfies the 1st law but this does not happen in actual practice.
- Let us consider an ideal gas undergoing free expansion process through a valve into an evacuated vessel, we notice that the temperature remains constant but there is an increase in volume and fall in pressure. Now, suppose the reverse process takes place i.e. gas at low pressure should rush back through the valve and compresses itself into its original states spontaneously. Though this reverse process is true according to the 1st law, it is not possible in actual practice.

5.2 Thermal Reservoir

A thermal reservoir is defined as sufficiently large system in stable equilibrium to which and from which finite amount of heat can be transferred without any change in its temperature. Generally, the high temperature reservoir is called the *heat source* and the low temperature reservoir is called the *heat sink*. Boiler-furnace, combustion chamber, nuclear reactor, sun etc., may be treated as heat source while atmospheric air, river and ocean may be treated as heat sink.

5.3 Heat Engine

A heat engine is defined as a thermodynamic system operating in a cycle in which net heat is transferred and from which net work is delivered. A combination of boiler, turbine, condenser and pump can be treated as a heat engine since they complete a cycle as shown in figure (a). Steam generated in a boiler

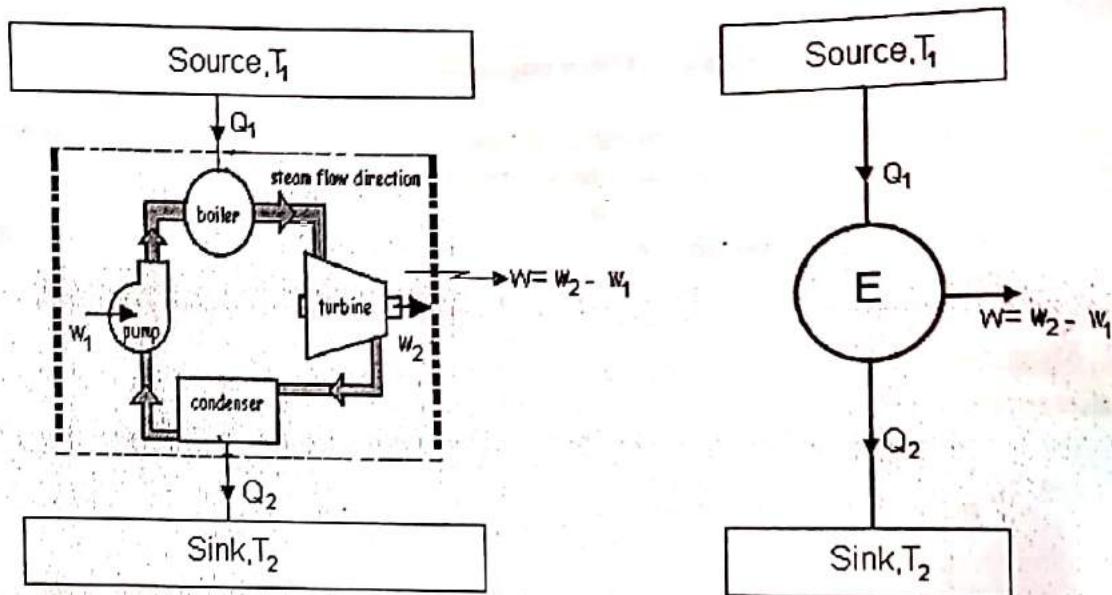


Fig 5.1 Schematic diagram of a heat engine

enters the turbine and does work, the exhaust steam is condensed in a condenser and return to the boiler by a feed pump. In this way, the working fluid is returned to its original states and thus the cycle is completed.

5.3.1 Thermal efficiency of Heat Engine

It is defined as the ratio of net work output to the heat input. Only a fraction of the heat input is converted into work and the rest is rejected.

Let, Q_1 =Heat transferred to the system

Q_2 =Heat reected from the system

W_{net} = Net work done by the system

Then,

$$\text{Thermal efficiency, } \eta = \frac{\text{Output}}{\text{Input}} = \frac{W_{net}}{Q_1} \quad \dots \dots \dots \text{(i)}$$

But from 1st law of thermodynamics applied to closed cycle , we have

$$\oint \delta W = \oint \delta Q$$

$$\therefore W_{net} = Q_1 - Q_2$$

Putting in eqⁿ(i), we get

$$\begin{aligned} \eta &= \frac{Q_1 - Q_2}{Q_1} = \frac{Q_1}{Q_1} - \frac{Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \\ \text{i.e. } \eta &= 1 - \frac{Q_2}{Q_1} \end{aligned}$$

Thus, the efficiency of a heat engine operating between two thermal reservoirs is always less than unity

5.4 Refrigerator and Heat Pump

A refrigerator or a heat pump is defined as a thermodynamic system operating in a cycle which removes heat from a low temperature body and delivers it to a high temperature

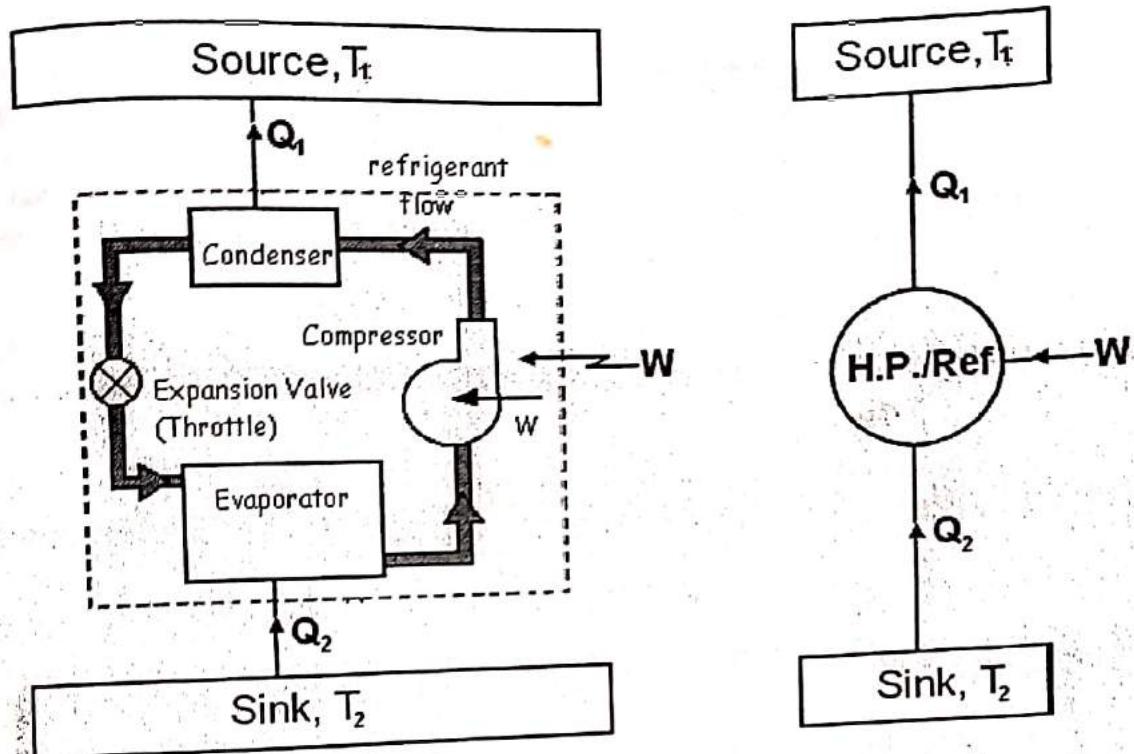


Fig 5.2 Schematic diagram of a heat pump / a refrigerator

body taking external energy in the form of work input. The prime function of a refrigerator is to maintain a body at a temperature lower than that of the surrounding whereas a heat pump maintains a body at a temperature higher than that of the surrounding. The two differs in their desired effect only. A refrigerator produces *refrigerating effect* by Q_2 whereas a heat pump produces *heating effect* by Q_1 . More the desired effect is, more the performance will be.

Heat Q_2 is transferred to the refrigerant from low temperature reservoir. As a result, the refrigerant gets vaporized at a low-pressure low-temperature state which then enters the compressor. Work W is supplied to the compressor from electrical mains to increases the pressure and temperature of the refrigerant vapour. This compressed vapour passes to the air-cooled condenser where it is steadily condensed to a liquid at high pressure. In condenser, heat Q_1 is transferred from the refrigerant to the surrounding. And, then, it is throttled as it passes through an expansion valve. Low-pressure low-temperature liquid from the expansion valve enters the evaporator to complete the cycle. The figure(b) shows the schematic block diagram of refrigerator or a heat pump.

5.4.1 Coefficient of Performance(COP)

Coefficient of performance, an index of performance of refrigerator or heat pump, is defined as the ratio of desired effect to the work input.

Let, Q_1 =Heat transferred to the system

Q_2 =Heat rejected from the system

W_{net} = Net work done by the system

Applying 1st law of thermodynamics to the above cycle, we get

$$\oint \delta W = \oint \delta Q$$

$$\therefore W_{net} = Q_1 - Q_2$$

$$(COP)_{Ref} = \frac{\text{Refrigerating effect}}{\text{Work input}} = \frac{Q_2}{W_{net}} = \frac{Q_2}{Q_1 - Q_2} \quad \dots \dots \dots \text{(i)}$$

$$(COP)_{HP} = \frac{\text{Heating effect}}{\text{Work input}} = \frac{Q_1}{W_{net}} = \frac{Q_1}{Q_1 - Q_2} \quad \dots \dots \dots \text{(ii)}$$

Subtracting eqⁿ (i) from (ii), we get

$$(COP)_{HP} - (COP)_{Ref} = \frac{Q_1}{Q_1 - Q_2} - \frac{Q_2}{Q_1 - Q_2} = \frac{Q_1 - Q_2}{Q_1 - Q_2} = 1$$

$$\therefore (COP)_{HP} - (COP)_{Ref} = 1 \Rightarrow (COP)_{HP} = (COP)_{Ref} + 1$$

Thus,

$$(COP)_{HP} = (COP)_{Ref} + 1$$

5.5 2nd law of thermodynamics

5.5.1 Kelvin-Plank Statement of 2nd Law

"It is impossible for a device operating in a thermodynamic cycle to produce work while having heat interaction with a single system at a fixed temperature (reservoir)"

Mathematically,

$$\oint \delta W \leq 0$$

single reservoir

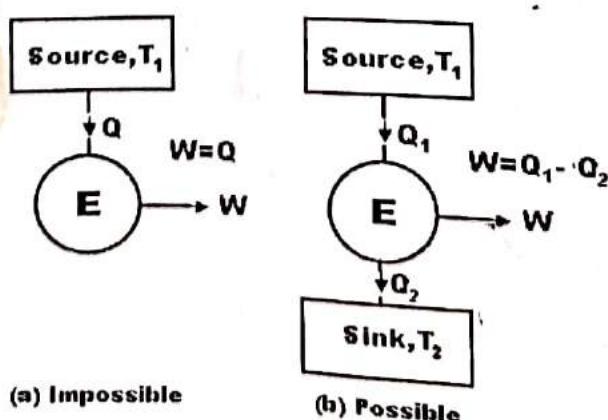


Fig 5.3 Feasibility of a heat engine

The above statement implies that no heat engine is possible which receives heat energy from a reservoir and converts all of it into work. If possible, $W=Q$ and the thermal efficiency of such an engine

$$\eta = \frac{W}{Q} = \frac{Q}{Q} = 1 = 100\%. \text{ Such an engine satisfies 1st law of thermodynamics but violates}$$

Kelvin-Plank statement and is called Perpetual Motion Machine of 2nd Kind (PMM II) and is represented in above figure (a). Since an engine must not be 100% efficient i.e. $W_{net} < Q_1 \Rightarrow Q_1 - Q_2 < Q_1 \Rightarrow Q_1 - Q_1 < Q_2 \Rightarrow Q_2 > 0$, there has always to be heat rejection. A heat engine has thus to exchange heat with two reservoirs. The figure(b) shows possible mode of a heat engine.

5.5.2 Clausius Statement of the 2nd Law

"It is impossible to construct a device which, operating in a cycle, will produce no other effect other than the transfer of heat from a cooler body to a hotter body."

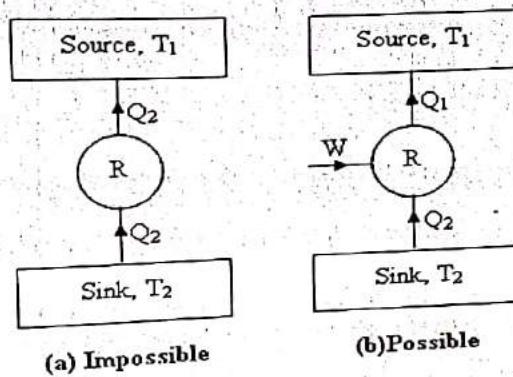


Fig 5.4 Feasibility of heat pump or refrigerator

Clausius statement implies that heat can not flow itself from lower temperature to higher temperature zone. A heat pump or refrigerator can not operate unless external work is supplied to it. If such a heat pump were to be possible, $W=0$ and COP of such pump would

be: $COP = \frac{Q_1}{W} = \frac{Q_1}{0} = \infty$. Thus, a reversed heat engine of infinite COP is not possible. Thus, refrigerator shown in figure(a) is impossible.

5.6 Equivalence of Kelvin-Plank and Clausius Statement

5.6.1 Violation of Clausius statement leads to the violation of Kelvin-Plank statement

Let us consider a refrigerator R that operates in a cycle and transfers heat from sink at T_2 to source at T_1 without any work input from external agency. This is in violation with Clausius statement. Indicated along with is a heat engine E that too operates in a cycle. This engine takes heat Q_1 from source and rejects heat Q_2 to the sink. Thus, it delivers work equivalent to $Q_1 - Q_2$ in compliance with both 1st law. Obviously, this engine conforms to the Kelvin-Plank statement of 2nd law.

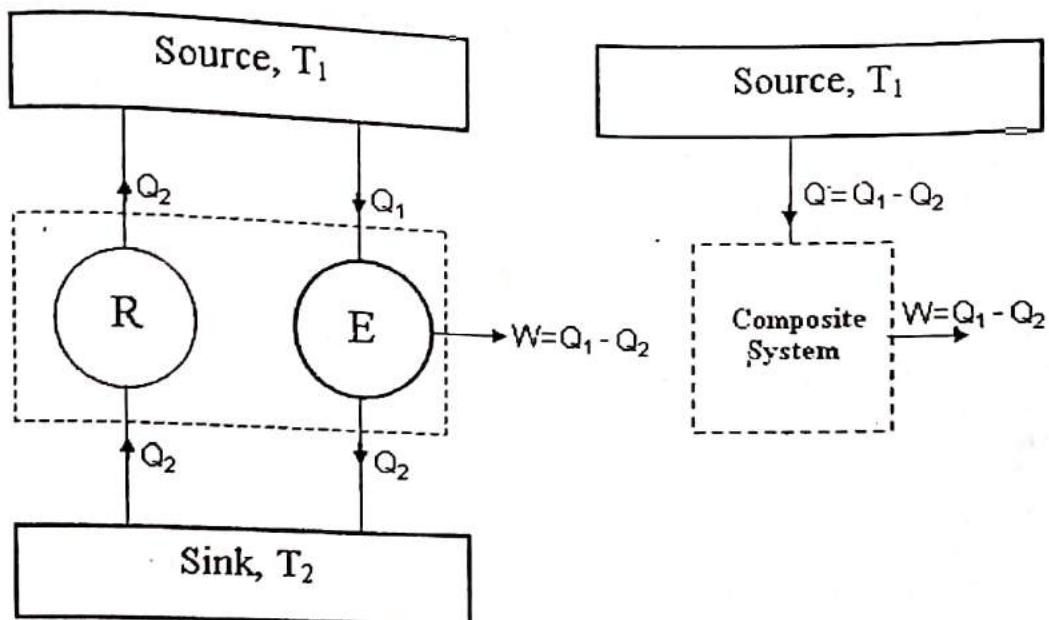


Fig 5.5 Violation of Clausius statement

Now, let us consider a system shown in figure by dotted boundary that encloses both engine and refrigerator under consideration in their coupled condition. The sink receives heat Q_2 from the engine and rejects the same to the refrigerator. Thus, no net heat transfer takes place from the sink and can be placed under no consideration to the composite system. This composite system receives heat $(Q_1 - Q_2)$ from the source and converts it completely into same amount of work $(Q_1 - Q_2)$ without rejecting any net heat to the sink. Thus the Kelvin-Plank statement is violated.

5.6.2 Violation of Kelvin-Plank statement leads to the violation of Clausius statement
 Let us consider a heat engine E which operates from a single heat reservoir at temperature T_1 . It receives heat Q_1 from source and converts it completely into work ($W = Q_1$) without rejecting any heat to the sink ($Q_2 = 0$). This is in violation with Kelvin-Plank statement. Indicated along with is a refrigerator R which extracts heat Q_2 from sink at T_2 consuming work $W = Q_1$ from external source. Hence, the refrigerator works in conformity with Clausius statement.

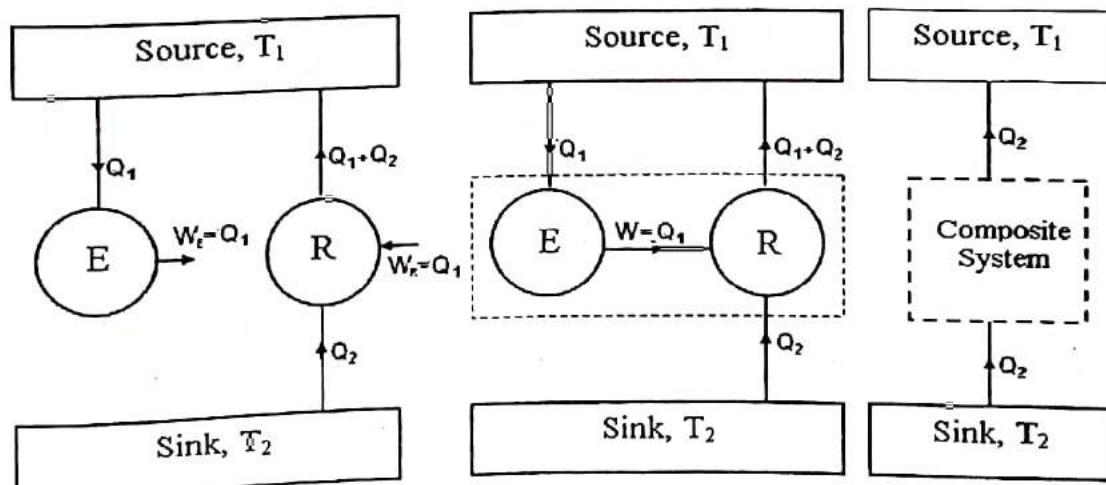


Fig 5.6 Violation of Kelvin-Plank statement

Now, let us consider a composite system indicated in figure by dotted boundary that encloses both the engine and refrigerator when coupled together. The work input required by the refrigerator is fulfilled by the work output by the engine within the composite itself. The composite system rejects $Q_1 + Q_2$ but receives heat Q_1 only thereby transferring net heat Q_2 to the source. Similarly, only heat Q_2 is received by the composite system. All the work interactions take place internally and there is no net work transfer to the composite system from outside. Thus, the composite system receives heat from sink at low temperature and transfer it all to the source at high temperature without any external work input. Thus, Clausius statement is also violated with the violation of Kelvin-Plank statement.

5.7 Carnot Theorem

"No heat engine operating between two given thermal reservoirs can be more efficient than a reversible engine operating between the same two reservoirs."

Let us consider a reversible heat engine E_B and any other heat engine E_A operating between a heat source at T_1 and a heat sink at T_2 as shown in figure. Let the heat transfer to each engine from the source at T_1 be Q_1 .

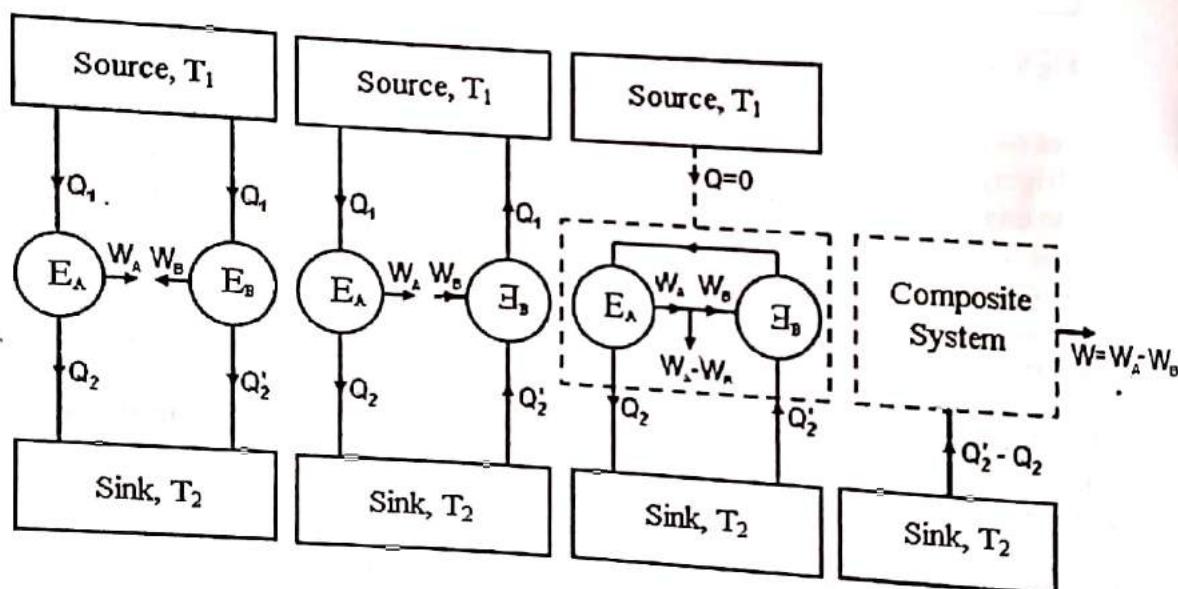


Fig 5.7 Carnot Theorem

In order to prove $\eta_B \geq \eta_A$, let us assume that this is not true i.e.

$$\begin{aligned} \eta_A &> \eta_B \\ \text{or, } \frac{W_A}{Q_1} &> \frac{W_B}{Q_1} \\ \Rightarrow W_A &> W_B \end{aligned}$$

Now, let E_B be reversed. Since it is reversible, the direction will only be reversed but magnitude remains same. Since $W_A > W_B$, part of $W_A (=W_B)$ may be fed to drive the reversed heat engine E_B . Now, let us consider a composite system shown by dotted boundary that encloses the coupling of E_A and reversed E_B . The composite system receives heat Q_1 from the source and rejects the same heat Q_1 to the source. Thus, the composite system does not have any net heat interaction with the source and as such the source can be eliminated. Since $W_A > W_B$, the composite system yields a net work output of $(W_A - W_B)$.

Now, our assumption that,

$$\eta_A > \eta_B \Rightarrow 1 - \frac{Q_2}{Q_1} > 1 - \frac{Q'_2}{Q_1} \Rightarrow -\frac{Q_2}{Q_1} > -\frac{Q'_2}{Q_1} \Rightarrow \frac{Q_2}{Q_1} < \frac{Q'_2}{Q_1} \Rightarrow Q_2 < Q'_2$$

Thus, we see that the composite system rejects heat energy Q_2 to the sink. So, the composite system receives net heat energy $Q'_2 - Q_2$ from thermal reservoir at T_2 . Also, from 1st law of thermodynamics, we have

$$W_A = Q_1 - Q_2$$

$$W_B = Q_1 - Q'_2$$

$$\therefore W_A - W_B = Q'_2 - Q_2$$

With the help of the above results, we conclude that the composite system receives net heat $Q'_2 - Q_2$ from thermal reservoir at T_2 and converts it completely into work without any interaction with other reservoir. This shows the violation of Kelvin-Plank statement. Hence, our assumption that $\eta_A > \eta_B$ can not be true.

$$\therefore \eta_A \leq \eta_B$$

$$i.e \quad \eta_{\text{any engine}} \leq \eta_{\text{reversible engine}}$$

5.8 Corollary to Carnot's Theorem

"All reversible heat engines operating between two given temperatures have same efficiency"

Let us consider two reversible heat engines E_A and E_B operating between a heat source at T_1 and a heat sink at T_2 as shown in figure. Let the heat transfer to each engine from the source at T_1 be Q_1

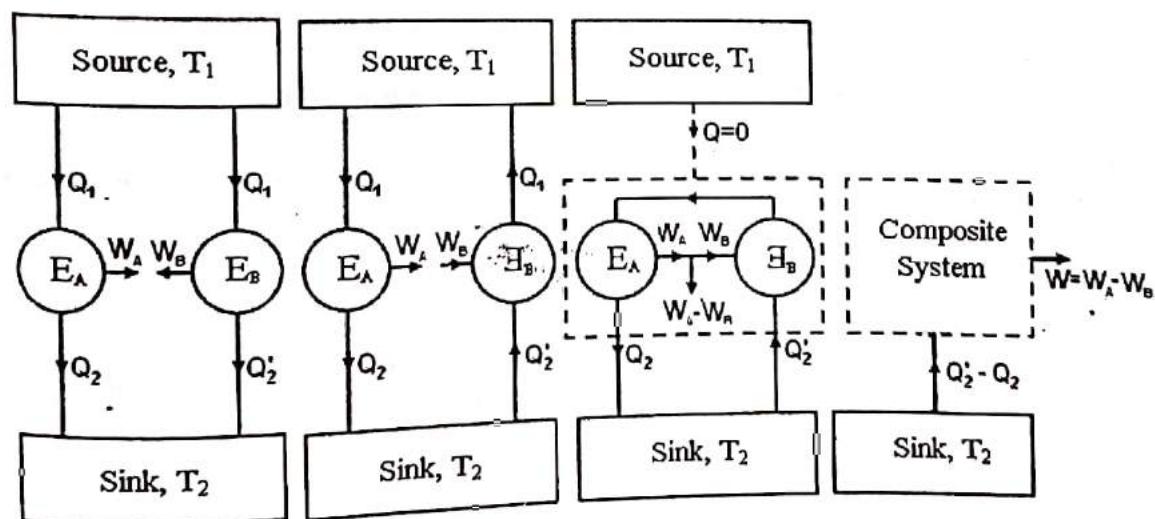


Fig 5.8 Corollary to Carnot Theorem

In order to prove this, let us assume:

$$\eta_A > \eta_B$$

$$\text{or, } \frac{W_A}{Q_1} > \frac{W_B}{Q_1} \Rightarrow W_A > W_B$$

Now, let E_B be reversed. Since it is reversible, the direction will only be reversed but magnitude remains same. Since $W_A > W_B$, part of $W_A (=W_B)$ may be fed to drive the reversed heat engine E_B . Now, let us consider a composite system shown by dotted boundary that encloses the coupling of E_A and reversed E_B . The composite system receives heat Q_1 from the source and rejects the same heat Q_1 to the source. Thus, the composite system does not have any net heat interaction with the source and as such the source can be eliminated. Since $W_A > W_B$, the composite system yields a net work output of $(W_A - W_B)$.

Now, our assumption that,

$$\eta_A > \eta_B \Rightarrow 1 - \frac{Q_2}{Q_1} > 1 - \frac{Q'_2}{Q_1} \Rightarrow -\frac{Q_2}{Q_1} > -\frac{Q'_2}{Q_1} \Rightarrow \frac{Q_2}{Q_1} < \frac{Q'_2}{Q_1} \Rightarrow Q_2 < Q'_2$$

Thus, we see that the composite system receives more heat energy Q'_2 from the sink than it rejects heat energy Q_2 to the sink. So, the composite system receives net heat energy $Q'_2 - Q_2$ from thermal reservoir at T_2 .

Also, from 1st law of thermodynamics, we have

$$W_A = Q_1 - Q_2$$

$$W_B = Q_1 - Q'_2$$

$$\therefore W_A - W_B = Q'_2 - Q_2$$

With the help of the above results, we conclude that the composite system receives net heat $Q'_2 - Q_2$ from thermal reservoir at T_2 and converts it completely into work without any interaction with other reservoir. This shows the violation of Kelvin-Plank statement.

Hence, our assumption that $\eta_A > \eta_B$ can not be true.

$$\therefore \eta_A \leq \eta_B \dots \dots \dots \text{(i)}$$

Similarly, if we assume $\eta_A < \eta_B$ and reverse the engine E_A , we will have same result i.e.

$$\eta_A \geq \eta_B \dots \dots \dots \text{(ii)}$$

The only possibility for (i) and (ii) to be simultaneously satisfied is that $\eta_A = \eta_B$ (*proved*)

Also, since the efficiency of all reversible heat engines operating between same temperature levels is same, it is independent of the nature of the working substance.

5.9 Thermodynamic Temperature Scale

Let a reversible engine operate between a source at t_H and sink at t_L . Let the heat transfer to and out of the engine be Q_H and Q_L . According to the Carnot's theorem, the efficiency of such an engine is maximum and its value depends only on the temperatures between which it operates and not on the properties of the substance that undergoes the cycle.

Hence, the efficiency has to be a function, say ψ , of the two temperatures only.

That is,

$$\eta_{rev,engine} = 1 - \frac{Q_L}{Q_H} = \psi(t_H, t_L) \Rightarrow \frac{Q_L}{Q_H} = 1 - \psi(t_H, t_L)$$

$$\therefore \frac{Q_H}{Q_L} = [1 - \psi(t_H, t_L)]^{-1} = \phi(t_H, t_L) \dots \dots \dots \text{(i)}$$

Where, ϕ is another function related to ψ .

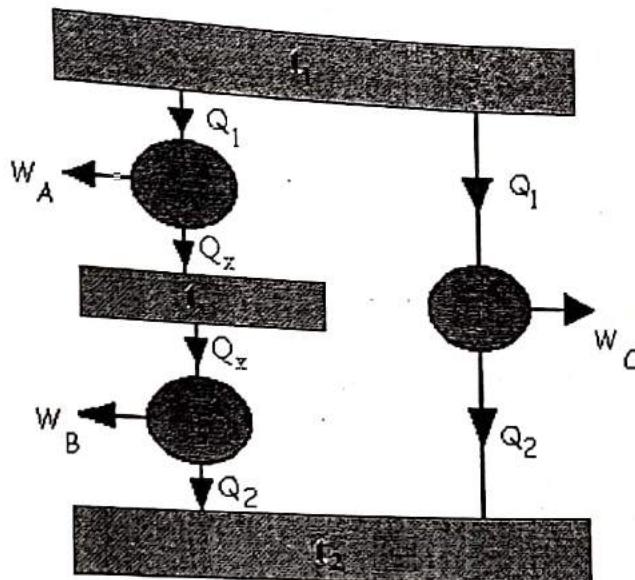


Fig 5.9 Thermodynamic Temperature Scale

Now, let us consider three heat engines A,B and C working between source at t_1 and sink at t_2 as shown in figure above. Engine A operates between reservoir at t_1 and an intermediate system at a fixed temperature of t_X , where $t_1 > t_X > t_2$. Let the heat transfer from the reservoir at t_1 to the engine A be Q_1 and the heat transferred by it to the system at t_X be Q_X . Let the work produced by A be W_A . Engine B operates between the system at t_X and the reservoir at t_2 . Let the heat transferred by the engine A goes to the engine B through the system at t_X . Let the heat transferred by the engine B to the sink at t_2 be Q_2 and work produced by it be W_B . Engine C operates between the same two reservoirs at t_1 and t_2 . Let the heat transfer to the engine C be Q_1 which is equal to the heat transferred to engine A. The heat transferred out of this engine must be Q_2 as, according to the Carnot's proposition, efficiency of engine C must be same as the efficiency of combined engine A and B. Let the work done by it be W_C . According to the relation(i), the following relations hold good for engines A,B and C i.e.

Replacing the ratios of heat transfers in (v) by the corresponding functions of temperatures from (ii), (iii) and (iv), we have

While the left-hand side of (vi) is a function of three temperatures t_1, t_x and t_2 , the right-hand side is a function of two temperatures viz, t_1 and t_2 only. Hence the value of the left-side product has to be independent of the value of temperature t_x . This gives an insight into the nature of the function ϕ which can satisfy such a condition. Therefore,

$$\phi(t_1, t_x) = \frac{f(t_1)}{f(t_x)}, \text{ and} \dots \dots \dots \quad (\text{vii})$$

$$\phi(t_x, t_2) = \frac{f(t_x)}{f(t_2)} \dots \dots \dots \quad (\text{viii})$$

Where f is some other function.

The right-hand side of (vi), from (iv), is equal to $\left(\frac{Q_1}{Q_2}\right)$

$$\text{Therefore, } \left(\frac{Q_1}{Q_2}\right) = \phi(t_1, t_x) \times \phi(t_x, t_2) \dots \dots \dots \quad (\text{ix})$$

Combining (vii), (viii) and (ix), we get

$$\left(\frac{Q_1}{Q_2}\right) = \left[\frac{f(t_1)}{f(t_x)}\right] \times \left[\frac{f(t_x)}{f(t_2)}\right]$$

$$\text{That is, } \left(\frac{Q_1}{Q_2}\right) = \frac{f(t_1)}{f(t_2)} \dots \dots \dots \quad (\text{x})$$

Here, f , in general can be any function; however, it is universally accepted, as suggested by Lord Kelvin, to consider $f(t)=T$, where T is in thermodynamic or Kelvin temperature scale.

$$\text{Therefore, } \left(\frac{Q_1}{Q_2}\right) = \frac{T_1}{T_2} \dots \dots \dots \quad (\text{xi})$$

Solved Examples

- Using an engine of 30% thermal efficiency to drive a refrigerator having a COP of 5, what is the heat input into the engine for each MJ removed from the cold body by the refrigerator?
If this system is used as a heat pump, how many MJ of heat would be available for heating for each MJ of heat input to the engine?

Solution

Referring to the adjoining figure, we have

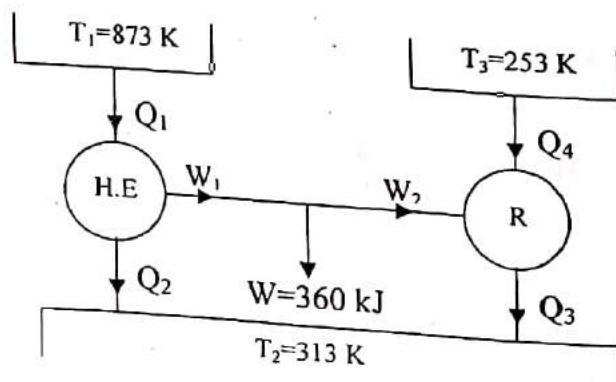
$$T_1 = 273 + 600 = 873 \text{ K}$$

$$T_2 = 273 + 40 = 313 \text{ K}$$

$$T_3 = 273 - 20 = 253 \text{ K}$$

$$Q_1 = 2000 \text{ kJ}$$

$$W = 360 \text{ kJ}$$



(a) Maximum efficiency of the heat engine cycle is given by

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{313}{873} \approx 1 - 0.358 = 0.642$$

But the efficiency of the engine is also given by

$$\eta = \frac{W_1}{Q_1} \Rightarrow W_1 = \eta \times Q_1 = 0.642 \times 2000 = 1284 \text{ kJ}$$

Maximum COP of the refrigerator cycle is

$$(COP)_{\max} = \frac{T_3}{T_2 - T_3} = \frac{253}{313 - 253} = 4.22$$

But we have,

$$(COP)_r = \frac{Q_4}{W_2} \Rightarrow Q_4 = 4.22 \times W_2 \dots\dots\dots(I)$$

$$\text{Since, } W_1 - W_2 = W = 360 \text{ kJ}$$

$$\therefore W_2 = W_1 - W = 1284 - 360 = 924 \text{ kJ}$$

$$\therefore Q_4 = 4.22 \times 924 = 3899 \text{ kJ}$$

$$\therefore Q_3 = Q_4 + W_2 = 924 + 3899 = 4823 \text{ kJ}$$

$$\therefore Q_2 = Q_1 - W_1 = 2000 - 1284 = 716 \text{ kJ}$$

$$\text{Heat rejection to the } 40^\circ\text{C reservoir} = Q_2 + Q_3 = 716 + 4823 = 5539 \text{ kJ}$$

(b) Efficiency of the actual heat engine cycle is

$$\eta = 0.4 \times \eta_{\max} = 0.4 \times 0.642$$

$$\text{Or, } \frac{W_1}{Q_1} = 0.4 \times 0.642$$

$$\therefore W_1 = 0.4 \times 0.642 \times 2000 = 513.6 \text{ kJ}$$

$$\therefore W_2 = W_1 - W = 513.6 - 360 = 153.6 \text{ kJ}$$

COP of the actual refrigerator cycle is

$$COP = \frac{Q_4}{W_2}$$

$$\therefore Q_4 = W_2 \times COP = 153.6 \times 1.69 \approx 259.6 \text{ kJ}$$

$$Q_3 = Q_4 + W_2 = 259.6 + 153.6 = 413.2 \text{ kJ}$$

$$Q_2 = Q_1 - W_1 = 2000 - 513.6 = 1486.4 \text{ kJ}$$

$$Q_2 + Q_3 = 1486.4 + 413.2 = 1899.6 \text{ kJ}$$

$$\text{Heat rejected to the } 40^\circ\text{C reservoir} = Q_2 + Q_3 = 413.2 + 1486.4 = 1899.6 \text{ kJ}$$

2. If 20 kJ are added to a Carnot cycle at a temperature of 100°C and 14.6 kJ are rejected at 0°C , determine the location of absolute zero on the Celsius scale.

Solution:
Referring to the figure, we have

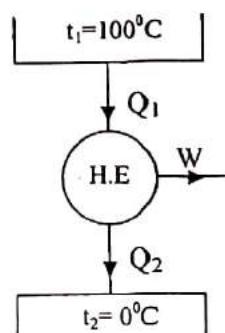
$$Q_1 = 20 \text{ kJ}$$

$$Q_2 = 14.6 \text{ kJ}$$

$$t_1 = 100^\circ\text{C}$$

$$t_2 = 0^\circ\text{C}$$

Let θ be the temperature on Kelvin scale corresponding to 0°C



$$T_1 = t_1 + \theta = 100 + \theta$$

$$T_2 = t_2 + \theta = 0 + \theta = \theta$$

From the concept of absolute temperature scale, we have

$$\frac{T_1}{T_2} = \frac{Q_1}{Q_2}$$

$$\text{Or, } \frac{100 + \theta}{\theta} = \frac{20}{14.6}$$

$$\text{Or, } 14.6(100 + \theta) = 20\theta$$

$$\text{Or, } 1460 + 14.6\theta = 20\theta$$

$$\text{Or, } 20\theta - 14.6\theta = 1460$$

$$\text{Or, } 5.4\theta = 1460$$

$$\therefore \theta = \frac{1460}{5.4} = 270.37$$

Hence for absolute zero point, 0K, Celsius scale point = -270.37°C

3. A reversible engine operates between temperatures T_1 and T ($T_1 > T$). The energy rejected from this engine is received by a second reversible engine at the same temperature T . The second engine rejects energy at temperature T_2 ($T_2 < T$). Show that (a) temperature T is the arithmetic mean of temperatures T_1 and T_2 if the engines produce the same amount of work output, and (b) the temperature T is the geometric mean of temperatures T_1 and T_2 if the engines have the same cycle efficiencies.

Solution:

(a) For the same work output of both the engines i.e. $W_1 = W_2$

For 1st Heat engine, we have

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T}{T_1} \quad \text{and also, } \eta = \frac{W_1}{Q_1}$$

$$\therefore \frac{W_1}{Q_1} = \left(1 - \frac{T}{T_1}\right) \Rightarrow W_1 = \left(\frac{T_1 - T}{T_1}\right) Q_1$$

For 2nd Heat engine, we have

$$\eta = 1 - \frac{T_2}{T} \quad \text{and also, } \eta = \frac{W_2}{Q_2} = \frac{W_2}{Q_3} = \frac{W_2}{Q_2}$$

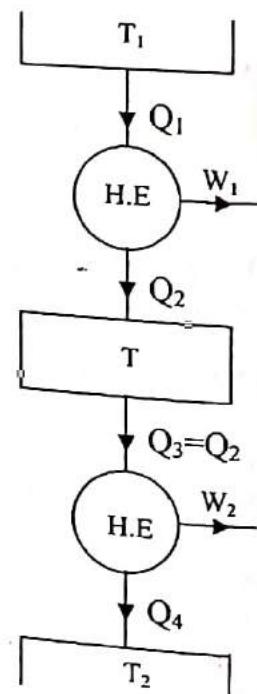
$$\therefore \frac{W_2}{Q_2} = \left(1 - \frac{T_2}{T}\right) \Rightarrow W_2 = \left(\frac{T - T_2}{T}\right) Q_2$$

But by the question, we have

$$W_1 = W_2$$

$$\text{Or, } \left(\frac{T_1 - T}{T_1}\right) Q_1 = \left(\frac{T - T_2}{T}\right) Q_2$$

$$\text{Or, } \frac{T_1 - T}{T_1} = \left(\frac{T - T_2}{T}\right) \frac{Q_2}{Q_1}$$



$$Or, \frac{T_1 - T}{T_1} = \left(\frac{T - T_2}{T} \right) \frac{T}{T_1}$$

$$Or, T_1 - T = T - T_2$$

$$Or, 2T = T_1 + T_2$$

$$\therefore T = \frac{T_1 + T_2}{2}$$

(b) For same efficiency, we have

$$\eta_1 = \eta_2$$

$$Or, 1 - \frac{T}{T_1} = 1 - \frac{T_2}{T}$$

$$Or, \frac{T_2}{T} = 1 - 1 + \frac{T}{T_1}$$

$$Or, \frac{T_2}{T} = \frac{T}{T_1}$$

$$Or, T^2 = T_1 T_2$$

$$\therefore T = \sqrt{T_1 T_2}$$

Problems

- If a refrigerator is used for heating purpose in winter so that the atmosphere becomes the cold body and the room to heated becomes the hot body, how much heat would be available for heating for each kW input to the driving motor? The COP of the refrigerator is 5, and the electromechanical efficiency of the motor is 90%. How does this compare with resistance heating?
- An electric storage battery which can exchange heat only with a constant temperature atmosphere goes through a complete cycle of two processes. In process 1-2, 2.8 kWh of electrical work flow into the battery while 732 kJ of heat flow out to the atmosphere. During the process 2.4 kWh of work flow out of the battery. A0 find the heat transfer in process 2-1 (b) if the process 1-2 has occurred as above, does the first law or the second law limit the maximum possible work of process 2-1 ? What is the maximum possible work? (c) if the maximum possible work were obtained in the process 2-1, what will be the heat transfer in the process ?
- A heat pump working on the Carnot cycle takes in heat from a reservoir at 5°C and delivers heat to a reservoir at 60°C . The heat pump is driven by a reversible heat engine which takes in heat from a reservoir at 840°C and rejects heat to a reservoir at 60°C . The reversible heat engine also drives a machine that absorbs 30 kW. If the heat pump extracts 17 kJ/s from the 5°C reservoir, determine (a) the rate of heat supply from the 840°C source, and (b) the rate of heat rejection to the 60°C sink.
- A heat engine is used to drive a heat pump. The heat transfers from the heat engine and from the heat pump are used to heat the winter circulating through the radiators of a building. The efficiency of the heat engine is 27% and the COP of the heat pump is 4. Evaluate the ratio of heat transfer to the circulating water to the heat transfer to the heat engine.

5. A heat engine operates between the maximum and the minimum temperatures of 671°C and 60°C respectively, with an efficiency of 50% of the appropriate Carnot efficiency. It drives a heat pump which uses river water at 4.4°C to heat a block of flats in which the temperature is to be maintained at 21.1°C . Assuming that a temperature difference of 11.1°C exists between the working fluid and the river water, on the one hand, and the required room temperature on the other, and assuming the heat pump to operate on the reversed Carnot cycle, but with a COP of 50% of ideal COP, the heat input to the engine per unit heat output from the heat pump. Why is direct heating thermodynamically more useful?
6. Two Carnot engines A and B are connected in series between thermal reservoirs maintained at 1000 K and 100 K respectively. Engine A receives 1680 kJ of heat from the high temperature reservoir and rejects heat to the Carnot engine B. Engine B takes in heat rejected by engine A and rejects heat to the low temperature reservoir. If engines A and B have equal efficiencies, determine (a) the heat rejected by the engine B, (b) the temperature at which heat is rejected by the engine A, and (c) the work done during the processes by engines A and B respectively, (d) the amount of heat taken in by the engine B, and (e) the efficiencies of engines A and B.
7. A heat pump is to be used to heat a house in winter and then reversed to cool the house in summer. The interior temperature is to be maintained at 20°C . Heat transfer through the walls and roof is estimated to be 0.525 kJ/s per degree temperature difference between the inside and outside. (a) if the outside temperature in winter is 5°C , what is the minimum power required to drive the heat pump? (b) if the power output is same as in part (a), what is the maximum outer temperature for which the inside can be maintained at 20°C ?
8. Consider an engine in outer space which operates on Carnot cycle. The only way in which heat can be transformed from the engine is by radiation. The rate at which is radiated is proportional to the fourth power of the absolute temperature and to the area of the radiating surface. Show that for a given power output and a given T_1 , the area of the radiator will be a minimum when $\frac{T_2}{T_1} = \frac{3}{4}$
9. A reversible power cycle is used to drive a reversible heat pump. The power cycle takes in Q_1 heat units at T_1 and rejects heat Q_2 at T_2 . The heat pump extracts Q_4 from the sink at T_4 and discharges Q_3 at T_3 . Develop an expression for the ratio Q_4/Q_1 in terms of the four temperatures.
10. A heat engine drives a heat pump at COP equal to 5 by supplying 70% of the work output. The heat rejected by the heat engine and the heat pump are used to heat water in a certain chemical plant. The total heat received by the chemical plant is twice the amount of heat supplied by the engine. Calculate the efficiency of the heat engine.
11. A reversible heat engine operates between 600°C and 40°C and drives a reversible refrigerator operating between 40°C and -18°C . Still there is net output of work equal to 370 kJ while the heat received by the engine is 2100 kJ . Determine the cooling effect.
12. A reversible engine receives heat from a mixture of water vapour and liquid water at 1 atm and rejects 3775 kJ/hr of heat at 100 K below temperature of a mixture of ice and liquid water at 1 atm. It delivers 0.386 kW power. Find the number of degrees separating absolute zero and ice point on Kelvin scale.

13. A refrigerator operating on a reversed Carnot cycle consumes 150 W power in summer when the ambient atmosphere is at 310 K. The heat leakage into the refrigerator through the doors is estimated to be at the continuous rate of 15 W per degree temperature difference between the ambient air and the cold space of the refrigerator. For continuous operation of the refrigerator, determine the temperature at which the cold space is maintained.
14. A perfect reversed heat engine is used for making ice at -5°C from water available at 25°C . The temperature of freezing mixture is -10°C . Calculate the quantity of ice formed per kWh. For ice: specific heat capacity = 2.1 kJ/kgK and latent heat = 335 kJ/kg .
15. An engine working on Carnot cycle absorbs Q_1 units of heat from a source at T_1 and rejects Q_2 units of heat to a sink at T_2 . Because of imperfect heat transfer, the maximum and the minimum absolute temperatures θ_1 and θ_2 attained during the cycle are given by

$$\theta_1 = T_1 - KQ_1 \quad \text{and} \quad \theta_2 = T_2 + KQ_2$$

where K is a constant. Show that efficiency of the plant is given by:

$$\eta = 1 - \frac{T_2}{T_1 - 2KQ_1}$$

Entropy

6.1 Clausius Inequality

6.1 Clausius Inequality
 “When a system undergoes a complete cyclic process, the integral of $\frac{\delta Q}{T}$ around the cycle is less than or equal to zero.”

Let the system enclosed by the system boundary S as shown in figure undergoes a cyclic process. At a given instant, let δQ and δW be the heat and work interactions the system has with the surroundings. Let T be the temperature of the system at this instant. Let us consider a heat engine E which receives heat δQ_0 from the reservoir at T_0 and transfers heat δQ to the part of the system at temperature T . Under these conditions, let the work output of E be δW_E .

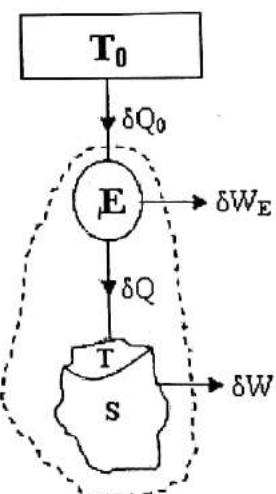


Fig 6.1 Clausius Inequality

Applying 1st law of thermodynamics to the system S, we get

$$\oint \delta Q = \oint \delta W \dots \dots \dots \quad (i)$$

Now, let us consider the system inside the boundary shown by the dotted lines. This system also undergoes a cyclic process as it is assumed that engine E undergoes a finite number of cycles for each cycle undergone by S. This system interacts with only one reservoir at T_0 . But according to the Kelvin-Plank statement, a heat engine having interaction with single reservoir can not produce work and mathematically it implies that net work from such an engine can not be positive i.e.

$$\oint \delta W \leq 0$$

Single Reservoir

So, for the system enclosed in the dotted boundary, it yields

$$\oint (\delta W_E + \delta W) \leq 0$$

Eliminating δW between (i) and (ii), we have

$$\begin{aligned} \oint \delta W_E + \oint \delta Q &\leq 0 \\ \oint (\delta W_E + \delta Q) &\leq 0 \quad \dots \dots \dots \text{(iii)} \end{aligned}$$

Applying 1st law to the engine E, we get

$$\delta Q_0 - \delta Q = \delta W_E \quad \dots \dots \dots \text{(iv)}$$

As E is reversible engine, by the definition of thermodynamic temperature scale, we have

$$\frac{\delta Q_0}{\delta Q} = \frac{T_0}{T} \Rightarrow \delta Q_0 = \delta Q \left(\frac{T_0}{T} \right) \quad \dots \dots \dots \text{(v)}$$

Eliminating δQ_0 between (iv) and (v), we have

$$\delta Q \left(\frac{T_0}{T} \right) - \delta Q = \delta W_E \quad \dots \dots \dots \text{(vi)}$$

Substituting the value of δW_E from (vi) in (iii), we get

$$\oint \left[\delta Q \left(\frac{T_0}{T} \right) - \delta Q + \delta Q \right] \leq 0$$

$$\text{Or, } \oint \delta Q \left(\frac{T_0}{T} \right) \leq 0$$

Since T_0 is always positive, we have

$$\oint \frac{\delta Q}{T} \leq 0$$

6.2 Clausius Theorem

"The cyclic integral of $\frac{\delta Q}{T}$ for a reversible cycle is equal to zero"

Mathematically,

$$\oint_{\text{Rev}} \frac{\delta Q}{T} = 0$$

Let us consider a system undergoing a reversible cycle 1-A-2-B-1 as shown in figure. If δQ be the heat transfer in this process, from Clausius inequality, we have

$$\oint \frac{\delta Q}{T} \leq 0 \quad \dots \dots \dots \text{(i)}$$

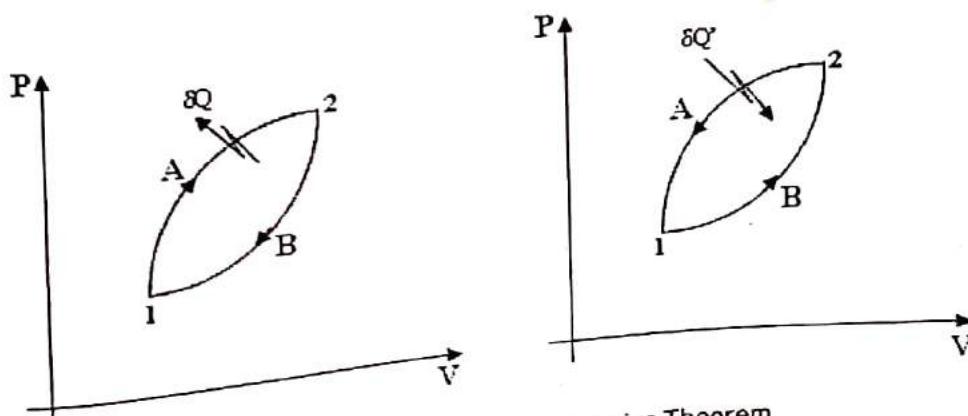


Fig 6.2 Diagram to prove Clausius Theorem

Now, let the direction of the above cycle be reversed so that the heat transfer now becomes $\delta Q'$. Again, from Clausius inequality, for the cycle 1-B-2-A-1, we have

$$\oint \frac{\delta Q'}{T} \leq 0 \dots \dots \dots \text{(ii)}$$

On reversing the direction of reversible process, the magnitude of heat transfer does not change but its direction of flow or transfer becomes opposite.

Therefore, $\delta Q' = -\delta Q \dots \dots \dots \text{(iii)}$

Substituting for $\delta Q'$ from (iii) in (ii), we get

$$\oint \frac{\delta Q}{T} \leq 0$$

$$\text{i.e. } \oint \frac{\delta Q}{T} \geq 0 \dots \dots \dots \text{(iv)}$$

For (i) and (iv) to be simultaneously true, the only possibility is that

$$\oint \frac{\delta Q}{T} = 0$$

6.3 Entropy: a property of a system

Let us consider a system that changes state from 1 to 2 as the system is subjected to reversible processes A or B. Let us also assume that there exists another reversible process C which restores the system back to its initial state.

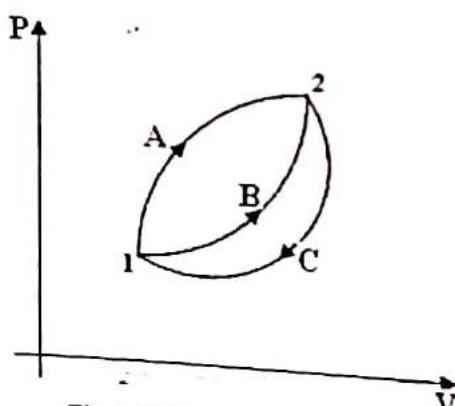


Fig 6.3 Entropy- a property

We know that the cyclic integral of $\frac{\delta Q}{T}$ for a reversible cycle is zero i.e.

$$\oint \frac{\delta Q}{T} = 0$$

Thus, for cycles 1-A-2-C-1 and 1-B-2-C-1, we have

$$\oint \frac{\delta Q}{T} = \int_1^2 \left(\frac{\delta Q}{T} \right)_A + \int_2^1 \left(\frac{\delta Q}{T} \right)_C = 0 \dots \dots \dots \text{(i)}$$

And,

$$\oint \frac{\delta Q}{T} = \int_1^2 \left(\frac{\delta Q}{T} \right)_B + \int_2^1 \left(\frac{\delta Q}{T} \right)_C = 0 \dots \dots \dots \text{(ii)}$$

Subtracting (ii) from (i), we get

$$\int_1^2 \left(\frac{\delta Q}{T} \right)_A - \int_1^2 \left(\frac{\delta Q}{T} \right)_B = 0 \dots \dots \dots \text{(iii)}$$

$$\text{i.e. } \int_1^2 \left(\frac{\delta Q}{T} \right)_A = \int_1^2 \left(\frac{\delta Q}{T} \right)_B$$

Obviously, the integral of $\left(\frac{\delta Q}{T} \right)_{\text{Rev}}$ is independent of path whether it is A or B and depends only on the end states (1 or 2). Hence, it is a property known as entropy represented as S. Thus,

$$dS = \left(\frac{\delta Q}{T} \right)_{\text{Rev}} \text{ or } \left(\frac{\delta Q_R}{T} \right)$$

For reversible adiabatic process,

$$\delta Q = 0$$

$$\therefore dS = 0 \Rightarrow S = \text{Constant}$$

Thus, entropy is a property of a system that remains constant during adiabatic reversible process.

Fig 6.4

6.4 Entropy Change during an irreversible Process (Entropy Generation)

Let us consider an irreversible process 1-2 along path A as shown in figure by dotted line.

Let a cycle be completed by a reversible process 2-1 along path B as shown in figure by bold line

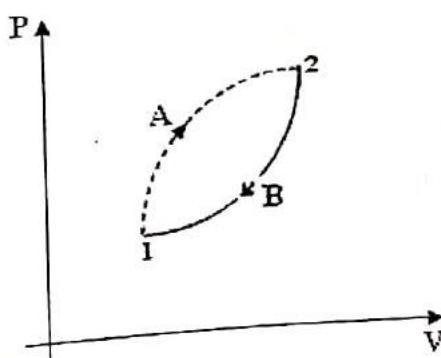


Fig 6.4 Entropy change in an irreversible process

From the Clausius Inequality, we have

$$\oint \frac{\delta Q}{T} < 0, \quad \text{for an irreversible cycle}$$

So, for cycle 1-A-2-B-1, we have

$$\int_1^2 \left(\frac{\delta Q}{T} \right)_A + \int_2^1 \left(\frac{\delta Q}{T} \right)_B < 0 \dots \dots \dots \text{(I)}$$

Since B is reversible,

$$\int_2^1 \left(\frac{\delta Q}{T} \right)_B = \int_1^2 \left(\frac{-\delta Q}{T} \right)_B = - \int_1^2 \left(\frac{\delta Q}{T} \right)_B$$

Thus, inequality (I) becomes

$$\int_1^2 \left(\frac{\delta Q}{T} \right)_A - \int_1^2 \left(\frac{\delta Q}{T} \right)_B < 0$$

$$\text{Or, } \int_1^2 \left(\frac{\delta Q}{T} \right)_A < \int_1^2 \left(\frac{\delta Q}{T} \right)_B$$

As the path B is reversible, we have

$$\int_1^2 \left(\frac{\partial Q}{T} \right)_B = \int_1^2 dS_B$$

Thus, relation (II) becomes

$$\int_1^2 dS_B > \int_1^2 \left(\frac{\delta Q}{T} \right), \quad \dots \dots \dots \quad (\text{III})$$

But, entropy is a property and does not depend upon the path or process i.e. the change of entropy between two states (viz. 1 and 2) must be same irrespective of the two paths A or B.

$$\therefore \int\limits_1^2 dS_B = \int\limits_1^2 dS_A$$

Thus inequality (III) becomes

$$\Rightarrow dS > \frac{\delta Q}{T} \quad \dots \dots \text{for irreversible process}$$

But for reversible process, we know that

$$dS = \frac{\delta Q}{T} \quad \text{for reversible process}$$

Thus, for any process, we have

$$dS \geq \frac{\delta Q}{T}$$

Obviously, the value of $\frac{\delta Q}{T}$ is less than the change in entropy in case of irreversible process. But the change in entropy remains same for both reversible and irreversible process if the end states are same. So, the system undergoing irreversible process must have some additional entropy apart from $\frac{\delta Q}{T}$, which is due to the interaction of heat between the system and surrounding. This additional entropy, thus, must generate within the system and is called entropy generation dS_{gen} .

$$\therefore dS = \frac{\delta Q}{T} + dS_{gen}$$

On its integration yields to

$$\Delta S_{gen} = (S_2 - S_1) - \int \frac{\delta Q}{T}$$

This generated entropy is due to the irreversibility within the system.

6.5 Principle of Increase of Entropy or Entropy Principle

"The entropy, an extensive property, must always increase or remain constant for an isolated system." Fig 13

We know that the following relation hold good to any process,

$$dS \geq \frac{\delta Q}{T} \dots \dots \dots \text{(i)}$$

But an isolated system does not undergo any heat interaction with the surrounding i.e.
 $\delta Q = 0$

So, inequality (i) yields to

$$dS_{iso} \geq 0$$

For reversible process,

$$dS_{iso} = 0 \Rightarrow S_{iso} = \text{Constant}$$

For irreversible process,

$$dS_{iso} > 0 \Rightarrow S_{iso} \text{ must increase.}$$

Let us verify this by considering a process in which a quantity of heat δQ is transferred from the surrounding at temperature T_0 to a system at temperature T i.e. $T_0 > T$. Let the work done in this process be δW

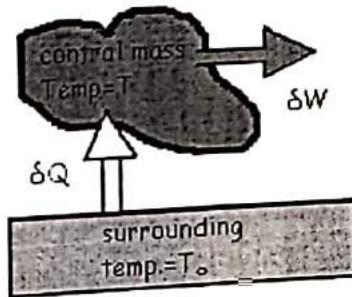


Fig 6.5 Entropy Principle

Change in entropy of the surrounding and system due to δQ transfer are given by

$$ds_{sur} = \frac{-\delta Q}{T_0}$$

$$ds_{sys} = \frac{+\delta Q}{T}$$

-ve sign is placed as heat has been rejected by the surrounding

Now, net change in the entropy of the universe is given by:

$$ds_{univ} = ds_{sys} + ds_{sur}$$

$$= \frac{\delta Q}{T} - \frac{\delta Q}{T_0}$$

$$\text{i.e. } ds_{univ} = \delta Q \left(\frac{1}{T} - \frac{1}{T_0} \right) \dots \dots \dots \text{(i)}$$

Since heat flows from the surrounding to the system,
 $T < T_0$

$$\text{or, } \frac{1}{T} > \frac{1}{T_0}$$

Also, when $T=T_0$, we have

Combining (I) and (II), we get

From (i) and (C), we conclude that

$$ds_{\text{univ}} \geq 0$$

$ds_{univ} \geq 0$
Obviously, universe being an ideal isolated system obeys the principle of increase of entropy. Hence, this principle is verified to be true.

6.6 Entropy Change for an Ideal Gas

From the 1st law of thermodynamics, we have

$$\delta Q = dU + \delta W$$

On unit mass basis, the same can be written as

$$\delta q = du + \delta w$$

For reversible process, $\delta q_{rev} = pdv$

But we know that, $ds = \frac{\delta q_{rev}}{T}$ and $c_v = \frac{du}{dT}$

So that, $\delta q = Tds$ and $du = c_v dT$

Now eqⁿ (I) becomes

$$Tds \equiv c_v dT + pdv$$

$$\text{Or, } ds = c_v \frac{dT}{T} + \frac{p}{T} dv \quad \dots \dots \dots \text{(II)}$$

But, we know that.

$$pV = RT \Rightarrow \frac{P}{T} = \frac{R}{V}$$

On substitution, eqⁿ (II) yields to

$$ds = c_v \frac{dT}{T} + \frac{R}{v} dv$$

Now, integrating both sides between states i and j ,

$$\int_{s_1}^{s_2} ds = \int_{T_1}^{T_2} c_v \frac{dT}{T} + \int_{v_1}^{v_2} R \frac{dv}{v}$$

$$\text{Or, } [s]_{s_1}^{s_2} = c_v \int_{T_1}^{T_2} \frac{dT}{T} + R \int_{v_1}^{v_2} \frac{dv}{v}$$

$$\text{Or, } s_2 - s_1 = c_v [\ln T]_{T_1}^{T_2}$$

$$\text{Or, } s_2 - s_1 = c_v [\ln T]_{T_1}^{T_2} + R [\ln \nu]_{\nu_1}^{\nu_2}$$

$$z = z_1 - c_v [\ln T_2 - T_1] + R [\ln \nu_2 - \ln \nu_1]$$

$$\therefore s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

6.7 Entropy Change of an Ideal Gas for different processes

6.7.1 Isochoric (Constant volume) Process

If the isochoric process is reversible one, we have

$$\delta Q_{rev} = mc_v dT$$

$$\Rightarrow \delta q_{rev} = \frac{\delta Q_{rev}}{m} = c_v dT$$

Dividing both the sides by T, we get

$$\frac{\delta q_{rev}}{T} = c_v \frac{dT}{T}$$

$$\therefore ds = c_v \frac{dT}{T}$$

Integrating both sides between states 1 and 2, we get

$$\int_{s_1}^{s_2} ds = \int_{s_1}^{s_2} c_v \frac{dT}{T}$$

$$\text{Or, } [s]_{s_1}^{s_2} = c_v \int_{T_1}^{T_2} \frac{dT}{T} = c_v [\ln T]_{T_1}^{T_2} = c_v [\ln T_2 - \ln T_1]$$

$$\therefore s_2 - s_1 = c_v \ln \frac{T_2}{T_1} \dots\dots\dots\dots (A)$$

$$s_2 - s_1 = c_v \ln \frac{p_2}{p_1} \dots\dots\dots\dots (B) \quad \left[\because p_2 v = RT_2 \text{ & } p_1 v = RT_1 \Rightarrow \frac{p_2}{p_1} = \frac{T_2}{T_1} \right]$$

6.7.2 Isobaric (Constant Pressure) Process

If the isobaric process is reversible one, we have

$$\delta Q_{rev} = mc_p dT$$

$$\Rightarrow \delta q_{rev} = \frac{\delta Q_{rev}}{m} = c_p dT$$

Dividing both the sides by T, we get

$$\frac{\delta q_{rev}}{T} = c_p \frac{dT}{T}$$

$$\therefore ds = c_p \frac{dT}{T}$$

Integrating both sides between states 1 and 2, we get

$$\int_{s_1}^{s_2} ds = \int_{s_1}^{s_2} c_p \frac{dT}{T}$$

$$\text{Or, } [s]_{s_1}^{s_2} = c_p \int_{T_1}^{T_2} \frac{dT}{T} = c_p [\ln T]_{T_1}^{T_2} = c_p [\ln T_2 - \ln T_1]$$

$$\therefore s_2 - s_1 = c_p \ln \frac{T_2}{T_1} \dots\dots\dots\dots (A)$$

$$s_2 - s_1 = c_p \ln \frac{v_2}{v_1} \dots\dots\dots\dots (B) \quad \left[\because p v_2 = RT_2 \text{ & } p v_1 = RT_1 \Rightarrow \frac{v_2}{v_1} = \frac{T_2}{T_1} \right]$$

6.7.3 Isothermal Process

The 1st law of thermodynamics can, on unit mass basis, be stated , as

$$\delta q = du + \delta w$$

For reversible process, $\delta q_{rev} = pdv$

Since, for ideal gas, we know that

$$u = f(T)$$

And T is constant whereby,

$$u = \text{constant} \Rightarrow du = 0$$

Substituting for du in eqⁿ (i), we get

But

$$pV = RT \Rightarrow p = \frac{RT}{V}$$

So that eqⁿ (ii) becomes,

$$\delta q_{rev} = \frac{RT}{v} dv = RT \frac{dv}{v}$$

$$\text{Or, } \frac{\delta q_{rev}}{T} = R \frac{dv}{v}$$

$$\therefore ds = R \frac{dv}{v}$$

Integrating both sides between states 1 and 2, we get

$$\int_{s_1}^{s_2} ds = \int_{v_1}^{v_2} R \frac{dv}{v} = R \int_{v_1}^{v_2} \frac{dv}{v}$$

$$or, [s]_{v_1}^{v_2} = R \int_{v_1}^{v_2} \frac{dv}{v} = R [\ln v]_{v_1}^{v_2}$$

$$or, s_2 - s_1 = R[\ln \nu_2 - \ln \nu_1]$$

$$\therefore s_2 - s_1 = R \ln \frac{v_2}{v_1} \dots \dots \dots (A)$$

$$s_2 - s_1 = R \ln \frac{p_1}{p_2} \dots \dots \dots (B)$$

$$\left[\because p_2 v_2 = p_1 v_1 = k \Rightarrow \frac{v_2}{v_1} = \frac{p_1}{p_2} \right]$$

6.8 Availability

The availability of a system is defined as the maximum useful work that can be obtained from the system-surrounding combination as the system goes from a given state to the dead state. The dead state is the state at which the system and the surrounding are in equilibrium. Availability is not a property of the system alone as its value depends on p_0 and T_0 also. Let us consider a system S which is at state (p_1, T_1) . Let T be the temperature of a part of this system which interacts with the surrounding atmosphere at (p_0, T_0) where $T < T_0$. The heat δQ_0 flows towards the system from the surrounding.

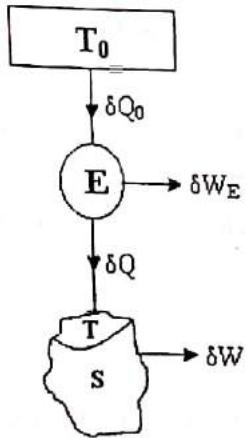


Fig 6.6 System for the study of Availability / Irreversibility

Now, let us place a reversible engine in between that converts part of δQ_0 into work δW_E and rejects δQ to the system. Let the system performs work δW .

Applying 1st law of thermodynamics to the engine E, we have

$$\delta W_E = \delta Q_0 - \delta Q \dots \dots \dots \text{(I)}$$

According to the thermodynamic temperature scale, we have

$$\frac{\delta Q_0}{\delta Q} = \frac{T_0}{T} \Rightarrow \delta Q_0 = T_0 \left(\frac{\delta Q}{T} \right)$$

So that eqⁿ (I) becomes,

$$\delta W_E = T_0 \left(\frac{\delta Q}{T} \right) - \delta Q \dots \dots \dots \text{(II)}$$

The maximum work that can be achieved is the sum of δW_E and δW

$$\therefore \delta W_{\max} = \delta W_E + \delta W \dots \dots \dots \text{(III)}$$

Substituting for δW_E from (II) in (III), we get

$$\begin{aligned} \delta W_{\max} &= T_0 \left(\frac{\delta Q}{T} \right) - \delta Q + \delta W \\ &= T_0 dS + (\delta W - \delta Q) \dots \dots \dots \text{(IV)} \end{aligned}$$

Now, applying 1st law to the system, we get

$$\delta Q = dE + \delta W \Rightarrow \delta W - \delta Q = -dE$$

On substitution reduces (IV) to

$$\delta W_{\max} = T_0 dS - dE = d(T_0 S - E)$$

$$\begin{aligned} \therefore W_{\max} &= \int_1^0 \delta W_{\max} = \int_1^0 d(T_0 S - E) \\ &= [T_0 S - E]_1^0 \\ &= (T_0 S_0 - E_0) - (T_0 S_1 - E_1) \dots \dots \dots \text{(V)} \end{aligned}$$

The system does work against the atmosphere at p_0 when it expands and it is just waste

$$W_{\text{sur}} = p_0 (V_0 - V_1)$$

So that,

$$\begin{aligned}W_{\max, \text{useful}} &= W_{\max} - W_{\text{sur}} \\&= (T_0 S_0 - E_0) - (T_0 S_1 - E_1) - p_0(V_0 - V_1) \\&= T_0 S_0 - E_0 - T_0 S_1 + E_1 - p_0 V_0 + p_0 V_1 \\&= (E_1 + p_0 V_1 - T_0 S_1) - (E_0 + p_0 V_0 - T_0 S_0) \\&= A_1 - A_0\end{aligned}$$

Where, $A = E + p_0 V - T_0 S$ is known as availability function

6.9 Irreversibility

Irreversibility is defined as the difference of maximum work obtainable from system-surrounding interaction and the actual work done by the system and is represented by I .

$$I = W_{\max} - W$$

In differential form, we have

$$\delta I = \delta W_{\max} - \delta W$$

Substituting for δW_{\max} from (III), we get

$$\delta I = \delta W_E + \delta W - \delta W = \delta W_E \dots \dots \dots \text{(VI)}$$

From (I) and (VI), we have

$$\delta I = \delta Q_0 - \delta Q = \delta Q_0 \left(1 - \frac{\delta Q}{\delta Q_0} \right) = \delta Q_0 \left(1 - \frac{T}{T_0} \right)$$

But,

$$T_0 > T \Rightarrow 1 > \frac{T}{T_0} \Rightarrow 1 - \frac{T}{T_0} > 0$$

So that,

$$\delta I > 0$$

When $T \neq T_0$,

$$\delta I = 0$$

Hence, for any process, $\delta I \geq 0$

6.10 Lost Work

For an infinitesimal reversible process by a closed system,

$$\delta Q_R = dU_R + pdV$$

If the process is reversible,

$$\delta Q_I = dU_I + \delta W$$

Since U is a property, $dU_R = dU_I$,

$$\delta Q_R - pdV = \delta Q_I - \delta W$$

$$\therefore \left(\frac{\delta Q}{T} \right)_R = \left(\frac{\delta Q}{T} \right)_I + \frac{pdV - \delta W}{T}$$

The difference $(pdV - \delta W)$ indicates the work that is lost due to irreversibility, and is called the lost work, which approaches zero as the process approaches reversibility as a limit.

Chapter 7

Thermodynamic Cycle

7.1 Vapor power cycles

A power cycle continuously converts heat into work, in which a working fluid repeatedly performs a succession of processes.

In the vapor power cycles, the working fluid, which is water, undergoes a change of phase.

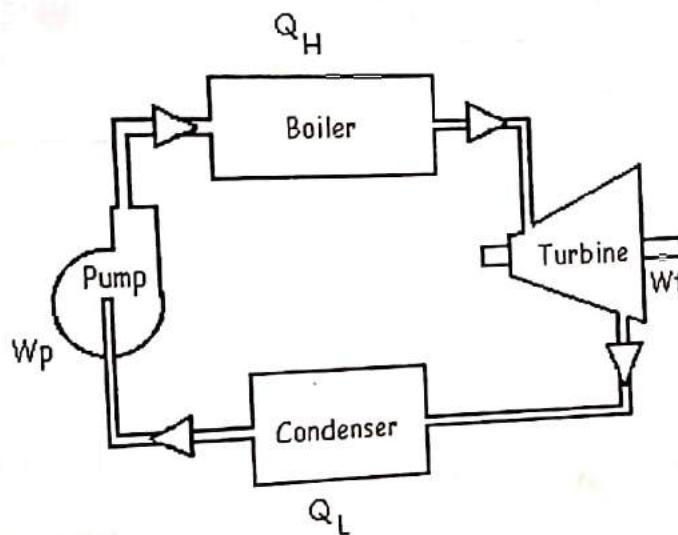


Fig 7.1 A power cycle

Heat is transferred to water in the boiler from an external source to raise steam, the high pressure, high temperature steam leaving the boiler expands in the turbine to produce shaft work, the steam leaving the turbine, condenses into water in the condenser rejecting heat and then the water is pumped back to boiler. In these vapor power cycle a fixed mass of water undergoes a thermodynamic cycle composed of four processes in the four SSSF devices and continuously converts heat to work.

From first law, we can write, for a control mass flowing in this power plant,

$$\oint \delta Q = \oint \delta W$$

or, $Q_H - Q_L = W_T - W_P$

Hence, the efficiency of the vapor power cycle can be given by,

$$\eta_{cycle} = \frac{W_{net}}{Q_H} = \frac{W_T - W_P}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

Thus, the efficiency of any thermodynamic cycle comprised of reversible processes is the function of the heat transfer from and to the cycle.

7.2 Rankine Cycle

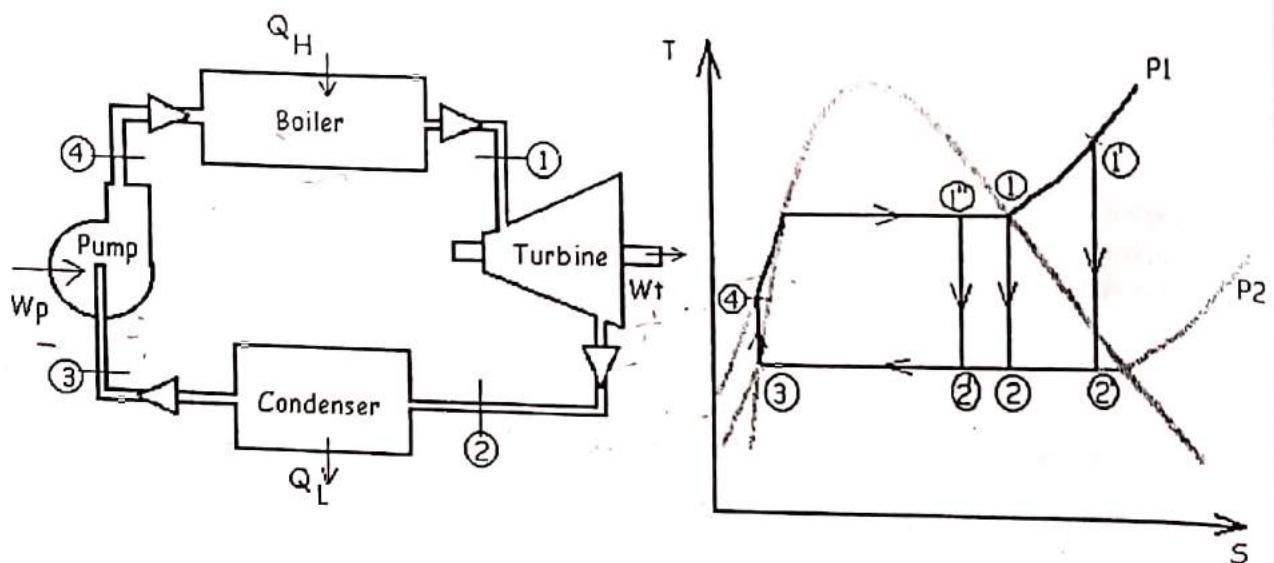


Fig 7.2 A Rankine Cycle

7.2.1 Working Principle

When all four processes in the vapor power cycle are assumed to be ideal, the cycle is called a Rankine cycle, in which the four processes are

Process 1-2: The steam from the boiler undergoes reversible and adiabatic expansion while it passes through the turbine. Pressure of the steam drops from boiler pressure P_1 to condenser pressure P_2 . Expansion work W_T is produced here by the turbine.

Process 2-3: The exhaust steam from the turbine undergoes condensation at constant pressure and temperature. The heat Q_2 is rejected by the steam to change into water.

Process 3-4: Reversible and adiabatic pumping of condensed water at state 3 from condenser pressure P_2 to boiler pressure P_1 takes place.

Process 4-1: The compressed water is first heated sensibly to point 5 and then evaporation takes place in the boiler to state 1.

7.2.2 Efficiency of Rankine Cycle

The steam entering the turbine can be dry (1), wet (1'), or superheated (1'). Since, all the devices used are, SSSF devices for 1 kg of steam.

$$h_i + \frac{V_i^2}{2} + gz_i + q = h_o + \frac{V_o^2}{2} + gz_2 + w_x$$

Assuming no change in KE and PE, we get

Applying eqⁿ(i) to Boiler, Turbine, Condenser and Pump, we have :

For Boiler:

$$h_4 + Q_1 = h_1 \Rightarrow Q_1 = h_1 - h_4$$

for turbine:

$$h_1 = h_2 + W_T \Rightarrow W_T = h_1 - h_2$$

for condenser:

$$h_2 - Q_2 = h_3 \Rightarrow Q_2 = h_2 - h_3$$

for pump:

$$h_3 = h_4 - W_P \Rightarrow W_P = h_4 - h_3$$

hence efficiency of Rankine Cycle is

$$\eta = \frac{W_{net}}{Q_1} = \frac{W_T - W_P}{Q_1} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$$

Usually, the pump work is quite small compared to the turbine work and is sometimes neglected. Then $h_4 - h_3 \approx 0 \Rightarrow h_4 = h_3$, and the approx. cycle efficiency becomes,

$$\eta \equiv \frac{h_1 - h_2}{h_1 - h_3}$$

7.3 Effects of Temperature and Pressure on Rankine cycle efficiency

In order to analyze the effect of Superheat or pressure on the efficiency, let us first understand the mean temperature of heat addition.

For reversible process, we know that,

$$ds = \frac{\delta Q}{T} \Rightarrow \delta Q = T ds$$

Hence, for heating process 4-1', we can write

$$\int_4^1 \delta Q = \int_4^1 T ds \Rightarrow Q_1 = \int_4^1 T_m ds = T_m \int_4^1 ds = T_m (s_1 - s_4)$$

Or, $Q_1 = T_m (s_1 - s_4)$ (i)

Where T_m is the mean temperature of heat addition. It can be taken to be uniform over entire heat addition. So, it has been taken out of the integral sign.

Similarly, for process 2' to 3, we have

$$\int_2^3 -\delta Q = \int_2^3 T ds \Rightarrow Q_3 = - \int_2^3 T_L ds = -T_L \int_2^3 ds = -T_L (s_3 - s_2) = T_L (s_2 - s_3)$$

Or, $Q_3 = T_L (s_2 - s_3)$

$$\therefore \eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_L (s_2 - s_3)}{T_m (s_1 - s_4)}$$

But, $s_1 = s_2$ and $s_3 = s_4$

$$\eta = 1 - \frac{T_L (s_1 - s_4)}{T_m (s_1 - s_4)} = 1 - \frac{T_L}{T_m} \quad \text{.....(i)}$$

Since T_L is constant, $\eta = f(T_m)$ i.e. it increases with T_m

7.3.1 Effect of Superheat or Boiler Temperature

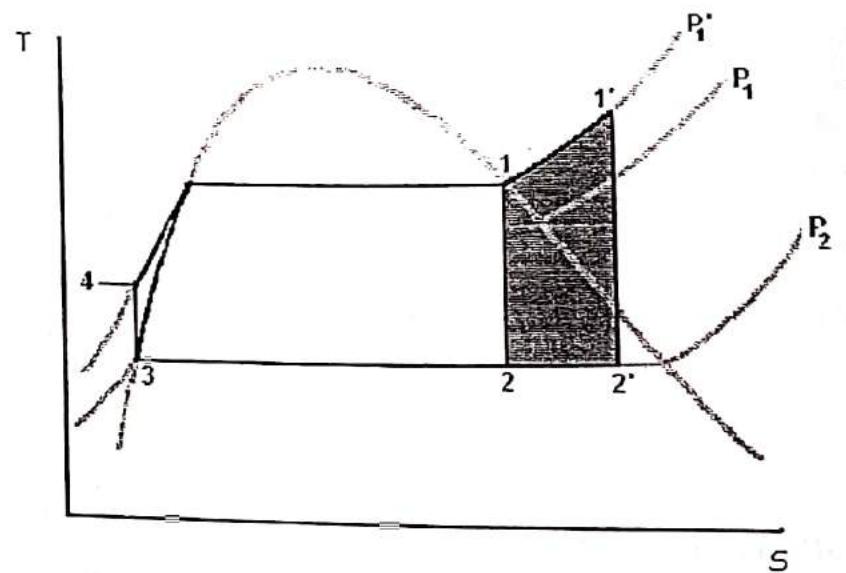


Fig 7.3 Effect of superheat on Rankine cycle efficiency

The above figure shows two Rankine cycles viz. 1-2-3-4-1 and 1'-2'-3-4-1 corresponding to the boilers producing dry saturated and superheated steam respectively. In both the cases, it is obvious that the temperature of heat rejection during the process 2-3 or 2'-3 remain uniform and same. But the temperature of heat addition varies. The mean temperature of heat addition, in the later cycle, is higher while the temperature of heat rejection remains same thereby increasing the efficiency of the cycle as is obvious in eqⁿ(i)

7.3.2 Effect of boiler pressure

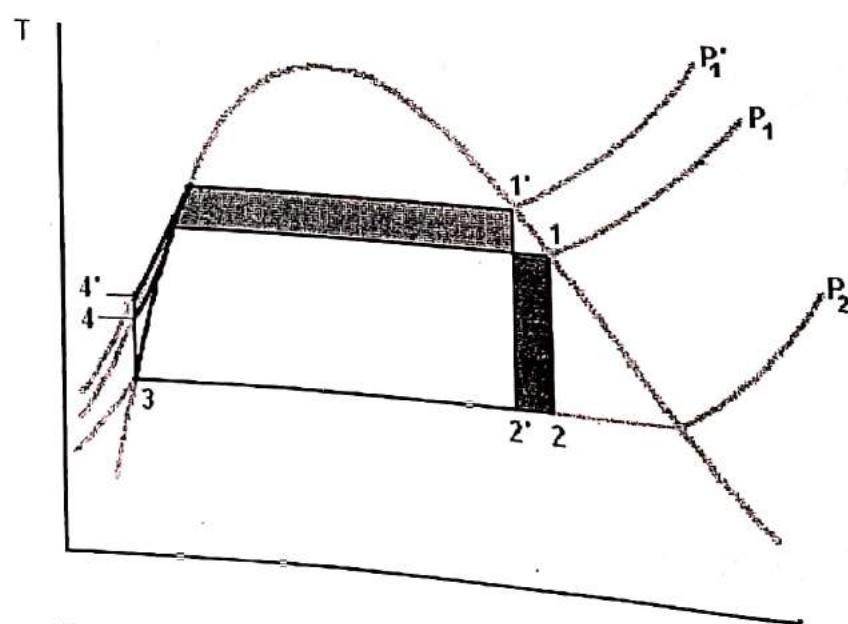


Fig 7.4 Effect of pressure on Rankine cycle efficiency

The above figure shows two Rankine cycles viz. 1-2-3-4-1 and 1'-2'-3-4'-1' corresponding to the boilers producing dry saturated steam at two pressures (P_1 and P_1') respectively. In both the cases, it is obvious that the temperature of heat rejection during the process 2-3 or 2'-3 remain uniform and same. But the temperature of heat addition varies. The mean temperature of heat addition, in the later cycle, is higher while the temperature of heat rejection remains same thereby increasing the efficiency of the cycle as is obvious in eq"(i)

7.4 Air standard cycles

7.4.1 Carnot cycle

It consists of two reversible isotherms and two reversible adiabatics.

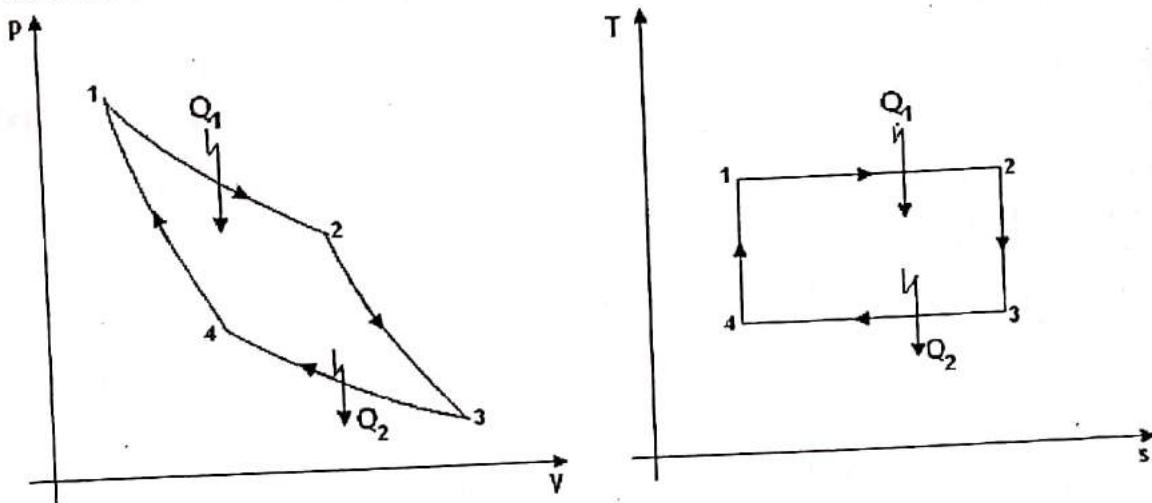


Fig 7.5 Carnot cycle on P-v and T-s diagram

From the 1st law of thermodynamics, we have

From the first law of thermodynamics, $Q_f = U_f - U_i + W_f$, we get

Applying the above law for the two isothermal processes for which $U_f - U_i = 0$, we get

$$Q_2 = -_3 Q_4 = -_3 W_4 = -mRT_3 \ln \frac{V_4}{V_3} = mRT_3 \ln \frac{V_3}{V_4} \dots \dots \dots \text{(ii)}$$

In a closed system undergoing a cycle, we have

For a closed

For the Carnot cycle 1-2-3-4-1, we have

For the Carnot

$$W = Q_1 - Q_2$$

$W_{net} = Q_1 - Q_2$
So, the efficiency of the cycle is

$$\eta = \frac{W_{net}}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

From from (i) and (ii), we get

$$\eta = 1 - \frac{mRT_3 \ln \frac{V_3}{V_4}}{mRT_1 \ln \frac{V_2}{V_1}} = 1 - \frac{T_3}{T_1} \frac{\ln \frac{V_3}{V_4}}{\ln \frac{V_2}{V_1}} \dots \dots \dots \text{(iii)}$$

Now, applying adiabatic gas eqⁿ $TV^{\gamma-1} = C$ to the process 2-3, we get

$$T_3 V_3^{r-1} = T_2 V_2^{r-1} \Rightarrow \frac{T_3}{T_2} = \left(\frac{V_2}{V_3} \right)^{r-1}$$

But, $T_2 = T_1 = T_H$ & $T_3 = T_4 = T$

Similarly, for adiabatic process 4-1, we have

$$T_4 V_4^{r-1} = T_1 V_1^{r-1} \Rightarrow \frac{T_4}{T_1} = \left(\frac{V_1}{V_4} \right)^{r-1}$$

$$[\because T_2 = T_1 = T_H \text{ & } T_4 = T_3 = T_L]$$

From (iv) and (v), we have

$$\left(\frac{V_2}{V_3}\right)^{r-1} = \left(\frac{V_1}{V_4}\right)^{r-1} \Rightarrow \frac{V_2}{V_3} = \frac{V_1}{V_4} \Rightarrow \frac{V_3}{V_4} = \frac{V_2}{V_1} \quad \dots \dots \dots \text{(vi)}$$

$$\eta = 1 - \frac{T_L}{T_H} \frac{\ln \frac{V_2}{V_1}}{\ln \frac{V_2}{V_1}} \quad [\because T_2 = T_1 = T_H \text{ & } T_4 = T_3 = T_L]$$

$$\therefore \eta = 1 - \frac{T_L}{T_H}$$

7.4.2 Brayton Cycle

This cycle is the for a simple gas turbine power plant. This is the same kind of cycle as Rankine but for this case air is used as the working fluid. Gas turbine power plants are used to generate power in places where air and fuel are abundant, not fresh water. Also, the popular jet engines operate in this cycle.

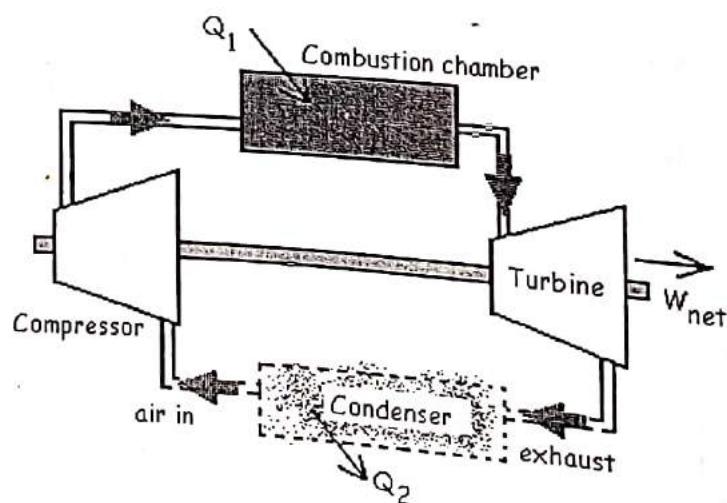


Fig 7.6 A simple gas turbine plant

A gas turbine power plant consists of a compressor where air is first compressed adiabatically. The air then enters the combustion chamber where fuel is burnt at constant pressure and then the products of combustion is expanded in the turbine adiabatically to ambient pressure and thrown out to the surrounding, if it is an open cycle gas turbine, or passed through a condenser for isobaric heat rejection and again to be sent back to the compressor.

If all these processes are done ideally in a closed cycle, then it is called the Brayton Cycle. Therefore, a Brayton cycle consists of two reversible isobars and two reversible adiabatics.

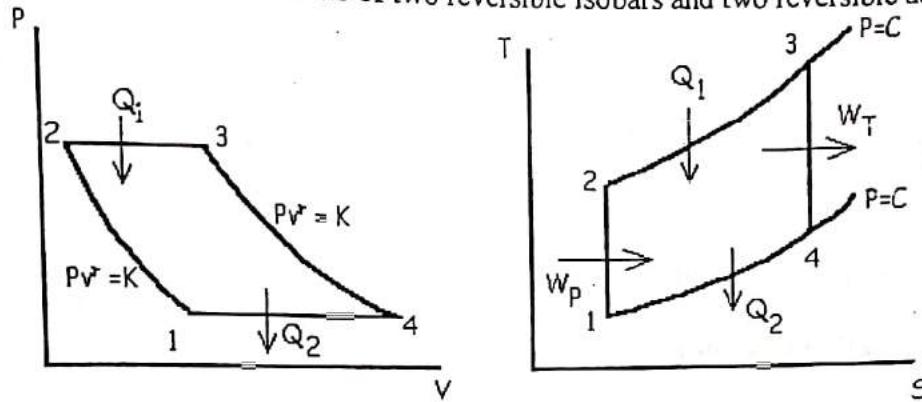


Fig 7.7 A Brayton cycle on P-v and T-s diagram

$$\text{so, heat supplied, } Q_1 = Q_3 = mC_p(T_3 - T_2)$$

$$\text{and heat rejected, } Q_2 = Q_4 = mC_p(T_4 - T_1)$$

Therefore, Cycle efficiency,

$$\begin{aligned}\eta &= 1 - \frac{Q_2}{Q_1} \\ &= 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}\end{aligned}$$

For processes 1-2 and 3-4, we have

$$Pv^r = k \text{ and } Pv = RT$$

Now on solving we get,

$$P \left(\frac{RT}{P} \right)^r = k$$

$$\therefore \frac{T^r}{P^{r-1}} = \text{const}$$

Applying the same to process 1-2, we get

$$\left(\frac{T_2}{T_1} \right)^r = \left(\frac{P_2}{P_1} \right)^{r-1} \dots \dots \dots \text{(i)}$$

Similarly for the process 3-4, we have

$$\left(\frac{T_3}{T_4} \right)^r = \left(\frac{P_3}{P_4} \right)^{r-1}$$

But, $P_2 = P_3$ & $P_4 = P_1$

$$\left(\frac{T_3}{T_4} \right)^r = \left(\frac{P_2}{P_1} \right)^{r-1} \dots \dots \dots \text{(ii)}$$

From (i) and (ii), we have

$$\therefore \frac{T_2}{T_1} = \frac{T_3}{T_4} \Rightarrow \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

$$\therefore \frac{T_4}{T_1} - 1 = \frac{T_3}{T_2} - 1$$

$$\text{or}, \frac{T_4 - T_1}{T_3 - T_2} = \frac{T_1}{T_2} = \left(\frac{P_1}{P_2} \right)^{\frac{r-1}{r}} = \frac{1}{r_p^{\frac{r-1}{r}}}$$

$$\therefore \eta = 1 - \frac{1}{r_p^{\frac{r-1}{r}}}$$

Where, $r_p = P_2 / P_1$ is pressure ratio

7.5 Internal Combustion Engine

7.5.1 Working principle or process of a spark ignition engine

Process 1-2, Intake: The inlet valve is open and the piston moves to the right when the fuel-air mixture is admitted into the cylinder at constant pressure.

Process 2-3, Compression: Both the valves are closed and the piston compresses the combustible mixture to the minimum volume.

Process 3-4, Combustion: The mixture is then ignited by means of a spark causing combustion which increases the temperature and pressure.

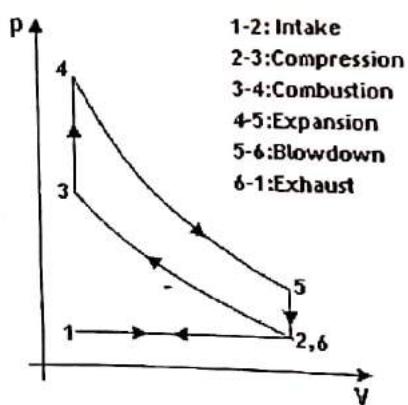


Fig 7.8 A S.I. Engine Indicator Diagram

Process 4-5, Expansion: The products of combustion do work on the piston which moves to the right causing the decrease in the pressure and temperature.

Process 5-6, Blow-down: The exhaust valve opens and the pressure drops to the initial pressure.

Process 6-1, Exhaust: With the exhaust valve open, the piston moves inwards to the initial pressure.

7.5.2 Otto Cycle

The air standard otto cycle is composed of four reversible process which are:

Process 1-2: Isentropic compression of air raising its pressure and temperature.

Process 2-3: Addition of heat at constant volume causing rise in pressure and temperature of air.

Process 3-4: Isentropic expansion of air causing considerable decrease in pressure and temperature.

Process 4-1: Rejection of heat at constant volume causing further drop in pressure and temperature of air restoring initial state.

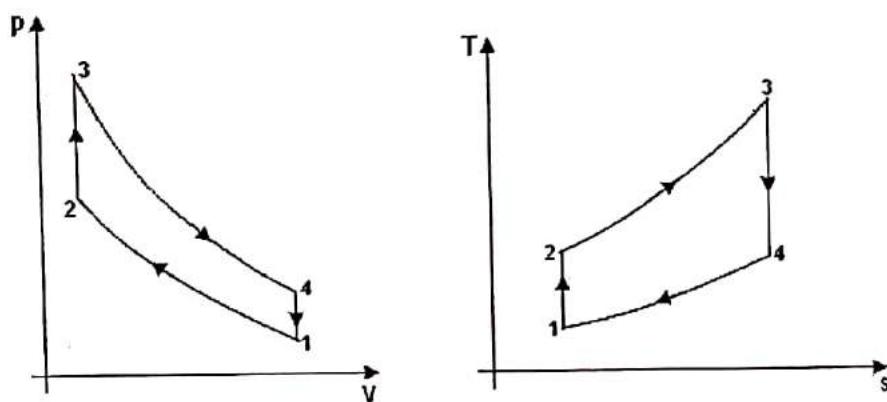


Fig 7.9 Otto cycle on P-v and T-s diagram

7.5.2.1 Efficiency of Otto cycle

Let m be the mass of air undergoing the Otto cycle.

Heat supplied,

$$Q_1 = Q_3 = mc_v(T_3 - T_2)$$

Heat rejected,

$$Q_2 = Q_4 = mc_v(T_4 - T_1)$$

∴ Efficiency,

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{mc_v(T_4 - T_1)}{mc_v(T_3 - T_2)}$$

$$\therefore \eta = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} \dots \dots \dots (1)$$

For the adiabatic process; 1-2,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2} \right)^{r-1} \dots \dots \dots (i) \quad (\because T v^{r-1} = K)$$

For the adiabatic process; 3-4,

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3} \right)^{r-1}$$

But, from diagram, $v_4 = v_1$ & $v_2 = v_3$

$$\therefore \frac{T_3}{T_4} = \left(\frac{v_1}{v_2} \right)^{r-1} \dots\dots\dots(ii)$$

From (i) & (ii), we have

$$\therefore \frac{T_2}{T_1} = \frac{T_3}{T_4}$$

$$\text{or}, \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

$$\text{or}, \frac{T_4}{T_1} - 1 = \frac{T_3}{T_2} - 1$$

$$\text{or}, \frac{T_4 - T_1}{T_1} = \frac{T_3 - T_2}{T_2}$$

$$\text{or}, \frac{T_4 - T_1}{T_3 - T_2} = \frac{T_1}{T_2} = \left(\frac{v_2}{v_1} \right)^{r-1}$$

$$\therefore \eta = 1 - \left(\frac{v_2}{v_1} \right)^{r-1} = 1 - \frac{1}{r_k^{r-1}}$$

Where, r_k is the compression ratio and $r_k = V_1/V_2 = v_1/v_2$.

7.5.3 Working Process for Compression Ignition (C.I.) Engine

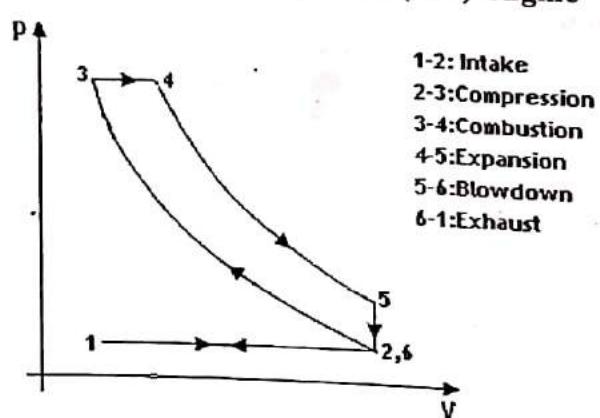


Fig 7.10 C.I. Engine Indicator Diagram

Process 1-2, Intake: The air valve is open. The piston moves out admitting air into the cylinder at constant pressure.

Process 2-3, Compression: The air is then compressed by the piston to the minimum volume with all the valves closed.

Process 3-4, Fuel injection and combustion: The fuel valve is open, fuel is sprayed into the hot air and combustion takes place at constant pressure.

Process 4-5, Expansion: The combustion products expand doing work on the piston which moves out to the maximum volume.

Process 5-6, Blow-down: The exhaust valve opens and the pressure drops to the initial pressure.

Process 6-1, Exhaust: With the exhaust valve open, the piston moves towards the cylinder cover driving away the combustion products from the cylinder at constant pressure.

7.5.4 Diesel Cycle

The air standard Diesel Cycle is composed of four reversible processes as stated below:

Process 1-2: Isentropic compression of air raising its pressure and temperature.

Process 2-3: Reversible isobaric heat addition to the air from an external source.

Process 3-4: Isentropic expansion of air causing considerable decrease in pressure and temperature.

Process 4-1: Rejection of heat at constant volume causing further drop in pressure and temperature of air restoring initial state.

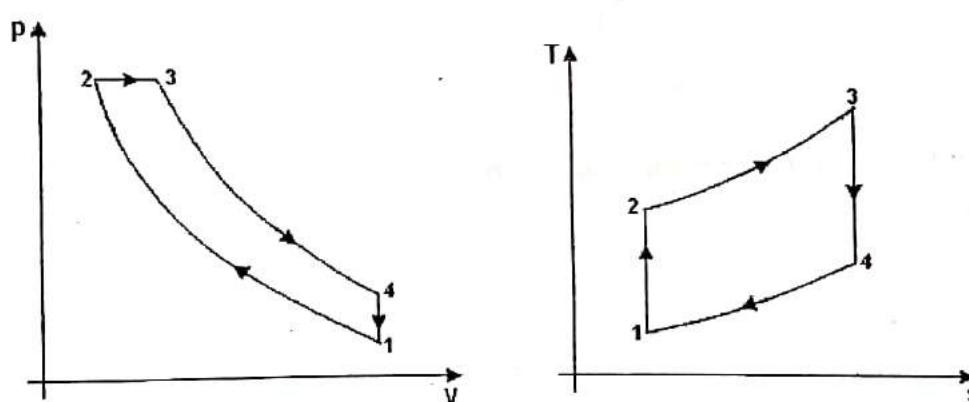


Fig 7.11 Diesel cycle on P-v and T-s diagram

7.5.4.1 Efficiency of Diesel Cycle

For m kg of air in the cylinder,

Heat supplied,

$$Q_1 = Q_2 = mc_p(T_3 - T_2)$$

Heat rejected,

$$Q_2 = Q_4 = mc_v(T_4 - T_1)$$

\therefore Efficiency,

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{mc_v(T_4 - T_1)}{mc_p(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{\frac{c_p}{c_v}(T_3 - T_2)}$$

$$\therefore \eta = 1 - \frac{(T_4 - T_1)}{\gamma(T_3 - T_2)} \quad \dots \dots \dots \text{(I)} \left(\because \gamma = \frac{c_p}{c_v} \right)$$

The efficiency may be expressed in terms of any of the following three ratios:

$$\text{compression ratio, } r_k = \frac{v_1}{v_2}$$

$$\text{Expansion ratio, } r_e = \frac{v_4}{v_3}$$

$$\text{cut off ratio, } r_c = \frac{v_3}{v_2}$$

Now, we see that

$$r_e r_c = \frac{v_4}{v_3} \times \frac{v_3}{v_2} = \frac{v_4}{v_2}$$

But, $v_4 = v_1$

$$\therefore r_e r_c = \frac{v_1}{v_2} = r_k$$

Applying $Tv^{\gamma-1} = \text{const}$ to the process 3-4, we have

$$T_4 v_4^{\gamma-1} = T_3 v_3^{\gamma-1}$$

$$\frac{T_4}{T_3} = \left(\frac{v_3}{v_4} \right)^{\gamma-1} = \left(\frac{1}{v_4/v_3} \right)^{\gamma-1} = \frac{1}{r_e^{\gamma-1}} = \frac{1}{(r_k/r_c)^{\gamma-1}}$$

$$\therefore T_4 = T_3 \frac{r_c^{\gamma-1}}{r_k^{\gamma-1}}$$

Applying $Pv = RT$ to the process 2-3, We have

$$\frac{RT_2}{RT_3} = \frac{P_2 v_2}{P_3 v_3} = \frac{P_2 v_2}{P_2 v_3} = \frac{v_2}{v_3}$$

$$\text{or, } \frac{T_2}{T_3} = \frac{1}{v_3/v_2} = \frac{1}{r_c}$$

$$\therefore T_2 = T_3 \cdot \frac{1}{r_c}$$

Applying $Tv^{\gamma-1} = \text{const}$ to the process 1-2, we have

$$T_1 v_1^{\gamma-1} = T_2 v_2^{\gamma-1}$$

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1} \right)^{\gamma-1} = \frac{1}{(v_1/v_2)^{\gamma-1}} = \frac{1}{r_k^{\gamma-1}}$$

$$\therefore T_1 = T_2 \cdot \frac{1}{r_k^{\gamma-1}} = T_3 \cdot \frac{1}{r_c} \cdot \frac{1}{r_k^{\gamma-1}}$$

Now, substituting the value of T_4, T_2, T_1 in equation.1, we get

$$\eta = 1 - \frac{1}{\gamma} \left(\frac{T_3 \left(\frac{r_c}{r_k} \right)^{\gamma-1} - T_3 \cdot \frac{1}{r_c} \cdot \frac{1}{r_k^{\gamma-1}}}{T_3 - T_3 \cdot \frac{1}{r_c}} \right)$$

$$\text{or, } \eta_{\text{diesel}} = 1 - \frac{1}{\gamma \cdot r_k^{\gamma-1}} \left(\frac{r_c^\gamma - 1}{r_c - 1} \right)$$

7.5.5 Comparison between Otto and Diesel Cycles

Fig 7.12

1. On the basis of same compression ratio and heat rejection

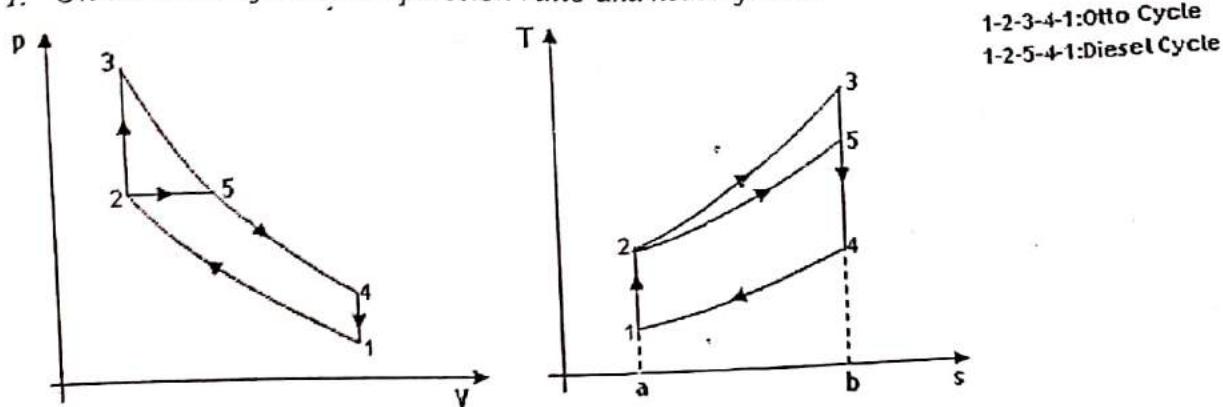


Fig 7.12 Comparison of Otto and Diesel cycles for same compression ratio

For the same Q_2 , the higher the value of Q_1 , the higher will be the cycle efficiency. We know that the area under T-s curve gives the amount of heat interaction. Thus, in the T-s diagram, area a-2-3-b and area a-2-5-b represent respective value of Q_1 for Otto cycle and Diesel cycle and Obviously, from figure , we have

$$\text{area}(a-2-3-b) > \text{area}(a-2-5-b)$$

$$\text{i.e. } Q_{1,\text{Otto}} > Q_{1,\text{Diesel}}$$

But, Process 4-1 is same to both the cycles and thus Q_2 is same.

$$\therefore \eta_{\text{Otto}} > \eta_{\text{Diesel}}$$

2. On the basis of same maximum Pressure and Temperature

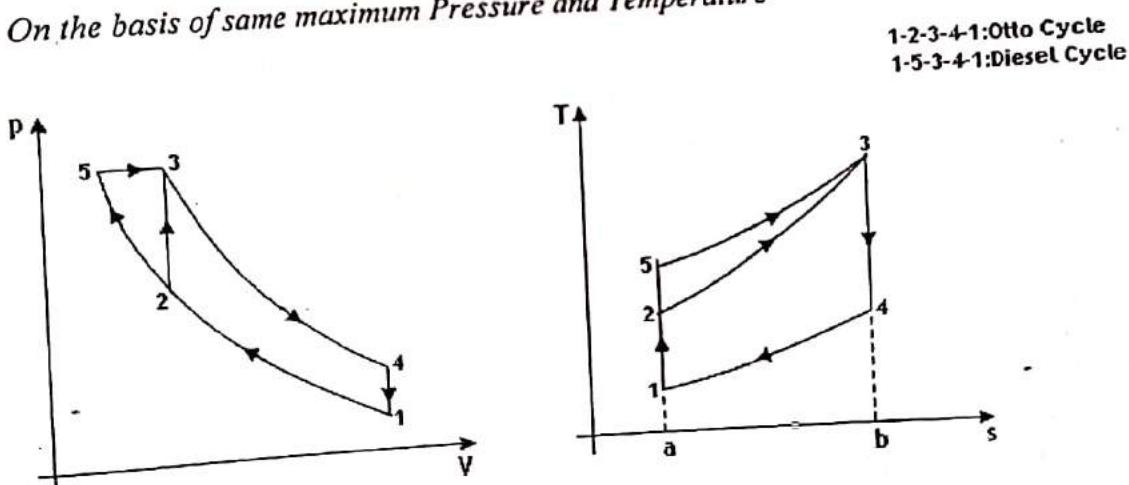


Fig 7.13 Comparison of Otto and Diesel cycles for same maximum pressure and temperature

For the same Q_2 , the higher the value of Q_1 , the higher will be the cycle efficiency. We know that the area under T-s curve gives the amount of heat interaction. Thus, in the T-s diagram, area a-2-3-b and area a-2-5-b represent respective value of Q_1 for Otto cycle and Diesel cycle and Obviously, from figure , we have

$$\text{area}(a-2-3-b) < \text{area}(a-2-5-b)$$

$$\text{i.e. } Q_{1,\text{Diesel}} > Q_{1,\text{Otto}}$$

But, Process 4-1 is same to both the cycles and thus Q_2 is same.

$$\therefore \eta_{\text{Diesel}} > \eta_{\text{Otto}}$$

Solved Examples

1. Steam at 20 bar, 360°C is expanded in a steam turbine to 0.08 bar. It then enters a condenser, where it is condensed to saturated liquid water. The pump feeds back the water into the boiler. (a) Assuming ideal processes, find the net work per kg and the cycle efficiency (b) if the turbine and the pump have each 80% efficiency, find the percentage reduction in the net work and cycle efficiency.

Solution:

At 20 bar

We have from steam table,
 $t_s = 212.42^{\circ}\text{C}$

But at point 1, we have
 $t_1 = 360^{\circ}\text{C}$

Since $t_1 > t_s$, the steam at state 1 must be superheated.

From steam table for superheated steam, we have

$$h_1 = 3159.3 \text{ kJ/kg} \quad \text{and} \quad s_1 = 6.9917 \text{ kJ/kg}$$

At $p_2 = 0.08 \text{ bar}$

From steam table, we have

$$h_{f|p_2} = 173.88 \text{ kJ/kg} ; h_{fg|p_2} = 2403.1 \text{ kJ/kg} ; s_{f|p_2} = 0.5926 \text{ kJ/kgK}$$

$$s_{fg|p_2} = 7.6361 \text{ kJ/kgK} ; s_{g|p_2} = 8.2287 \text{ kJ/kg} ; v_{f|p_2} = 0.001008 \text{ m}^3 / \text{kg}$$

State 3 must be for saturated water at 0.08 bar.

$$\therefore h_3 = h_{f|p_2} = 173.88 \text{ kJ/kg} ; s_3 = s_{f|p_2} = 0.5926 \text{ kJ/kgK}$$

Process 1-2 is an isentropic process

$$s_2 = s_1 = 6.9917 \text{ kJ/kgK}$$

Since $s_2 < s_{g|p_2}$, the steam at state 2 must be wet.

For wet steam, we know that

$$s_2 = s_{f|p_2} + X_2 s_{fg|p_2}$$

$$\text{Or, } 6.9917 = 0.5926 + X_2 (7.6361)$$

$$\therefore X_2 = \frac{6.9917 - 0.5926}{7.6361} = \frac{6.3991}{7.6361} = 0.838$$

$$\text{So, } h_2 = h_{f|p_2} + X_2 h_{fg|p_2} = 173.88 + (0.838)(2403.1) = 2187.68 \text{ kJ/kg}$$

$$(a) W_p = h_4 - h_3 = v_{f|p_2} (p_1 - p_2) = 0.001008 \times (20 - 0.08) \times 10^5 = 2007.936 \text{ J/kg} = 2.008 \text{ kJ/kg}$$

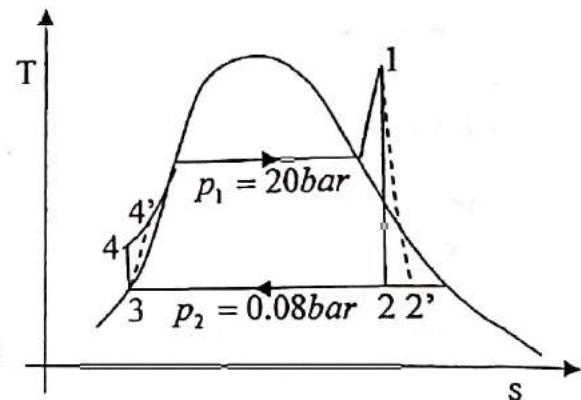
$$\therefore h_4 = h_3 + 2.008 \text{ kJ/kg.} = 173.88 + 2.008 = 175.89 \text{ kJ/kg}$$

$$W_r = h_1 - h_2 = 3159.3 - 2187.68 = 971.62 \text{ kJ/kg}$$

$$\therefore W_{net} = W_r - W_p = 971.62 - 2.008 = 969.61 \text{ kJ/kg}$$

$$Q_1 = h_1 - h_4 = 3159.3 - 175.89 = 2983.41 \text{ kJ/kg}$$

$$\therefore \eta_{cycle} = \frac{W_{net}}{Q_1} = \frac{969.61}{2983.41} = 0.325 = 32.5\%$$



(b) If $\eta_p = 80\%$, and $\eta_T = 80\%$, we have

$$W_p = \frac{2.008}{0.8} = 2.51 \text{ kJ/kg}$$

$$W_T = 0.8 \times 971.62 = 777.3 \text{ kJ/kg}$$

$$W_{net} = W_T - W_p = 777.3 - 2.51 = 774.8 \text{ kJ/kg}$$

$$\text{So, \% reduction in work output} = \frac{969.61 - 774.8}{969.61} \times 100\% = 20.1\%$$

Also,

$$W_p = h_4' - h_3 = 2.51 \text{ kJ/kg}$$

$$\therefore h_4' = h_3 + 2.51 = 173.88 + 2.51 = 176.39 \text{ kJ/kg}$$

$$\therefore Q_1 = h_1 - h_4' = 3159.3 - 176.39 = 2982.91 \text{ kJ/kg}$$

$$\therefore \eta'_{cycle} = \frac{774.8}{2982.91} = 0.2597 = 25.97\%$$

$$\therefore \% \text{ reduction in cycle efficiency} = \frac{0.325 - 0.2597}{0.325} \times 100\% = 20.1\%$$

2. In a gas turbine plant working on the Brayton cycle the air at the inlet is at 27°C , 0.1 MPa. The pressure ratio is 6.25 and the maximum temperature is 800°C . The turbine and compressor efficiencies are each 80%. Find (a) the compressor work per kg of air (b) the turbine work per kg of air (c) the heat supplied per kg of air (d) the cycle efficiency , and (e) the turbine exhaust temperature.

Solution: Given data

$$T_1 = 27^{\circ}\text{C} = 300 \text{ K}$$

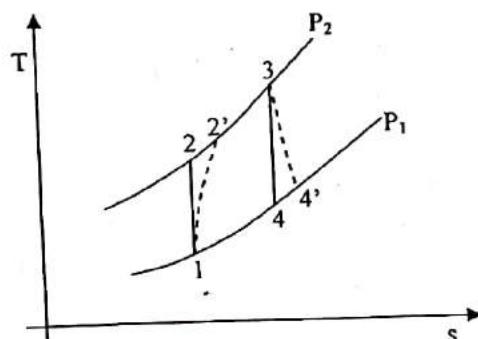
$$p_1 = 0.1 \text{ MPa}$$

$$\underline{p_2 = 6.25}$$

$$\underline{p_1}$$

$$T_3 = 800^{\circ}\text{C} = 1073 \text{ K}$$

$$\eta_c = \eta_T = \text{isentropic efficiency} = 80\%$$



For the process 1-2, we have

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{r-1}{r}}$$

$$\Rightarrow T_2 = T_1 \times \left(\frac{p_2}{p_1} \right)^{\frac{r-1}{r}} = 300 \times (6.25)^{\frac{1.4-1}{1.4}} = 506.7 \text{ K}$$

Also we know that,

$$\eta_c = \frac{h_2 - h_1}{h_{2'} - h_1} = \frac{c_p(T_2 - T_1)}{c_p(T_{2'} - T_1)} = \frac{T_2 - T_1}{T_{2'} - T_1}$$

$$\Rightarrow T_2 - T_1 = \frac{T_2 - T_1}{\eta_c} = \frac{506.7 - 300}{0.8} = 258.4 \text{ K}$$

$$\therefore T_2 = T_1 + 258.4 = 300 + 258.4 = 558.4 \text{ K}$$

For the process 3-4, we have

$$\frac{T_4}{T_3} = \left(\frac{p_1}{p_2} \right)^{\frac{r-1}{r}}$$

$$\Rightarrow T_4 = \frac{T_3}{(p_2/p_1)^{\frac{r-1}{r}}} = \frac{1073}{(6.25)^{\frac{1.4-1}{1.4}}} = 635.3 \text{ K}$$

Now,

$$\eta_r = \frac{h_3 - h_{4'}}{h_3 - h_4} = \frac{c_p(T_3 - T_{4'})}{c_p(T_3 - T_4)} = \frac{T_3 - T_{4'}}{T_3 - T_4}$$

$$\Rightarrow T_3 - T_{4'} = \eta_r \times (T_3 - T_4) = 0.8 \times (1073 - 635.3) = 350.2 \text{ K}$$

$$\therefore T_{4'} = T_3 - 350.2 = 1073 - 350.2 = 722.84 \text{ K}$$

(a) Compressor work, $W_c = h_{2'} - h_1 = c_p(T_{2'} - T_1) = 1.005 = 259.7 \text{ kJ/kg}$

(b) Turbine work, $W_T = h_3 - h_{4'} = c_p(T_3 - T_{4'}) = 1.005(350.2) = 351.95 \text{ kJ/kg}$

(c) Heat supplied $Q_1 = h_3 - h_2 = c_p(T_3 - T_{2'}) = 1.005(1073 - 558.4) = 517.17 \text{ kJ/kg}$

(d) Turbine exhaust temperature, $T_{4'} = 722.84 \text{ K}$

(e) Cycle efficiency, $\eta = \frac{W_{net}}{\text{Heat supplied}} = \frac{W_T - W_c}{Q_1} = \frac{351.95 - 259.7}{517.17} = 0.1784 = 17.84\%$

3. An engine working on the Otto cycle is supplied with air at 0.1 MPa, 35°C. The compression ratio is 8. Heat supplied is 2100 kJ/kg. Calculate the maximum pressure and temperature of the cycle, the cycle efficiency, and the mean effective pressure.

Solution:

Referring to the figure given below, we get

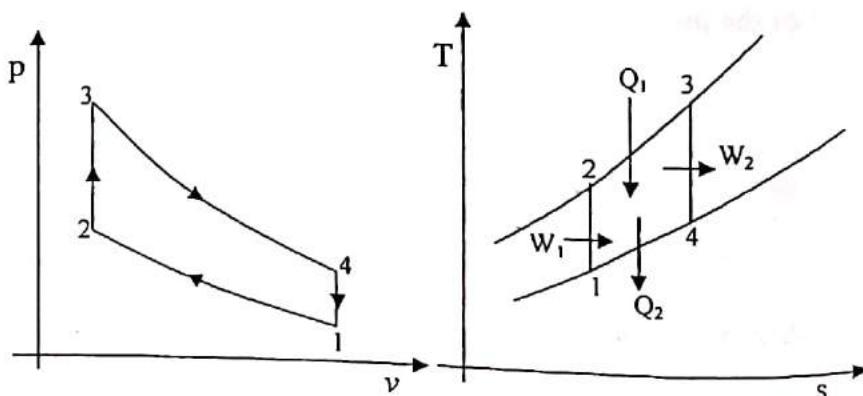
Given:

$$T_1 = 273 + 35 = 308 \text{ K}$$

$$p_1 = 0.1 \text{ MPa} = 100 \text{ kPa}$$

$$Q_1 = 2100 \text{ kJ/kg}$$

$$r_k = 8$$



$$\therefore \eta_{O_{max}} = 1 - \frac{1}{r_k^{r-1}}$$

$$\therefore \eta_{O_{max}} = 1 - \frac{1}{8^{1.4-1}} = 0.565 = 56.5\%$$

Applying eqⁿ of state to the process 1-2, we get

$$p_1 v_1 = RT_1 \Rightarrow v_1 = \frac{RT_1}{p_1} = \frac{0.287 \times 308}{100} = 0.884 \text{ m}^3 / \text{kg}$$

We know that,

$$\frac{v_1}{v_2} = r_k = 8$$

$$\therefore v_2 = \frac{v_1}{8} = \frac{0.884}{8} = 0.11 \text{ m}^3 / \text{kg}$$

Now applying adiabatic gas eqⁿ $Tv^{r-1} = c$ to the process 1-2, we get

$$T_2 v_2^{r-1} = T_1 v_1^{r-1} \Rightarrow \frac{T_2}{T_1} = \left(\frac{v_1}{v_2} \right)^{r-1} = r_k^{r-1}$$

$$\therefore T_2 = T_1 \times r_k^{r-1} = 308 \times 8^{1.4-1} = 708.4 \text{ K}$$

$$\therefore Q_1 = c_v(T_3 - T_2)$$

$$\Rightarrow T_3 - T_2 = \frac{Q_1}{c_v} = \frac{2100}{0.718} = 2924.79$$

$$\therefore T_3 = T_{\max} = T_2 + 2924.79 = 708.4 + 2924.79 = 3633.19 \text{ K}$$

Again applying $pv^r = c$ to the process 1-2, we get

$$p_2 v_2^r = p_1 v_1^r \Rightarrow \frac{p_2}{p_1} = \left(\frac{v_1}{v_2} \right)^r = r_k^r$$

$$\therefore p_2 = p_1 \times r_k^r = 0.1 \times 8^{1.4} = 1.873 \text{ MPa}$$

Now, applying the eqⁿ of state to the process 2-3, we get

$$\frac{p_3 v_3}{T_3} = \frac{p_2 v_2}{T_2} \Rightarrow \frac{p_3}{T_3} = \frac{p_2}{T_2}$$

$$\therefore p_3 = p_{\max} = p_2 \times \frac{T_3}{T_2} = 1.873 \times \frac{3633.19}{708.4} = 9.606 \text{ MPa}$$

$$\therefore \eta_{O_{max}} = \frac{W_{net}}{Q_1}$$

$$\therefore W_{net} = Q_1 \times \eta_{O_{max}} = 2100 \times 0.565 = 1186.5 \text{ kJ/kg}$$

Mean effective pressure, p_m is given by

$$p_m = \frac{W_{net}}{v_1 - v_2} = \frac{1186.5}{0.884 - 0.11} = 1532.95 \text{ kPa} = 1.533 \text{ MPa}$$

4. A Diesel engine has a compression ratio of 14 and cut-off takes place at 6% of the stroke. Find the air standard efficiency.

Solution:

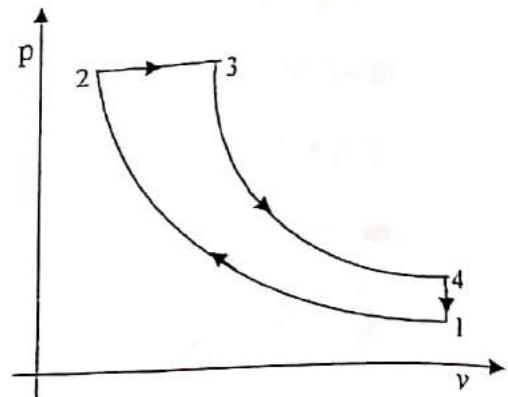
Referring to the adjoining figure, we have

$$r_k = \frac{v_1}{v_2} = 14 \Rightarrow v_1 = 14v_2$$

Also by question, we have

$$\begin{aligned} v_3 - v_2 &= 6\% \text{ of } (v_1 - v_2) \\ &= 0.06(14v_2 - v_2) \\ &= 0.06 \times 13v_2 \\ &= 0.78v_2 \end{aligned}$$

$$\therefore v_3 = v_2 + 0.78v_2 = 1.78v_2$$

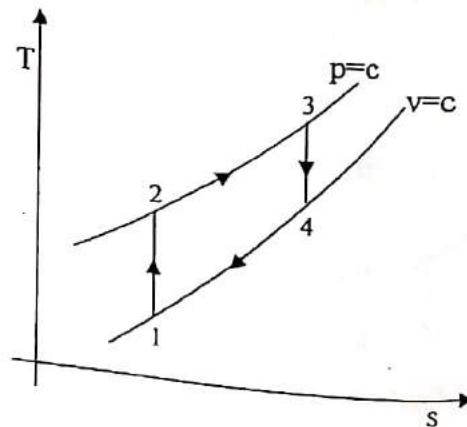
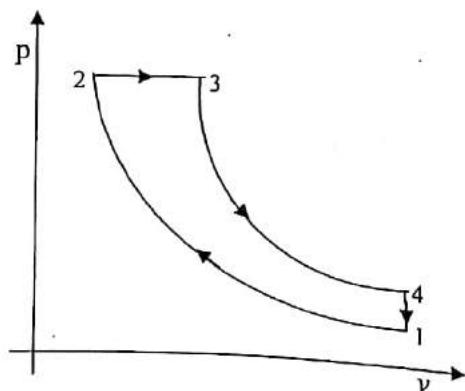


$$\therefore \text{Cut-off ratio, } r_c = \frac{v_3}{v_2} = 1.78$$

And efficiency of this cycle η_{Diesel} is given by

$$\begin{aligned} \eta_{Diesel} &= 1 - \frac{1}{\gamma} \frac{1}{r_k^{\gamma-1}} \frac{r_c^\gamma - 1}{r_c - 1} \\ &= 1 - \frac{1}{1.4} \frac{1}{(14)^{1.4-1}} \frac{(1.78)^{1.4} - 1}{1.78 - 1} \\ &= 1 - 0.248 \times \frac{1.24}{0.78} \\ &= 0.605 = 60.5\% \end{aligned}$$

5. In an air standard Diesel cycle, the compression ratio is 16, and at the beginning of isentropic compression, the temperature is 15°C and the pressure is 0.1 MPa. Heat is added until the temperature at the end of the constant pressure process is 1480°C . Calculate (a) the cut-off ratio, (b) the heat supplied per kg of air, (c) the cycle efficiency,



Solution:

Referring to the above figure, we have

$$r_t = \frac{v_1}{v_2} = 16$$

$$T_1 = 273 + 15 = 288 \text{ K}$$

$$p_1 = 0.1 \text{ MPa} = 100 \text{ kPa}$$

$$T_3 = 1480 + 273 = 1735 \text{ K}$$

Applying the adiabatic gas eqⁿ $Tv^{r-1} = c$ to the process 1-2, we get

$$T_2 v_2^{r-1} = T_1 v_1^{r-1} \Rightarrow \frac{T_2}{T_1} = \left(\frac{v_1}{v_2} \right)^{r-1}$$

$$\therefore T_2 = T_1 \times \left(\frac{v_1}{v_2} \right)^{r-1} = 288 \times (16)^{1.4-1} = 873 \text{ K}$$

Now applying equation of state to the process 1-2 (*for which $p_2 = p_3 = p$*), we get

$$\frac{p_3 v_3}{T_3} = \frac{p_2 v_2}{T_2} \Rightarrow \frac{v_3}{v_2} = \frac{p_2}{p_3} \frac{T_3}{T_2} = \frac{p T_3}{P T_2} = \frac{T_3}{T_2}$$

Hence,

$$(a) \text{ Cut-off ratio, } r_c = \frac{v_3}{v_2} = \frac{T_3}{T_2} = \frac{1753}{273} = 2.01$$

$$(b) \text{ Heat supplied, } Q_1 = c_p (T_3 - T_2) = 1.005(1753 - 873) = 884.4 \text{ kJ/kg}$$

Now applying the adiabatic gas eqⁿ $Tv^{r-1} = c$ to the process 3-4, we get

$$T_4 v_4^{r-1} = T_3 v_3^{r-1} \Rightarrow \frac{T_4}{T_3} = \left(\frac{v_3}{v_4} \right)^{r-1} = \left(\frac{\frac{v_3}{v_2}}{\frac{v_4}{v_2}} \right)^{r-1} = \left(\frac{r_c}{r_k} \right)^{r-1}$$

$$\therefore T_4 = T_3 \times \left(\frac{r_c}{r_k} \right)^{r-1} = 1753 \times \left(\frac{2.01}{16} \right)^{1.4-1} = 766 \text{ K}$$

$$\text{So, Heat rejected, } Q_2 = c_p (T_4 - T_1) = 1.005(766 - 288) = 343.2 \text{ kJ/kg}$$

$$(c) \text{ Cycle efficiency, } \eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{343.2}{884.4} = 0.612 = 61.2\%$$

$$\text{Net work, } W_{net} = \eta \times Q_1 = 0.612 \times 884.4 = 541.3 \text{ kJ/kg}$$

$$v_1 = \frac{RT_1}{p_1} = \frac{0.287 \times 288}{100} = 0.827 \text{ m}^3/\text{kg}$$

$$\therefore \frac{v_1}{v_2} = 16 \Rightarrow v_2 = \frac{v_1}{16} = \frac{0.827}{16} = 0.052 \text{ m}^3/\text{kg}$$

$$(d) \text{ m.e.p.} = \frac{W_{net}}{v_1 - v_2} = \frac{541.3}{0.775} = 698.45 \text{ kPa}$$

Problems

1. Dry and saturated steam at pressure of 11 bar is supplied to a turbine and expanded isentropically to a pressure 0.07 bar. Calculate : (a) heat supplied (b) change of entropy during heat rejection (c) heat rejected (d) theoretical thermal efficiency (e) overall thermal efficiency if $\eta_t = 80\%$, $\eta_m = 95\%$, $\eta_g = 96\%$, $\eta_b = 90\%$
2. A steam turbine receives steam at a pressure of 20 bar and superheated to 88.6°C . The exhaust pressure is 0.07 bar and the expansion of steam takes place isentropically. Using steam table, calculate the followings:
 - a) heat supplied, assuming that the feed pump supplies water to the boiler at 20 bar
 - b) heat rejected (c) net work done (d) work done by the turbine (e) thermal efficiency
3. A steam turbine plant is supplied with steam at pressure of 17 bar and superheated to 100°C . The exhaust pressure is 0.06 bar. The temperature of the condensate in the hotwell is actually 33°C ($v_w=0.001 \text{ m}^3/\text{kg}$) if the measured steam condensate is 5 kg/kWh, and if the boiler efficiency is 82% and generator efficiency is 96%, what is the thermal efficiency of the plant?
4. A Rankine cycle has steam entering at 140 bar and 550°C . If the exhaust pressure is 0.06 bar and all the processes are reversible, (a) find thermal efficiency of the cycle (b) if the efficiency of turbine is 80% and the efficiency of the pump is 50%, what is the thermal efficiency?
5. A steam turbine working on Rankine cycle is supplied with dry saturated steam at 25 bar and the exhaust takes place at 0.2 bar. For a steam flow rate of 10 kg/s, determine: (a) quality steam at the end of expansion, (b) turbine shaft work, (c) power required to drive the pump, (d) work ratio , (e) Rankine efficiency , and (f) heat flow in the condenser.
6. A cyclic steam power plant is to be designed for a steam temperature at turbine inlet of 360°C and an exhaust pressure of 0.08 bar. After isentropic expansion of steam in the turbine, the moisture content at the turbine exhaust is not to exceed 15%. Determine the greatest allowable steam pressure at the turbine inlet, and calculate the Rankine cycle efficiency for these steam conditions. Estimate also the mean temperature of heat addition.
7. In an ideal Brayton cycle, air from the atmosphere at 1 atm, 300 K is compressed to 6 atm and the maximum temperature is limited to 1100 K by using a large air-fuel ratio. If the heat supply is 100 MW, find (a) the thermal efficiency of the cycle (b) work ratio (c) power output.
8. In a gas turbine plant working on the Brayton cycle, the air at the inlet is at 27°C , 0.1 MPa. The pressure ratio is 6.25 and the maximum temperature is 800°C . The turbine and compressor efficiencies are each 80%. Find (a) the compressor work per kg of air, (b) the turbine work per kg of air, (c) the heat supplied per kg of air, (d) the cycle efficiency, and (e) the turbine exhaust temperature.

9. A simple gas turbine power plant based on open Brayton cycle takes air at 1 bar and 15°C and compresses the air to 5 bar. This is then heated in a combustion chamber to a temperature of 800°C . The hot air expands in the turbine to 1 bar. Find (a) the power developed (b) thermal efficiency of the plant.
10. In a simple Rankine cycle steam leaves the boiler and enters the turbine at 4 MPa and 400°C . The condenser pressure is 19 kPa. Determine the cycle efficiency and the Carnot cycle efficiency for the same temperature limits.
11. Compare the changes in the efficiency of an Otto cycle when the compression ratio changes from 4 to 5. Take $\gamma=1.4$ for air.
12. The pressure limits in an Otto cycle are 100 kN/m^2 respectively. The compression ratio is 4. Calculate (a) thermal efficiency and (b) mean effective pressure. Take $\gamma=1.4$ for air.
13. An air standard Otto cycle has a compression ratio of 8. At the start of the compression process, the temperature is 26°C and the pressure is 1 bar. If the maximum temperature of the cycle is 1080°C , calculate (a) the heat supplied per kg of air (b) the net work done per kg of air and (c) the thermal efficiency of the cycle.
14. Show that, for maximum work to be done per kg of air in Otto cycle between given upper and lower limits of absolute temperature T_3 and T_1 respectively, the compression ratio should have the value
- $$r = \left(\frac{T_3}{T_1} \right)^{1.25} \quad \text{When } \gamma = 1.4$$
15. A Fiat car working on Otto cycle has initial pressure of 1 bar and pressure after compression as 10 bar. Calculate (a) the compression ratio (b) the percentage clearance and (c) the thermal efficiency. Also calculate (d) the temperature at the end of compression if the initial temperature is 26°C (ii) the mean effective pressure if the maximum pressure in the cycle is 20 bar.
16. The bore and stroke of the cylinder of an engine working on Otto cycle are 18 cm and 32 cm respectively. The clearance volume is 0.002 m^3 . Find the compression ratio and the thermal efficiency of the engine. Take $\gamma=1.4$ for air
17. In an air standard Otto cycle the compression ratio is 10, the compression begins at 37.8°C , 1 bar and the maximum temperature of the cycle is 1060°C . Determine (a) the heat supplied per kg of air, (b) the work done per kg of air, (c) the maximum pressure of the cycle ,and (d) the thermal efficiency.
18. At the beginning of compression in an air standard Otto cycle engine cylinder the temperature is 37.8°C , the pressure is 0.99 bar and volume is 0.000707 m^3 . At the end of compression, the pressure is 10.55 bar. The heat supplied to the cycle is 1.5 kJ. Calculate (a) the compression ratio, (b) the net work per cycle, (c) the mean effective pressure.
19. The compression ratio of an Otto cycle is 8:1. Before the compression stroke of the cycle begins, the pressure is 1 bar and the temperature is 26.7°C . The heat added to the air per cycle is 2079 kJ/kg of air. Determine (a) the pressure and temperature at the end of each process of the cycle, (b) the thermal efficiency and the mean effective pressure.

20. An air standard Otto cycle operates with a compression ratio of 8.5. At the beginning of the compression the air is at 1 bar and 32°C , and during the heat addition process the pressure is tripled. Calculate (a) the thermal efficiency of the cycle and (b) the efficiency of the Carnot engine operating between the same overall temperature limits.
21. In a gas engine working on Otto cycle, the compression ratio is 10 and the suction conditions are 1 bar and 50°C . If the heat rejection equals 3517 kJ/kg, calculate the air standard efficiency and work ratio. How these quantities would compare with those of a Diesel cycle working under similar conditions?
22. An air standard Diesel cycle has a compression ratio of 16. The pressure at the beginning of the compression stroke is 1 bar and the temperature is 20°C . The maximum temperature is 1430°C . Determine the thermal efficiency and the mean effective pressure for this cycle.
23. A Diesel engine operates on the air standard Diesel cycle. The engine has 6 cylinders of 11 cm bore and 13 cm stroke. The engine runs at 2000 rpm. At the beginning of compression the air is at 1 bar and 26°C . If the clearance volume is 12.5% of the stroke volume, find (a) the compression ratio, (b) pressure and temperature of the air after compression, (c) thermal efficiency if the air is heated to 1370°C .
24. A Diesel engine working on air standard Diesel cycle takes in air at 1 bar and 25°C . The specific volume of air at the inlet is $0.8 \text{ m}^3/\text{kg}$. The compression ratio is 14 and 480 kJ/kg of heat is added at constant pressure. Determine (a) pressure, specific volume and temperature at each point of the cycle, (b) thermal efficiency, (c) mean effective pressure.
25. The German Mercedes "1900" car has a four stroke cylinder in line Diesel engine with compression ratio 20:1 and expansion ratio 10:1. Calculate the (a) cut-off ratio (b) air standard efficiency.
26. An air standard Diesel cycle has a compression ratio of 16 and a cut-off ratio of 2. At the beginning of compression the cylinder volume is 0.001415 m^3 , the air pressure is 1 bar and the temperature is 4.45°C . Calculate the thermal efficiency.
27. A Diesel engine with cut-off ratio 1.75 and compression ratio 13.5 has a polytropic exponent 'n' for both expansion and compression of 1.35. The pressure at beginning of compression is 1 bar. Calculate the mean effective pressure and the maximum pressure of the cycle.
28. An engine operates on the theoretical Diesel cycle with a compression ratio of 12:1 and fuel is injected for 10% of the stroke. The pressure of the air entering the cylinder is 0.98 bar and its temperature is 15°C . Calculate (a) cut-off ratio (b) the temperature at the end of compression process, and (c) the heat input.
29. A Diesel cycle operating on an standard cycle has a compression ratio of 15. The pressure and temperature at the beginning of the compression are 1.04 bar and 15°C . If the maximum temperature is 2330 K, determine (a) the thermal efficiency , and (b) the mean effective pressure.
30. In an air standard Diesel cycle the compression ratio is 15 and the pressure and temperature of the air at the beginning of the compression is 1 bar and 288 K. The peak temperature in the cycle is 2700 K. Calculate (a) heat supplied, (b) work done, (c) cycle efficiency, (d) peak pressure of the cycle, (e) cut-off ratio, and (f) m.e.p

Heat Transfer

8.1 Conduction

Thermal conduction is the mechanism of heat propagation from a region of higher temperature to a region of low temperature within a medium (solid, liquid or gaseous) or between different mediums in direct physical contact. Conduction does not involve any movement of macroscopic portions of matter relative to one another. The thermal energy may be transferred by means of electrons which are free to move through the lattice structure of the material. Alternatively, it may be transferred as vibrational energy in lattice structure.

Examples:

- Heat transfer in metal rods
- Heat transfer through the walls of heat exchange equipments.

8.1.1 Fourier's law of heat conduction

"The rate of heat flow through a medium by conduction is directly proportional to the surface area normal to the direction of its flow and the temperature gradient in the direction of flow."

If \dot{Q} is the rate of conduction heat flow through surface of area A which is normal to it, we can write the Fourier's law as:

$$\dot{Q} \propto A \quad \dots \dots \dots \text{(i)}$$

$$\propto \frac{dt}{dx} \quad \dots \dots \dots \text{(ii)}$$

Where, $\frac{dt}{dx}$ is temperature gradient.

Combining (i) and (ii), we get

$$\dot{Q} \propto A \frac{dt}{dx}$$

$$\therefore \dot{Q} = -kA \frac{dt}{dx} \quad \dots \dots \dots \text{(iii)}$$

The negative sign shows that temperature decreases as flow of heat progresses. k is called *thermal conductivity* and it depends on the material.

Assumptions for Fourier's law

- The heat flow is steady one i.e. the time-rate of heat flow is constant with respect to time.
- The heat flow is one dimensional.
- The material of the medium is isotropic and homogeneous.
- There is no source of internal heat generation.

8.1.2 Conduction Heat Transfer through a Plane Slab

Let us consider a plane slab of thickness 'x' having material of thermal conductivity k and face area A . One face of it is at temperature t_1 while the other face is at t_2 where $t_1 > t_2$. If heat flows through it at rate \dot{Q} , we have from Fourier's law:

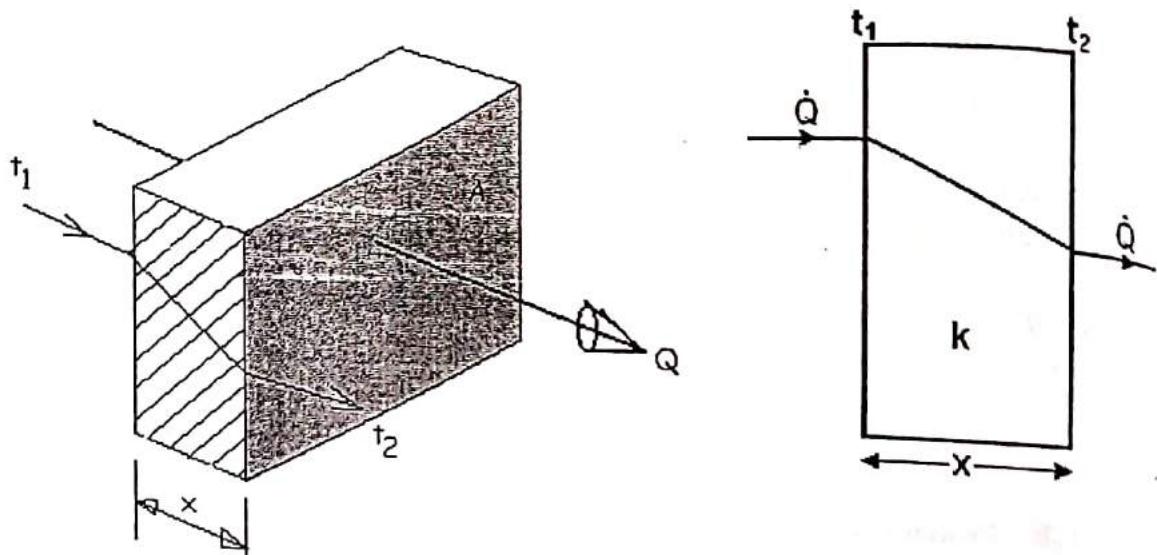


Fig 8.1 Conduction through a plane slab

$$\dot{Q} = -kA \frac{dt}{dx}$$

$$\Rightarrow dt = -\frac{\dot{Q}}{kA} dx$$

Integrating both sides, we get

$$or, [I]_1^2 = -\frac{\dot{Q}}{kA} \int_0^x dx = -\frac{\dot{Q}}{kA} [x]_0^x$$

$$or, t_2 - t_1 = -\frac{\dot{Q}}{kA}(x - 0)$$

$$or, -(t_1 - t_2) = -\frac{\dot{Q}}{k_A} x$$

$$or, t_1 - t_2 = \frac{\dot{Q}}{k_A} x$$

$$\therefore \dot{Q} = \frac{kA(t_1 - t_2)}{x}$$

8.1.3 Electrical Analogy

And the heat flow-rate through a plane slab of thickness 'x' with thermal conductivity k when there is temperature difference of $t_1 - t_2$ across it is given by Fourier's law as

There is analogy in eqⁿ(A) and (B) in the sense that the LHS of these two contain flow-rate terms whereas the numerator of RHS contain difference of driving potential or temperature. Both of these differences drive the corresponding flow. So, we can conclude that the 3rd term must be analogous i.e. it must be thermal resistance similar as electrical resistance.

$$\therefore R_{sh, slab} = \frac{x}{kA}$$

8.1.4 Conduction through a Composite Plane Slab (Multilayer Slab)

8.1.4 Conduction through a Composite Plane Slab (Multilayer Slab)
 Let us consider a composite multilayer slab having three layers of different materials tightly fitted to one another as shown in figure. The layers have thicknesses X_1 , X_2 and X_3 and their respective thermal conductivities are k_1, k_2 , and k_3 . The surface temperatures of the slab are t_1 , and t_4 whereas the temperatures at the interfaces are t_2 , and t_3 .

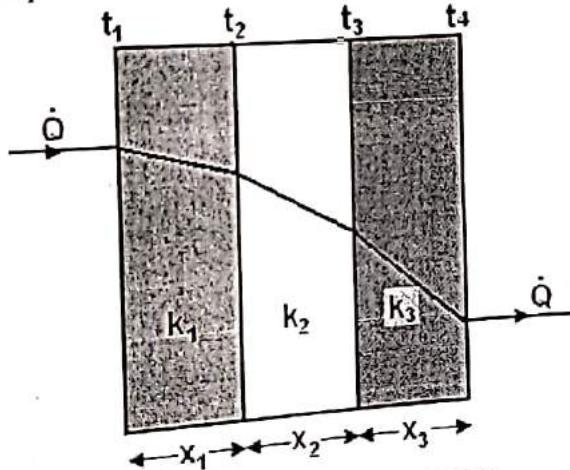


Fig 8.2 A multilayer plane slab

If the heat flow is steady i.e. \dot{Q} is constant, we have from Fourier's law

$$k_1 A(t_1 - t_2) = \frac{k_2 A(t_2 - t_3)}{X} = \frac{k_3 A(t_3 - t_4)}{X}$$

$$\dot{Q} = \frac{k_1 A(t_1 - t_2)}{X_1} = \frac{k_2 A(t_2 - t_3)}{X_2} = \frac{k_3 A(t_3 - t_4)}{X_3}$$

$$\Rightarrow t_1 - t_2 = \frac{\dot{Q}X_1}{k_1 A}$$

$$t_2 - t_3 = \frac{\dot{Q}X_2}{k_2 A}$$

$$t_3 - t_4 = \frac{QX_3}{k_3 A}$$

Summing all the above equations, we get

$$t_1 - t_2 + t_2 - t_3 + t_3 - t_4 = \frac{\dot{Q}X_1}{k_1 A} + \frac{\dot{Q}X_2}{k_2 A} + \frac{\dot{Q}X_3}{k_3 A}$$

$$or, t_1 - t_4 = \dot{Q} \left(\frac{X_1}{k_1 A} + \frac{X_2}{k_2 A} + \frac{X_3}{k_3 A} \right)$$

$$\therefore \dot{Q} = \frac{t_1 - t_4}{\frac{X_1}{k_1 A} + \frac{X_2}{k_2 A} + \frac{X_3}{k_3 A}}$$

Thus, we see that the equivalent thermal resistance for the multislab layers in series is

$$R_{eq} = R_1 + R_2 + R_3$$

And the heat flow-rate in the case of series combination can be obtained by

$$\dot{Q} = \frac{\Delta t_{net}}{R_{eq}}$$

8.1.5 Heat conduction through a Hollow Cylinder

Consider a hollow cylinder of length l , having inner radius r_1 and outer radius r_2 . The inner and outer surface temperature of the cylinder is maintained at t_1 and t_2 ($t_1 > t_2$) respectively. The thermal conductivity of the material is k .

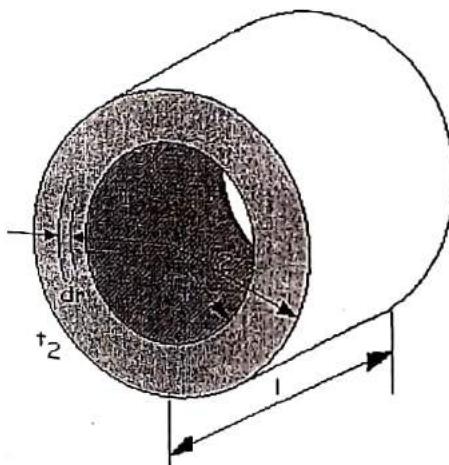


Fig 8.3 Conduction through a cylinder

The rate of heat transfer through a thin cylinder of thickness dr with inner radius r is given by,

$$\dot{Q} = -kA \frac{dt}{dr}$$

$$\dot{Q} = -k \cdot 2\pi l \frac{dt}{dr}$$

$$\text{or, } dt = -\frac{\dot{Q}}{2\pi k l} \frac{dr}{r}$$

On integration, we get

$$\int_{r_1}^{r_2} dt = \int_{r_1}^{r_2} -\frac{\dot{Q}}{2\pi k l} \cdot \frac{dr}{r}$$

$$[t]_{r_1}^{r_2} = -\frac{\dot{Q}}{2\pi k l} \int_{r_1}^{r_2} \frac{dr}{r} = -\frac{\dot{Q}}{2\pi k l} [\ln r]_{r_1}^{r_2} = -\frac{\dot{Q}}{2\pi k l} [\ln r_2 - \ln r_1]$$

$$\text{or, } t_2 - t_1 = -\frac{\dot{Q}}{2\pi k l} \ln \frac{r_2}{r_1}$$

$$\text{or, } -(t_1 - t_2) = -\frac{\dot{Q}}{2\pi k l} \ln \frac{r_2}{r_1}$$

$$\text{or}, t_1 - t_2 = \frac{\dot{Q}}{2\pi k l} \ln \frac{r_2}{r_1}$$

$$\text{or}, \dot{Q} = \frac{2\pi k l (t_1 - t_2)}{\ln \frac{r_2}{r_1}} = \frac{t_1 - t_2}{\frac{\ln \frac{r_2}{r_1}}{2\pi k l}}$$

Comparing with $\dot{Q} = \frac{t_1 - t_2}{R_{th}}$, we get

$$R_{th} = \frac{\ln r_2/r_1}{2\pi k l}$$

In case of multilayer cylindrical wall, thermal resistance of each layer should be calculated individually and summed to get the total thermal resistance of the composite system. If n is the number of concentric cylindrical shells enveloping each other, heat flow rate is given by,

$$\dot{Q} = \frac{t_1 - t_{n+1}}{\sum_{i=1}^n R_{th_i}}$$

8.1.6 Conduction through a hollow sphere

Consider a hollow sphere having inner radius r_1 and outer radius r_2 . The inner and outer surface temperature of the sphere is maintained at t_1 and t_2 ($t_1 > t_2$) respectively.

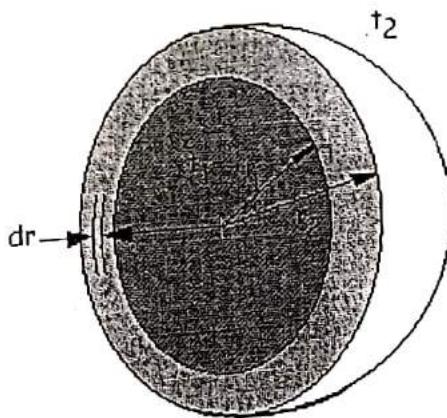


Fig 8.4 Conduction through a sphere

Now for a thin cylinder of thickness dr , and temperature difference dt across it, from Fourier's law ,we can write

$$\dot{Q} = -k \cdot A \frac{dt}{dr}$$

But,

$$A = 4\pi r^2$$

$$\therefore \dot{Q} = -k(4\pi r^2) \frac{dt}{dr} \Rightarrow dt = -\frac{\dot{Q}}{4\pi k r^2} dr$$

In order to calculate the total rate of heat transfer across the cylinder, we have to integrate the above equation.

$$\int_1^2 dt = - \int_1^2 \frac{\dot{Q}}{4\pi k} \frac{1}{r^2} dr$$

$$\text{or, } t_2 - t_1 = - \frac{\dot{Q}}{4\pi k} \int_1^2 \frac{1}{r^2} dr$$

$$\text{or, } t_2 - t_1 = - \frac{\dot{Q}}{4\pi k} \left[-\frac{1}{r} \right]_1^{r_2}$$

$$\text{or, } -(t_1 - t_2) = - \frac{\dot{Q}}{4\pi k} \left[-\frac{1}{r_2} + \frac{1}{r_1} \right]$$

$$\text{or, } t_1 - t_2 = \frac{\dot{Q}}{4\pi k} \left[\frac{1}{r_1} - \frac{1}{r_2} \right] = \frac{\dot{Q}}{4\pi k} \frac{r_2 - r_1}{r_1 r_2}$$

$$\text{or, } \dot{Q} = \frac{4\pi k r_1 r_2 (t_1 - t_2)}{r_2 - r_1}$$

$$\therefore \dot{Q} = \frac{t_1 - t_2}{\frac{r_2 - r_1}{4\pi k r_1 r_2}}$$

Comparing with $i = \frac{V_1 - V_2}{R}$, we get the thermal Resistance of hollow sphere as

$$R_{th} = \frac{(r_2 - r_1)}{4\pi k r_2 r_1}$$

8.2 Convection

Thermal convection is a process of energy transport affected by the circulation or mixing of a fluid medium (gas, liquid or powdery substance). Convection is possible on in a fluid medium and is directly linked with the transport of medium itself. Macroscopic particles of a fluid moving in space cause the heat exchange, and thus convection constitutes the macroform of the heat transfer. The effectiveness of heat transfer by convection depends largely upon the missing motion of the fluid.

With respect to origin, two types of convection are distinguished; forced and natural or free convection.

8.2.1 Natural Convection

In natural or free convection, the circulation of the fluid medium is caused by buoyancy effects, ie., by the difference in the densities of the cold and heated particles. Consider heat flow from a hot plate to atmosphere. The stagnant layer of air in the immediate vicinity of the plate gets thermal energy by conduction. The energy thus transferred serves to increase the temperature and internal energy of the air particles. Because of temperature rise these particles become less dense (and therefore lighter) than the surrounding air,. The lighter air moves upwards to a region of low temperature where they mix with and transfer a part of their energy to the cold particles. Simultaneously the cold air particles descend downwards to fill the space vacated by the hot air particles. The circulation pattern, upward movement of the warm air and the downward movement of the cold air is called the convection currents.

Some other examples of free convection are:

- > Chilling effect of a cold wind on a warm body,
- > Heat flow from a hot pavement to surrounding atmosphere and heating of air in a room by a stove,
- > Cooling of billets in the atmosphere,
- > Heat exchange of the outside of cold and warm pipes.

8.2.2 Forced Convection *Duy P*

In forced convection, the flow of fluid is caused by a pump, fan or by atmospheric winds. These mechanical devices provide a definite circuit for the circulating currents and that speeds up the heat transfer rate. Example of forced convection are:

- > Flow of water in condenser tubes,
- > Fluid passing through the tubes of a heat exchanger,
- > Cooling of internal combustion engine,
- > Air conditioning installation and nuclear reactors.

8.2.3 Newton's Law for Convective Heat Transfer

The convective heat transfer rate between a surface and an adjacent fluid is given by:

$$\dot{Q} = hA(t_s - t_f)$$

Where \dot{Q} is the convective heat transfer rate, A is the area exposed to heat transfer, t_s and t_f are the surface and fluid temperatures respectively. The heat transfer coefficient h depends upon the thermodynamic and transport properties (e.g. density, viscosity, specific heat and thermal conductivity of the fluid), the geometry of the surface, the nature of fluid flow, and the prevailing thermal conditions.

8.3 Heat Transfer through a Plane Slab Separating Two Fluids

Let us consider a plane slab of thickness x and thermal conductivity k with conducting face area A. The face temperatures of the slab are t_1 and t_2 where $t_1 > t_2$. Adjacent to the face at area A. The face temperatures of the slab are t_1 and t_2 where $t_1 > t_2$. Adjacent to the face at area A. The face temperatures of the slab are t_1 and t_2 where $t_1 > t_2$. Adjacent to the face at area A. The face temperatures of the slab are t_1 and t_2 where $t_1 > t_2$. Adjacent to the face at area A. The face temperatures of the slab are t_1 and t_2 where $t_1 > t_2$. Obviously, $t_s > t_1 > t_2 > t_o$.

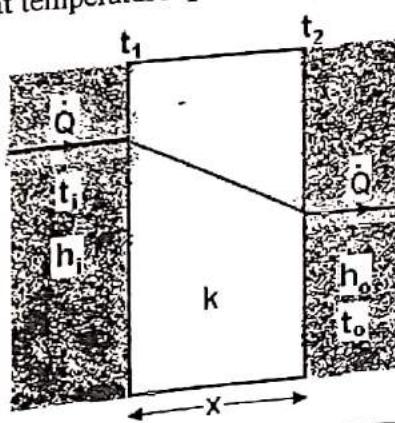


Fig. 8.5 Plane slab separating two fluid media

From Newton's law, we have

$$\dot{Q} = h_1 A(t_s - t_1) \Rightarrow t_s - t_1 = \frac{\dot{Q}}{h_1 A} \dots \dots \dots (i)$$

From Fourier's law, we have

$$\dot{Q} = \frac{kA(t_1 - t_2)}{x} \Rightarrow t_1 - t_2 = \frac{\dot{Q}x}{kA} \dots\dots\dots(i)$$

Again, from Newton's law, we have

$$\dot{Q} = h A(t_2 - t_o) \Rightarrow t_2 - t_o = \frac{\dot{Q}}{h_o A} \dots\dots\dots(ii)$$

Adding all the equations (i), (ii) and (iii), we get

$$t_1 - t_o + t_1 - t_2 + t_2 - t_o = \frac{\dot{Q}}{h_i A} + \frac{\dot{Q}x}{kA} + \frac{\dot{Q}}{h_o A}$$

$$\text{Or, } t_1 - t_o = \dot{Q} \left(\frac{1}{h_i A} + \frac{x}{kA} + \frac{1}{h_o A} \right)$$

$$\therefore \dot{Q} = \frac{t_1 - t_o}{\frac{1}{h_i A} + \frac{x}{kA} + \frac{1}{h_o A}} \dots\dots\dots(iv)$$

Eqⁿ (iv) presents \dot{Q} in terms of overall temperature difference. Let us correlate \dot{Q} with overall temperature difference in another form :

$$\dot{Q} = UA(t_1 - t_o) = \frac{t_1 - t_o}{1/UA} \dots\dots\dots(v)$$

Where U is a coefficient known as *Overall Heat Transfer Co-efficient*.

From (iv) and (v), we have

$$\frac{1}{UA} = \frac{1}{h_i A} + \frac{x}{kA} + \frac{1}{h_o A}$$

$$\text{i.e. } \frac{1}{U} = \frac{1}{h_i} + \frac{x}{k} + \frac{1}{h_o}$$

Overall Heat Transfer Coefficient depends upon the geometry of the separating wall, its thermal properties and convective coefficient at the two surfaces. It is particularly useful in the case of composite walls such as in the design of structural walls for boilers, refrigerators, air-conditioned buildings and in the design of heat exchangers.

8.4 Some Dimensionless Numbers

8.4.1 Reynold's Number Re:

It is the ratio of the inertia force to the viscous force given by

$$Re = \frac{\text{Inertia Force}}{\text{Viscous Force}} = \frac{\rho V^2 l^2}{\mu V l} = \frac{\rho V l}{\mu}$$

Reynold's number is indicative of the relative importance of inertial and viscous effects in a fluid motion. At low Reynold's number, the viscous effects dominate and the fluid motion is laminar. At high Reynold's number, the inertial effects lead to turbulent flow and the associated turbulence level dominates the momentum and energy flux. Reynold's number constitutes an important criterion of kinematic and dynamic similarity in forced convection heat transfer. Velocity within the given fields would be similar in magnitude, direction and turbulence pattern when their Reynolds number are same.

8.4.2 Grashof Number Gr:

It indicates the relative strength of the buoyant to viscous forces.
From its mathematical formulation,

$$\begin{aligned} Gr &= \frac{l^3 \rho^2 \beta g \Delta T}{\mu^2} \\ &= (l^3 \rho \beta g \Delta T) \frac{\rho}{\mu^2} \\ &= (l^3 \rho \beta g \Delta T) \frac{\rho V^2 l^2}{(\rho V l)^2} \\ &= \text{buoyant force} \times \frac{\text{inertia force}}{(\text{viscous force})^2} \end{aligned}$$

Grashof number represents the ratio of the product of buoyant and inertia forces to the square of the viscous forces. Grashof number has a role in free convection similar to that played by Reynold's number in forced convection. Free convection is usually suppressed at sufficiently small Gr, begins at some critical value of Gr (depending upon the arrangement) and then becomes more and more effective as Gr increases.

8.4.3 Prandtl Number Pr:

It is indicative of the relative ability of the fluid to diffuse momentum and internal energy by molecular mechanisms. From its mathematical formulation,

$$Pr = \frac{\mu c_p}{k} = \frac{\rho v c_p}{k} = \frac{v}{(k/\rho c_p)}$$

Recalling that the parameter $(k/\rho c_p)$ is thermal diffusivity α of the fluid,

$$Pr = \frac{v}{\alpha} = \frac{\text{kinematic viscosity}}{\text{thermal diffusivity}}$$

Apparently Pr is the ratio of the kinematic viscosity to thermal diffusivity of the fluid. The kinematic viscosity indicates the momentum transport by molecular friction and thermal diffusivity represents the heat energy transport through conduction. Obviously Pr provides a measure of the relative effectiveness of momentum and energy transport by diffusion. For highly viscous oils, Pr is quite large (100 to 10000) and that indicates rapid diffusion of momentum by viscous action compared to the diffusion of energy. Prandtl number for gases is near unity and accordingly the momentum and energy transfer by diffusion are comparable. In contrast, the liquid metals have $Pr=0.003$ to 0.01 and that indicates more rapid diffusion of energy compared to the momentum diffusion rate.

8.4.4 Nusselt Number:

It establishes the relation between the convective film coefficient h , thermal conductivity of the fluid k and a significant length parameter l of physical system:

$$Nu = \left(\frac{hl}{k} \right)$$

An energy balance at the surface of a heated plate stipulates that energy transport by conduction must equal the convective heat transfer into the fluid flowing past the plate.

Thus,

$$Q = -kA \left(\frac{\partial t}{\partial y} \right)_{y=0} = hA(t_s - t_\infty)$$

$$h = \frac{-k(\partial t / \partial y)_{y=0}}{(t_s - t_\infty)}$$

$$\frac{hl}{k} = \frac{-k(\partial T/\partial y)_{y=0}}{(T_s - T_\infty)/l}$$

Apparently the Nusselt number $\frac{hl}{k}$ may be interpreted as the ratio of temperature gradient at the surface to an overall reference temperature gradient. The Nusselt number is a convective measure of the convective heat transfer coefficient. For a given value of the Nusselt number, the convective heat transfer coefficient is directly proportional to thermal conductivity of the fluid and inversely proportional to the significant length parameter.

8.4.5 Stanton Number St :

It is the ratio of heat transfer coefficient to the flow of heat per unit temperature rise due to the velocity the fluid.

$$St = \frac{h}{\rho V c_p}$$

$$\frac{hl/k}{(\rho V l / \mu) \times (\mu c_p / k)} = \frac{Nu}{Re \times Pr}$$

Thus the Stanton number can be expressed in terms of other dimensionless number. Through dimensional analysis, we can reach the following possible relations

$$St = f(Re, Pr)$$

8.4.6 Peclet Number Pe :

It is the ratio of heat flow rate by convection to the flow rate by conduction under a unit temperature gradient and through thickness l

$$Q_{conv} = mc_p = (\rho A V) c_p \quad ; \quad Q_{cond} = \frac{kA \Delta T}{l} = \frac{kA}{l}$$

$$\therefore Pe = \frac{Q_{conv}}{Q_{cond}} = \frac{(\rho A V) c_p}{\frac{kA}{l}} = \frac{\rho c_p}{k} l V = \frac{lV}{\alpha}$$

The Peclet number can be recast as

$$Pe = \frac{\rho c_p}{k} l V = \frac{\rho l V}{\mu} \times \frac{\mu c_p}{k} = Re \times Pr$$

i.e. the Peclet number is a function of Reynold's number and Prandtl number.

8.5 Thermal Radiation

Thermal radiation is the transmission of heat in the form of electromagnetic wave from one body to another. Unlike heat transfer by conduction and convection, transport of thermal radiation does not necessarily affect the material medium between the heat source and the receiver. An intervening is not even necessary and the radiation can be affected through vacuum or a space devoid of any matter. Radiation exchange, in fact, occurs most effectively in vacuum. A material present between the heat source and the receiver would either reduce or eliminate entirely the propagation of radiation energy. The mechanism of the heat flow by radiation consists of three distinct phases:

* Conversion of thermal energy of the hot source into electromagnetic waves:

All bodies above absolute zero temperature are capable of emitting radiant energy. Energy released by a radiating surface is not continuous but is in the form of successive and discrete packets or quanta of energy called photons. The photons are propagated through the space as rays; the movement of swarm of photons is described as electromagnetic waves.

* Passage of wave motion through intervening space

The photons, as carriers of energy, travel with unchanged frequency in straight paths with speed equal to that of light.

* Transformation of waves into heat

When the photons approach the cold receiving surface, there occurs re-conversion of wave motion into thermal energy which is partly absorbed, reflected or transmitted through the receiving surface.

Thermal radiation is limited to range of wavelength between 0.1 and $100\mu\text{m}$ of the electromagnetic spectrum. Thermal radiations thus include the entire visible and infrared, and a part of ultraviolet spectrum.

8.5.1 Salient Features and Characteristics of radiation

- The electromagnetic waves are emitted as a result of vibrational and rotational movements of the molecular, atomic or sub-atomic particles comprising matter. The emission occurs when the body is excited by an oscillating electrical signal, electronic or neutronic bombardment etc., The emission of thermal radiation depends on the nature of the surface and its absolute temperature.
- The distinction between one form of radiation and another lies only in its frequency and wavelength which are related by:

$$c(\text{speed of light}) = \lambda(\text{wavelength}) \times f(\text{frequency})$$

- Each photon carries energy equivalent to hf i.e. $E=hf$ where h is Plank's constant
- The general phenomenon of radiation covers the propagation of electromagnetic waves of all the wavelengths, from short wavelength gamma rays, X-rays and ultraviolet radiation to the long wavelength microwaves. Thermal radiation is limited to range of wavelength between 0.1 and $100\mu\text{m}$; it thus includes the entire visible and infrared and a part of the ultraviolet spectrum.

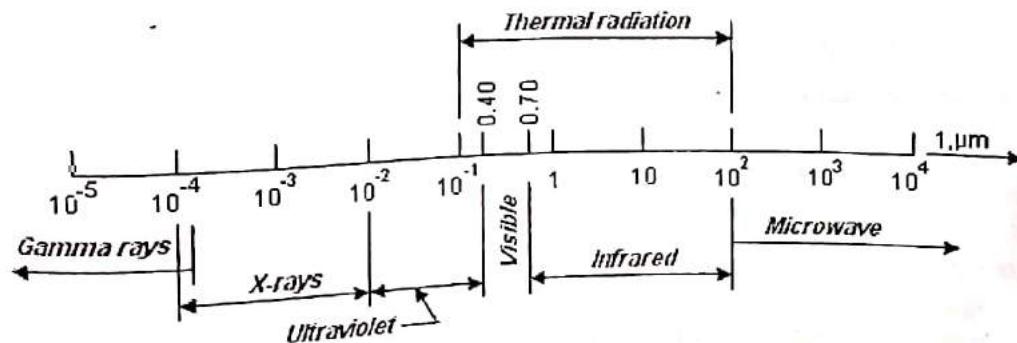


Fig 8.6 Spectrum of electromagnetic radiation

- Thermal radiations exhibit characteristics similar to those of visible light, and follow optical laws. They can undergo reflection, refraction, diffraction, polarization etc.,
- All bodies above zero Kelvin can emit radiation and those at higher temperature emit at a greater rate than those at lower temperature.

8.5.2 Reflection, Absorption and Transmission:

Radiation impinging on the surface of a body may be partly absorbed, partly reflected and partly transmitted as shown in figure.

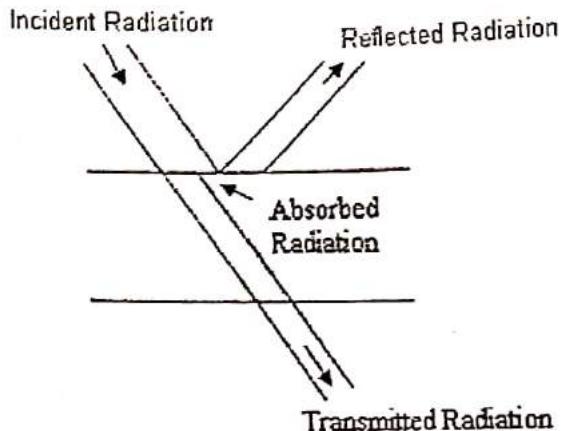


Fig 8.7 Absorption, reflection and transmission

The fraction of the incident radiation energy absorbed is called *absorptivity* (α). Similarly the fraction of the incident radiation reflected is called the *reflectivity* (ρ) and the fraction of radiation transmitted through a body is called *transmissivity* (τ).

If Q denotes the total incident radiation per unit time per unit area of a surface, and Q_α, Q_ρ , and Q_τ represent respectively the amount of radiation absorbed, reflected and transmitted, then

$$\alpha = \frac{Q_\alpha}{Q}$$

$$\rho = \frac{Q_\rho}{Q}$$

$$\tau = \frac{Q_\tau}{Q}$$

Also,-

$$Q_\alpha + Q_\rho + Q_\tau = Q$$

Dividing both sides by Q , we get

$$\frac{Q_\alpha}{Q} + \frac{Q_\rho}{Q} + \frac{Q_\tau}{Q} = \frac{Q}{Q}$$

$$\therefore \alpha + \rho + \tau = 1$$

(i) *Transparent Body*: It transmits part of the radiation falling on its surface.
For this: $\tau = 1$

(ii) *Opaque Body*: If a body does not transmit any radiation at all, it is called opaque body.
For this: $\tau = 0$
i.e. $\alpha + \rho = 1$

(iii) *Black body*: A body which neither reflects nor transmits any part of the incident radiation but absorbs all of it is called black body. In reality, a perfectly black body does not exist.

For this: $\rho=0; \tau=0$ and $\alpha=1$

(iv) *White body*: The body which reflects all the incident radiation and does not absorb or transmit any part of it is called white body.

For this: $\alpha=0; \tau=0$ and $\rho=1$

(v) *Grey body*: If the absorptivity of the surface of a body does not vary with temperature and wavelength of the incident radiation, the body is called grey body.

For this: $\alpha=(\alpha)_\lambda=\text{Constant}$

8.5.3 Emissive Power

The emissive power of a surface is defined as the total radiant energy emitted by the surface in all directions over the entire wavelength range per unit surface area per unit time.

8.5.4 Stefan-Boltzmann's Law

The total emissive power of the radiating surface of a black body is proportional to the fourth power of the absolute temperature of the surface.

If E_b is the emissive power and T the absolute temperature of a black body surface, we have

$$E_b \propto T^4$$

i.e. $E_b = \sigma T^4$

Where,

σ is proportionality constant known as Stefan-Boltzmann constant.
 $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$

8.5.5 Emissivity

The emissivity, ϵ , of a surface is defined as the ratio of the emissive power of the surface to the emissive power of a hypothetical black body at the same temperature. It depends on the nature or characteristics of the surface and is independent of wavelength of radiation.

$$\epsilon = \frac{E}{E_b}$$

So, emissive power of a real body is,
 $E = \epsilon \sigma T^4$

Energy radiated per unit time is given by
 $Q = \epsilon A \sigma T^4$

8.5.6 Kirchoff's Law

"It states that at any temperature, the ratio of the total emissive power E to the absorptivity α is a constant for all substances which are in thermal equilibrium with their environment."

Let us consider a plane black body and a plane non-black body facing parallel to each other such that the radiation of one falls totally on the other. The radiant energy E emitted by the non-black body surface impinges on the black surface and fully absorbed. Likewise the radiant energy E_b emitted by the black surface strikes the non-black surface. If the non-black

surface has absorptivity α , it will absorb αE_b of the radiation and the remainder $(1 - \alpha)E_b$ will be reflected back.

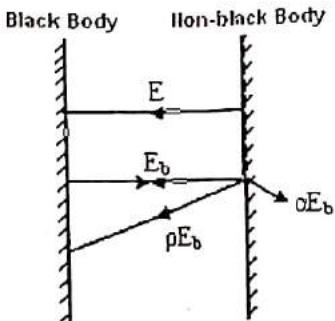


Fig 8.8 Exchange of heat between a black and a non-black surface

Net exchange of energy by the non-black body = $E - \alpha E_b$
 If the surface is at thermal equilibrium, we have

$$Or, \frac{E}{\alpha} = E_b \dots \dots \dots (i)$$

By Stefan-Boltzman's law, we have

$$E_b = \sigma T^4 = f(T)$$

So, eqⁿ(i) becomes

$$\frac{E}{\alpha} = E_b = f(T)$$

At thermal equilibrium, T remains constant i.e $f(T) = \text{Constant}$

$$\frac{E}{\alpha} = \text{Constant}$$

From equation (i), we have

$$\frac{E}{E_0} = \alpha \dots \dots \dots \text{(ii)}$$

But by the definition of emissivity we have

$$\frac{E}{E_0} = \varepsilon \dots \dots \dots \text{(iii)}$$

From (ii) and (iii), we conclude

$$\varepsilon = \alpha$$

Kirchoff's law can also be stated as: "The emissivity ϵ and absorptivity α of a real surface are equal for radiation with identical temperature and wavelength i.e a perfect absorber is a perfect emitter."

8.5.7 Wien's Displacement Law

The absolute temperature of a black body and the wavelength at which the maximum value of monochromatic emissive power occurs are inversely proportional.

$$\lambda_{\max} \propto \frac{1}{T}$$

$$\text{or } \lambda_{\max} T = \text{constant} = 2.898 \times 10^{-3}$$

The wavelength of radiation from the sun for which the energy is maximum is 0.49μ . The temperature of the sun is thus,

$$T_{\max} = \frac{2.898 \times 10^{-3}}{0.49 \times 10^{-6}} = 5914K$$

8.5.8 Radiant Heat Exchange between Infinite Parallel Planes

Let us consider two parallel plane surfaces 1 and 2 arranged at a small distance from each other so that all radiation emitted by one surface falls entirely on the other. The surface 1 emits radiant energy E_1 which strikes the surface 2. From it a part $\alpha_2 E_1$ is absorbed by the surface 2 and the remainder $(1-\alpha_2)E_1$ is reflected back to the surface 1. On reaching surface 1, a part $\alpha_1(1-\alpha_2)E_1$ is absorbed and the remainder $(1-\alpha_1)(1-\alpha_2)E_1$ is reflected and so on. The amount of radiant energy which left surface 1 per unit time is:

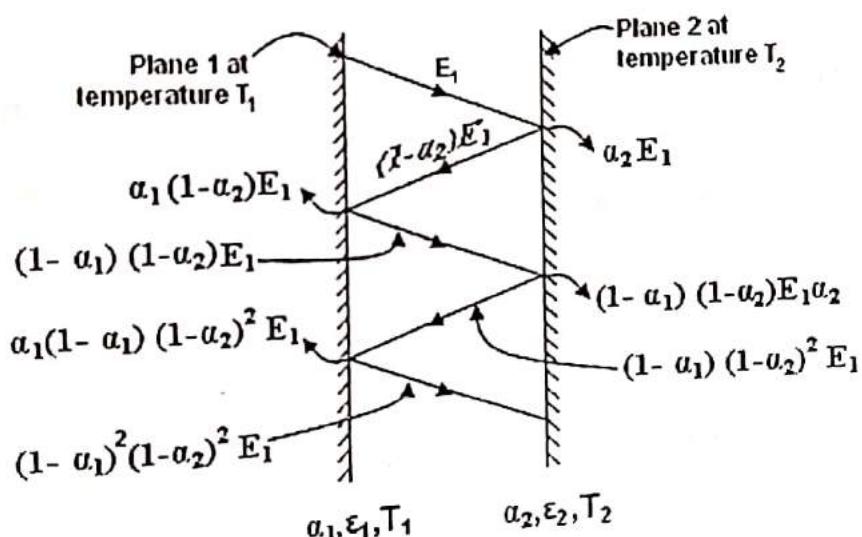


Fig 8.9 Radiant heat exchange between two non-black parallel surfaces

$$\begin{aligned} Q_1 &= E_1 - [\alpha_1(1-\alpha_2)E_1 + \alpha_1(1-\alpha_1)(1-\alpha_2)^2 E_1 + \alpha_1(1-\alpha_1)^2(1-\alpha_2)^3 E_1 + \dots] \\ Q_1 &= E_1 - \alpha_1(1-\alpha_2)E_1 [1 + (1-\alpha_1)(1-\alpha_2) + (1-\alpha_1)^2(1-\alpha_2)^2 + \dots] \\ Q_1 &= E_1 - \alpha_1(1-\alpha_2)E_1 [1 + P + P^2 + \dots] \\ \text{Where } P &= (1-\alpha_1)(1-\alpha_2) \end{aligned}$$

Since P is less than unity, we have

$$1 + P + P^2 + \dots$$

$$\begin{aligned} &= \frac{1}{1-P} \\ \therefore Q_1 &= E_1 - \frac{\alpha_1(1-\alpha_2)E_1}{1-P} = E_1 \left[1 - \frac{\alpha_1(1-\alpha_2)}{1-(1-\alpha_1)(1-\alpha_2)} \right] \end{aligned}$$

From Kirchoff's law, emissivity and absorptivity of a surface are equal i.e. $\alpha = \epsilon$

$$\begin{aligned}\therefore Q_1 &= E_1 \left[1 - \frac{\epsilon_1 (1 - \epsilon_2)}{1 - (1 - \epsilon_1)(1 - \epsilon_2)} \right] \\ &= E_1 \left[1 - \frac{\epsilon_1 - \epsilon_1 \epsilon_2}{1 - (1 - \epsilon_1 - \epsilon_2 + \epsilon_1 \epsilon_2)} \right] \\ &= E_1 \left[1 - \frac{\epsilon_1 - \epsilon_1 \epsilon_2}{1 - 1 + \epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2} \right] \\ &= E_1 \left[1 - \frac{\epsilon_1 - \epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2} \right] \\ &= E_1 \left[\frac{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2 - \epsilon_1 + \epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2} \right] \\ &= E_1 \left[\frac{\epsilon_2}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2} \right]\end{aligned}$$

Similarly, the surface 2 emits radiation of emissive power E_2 . From it a part $\alpha_1 E_2$ is absorbed by the surface 1 and the remainder $(1 - \alpha_1)E_2$ is reflected back to it. On reaching the surface 2, a part $\alpha_2(1 - \alpha_1)E_2$ is absorbed and the rest $(1 - \alpha_1)(1 - \alpha_2)E_2$ is reflected and so on. Proceeding in the same way as above, we can calculate the amount of radiant energy which left surface 2 per unit time as:

$$Q_2 = E_2 \left[\frac{\epsilon_1}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2} \right]$$

The net heat flow from surface 1 to surface 2 per unit time is then given by

$$\begin{aligned}Q_{12} &= Q_1 - Q_2 \\ &= \frac{E_1 \epsilon_2}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2} - \frac{E_2 \epsilon_1}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2} \\ &= \frac{E_1 \epsilon_2 - E_2 \epsilon_1}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2}\end{aligned}$$

From Stefan-Boltzmann law for non-black surfaces, we have

$$\begin{aligned}E_1 &= \epsilon_1 \sigma_b T_1^4 \quad \text{and} \quad E_2 = \epsilon_2 \sigma_b T_2^4 \\ \therefore Q_{12} &= \frac{\epsilon_1 \sigma_b T_1^4 \epsilon_2 - \epsilon_2 \sigma_b T_2^4 \epsilon_1}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2} \\ &= \frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2} \sigma_b (T_1^4 - T_2^4) \\ &= f_{12} \sigma_b (T_1^4 - T_2^4)\end{aligned}$$

Where, $f_{12} = \frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2} = \frac{1}{1/\epsilon_1 + 1/\epsilon_2 - 1}$, also known as interchange factor for the radiation from surface 1 to 2.

Solved Examples

1. One side of a plane wall is maintained at 100°C while the other side is exposed to a convection environment having $T=10^{\circ}\text{C}$ and $h=6 \text{ W/m}^2\text{K}$ and is 40 cm thick. Calculate (i) the rate of heat transfer through the wall and (ii) the temperature of the exposed surface.

Solution:

Given

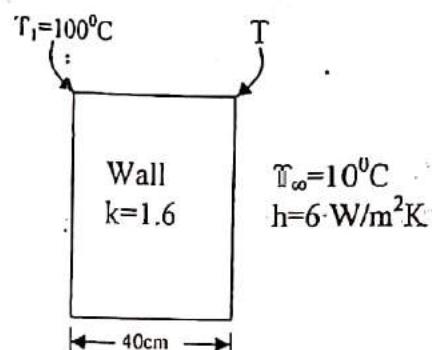
Referring to the adjoining figure, we have
 $T_1 = 100^{\circ}\text{C}$

$$T_\infty = 10^{\circ}\text{C}$$

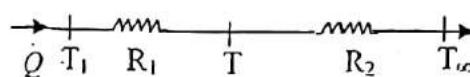
$$k = 1.6 \text{ W/mK}$$

$$x = 40\text{cm} = 0.40\text{m}$$

$$h = 6 \text{ W/m}^2\text{K}$$



Since surface area is not given, we have to calculate the heat flux, q



But we know that,

$$\text{Heat flow rate, } \dot{Q} = \frac{T_1 - T_\infty}{R_h}$$

$$\text{Where, } R_{th} = R_1 + R_2 = \frac{x}{kA} + \frac{1}{hA} = \frac{1}{A} \left(\frac{0.40}{1.6} + \frac{1}{6} \right) = \frac{0.4167}{A}$$

$$\therefore \dot{Q} = \frac{100 - 10}{0.4167} = \frac{90 \times A}{0.4167}$$

$$\text{Or, } \frac{\dot{Q}}{A} = \frac{90}{0.4167} = 215.98 \text{ W/m}^2$$

$$\text{i.e. } q = 215.98 \text{ W/m}^2$$

We also know that,

$$\dot{Q} = \frac{T_1 - T}{R_1} = \frac{T - T_\infty}{R_2}$$

$$\text{Or, } \frac{100 - T}{\frac{x}{kA}} = \frac{T - 10}{\frac{1}{hA}}$$

$$\text{Or, } \frac{kA(100 - T)}{x} = hA(T - 10)$$

$$Or, \frac{1.6(100 - T)}{0.4} = 6(T - 10)$$

$$Or.4(100 - T) = 6(T - 10)$$

$$Or.400 - 4T = 6T - 60$$

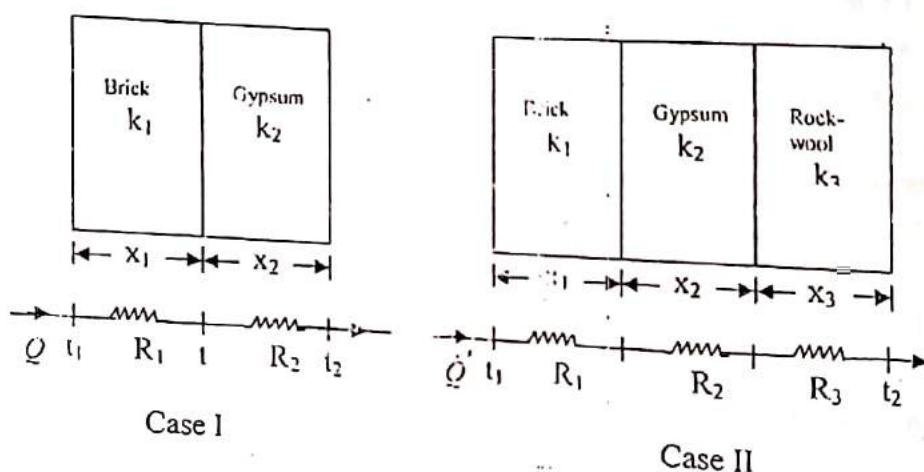
$$Or. -4T - 6T = -60 - 400$$

$$Or_{\perp} - 107 = -460$$

$$\therefore T = \frac{460}{10} = 46^{\circ}C$$

2. An exterior wall of a house may be approximated by a 100 mm layer of common brick ($k=0.7 \text{ W/mK}$) followed by a 40 mm layer of gypsum plaster ($k=0.48 \text{ W/mK}$). What thickness of loosely packed rock-wool insulation ($k=0.065 \text{ W/mK}$) should be added to reduce the heat loss (or gain) through the wall by 80%?

Solution:



Case I: Considering no rock-wool insulation, we have

$$\dot{Q} = \frac{T_1 - T_2}{R_1 + R_2} \dots \dots \dots (I)$$

Case II: Considering also the rock-wool insulation.

$$Q' = \frac{I_1 - I_2}{R_1 + R_2 + R} \dots \dots \dots (II)$$

Dividing (1) by (1), we get

$$\frac{Q}{Q'} = \frac{t_1 - t_2}{R_1 + R_2} \times \frac{R_1 + R_2 + R_3}{t_1 - t_2} = \frac{R_1 + R_2 + R_3}{R_1 + R_2} \quad , \quad (III)$$

But by question, we have

$$Q' = Q - 80\% \text{ of } Q = Q - 0.8Q = 0.2Q$$

From (III) & (IV), we have

$$\frac{R_1 + R_2 + R_3}{R_1 + R_2} = 5$$

$$Or, R_1 + R_2 + R_3 = 5R_1 + 5R_2$$

$$Or, R_3 = 5R_1 - R_1 + 5R_2 - R_2 = 4R_1 + 4R_2$$

$$Or, R_3 = 4(R_1 + R_2)$$

$$Or, \frac{x_3}{k_1 A} = 4 \left(\frac{x_1}{k_1 A} + \frac{x_2}{k_2 A} \right) = \frac{4}{A} \left(\frac{x_1}{k_1} + \frac{x_2}{k_2} \right)$$

$$Or, \frac{x_3}{k_1} = 4 \left(\frac{x_1}{k_1} + \frac{x_2}{k_2} \right)$$

$$\therefore x_3 = 4 \left(x_1 \frac{k_1}{k_1} + x_2 \frac{k_2}{k_2} \right) = 4 \left(10 \times \frac{0.065}{0.7} + 40 \times \frac{0.065}{0.48} \right) = 58.8 \text{ mm}$$

3. Obtain an expression for the heat transfer through a plane slab of thickness μ . The temperatures on the two sides of the slab are t_1 and t_2 where ($t_1 > t_2$). The thermal conductivity of the slab material varies according as $k = k_0(1 + \alpha t + \beta t^2)$.

Solution:

From Fourier's law of conduction, we have

$$\dot{Q} = -kA \frac{dt}{dx}$$

$$Or, -\frac{\dot{Q}}{A} = k \frac{dt}{dx}$$

$$Or, -\dot{q} = k_0 (1 + \alpha t + \beta t^2) \frac{dt}{dx}$$

$$Or, -\frac{\dot{q}}{k_0} dx = (1 + \alpha t + \beta t^2) dt$$

Integrating both sides, we get

$$\int_0^\mu -\frac{\dot{q}}{k_0} dx = \int_{t_1}^{t_2} (1 + \alpha t + \beta t^2) dt$$

$$Or, -\frac{\dot{q}}{k_0} \int_0^\mu dx = \int_{t_1}^{t_2} (dt + \alpha t dt + \beta t^2 dt)$$

$$Or, -\frac{\dot{q}}{k_0} [x]_0^\mu = \left[t + \frac{\alpha t^2}{2} + \frac{\beta t^3}{3} \right]_{t_1}^{t_2}$$

$$Or, -\frac{\dot{q}}{k_0} (\mu - 0) = t_2 + \frac{\alpha t_2^2}{2} + \frac{\beta t_2^3}{3} - t_1 - \frac{\alpha t_1^2}{2} - \frac{\beta t_1^3}{3}$$

$$Or, -\frac{\dot{q}}{k_0} (\mu - 0) = (t_2 - t_1) + \frac{\alpha}{2} (t_2^2 - t_1^2) + \frac{\beta}{3} (t_2^3 - t_1^3)$$

$$Or, -\frac{\dot{q}}{k_0} \mu = (t_2 - t_1) + \frac{\alpha}{2} (t_2 - t_1)(t_2 + t_1) + \frac{\beta}{3} (t_2 - t_1)(t_2^2 + t_2 t_1 + t_1^2)$$

$$Or, -\frac{\dot{q}}{k_0} \mu = (t_2 - t_1) + \left[1 + \frac{\alpha}{2} (t_2 + t_1) + \frac{\beta}{3} (t_2^2 + t_2 t_1 + t_1^2) \right]$$

$$Or, -\frac{\dot{q}}{k_0} \mu = -(t_1 - t_2) \left[1 + \frac{\alpha}{2} (t_2 + t_1) + \frac{\beta}{3} (t_2^2 + t_2 t_1 + t_1^2) \right]$$

$$\therefore \dot{q} = \frac{k_0}{\mu} (t_1 - t_2) \left[1 + \frac{\alpha}{2} (t_2 + t_1) + \frac{\beta}{3} (t_2^2 + t_2 t_1 + t_1^2) \right]$$

4. A steam pipe 18 cm inside diameter and 20 cm outside diameter is covered with two layers of insulation. The thickness of the first and the second layers is 4 cm and 8 cm respectively and their corresponding thermal conductivities are 0.15 and 0.08 W/mK respectively. The conductivity of the pipe material is 50 W/mK. The temperature of the inner surface of the steam pipe is 350°C and that of the outer surface of insulation is 30°C . Find the quantity of heat lost per metre length of the pipe. Also find out the percentage decrease in the heat loss with two layers of insulation compared to only first layer of insulation.

Solution:

Given:

Referring to the adjacent figure, we have

$$r_1 = \frac{18}{2} = 9 \text{ cm}$$

$$r_2 = \frac{20}{2} = 10 \text{ cm}$$

$$r_3 = r_2 + 4 = 10 + 4 = 14 \text{ cm}$$

$$r_4 = r_3 + 8 = 14 + 8 = 22 \text{ cm}$$

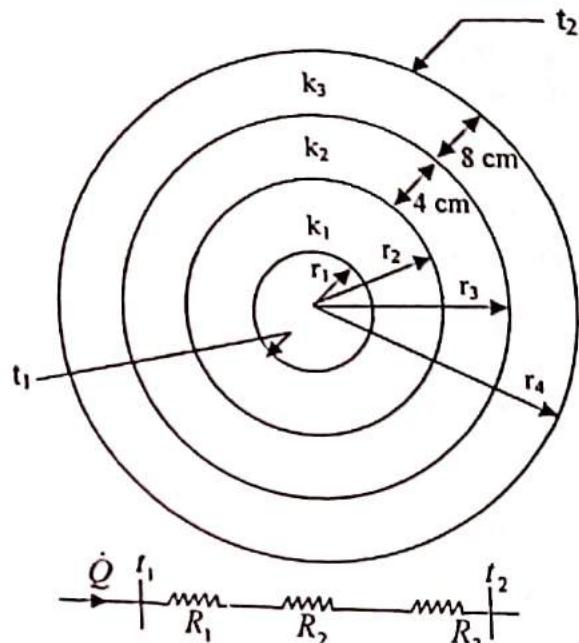
$$k_1 = 50 \text{ W/mK}$$

$$k_2 = 0.15 \text{ W/mK}$$

$$k_3 = 0.08 \text{ W/mK}$$

$$t_1 = 350^{\circ}\text{C}$$

$$t_2 = 30^{\circ}\text{C}$$



Case I: Considering both layers of insulation

We know that

$$\dot{Q} = \frac{t_1 - t_2}{R_{th}}$$

But,

$$R_{th} = R_1 + R_2 + R_3$$

$$\begin{aligned} &= \frac{\ln \frac{r_2}{r_1}}{2\pi k_1 l} + \frac{\ln \frac{r_3}{r_2}}{2\pi k_2 l} + \frac{\ln \frac{r_4}{r_3}}{2\pi k_3 l} \\ &= \frac{1}{2\pi l} \left(\frac{\ln \frac{10}{9}}{50} + \frac{\ln \frac{14}{10}}{0.15} + \frac{\ln \frac{22}{14}}{0.08} \right) \\ &= \frac{7.895}{2\pi l} \end{aligned}$$

So that,

$$\dot{Q} = \frac{350 - 30}{7.895} = \frac{2\pi l \times 320}{7.895}$$

$$\therefore \left(\frac{\dot{Q}}{l} \right) = \frac{2\pi \times 320}{7.895} = 254.67 \text{ W/m}$$

i.e. $q = 254.67 \text{ W/m}$

Case II: considering single layer insulation only

In this case, the temperature of outer surface will remain same.

Then,

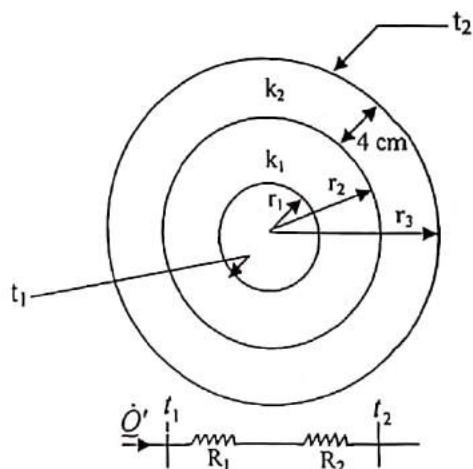
$$\dot{Q}' = \frac{t_1 - t_2}{R_1 + R_2}$$

$$= \frac{350 - 30}{\frac{\ln(r_2/r_1)}{2\pi k_1 l} + \frac{\ln(r_3/r_2)}{2\pi k_2 l}}$$

$$= \frac{2\pi l \times 320}{\frac{\ln(10/9)}{50} + \frac{\ln(14/10)}{0.15}}$$

$$\therefore \left(\frac{\dot{Q}'}{l} \right) = \frac{2\pi \times 320}{\frac{\ln(10/9)}{50} + \frac{\ln(14/10)}{0.15}} = 895.497 \text{ W/m}$$

i.e. $q' = 895.497 \text{ W/m}$



$$\therefore \% \text{ decrease in heat loss} = \frac{\Delta q}{q'} \times 100\% = \frac{q' - q}{q'} \times 100\%$$

$$= \frac{895.497 - 254.67}{895.497} \times 100\%$$

$$= 71.56\%$$

5. Air inside a thin-wall stainless steel tube with $h=65 \text{ W/m}^2\text{K}$ is kept at 120°C . The inside diameter of the tube is 2.5 cm and the wall thickness is 0.4 mm, $k=18 \text{ W/m}^2\text{K}$ for the steel. The tube is exposed to an environment with $h=6.5 \text{ W/m}^2\text{K}$ and $T_\infty=15^\circ\text{C}$. Calculate the overall heat transfer co-efficient and the heat loss per metre of length. What thickness of an insulation having $k=40 \text{ W/mK}$ should be added to reduce the heat loss by 90%?

Solution:

Referring to the adjoining figure, we have

$$r_1 = \frac{2.5}{2} = 1.25 \text{ cm}$$

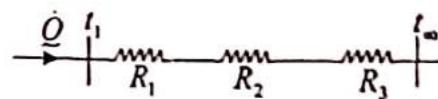
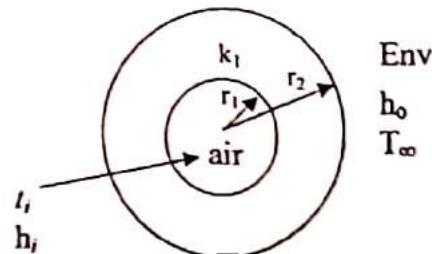
$$r_2 = 1.25 \text{ cm} + 0.4 \text{ mm}$$

$$= 1.25 \text{ cm} + 0.04 \text{ cm} = 1.29 \text{ cm}$$

$$t_i = 120^\circ C ; t_\infty = 15^\circ C$$

$$h_i = 65 \text{ W/m}^2\text{K} ; h_o = 6.5 \text{ W/m}^2\text{K}$$

$$k_1 = 18 \text{ W/mK}$$



Case I: Considering no insulation

For this case, we have

$$\dot{Q} = U_o A_o (t_i - t_\infty) = \frac{t_i - t_\infty}{R_1 + R_2 + R_3}$$

$$\text{Or, } U_o (2\pi r_2 l) = \frac{1}{R_1 + R_2 + R_3}$$

$$\text{Or, } U_o (2\pi r_2 l) = \frac{1}{\frac{1}{h_i A_i} + \frac{\ln(r_2/r_1)}{2\pi k_1 l} + \frac{1}{h_o A_o}}$$

$$\text{Or, } U_o (2\pi r_2 l) = \frac{1}{\frac{1}{h_i (2\pi r_1 l)} + \frac{\ln(r_2/r_1)}{2\pi k_1 l} + \frac{1}{h_o (2\pi r_2 l)}}$$

$$\text{Or } U_o (2\pi l) r_2 = \frac{(2\pi l)}{\frac{1}{h_i r_1} + \frac{\ln(r_2/r_1)}{k_1} + \frac{1}{h_o r_2}}$$

$$\text{Or, } U_o = \frac{1}{r_2 \left[\frac{1}{h_i r_1} + \frac{\ln(r_2/r_1)}{k_1} + \frac{1}{h_o r_2} \right]} = \frac{1}{0.0129 \left[\frac{1}{65 \times 0.0125} + \frac{\ln(1.29/1.25)}{18} + \frac{1}{6.5 \times 0.0129} \right]}$$

$$\therefore U_o = 5.89 \text{ W/m}^2\text{K}$$

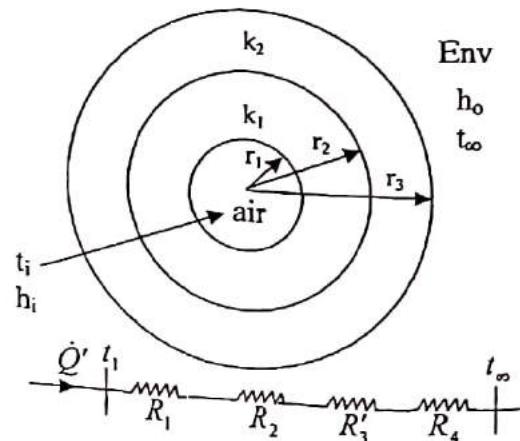
so,

$$\begin{aligned} \dot{Q} &= U_o A_o (t_i - t_\infty) = 5.89 \times 2\pi r_2 l \times (120 - 15) \\ \Rightarrow \left(\frac{\dot{Q}}{l} \right) &= 5.89 \times 2\pi \times 0.0129 \times (120 - 15) = 50.13 \text{ W/m} \end{aligned}$$

Case II: Considering the insulation

In this case, we have

$$\dot{Q}' = \frac{t_i - t_\infty}{R_1 + R_2 + R'_3 + R_4} \dots \dots \dots (I)$$



But, for the case with no insulation, we have

Now, by the question , we have

Substituting the value of \dot{Q} and \dot{Q}' , we get

$$\frac{t_i - t_\infty}{R_1 + R_2 + R_3} = 10 \frac{t_i - t_\infty}{R_1 + R_2 + R'_3 + R_4}$$

$$Or, R_1 + R_2 + R'_3 + R_4 = 10(R_1 + R_2 + R_3)$$

$$Or, R_4 = 10R_1 - R_2 + 10R_3 - R_4 + 10R_5$$

$$Or, \frac{\ln(r_3/r_2)}{2\pi k_b l} = 9 \frac{1}{k(2\pi r_1 l)} + 9 \frac{\ln(r_2/r_1)}{2\pi k_b l} + 10 \left[\frac{1}{(\frac{r_3}{r_1} - 1)} - \frac{1}{(\frac{r_2}{r_1} - 1)} \right]$$

$$Or, \ln\left(\frac{r_1}{r_2}\right) = k_2 \left[\frac{9}{h_i r_i} + \frac{9 \ln(r_2/r_1)}{k_i} + \frac{10}{h_i r_i} - \frac{1}{h_i r_i} \right]$$

$$Or, \ln\left(\frac{r_3}{0.0129}\right) = 40 \times 10^{-3} \left[\frac{9}{65 \times 0.0125} + \frac{9 \ln(1.29/1.25)}{18} + \frac{10}{6.5 \times 0.0129} - \frac{1}{6.5 \times r_3} \right]$$

Putting $r_3=1$, we get LHS=0.0062 and RHS=0.8635

Putting $r_3=2$, we get LHS=0.6962 and RHS=0.8635

Putting $r_3=3$, we get LHS=1.099 and RHS=0.8635

Thus, we see that r_3 must lie between 2 and 3. Let us put $r_3=2.5$.

Putting $r_3=2.5$, we get LHS=0.9208 and RHS=0.8635.

Again we see that r_3 lies between 2 and 2.5 Let us put $r_3=2.25$

Putting $r_3=2.25$, we get LHS=0.8109 and RHS=0.8635

We see that r_3 lies between 2.25 and 2.5. Let us put $r_3=2.376$.

Putting $\tau = 3.376$, we get LHS=0.8680 and RHS=0.8635.

Thus, we see that $r_2 = 2.376$ nearly satisfies the above logarithmic equation.

Hence, the required value of $r_3 = 2.376 \text{ m} = 237.6 \text{ cm}$

$$circumference = r_2 - r_1 = 237.6 - 1.29 = 236.31 \text{ cm}$$

Problems

1. A 150 mm steam pipe has inside diameter of 120 mm and outside diameter of 160 mm. It is insulated at the outside with asbestos. The steam temperature is 150°C and the air temperature is 20°C , $h(\text{steam side})=100 \text{ W/m}^2\text{K}$, $h(\text{air side})=30 \text{ W/m}^2\text{K}$, $k(\text{asbestos})=0.5 \text{ W/mK}$, and $k(\text{steel})=42 \text{ W/mK}$. How thick should the asbestos be provided in order to limit the heat loss to 2.1 kW/m^2 ?
2. A room has a brick wall 25 cm in thickness. The inside air is at 25°C and the outside air is at -25°C . The heat transfer co-efficient on the inside and outside are 8.72 and 28 $\text{W/m}^2\text{K}$ respectively. Find the rate of heat transfer through the wall and the inside surface temperature. [$k_{\text{brick}}=0.7 \text{ W/mK}$]
3. Sheets of brass and steel, each 1 cm thick, are placed in contact. The outer surface of brass is kept at 100°C and the outer surface of steel is kept at 0°C . What is the temperature of the common interface? The thermal conductivity of brass and steel are in the ratio of 2:1.
4. A hollow sphere ($k=35 \text{ W/mK}$), the inner and outer diameters of which are 28 cm and 32 cm respectively, is heated by means of a 20Ω coil placed inside the sphere. Calculate the current required to keep the two surfaces at a constant temperature difference of 50°C and also calculate the rate of heat supply.
5. One side of a copper block 4 cm thick is maintained at 175°C . The other side is covered with a layer of fibre glass 1.5 cm thick. The outside of the fibre glass is maintained at 80°C , and the total heat flow through the composite slab is 300 W. What is the area of the slab? Given: $k_{\text{cu}}=50 \text{ W/mK}$; $k_{\text{fib}}=0.065 \text{ W/mK}$.
6. A hot steam pipe having an inside surface temperature of 250°C has an inside diameter of 8 cm and a wall thickness of 5.5 mm. It is covered with a 9 cm layer of insulation having $k=0.5 \text{ W/mK}$, followed by a 4 cm layer of insulation having $k=0.25 \text{ W/mK}$. The outside temperature of the insulation is 20°C . Calculate the heat lost per metre of length. Assume $k=47 \text{ W/mK}$ for the pipe.
7. A house wall may be approximated as two 12 layers of fiber insulating board, a 8 cm layer of loosely packed asbestos, and a 10 cm layer of common brick. Assuming the overall heat transfer co-efficient for this arrangement. [$k_{\text{fib}}=0.7 \text{ W/mK}$; $k_{\text{asbestos}}=0.18 \text{ W/mK}$; $k_{\text{brick}}=0.7 \text{ W/mK}$].
8. A cold storage room has walls made of 0.23 m of brick on the outside, 0.08 m of plastic foam and finally 1.5 cm of wood on the inside. The outside and inside air temperatures are 20°C and -2°C respectively. If the inside and outside heat transfer co-efficients are respectively $29.12 \text{ W/m}^2\text{K}$ and the thermal conductivities of brick, foam and wood are 0.98, 0.02 and 0.17 W/mK respectively, determine (i) the rate of heat removed by refrigeration if the total wall area is 90 m^2 , and (ii) the temperature of the inside surface of the brick.