

Introduction :Introduction to Signal :

A signal is a function that conveys information about a phenomenon. In electronics, it refers to any time varying voltage, current or electromagnetic wave that carries information.

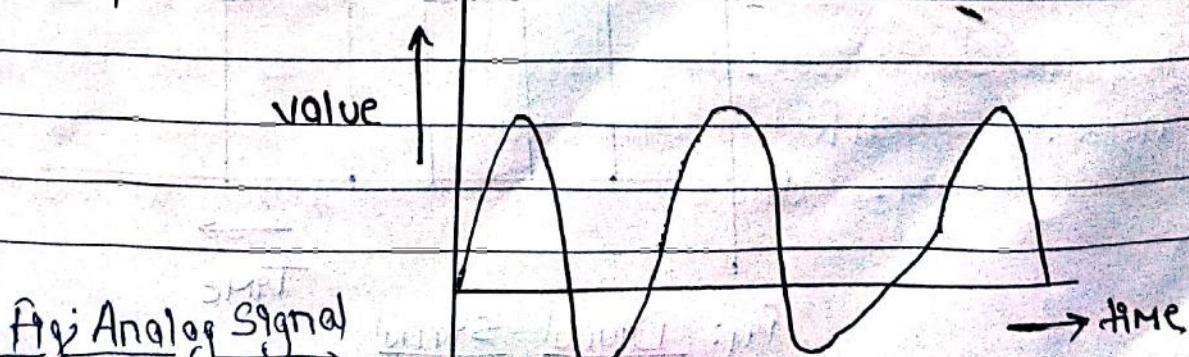
These are following two types of signals:

- ① Analog
- ② Digital.

① Analog Signal

Analog Signal is a kind of continuous wave form that changes over time. An analog signal is further classified into simple and composite signals. A simple analog signal is a sine wave that cannot be decomposed further. On the other hand, a composite analog signal can be further decomposed into multiple sine waves.

An analog signal is described using amplitude, period or frequency and phase. Amplitude marks the maximum height of the signal. Frequency marks the rate at which signal is changing. Phase marks the position of the wave with respect to time zero.



An analog signal is not immune to noise hence, if it faces distortion and decrease the quality of transmission. The range of value in an analog signal is not fixed.

b) Digital Signal

Digital signals also carry information like analog signals but is somewhat is different from analog signal. Digital signal is non-continuous, discrete time signal. Digital signal carries information or data in the binary form i.e. a digital signal represent information in the form of bits. Digital signal can be further decomposed into simple sine waves that are called harmonics. Each simple wave has different amplitude, frequency and phase. Digital signal is described with bit rate and bit interval. Bit interval describes the time required for sending a single bit. On the other hand, bit rate describes the frequency of bit interval.

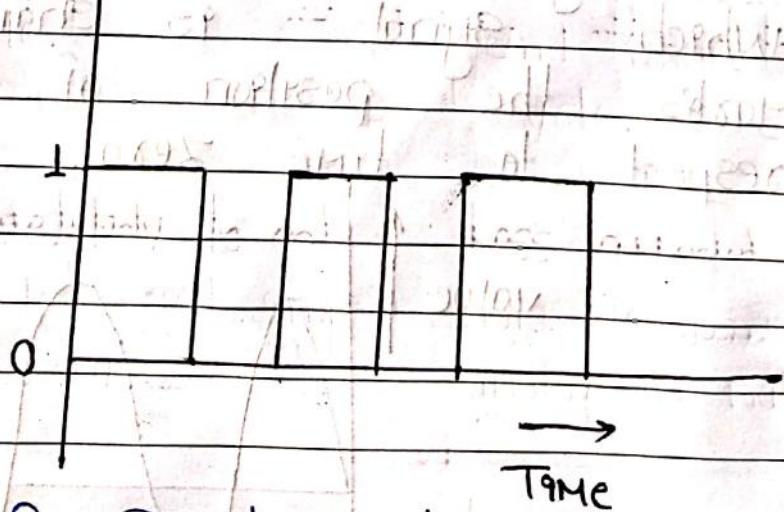


Fig: Digital Signal

A digital signal is more immune to the noise; hence, it hardly faces any distortion. Digital signals are easier to transmit and are more reliable when compared to Analog Signals. Digital signal has a finite range of values. The digital signal consists of 0s and 1s.

Comparison chart:

Basic for comparison	Analog Signal	Digital Signal
Basic	An analog system is a continuous wave that changes over a time period.	A digital signal is discrete wave that carries information in binary form.
Representation	An analog system is represented by a sine wave.	A digital signal is represented by square waves.
Description	An analog system is described by the amplitude, period of frequency, and phase.	A digital signal is described by bit rate and bit intervals.
Range	It has no fixed range.	It has a finite number i.e. 0 or 1.
Distortion	It is more prone to distortion.	It is less prone to distortion.

flexibility	Analog hardware is not flexible.	Digital hardware is flexible in implementation.
Uses	Can be used in analog devices only. Best suited for audio and video transmission.	Best suited for computing and digital electronics.
Application example	Thermometer.	PCs, PDAs.
Memory	Stored in the form of wave signal.	Stored in the form of binary bit.
Power	Analog instrument draws large power.	Digital instrument draws only negligible power.
Cost	Low cost and portable cost is high and not easily portable.	
Impedance	Low	High order of 10s Megohm
Errors	Analog instruments usually have a scale which is cramped at lower end and give considerable observational errors.	Digital instruments are free from like parallax and approximation errors.

Due to above comparisons the digital systems are more preferred rather than analog system.

- ## # Advantages of digital System;
- Have made possible many scientific, industrial, and commercial advances that would have been unattainable otherwise.
 - Less expensive.
 - More reliable.
 - Easy to manipulate.
 - Flexibility and compatibility.
 - Information storage can be easier in digital computer systems than in analog ones. New features can often be added to a digital system more easily too.

Disadvantage of digital System;

- Use more energy than analog circuits to accomplish the same task, thus producing more heat as well.
- Digital circuits are often fragile, in that if a single piece of data is lost or misinterpreted, the meaning of large blocks of related data can be completely change.
- Digital computer manipulates discrete elements of information by means of a binary code.
- Quantization error during analog signal sampling.

Number system can simply be defined as a way to represent numbers.

Types of number system:

1) Decimal number system

$$\text{Radix}(r) = 10$$

$$\text{Symbol} = 0 \text{ through } r-1 = \{0, 1, 2, \dots, 9\}$$

It is also known as base 10 number system as it uses just '10' symbols i.e. 0 to 9.

2) Binary number System.

$$\text{Radix}(r) = 2$$

$$\text{Symbol} = 0 \text{ through } r-1 = \{0, 1\}$$

Binary System refers to the number system that uses only two symbols i.e. 0 and 1. That's why it is called a base '2' number system.

This number system is especially used in the internal processing of Computer System.

3) Octal number System

$$\text{Radix}(r) = 8$$

$$\text{Symbol} = 0 \text{ through } r-1 = \{0, 1, 2, \dots, 7\}$$

If it is base '8' System.

4) Hexadecimal number System :

$$\text{Radix}(r) = 16$$

$$\text{Symbol} = 0 \text{ through } (r-1) = \{0, 1, 2, \dots, 9, A, B, C, D, E, F\}$$

Conversion of Number System: Binary

(1) \rightarrow binary to decimal (base 2 to base 10)

(1) Convert $(110011)_2$ to decimal.

$$\begin{aligned} &= 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ &= 32 + 16 + 2 + 1 \\ &= (51)_{10} \end{aligned}$$

(2) $(1011.101)_2 \rightarrow (?)_{10}$

$$\begin{aligned} &= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times \\ &= \frac{8}{2} + \frac{2}{2^2} + \frac{1}{2} + \frac{1}{2^3} + 1 \\ &= (11.625)_{10} \end{aligned}$$

(2) binary to octal:

(1) Convert $(10110101)_2$ into octal.

Here

$$\begin{array}{r} 10110101 \\ 2 \quad 6 \quad 5 \end{array}$$

$\therefore (10110101)_2 = (265)_8$

(2) $(11100)_2 \rightarrow (?)_8$

$$\begin{array}{r} 011 \quad 100 \\ 3 \quad 4 \end{array}$$

$$\therefore (11100)_2 = (34)_8$$

Q) $(76.2)_2 \rightarrow ()_8$

\rightarrow 7 6 2
 111 110 010

$$\therefore 76.2 \rightarrow (111110.010)_8$$

(3) binary to hexa-decimal:

@ $(1010.11)_2 \rightarrow ()_{16}$

$$(1010.1100)_2 \rightarrow ()_{16}$$

\rightarrow 1010 1100
 10(A) 12(C)

$$\therefore (1010.1100)_2 \rightarrow (A.C)_{16}$$

⑥ $(1010101101)_2 \rightarrow ()_{16}$

Here,

$$0 = 08 \quad 0101 \quad 0101 \quad 1101$$

$$1 = 09 \quad 0101 \quad 0101 \quad D$$

$$\therefore (1010101101)_2 \rightarrow (55D)_{16}$$

$$(10001.0001001) = (22.05)_{10}$$

Decimal

(4) Conversion of decimal to binary
(base 10 to 2)

Q) $(51)_{10} \rightarrow (?)_2$

2	51	1
2	25	1
2	12	0
2	6	0
2	3	1
2	1	1
0		

$$\therefore (51)_{10} \rightarrow (110011)_2$$

Q) $(72.55)_{10} \rightarrow (?)_2$

2	72	0	$.55 \times 2 = 1.10 = 1$
2	36	0	$.10 \times 2 = 0.20 = 0$
2	18	0	$.20 \times 2 = 0.40 = 0$
2	9	1	$.40 \times 2 = 0.80 = 0$
2	4	0	$.80 \times 2 = 1.60 = 1$
2	2	0	
2	01	1	
0			

$$\therefore (72.55)_{10} = (1001000.10001)_2$$

(5) Decimal to octal (base 10 to base 8)

Q)

8	177	1
8	22	6
8	2	2
0		

$$\therefore (177)_{10} \rightarrow (262)_8$$

Q)

$$(25.5)_{10} \rightarrow (?)_8$$

8	25	1
8	3	3
0		

$$\therefore (25.5)_{10} = (31.4)_8$$

(6) Decimal to hexa-decimal (base 10 to base 16)

$$(77)_{10} \rightarrow (?)_{16}$$

16	77	13	= D
16	4	4	= 4
0			

$$\therefore (77)_{10} = (4D)_{16}$$

Conversion of octal

(1) Octal to decimal:

$$\begin{aligned} @ & (632)_8 \rightarrow (?)_{10} \\ = & 6 \times 8^2 + 3 \times 8^1 + 2 \times 8^0 \\ = & 384 + 24 + 2 \\ = & (410)_{10} \end{aligned}$$

(2) Octal to binary:

$$\begin{aligned} (741)_8 & \rightarrow (?)_2 \\ = & \begin{array}{ccc} 7 & 4 & 1 \\ 111 & 100 & 001 \end{array} \end{aligned}$$

$$\therefore (741)_8 = (111100001)_2$$

(3) Octal to hexa-decimal:

$$(75)_8 \rightarrow (?)_{16}$$

$$\rightarrow \begin{array}{cc} 7 & 5 \\ 111 & 101 \end{array}$$

$$\text{Now, } 0011 \quad 1101$$

$$\begin{array}{r} 9 \\ \hline 10 \end{array} \quad \text{D}$$

$$\therefore (75)_8 = (3D)_{16}$$

1.e Conversion of hexa-decimal :
hexadecimal to decimal :

$$\begin{aligned}
 & (F4C)_{16} \rightarrow (?)_{10} \\
 = & F \times 16^2 + 4 \times 16^1 + C \times 16^0 \\
 = & 3840 + 64 + 12 \\
 = & (3916)_{10}
 \end{aligned}$$

$$\therefore (F4C)_{16} \rightarrow (3916)_{10}$$

(ii) hexadecimal to binary :

@ $(A1D)_{16} \rightarrow (?)_2$

$$\begin{array}{ccc}
 A & 1 & D \\
 \downarrow & & \\
 1010 & 0001 & 1101
 \end{array}$$

$$\therefore (A1D)_{16} = (101000011101)_2$$

Some examples

$$1) (75)_8 \rightarrow (?)_{16}$$

↓ ↓
111 101

Now, for hexadecimal,

0011

3

1101

D

$$\therefore (75)_8 = (3D)_{16}$$

$$2) (751)_8 \rightarrow (?)_{10}$$

$$= 7 \times 8^2 + 5 \times 8^1 + 1 \times 8^0$$

$$= 448 + 40 + 1$$

$$= (489)_{10}$$

$$3) (BCA)_{16} \rightarrow (?)_8$$

B

1011

C

1100

A

1010

Now

Binary no	101	111	001	010
Octal equivalent	5	7	1	2

$$(880.70)_{10} \rightarrow (?)_{16}$$

16	880	0
16	55	7
16	3	3
0		

Q)

$$10 \times 16 = 112 \rightarrow 8$$

$$2 \times 16 = 32 \rightarrow 3$$

$$2 \times 16 = 32 \rightarrow 3$$

$$2 \times 16 = 32 \rightarrow 3$$

$$\therefore (880.70)_{10} \rightarrow (370.B3)_{16}$$

$$(0110110.1101)_2 \rightarrow (?)_8$$

Here,

$$(000110110.110100)_2 \rightarrow (?)_8$$

000	110	110	110	100
0	6	6	6	4

$$\therefore (000110110.110100)_2 \rightarrow (066.64)_8$$

(S-X) (S-V)

11000

0010

10100

01100

11010

00101

10010

01011

11001

00111

Decimal to BCD conversion. (8421) (Binary coded decimal)

Decimal	BCD	Unused (don't care)
8	1000	1010
4	0100	1011
2	0010	1100
3	0011	1101
5	0101	1110
6	0110	1111
7	0111	
8	1000	
9	1001	

Decimal to excess-3 (x_{9-3}) ($x-3$)

Decimal	8421	excess-3
0	0000	0011
1	0001	0100
2	0010	0101
3	0011	0110
4	0100	0111
5	0101	1000
6	0110	1001
7	0111	1010
8	1000	1011
9	1001	1100

Self complementing

Ques $(11011)_2 \rightarrow (?)$ excess-3

$$\begin{array}{r} 11011 \\ + 0011 \\ \hline (11110) \times 3 - 3 \end{array}$$

Basic Term:

MSB (Most Significant Bit)

→ The left most bit of a number is called MSB.

LSB (Least Significant bit)

→ The right most bit of a number is called L.S.B.

Decimal to 2421 Code

L.S.B. ←

MSB →

Decimal	2421	2421
0	0000	0000
1	0001	0001
2	0010	1000
3	0011	1001
4	0100 (1010)	1010
5	0101	1011
6	0110	1100
7	0111	1101
8	1110	1110
9	1111	1111

Note: If the addition of the weight is equal to '9' then it become Self-complementing weight.

Qn. Prove 2421 is a self-complementing.

Decimal

0

1

2

3

4

5

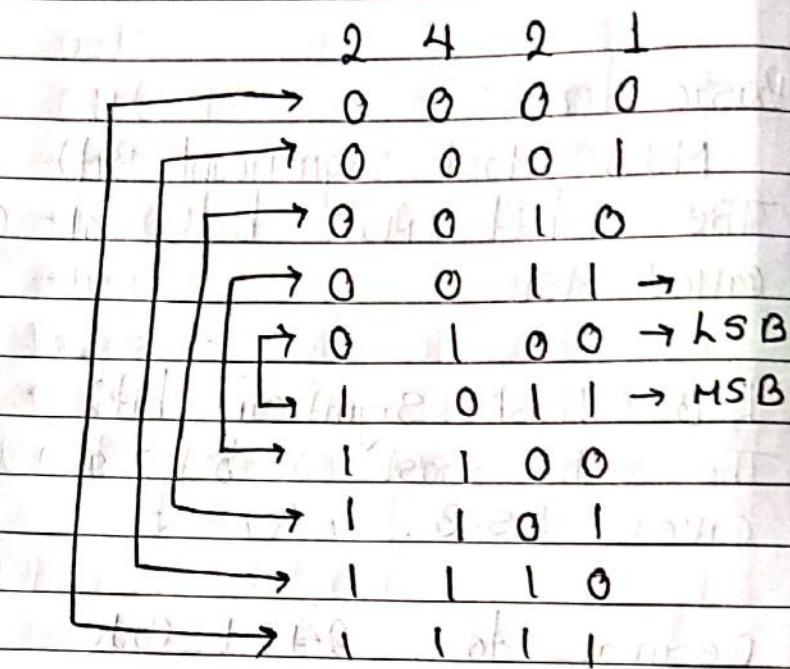
6

7

8

9

\leftarrow Weight



Hence 2421 is self-complementing.

0000

0000

1000

1000

0001

0100

1001

1100

0101

(0101) 0010

1101

1010

0011

11010

1011

1110

0111

0111

1111

1111

84-2-1 code

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Decimal

0
1
2
3
4
5
6
7
8
9

8	4	-2	-1
0	0	0	0
1	0	01	01
2	0	1	10
3	0	10	01
4	D	01	D
5	1	0	1
6	1	01	0
7	1	00	1
8	1	00	0
9	1	00	1

Hence, 84-2-1 is Self-complementing code.

Decimal to XS-3

④ $(6)_{10} \rightarrow (?)_{XS-3}$

0110

+ 11

1001

⑤ $(68)_{10} \rightarrow (?)_{XS-3}$

0110 1000

+ 11 + 11

1001 1011

$\therefore (68)_{10} = 10011011$

$$Q \rightarrow (2)_{10} + (7)_{10}$$

$$\begin{array}{r} 0010 \\ + 111 \\ \hline 01001 \end{array} \quad \begin{array}{r} 0111 \\ + 11 \\ \hline 1010 \end{array}$$

$$\begin{array}{r} 01001 \\ + 1010 \\ \hline 11101 \end{array}$$

Now,

$$\begin{array}{r} 11101 \\ - 0011 \\ \hline (1100)_{xs=3} \end{array}$$

$$\begin{aligned} \therefore (2)_{10} + (7)_{10} &= (1100)_{xs=3} \\ &= (12)_{10} \end{aligned}$$

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Binary to Gray code and vice versa

a) $(11010)_2 \rightarrow (?)$ graycode.

$$\begin{array}{cccccc} & \downarrow & \downarrow & \downarrow & \downarrow & \\ 1 & 1 & 0 & 1 & 0 & \\ \downarrow & & & & & \\ 1 & 0 & 1 & 1 & 1 & \end{array} \quad (\text{to remove } 1)$$

$\therefore (11010)_2 \rightarrow (10111)$ graycode.

b) (10111) graycode $\rightarrow (?)$ binary

$$\begin{array}{ccccc} 1 & 0 & 1 & 1 & 1 \\ \downarrow & \nearrow & \nearrow & \nearrow & \nearrow \\ 1 & 1 & 0 & 1 & 0 \end{array}$$

$\therefore (10111)$ graycode = $(11010)_2$

BCD to gray code.

Decimal	BCD	Gray code
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101

Que

@

$$(11001010)_2 \rightarrow (\text{ }) \text{gray code}$$



$$1\ 0\ 0\ 1\ 0\ 1\ 0$$

$$\downarrow$$

$$1\ 0\ 1\ 0\ 1\ 1$$

$$\therefore (11001010)_2 \rightarrow (10101111) \text{gray code.}$$

⑤ $(6)_{10} + (7)_{10}$ using X-S-3

$$\begin{array}{r}
 0110 \\
 + 11 \\
 \hline
 1001
 \end{array}
 \quad
 \begin{array}{r}
 0111 \\
 + 11 \\
 \hline
 1010
 \end{array}$$

$$\text{Now } 1001$$

$$+ 1010$$

$$10011$$

$$- 11$$

$$\underline{(10000)_{X-S-3}}$$

$$\therefore (6)_{10} + (7)_{10} = (16)_{10}$$

Complements :

There are two types of complements:
 r 's complement and $(r-1)$'s complements.
 Where ' r ' is the base of a number system.

In binary number system there are two types of complements: 1's complement & 2's complements.

Similarly, decimal number system has 9's and 10's complement.

#

1's complement:

We can get 1's complement by simply replacing '1' by '0' and '0' by 1.

eg: 1's complement of 1011 = 0100

Subtraction of binary number using 1's complement:
 Steps:

1) Make the both numbers having same number of bits.

2) Determine the 1's complement of the number to be subtracted (subtrahend).

3) Add the 1's complement to the given number from which we subtract (minuend).

4) If carry bit occurs; remove and add it to the results.

5) If there exist no carry determine 1's complement of the result and prefix by a negative sign to get the final result.

eg: Sub 1110000 from 1100000

$$\text{i.e., } 1100000 - 1110000$$

1's complement of subtrahend : 0001111

(a) Add it with minuend : 0001111

$$+ \underline{1100000}$$

$$1101111$$

Since, no any additional bits occur,
1's complement of 1101111 = 0010000

Hence difference will be -0010000.

(2)

$$111000 - 110000$$

1's complement of subtrahend 001111

Add it with minuend

$$001111$$

$$+ \underline{111000}$$

$$1000111$$

Since, there exist one additional bit,
difference

$$000111$$

$$+ \underline{111000}$$

$$001000$$

Hence, difference will be 001000.

$$001000$$

2's Complement

The 2's complement of binary number is obtained by adding 1 to the 1's complement.

eg: 2's complement of 1101101 = ?

$$\begin{array}{rcl} \text{1's complement} & = & 0010010 \\ \text{2's complement} & = & 0010010 \end{array}$$

$$\begin{array}{rcl} & + & 1 \\ \hline & & 0010011 \end{array}$$

Subtraction using 2's complement:

Steps:

- (1) Make the both numbers having the same number of bits.
- (2) Determine the 2's complement of the number to be subtracted. (Subtrahend)
- (3) If there exist no carry determine the 2's complement of the result and prefix by a negative sign to get a final result.
- (4) If carry bit occur remove (1) and the final is ans.

Eg: Sub 1110000 from 1100000

Here,

$$1100000 - 1110000$$

2's complement of Subtrahend 0001111

$$+ \underline{1}$$

$$0010000$$

Add it to minuend

$$0010000$$

$$+ 1110000$$

$$1000000 1110000$$

Since, there exists no additional bit,

2's complement of 1110000 = 0001111

$$+ \underline{1}$$

$$0010000$$

Hence difference will be -0010000.

*

$$111000 - 110000$$

2's complement of 110000 = 001111

$$+ \underline{1}$$

Add it to minuend

$$010000$$

$$+ 111000$$

$$1001000$$

Since, there exists additional bit.

Difference will be 001000 (neglect 1).

Examples :

$$\textcircled{1} \quad (1001)_2 - (1000)_2$$

→ For 1's complements,
 1's complement of subtrahend (0111),
 Add it to minuend 0111
 + 1001
 (10000)₂

Since, additional bit occur.

Difference 0000

+ 1 1 1

1000110

$(0001)_2$ Ans

→ For 2's complement,

2's complement of Subtrahend

三三一

卷之三

(1000),

Add it to menuend

1001

+ 1000

(1000),

Since additional bit occur.

difference (0001), Am

Kimberly Ann Hanibell

(91)

$$(10110)_2 - (011)_2$$

1st Complement:

At 1st balancing the eqn,



$$(10110)_2 - (0001)_2$$

1st complement of subtrahend $(11100)_2$ Add it to minuend 11100

$$+ 10110$$

$$(110010)_2$$

Since, additional bit occur,

difference 110010

$$+ 1$$

$$(10011)_2 \text{ Ans}$$



2's Complement

2's complement of subtrahend 11100

$$+ 1$$

Add it to minuend 11101

$$+ 10110$$

$$(110011)_2$$

Since additional bit occurs,

difference $(10011)_2$ Ans (neglect 1).

9's & 10's complements :

11 The 9's complement of decimal no. can be obtained by subtracting each digit of number from 9.

For example 9's complement of 3 is 6. ($9-3$).
and 234 is 765 ($999 - 234 = 765$).

Steps :

- 1) Make the both numbers having the same number of digits.
- 2) Determine the 9's complement of the number from which to be subtracted (subtrahend).
- 3) Add the 9's complement to the given number from which we subtract (minuend).
- 4) If additional bit occur remove it and add it to the complement of the result.
- 5) If additional bit doesn't occur determine 9's complement of result and prefix negative to get final result.

$$(1+8-3) \rightarrow 11-3 = 8$$

: Resulting 2nd value will be 0.

Value of minuend is 11 and value of subtrahend is 3.

Value of minuend is 11 and value of subtrahend is 3.

eg: Subtract $(123)_{10}$ from $(345)_{10}$

$$(345)_{10} - (123)_{10}$$

9's complement of subtrahend 999

$$\begin{array}{r} \underline{-123} \\ 876 \end{array}$$

Add it with minuend

$$\begin{array}{r} 876 \\ + 345 \\ \hline 1221 \end{array}$$

Since, additional bit occurs,
difference

221

+ 111

222

$(222)_{10}$ is the required ans.

10's complement

The 10's complement of decimal number can be obtained by adding 1 to the least significant digit of 9's complement.

eg: 10's complement of 3 98 7 ($9-3+1$)

Subtraction using 10's complement:

17. Make the both numbers having same number of digits.

97. Determine 10's complement of the number to be subtracted.

- 4) Add the 10's Complement to the given number from which we subtract.
- 47) If there exists any additional digit in the result after addition, remove it from the result and the remaining digits form the final result.
- 5) If there exists no any carry then determine 10's complement of the result and prefix by the negative sign to get final result.

eg: $(345)_{10} - (123)_{10}$

10's complement of Subtrahend $(999-123)$
 $= 876 + 1 = 877$

Add it to Minuend, 877
 $+ 345$
 1222

Since, additional bit occurs.

So, difference 1222 (neglect 1)

$(122)_{10}$ is the req ans.

(1010)

Some other examples:

Q7 $(20)_2 - (10)_2$ using (J-U)'s complement
changing into binary

$$20 = (11010)_2$$

$$10 = (1010)_2$$

$$\text{Here, } (11010)_2 - (1010)_2$$

balancing eqn,

$$(11010)_2 - (01010)_2$$

M

S

1's complement of Subtrahend $(1010)_2$

Add it to minuend

$$10101$$

$$+ 10100$$

$$(101001)_2$$

Since additional bit occur,
difference

$$01001$$

$$+ 1$$

$$(01010)_2$$

Hence, difference will be $(01010)_2$
i.e $(10)_10$

27) $(5)_{10} - (-7)_{10}$ using $(r-1)$'s complement.

9's complement of 7 $(9-7) = 2$
 Add 9t to minuend

$$\begin{array}{r} \text{minuend} \\ + 5 \\ \hline 7 \end{array}$$

Since, no additional bit occurs.
 9's complement of 7 $\rightarrow 9$

$$\begin{array}{r} 9 \\ - 7 \\ \hline 2 \end{array}$$

\therefore Difference will be $-(2)_{10}$

37) $(789)_{10} - (890)_{10}$ using r's and $(r-1)$'s complement.

r's complement

10's complement of 890, $999 - 890 = 109$

Add 9t to minuend

$$\begin{array}{r} 110 \\ + 789 \\ \hline 899 \end{array}$$

Since, no additional bit occurs,
 10's complement of 899 $9s - 899 = 100$

\therefore Difference will be $-(10)_{10}$.

→ $(\bar{r}-1)$'s complement.
 9's complements of 890 is 999
 $\begin{array}{r} 999 \\ - 890 \\ \hline 109 \end{array}$

Add it to minuend, $\begin{array}{r} 109 \\ + 789 \\ \hline 998 \end{array}$

Since no additional bit occur,
 diff 9's complement of 898 is 999
 $\begin{array}{r} 999 \\ - 898 \\ \hline 101 \end{array}$

∴ Difference is $-(101)_{10}$

(4) $(85)_2 - (70)_2$ using 7's complement.

Changing to binary

$$85 = (101010)_2$$

$$70 = (1000110)_2$$

Here,

$$(101010)_2 \oplus (1000110)_2$$

2's complement of Subtrahend

Add it to minuend $011100 + 011100$

$$\begin{array}{r} 011100 \\ + 011100 \\ \hline 10001110 \end{array}$$

Since additional bit occur.

$$(10001111)_2$$

$$\begin{array}{r} 101 \\ \downarrow \\ (15)_{10} \end{array}$$

(5)

0110010

011110110

1's complement

10010110
1001010110

Now, $10010110 + 1001010110 = 1000101110$

1's complement of subtraction : 0111010100
Add it to inverse : 0111010100
+ 1001011
1000101110

Since no odd bit occurs,
1's complement : 0111100000.
Difference = $(0111100000)_2$
= $(480)_{10}$

67 $(1101.10)_2 - (1101.00)_2$ using 1's and 2's complement
Using 2's complement

→ 1's complement of subtrahend $(0010.11)_2$
Add it to minuend $(110000.01)_2$

$$+ 1101.10$$

$$\text{Cin} = 0 \text{ because } 110000 + 1101.10 = 10000.01$$

Since additional bit occurs,

difference 0000.01_2

$$+ 1$$

$$(0000.10)_2$$

using 2's complement

2's complement of Subtrahend 0010.11_2

$$+ 1$$

$$0011.00$$

Add to minuend 0011.00

$$+ 1101.10$$

$$10000.10$$

Since additional bit occurs,
difference will be $(0000.10)_2$ (neglect 1).

(7) $(80.2)_10 - (70.2)_10$ using 1's and 10-1)'s complement
 \rightarrow using 10's complement

10's complements of 70.2

99.9

$- 70.2$

29.7

Add it to minuend

29.7

$+ 1$

$+ 80.2$

29.8

$\bullet 110.0$

Since add. bit occurs,
difference

$(10.0)_10$ Ans

\rightarrow Using 10-1)'s complements.

9's complement of 70.2

99.9

$- 70.2$

29.7

Add to minuend

29.7

$+ 80.2$

109.9

Since add. bit occurs.

difference,

09.9

$+ 1$

$(10.0)_10$ Ans

(523) x . Find the value of x .

Here,

$$x-1 = 5$$

$$x = 6$$

$x = 6, 7, 8 \dots$ So on.

If find only natural value then,
 $x = 6$.

#

Convert $(62)_x = (50)_{10}$

$$6x^1 + 2x^0 = 15 \times 10^1$$

$$\text{or, } 6x = 48$$

$$\therefore x = 8$$

$$\therefore (62)_8 = (50)_{10}$$

\rightarrow

$$(40)_x = (50)_8$$

$$4x^1 = 5 \times 8^1$$

$$4x = 40$$

$$\therefore x = 10$$

$$\therefore (40)_{10} = (50)_8$$

#

17

27

Parity
even parity
odd parity

17 even parity

A	B	C	P
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

27 odd parity

A	B	C	P
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

May 1910 and large flock
of young birds. The species of which
are not yet known, but they are mostly
with wings expanded.

Protein 16.4% 43 min. intake 600
Carbohydrate 45.0% 100% digest. 600
Dietary fiber 16.0% 100% digest. 600
Amino acids 10.0% 100% digest. 600
Glycogen 1.0% 100% digest. 600

Winter

三

三

四

2000, 1900 (1900)
1900, 1900 (1900)
1900, 1900 (1900)

Laws of Boolean Algebra:

	Name	AD/D FORM	OR FORM
1)	Identity law	$1 \cdot A = A$	$0 + A = A$
2)	Null law	$0 \cdot A = 0$	$1 + A = 1$
3)	Idempotent law	$A \cdot A = A$	$A + A = A$
4)	Inverse law	$A \bar{A} = 0$	$A + \bar{A} = 1$
5)	Commutative law	$AB = BA$	$A + B = B + A$
6)	Associative law	$(AB)C = A(BC)$	$(A+B)+C = A+(B+C)$
7)	Distributive law	$A+(BC) = (A+B) \cdot (A+C)$	$A(B+C) = AB+AC$
8)	Absorption law	$A(A+B) = A$	$A + AB = A$
		$A + (A\bar{B}) = A + B$	
9)	De-Morgan's law	$\bar{AB} = \bar{A} + \bar{B}$	$\bar{A+B} = \bar{A} \cdot \bar{B}$

Logic gate and truth table

A logic gate is an electronic circuit that operates on one or more input signals to produce an output signal. A logic gate is also known as building block of a digital circuit.

Truth table is a table of all possible combinations of the variables showing the relation between the values that variables may take and the result of the operation.

- It is divided into three parts;
- 17 Basic types (AND, OR, NOT).
 - 27 Universal gates (NAND, NOR).
 - 37 Combinational gates (EX-OR, EX-NOR).

#

Basic gates(1) AND gate:

→ AND gate performs logical multiplication.
 → Output of AND gate is high if all the inputs are high otherwise output will be low.

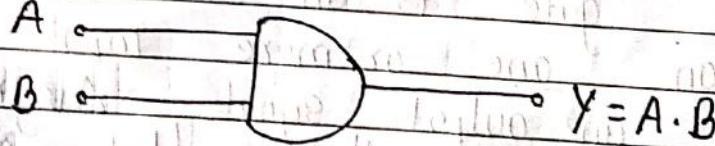
The graphical symbol, logical ckt, algebraic expression and truth table of AND gate is shown below;

①

Mathematical eqn or boolean eqn, is

$$Y = A \cdot B$$

②



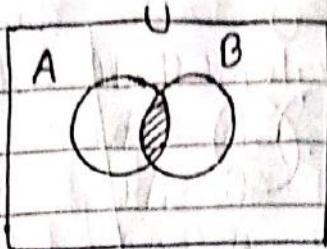
③

graphical symbol

Truth table

A	B	$Y = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

Venn-diagram



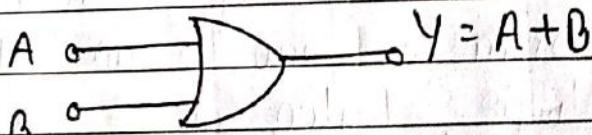
(ii) OR gate

- It performs logical addition.
- The OR gate contains two or more than two input values which produces only one output value.
- Its output will be high if any or all input are high.

→ The boolean equation of OR gate is

$$Y = A + B$$

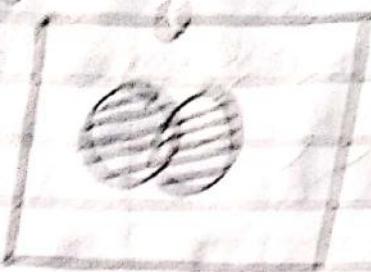
→ Symbol of OR gate is,



→ Truth table of OR gate,

A	B	$Y = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

Venn diagram:



iii) NOR gate

- It contains logical NOT operation.
- The NOR gate contains only one input AND gate which produces only one output value. This gate is also known as NOT OR gate.
- It's symbol is like OR gate except its output is inverted.
- The Boolean expression of NOR gate is, $y = \bar{A}$.

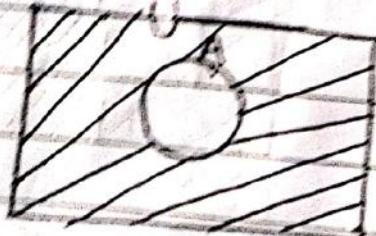
→ Symbol;

$$A - \square \rightarrow y = \bar{A}$$

→ Truth table;

A	$y = \bar{A}$
0	1
1	0

→ Venn diagram:



Universal gate:
 NAND and NOR gates are called universal gates because all basic gates can be realized from these gates.

(9) NOR gate

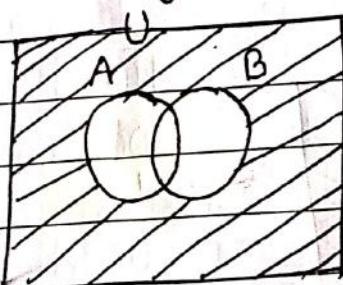
- This gate is combination of OR and NOT gate. This gate is complement of the OR function.
- The output is high if both input are low otherwise output are low.
- It's boolean expression is,

$$y = \overline{A+B}$$

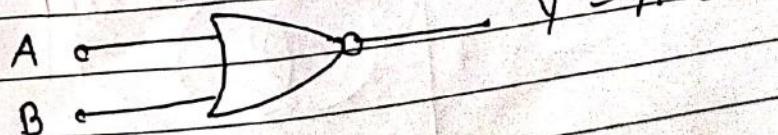
Truth table

A	B	$y = \overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

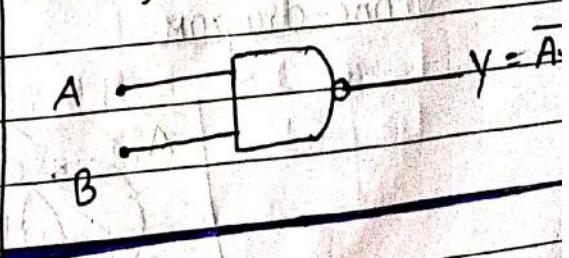
Venn diagram



Symbol NOR



Symbol NAND



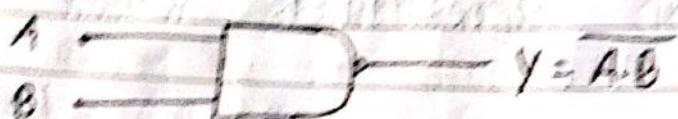
AND gate

- This gate is the combination of AND NOT gate. This gate is called as AND NOT Buffer.
- The output is high if any one of input or both inputs are low.

→ The Boolean expression:

$$Y = \overline{A} \cdot B$$

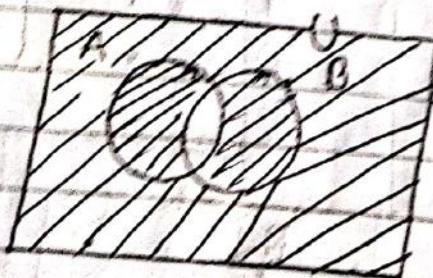
→ Symbol:



→ Truth table:

A	B	$Y = \overline{A} \cdot B$
0	0	1
0	1	0
1	0	0
1	1	0

Venn-diagram



Combination of gates

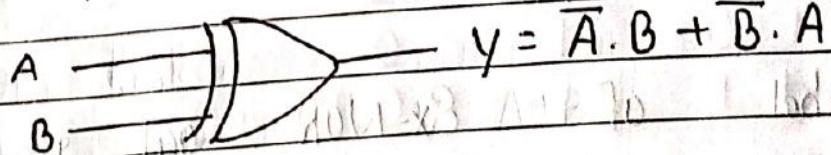
Ex-OR / X-OR gate

- It is called combination because all the basic gate functions are included in it.
- the output of exclusive OR gate is high if both inputs are ~~not~~ different.
- The boolean expression of Ex-OR gate is

$$y = \overline{A} \cdot B + \overline{B} \cdot A$$

or, $y = A \oplus B$

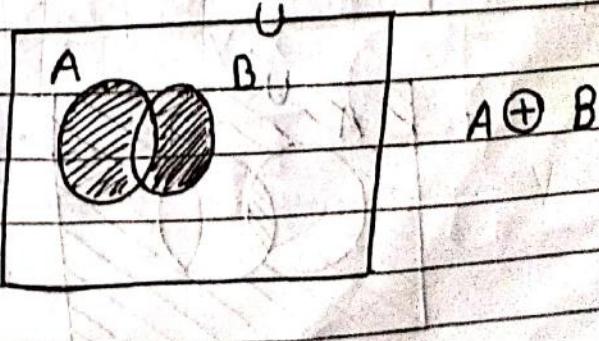
- The symbol of Ex-OR gate is,



The truth table for Ex-OR gate is,

A	B	\overline{A}	\overline{B}	$\overline{A} \cdot B$	$\overline{B} \cdot A$	$\overline{A} \cdot B + \overline{B} \cdot A$
0	1	1	0	1	0	1
0	0	1	1	0	0	0
1	1	0	0	0	0	0
1	0	0	1	0	1	1

Venn-diagram



27

Exclusive NOR (X-NOR) Gate:

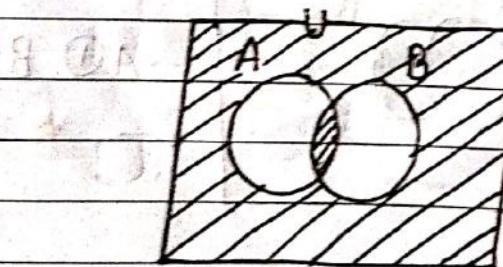
- The gate contains two or more than two input values which produce only one output value. The 'X-NOR' is the complement of 'X-OR', as indicated by small circle in the graphical symbol.
- The output of XNOR gate is high if both input are same (i.e. high-high or low-low)
- The boolean expression for Ex-NOR gate is

$$Y = A \cdot B + \overline{A} \cdot \overline{B}$$

- Symbol of Ex-NOR gate is,



A	B	\bar{A}	\bar{B}	$\bar{A}\bar{B}$	$y = A \cdot B + \overline{A} \cdot \overline{B}$
0	0	1	1	1	1 0 0
0	1	1	0	0	0 1 1
1	0	0	1	0	0 0 1
1	1	0	0	0	1



De Morgan's Theorem :

De Morgan's theorem is associated with Boolean algebra which was given by great logical and mathematician De Morgan. So, it is called "De Morgan's theorem". It consists of first and second theorems.

1) First Theorem (Addition theorem)

It states that "The complement of the sum is equal to the product of complements of individual variables".

If x and y are two variables then

$$\overline{x+y} = \overline{x} \cdot \overline{y}$$

for three variables,

$$\overline{A+B+C} = \overline{A} \cdot \overline{B} \cdot \overline{C}$$

Proof:

x	y	$x+y$	$\overline{x+y}$	\overline{x}	\overline{y}	$\overline{x} \cdot \overline{y}$	$\overline{x+y} = \overline{x} \cdot \overline{y}$
0	0	0	1	1	1	0	0
0	1	1	0	1	0	0	0
1	0	1	0	0	1	0	0
1	1	1	0	0	0	0	0

Hence, from above table,

$$\overline{x+y} = \overline{x} \cdot \overline{y}$$

graphically



$$\text{As we know: } x \cdot y + z = x \cdot y \cdot z.$$

Product	x	y	z	$x \cdot y \cdot z$	$x \cdot y + z$	x	y	z	$x \cdot y \cdot z$
0	0	0	0	0	1	1	1	0	0
0	0	0	1	0	0	1	0	1	0
0	0	1	0	0	1	0	0	0	0
0	0	1	1	0	1	0	0	0	0
0	1	0	0	0	0	1	1	0	0
0	1	0	1	0	0	0	1	0	0
0	1	1	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0

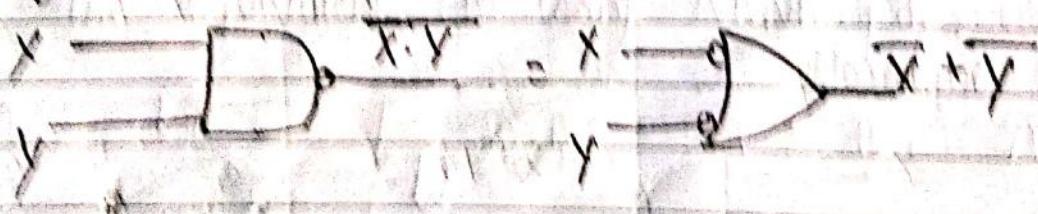
Product

87 2nd theorem (Multiplication theorem)

The complement of product term is equal to sum of individual complement, i.e

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

graphically,



proof:

	x	y	$\bar{x}y$	$\bar{x}\bar{y}$	x	\bar{y}	$\bar{x} + \bar{y}$
x	1	0	0	1	1	0	1
0	0	0	0	1	0	0	0
0	1	1	0	0	0	1	1
1	0	0	1	1	0	1	1

Hence, $\bar{x}\bar{y} = \bar{x} + \bar{y}$

for 3-variable

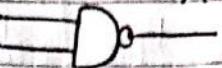
x	y	z	xyz	\bar{xyz}	\bar{x}	\bar{y}	\bar{z}	$\bar{x} + \bar{y} + \bar{z}$
0	0	0	0	1	1	1	0	1
0	0	1	0	1	1	0	1	1
0	1	0	0	1	1	0	0	1
0	1	1	0	1	0	1	1	1
1	0	0	0	1	0	1	0	1
1	0	1	0	1	0	1	1	1
1	1	0	0	1	0	0	1	1
1	1	1	1	0	0	0	0	0

Proved

$$\bar{xyz} = \bar{x} + \bar{y} + \bar{z}$$

(#) Realization of universal gate :

1) NAND gate

Symbol  $\overline{A \cdot B}$

→ NOT gate

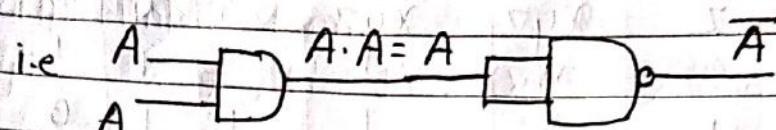
We know for NAND gate,

$$y = \overline{A \cdot B}$$

$$\text{if } A = B$$

$$y = \overline{A \cdot A} = \overline{A}$$

$$\therefore y = \overline{A}$$



→ AND gate

$$y = A \cdot B$$

For that the result of NAND gate is

$$y = \overline{\overline{A \cdot B}} = A \cdot B$$

→ OR gate; $y = A + B$

The equivalent circuit using NAND gate.

$$A \xrightarrow{\text{NAND}} \overline{A} \quad B \xrightarrow{\text{NAND}} \overline{B}$$

$$\overline{A \cdot B} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$$

(from deMorgan's law)

2) NOR gate

→ NOT gate $Y = \bar{A}$.



A.	B.	$A+B$
0	0	0
0	1	1
1	0	1
1	1	1

→ OR gate

$$\begin{array}{ccc} A & \xrightarrow{\text{NOR}} & \overline{A+B} \\ B & \xrightarrow{\text{NOR}} & \end{array} \quad \overline{A+B} = A+B$$

→ AND gate $Y = A \cdot B$

$$\begin{array}{ccc} A & \xrightarrow{\text{NOT}} & \overline{A} \\ B & \xrightarrow{\text{NOT}} & \overline{B} \end{array} \quad \overline{\overline{A} + \overline{B}} = \overline{\overline{A}} \cdot \overline{\overline{B}} = A \cdot B$$

(Using de-Morgan's law)

Input A	Input B	Output Y
0	0	0
0	1	0
1	0	0
1	1	1

Date _____ Page _____

II Minterms, Maxterms, SOP, POS, Complement and Standard form:

A minterm is a product (AND) of all variables in the function, in direct or complemented form.

Eg: $x'y'$, xy , $x'y$ and $y'z$.

A maxterm is a sum (OR) of all the variables in the function, in direct or complemented form.

Eg: $x+yz$, $x+y+z$, $x+y'$, $x'+y$.

The following table shows the representation of minterms and maxterms for 2 variables.

x	y	Minterms (SOP)	Max terms (POS)
0	0	$m_0 = x'y'$	$m_0 = x' + y$
0	1	$m_1 = x'y$	$m_1 = x + y$
1	0	$m_2 = xy'$	$m_2 = x + y'$
1	1	$m_3 = xy$	$m_3 = x' + y'$

11 Canonical form

Both Boolean function expressed as a sum of minterms or product of maxterm are called canonical form.

There are two ways of expressing function they are;

i) Canonical SOP form:

In this form each product term contains all literals. So, these product terms are nothing but min terms. Hence, Canonical SOP form is also called as sum of min terms form.

The 4 corner pending minterms are $p'q'r$, $pq'r$, pqr' , pqr . By doing logical OR of these four minterms, we will get Boolean function of output.

We can represent this function in following two notations;

$$f = m_3 + m_5 + m_6 + m_7$$

$$f = m_3 + m_5 + m_6 + m_7$$

$$f = \sum m(3, 5, 6, 7)$$

27 Canonical POS form:

In this form, each sum term contains all literals. Hence, Canonical POS is the product of max-term form.

$$(p+q).(p+q') = 1$$

classmate
Date _____
Page _____

Corresponding Max terms are $p'q'q'$, $p'q'q$, $p'qq'$,
 pqq' , pqq , pqq' .
By doing logical AND of these four terms we will get the Boolean function of output.

Notations:

M_0, M_1, M_2, M_4

$F = \sum (D, 1, 2, 4)$

Both Canonical SOP and Canonical POS forms are dual to each other. Functionally, these two forms are same. Based on requirements, we can use one of these two forms.

11. Standard SOP and POS Forms.

These are the simplified version of Canonical forms. The main advantage of Standard form is that the number of input applied to logic gates can be minimized. Sometime there will be reduction in total number of logic gates required.

Standard SOP form

In this form, each product term need not contain all literals. In the product terms may or may not be in minterms. Therefore the Standard SOP form is the simplified form of Canonical SOP form.

eg. Convert the following Boolean function
into Standard POS form.

$$P = \overline{Q}R + \overline{P}\overline{Q} + \overline{P}QR + P\overline{Q}R$$

$$= \overline{Q}R + \overline{P}R + \overline{P}R + P\overline{Q}R + P\overline{Q}R$$

$$= Q(R + \overline{P}) + \overline{P}(R + \overline{Q}) + P\overline{Q}R + P\overline{Q}R$$

$$= QR + \overline{P}R + \overline{Q}R$$

$$= PR - QP + PR$$

This is simplified Boolean function.

Standard POS form:

In this form, each sum term need not contain all literals. i.e., the sum terms may not be max terms. In the Standard POS form is the Simplified form of Canonical POS form.

Some questions:

To change
functions
In GDD
In DFG

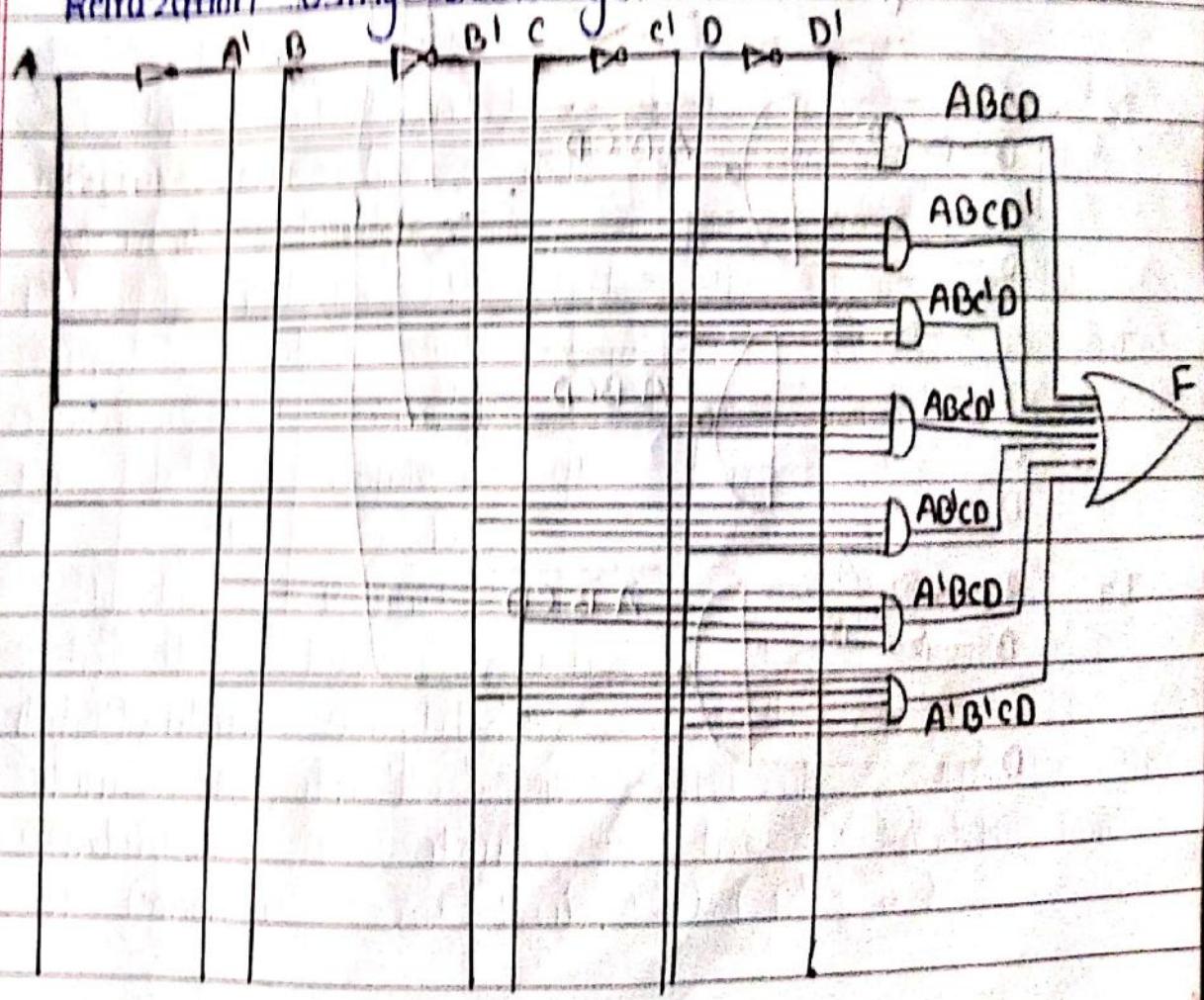
discrete
Bots
Page
canoncial
in sub or pos
use X1
use +1

Q) F = AB + CD change into Canonical GDD

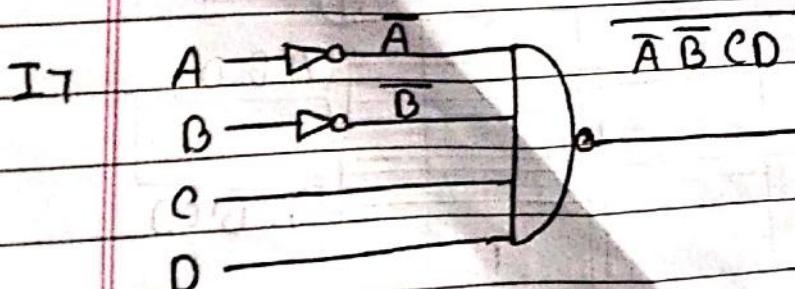
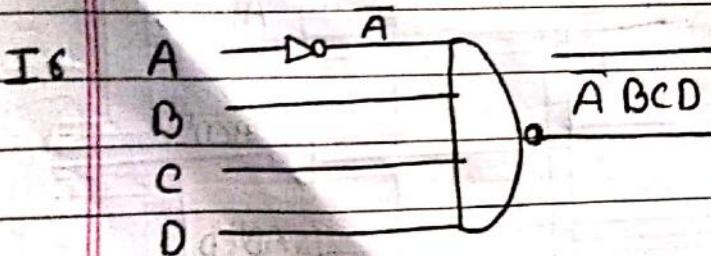
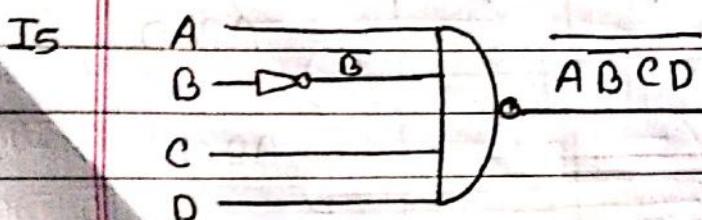
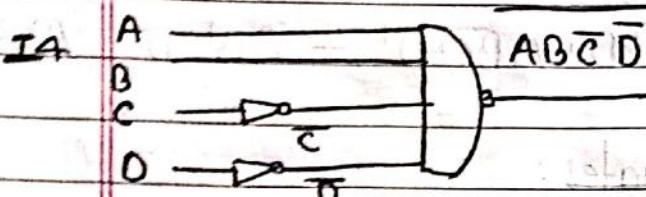
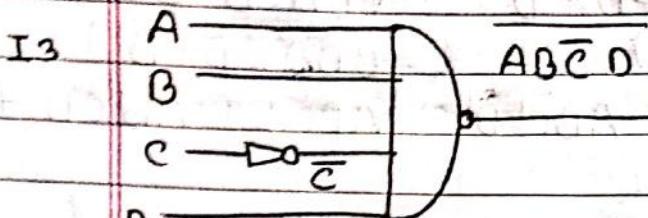
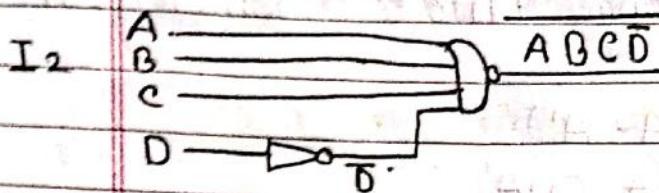
$$\begin{aligned}
 & ABCC + C' + A' + CDCA + A' \\
 & ABC + ABC' + CDA + CDA' \\
 & ABC(CD + D') + ABC'CD + D' + CDACB + B' \\
 & + CDA'(B + B') \\
 & ABCD + ABCD' + ABC'D + ABC'D' + ABCD + ABC'D + \\
 & A'BCD + B'C'DA'
 \end{aligned}$$

$$\begin{aligned}
 F = & ABCD + ABCD' + ABC'D + ABC'D' + AB'CD + A'BCD \\
 & + A'B'CD
 \end{aligned}$$

Realization using basic gates:



Realization using NAND gate:



$$I_1 \times I_2 \times I_3 \times I_4 \times I_5$$

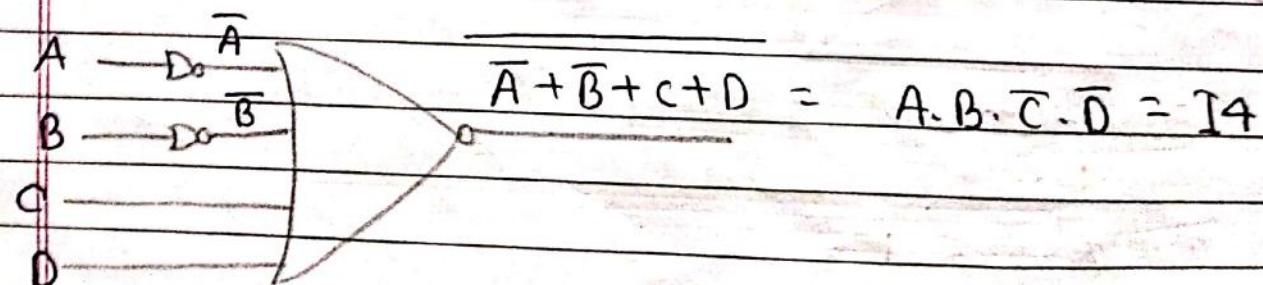
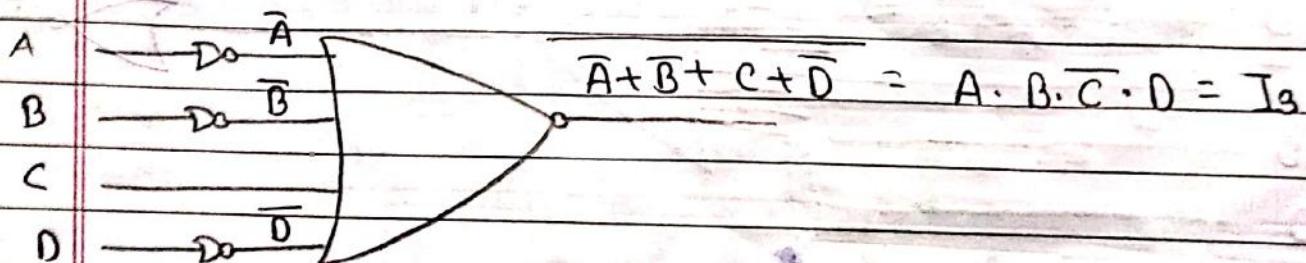
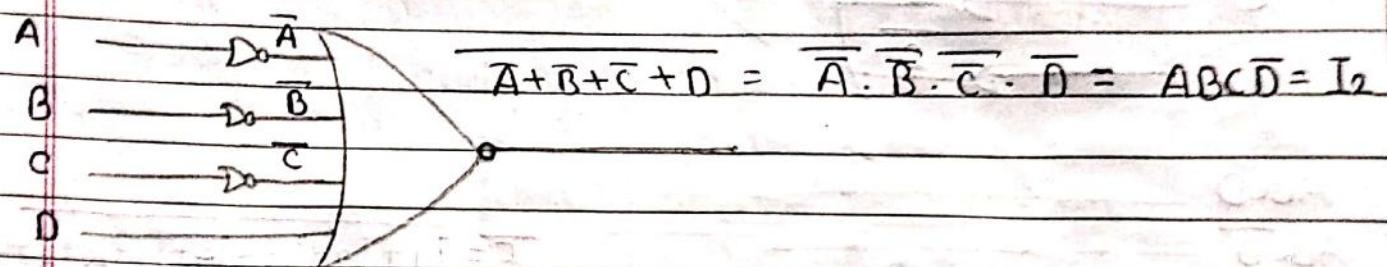
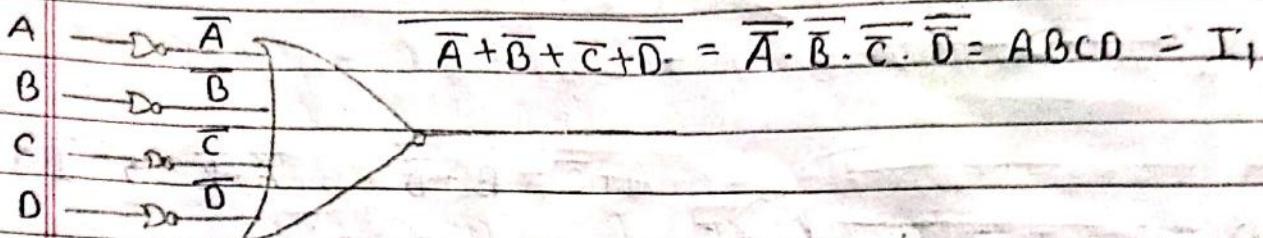
$$= \overline{ABCD} \cdot \overline{ABC\bar{D}} \cdot \overline{ABC\bar{D}} \cdot \overline{ABC\bar{D}} \cdot \overline{ABC\bar{D}} \cdot \overline{ABC\bar{D}} \cdot \overline{ABC\bar{D}}$$

using deMorgan's law,

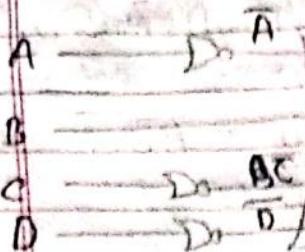
$$= \overline{ABCD} + \overline{ABC\bar{D}} + \overline{ABC\bar{D}} + \overline{ABC\bar{D}} + \overline{ABC\bar{D}} + \overline{ABC\bar{D}} + \overline{ABC\bar{D}}$$

$$= ABCD + ABC\bar{D} + ABC\bar{D} + ABC\bar{D} + ABC\bar{D} + ABC\bar{D} + ABC\bar{D}$$

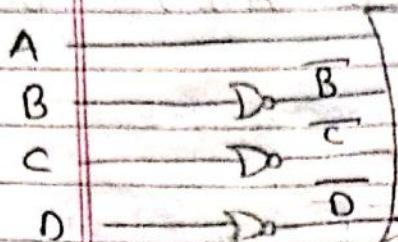
Realization using NOR gate



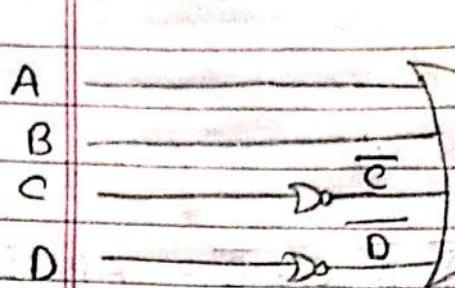
Realization using NOR Gate:



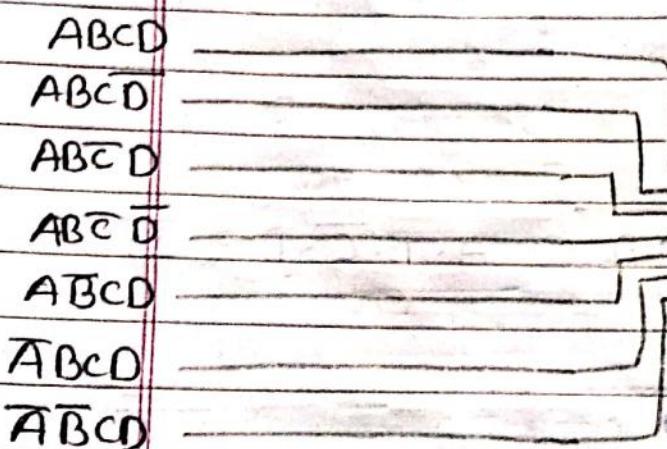
$$\bar{A} + \bar{B} + \bar{C} + \bar{D} = A \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} = I_5$$



$$A + \bar{B} + \bar{C} + \bar{D} = \bar{A} \cdot B \cdot C \cdot D = I_6$$



$$A + \bar{C} + B + \bar{D} = \bar{A} \cdot \bar{B} \cdot C \cdot D = I_7$$



$$F = \bar{I}_1 + I_2 + I_3 + I_4 + I_5 + I_6 + I_7 = F$$

Note: To convert 'SOP to POS' add '0' for complement
 To convert 'POS' to SOP ~~add~~ multiply '0'.

change $f = A + \bar{B} + C + \bar{D}$ into SOP.

$$\begin{aligned}
 &= A \cdot 1 + \bar{B} \cdot 1 + C \cdot 1 + \bar{D} \cdot 1 \\
 &= A(C + \bar{C}) + \bar{B}(A + \bar{A}) + C(A + \bar{A}) + \bar{D}(A + \bar{A}) \\
 &= AB + A\bar{B} + A\bar{B} + A\bar{B} + AC + \bar{A}C + AD + \bar{A}D \\
 &= AB + A\bar{B} + \bar{A}\bar{B} + AC + \bar{A}C + A\bar{D} + \bar{A}\bar{D} \\
 &= ABC(C + \bar{C}) + A\bar{B}(C + \bar{C}) + A\bar{B}(C + \bar{C}) + AC(B + \bar{B}) + \\
 &\quad \bar{A}CC(B + \bar{B}) + AD(C + \bar{C}) + \bar{A}\bar{D}CC(C + \bar{C}) \\
 &= \overbrace{ABC}^1 + \overbrace{ABC}^1 + \overbrace{A\bar{B}C}^1 + \overbrace{A\bar{B}C}^1 + \overbrace{\bar{A}\bar{B}C}^1 + \overbrace{\bar{A}\bar{B}C}^1 + \overbrace{ABC}^1 + \\
 &\quad \overbrace{A\bar{B}C}^1 + \overbrace{A\bar{B}C}^1 + \overbrace{A\bar{D}C}^1 + \overbrace{AC\bar{D}}^1 + \overbrace{\bar{A}C\bar{D}}^1 + \overbrace{\bar{A}C\bar{D}}^1 \\
 &= ABC(D + \bar{D}) + ABC(\bar{D} + D) + A\bar{B}C(D + \bar{D}) + \\
 &\quad A\bar{B}C(\bar{D} + D) + \bar{A}\bar{B}C(D + \bar{D}) + \bar{A}\bar{B}C(\bar{D} + D) + \\
 &\quad A\bar{B}C(D + \bar{D}) + ABC(D + \bar{D}) + \cancel{AD}CC(B + \bar{B}) \\
 &\quad + AC\bar{D}(B + \bar{B}) + \bar{A}C\bar{D}(B + \bar{B}) + \bar{A}C\bar{D}(B + \bar{B}) \\
 &= ABCD + \cancel{ABC}\bar{D} + \cancel{ABC}D + \cancel{AB}\bar{C}\bar{D} + \cancel{AB}\bar{C}D + \cancel{ABC}\bar{D} + \\
 &\quad \cancel{A}\bar{B}\bar{C}D + \cancel{ABC}\bar{D} + \cancel{A}\bar{B}\bar{C}D + \cancel{ABC}\bar{D} + \cancel{A}\bar{B}\bar{C}D + \\
 &\quad \cancel{ABC}\bar{D} + \cancel{ABC}\bar{D} + \cancel{ABC}\bar{D} + \cancel{ABC}\bar{D} + \cancel{ABC}\bar{D} + \\
 &\rightarrow ABC\bar{D} + \cancel{ABC}\bar{D} + \cancel{ABC}\bar{D} + \cancel{ABC}\bar{D} + \cancel{ABC}\bar{D} + \\
 &\quad \cancel{ABC}\bar{D} + \cancel{ABC}\bar{D} + \cancel{ABC}\bar{D} \\
 &= ABCD + ABC\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D} + \\
 &\quad + A\bar{B}\bar{C}D + A\bar{B}\bar{C}D + \cancel{ABC}D + \cancel{ABC}\bar{D} + \cancel{ABC}D + \\
 &\quad \cancel{ABC}\bar{D} + \cancel{ABC}\bar{D} + \cancel{ABC}\bar{D} + \cancel{ABC}\bar{D}
 \end{aligned}$$

→ change into POS
 $F = \sqrt{(A+B) \cdot (D+C)}$

$$= (A+C+D) \cdot (D+D+0)$$

$$= \{ (A+C) + (B, D) \} \cdot \{ (D+D) + (A, \bar{A}) \}$$

$$= (A+C+B) \cdot (A\bar{B}+C) \cdot (A+\bar{B}+D) \cdot (\bar{A}+\bar{B}+D)$$

$$= \{ (A+0+C) : (A+\bar{B}+C) \} \cdot \{ (D, D) \} \cdot \{ (\bar{A}+\bar{B}+D) \cdot (A+\bar{B}+D) \} (C, \bar{C})$$

$$= (A+B+C+D) \cdot (A+\bar{B}+C+\bar{D}) \cdot (A+\bar{B}+C+D) \cdot (\bar{A}+\bar{B}+\bar{C}+D) + \\ (A+\bar{B}+C+D) \cdot (A+\bar{B}+\bar{C}+D) \cdot (A+\bar{B}+\bar{C}+D) + (A+\bar{B}+\bar{C}+D) \cdot (\bar{A}+\bar{B}+\bar{C}+D)$$

change into POS.

$$F = A + BC$$

$$= \overline{A + BC}$$

$$= \overline{A} \cdot \overline{BC}$$

$$= \overline{A} \cdot (C\bar{B} + \bar{C})$$

$$= \overline{A}\bar{B} + \overline{A}\bar{C}$$

$$= \overline{A}\bar{B} \cdot \overline{A}\bar{C}$$

$$= (A+B) \cdot (A+c)$$

POS (product of sum)

$$F = (A+B) \cdot C$$

$$= (A+B+0) \cdot (C+0)$$

$$= ((A+B)+(C \cdot \bar{C})) \cdot (C+(B \cdot \bar{B}))$$

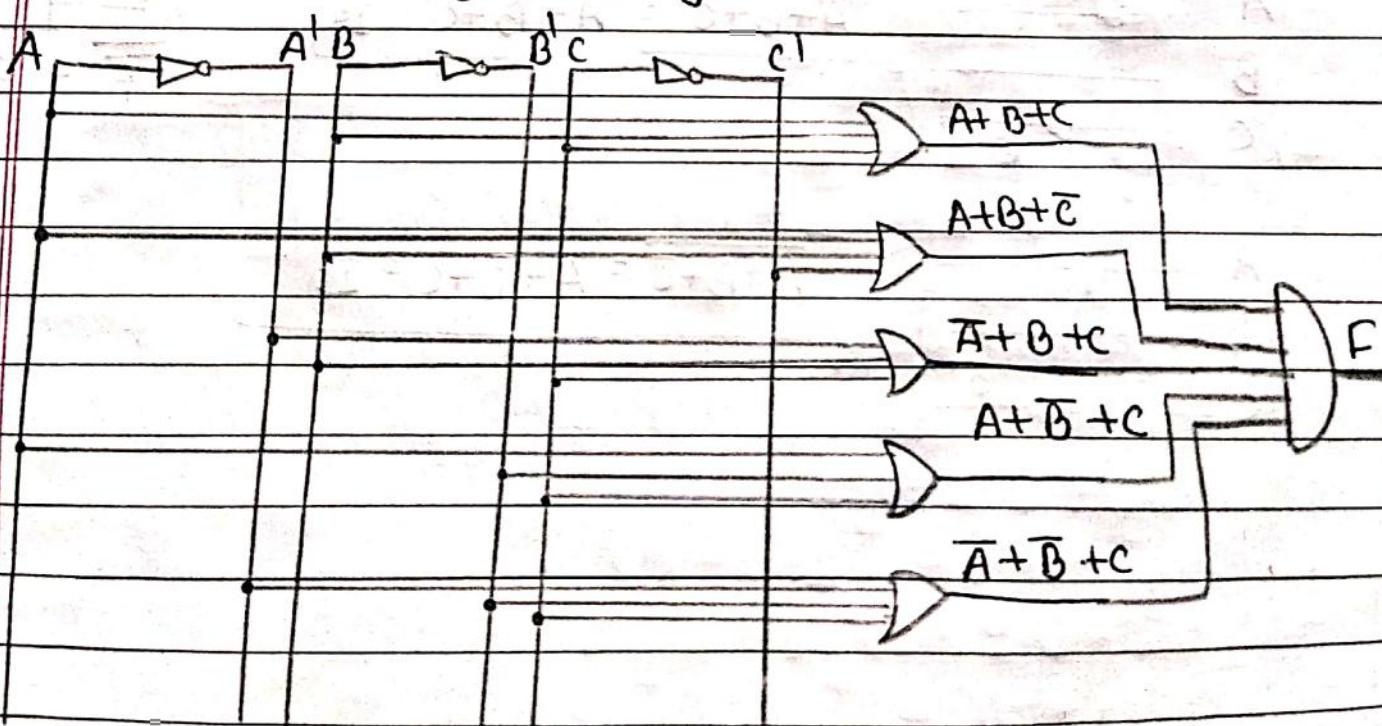
$$= (A+B+C) \cdot (A+B+\bar{C}) \cdot (C+B) \cdot (C+\bar{B})$$

$$= (A+B+C)(A+B+\bar{C}) \cdot \{(C+B) \cdot (\bar{B}+C) + (A \cdot \bar{A})\}$$

$$= (A+B+C)(A+B+\bar{C}) \cdot (B+C+A) \cdot (A+B+C) \cdot (A+\bar{B}+C)(\bar{A}+\bar{B}+C)$$

$$= (A+B+C)(A+B+\bar{C})(\bar{A}+B+C) \cdot (A+\bar{B}+C)(\bar{A}+\bar{B}+C)$$

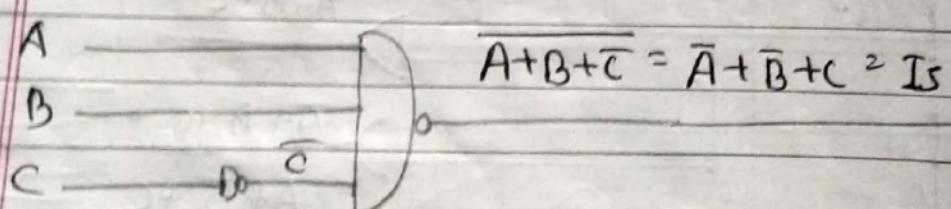
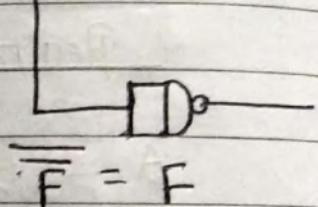
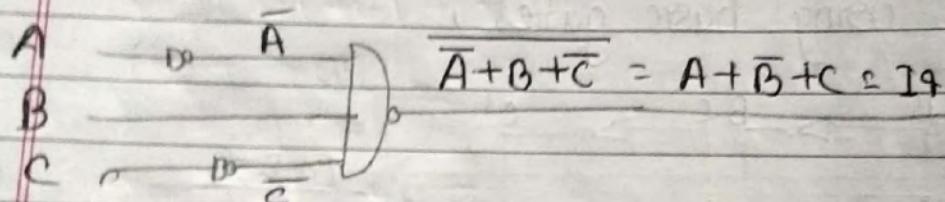
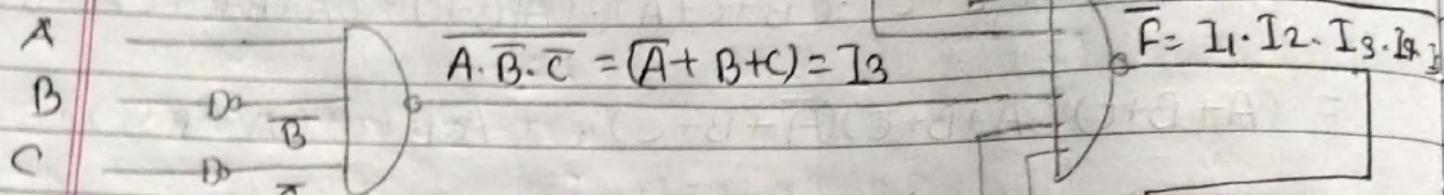
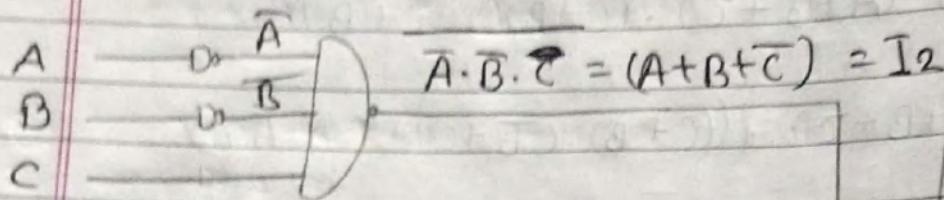
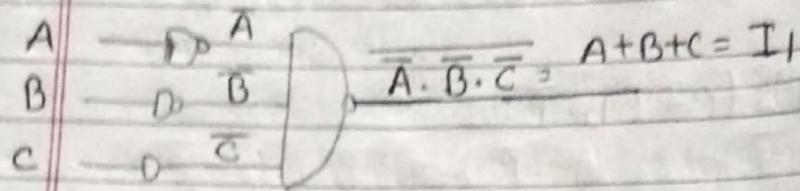
Realization using basic gates;



$$F = (A+B+C) \cdot (A+B+\bar{C}) \cdot \\ (\bar{A}+B+C) \cdot (A+\bar{B}+C) \cdot (\bar{A}+\bar{B}+C)$$

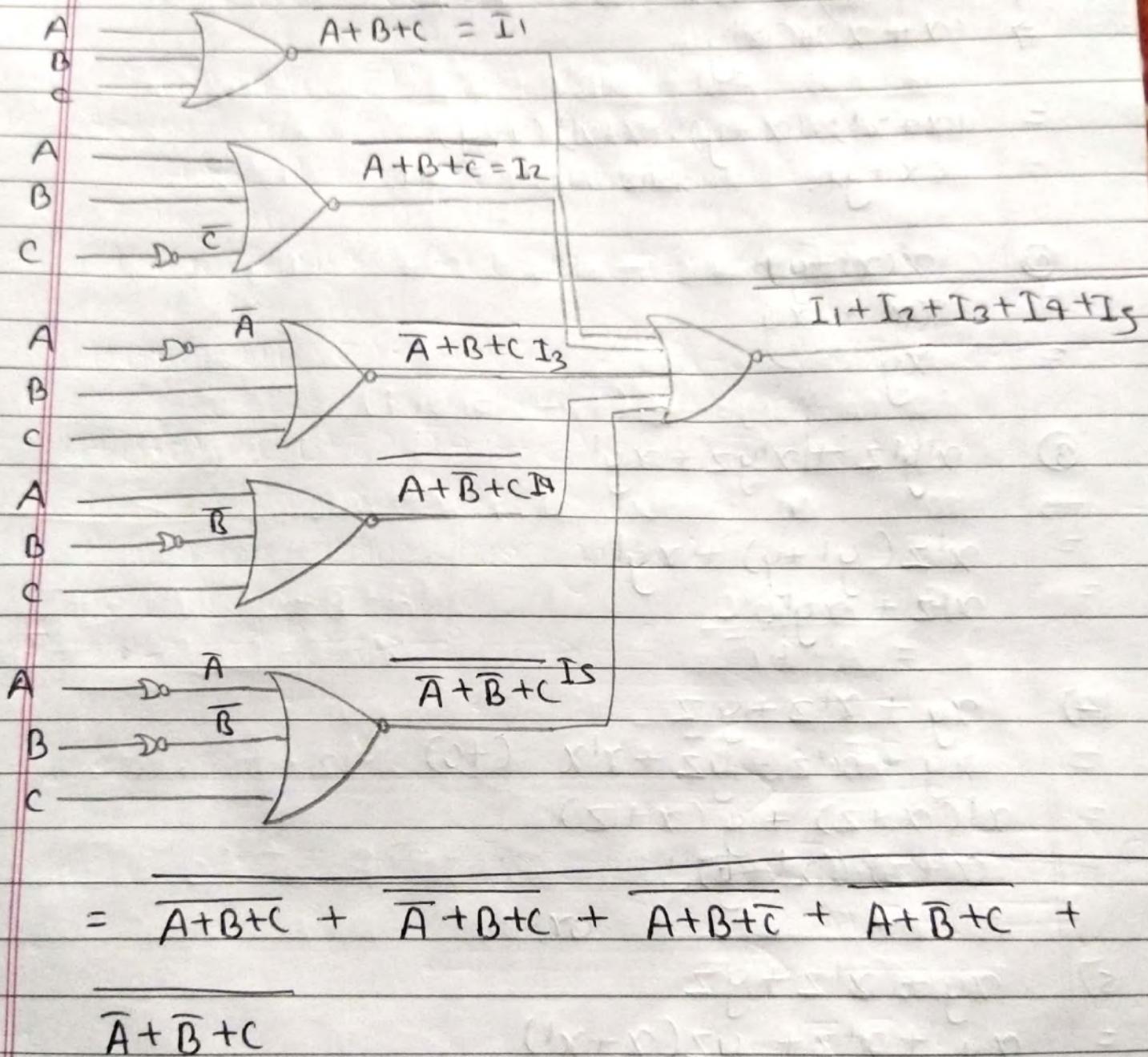
Using NAND gate

$$F = (A+B+C) \cdot (A+B+\bar{C}) \cdot (\bar{A}+B+C) \cdot (\bar{A}+B+\bar{C})$$



Using NOR gate :

$$F = (A+B+C) \cdot (A+B+\bar{C}) \cdot (\bar{A}+B+C) \cdot (A+\bar{B}+\bar{C}) \cdot (\bar{A}+\bar{B}+C)$$



$$\begin{aligned}
 &= (A+B+C) \cdot (A+B+\bar{C}) \cdot (\bar{A}+B+C) \cdot (A+\bar{B}+\bar{C}) \cdot (\bar{A}+\bar{B}+C) \\
 &= F
 \end{aligned}$$

Rx+

Simplify the following
minimum no of literals.

Boolean

function

to

$$7) \quad x + x'y \\ \Rightarrow (x + x') + (x'y)$$

$$= (x + x') \cdot (x + y) \cdot (x + x') (x + y) \\ = xy \quad \text{Ans}$$

$$8) \quad x(x' + y) \\ = xx' + xy \\ = xy$$

$$9) \quad x'y'z + x'y'z + xy' \\ = x'z(y' + y) + xy' \\ = x'z + xy'$$

$$10) \quad xy + x'z + yz \\ = xy + x'z + yz + x'x \quad (+0) \\ = x'(x+z) + y(x+z) \\ = (x+z)(x'+y)$$

$$11) \quad xy + x'z + yz \quad \text{or}, \\ = xy + x'z + yz(x+x') \\ = xy + x'z + xyz + x'y'z \\ = xy(1+z) + x'z(1+y) \quad (1+z=1) \\ = xy + x'z$$

$$\boxed{x + xy = x}$$

$$\begin{aligned} & 67 \quad (x+y) (x^1+z) (y+z) \\ & = (x+y) (x^1+z) (y+z+0) \\ & = (x+y) (x^1+z) (y+z \cdot (x^1 x^1)) \\ & = (x+y) (x^1+z) (x+y+z) \cdot (x^1 y + z) \\ & = (x+y) (x^1+z) \cdot \\ & = x(x+z) \cdot y(y+y) \quad [x+x^1 = x \\ & = x \cdot y \quad x \cdot (x+y) = x \\ & = (x+y) (x^1+z) \quad \text{absorption law} \end{aligned}$$

(7) $F = (x+y) \cdot (x^1+y) + (x+y^1)$.

$$\begin{aligned} & = xx^1 + x^1y + xy + y + x+y^1 \\ & = y(x+x^1) + y + x+y^1 \\ & = \cancel{xy} + x^1y + x \\ & = x \end{aligned}$$

Express the Boolean Function:

$F = A + B'C$ in a Sum of Minterms.

$$= A(B+B') + B'C(A+A')$$

$$= AB + AB' + AB'C + A'B'C$$

$$= ABC + ABC' + AB'C + AB'C' + AB'C + A'B'C$$

$$= ABC + ABC' + AB'C + AB'C' + A'B'C$$

$$= m_7 + m_6 + m_5 + m_4 + m_1$$

$$\therefore F(A, B, C) = \Sigma(1, 4, 5, 6, 7)$$

→ in a product of Maxterms.

$$F = \frac{A + B'C}{A + B'C}$$

$$(A + \bar{B} + C) \cdot (A + \bar{B} + \bar{C}) \cdot \\ (A + C + B) \cdot (A + C + \bar{B})$$

$$= A \cdot \bar{B}C$$

$$= M_2 \cdot M_3 \cdot M_0$$

$$= A \cdot (B + \bar{C})$$

$$\Rightarrow F = \prod(0, 2, 3)$$

$$= \bar{A}B + \bar{A}\bar{C}$$

$$= \bar{A}\bar{B} \cdot \bar{A}\bar{C}$$

$$= (A + \bar{B}) \cdot (A + C)$$

$$= (A + \bar{B} + 0) \cdot (A + C + 0)$$

$$= \{A + \bar{B} + (C\bar{C})\} \cdot (A + C + (B\bar{B}))$$

Minimum no of literals

$$\begin{aligned} & \rightarrow xyz + x'y + xy'z \\ & = xyz + x'y'z + xy'z + xyz \\ & = xy + x'y \\ & = y \end{aligned}$$

Converting exprs to TT.
(pos)

$$x = (A+B+C) \cdot (A+B'+C) \cdot (A+B+C') \cdot (A'+B+C) \\ (A'+B+C') \cdot (A'+B'+C)$$

input	output	Maxterms
0 0 0	0	$A+B+C$
0 0 1	1	
0 1 0	0	$A+B'+C$
0 1 1	0	$A+B+C'$
1 0 0	1	$A'+B+C$
1 0 1	0	$A'+B+C'$
1 1 0	0	$A'+B'+C$
1 1 1	1	

$$x = \pi(M_0, M_2, M_3, M_5, M_6)$$

(#)

SOP

$$\begin{aligned} F &= ABC + ABc + AB^lC + AB^lC + A^lB^lC \\ &= m_7 + m_6 + m_5 + m_4 + m_1 \\ &= \Sigma(1, 4, 5, 6, 7) \end{aligned}$$

Minterms

IP	o/D	Minterms
000	0	A^lB^lC
001	1	
010	0	
011	0	
100	1	
101	1	ABC
110	1	AB^lC
111	1	AB^lC

(*)

K-map or (Map) is a diagram made up of squares with each square representing one minterms of a function.

The map method provides a straight forward procedure for minimizing Boolean eqn.