→ an eq^n involving one dependent variable and it's derivative w.r.t one or ~~two~~ more independent variables.

Differential equation

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ODE	PDE.
→ If the derivative in the equation have reference to a single independent variable	When the derivative in the equation have two or more independent variables is known as PDE

Order :→ if the highest derivative of y in eq^n is n^{th} derivative

eg $\frac{d^3y}{dx^3} + 5\frac{d^2y}{dx^2} + 6y = 0$

↳ eq^n of order 3.

Degree ⇒ degree of the highest derivative in the equation after it has been freed from fractions and radical signs.

$\left(\frac{d^2y}{dx^2}\right)^3 + 5\frac{dy}{dx} + y = 0$ → degree of 3.

Solution of a differential eq^n

Is the functional relation among the variables, free from the derivatives, which along the way satisfy the given differential eq^n.

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Define

(1) Homogenous diff^n equation.

The equation of the form $y' = g\left(\frac{y}{x}\right)$ is known as homogenous eq^n of first order.

The eq^n of the form $y'' + py' + q = 0$ is the homogenous equation of 2nd order.

(2) Exact diff^n eq^n

The diff eq^n of the form $Mdx + Ndy = 0$, is called differential eq^n if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ is satisfied

if not satisfied, we multiply both by IF and proceed.

(3) linear diff^n eq^n

differential eq^n of the form $y' + Py = Q$ - where P & Q are constant or function of independent variables. of the eq^n is called linear diff^n eq^n of 1st order.

$\boxed{\text{Note}}$ if $Q = 0$ Then it's called homogenous linear diff eq^n.

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$y'' + Py' + Qy = R$ → homogenous linear diff^n eq^n of second order.

(4) Bernoulli's equation.

The differential equation of the form $y' + Py = Qy^n$ where P and Q are constant or function of independent variable of the equation is called Bernoulli's eq^n.

(5) What is auxiliary equation. How can you find the solution of 2^{nd} order differential eq^n?

→ An auxiliary eq^n is when we replace $m = d/dx$ when solving a homogenous eq^n of 2^{nd} order.

We get an auxiliary eq^n

The auxiliary eq^n forms two roots m_1 and m_2.
if $m_1 = \alpha$, $m_2 \neq \beta$ Then → $y = C_1 e^{\alpha} + C_2 e^{\beta}$

if $m_1 = m_2 = \alpha$ Then $y = (C_1 + C_2 x) e^{\alpha}$

if $m_1 = a + ib$, $m_2 = a - ib$ Then $y = A\cos x + B\sin x$.

In each case $y_1 = e^{m_1 x}$, $y_2 = e^{m_2 x}$.

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(6) If $m^2 + am + b = 0$, Then write different conditions obtained in solution

(6) (1) we may have

(i) two equal roots

(ii) two distinct roots

(iii) two complex conjugate roots.

(7)

General solution	Particular Solution
→ A general Solution of a differential equation is a solution that has arbitrary constant equal in number with the order of the equation.	Any Solution of a differential equation obtained from general solution by giving particular values of the constants
$y(h)$	y_p.

(9) if $R = x^3$ then $y_p = ?$

$\longrightarrow y_p = C_1 x^3 + C_2 x^2 + C_3 x + C_4$

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Define 2^{nd} order non-homogenous equation

$$y'' + Py' + Qy = R$$

~~1st~~ 2^{nd} order homogenous equation

$$y'' + Py' + Qy = 0$$

Wronskian → is a determinant used ~~to~~ in the study of differential eq^n where it can sometimes show linear independence in a set of solutions

formula : $W(y_1 + y_2) = y_1 y_2' + y_2 y_1'$

~~Where~~

y.

Laplace Transform.

let $f(t)$ be a given function defined for all $t>0$. Then the function $F(s)$, obtained by multiplying $f(t)$ by e^{-st} and integrating w.r.t t from $0 \to \infty$ is called Laplace transform.

It is defined as

$$L\{f(t)\} = F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

The operation by which $F(s)$ is obtained from $f(t)$ is called laplace transformation

The original $f(t)$ in (1) is called inverse transform or Inverse of $F(s)$.
It is denoted by $L^{-1}(F)$

Linearity property of laplace

let the laplace transformation of $f(t)$ and $g(t)$ exi
Then, for any two constant a and b.

$$L(af + bg) = aL(f) + bL(g)$$

Note The inverse of laplace transform is also linear

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1st Shifting Theorem.

If $L\{f(t)\} = F(s)$ Then for $a>0$,

$$L\{e^{at} f(t)\} = F(s-a) = (F(s))_{s \to s-a} \quad \text{for } s>a.$$

Note

for $L\{e^{-at} f(t)\} = (F(s))_{s \to s+a}$.

Unit step function

$$u_a(t) = u(t-a) = \begin{cases} 0 & \text{for } t<a \\ 1 & \text{for } t \geq a \end{cases}$$

2nd shifting theorem.

if $L\{f(t)\} = F(s)$ Then,

$$L\{f(t-a)\, u_a(t)\} = e^{-as} F(s)$$

Differentiation of laplace transform function.

(1) if $L\{f(t)\} = F(s)$

Then, $L\{t\, f(t)\} = -F'(s) = -\frac{d}{ds} F(s)$

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(11) if $L\{f(t)\} = F(S)$ Then

$$L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}(F(S))$$

for $n = 1, 2, 3, \ldots$

Convolution of two function

$$(f * g)(t) = \int_0^t f(T)\, g(t-T)\, dT = \int_0^t f(t-T)\, g(t)\, dT.$$

Convolution theorem for laplace transform

let $f(t)$ and $g(t)$ are piecewise continuous function in any finite interval for $t \geq 0$. Also, let $L\{f(t)\} = F(s)$ and

$$L\{g(t)\} = G(S) \quad \text{Then}$$

$$L(f * g) = F(S)\, G(S)$$

Existance theorem for laplace Trans form

let $f(t)$ is piecewise continuous function of any finite interval in $t \geq 0$ and $|f(t)| \leq Me^{kt}$ is $t \geq 0$ and $|f(t)| \leq Me^{kt}$ is satisfied for $t \geq 0$ and for some constants M and k Then

$$L\{f(t)\} = \int_0^\infty e^{-st} f(dt) \quad \text{for } S \geq k.$$

Partial differentiation

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(1) $f(x)$ = function of independent variable x
$f(y)$ = " " " y

(2) Homogenous function of two independent variables

a function $f(xy)$ is said to be homogenous of degree n if it can be expressed in the form

$$f(xy) = x^n \phi\left(\frac{y}{x}\right) \quad \text{or} \quad f(xy) = y^n \psi\left(\frac{y}{x}\right)$$

also,

if $f(tx, ty) = t^n f(xy)$; for all values of t.

(3) is $f_{xy} = f_{xy}'$. if not, why?

$\Rightarrow$

(4) State Euler's Theorem:

if $u = f(xy)$ be a homogenous function of degree n then

$$x\frac{du}{dx} + y\frac{du}{dy} = nu$$

&

$$x\frac{df}{dx} + y\frac{df}{dy} + z\frac{df}{dz} = nf$$

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If f_x and f_y are partial derivative of f then find it's total derivative.

Partial derivative

let $z = f(x, y)$ be a function of two variables x and y. Then the partial derivative of f w.r.t x is defined as

$$f_x = \frac{df}{dx} = \lim_{h \to 0} f(x+h, y) - f(x, y)$$

& partial derivative of f. w.r.t y is defined as

$$f_y = \frac{df}{dy} = \lim_{k \to 0} \frac{f(x, y+k) - f(x, y)}{k}$$

Total derivative

let u be a function of x and y. let $\frac{du}{dx}$ and $\frac{du}{dy}$ are continuous

Then, total derivative of u is defined as,

$$du = \frac{du}{dx} dx + \frac{du}{dy} dy.$$

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What are implicit function

Maxima and Minima

Maximum value

let $f(x,y)$ be a function of two variables, Then we say f has maximum value at the point (a,b) if $f(x\,y) < f(a,b)$

Minimum value

$f(xy) > f(a,b)$

for all $(x\,y)$ lies in some open disk containing $(a\,b)$

Criteria $\boxed{\text{Maxima}}$ ~~extreme point~~

$f_{xx} < 0$ ② $\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} > 0$

$\boxed{\text{Minimum}}$

① $f_{xx} > 0$ ② $\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} > 0$

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① Extreme point

point (a,b) is said to be extreme point of given function f(X,Y) if the function f(x,y) has maximum or minimum value at (a,b).

② Stationary point (~~critiale~~ critical point)

The point, at which the first differential function is zero, is known as stationary point of function.

$f_x = 0$ $f_y = 0$

③ critical point

let f(X,Y) be a function of two variables a pair (a,b) is a critical point of f if either

i) $f_x(a,b) = 0$ and $f_y(a,b) = 0$

ii) $f_x(a,b)$ or $f_y(a,b)$ does not exist.

④ Saddle point

A point (a,b) is called the saddle point of the function f(X,Y) if $f_{xx} f_{yy} - f^2_{xy}$ is negative where the suffixes indicates the derivative of the function with respect to the variable in the suffix.

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Double Integral

let $f(x,y)$ be a function of two variables defined in a rectangular region R; where
R : $a \le x \le b$, $c \le y \le d$
Then double integral of f over R is denoted by

$$\iint_R f(x,y)\, dA \quad \text{or} \quad \iint_R f(x,y)\, dy\, dx$$

Area and volumes by double integral.

If $f(x,y) \geq 0$ and f is continuous, Then the volume V of the solid that lies under the graph of $z = f(x,y)$ and over a Rx region in the xy plane is given by

$$V = \int_a^b A_x\, dx = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\, dy\, dx$$

where A_x is the area of typical cross section of solid.

Note if $f(x,y) = 1$, Then $\iint_R f(x,y)\, dA$ gives area bounded by the curve in region R otherwise it gives the volume of the solid bounded by the region R.

Jacobian

Changing Cartesian Integral to polar.

① substute $x = r\cos\theta$
$y = r\sin\theta$
$dy\,dx = r\,dr\,d\theta$

② Supply polar limits of integration.

Ch 1 -

direction cosine ⇒ The cosine of the angle made by a straight line with positive direction of the axes of coordinate are known as direction cosines of a line.

let α, β and γ are the angle made by a straight line with positive direction of axes, then $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are direction cosines of the line.

generally denoted as
$l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$.

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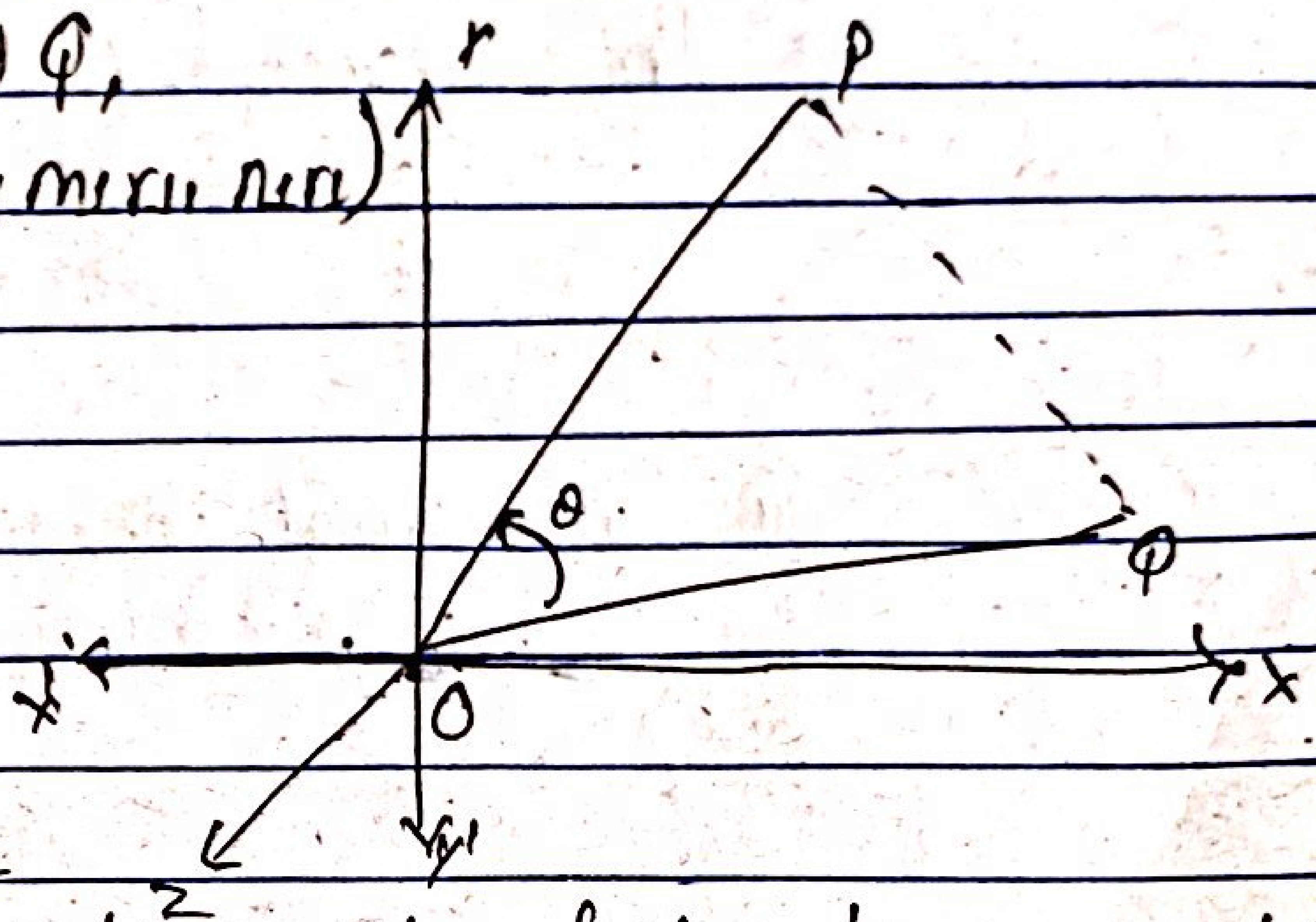
Direction ratio

The number that are proportional to the direction cosines of the line is called direction ratio.

Angle between two straight line

let l_1, m_1, n_1 and l_2, m_2, n_2 are two direction cosines of line OP and OQ,



Then the coordinates of $P(l_1r_1, m_1r_1, n_1r_1)$ and $Q(l_2r_2, m_2r_2, n_2r_2)$

Where $OP = r_1$ & $OQ = r_2$

Then

$\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$

Note

① if $a_1\ b_1\ c_1$ & $a_2\ b_2\ c_2$ are direction ratio of two lines l_1 & l_2 having angle θ

Then,

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{(\Sigma a_1^2)(\Sigma a_2^2)}$$

② if two line perpendicular

$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

③ if two lines are parallel, then

$l_1 = l_2 \qquad m_1 = m_2 \qquad n_1 = n_2$

or

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

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Plane → A surface such that every point on the line joining any two points on the surface lies on the surface.

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A surface represented by an equation $ax+by+cz+d=0$ is known as plane.

it's different form

(i) General form:

$$ax+by+cz+d=0$$

(ii) In Intercept form:

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \quad \text{for } d\neq 0$$

(iii) In normal form:

$$lx+my+nz=p.$$

Suppose $ax+by+cz+d=0$ is the equation of the plane.
What are the dr's of the normal?

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eq^n of line $\Rightarrow \dfrac{x-x_1}{a} = \dfrac{y-y_1}{b} = \dfrac{z-z_1}{c}$

a, b, c → direction ratios
Through (x_1, y_1, z_1)

(ii) Through (x_1, y_1, z_1) and (x_2, y_2, z_2)

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

General point of line
let, $\dfrac{x-x_1}{a} = \dfrac{y-y_1}{b} = \dfrac{z-z_1}{c} = r$

$x = ar + x_1$ $\quad y = br + y_1$, $\quad z = cr + z_1$
be the general point of the line.

Angle betn a plane and a line

$$\sin\theta = \frac{al + bm + cn}{\sqrt{a^2+b^2+c^2}\ \sqrt{l^2+m^2+n^2}}$$

Condition to represent a pair of planes
an eq^n $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ represent a pair of planes if the condition
$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$
is satisfied.

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plane containing a line

plane containing a line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ is

$$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$$

where $al+bm+cn=0$

Condition for a line to lie on the plane.

A line $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ lie on a plane $a_2x+b_2y+c_2z+d=0$ if

$a_1a_2+b_1b_2+c_1c_2=0$ and $a_2x_1+b_2y_1+c_2z_1+d=0$

Eqn of plane containing a line

$(a_1x+b_1y+c_1z+d_1)+k(a_2x+b_2y+c_2z+d_2)=0$

where k is constant

Coplanar

two lines are said to be coplanar if they lie in same plane

Condition

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0 \text{ is satisfied}$$

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Shortest distance

If two lines are neither parallel nor Intersect to each other then such lines are called skew lines

Shortest distance between two skew lines is the perpendicular distance between them.

formula

$$SD = (x_2 - x_1)l + (y_2 - y_1)m + (z_2 - z_1)n$$

$\boxed{\text{Sphere}}$ The collection of points in a three dimensional space such that the distance them to a fixed point is constant

The fixed point is said to be center and the fixed distance is called the radius of sphere.

eqn $\quad (x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2 = r^2$

formula of sphere in diameter form

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