Current Electricity:

Electric current (I):

It is the rate of flow of charge or the net flow of charge.

i.e. $I = \frac{dq}{dt}$

Current density (J):

It is defined as the current per unit area.

i.e. $J = \frac{I}{A}$

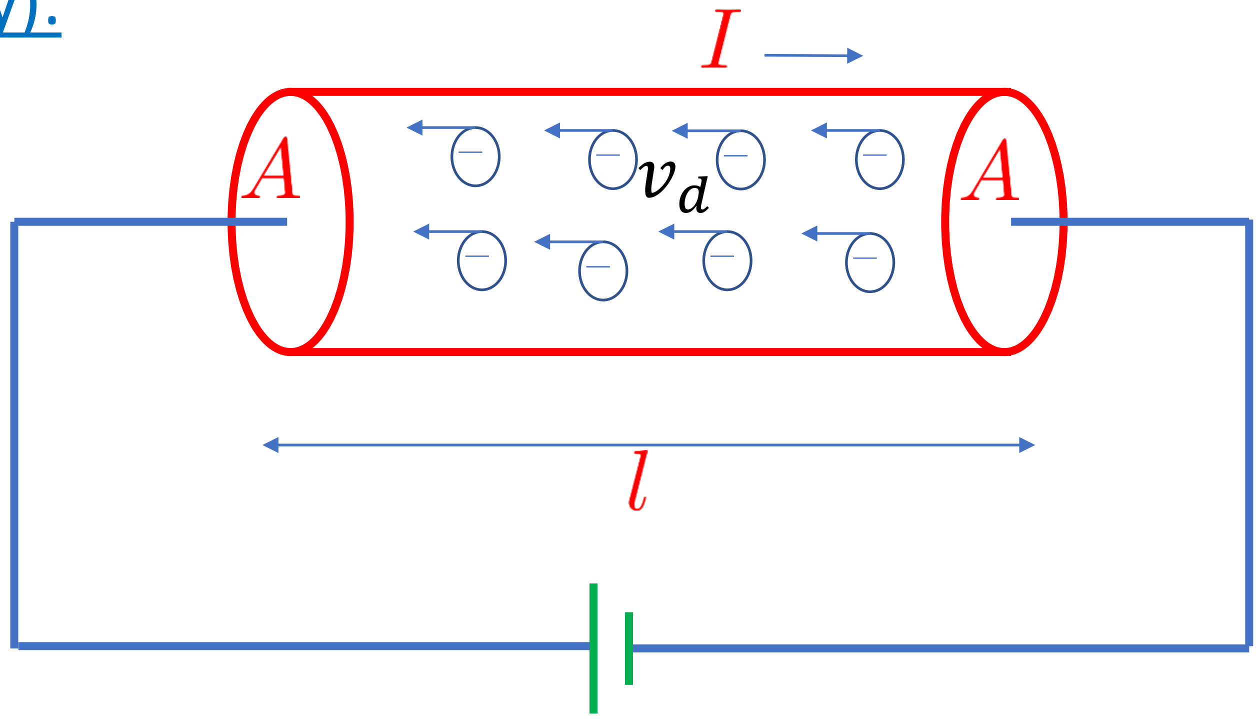
Mathematically,

$$I = \oint \vec{J}.\ \overrightarrow{dA}$$

Note:

(i) I is a scalar.
(ii) J is a vector.
(iii) I is a macroscopic quantity and J is a microscopic quantity.

Expression of current through a conductor (Relation between current and the drift velocity):



Consider a conductor of length ‘l’ and cross-sectional area ‘A’. Let a source of emf be applied to the two ends of the conductor. Then , charges (in this case, the electrons) move in the opposite direction with drift velocity v_d as shown in the figure above.

Then the current is given by

$$I = \frac{q}{t}$$

$$= \frac{Ne}{t}$$

Where, N = total no. of electrons in the conductor.

and $n = \frac{N}{V}$, is the concentration of electrons. V = volume of the conductor = A.l

So, N = nAl

$$I = \frac{q}{t} = \frac{nA\cancel{l}e}{t} = nA\left(\frac{l}{t}\right)e$$

Here, $\frac{l}{t} = v_d$, drift velocity

$\therefore$ $\boxed{I = v_d e n A}$, which is the required relation between I & v_d.

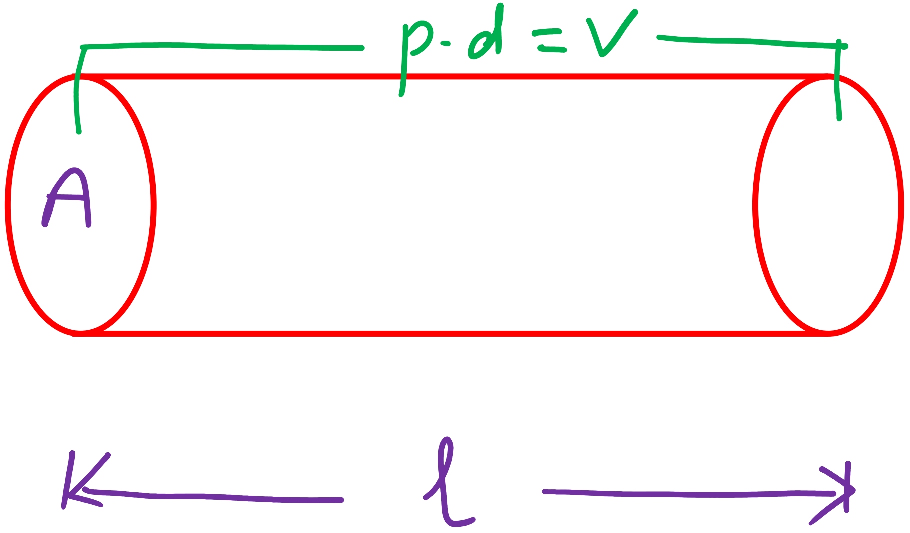
<u>Current density (J)</u> :-

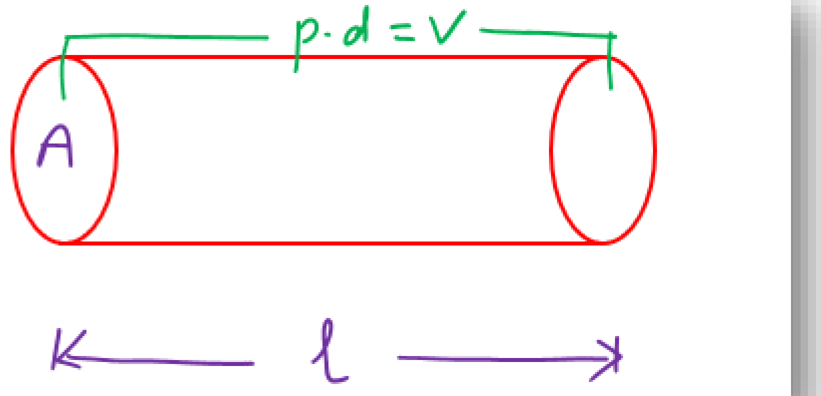
Since $J = \frac{I}{A} = \frac{v_d e n \cancel{A}}{\cancel{A}} = v_d e n$

$$\therefore \boxed{J = v_d e n}$$

Relation between J & E :-

(E = electric field)





Let V be the p.d. between the two ends of the conductor of length l.

Then,

$$E = \frac{V}{l}, \quad E \text{ being the electric field.}$$

By Ohm's law,

$$V \propto I$$

$\Rightarrow V = IR$, R being the resistance.

Again, $R \propto l$

& $R \propto \frac{1}{A}$

$\Rightarrow \boxed{R = \rho \frac{l}{A}}$, ρ being the resistivity.

Note: $R \xrightarrow{\text{Unit}} \text{ohm} (\Omega)$

$$\rho \longrightarrow \Omega m$$

$$1\,\Omega = 1\,VA^{-1} \quad \left[\because R = \frac{V}{I}\right]$$

Combing the equations:

$V = IR$ & $R = \dfrac{\rho l}{A}$, we get

$$V = I \rho \frac{l}{A}$$

$$\Rightarrow \frac{V}{l} = \left(\frac{I}{A}\right) \rho$$

$$\frac{V}{l} = \left(\frac{I}{A}\right)\rho$$

$\Rightarrow \boxed{E = J\rho}$, since $E = \frac{V}{l}$ & $J = \frac{I}{A}$

$$\therefore \quad J = \frac{1}{\rho} E$$

where $\frac{1}{\rho} = \sigma$, conductivity

$$\therefore \quad \boxed{J = \sigma E}$$

In vector notation,

$$\boxed{\vec{J} = \sigma \vec{E}}$$

Mobility (μ) :-

The drift velocity (v_d) is directly proportional to the electric field applied.

i.e. $v_d \propto E$

$$\Rightarrow \boxed{v_d = \mu E}$$

where $\boxed{\mu = \frac{v_d}{E}}$, is the mobility

$\because J = \sigma E$

$v_d e n = \sigma E$

$\Rightarrow v_d \propto E$

Defn :- Mobility is defined as the drift velocity per unit electric field applied.

Alternatively,

$v = \cancel{u}^{0} + at$

$= at = \frac{F}{m} t$

$= \frac{eE}{m} t \Rightarrow v_d \propto E$

Unit of μ :-

$$\left[v_d = \mu E \Rightarrow \mu = \frac{v_d}{E} \to \frac{m s^{-1}}{V m^{-1}} = m^2 V^{-1} s^{-1} \right.$$

$$\left. \Rightarrow v_d \xrightarrow{\text{Unit}} m^2 V^{-1} s^{-1} \right]$$

Relation between σ & μ :-

Since $J = v_d e n = \sigma E$

$$\Rightarrow v_d e n = \sigma E$$

$$\Rightarrow \mu \cancel{E} e n = \sigma \cancel{E}$$

$$\Rightarrow \boxed{\sigma = \mu e n}$$

, σ = conductivity

μ = mobility

Atomic View of resistivity: (Th / numerical / short note)

Conductivity in a metal arises due to the unidirectional flow of conduction electrons in the presence of the external electric field (E). The electrons move within the metal with the velocity of order of 10^6 m/s. However, they encounter the positive ion cores in the course of their motion. As a result, their velocity is reduced to zero. Again, they gain acceleration due to the effect of the electric field 'E' which is given by

$$\text{acceleration}(a) = \frac{\text{Force}(F)}{\text{mass}(m)}$$

$$\therefore \boxed{a = \frac{eE}{m}} \quad \text{—— (1)}$$

where e is the charge of the electron.

Now, the drift velocity is given by

$$\boxed{v_d = a\tau} \quad \text{--- (2)} \qquad \left[\because v = \cancel{u}^{\,0} + at = at\right]$$

where τ is called the relaxation time i.e. the time between the two successive collisions.

Using (1) in (2), we get

$$\boxed{v_d = \frac{eE}{m}\tau} \quad \text{--- (3)}$$

Also, $\boxed{\tau = \dfrac{\lambda}{\bar{v}}}$ $\left[\because \text{time} = \dfrac{\text{distance}}{\text{velocity}}\right]$

where λ is called the mean free path i.e. the distance between the two successive collisions. and $\bar{v}$ is the average (free) speed of the electron (also denoted as V_{avg}).

Now, the current density is given by

$$\boxed{J = V_d e n} \quad \text{—— (4)}$$

where n = concentration of electrons.

Using eqn (3) in (4),

$$J = \left(\frac{eE}{m}\tau\right) en$$

or, $$J = \left(\frac{ne^2\tau}{m}\right) E$$

Since $J = \sigma E$, σ = conductivity

Comparing, we get

$$\boxed{\sigma = \frac{ne^2\tau}{m}} \Rightarrow \text{V.V. Imp}$$

$$\therefore \text{ resistivity } (\rho) = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$$

This shows that the resistivity depends upon m(mass of the electron), n (no. of electrons per unit volume), e (charge of electron & τ (relaxation time, all terms being the atomic quantities. Hence, this is the atomic view of resistivity.

V.V. Imp (Numerical) :-

Q. What are (a) the mean time (τ) between the collisions & (b) the mean free path for free electrons in copper? Given: $n = 8.4 \times 10^{28}\ m^3$, $\rho = 1.7 \times 10^{-8}\ \Omega$, $\bar{v} = 1.6 \times 10^{6}\ ms^{-1}$.

[Ans: (a) 2.48×10^{-14} sec.

(b) 3.98×10^{-8} m]

Temperature dependence of resistance/resistivity:-

The resistance for most material changes with temperature. For $\theta_1\,°C$ & $\theta_2\,°C$,

$$\boxed{R_2 = R_1\left[1 + \alpha\Delta\theta\right]}$$

where R_1 = resistance at $\theta_1\,°C$

R_2 = " " $\theta_2\,°C$

& $\Delta\theta = \theta_2 - \theta_1$

Also, α = temperature coefficient of resistance

Also, $R_\theta = R_0(1 + \alpha\theta)$

where R_θ = Resistance at θ °C

R_0 = " " 0 °C

$\Delta\theta = \theta$

For resistivity, $\rho_2 = \rho_1 [1 + \alpha\Delta\theta]$

$\rho_\theta = \rho_0 [1 + \alpha\theta]$

<u>Note</u> :- α is positive for metals.

while it is negative for semiconductor

$\therefore$ For semiconductor,

$$\rho_\theta = \rho_0 [1 - \alpha\theta]$$

$$\& \quad \rho_2 = \rho_1 [1 - \alpha \Delta\theta]$$

$\Rightarrow$ Resistance & resistivity increase with temperature for metals (metallic/ohmic conductor) while it decreases for semi-conductor.

Joule's law of heating :-

OR

Transfer of energy in electric circuit :-

If a charge dq moves through the circuit from one point to another point, the energy transferred in doing so is given by

$$dU = V\,dq \quad \left[\because V = \frac{W}{q} = \frac{dW}{dq} \right]$$

Here, $dq = I\,dt$ $\left[\because I = \frac{dq}{dt}\right]$

$\therefore dU = VI\,dt$

$\Rightarrow \frac{dU}{dt} = IV$

$\Rightarrow$ power $(P) = IV$

Also, Ohm's law, $V = IR$

$\therefore \frac{dU}{dt} = I^2R$

$$\Rightarrow \boxed{P = I^2R}$$

Also, $\frac{dU}{dt} = IV = I^2R$

$\Rightarrow dU = I^2 R\, dt$

$\Rightarrow \boxed{H = I^2 R t}$ [since upon

integrating, $\int dU = I^2 R \int_0^t dt$

$\Rightarrow U = H = I^2 R t$]

H.W / Assignment

1. Book examples Q.n. 1 to 10.
   (page number 284 - 287)

2. Q.n. 19, 24, 25, 26

Numerical Problems:

1. A wire of resistance 16 ohm is melted and drawn into a wire of half its length. Calculate the resistance of the new wire and the percentage change in resistance.
2. Using Ohm's law, prove J = σE, where J, E and σ are the current density, electric field and conductivity.
3. Two conductors are made of the same material and are of the same length. The first conductor is a solid wire of diameter 1 mm and the second conductor is a hollow wire of diameters 2 mm and 1 mm. Determine their resistance ratio.
4. A copper wire is stretched to make it 0.1% longer. What is the % change in resistance?

5. A rectangular carbon block has dimensions 1 cm x 1cm x 50 cm. What is the resistance measured

(i) between the square ends.

(ii) between the rectangular faces. Given: resistivity of carbon at 20^0 C is 3.54 x 10^{-5} Ωm.