Polarization

Picture Source:

https://www.sunglassesforsport.com/

Better to watch:

1. https://www.amazon.co.uk/Polaroid-P32RDP0119U-Freeview-Playback-Renewed/dp/B07YST7FJK
2. https://info.uvex-safety.com.au/polarised-safety-glasses
3. https://www.youtube.com/watch?v=gP751qpm4n4
4. flickr.com/photos/finlap/50897399142/in/photostream/

Polarization:

The phenomenon of interference , diffraction, reflection and refraction are the characteristics of all waves. However, the polarization is the characteristic of the transverse wave.

Definition:

The phenomenon of restriction of vibration of light in a particular plane or direction is called polarization.

The plane in which the vibration of polarized light takes place is called the plane of vibration. And the plane perpendicular to the plane of vibration where no vibration occurs is called the plane of polarization.

The material which polarizes the transverse wave is called the polaroid. Examples are tourmaline crystal, Nicol prism, calcite crystal, quartz crystal.

Representation of Polarized and Unpolarized Light:

Light is not necessarily polarized. Light can be unpolarized. That means the source has vibrations in many planes at once as shown in the figure.

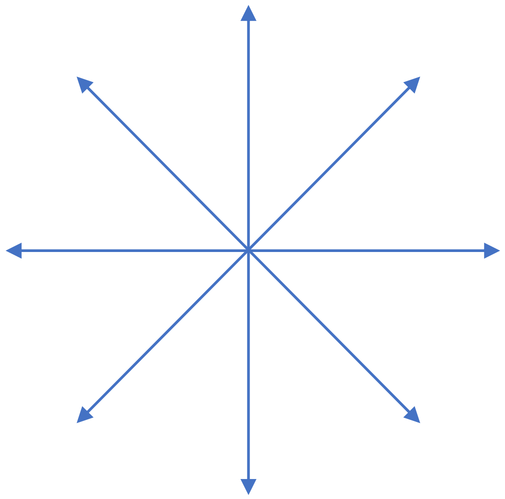


Fig: Vibrations of the electric field vectors in unpolarized light.

An ordinary light bulb emits unpolarized light as the sun does. Polarized Light can be obtained from unpolarized light using the crystal like tourmaline. Nowadays with modern technology polarized light can be obtained with polarizing sheets in which there are complicated long molecules arranged with their axes parallel.

This Polaroid acts as a series of parallel slits to allow one orientation of polarization to pass through whereas a perpendicular polarization is almost completely absorbed.

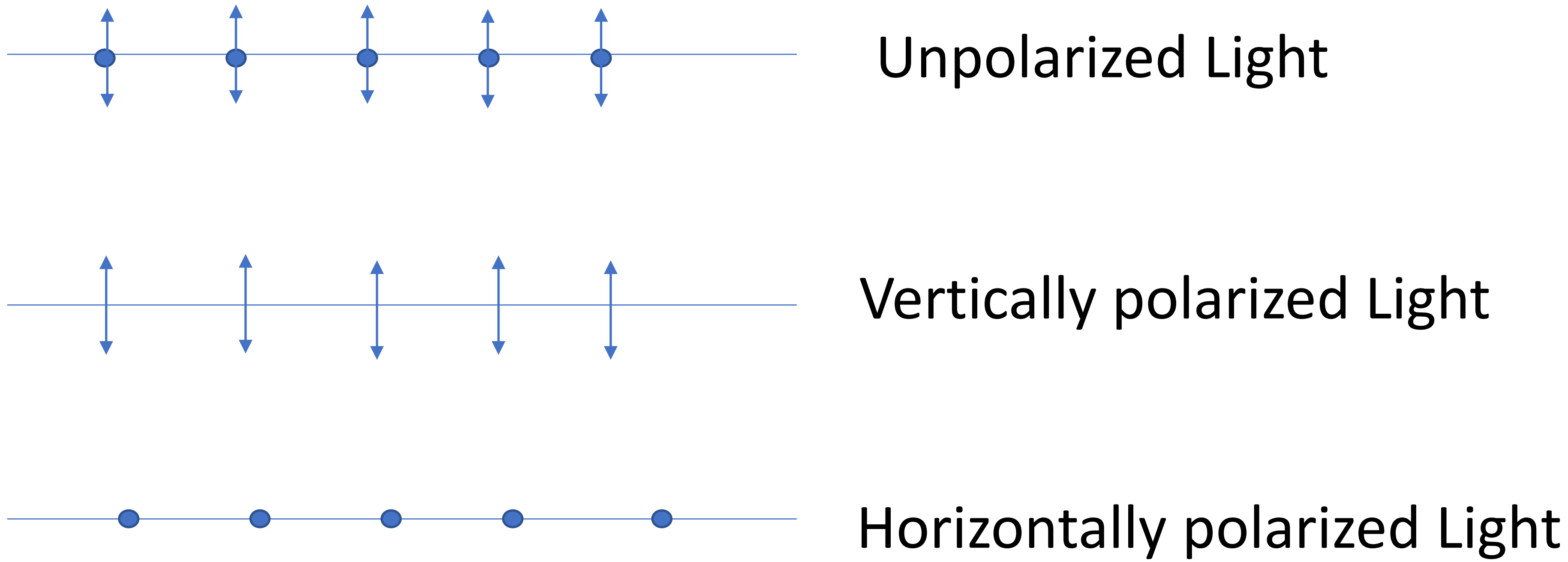


Fig: Pictorial representation of polarized and unpolarized light

What is polarisation?

Polarisation refers to light waves that form different properties in various directions, particularly when vibration takes place in a single plane. Polarised light is most useful after it has been filtered. In photography, polarising filters reduce polarised light, to increase contrast, darken skies, and reduce reflections and glare.

Source:

https://urth.co/magazine/polarisation-history/

Polarization by reflection (Brewster's law):

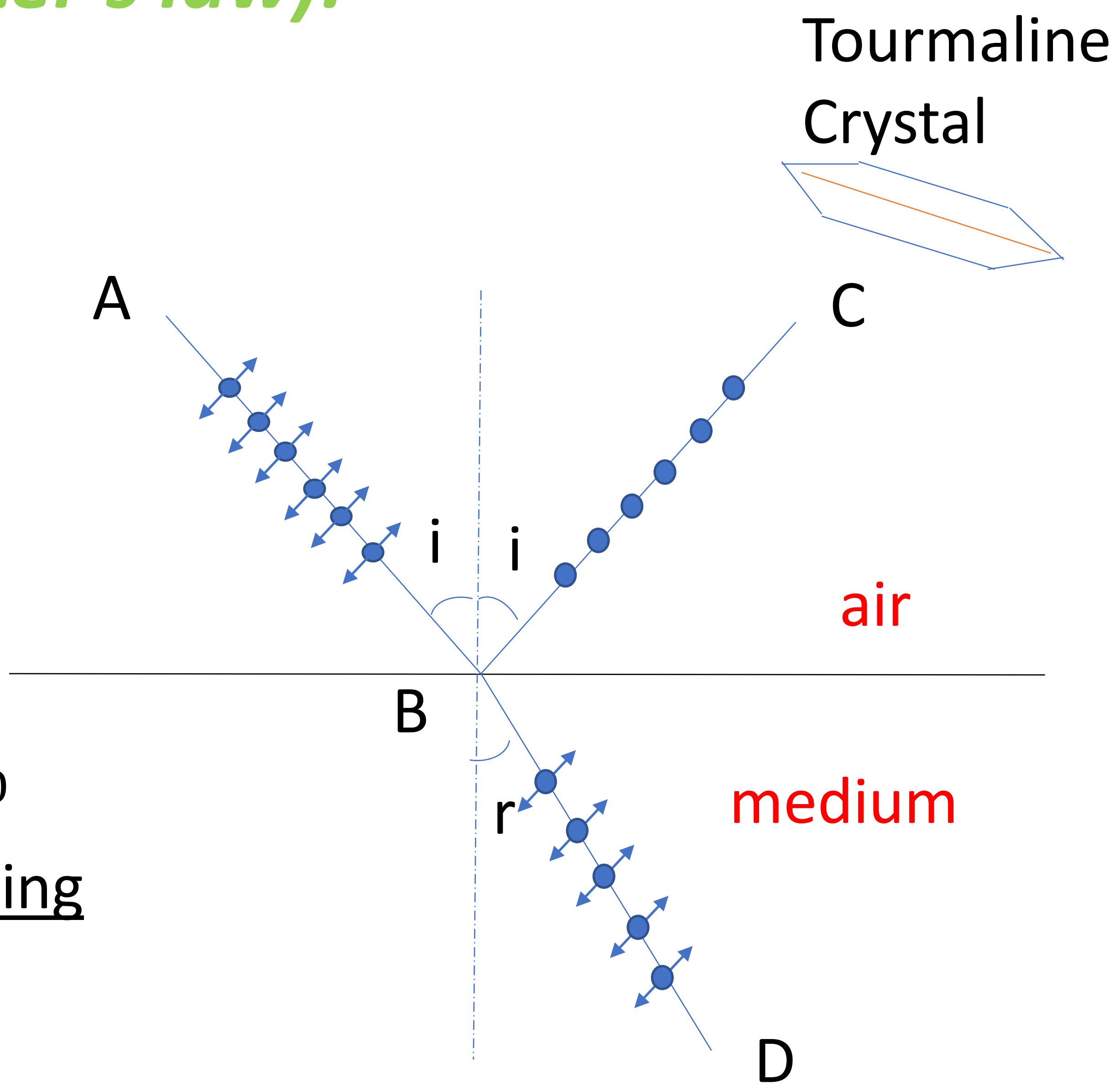
Consider the light incident along the path AB on a transparent surface.

Light is reflected along BC.

In the path of BC, place a tourmaline crystal and rotate it slowly.

It will be observed that light is completely *extinguished* only at one particular angle of incidence.

This angle of incidence (is equal to 57.5^0 for glass surface) and is known as polarizing angle.



Explanation: The production of polarized light can be explained as follows.

The vibrations of the incident light can be resolved into components parallel to the glass or water surface and perpendicular to the surface. Light due to the components parallel to the surface is reflected whereas light due to the components perpendicular to the glass surface is transmitted.

Thus the light reflected by glass is plane polarized and can be detected by a tourmaline crystal.

Brewster's law:

It states that when light is incident at polarizing angle, denoted by 'p', the reflected and refracted rays are perpendicular to each other. i.e. BC ⊥ BD. Then,

$$\mathrm{i} + \mathrm{r} + 90^0 = 180^0$$

$$\Rightarrow \mathrm{r} = 90^0 - i = 90^0 - p$$

So, $$\mu = \frac{\sin i}{\sin r}$$

$$\Rightarrow \quad \mu = \frac{\sin p}{\sin(90^0 - p)}$$

$$\Rightarrow \quad \mu = \frac{\sin p}{\cos p}$$

$$\Rightarrow \quad \boxed{\mu = \tan p}$$

This is Brewster's law.

Malus' law:

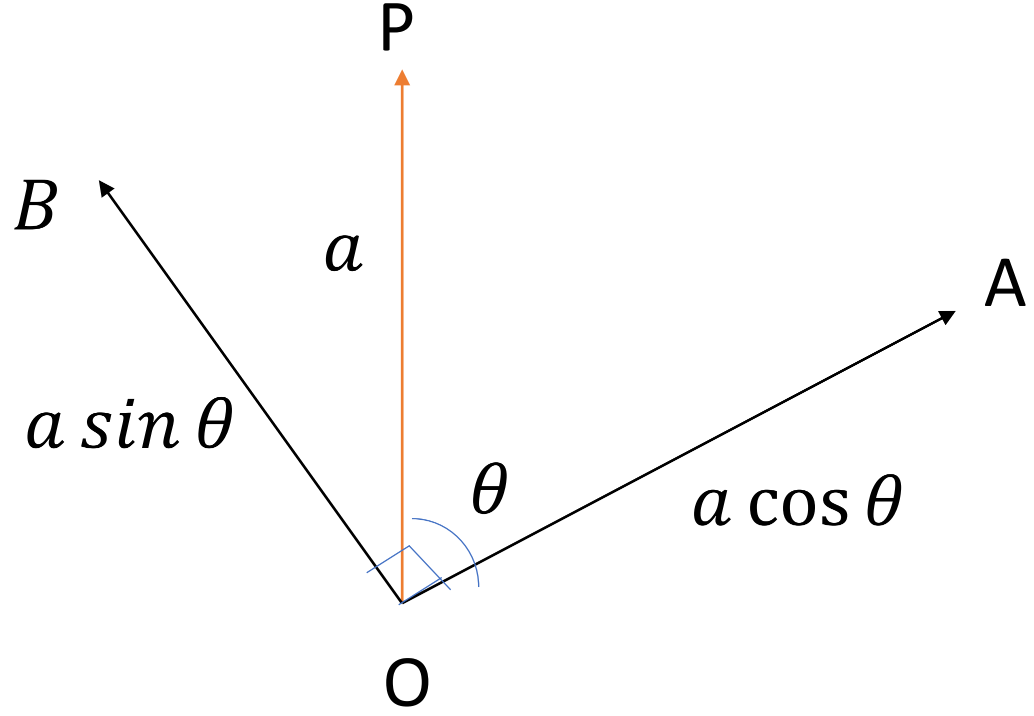
It states that the intensity of light passing through the analyzer is directly proportional to the square of the cosine of the angle between the polarizer and the analyzer.

$ie. I \alpha \cos^2\theta$

Proof:

Let the angle between the polarizer and the analyzer be θ and the amplitude of the light through the polarizer be a.

Then, the intensity of light through the analyzer is



$$I = (a\cos\theta)^2$$

$$\Rightarrow I = I_0\cos^2\theta$$

$$\Rightarrow I \alpha \cos^2\theta$$, where $I_0 = a^2$ is the maximum intensity.

Double refraction:

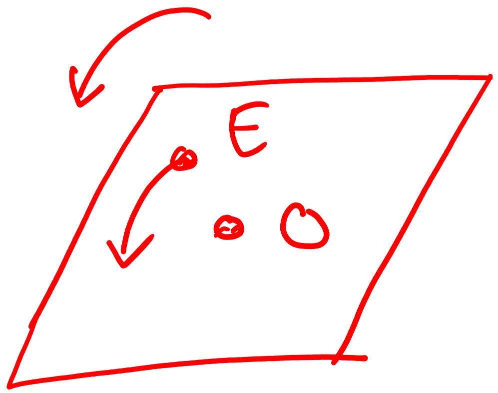
Picture source: https://www.quirkyscience.com/i-see-double-the-birefringence-or-double-refraction-of-calcite/

For the comprehension of this topic, let's first watch the video below:

https://www.youtube.com/watch?v=MoZar-gCj3E

Exam

Double refraction :-



(i)

When an ink mark is made on a piece of paper and place a calcite ($CaCO_3$) or quartz crystal over it, two images are produced.

This phenomenon is called double refraction.

Now, rotate the crystal slowly. One image rotates while the other image does not rotate. The image which rotates is called the extraordinary image & the image which does not rotate is called ordinary image.

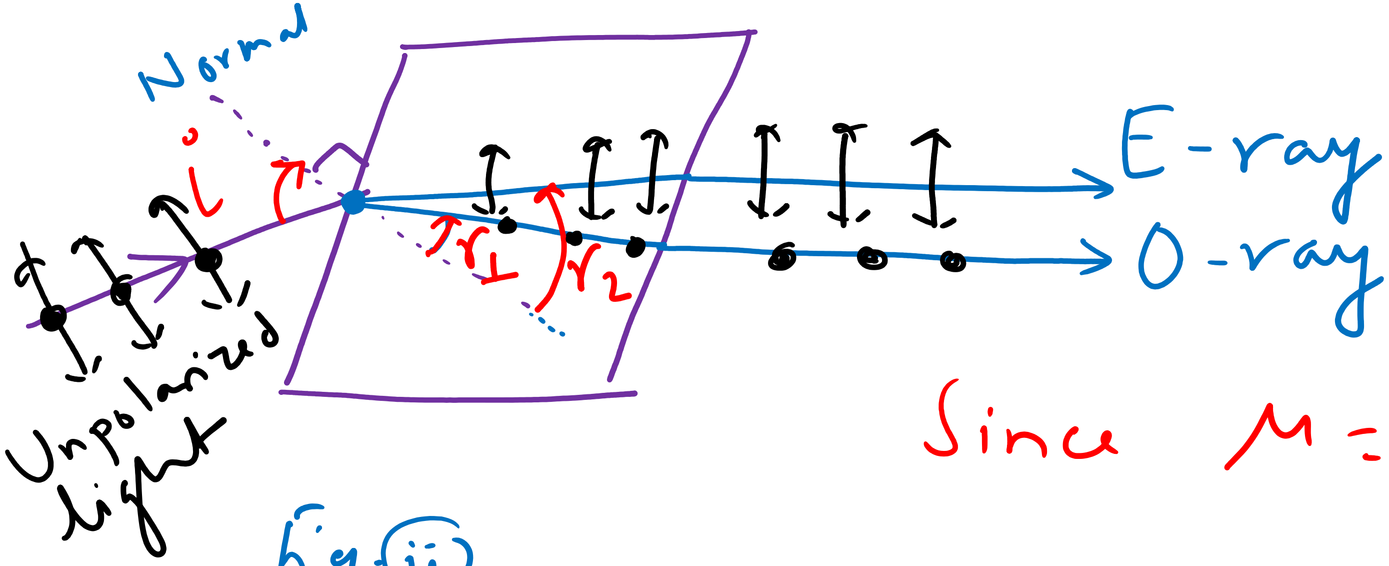


fig (ii)

Note:
dot → ⊥r
↓
O-ray

Since $\mu = \frac{\sin i}{\sin r}$,

$\mu_o = \frac{\sin i}{\sin r_1}$, refractive index of calcite for O-ray

& $\mu_E = \frac{\sin i}{\sin r_2}$, refractive index of calcite for E-ray

Since $r_2 > r_1$, $\boxed{\mu_o > \mu_E}$: (for calcite)

Also, $\mu = \dfrac{c}{v} = \dfrac{\text{velocity of light in vacuum}}{\text{" " " medium}}$

$\Rightarrow v = \dfrac{c}{\mu}$

Since $\mu_o > \mu_E$, $v_o < v_E$ (for calcite)

<u>Cases</u> :-

(i) When incident light is <u>parallel</u> to the optic axis, O-ray & E-ray <u>coincide</u> (i.e. they travel along the same direction with same velocity).

(ii) When the incident light is perpendicular to the optic axis, they travel along the same direction with different velocities.

Note :-

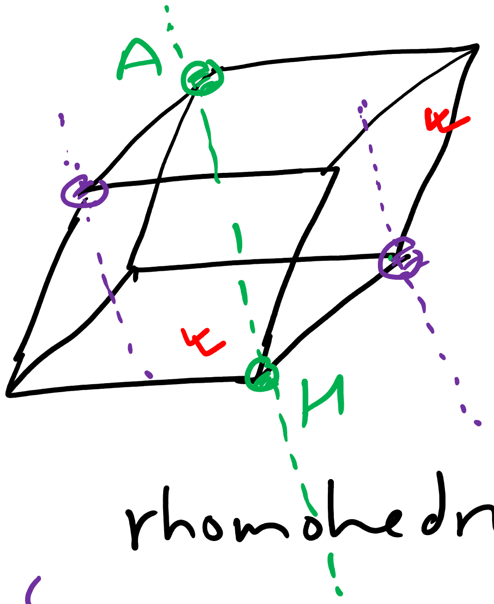
1. The vibration of O-ray is perpendicular to the optic axis of the crystal.
2. The vibration of E-ray is parallel to the optic axis of the crystal.

[Recall :- perpendicular $\xrightarrow{\text{notation}}$ dot [⊙]
O-ray]

Optic axis :-

A, H ⇒ blunt corners

The line joining the blunt corners is called the <u>optic axis</u>.



rhombohedron

Any line parallel to the optic axis is also regarded as the optic axis.

<u>Principal section</u> :- The plane containing the <u>optic axis</u> and is perpendicular to the opposite sides of the crystal is called the principal section.

Nicol Prism :- It is an optical device used for ~~producid~~ producing & analyzing the plane polarized light.

Construction :- A calcite crystal, whose length is 3 times its breadth, is cut into two pieces. The two pieces are grounded & polished optically flat and then cemented together by canada balsam layer, whose refractive index lies in betn the refractive indices of calcite for O-ray & E-ray (i.e. $\mu_O > \mu_B > \mu_E$) For calcite, $\mu_O = 1.658$

$\mu_B = 1.55$

$\mu_E = 1.486$

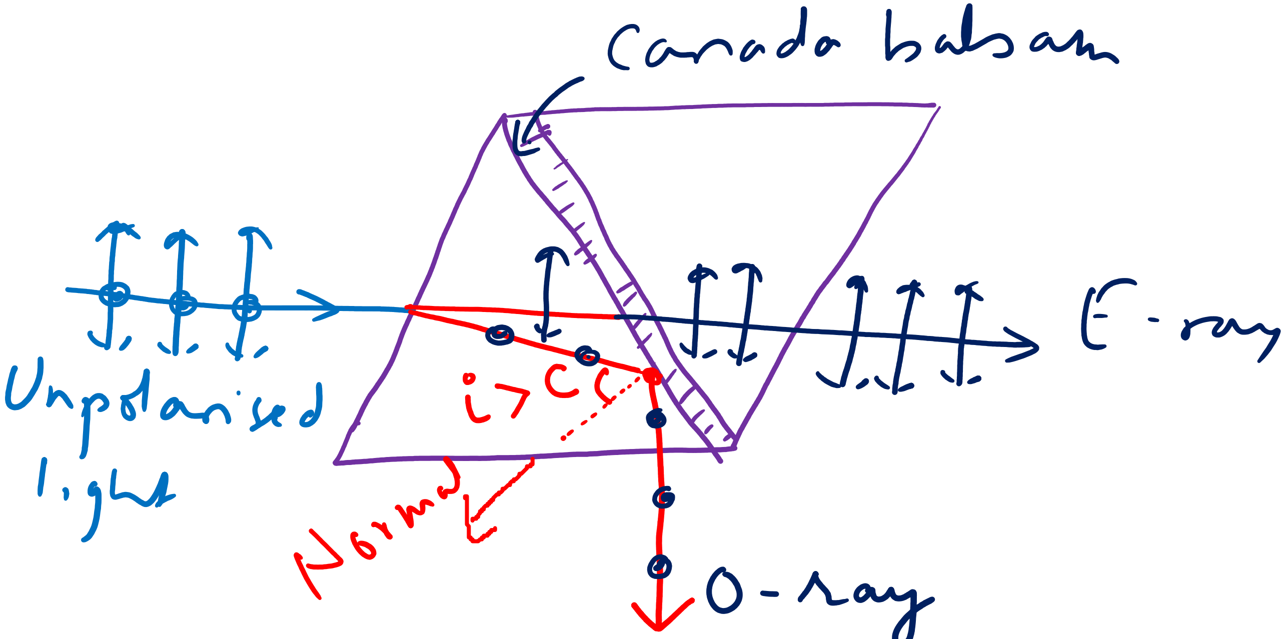
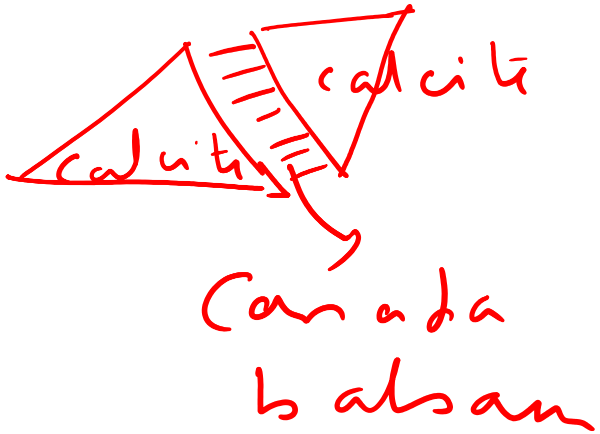
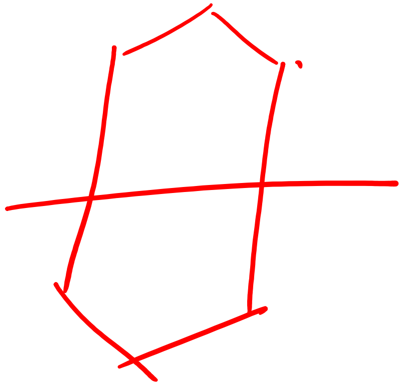


Fig. Nicol prism





Working :- When incident light ~~is~~ strikes Nicol prism, O-ray & E-ray are produced (due to double refraction by calcite). O-ray further strikes the canada balsam layer at an angle of incidence greater than the critical angle (69°) and is eliminated by total internal reflection. On the other hand, E-ray passes through the prism. In this way, Nicol Prism produces plane polarized light.

Q. How can Nicol Prism be used as a polarizer & analyzer?

Soln:

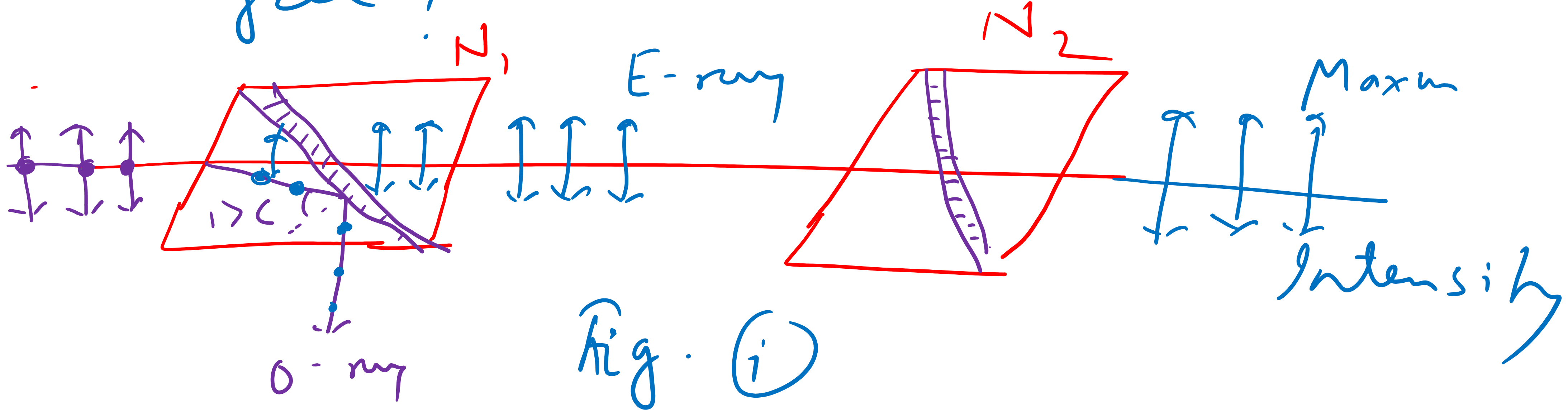


Fig. (i)

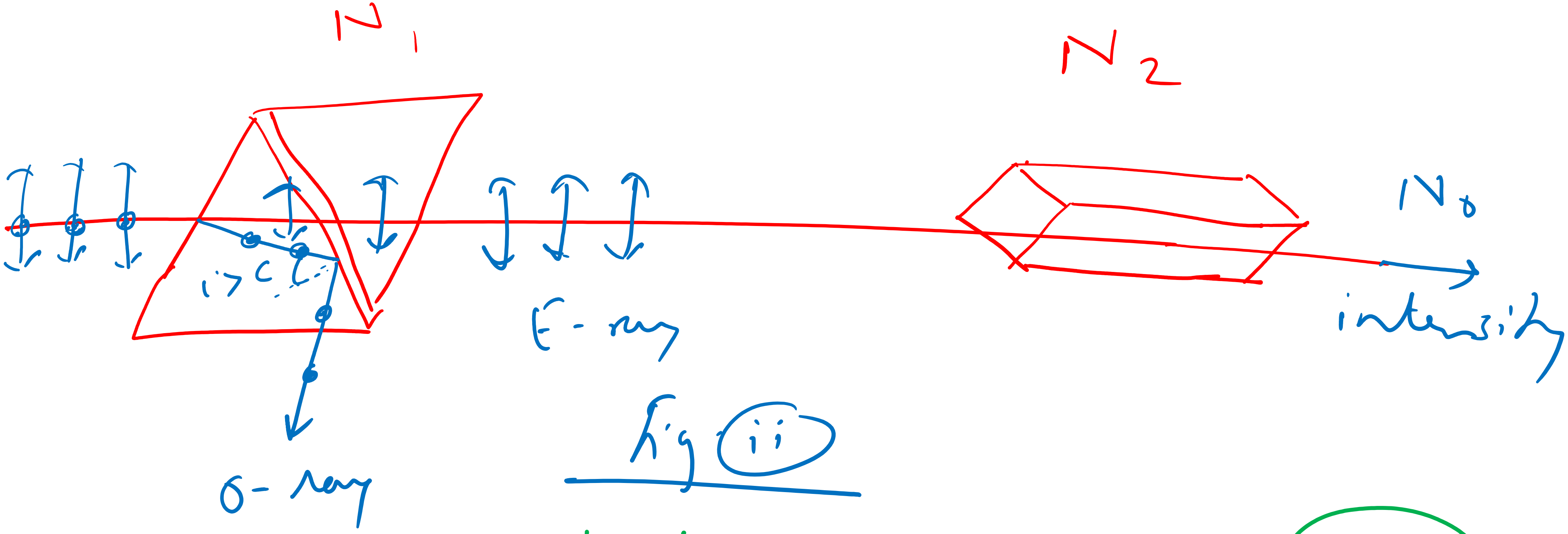
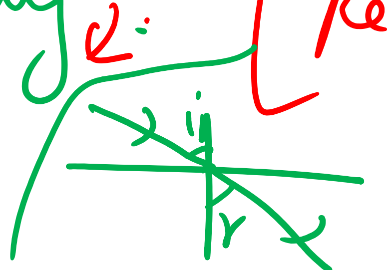
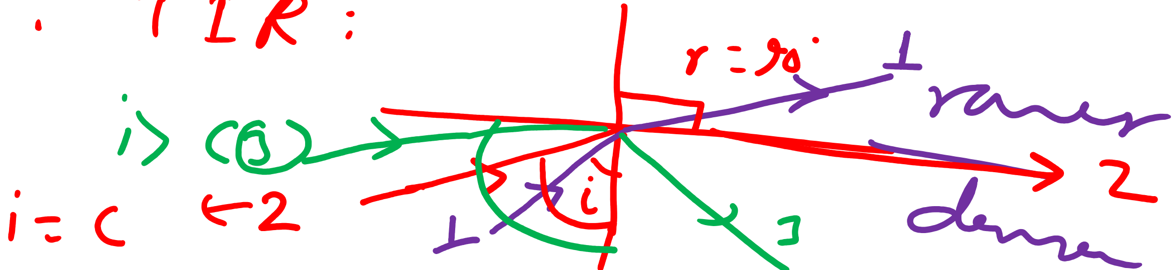


fig (ii)

Note :- Canada balsam acts as rarer medium for O-ray & denser medium for E-ray.

Recall : TIR :



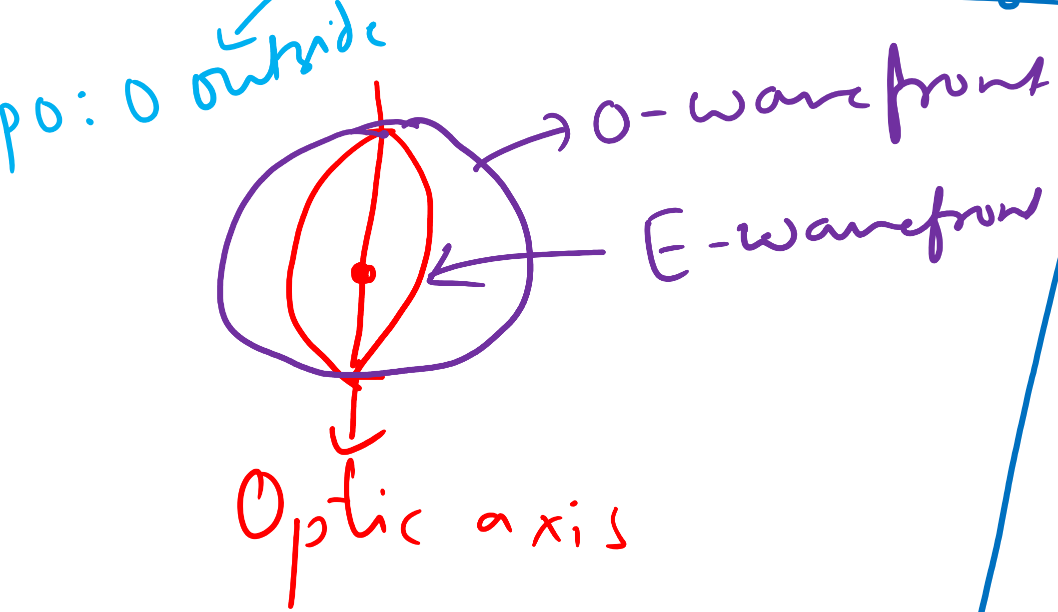
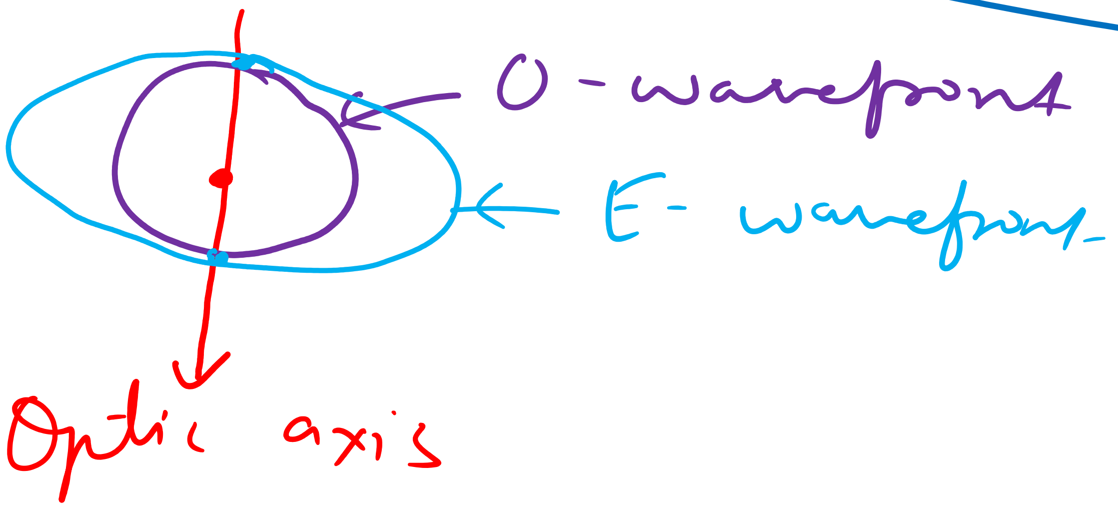


Note: (1) If the $\angle$ of incidence is less than the critical angle for O-ray, it is not reflected (by total internal reflection) & is transmitted ~~along~~ through the prism as does E-ray.

(2) If E-ray travels along optic axis, its refractive index is same as that of O-ray & it will also be totally internally reflected.

Q. Huygen's explanation of double refraction :-

Huygen explained double refraction on the basis of his law of secondary wavelets. A point source of light in a double refracting medium is the source or origin of two wavefronts. For O-ray, for which velocity is same in all directions, the wavefront is spherical. For E-ray, for which velocity is different in different directions, the wavefront is ellipsoid. However, the velocities of O-ray & E-ray is same along the optic axis. The differences between positive & negative crystals are given below:-

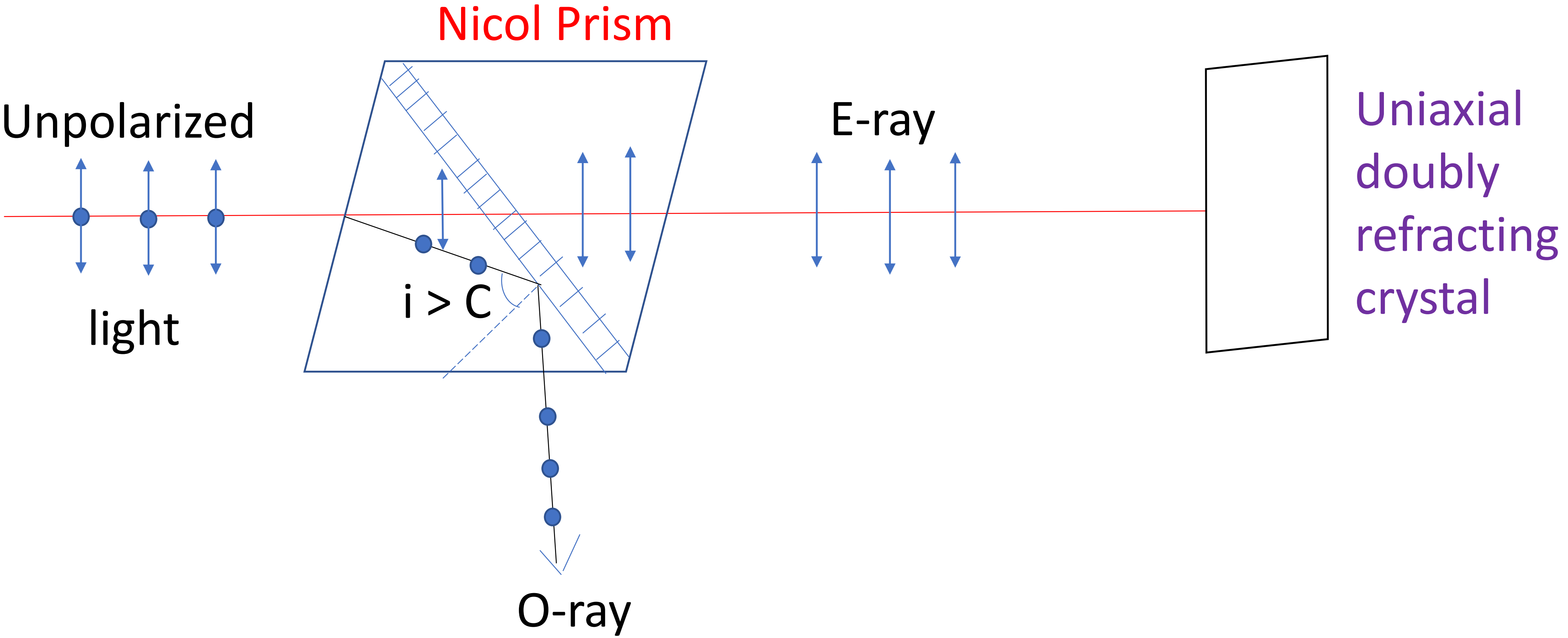
Positive Uniaxial crystal	Negative un. cryst
	

p9

Positive	Negative
1) Eg. quartz	1) calcite
2) $\mu_E > \mu_0$	2) $\mu_0 > \mu_E$
3) O-wavefront or wavesurface lies outside E-wavefront	3) E-wavefront lies outside O-wavefront
4) Velocity of E-ray is maxm along optic axis & minm along $\perp$r to optic axis	4) Velocity of E-ray is maxm along $\perp$r to optic axis & minm along optic axis

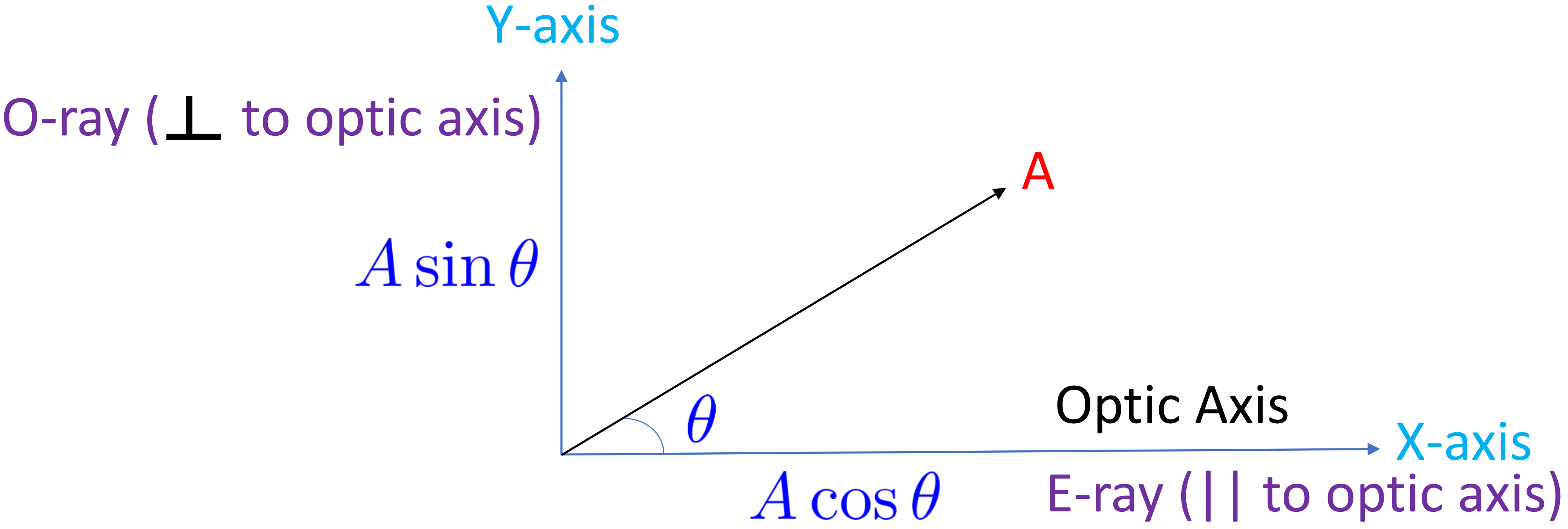
Long question:

<u>How can you produce linearly, circularly and elliptically polarized light. Explain with mathematical treatment. (9 marks)</u>



Let an unpolarized light be passed through a Nicol Prism. It eliminates O-ray and transmits E-ray. We place a uniaxial doubly refracting crystal as shown in the figure. This crystal further splits the incident ray into O-ray and E-ray which have a phase difference ϕ.

Let the incident direction of ray make an angle θ with the optic axis of the crystal. Then,



For E-ray,

$$x = (A\cos\theta)\sin(\omega t + \phi) \tag{1}$$

For O-ray,

$$y = (A\sin\theta)\sin\omega t \tag{2}$$

Let $A\cos\theta = a$ and $A\sin\theta = b$

Then equations (1) & (2) become

$$x = a\sin(\omega t + \phi)$$

or, $$\frac{x}{a} = \sin(\omega t + \phi)$$

or, $$\frac{x}{a} = \sin\omega t\cos\phi + \cos\omega t\sin\phi \tag{3}$$

And $y = b\sin\omega t$

or, $\dfrac{y}{b} = \sin\omega t$

So, $\cos\omega t = \sqrt{1 - \dfrac{y^2}{b^2}}$

Then,

$$\frac{x}{a} = \frac{y}{b}\cos\phi + \sqrt{1 - \frac{y^2}{b^2}}\sin\phi$$

Or,

$$\frac{x}{a} - \frac{y}{b}\cos\phi = \sqrt{1 - \frac{y^2}{b^2}}\sin\phi$$

Squaring both sides, we get

$$\frac{x^2}{a^2} - 2.\frac{x}{a}.\frac{y}{b}\cos\phi + \frac{y^2}{b^2}\cos^2\phi = (1 - \frac{y^2}{b^2})\sin^2\phi$$

Or, $$\frac{x^2}{a^2} - 2.\frac{x}{a}.\frac{y}{b}\cos\phi + \frac{y^2}{b^2}\cos^2\phi = \sin^2\phi - \frac{y^2}{b^2}\sin^2\phi$$

Or, $$\boxed{\frac{x^2}{a^2} - 2.\frac{x}{a}.\frac{y}{b}\cos\phi + \frac{y^2}{b^2} = \sin^2\phi}$$

This is the general equation of ellipse.

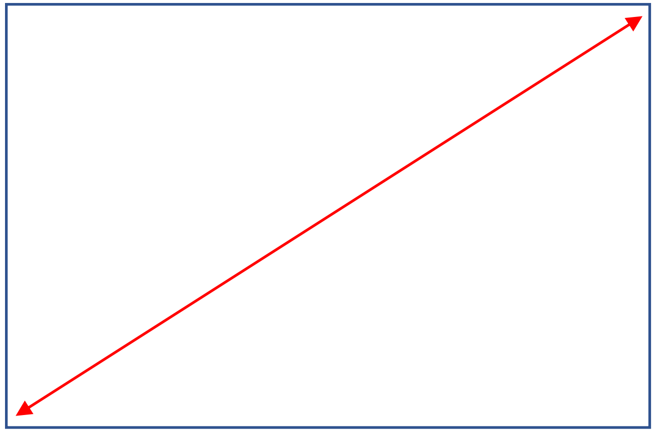
<u>Special case:</u> $(i)\ For\ \phi = 0^\circ,$

$$\frac{x^2}{a^2} - 2.\frac{x}{a}.\frac{y}{b} + \frac{y^2}{b^2} = 0$$

$$\rightarrow\ (\frac{y}{b} - \frac{x}{a})^2 = 0$$

$$\rightarrow\ \frac{y}{b} = \frac{x}{a}$$

$$\rightarrow \;\; y = \left(\frac{b}{a}\right)x$$

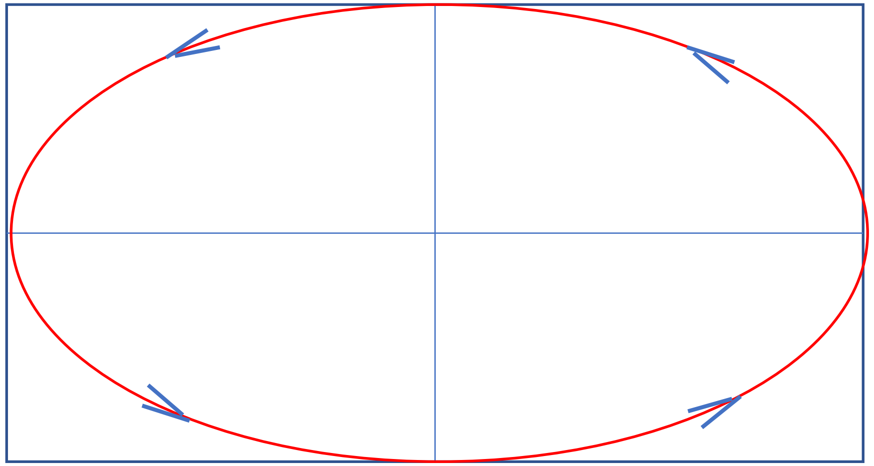


which is the equation of a straight line. Hence the light is said to be linearly polarized.

$$(ii)\;\; For\;\phi = 90^{\circ}, a \neq b,$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

which is the equation of the ellipse. Hence the light is said to be elliptically polarized.

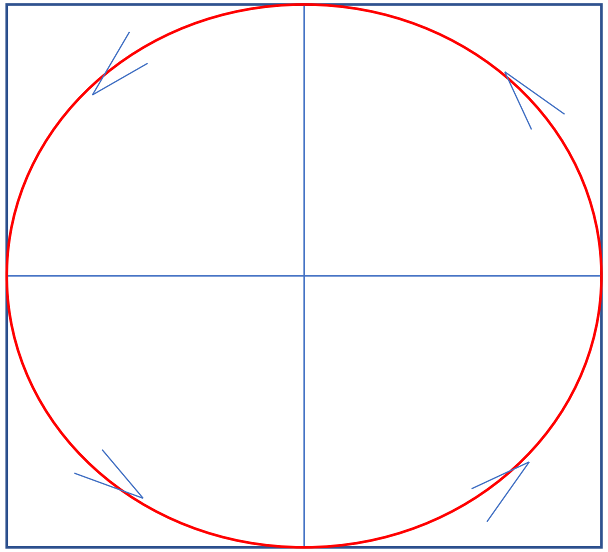


$$(iii) For\ \phi = 90^\circ, a = b,$$

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$$

$$so,\ x^2 + y^2 = a^2$$

which is the equation of the circle. Hence the light is said to be circularly polarized.



Wave plate: (Short note or numerical):

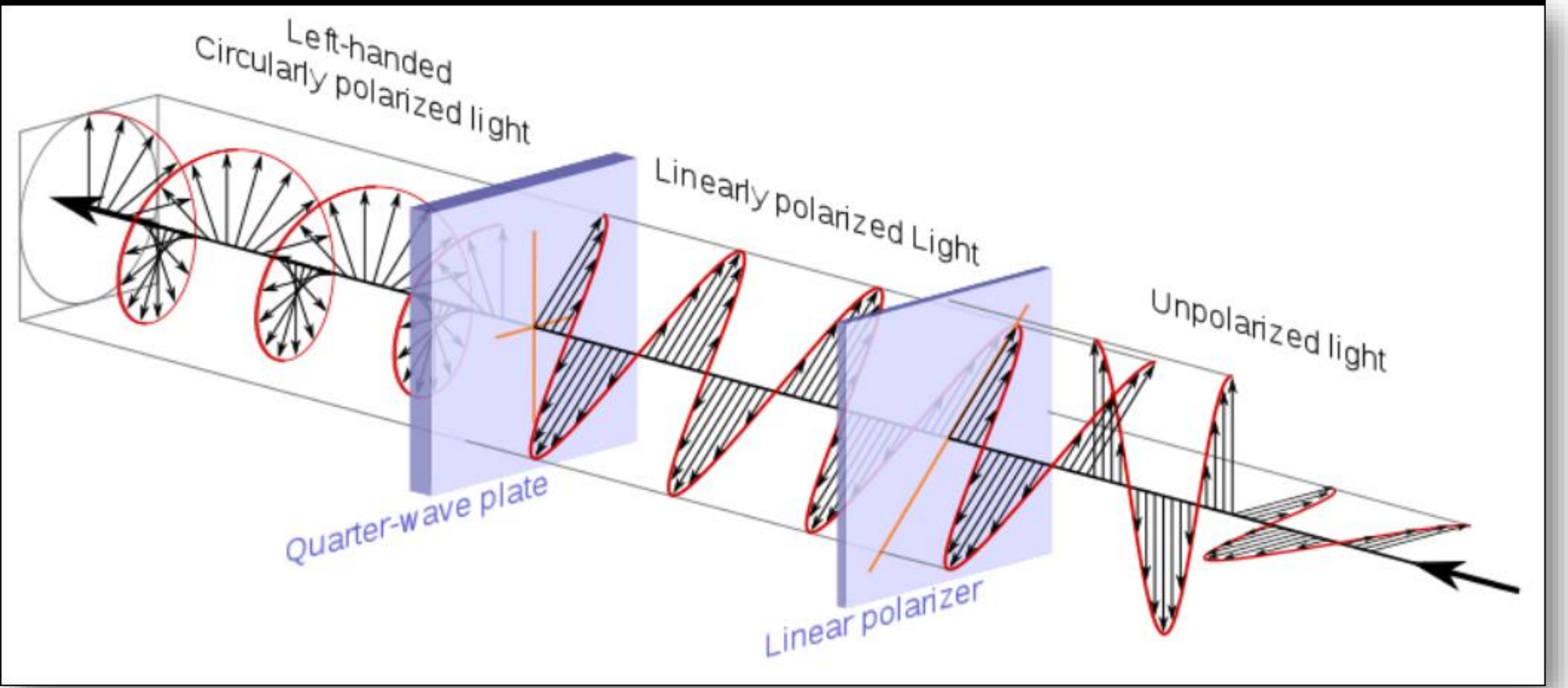
It is a plate of doubly refracting uniaxial crystal of calcite or quartz of suitable thickness whose refracting faces are cut parallel to the direction of optic axis.

The incident plane polarized light is perpendicular to its surface and the ordinary and extraordinary rays travel along the same direction with different velocities. If the thickness of the plate is t and the refractive indices of O-ray and E-ray are $\mu_O \; and \; \mu_E$ respectively, then the path difference between these two rays is $(\mu_O - \mu_E)t,$ for negative crystals (calcite) and $(\mu_E - \mu_O)t,$ positive crystals (quartz).

Types of wave plate: There are two types of wave plate, viz.

(i) Half wave plate

(ii) Quarter wave plate



Source: https://www.wikiwand.com/en/Waveplate

(i) Half wave plate:

The thickness of half wave plate is such that the ordinary and the extraordinary rays have a path difference of $\lambda/2$.

Then, for negative crystals (calcite),

$$\boxed{(\mu_O - \mu_E)t = \frac{\lambda}{2}}$$

& for positive crystals (quartz),

$$\boxed{(\mu_E - \mu_O)t = \frac{\lambda}{2}}$$

(ii) Quarter wave plate:

The thickness of half wave plate is such that the ordinary and the extraordinary rays have a path difference of $\lambda/4$.

Then, for negative crystals (calcite),

$$\boxed{(\mu_O - \mu_E)t = \frac{\lambda}{4}}$$

& for positive crystals (quartz),

$$\boxed{(\mu_E - \mu_O)t = \frac{\lambda}{4}}$$

Note:

If the plane polarized light, whose plane of vibration is inclined at an of 45^0 to the optic axis, is incident on a quarter wave plate, the emergent light is circularly polarized.

Optical Activity:

The phenomenon of rotation of the plane of vibration by certain crystals or substances is known as optical activity and the substance is called the optically active substance.

Calcite does not show this property, hence it is not the optically active substance.

Substances like sugar crystals, sugar solution, turpentine, sodium chlorate etc. are optically active.

Some substances rotate the plane of vibration to the left (anti-clockwise) and are called laevorotatory or left handed while the substances that rotate the plane of vibration to the right (clockwise) are called dextrorotatory or right handed.

Eg. Quartz crystals are dextrorotatory while others are laevorotatory.

Specific Rotation:

The angle through which the plane polarized light is rotated depends upon:

(i) thickness of the medium

(ii) concentration of the solution or the density of the active substance in the solvent

(iii) wavelength of light and

(iv) the temperature

The specific rotation is defined as the rotation produced by a decimeter (10 cm) long column of liquid containing 1 gm of the active substance in 1 cc of the solution. It is denoted by $S \ or \ S_\lambda{}^t \ or \ S_\lambda{}^\theta$ and is given by

$$\boxed{S = \frac{10\ \theta}{LC}}$$

where S = specific rotation (in degree)

$\theta =$ Angle of rotation (in degree)

L = Length of solution (in cm)

C = Concentration or the density of the substance(in g/cc)

Remember: Calculate the numerical values in CGS

Numericals:

1. Find the thickness of a quarter wave plate when the wavelength of light is 5890 A^0. $\mu_E = 1.553\ and\ \mu_O = 1.544$ (Ans: 1.636 x 10 ^-3 cm)

2. A 200 mm long tube containing 48 cm^3 of sugar solution produces an optical rotation of 11^0 When placed on a saccharimeter. If the specific rotation of sugar solution is 66^0 , calculate the quantity of sugar contained in the tube in the form of a solution. (Ans: 4 grams)

3. Calculate the thickness of a double refracting plate capable of producing a path difference of $\lambda/4$ between the extraordinary and ordinary waves. λ = 5890 A^0, $\mu_E = 1.54\ and\ \mu_O = 1.53$. (Ans: t = 1.47x10^-2 mm)

4. A length of 25 cm of a solution, containing 50 gm of solute per liter causes a rotation of the plane of polarization by 5^0 . Find the rotation of the plane of polarization by a length of 75 cm of a similar solution containing 100 gm solute per liter. (Ans: 30^0)

5. Determine the specific rotation of the given sample of sugar solution if the plane of polarization is turned through 13.2^0. The length of the tube containing 10% sugar solution is 20 cm. (Ans: 66^0)

6. A sugar solution in a tube of length 20 cm produces optical rotation of 13^0 . The Solution is then diluted to 1/3 of its previous concentration. Find the optical rotation produced by 30 cm long tube containing the diluted solution. (Ans: 6.5^0)

7. Plane polarized light is incident on a piece of quartz cut parallel to the axis. Find the least thickness for which the ordinary and extra-ordinary rays combine to form plane polarized light. $\mu_E = 1.553$ *and* $\mu_O = 1.5442$ & λ = 5x10^-5 cm

(Ans: 2.75x10^-3 cm)