Problem solving Technique

2013 (Fall)

1.a)

Sol^n

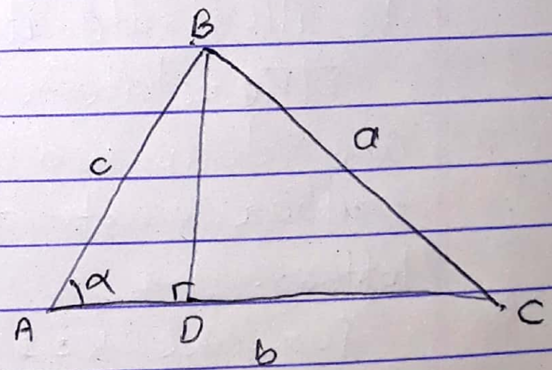
The number of diagonals in a polygon = $N(N-3)/2$
where N is the number of polygon sides

For a convex n-sided polygon, there are n vertices and from each vertex ~~yo~~ we can drawn (N-3) diagonals, so the total number of diagonals that can be drawn is N(N-3). However, this would mean that each diagonal would be drawn twice (to and from each vertex), so, the expression must be divided by 2.

OR,

1.a)

Sol^n



In rt. $\triangle BDA$,

$\cos\alpha = \frac{AD}{AB}$

or, $AD = AB\cos\alpha$

$DC = AC - AD$

$= \cancel{b} \; AC - AB\cos\alpha$

P.T.O

In $\therefore$ rt $\triangle BDA$,

$$\sin\alpha = \frac{BD}{AB}$$

$$BD = AB\sin\alpha$$

In $\triangle BDC$, applying pythagorean theorem

$$BC^2 = BD^2 + DC^2$$
$$= (AB\sin\alpha)^2 + (AC - AB\cos\alpha)^2$$

$$BC^2 = AB^2\sin^2\alpha + AC^2 - 2AB \cdot AC\cos\alpha + AB^2\cos^2\alpha$$

$$\boxed{BC^2 = AB^2 + AC^2 - 2AB \cdot AC\cos\alpha}$$

1.b) Sol^n

Although many solution are available we will use induction method just to illustrate it.

The statement $P(K)$ is " If $(K+1)$ letter are delivered to K mailbox, then same mailbox must recieved at least two letters.

for case $K=1$, noticed that if $K+1=2$ letters are delivered to $K=1$ mailbox, then some mailbox will recieved two letter.

Assume that $P(K)$ has been proved. we have to prove that $(K+1)+1$ letter have been delivered to $(K+1)$ mailbox.

P.T.O

If the last mailbox is empty, then all the letter have been delivered to first K mailbox. In particular case, at least (K+1) letter have been deliver to first K mailbox. so inductive hypothesis apply and one of this first K mailbox contain at least two letter.

If the last mailbox contain one letter, then the remaining (K+1) letter have been deliver to first 'K' mailbox. Once again, inductive hypothesis apply to K mailbox. so, one of them contain at least two letter.

If the last mailbox contain two or more letter, then we are done because some mailbox contain at least two letter.

Hence, we proved the statement " K is positive integer, if (K+1) letters are delivered to 'K' mailboxes, then show that one mailbox must contain at least two letter using induction method.

2.a) $$\sum_{i=1}^{K} i^2 = \frac{2K^3 + 3K^2 + K}{6}$$

we can write,

$$P(K) = 1^2 + 2^2 + 3^2 + \cdots + K^2 = \frac{2K^3 + 3K^2 + K}{6}$$

Basic step

for $K = 1$

$$P(1) = K^2 = 1^2 = 1$$

$$= \frac{2 \cdot 1^3 + 3 \cdot 1^2 + 1}{6} = 1$$

so, $P(1)$ is true.

Inductive step

Now, we assumed that $P(K)$ is true for some natural number N

i.e

$$P(N) = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{2N^3 + 3N^2 + N}{6}$$

Now, we have to show that $P(N)$ is true for $N+1$

i.e

$$P(N+1) = 1^2 + 2^2 + 3^2 + \cdots + N^2 + (N+1)^2 = \frac{2(N+1)^3 + 3(N+1)^2 + (N+1)}{6}$$

$$= \frac{2N^3 + 3N^2 + N}{6} + (N+1)^2$$

$$= \frac{1}{6}\left(\frac{2N^3 + 3N^2 + N + 6(N+1)^2}{6}\right)$$

$$= \frac{1}{6} N(N+1)(2N+1) + (N+1)^2$$

$$= \frac{1}{6}(N+1)\{N(2N+1) + 6(N+1)\}$$

$$= \frac{1}{6}(N+1)\{2N^2+7N+6)$$

$$= \frac{1}{6}(N+1)(N+2)(2N+3)$$

$$= \frac{1}{6}(N+1)(N+1+1)\{2(N+1)+1\}$$

$$= \frac{1}{6}[\{(N+1)^2+(N+1)\}\{2(N+1)+1\}]$$

$$= \frac{1}{6}[2(N+1)^3 + 3(N+1)^2 + (N+1)]$$

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{n}{(n+1)!}$$

Sol^n

$$\frac{2-1}{2!} + \frac{3-1}{3!} + \frac{4-1}{4!} + \cdots + \frac{(n+1)-1}{(n+1)!}$$

$$= \frac{2}{2!} - \frac{1}{2!} + \frac{3}{3!} - \frac{1}{3!} + \frac{4}{4!} - \frac{1}{4!} + \cdots + \frac{(n+1)-1}{(n+1)!}$$

$$= \frac{\cancel{2}}{\cancel{2}\times 1!} - \frac{1}{2!} + \frac{\cancel{3}}{\cancel{3}\times 2!} - \frac{1}{3!} + \frac{\cancel{4}}{\cancel{4}\times 3!} - \frac{1}{4!} + \cdots + \frac{(n+1)-1}{(n+1)!}$$

$$= \frac{1}{1!} - \frac{1}{2!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{3!} - \frac{1}{4!} + \cdots + \frac{(n+1)-1}{(n+1)!}$$

$$= \frac{1}{1!} - \frac{n}{(n+1)!}$$

$$= 1 - \frac{n}{(n+1)!}$$

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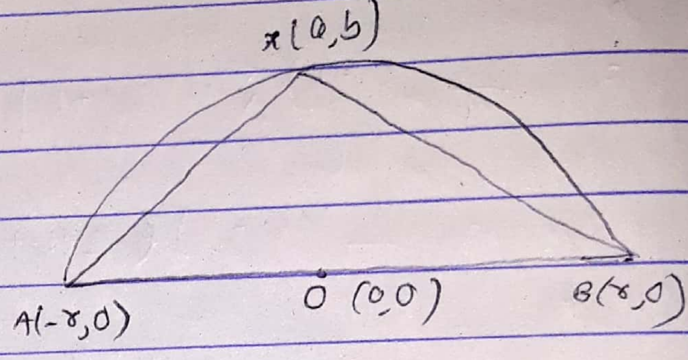
3.a)



sol^n

We assume centre of semi-circle is the origin, the radius 'r' and vertex of angle in the curve is $x(a,b)$

Now,

slope of segment Ax is $\frac{b-0}{a-(-r)} = \frac{b}{a+r}$

slope of segment Bx is $\frac{b-0}{a-r} = \frac{b}{a-r}$

The product of slope is

$$\frac{b}{(a+r)} \times \frac{b}{(a-r)}$$

$$= \frac{b^2}{a^2-r^2}$$

we assume that (a,b) lies in the circle with centre $(0,0)$

$$a^2+b^2=r^2$$

$$r^2-a^2=b^2$$

$$\therefore \frac{r^2-a^2}{a^2-r^2} = -1$$

Hence, any angle subtended by semi-circle is a right angle.

3.3) Soln

Let V denotes no. of vertices

F denote no. of face

and E denote no. of edge

There are 3 edges meeting at each vertex but we counted each edges twice therefore the relation is

$$2E = 3V$$

$$E = \frac{3}{2}V$$

Again, 3 square faces meeting at each vertex. so, the relation is

$$4F = 3V$$

$$F = \frac{3}{4}V$$

Now, using Euler formula

$$V - E + F = 2$$

$$V - \frac{3V}{2} + \frac{3V}{4} = 2$$

$$4V - 6V + 3V = 8$$

$$V = 8$$

Then,

$$E = \frac{3 \times 8}{2} = 12$$

$$F = \frac{3 \times 8}{4} = 6$$

4.a)

$$\begin{array}{r} SEND \\ +\ MORE \\ \hline MONEY \end{array}$$

sol^n

Addition of two numbers with 'n' digits results in a n+1 digits, then the left most place always = 1.

So, M=1, substitute this value.

Now, 'O' cannot be 1 as M already 1. It may not be 2 either as S+1 = 12 or 1+S+1 = 12 in both cases S is a two digit number. So, 'O' is nothing but zero put 'O' = 0.

Now, 'S' can be either 8 or 9. if S=8, then there must be a carry over E+O = 10+N or 1+E+O = 10+N

In the above two cases, E-N=10 is not possible as 'N' cannot be zero.

So, E=9.

Now, E+O = N is not possible E=N. So, 1+E = N possible.

$$\begin{array}{r} 1 \\ 9\ E\ N\ D \\ +\ 1\ 0\ R\ E \\ \hline 1\ 0\ N\ E\ Y \end{array}$$

P.T.O

The possible cases are, $N + R = 10 + E$ — (1) or

$1 + N + R = 10 + E$ — (2)

substituting $E = N - 1$ in the first equation,

$N + R = 10 + N - 1$, we get $R = 9$ which is not possible.

substituting $E = N - 1$ in the second equation,

$1 + N + R = 10 + N - 1$, we get $R = 8$

We know that N and E are consecutive and N is larger. Take $(N, E) = (7, 6)$ check and substitute you want get any unique value for D.

Take $(N, E) = (6, 5)$, Now you get $D = 7$, $Y = 2$

$$
\begin{array}{r}
9\ 5\ 6\ 7 \\
+\ 1\ 0\ 8\ 5 \\
\hline
1\ 0\ 6\ 5\ 2
\end{array}
$$

4.b) sol^n

31	17	18	28
20	26	25	23
24	22	21	27
19	29	30	16

5.a)

sol^n

Device A result in using only 0.75 of the fuel that the machine would use without ~~at~~ any devices.

Device B applied on top of device A, result in 0.55 of the fuel being used to achieved the same milage.

and device C on top of these two result 0.70 of that fuel being used to achieve the same milage.

All three devices equipped (A, B and C) at that time, we need only (0.75 × 0.55 × 0.70) = 0.288 fuel being used to achieve same milage.

Here, when these three device equipped, we need to use 0.288 fuel. So, we save

$(1-0.288) = 0.71$ fuel

i.e. 71% fuel.

5.b) suppose boy have x ~~no. of~~ pen and girl have y ~~pe~~ ~~no of~~ pen.

According to first condition,

$$x-1 = y+1$$

or, $x = y+2$ — (i)

According to 2nd condition,

$$x+1 = 3(y-1)$$

or, $x+1 = 3y-3$

or, $y+2+1 = 3y-3$ [from eq^n (i)]

or, $y+3 = 3y-3$

or, $3y-y = 3+3$

or, $2y = 6$

$y = 3$

Using value of y in eq^n (i),

$x = 3+2$

$\therefore x = 5$

Hence, boy have 5 pen and girl have 3 pen.

5.c)

Soln

we can use the binomial theorem as follows.

$11^{10}-1$

$=(10+1)^{10}-1$

$= \left[\left({}^{10}C_0\right).10^{10} + \left({}^{10}C_1\right) 10^9 + ---- + \left({}^{10}C_8\right).100 + \left({}^{10}C_9\right)10+1 \right] - 1$

$= \left[\left({}^{10}C_0\right)10^{10} + \left({}^{10}C_1\right)10^9 + --- + \left({}^{10}C_8\right)100 + 10.10 \right]$

$= 100 \left[\left({}^{10}C_0\right)10^8 + \left({}^{10}C_1\right)10^7 + --- + \left({}^{10}C_8\right) + 1 \right]$

Here, it is the multiple of 100, so, $11^{10}-1$ is divisible by 100.

$(x+1)^n = \left({}^nC_0\right)x^n + \left({}^nC_1\right)x^{n-1} + \left({}^nC_2\right)x^{n-2} + ...$

6.a)

(b)
$$\cos(a/2)\cdot\cos(a/4)\cdot\cos(a/8) = \frac{\sin(a)}{8\sin(a/8)}$$

$$L.H.S = \cos(a/2)\cdot\cos(a/4)\cdot\cos(a/8)$$

$$= \frac{\cancel{\cos(a}\left(2\sin\left(\frac{a}{2}\right)\cdot\cos\left(\frac{a}{2}\right)\right)\}\cos(a/4)\cos(a/8)}{2\sin(a/2)}$$

$$= \frac{\sin a\cdot\cos(a/4)\cdot\cos(a/8)}{2\sin(a/2)}$$

$$\frac{\sin a\cdot\{2\sin(a/4)\cdot\cos(a/4)\}\cdot\cos(a/8)}{2\sin\left(\frac{a}{4}\right)\cdot 2\sin(a/2)}$$

$$= \frac{\sin a\cdot\cancel{\sin a/2}\cdot\cos(a/8)}{2\sin(a/4)\cdot\cancel{2\sin(a/2)}}$$

$$= \frac{\sin a\cdot\{2\sin\left(\frac{a}{8}\right)\cdot\cos(a/8)\}}{4\sin(a/4)\cdot 2\sin(a/8)}$$

$$= \frac{\sin a\cdot\cancel{\sin(a/4)}}{8\cancel{\sin(a/4)}\cdot\sin(a/8)}$$

$$= \frac{\sin a}{8\sin(a/8)}$$

$= R.H.S$ proved//

6.c) soln

Let x and y be two +ve real number.

$\therefore x+y=100$ — (i)

If the product of these number is 3000

i.e $x*y=3000$ — (ii)

$x(100-x)=3000$

$100x-x^2=3000$

or, $x^2-100x+3000=0$

$$\therefore x=\frac{-(-100)\pm\sqrt{(-100)^2-4.1.3000}}{2\times1}$$

$$=\frac{100\pm\sqrt{-200}}{2}$$

which is imaginary number but by question x and y are real number

Hence, by contradiction, product of these two number never be 3000.

7.a) Use of problem solving technique in computer fields

A computer is a very powerful and versatile machine capable of performing a multitude of different tasks, yet it has no intelligence or thinking power. The intelligence quotient (I.Q) of a computer is zero. A computer performs many tasks exactly in the same manner as it is told to ~~you~~ do. This places responsibility on the user to instruct the computer in a correct and precise manner, so that the machine is able to perform the required job in a proper way. A wrong or ambiguous instruction may sometimes prove disastrous.

In order to instruct a computer correctly, the user must have clear understanding of the problem to be solved. A part from the this he should be able to develop a method, in the form of series of sequential steps, to solve it. Once the problem is well-defined and a method of solving it is developed, then instructing the computer to solve the problem becomes relatively easier task.

2017 (Fall)

1.a) $310! \times 2^{12} \times 5^{8} + 780!$

Soln

for 310!

Multiple of 5 = $\frac{310}{5} = 62$

Multiple of 25 = $\frac{310}{25} = 12.4 \simeq 12$

Multiple of 125 = $\frac{310}{125} = 2.48 \simeq 2$

we get $62 + 12 + 2 = 76$ no. of 5

In $2^{12} \times 5^{8}$,

we can make maximum 8 pairs of 2 and 5 which means, we have 8 zeroes.

$\therefore 310! \times 2^{12} \times 5^{8} = 76 + 8 = 84$ zeroes at the end.

for 780!,

Multiple of 5 = $\frac{780}{5} = 156$

multiple of 25 = $\frac{780}{25} = 31.2 \simeq 31$

Multiple of 125 = $\frac{780}{125} = 6.24 \simeq 6$

Multiple of 625 = $\frac{780}{625} = 1.248 \simeq 1$

we get $156 + 31 + 6 + 1 = 194$ no. of 5

$\therefore$ Hence, $310! \times 2^{12} \times 5^{8} + 780!$, have 84 zeroes at the end of the product.

1.5) sol^n

$2^n > n^2$ for integer $n \geq 5$

Basic step

for $n=5$

$2^5 > 5^2$

$32 > 25$ which is true

Inductive step

We assumed that given statement is true for some K

$2^K > K^2$

~~We~~

Now, we have to show that given statement is true for integer $(K+1)$

$\therefore 2^{K+1} > (K+1)^2$

$2 \times 2^K > K^2 + 2K + 1$

$2 \times K^2 > (K+1)^2$

The first inequality follows from the induction hypothesis and as for the second, we know that $(n-1)^2 \geq 4^2 > 2$, since $n \geq 5$. We can expand this inequality $(n-1)^2 > 2$ as follows

$n^2 - 2n + 1 > 2$

$n^2 - 2n - 1 > 0$

$2n^2 > n^2 + 2n + 1 = (n+1)^2$

1.c) sol^n

$$\frac{(a^2+1)(b^2+1)(c^2+1)(d^2+1)}{abcd}$$

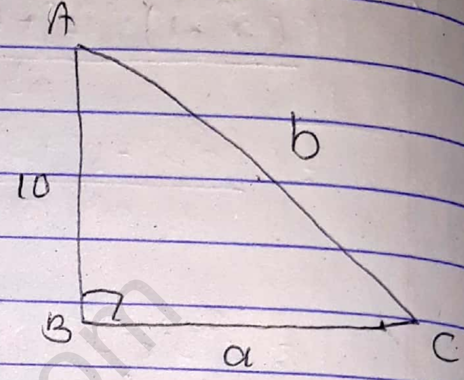
$$= \frac{(a^2b^2+a^2+b^2+1)(c^2d^2+c^2+d^2+1)}{abcd}$$

$$= \frac{a^2b^2c^2d^2+a^2b^2c^2+a^2b^2d^2+a^2b^2+a^2c^2d^2+a^2c^2+a^2d^2+a^2+b^2c^2d^2+b^2c^2+b^2d^2+b^2+c^2d^2+c^2+d^2+1}{abcd}$$

In numerator, there are 16 term, if put any ~~integ~~ positive real value for a, b, c and d, ~~it gives value for~~ in above expression, it always gives value more than 16 which we have to prove.

$$\therefore \frac{(a^2+1)(b^2+1)(c^2+1)(d^2+1)}{abcd} \geq 16$$ proved//

2.a)



Since, the given side of right angled triangle is even.

So,

$$\frac{a^2}{4} = \frac{10^2}{4} = 25.$$

The value before and after 25 are the other two side of right angled triangle i.e 24 and 26 are two side of right angled triangle.

Using Pythagorus theorem

$$26^2 = 24^2 + 10^2$$

$$676 = 676 \text{ which is true.}$$

$\therefore a = 24$ and $b = 26$.

2.b) Soln

Let v denotes no. of vertices, E denotes no. of edges and F denotes no. of faces

There are 3 edges meeting at each vertex twice,

$$\therefore E = \frac{3v}{2} \quad \text{— (i)}$$

Again, 6 triangular face meet at each vertex.

$$6F = 3v$$

$$F = \frac{3v}{6} = \frac{v}{2} \quad \text{— (ii)}$$

By using eular formula,

$$V - E + F = 2$$

$$V - \frac{3V}{2} + \frac{V}{2} = 2$$

$$2V - 3V + V = 4$$

$0 = 4$ which is impossible.

if 5 triangular face meet at each vertex

$$5F = 3V$$

$$F = \frac{3}{5} V$$

Using eular formula

$$V - \frac{3V}{2} + \frac{3V}{5} = 2$$

$$10V - 15V + 6V = 20$$

$$V = 20$$

$$E = \frac{3 \times 20}{2} = 30$$

$$F = \frac{3 \times 20}{5} = 12$$

3.b) sol^n

Let's abbreviate "the quart container" as "the 9qc" and the 4 quart container as "the 4qc"

i) Fill the 9qc from the river
ii) Fill the 4qc from the 9qc, leaving 5qts. in the 9qc.
iii) Empty the 4qc
iv) Fill the 4qc from the 9qc, leaving 1qt. in the 9qc.
v) Empty the 4qc.
vi) Pour the 1qt. from the 9qc into the 4qc, leaving room in the 4qc for 3qts.
vii) Fill the 9qc from the river
viii) Finish filling the 4qc from the 9qc which will amount to pouring off 3qts. from the 9qc leaving 6 qts. in the 9qc.

3.c) sol^n

31	17	18	28
20	26	25	23
24	22	21	27
19	29	30	16

Q.b)

sol^n

Divide 9 pearls into 3 group. label the group by G1, G2 and G3. Weight G1 against G2.

case 1

If they happen balance then G1 and G2 are control pearls. The odd is one of the pearls in G3.

case 2

If they do not balance, all the pearls in G3 are control pearls. The odd pearls in either in G1 or G2 but we don't know which one is

for case 1

Weight G1 against G3, they will not balance, G3 will either be lighter or heavier. Let G3 is heavier i.e. odd pearls is heavier than other. select any two pearls from G3 and weight them against each other. If they balance then odd pearls is third ~~pea~~ pearls in G3 and it is heavier if they do not balance, then heavier of two is odd pearls.

for case 2

Whether G1 or G2 is heavier for specified say G1 is heavier. Now, weight G1 against G3. If they balance, then odd pearls is G2.

and it is lighter from G2, weight them against each other. If they balance, then odd pearls the third one from G2 and it is lighter. If they do not balance, then lighter of 2 is odd.

If G1 and G3 do not balance, then only possibilities is that G1 is heavier than G3. so, odd pearls is in G1 and it is heavier. Again, weight any two pearls from G1 against each other. If they are balance, then third one is odd and heavier, else if they are balance, then one from two of G1 is odd.

5.a) sol^n

Let a, b be two unequal rational numbers and let $a<b$. suppose to the contrary that there was an interval $[a,b]$ with a, b ~~interval~~ rational which contained no irrational numbers. That would imply that the interval contained only rational numbers since the reals are composed of rationals and irrational numbers. Furthermore, this interval has measure b-a, a contradiction since this is a subset of Q which has measure zero.

let $a = \frac{m}{n}$, $b = \frac{p}{q}$ $a > b$ - then

$a-b = \frac{m}{n} - \frac{p}{q} = \frac{mq-pn}{nq}$. since $mq-np \geq 1$,
we can construct an irrational number $a + \frac{1}{nq\sqrt{2}}$ which is between a and b.

c)

sol^n

Basic step

for $n = 1$,

$p(1) = 5^1 + 2 \cdot 3^{1-1} + 1 = 8$ which is divisible by 8, so true.

Inductive step

we assumed that $p(n)$ is true for any arbitary k such that

$p(k) = 5^k + 2 \cdot 3^{k-1} + 1$ which is divisible by 8

we have to show that $p(n)$ is true for $(k+1)$

i.e. $p(k+1) = 5^{k+1} + 2 \cdot 3^{k-1+1} + 1$ is divisible by 8

$5^{k+1} + 2 \cdot 3^k + 1$

Now,

$5^{k+1} + 2 \cdot 3^{k-1+1} + 1$

$5^{k+1} + 2 \cdot 3^k + 1$

$= 5^k \cdot 5 + 2 \cdot 3^k + 1$

$= 5^k \cdot (8-3) + 2 \cdot 3^k + 1$

$= 8 \cdot 5^k - 3 \cdot 5^k + 2 \cdot 3^k + 1$

$= 8 \cdot 5^k - 3(5^k - 2 \cdot 3^{k-1}) + 1$

Here,

$8 \cdot 5^k$ is multiple of 8 and $5^k - 2 \cdot 3^{k-1}$ is divisible by 5 as we have assumed.

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2018 (spring)

1.a)

Sol^n

for a instance,

$$g(k) = k^2$$

$$g(k+1) - g(k) = (k+1)^2 - k^2$$
$$= 2k+1$$

Here,

$$2^2 - 1^2 = 2\cdot1+1$$
$$3^2 - 2^2 = 2\cdot2+1$$
$$4^2 - 3^2 = 2\cdot3+1$$
$$\vdots \quad \vdots \qquad \vdots \quad \vdots \quad \vdots$$

$$k^2 - (k-1)^2 = 2(k-1)+1$$
$$(k+1)^2 - k^2 = 2k+1$$

Now, adding each column,

$$(k+1)^2 - 1 = 2(1+2+3+\cdots+k) + [1+1+1\cdots 1]$$

or $$k^2 + 2k = 2\cdot S + k$$

where 'S' is sum we wish to calculate

$$S = \frac{k^2+k}{2}$$

$$S = \frac{k(k+1)}{2}$$

1.b) Sol^n

$$\frac{6^{300} \times 15^{800}}{150!} - 10^{99}$$

For $6^{300} \times 15^{800}$

$= 2^{300} \times 3^{300} \times 3^{800} \times 5^{800}$

we have 300 number of 2 and 800 no. of 5. we can make maximum 300 pairs of 2 and 5. This means we have 300 times 10 in our multiplication which means we have 300 zeroes at the end of the product.

For 150!

Multiple of 5 = $\frac{150}{5} = 30$

Multiple of 25 = $\frac{150}{25} = 6$

Multiple of 125 = $\frac{150}{125} = 1.2 \simeq 1$

we get $30+6+1 = 37$ no. of 5

Now,

$\frac{6^{300} \times 15^{800}}{150!} = 263$ no. of zeroes

For 10^{99}

Here, we have 99 zeroes

finally,

$263 - 99 = 164$ no. of zeroes.

2.a) sol^n

If no. of student $k=0 \rightarrow$ no. of handshake $=0$ "even"
If no. of students $k=1 \rightarrow$ no. of handshake $=0$ "even"
If no. of students $k=2 \rightarrow$ no. of handshake $=1$ "odd"
If no. of student $k=3 \rightarrow$ no. of handshake $=3$ "odd"
If no. of student $k=4 \rightarrow$ no. of handshake $=6$ "even"
If no. of student $k=5 \rightarrow$ no. of handshake $=10$ "even"
If no. of student $k=6 \rightarrow$ no. of handshake $=15$ "even"
If no. of student $k=7 \rightarrow$ no. of handshake $=21$ "odd"
If no. of student $k=8 \rightarrow$ no. of handshake $=28$ "even"
If no. of student $k=9 \rightarrow$ no. of handshake $=36$ "even"

We see that the first two no. for handshakes are even, then there are two odd, then there are two even and so fourth.

2.b) sol^n

Let V denote no. of vertices, E denote no. of edges and F denote no. of face (or region). There are 3 edges meeting at each vertex but we counted each edges twice, therefore, we can write

$$E = \frac{3V}{2} \quad \text{--- (i)}$$

Since 5 triangular face meet at each vertex. so

$$5f = 3V$$

$$f = \frac{3}{5}V \quad \text{--- (ii)}$$

By using eular formula

$$v - e + f = 2$$

$$v - \frac{3v}{2} + \frac{3v}{5} = 2$$

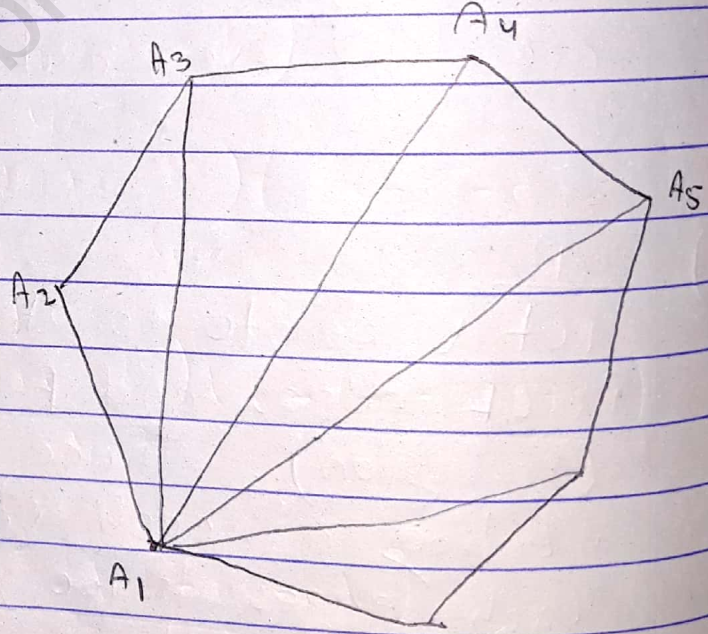
$$10v - 15v + 6v = 20$$

$$v = 20$$

$$e = \frac{3 \times \cancel{20}^{10}}{\cancel{2}} = 30$$

$$f = \frac{3 \times \cancel{20}^{4}}{\cancel{5}} = 12$$

3.a)



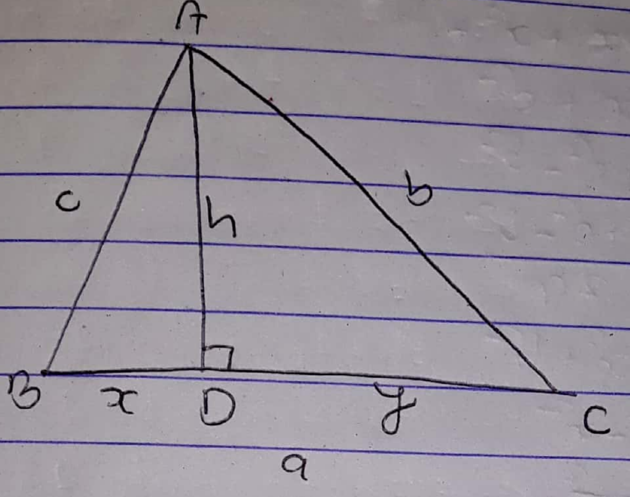
from any one of the vertices, say A_1, construct diagonals to other vertices

There are altogether $(n-2)$ triangles

sum of angles of each triangle $= 180°$

sum of interior angles of k-sided polygon $= (k-2) \times 180°$

3.b)



consider a $\triangle ABC$ such that $AB = c$, $BC = a$, $AC = b$

Let $AD \perp BC$ and $AD = h$

Let $BD = x$, $DC = y$

The perimeter of triangle is $a+b+c$

i.e $2s = a+b+c$ — (i)

We know that area of triangle

$\frac{1}{2} \times \text{base} \times \text{height}$

or, $= \frac{1}{2} \times a \times h$ — (ii) $h = \cancel{2a}$

In $\triangle ADB$, $\angle ADB = 90^\circ$

$\therefore c^2 = \cancel{g}\ x^2 + h^2$ [By pythagoras theorem]

Similarly,

in $\triangle ADC$, $\angle ADC = 90^\circ$

$\therefore b^2 = y^2 + h^2$ [By pythagoras theorem]

From the figure

$x + y = a$

$y = a - x$

$\Rightarrow y^2 = (a-x)^2 = a^2 - 2ax + x^2$

Adding h^2 on both sides

$$y^2+h^2 = a^2-2ax+x^2+h^2$$

$$b^2 = a^2-2ax+c^2$$

$$2ax = a^2+c^2-b^2$$

$$x = \frac{a^2+c^2-b^2}{2a}$$

consider, $c^2 = x^2+h^2$

$$\Rightarrow h^2 = c^2-x^2$$

$$\cancel{\Rightarrow h^2 = c^2}$$

$$\Rightarrow h^2 = c^2-\left(\frac{a^2+c^2-b^2}{2a}\right)^2$$

$$= \frac{4a^2c^2-(a^2+c^2-b^2)^2}{4a^2}$$

$$= \frac{(2ac)^2-(a^2+c^2-b^2)^2}{4a^2}$$

$$= \frac{(2ac+a^2+c^2-b^2)(2ac-a^2-c^2+b^2)}{4a^2}$$

$$\cancel{= [(a+c)^2}$$

$$= \frac{(a^2+2ac+c^2-b^2)[b^2-(a^2-2ac+c^2)]}{4a^2}$$

$$= \frac{[(a+c)^2-b^2][b^2-(a-c)^2]}{4a^2}$$

$$= [(a+c+b)(a+c-b)][(b+a-c)(b-a+c)]$$

$$= \frac{2s(2s-b-b)(2s-c-c)(2s-a-a)}{4a^2}$$

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$$= \frac{2s(2s-2b)(2s-2c)(2s-2a)}{4a^2}$$

$$= \frac{\cancel{16}^{4} s(s-a)(s-b)(s-c)}{\cancel{4}a^2}$$

$$h^2 = \frac{4s(s-a)(s-b)(s-c)}{a^2}$$

$$\therefore h = \frac{2\sqrt{s(s-a)(s-b)(s-c)}}{a}$$

from ①, we have

$$\text{area of } \triangle ABC = \frac{1}{2} \times a \times \frac{2\sqrt{s(s-a)(s-b)(s-a)}}{a}$$

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

This is known as heron's formula

a_1

Fibonacci sequence

$$F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$$

Proof :-

Let,
$$f(x) = a_0 + a_1 x + a_2 x^2 + \cdots$$
$$x f(x) = a_0 x + a_1 x^2 + a_2 x^3 + \cdots$$
$$x^2 f(x) = a_0 x^2 + a_1 x^3 + a_2 x^4 + \cdots$$

$$F(x) - xF(x) - x^2 F(x)$$
$$= \cancel{a_0 + a_1 x + a_2}$$
$$= a_0 + (a_1 - a_0)x + (a_2 - a_1 - a_0)x^2 + (a_3 - a_2 - a_1 - a_0)x^3 + \cdots$$

But basic property of fibonacci sequence,
$a_2 - a_1 - a_0 = 0$, $a_3 - a_2 - a_1 - a_0 = 0$, $a_1 - a_0 = 0$, etc.
and $a_0 = a_1 = 1$

$$\therefore F(x) - xF(x) - x^2 F(x) = 1$$
$$(1 - x - x^2) F(x) = 1$$
$$F(x) = \frac{1}{1 - x - x^2}$$

Now, $1 - x - x^2$ is a quadratic equation, so its factor are

$$x = \frac{1 \pm \sqrt{1 - 4.1.(-1)}}{2.(-1)}$$
$$= \frac{1 \pm \sqrt{5}}{-2}$$

Let the roots of our original quadratic be

P.T.O

$\alpha = \dfrac{1+\sqrt{5}}{-2}$ and $\beta = \dfrac{1-\sqrt{5}}{-2}$

Since, both α and β are roots of the quadratic, the must both satisfy

$$x^n = f_n x + f_{n-1}. \text{ so,}$$

$$\alpha^n = f_n \alpha + f_{n-1}$$

and,

$$\beta^n = f_n \beta + f_{n-1}$$

subtracting the 2nd equation from the first equation yields

$$\alpha^n - \beta^n = f_n \alpha + f_{n-1} - f_n \beta - f_{n-1}$$

$$= f_n(\alpha - \beta)$$

$$\left(\frac{1+\sqrt{5}}{-2}\right)^n - \left(\frac{1-\sqrt{5}}{-2}\right)^n = f_n\left(\frac{1+\sqrt{5}}{-2} - \frac{1-\sqrt{5}}{-2}\right)$$

$$\left(\frac{1+\sqrt{5}}{-2}\right)^n - \left(\frac{1-\sqrt{5}}{-2}\right)^n = f_n\left(\frac{\cancel{1}+\sqrt{5}-\cancel{1}+\sqrt{5}}{-2}\right)$$

$$\left(\frac{1+\sqrt{5}}{-2}\right)^n - \left(\frac{1-\sqrt{5}}{-2}\right)^n = f_n\left(\frac{2\sqrt{5}}{-2}\right) = -f_n\sqrt{5}$$

$$f_n = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n}{\sqrt{5}}$$

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3x3

8	1	6
3	5	7
4	9	2

4x4

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

sol^n

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

34

5x5

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

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Page ____

6x6

Solve by 3x3

I ~~III~~ III

8	1	6	26	19	24
3	5	7	21	23	25
4	9	2	22	27	20
35	28	33	17	10	15
30	32	34	12	14	16
31	36	29	13	18	11

II

IV

now,

35	1	6	26	19	24
3	32	7	21	23	25
31	9	2	22	27	20
8	28	33	17	10	15
30	5	34	12	14	16
4	36	29	13	18	11

~~111~~ 111

7x7

30	39	48	1	10	19	28
38	47	7	9	18	27	29
46	6	8	17	26	35	37
5	14	16	25	34	36	45
13	15	24	33	42	44	4
21	23	32	41	43	3	12
22	31	40	49	2	11	20

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8x8

1	63	62	4	5	59	58	8
56	10	11	53	52	14	15	49
48	18	19	45	44	22	23	41
25	39	38	28	29	35	34	32
33	31	30	36	37	27	26	40
24	42	43	21	20	46	47	17
16	50	51	13	12	54	55	9
57	7	6	60	61	3	2	64

1	63	62	4	5	59	58	8
56	10	11	53	52	14	15	49
48	18	19	45	44	22	23	41
25	39	38	28	29	35	34	32
33	31	30	36	37	27	26	40
24	42	43	21	20	46	47	17
16	50	51	13	12	54	55	9
57	7	6	60	61	3	2	64

260

~~for Tonor~~

Tower of Hanoe

3 disk

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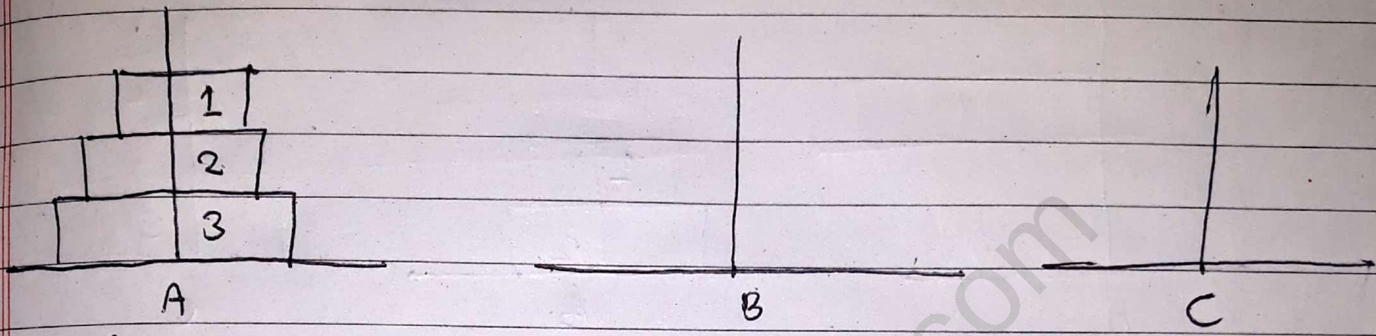
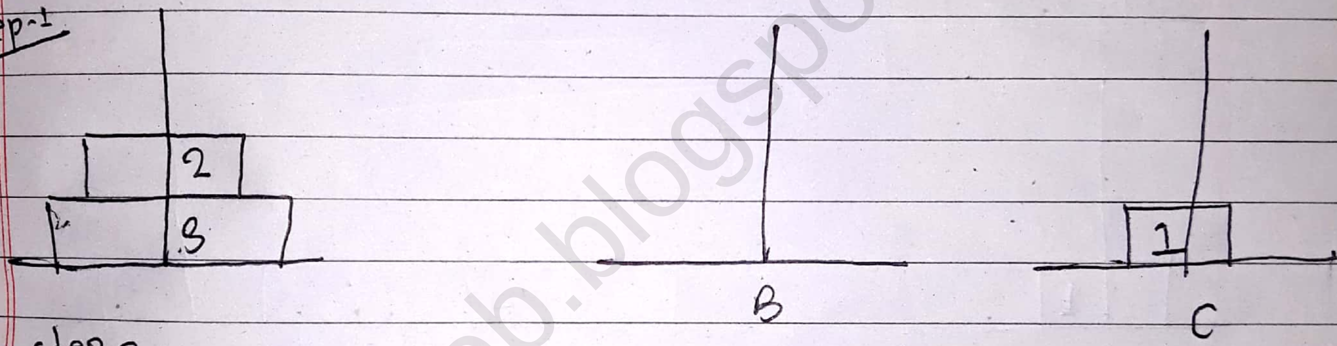
Page ______

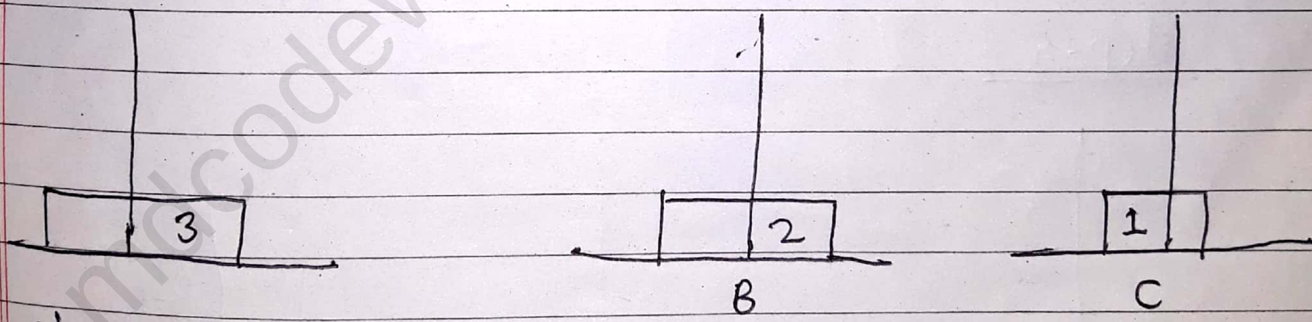


sol^n

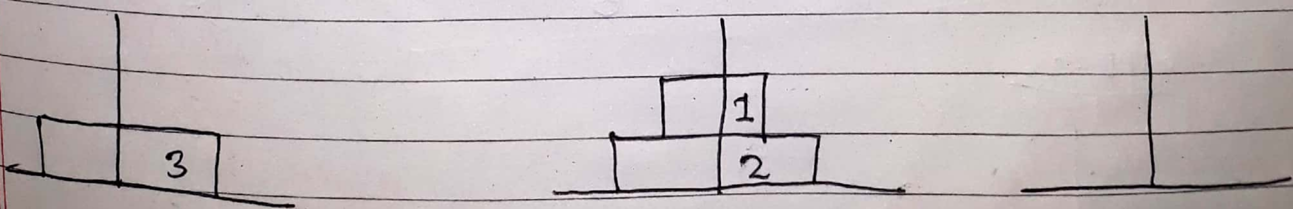
step-1



step.2



step-3



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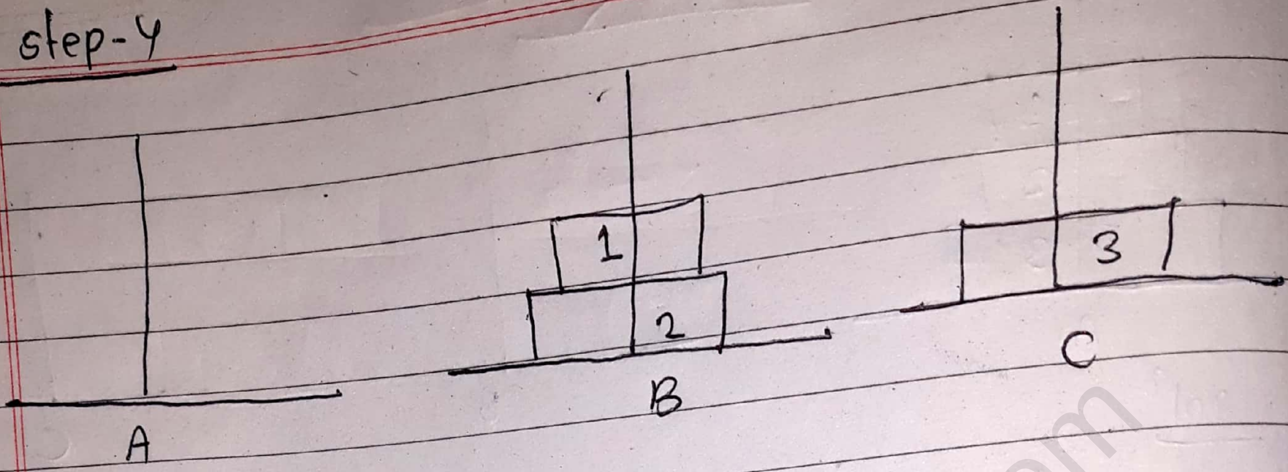
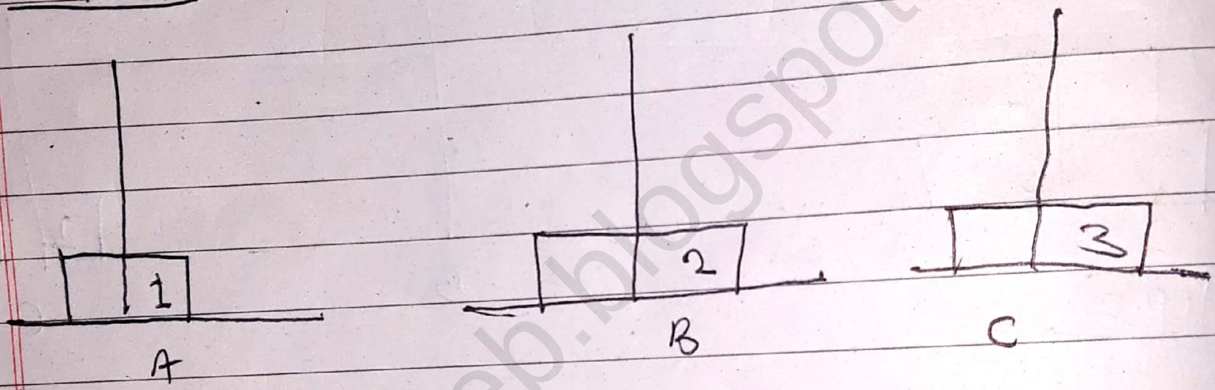
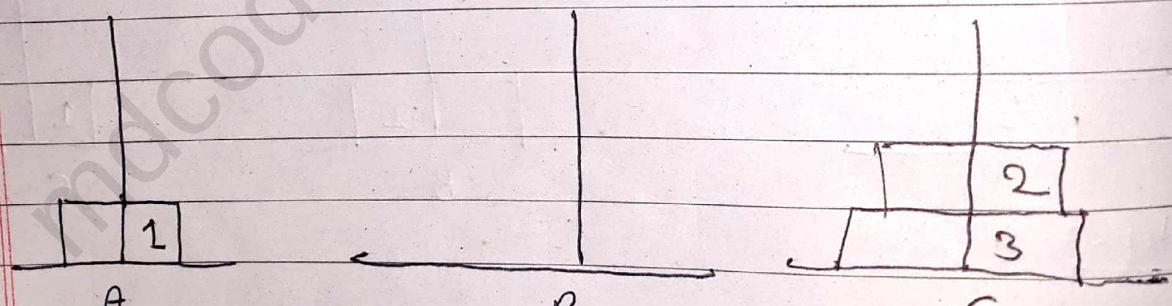
step-4



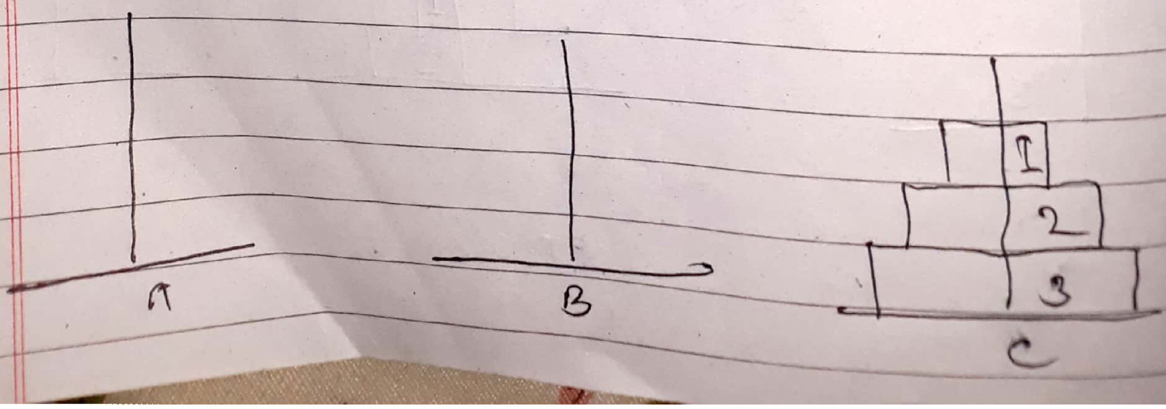
step-5



step-6



step-7



4 disk

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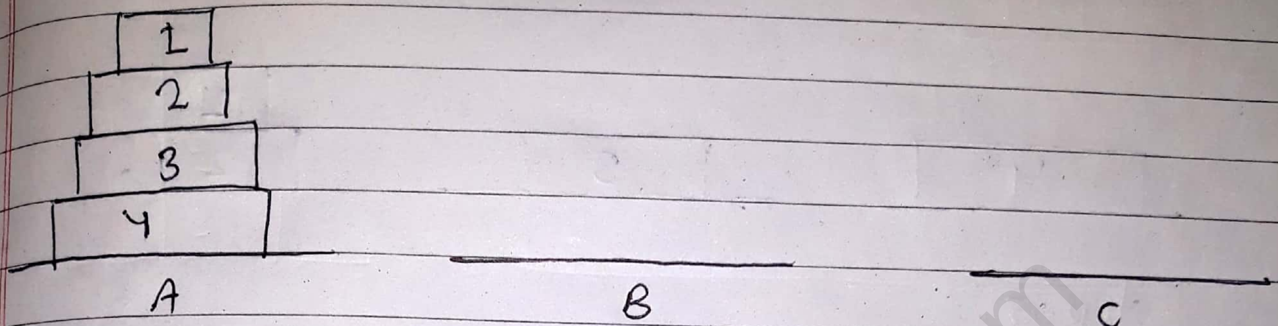
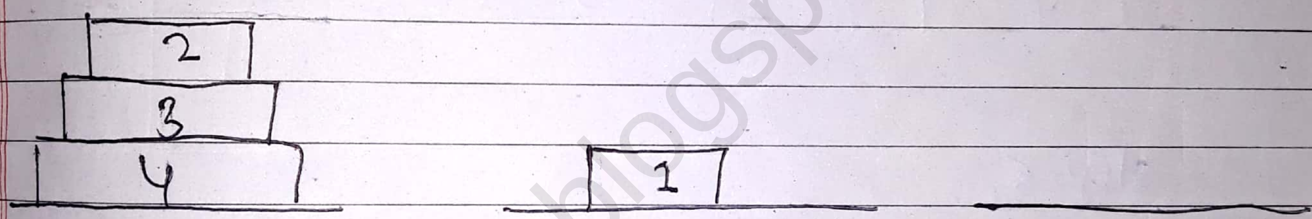
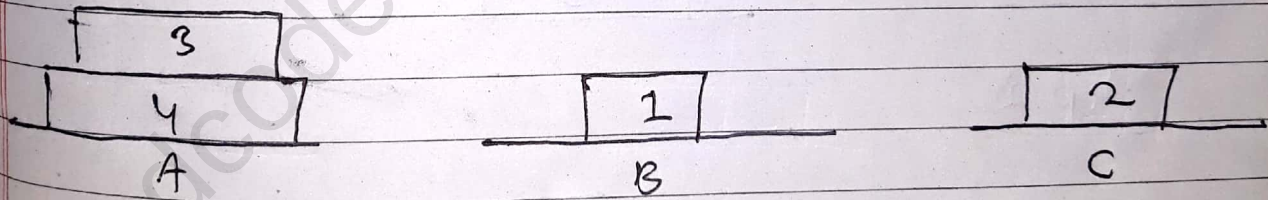


step-1



step 2



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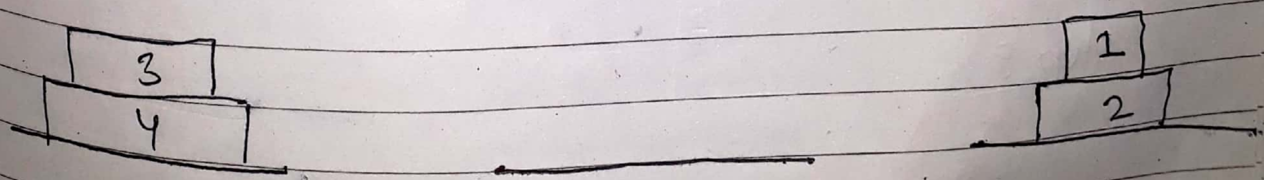

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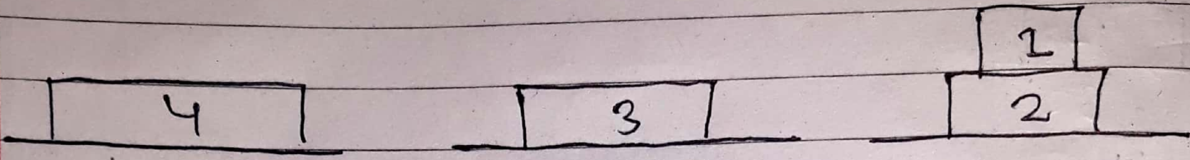
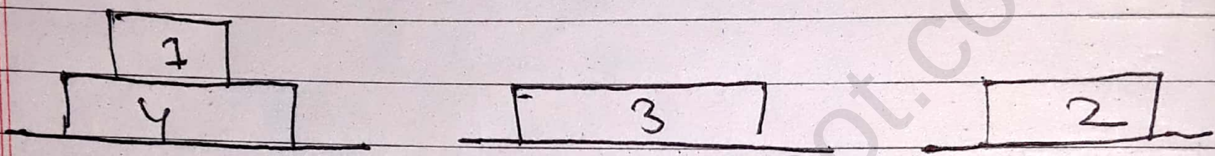
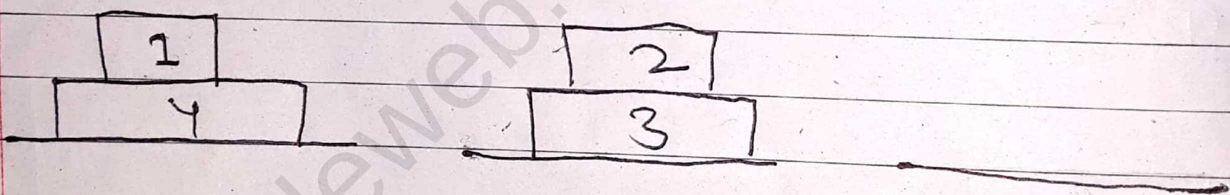


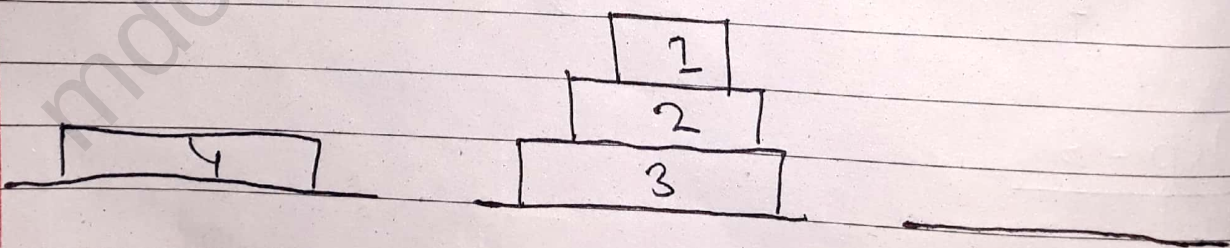
step-5



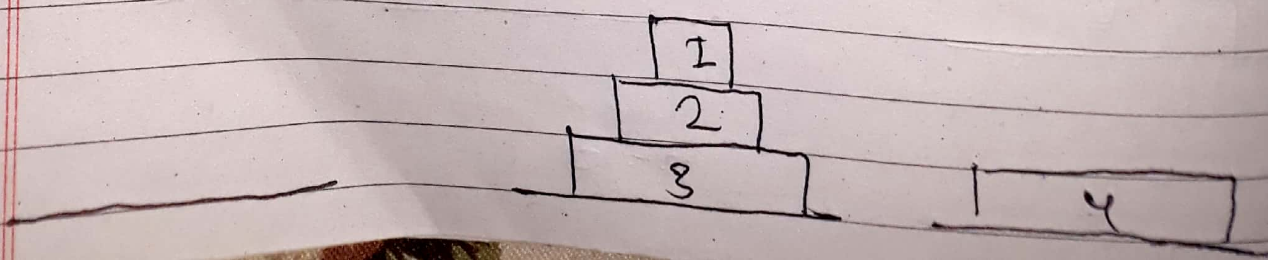
step-6



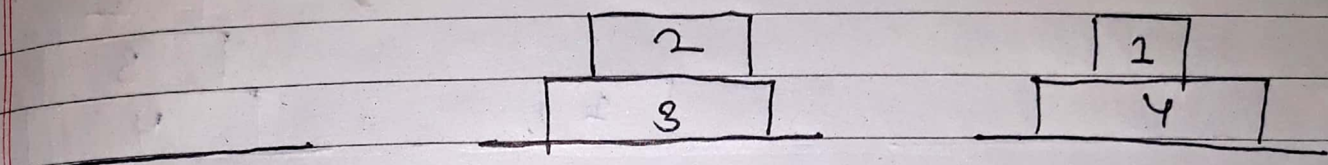
step 7



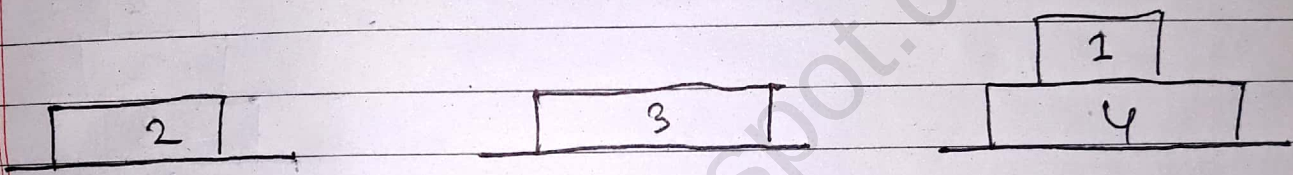
step 8



step-9



step-10



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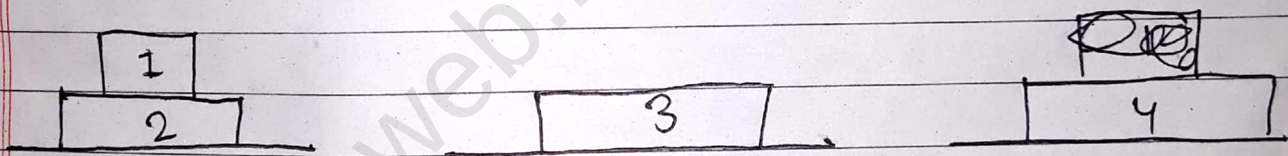
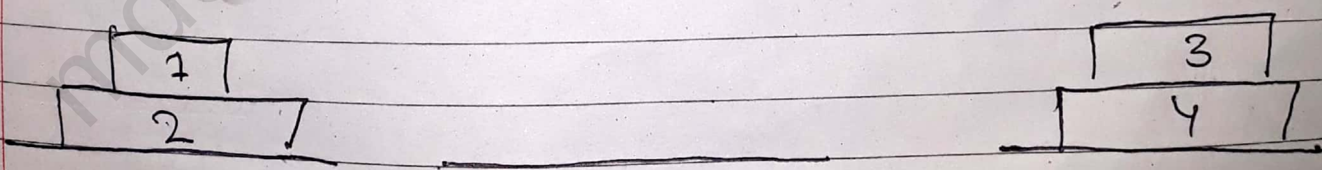
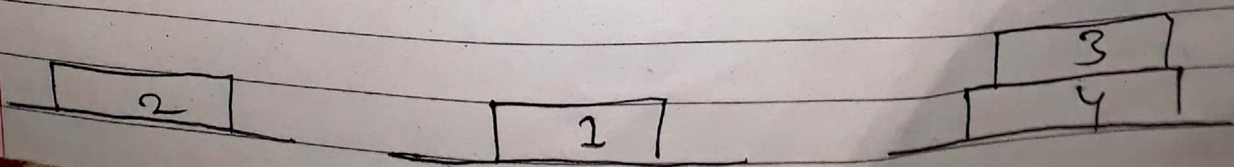


step-12



step-13



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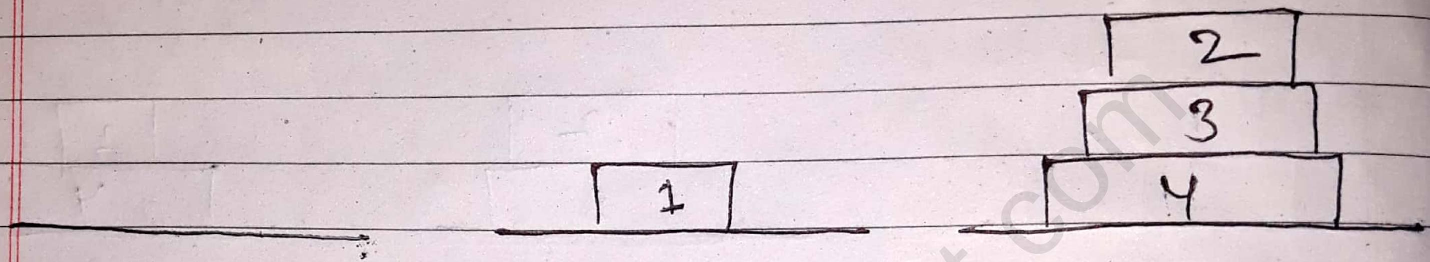
step-14



step-15

2014 (fall)

~~213213~~

6.b) $\frac{215215}{7} = 30745$

$\frac{30745}{11} = 2795$

$\frac{2795}{13} = 215$

This works because $7 \times 11 \times 13 = 1001$

Multiplying any three-digit number by 1001 causes it to repeat its digits.

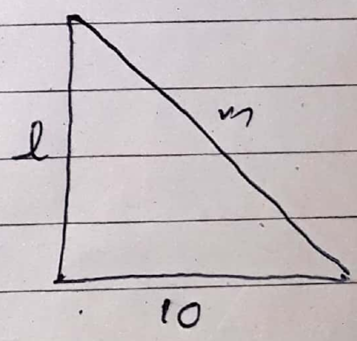
$215 \times 1001 = 215215$

Because $1000 \times 215 = 215000 + 1 \times 215 = 215215$

Therefore this work for all three digits numbers.

6.b

Here given one side of right ~~angle~~ triangle which is not hypotenuse so it may be base or ~~height~~ perpendicular



Given length of side is 10 which is even.

so. $\frac{n^2}{4} = \frac{10^2}{4} = \frac{100}{4} = 25$

Now, the number before and after 25 are the ~~two remaining~~ two side of right triangle i.e 24 and 26

~~let~~ say $m = 26$ and $l = 24$

P.T.O

~~x+10y~~

Now, using pythagoras theorem.

$$l^2 = m^2 + 10^2$$

$$26^2 = 24^2 + 10^2$$

$$676 = 576 + 100$$

$676 = 676$ which is true

Hence, the value of m and l are 24 and 26 or, 26 or 24

Equilateral triangle

$$P = 3a$$

$$s = 3a/2$$

$$A = \frac{1}{4} \times \sqrt{3} \times a^2$$

$$h = \frac{1}{2} \times \sqrt{3} \times a$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{\frac{3a}{2}\left(\frac{3a}{a}-a\right)\left(\frac{3a}{a}-a\right)\left(\frac{3a}{a}-a\right)}$$

$$= \sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)^3} = \sqrt{\frac{3a}{2}\left(\frac{a^3}{8}\right)}$$

$$= \sqrt{\frac{3a^4}{16}}$$

2016 (fall)

Date ____
Page ____

5.b) Sol^n

Given,

$P = \$100$

Since, she is 10 yrs old and she want to withdraw the money on her 21^{st} birthday.

so, $T = 11$ years

i) an account with 5% interest compounded daily

$\therefore R = 5\%$

for compounded daily,

~~$\therefore n =$~~ $R = (5/100)/365 = \frac{1}{7300}$ per day

$n = 11 \text{ yrs} \times 365 \text{ days} = 4015$ days

$C.A = P * (1+R)^n$

$= \$100\left(1 + \frac{1}{7300}\right)^{4015}$

$= \$173.31$

ii) an account with 5.1% interest compounded weekly

$R = 5.1\%$

for compounded weekly

$R = (5.1/100)/52 = \frac{51}{52000}$ per week

$n = 11 \text{ yrs} \times 52 \text{ weeks} = 572$ week

$C.A = P * (1+R)^n$

$\$100 * \left(1 + \frac{51}{52000}\right)^{572} = \$175.194$

compound amount weekly is higher than compound amount daily. So, an account with 5.1% interest comp. weekly is better scheme snow ~~pr~~ plow problem

Q. Before noon, it started

.2015 (Spring)
5.b

Sol^n

Let t be the time measured in hours after noon.
Let $x(t)$ be the distance the snowplow has travelled
Let $h(t)$ be the height of the snow at time t.
Let α be the constant rate of snow removal. (~~in any~~
Let k be the constant rate at which snow ~~comes~~ falls.
Let b be the (unknown) ~~hours~~ number of hours before noon that it started snowing.

The change in height is given by the rate the snow falls,

$$\frac{d}{dt} h(t) = k$$

$$\Rightarrow h(t) = kt + c \qquad h(-b) = 0, \; c = kb$$

$$h(t) = k(t+b)$$

The rate α is proportional to the cross-sectional of the snow being plowed and the speed of the

P.T.O

road is

truck. Let's assume the width of the road is $h \propto d$

a constant w. Then $\alpha = w h(t) \frac{d}{dt} x(t)$

$$C = \frac{\alpha}{k w}$$

$$\Rightarrow \frac{d}{dt} x(t) = \frac{C}{t+b}$$

which is separable differential equation,
Integrating both sides yields

$$\cancel{x(t) = C\ln(t)}$$
$$x(t) = C\ln|t+b| + D$$

plugging in the condition $x(0) = 0$ gives the constant of integration $D = -C\ln(b)$. plugging in the other two condition $x(1) = 1$ and $x(2) = 2$, then lets us solve for $b = 0.618$ hour. So, the snow began 37 minutes before noon. That is at about 11:23 am.

a

Q. what is the last digit of 3^{4789}.

⇒ soln

Here, we have,

$3^1 = 3$

$3^2 = 9$

$3^3 = 27$

$3^4 = 81$

$3^5 = 243$ & so on.

~~The possible number at the least digit of 3 are {3, 9, 27, 81 ...} and repeat.~~
~~Here one is the special case.~~
~~i.e 1.1 = 1~~
~~which get from 4th power of 3.~~

The last digit pattern is 3, 9, 7, 1, 3, 9, 7, 1 ...
The last digit is repeated after every fourth power
If we do

$\frac{4789}{4}$, we get remainder 1,

∴ The last digit is 1st term of last digit pattern
i.e last digit is 3 Ans

Q. Prove by induction method that

$$\sum 3^n = \frac{1}{2}(3^{n+1} - 1), \quad n \geq 0$$

Sol^n

$$\sum 3^n = 3^1 + 3^2 + 3^3 + \cdots + 3^n$$

L.H.S of $p(0) = 3^0 = 1$

R.H.S of $p(0) = \frac{1}{2}(3^{0+1} - 1)$

$= 1$

So, $p(0)$ is true

Suppose $P(K)$ is true for some integer K.

i.e. $3^1 + 3^2 + 3^3 + \cdots + 3^K = \frac{1}{2}(3^{K+1} - 1)$

We have to show that $P(K)$ is true for $K+1$

i.e $3^1 + 3^2 + 3^3 + \cdots + 3^K + 3^{K+1} = \frac{1}{2}(3^{\{(K+1)+1\}} - 1)$

$$= \frac{1}{2}(3^{K+1} - 1) + 3^{K+1}$$

$$= \frac{1}{2}3^{K+1} - \frac{1}{2} + 3^{K+1}$$

$$= \left\{\left(\frac{1}{2} + 1\right)3^{K+1}\right\} - \frac{1}{2}\}$$

$$= \left\{\left(\frac{3}{2}\, 3^{K+1}\right) - \frac{1}{2}\right\}$$

$$= \frac{1}{2} \times 3^{K+1+1} - \frac{1}{2}$$

$= \frac{1}{2}(3^{\{(K+1)+1\}} - 1)$ which is true.

Proof by contradiction

Q-1. The difference of any rational number and any irrational number is irrational.

Sol^n

Let a rational number 'x' and an irrational number 'y' such that $(x-y)$ is rational.

By defination of rational, we have

$x = a/b$ for some integers a and b with $b \neq 0$

$x - y = c/d$ for some integers c and d with $d \neq 0$

By substitution, we have,

$$x - y = c/d$$

$$\frac{a}{b} - y = c/d$$

$$y = a/b - c/d$$

$$= (ad-bc)/bd$$

But $(ad-bc)$ are integers [because a, b, c, d are all integers and products and differences of integers are integers] and $bd \neq 0$ [by zero product property]

Therefore, by definition of rational y is rational. Hence, the supposition is false and the theorem is true.

Q.2. The negative of any irrational number is irrational.

Sol^n

suppose irrational number x such that $-x$ is rational.

By definition of rational,

$-x = a/b$ for some integers a and b with $b \neq 0$

~~Multiply both integers a and b with $b \neq 0$~~

Multiplying both sides by -1.

$$x = -(a/b)$$
$$= -a/b$$

But -a and b are integers [since a and b are integers] and $b \neq 0$ [by zero product property]. Thus, x is a ratio of the two integers -a and b with $b \neq 0$. Hence, by definition of rational, x is rational, which is a contradiction. This contradiction shows that the supposition is false and so the given statement is true.

Q.3. For all integer n, if n^2 is odd, then n is odd.

Proof:-
Suppose an integer n such that n^2 is odd and n is even.
By definition of even, we have,

$n = 2k$ for some integer k

so, by ^

or, $n.n = (2k).(2k)$

$= 2(2.k.k)$

Now, $(2.k.k)$ is an integer because products of integers are integer and 2 and k are integers.
Hence,

$n.n = 2$

$n^2 = 2$

so by definition of n^2 ~~is~~ even, is even.
So, the conclusion is since n is even, n^2, which is the product of n with itself, is also even. This contradicts the supposition that n^2 is odd. Here, the supposition is false and the ~~pro~~ theorem is true.