$J = nev$

Chapter-2

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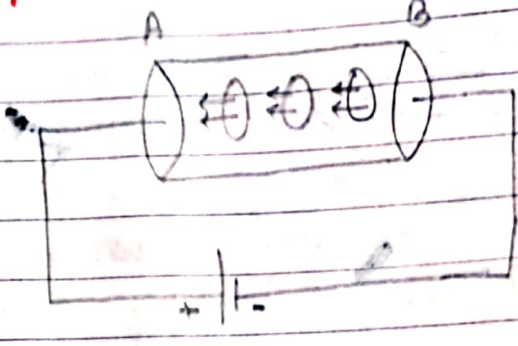
Free Electron Theory of Conduction in metals

Free electron theory of Conduction in metals



(Ohm's law →)

Let,

E = Applied Electric field

e = Charge of an electron

m = mass of Electron

Now,

From Kinetic Theory,

$$\frac{1}{2}mv^2 = \frac{3}{2}k_B T \quad \text{—— (1)}$$

where, m = mass of electron

v = velocity of electron

k_B = Boltzman constant

T = Temperature

And, the acceleration is given by,

$$a = \frac{F}{m} = \frac{Ee}{m} \quad \leftarrow (2)$$

If λ be the mean free path then,

$$v = \frac{\lambda}{t} \quad \text{—— (3)}$$

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Now, the average distance travelled by the electron in time 't' is given by,

$$S = \frac{1}{2} a t^2$$

$$\therefore S = \frac{1}{2} \frac{Ee}{m} \cdot \frac{\lambda^2}{V^2} \quad — 4$$

Thus, average velocity of electron is,

$$\bar{V} = \frac{S}{t} = \frac{1}{2} \frac{Ee}{m} \cdot \frac{\lambda^2}{V^2} \times \frac{V}{\lambda}$$

$$= \frac{1}{2} \frac{Ee}{3k_BT} V^2 \cdot \frac{\lambda}{V}$$

$$\therefore \bar{V} = \frac{1}{6} \frac{Ee}{k_BT} V\lambda$$

Now;

If n = no. of electron per unit volume in the conduction

J = Current density (current per unit area)

Then,

$$J = ne\bar{V}$$

$$= ne \frac{1}{6} \frac{Ee}{k_BT} V\lambda$$

$$\boxed{\therefore J = \frac{ne^2 \lambda V}{6 K_B T} \cdot E}$$

R : S R/A   RA/T

$$\therefore \boxed{J = \sigma E} \quad -⑤$$

where, $\sigma = \dfrac{ne^2\lambda v}{6k_B T}$

is the required expression of ohm's law.

Electrical conductivity (σ)

It is defined as the quantity of electricity that flow in unit time per unit area of cross-section of the conductor per unit potential gradient. The quantity of electricity flowing through conductor is,

$$q = \sigma n e T \quad \text{or,} \quad \sigma A E T$$

where, A = Area of the cross-section
T = time duration
E = Potential gradient
n = no. of electron per unit volume

If T = 1 sec then,

$$\sigma = \frac{q}{AE}$$

$$\Rightarrow J = \sigma E \quad -①$$

Now, from ohm's law,

$$\sigma = \frac{ne^2\lambda v}{6k_BT} \quad — (2)$$

Now,

$$\sigma = \frac{ne^2\lambda v}{6mv^2} \times 3 \qquad \left[\because k_BT = \frac{mv^2}{3}\right]$$

or, $$\sigma = \frac{ne^2\lambda}{2mv}$$

or, $$\sigma = \frac{ne^2}{2m} \cdot t$$

$$\boxed{\therefore \sigma = \frac{ne^2\tau}{m}}$$

where, τ = mean free time [half of time period

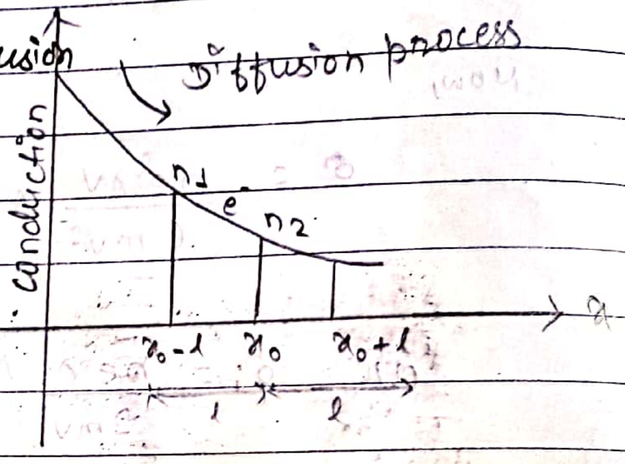
or, $\boxed{\sigma = ne\mu}$

where, $\mu = \frac{e\tau}{m}$, mobility of electron

or, $\boxed{\rho = \frac{1}{\sigma} = \frac{1}{ne\mu}}$

Diffusion of electron.

Let, l = distance travelled by diffusion electron in time τ



and concentration of electron at $x = x_0 - l$ is n_1 and $x = x_0$ is n_2.

Now,

The no. of electron crossing x_0 from $x_0 - l$ will be half of $n_1 l$ i.e $\frac{1}{2} n_1 l$

no. of electron crossing $x_0 + l$ from x_0 will be half of $n_2 l$. i.e $\frac{1}{2} n_2 l$.

Here,

n_1 = concentration of electron in the position of $x_0 - l$

n_2 = Concentration of electron in the position x_0.

Now, the number of electrons crossing x_0 point per unit time in +ve x- direction is given by,

$$\frac{\frac{1}{2} n_1 l - \frac{1}{2} n_2 l}{\tau}$$

$$= \frac{1}{2\tau}(n_1 - n_2) l \quad \text{--- (i)}$$

$$= -\frac{l}{2\tau}(n_2 - n_1) \quad \text{--- (ii)}$$

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$$= -\frac{l}{2\tau}\Delta n \quad \text{--- (iii)}$$

where, $\Delta n = n_2 - n_1$

Then, we can write,

$$\Delta n = \frac{\Delta n}{\Delta x} \cdot \Delta x$$

Thus, Δx is given by,

$$x_0 + l - (x_0 - l)$$
$$= 2l.$$

So,

$$\Delta n = \frac{\Delta n}{\Delta x} \cdot 2l \quad \text{--- (iv)}$$

Now, from eq^n (iii) and (iv), we get;

no. of electron crossing x_0 point $= -\frac{l}{2\tau}\frac{\Delta n}{\Delta x} \cdot 2l$

$$= -\frac{l^2}{\tau} \cdot \frac{dn}{dx}$$

where, $\frac{dn}{dx}$ = concentration gradient.

$\frac{l^2}{\tau}$ = Diffusion coefficient of electron represented by

$$\boxed{D_e = \frac{l^2}{\tau}}$$

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Einstein's Relationship between mobility and diffusion coefficie[nt]

The diffusion coefficient for electron in one dimension along x- direction is given by,

$$De = \frac{l^2}{\tau} \quad -(1)$$

Let, l be the mean free path and τ be the mean free time.
Then,

$$l = V_x . \tau \quad -(2).$$

Now, eq^n (1) becomes,

$$De = V_x^2 \tau \quad -(3)$$

So, energy of free electron in conduction band is eq[ual to] $\frac{1}{2}kT$ and this energy is equal to the kinetic energy of fre[e] electron.

So, we can write,

$$\frac{1}{2} m_e V_x^2 = \frac{1}{2} kT \quad -(4)$$

$$\therefore \quad V_x^2 = \frac{kT}{me}$$

Now, eq^n (3) becomes,

Annie → Italy

classmate

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$$D_e = \frac{kT}{m_e} \cdot \tau$$

$$= \frac{kT}{e\,m_e} \cdot e\tau$$

$$= \frac{kT}{e}\left(\frac{e\tau}{m_e}\right)$$

$$\boxed{\therefore D_e = \frac{kT}{e}\mu_e}$$ $\left[\because \mu_e = \frac{e\tau}{m_e}\text{, mobility of electron}\right]$

which is required expression for Einstein's relation.

From above Einstein's relation, it is clear that the diffusion coefficient of electron is proportional to the absolute temperature and drift mobility of the electron.

Q. Calculate the diffusion coefficient of electrons at 27°c if the electron drift mobility is 1300 cm²/Vs.

soln,

Given, $T = 27°c = 27+273 = 300k$

$\mu_e = 1300\ cm^2/Vs$

$k = 1.38 \times 10^{-23}\ J/k$

$e = 1.6 \times 10^{-19}\ c$

so,

$$D_e = \frac{kT}{e}\mu_e$$

$$= \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \times 1300$$

$$\therefore D_e = 33.63\ \text{Ar}$$

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$V = \frac{M}{D}$, 6.5 given $\frac{A}{N} = D$.

6.023×10^{23}

Q. Calculate the drift mobility in copper at room temperature (2...). Given that the conductivity of copper is $5.9 \times 10^5 \Omega^{-1} m^{-1}$. The density of copper = $8.93 gm/cm^3$ and its ~~assume~~ atomic mass is 63.5 gm/mole.

Soln,

Conductivity of cu (σ) = $5.9 \times 10^5 \Omega^{-1} m^{-1}$

density of cu (ρ) = $8.93 \ gm/cm^3$

Atomic mass (m) = 63.5 gm/mole

We have,

$$\text{drift mobility} (\mu_d) = \frac{\sigma}{en}$$

where, $e = 1.6 \times 10^{-19} c$.

Now,

$$n = \frac{\rho N A}{m} = \frac{5.9 \times 10^5 \Omega^{-1} m^{-1} \times 6.023 \times 10^{23}}{63.5}$$

$$\frac{\sigma N_A}{m}$$

$$= 8.5 \times 10^{22} cm^3.$$

Then,

$$\text{drift mobility} (\mu_d) = \frac{\sigma}{en}$$

$$= \frac{(5.9 \times 10^5)}{1.6 \times 10^{-19} \times 8.5 \times 10^{22}}$$

$$= 43.4 \ cm^2 V^{-1} s^{-1}$$

Chemical & physical properties of common conduction of materials.

Physical properties are those that can be observed without changing the identity of the substances. The general properties of materials such as color, density, hardness etc are the examples of physical properties. Properties that describe how a substance changes into a completely different substance are caued chemical properties.

The difference between the physical and chemical properties is straight forward until the phase of the material is considered. When a material changes from a solid to a liquid and to a vapour, it seems like them become a different substance. However, when a material melts, solidifies, vapourizes and condenses, only the state of the substance changes.

Consider ice, liquid - water and water vapour, they all are simply H_2O. Phase is a physical property of the material and material can exist in three phases : solid, liquid and gas.

Some of the more important physical and chemical property ~~for~~ from an engineering material view point. can be discussed in the following section.

→ Phase transformation temperature
→ density
→ specific gravity
→ Thermal conductivity

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Arsenicum (latin).
Aurum (latin)
Argentum (latin)
Cuprum

Mag Manganese

→ Linear coefficient of thermal expansion
→ Electrical conductivity & resistivity
→ Magnetic permeability etc.

Some of the commonly used materials are As (Arsenic), Au (Gold), Ag (silver), Al, Mn, Cu etc.

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(x) Lattice Scattering:-

Lattice scattering is the scattering of ions by interaction with atoms in a lattice. In the quantum understanding, an electron is viewed as a wave travelling through a medium. When the wavelength of the electrons is larger than the crystal spacing, the electrons will propagate freely throughout the metal without collision.

Since, a crystal is essentially a periodic repetition of small volume of atoms in three dimensions, it is useful to identify the repeating unit. So that the crystal properties can be described through this unit. The most convenient small cell in the crystal structure that carries the properties of the crystal is called unit cell. The repetition of unit cell in three dimensions generates the whole crystal.

There are different types of unit cell geometries of crystals of different materials. In cubic type geometries, there are basically three types of unit cells, namely;

(i) Simple cubic structure
(ii) Face centered cubic (FCC)
(iii) Body centered cubic (BCC).

(i) Simple cubic structure.

Atoms are placed at the corners of the unit cell geometry and by repeating this in all directions crystal is formed.

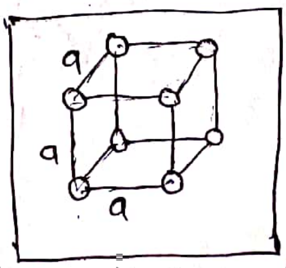

fig:- Unit cell of SCS.

The length of a side is equal to the distance between two atoms. Each atom touches along the cube edges only. The structure is loosely packed.

(ii) Face centered cubic (FCC)

Here, the atoms are placed as the case of SCS at all the corners of the unit cell and additional atoms are placed at the intersection of the face diagonals in all faces. Half of the atom at each face diagonals belongs to a unit cell.

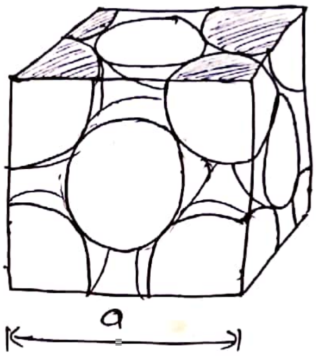

Fig:- FCC unit cell geometry

(iii) Body centered cubic (BCC)

In this type of unit cell structure, in addition to atoms at the corner of the cell geometry, there is an atom occupying the central place within the unit cell. All the corners atoms are touching the central atom.

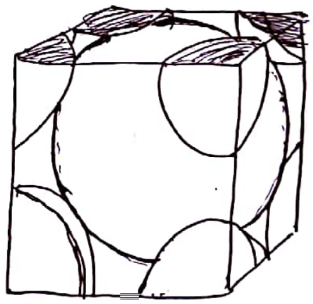

Fig:- BCC unit cell geometry

Note:- Not all the elements and compounds have cubic structures; there are other unit cell geometries like orthorhombic, monoclinic, hexagonal, tetragonal etc.

Thermal velocity of Electron.

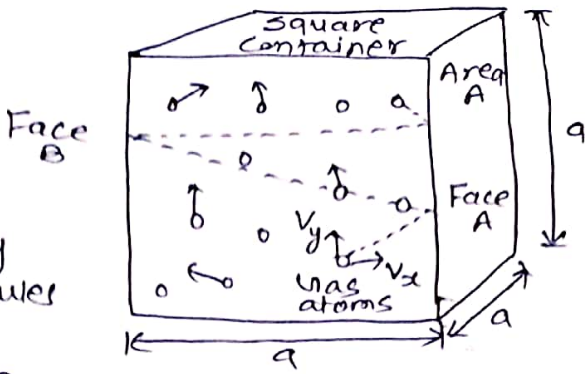

Fig. The gas molecules in the Container.

Applying kinetic molecular theory of gases ~~for~~ molecules contained in a container, we can write the final theoritical derivation in the form,

$$\boxed{PV = \frac{Nmv^2}{3}} \qquad \text{——— (i)}$$

where,

P = pressure of gas inside the container.
V = volume of the container.
N = Number of gas molecules in container.
m = Mass of the gas molecules.
v^2 = mean square velocity of molecules.

The experimental observation is,

$$\boxed{PV = \frac{N}{N_A} RT = nRT} \qquad \text{——— (ii)}$$

Where,

N_A = Avogadro's number.
R = universal gas constant.
T = Temperature.
$n = N/N_A$ = Number of moles of gas.

For N-molecule gas, every molecule will have its random velocity. The change in momentum of the molecule following collision with the wall of the container is $\Delta P_x = 2mv_x$ where m is the molecular mass. Time interval between collision will be $\Delta t = 2a/v_x$ where a is the side of container.

Force exerted by this molecule on the face A of the container is $F_x = \frac{\Delta P}{\Delta t} = \frac{m v_x^2}{a}$

And the total pressure exerted by N molecules on face A due to total force is,

$$P = \frac{\text{Total Force}}{\text{Area}} = \frac{m v_{x_1}^2 + m v_{x_2}^2 + \cdots + m v_{x_N}^2}{a^3}$$

$$= \frac{m}{a^3}\left(v_{x_1}^2 + v_{x_2}^2 + \cdots + v_{x_N}^2\right).$$

or, $P = \frac{mN}{V} v_x^2$

where, $N v_x^2 = v_{x_1}^2 + v_{x_2}^2 + \cdots v_{x_N}^2$.

Since, the molecules are in random motion and collide randomly with each other, mean square velocity in x-direction will be equal to that in y & z directions.

So, $v_x^2 = v_y^2 = v_z^2$.

So, The velocity of any molecule is given by,

$$v^2 = v_x^2 + v_y^2 + v_z^2 = 3v_x^2.$$

v^2 = mean square velocity of the molecule.

We will find that the average kinetic energy per degree of freedom is (can be derived) $KT/2$. So for three degrees of freedom, this is $3KT/2$. Average Kinetic energy per mole is $mv^2/2$.

So, $KE = \frac{mv^2}{2} = \frac{3}{2}KT$.

$$\therefore \boxed{v_{th} = \left(\frac{3KT}{m}\right)^{1/2}}.$$

Where v_{th} is the thermal velocity of electrons and is the required expression.

//.