Chapter 1

Mechanical Oscillation

Introduction

Oscillation is a very general term, it indicates any kind of to and fro motion. Oscillation can repeat at fixed intervals or not.

Periodic motion is more specific type of oscillation, where the body returns to each state after a fixed period that is it repeats at fixed intervals.

Simple harmonic motion is a very exact term with a very specific meaning. It is a kind of periodic motion in which the body's acceleration at each point is directly proportional to its displacement from mean point of its motion and the acceleration is always directed towards the mean position.

For SHM to take place.

1. The motion should be periodic.
2. Acceleration is directly proportional to the displacement of the particle and directed towards mean position.
3. The restoring force should be directly proportional to displacement of the body from its mean position.

In SHM, since the motion is periodic, so this is expressed in terms of harmonic (periodic) functions sine, cosine, or combination of both.

If there are frictional or damping forces present, as there almost always are, these will generally cause the amplitude of the motion gradually decrease, which is not exactly simple harmonic. So, simple harmonic motions are rare in nature.

Characteristics of SHM

1. **Displacement**

   In SHM the motion of the particle is periodic (harmonic). So it's displacement can be expressed as,

   $$x = x_m \sin(\omega t + \phi)$$

Where, x = displacement of the particle at a time t.

It is defined as the distance of the particle from mean position at that time.

→ The quantity x_m is maximum displacement called amplitude of motion and simply represented by A.

→ The time varying quantity $(\omega t + \phi)$ is called phase of the motion and the constant ϕ is called phase constant.

→ And the term 'ω' is called angular frequency.

2. Velocity

We have, $x = x_m \sin(\omega t + \phi)$

$$v = \frac{dx}{dt} = x_m \omega \cos(\omega t + \phi)$$

$$v = \omega x_m \cdot \sqrt{1 - \sin^2(\omega t + \phi)} = \omega x_m \sqrt{1 - \frac{x^2}{x_m^2}}$$

$$v = \omega x_m \cdot \sqrt{\frac{x_m^2 - x^2}{x_m^2}}$$

$$v = \omega \sqrt{A^2 - x^2} \quad (\text{where } x_m = A)$$

3. Acceleration

We have, $v = \omega x_m \cos(\omega t + \phi)$

$$\text{Acceleration, } a = \frac{dv}{dt} = -\omega^2 x_m \sin(\omega t + \phi)$$

$$a = -\omega^2 x$$

-ve sign indicates 'a' is directed towards mean position.

4. Time period

The time taken by particle to complete a cycle of oscillation is called time period 'T'.

This means the displacement 'x' returns to its initial value after one complete oscillation in time period T and the sine function repeat in angle 2π.

Therefore, $x = x_m \sin \omega(t + T) = x_m \sin(\omega t + 2\pi)$

$$\Rightarrow \omega(t + T) = \omega t + 2\pi$$

$$\Rightarrow \omega t + \omega T = \omega t + 2\pi$$

$$\Rightarrow \omega T = 2\pi$$

$$\Rightarrow T = \frac{2\pi}{\omega}$$

Since, $\omega = 2\pi f$, where, f is the frequency

$$\text{Therefore, } T = \frac{2\pi}{2\pi f} = \frac{1}{f}.$$

5. Frequency

The number of complete oscillations made by an oscillating particle in one second is called frequency. It is given by

$$f = \frac{1}{T}$$

6. Phase and phase constant

In equation $x = x_m \sin(\omega t + \phi)$, $(\omega t + \phi)$ is called phase and constant ϕ is called phase constant. The value of ϕ depends on the displacement and velocity of particle at time t = 0.

Differential Equation of Simple Harmonic Motion

Consider a particle of mass m is in simple harmonic motion and the displacement of the particle at any instant t be x.

Then from Hook's law (According to which the restoring force is directly proportional to the displacement from mean position).

i.e. $F \propto x$

or, $F = -k x$ ....(1) where k is constant of proportionality called force constant. Negative sign shows that restoring force and displacement are in opposite direction.

Also from Newton's second law of motion

$$F = ma = \frac{md^2x}{dt^2} \quad ......(2)$$

$$\text{From (1) and (2), } \frac{md^2x}{dt^2} = -kx$$

$$\Rightarrow \frac{md^2x}{dt^2} + kx = 0$$

$$\Rightarrow \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\text{or, } \frac{d^2x}{dt^2} + \omega^2 x = 0 \quad ...(1)$$

This is the differential equation of motion of simple harmonic motion.

Where $\omega = \sqrt{\frac{k}{m}}$ is the angular velocity.

Here d^2x/dt^2 represent acceleration of the particle. From equation (3)

$$a = \frac{d^2x}{dt^2} = -\omega^2 x.$$

Which shows that acceleration is proportional to the displacement from mean position and the -ve sign shows that acceleration is directed towards mean position.

The solution of equation (1) is

$$x = A \sin(\omega t + \phi)$$

Energy is Simple Harmonic Motion

The energy of linear oscillator transfers back and forth between kinetic energy and potential energy. But the sum of two energy remains constant. Consider a spring block system as shown in figure. Let the mass of block is 'm' and it's velocity be v.

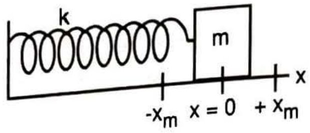


Figure (1)

Kinetic energy of the system is entirely associated with block and is given by

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 = \frac{1}{2}m\left[\frac{d}{dt}\{A\sin(\omega t + \phi)\}\right]^2$$

$$= \frac{1}{2}m\omega^2A^2\cos^2(\omega t + \phi)$$

The potential energy of the system is given by amount of work done to move the system from the position 0 to x by applying a force.

$$P.E = -\int_0^x F.\,dx = -\int_0^x (-kx)\,dx = k\frac{x^2}{2} = \frac{1}{2}kx^2$$

$$P.E = \frac{1}{2}kA^2\sin^2(\omega t + \phi) = \frac{1}{2}m\omega^2A^2\sin^2(\omega t + \phi)$$

Therefore, total mechanical energy E = K.E + P.E

$$= \frac{1}{2}m\omega^2A^2[\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)]$$

$$E = \frac{1}{2}m\omega^2A^2$$

Therefore, the total energy is constant and independent of time

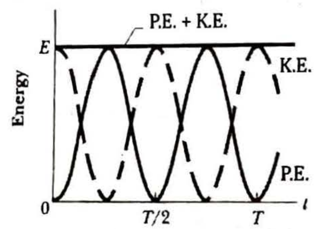


Figure 2: P.E., K.E. and total energy E as a function of time

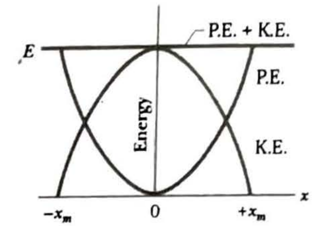


Figure 3: P.E., K.E. and total energy E as a function of position

Figure (2) shows that the P.E. and K.E. become maximum twice during a time period, and become minimum twice during a period. Figure (3) shows that for x = 0 the energy is all kinetic, and for $x = \pm x_m$ it is all potential. At mean position (x = 0), v is maximum so K.E. $= \frac{1}{2}mv^2$ is maximum. At extreme position ($x = x_m$), v is zero so K.E. is zero and whole energy is potential.

Examples of SHM

1. Simple Pendulum

A simple pendulum is an ideal system in which a point mass is suspended at one end of mass-less string. Consider a simple pendulum of length l and a bob of mass m is suspended at the free end of the pendulum as shown in figure. The other end of the string is suspended at the rigid support. If the bob is displaced by an angle θ, and released, it oscillates simple harmonically about the mean position. The weight of the mass, mg acts vertically downward. This force due to gravity can be resolved into two components; mg cosθ is the force along the string and mg sinθ perpendicular to the string. The tension of the string (T) is balanced by mg cosθ. The component mg sinθ provides the restoring force which tends to bring the pendulum back to its mean position. That is

Restoring force (F) = – mg sinθ

where the negative sign shows that the restoring force acts opposite to the displacement (x) of the bob.

If a is the acceleration of the bob, then

$$F = ma = -mg\sin\theta$$

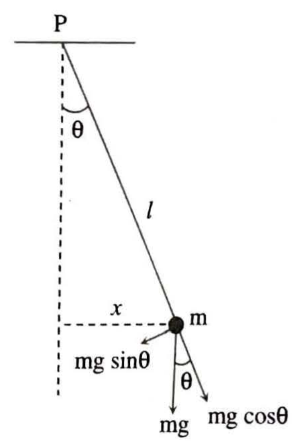
$$ma = -mg\sin\theta$$

$$\frac{d^2x}{dt^2} = -g\sin\theta$$

$$\frac{d^2x}{dt^2} + g\sin\theta = 0$$

For small θ, sinθ = θ

$$\frac{d^2x}{dt^2} + g\theta = 0 \quad \text{....(1)}$$



If θ is small, x can be considered to be nearly equal to the length of arc, therefore,

$$x = l\theta$$

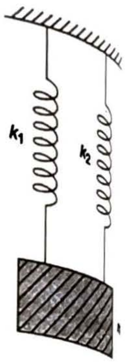
Substituting for x in equation (1)

$$l\frac{d^2\theta}{dt^2} + g\theta = 0$$

or, Frequency, $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k_1k_2}{m(k_1+k_2)}} = \frac{1}{2\pi}\sqrt{\frac{k}{2m}}$ for $k_1 = k_2$

b. Parallel Combination of Spring

Two springs S_1 and S_2 having spring constants k_1 and k_2 are joined to a mass 'm'. The other ends of both are fixed at finite distance on a rigid support as shown in figure. Let the extension produced on both spring be x.



Then for first spring, $F_1 = -k_1 x$

And for second spring, $F_2 = -k_2 x$

Total restoring force, $F = F_1 + F_2$

or, $F = -(k_1 + k_2)\,x$

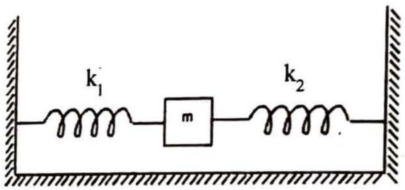
$$m\frac{d^2x}{dt^2} + (k_1 + k_2)\,x = 0$$

or, $\frac{d^2x}{dt^2} + \frac{(k_1+k_2)}{m}.x = 0$, or $\frac{d^2x}{dt^2} + \omega^2 x = 0$

Where, $\omega^2 = \frac{k_1+k_2}{m} \Rightarrow \omega = \sqrt{\frac{k_1+k_2}{m}}$

Therefore, time period $= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k_1+k_2}} = 2\pi\sqrt{\frac{m}{2k}}$ for $k_1 = k_2$

and frequency, $\frac{1}{f} = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ for $k_1 = k_2$



3. Physical or Compound Pendulum

A compound pendulum is just a rigid body of what ever shape, capable of oscillating about a horizontal axis passing through it. All real pendulums are physical pendulum.

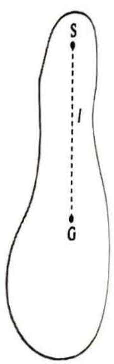


Figure (1) shows a compound pendulum, free to rotate about a horizontal axis passing through the point of suspension S. In its normal position of rest, it's c.g. G, lies vertically below S. The distance between S and G is the length 'l' of the pendulum.

Let the pendulum be given small angular displacement 'θ', so that it's c.g. takes new position G'. Due to the weight mg acting vertically downward at G, it constitutes a restoring torque whose action is to tend to bring the pendulum back into its original position.

The restoring torque is

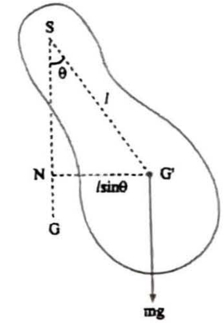
$$\tau = -\,mg\,(G'N)$$

$$= -\,mg\,l\sin\theta \qquad ....(1)$$

-ve sign indicates that torque is oppositely directed to the displacement θ.

If I is the moment of inertia of the pendulum about the axis of suspension and α, its angular acceleration then,

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2} \qquad ......(2)$$



From (1) and (2), $I\frac{d^2\theta}{dt^2} = -\,mgl\sin\theta$

Since, $\sin\theta = \theta - \theta^3/3! + \theta^5/5! + ...$, if θ is small, $\sin\theta \approx \theta$.

Therefore, $I\frac{d^2\theta}{dt^2} = -\,mg\,l\theta$ or $\frac{d^2\theta}{dt^2} = -\frac{mgl}{I}\theta$

The pendulum thus executes SHM

[Since, $\alpha = \frac{d^2\theta}{dt^2} \propto \theta$ i.e acceleration is directly proportional to displacement]. Therefore for large θ the motion will not be SHM.

or, $$\frac{d^2\theta}{dt^2} + \frac{mgl}{I}\theta = 0.$$

or $$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$$

Here,

$$\omega^2 = \frac{mgl}{I} \Rightarrow \omega = \sqrt{\frac{mgl}{I}}$$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{mg\,l}{I}}$$

Time period, $T = \frac{1}{f} = 2\pi\sqrt{\frac{I}{mgl}}$

If k is the radius of gyration of the pendulum. Then from theorem of parallel axis, the total moment of inertia of the pendulum about the axis through point of suspension is,

$$I = I_o + ml^2 = mk^2 + ml^2$$

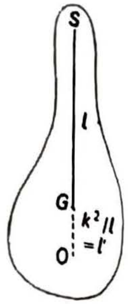
Therefore, $T = 2\pi\sqrt{\frac{m(k^2+l^2)}{mgl}} = 2\pi\sqrt{\frac{k^2/l + l}{g}}$

Thus the time period of compound pendulum is same as that of a simple pendulum of length $L = (k^2/l + l)$. This length 'L' is therefore called the length of an equivalent simple pendulum. Since $k^2 > 0$ i.e. $k^2/l > 0$ (positive), therefore, $L > l$. i.e. the length of equivalent simple pendulum (L) is always greater than l, the length of compound pendulum.

Centre of Suspension and Centre of Oscillation:

Centre of suspension

The point at which the vertical plane passing through c.g. of the pendulum meets the axis of rotation is called its point or centre of suspension.



Centre of oscillation

A point 'O' on the other side of the pendulum, in the vertical plane passing through c.g. of the pendulum at a distance k^2/l from the c.g. is called centre or point of oscillation.

Inter changeability of centre of suspension and oscillation

The time period of compound pendulum is

$$T = 2\pi\sqrt{\frac{k^2/l + l}{g}}$$

Here, l = length of pendulum = distance between point of suspension and centre of gravity

And k^2/l = distance between point of oscillation and c.g.

Let, $\frac{k^2}{l} = l' \Rightarrow k^2 = l\,l'$

Then, $T = 2\pi\sqrt{\frac{l' + l}{g}}$

If we now invert the pendulum, so that it oscillate about the axis of oscillation, then.

l' = length of pendulum = distance between point of suspension and c.g. and l = distance between point of oscillation and c.g.

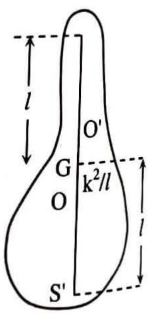
The new time period, $T' = 2\pi\sqrt{\frac{k^2/l' + l'}{g}} = \sqrt{\frac{\frac{ll'}{l'} + l'}{g}}$ $[\because \frac{k^2}{l} = l']$

Therefore, $T' = 2\pi\sqrt{\frac{l + l'}{g}} = T$

Thus the centre of suspension and oscillation are reciprocal to each other. i.e. the time period of the pendulum is the same about either.

Existence of four co - linear points

Let the pendulum is in position as shown in figure. Here S and O are point of suspension and oscillation at a distance l and k^2/l from the c.g. Now take radius equal to SG = l with G as centre so as to cut the vertical axis at S' such that S' G = l.



Again take radius equal to GO = k^2/l and cut the vertical axis on the other side at O' such that GO' = k^2/l

Therefore the time period of pendulum about S' and O' be same as that of axis through S and O.

Thus there are four points S, O, S' and O' collinear with the c.g. of the pendulum about which its time period is same.

Maximum and minimum time periods of compound pendulum

The time period of compound pendulum is

$$T = 2\pi\sqrt{\frac{k^2/l + l}{g}} \quad ....(1)$$

Squaring both side $T^2 = \frac{4\pi^2}{g}(k^2/l + 1)$

Differentiating w.r.t 'l'.

$$2T\frac{dT}{dl} = \frac{4\pi^2}{g}\left(-\frac{k^2}{l^2} + 1\right) \quad .....(2)$$

T will be minimum or maximum when $dT/dl = 0$

or, $4\pi^2/g\left(\frac{-k^2}{l^2} + 1\right) = 0 \Rightarrow (-k^2/l^2) + 1 = 0$

$\Rightarrow$ $k^2/l^2 = 1$

or, $\frac{k^2}{l} = l$

or, $k = \pm l$

or, $k = l$ [Since, k can't be negative]

Again differentiating equation (2) w.r.t. l

$$2T\frac{d^2T}{dl^2} + 2.\frac{dT}{dl}\cdot\frac{dT}{dl} = \frac{4\pi^2}{g}\left(\frac{2k^2}{l^3}\right)$$

$$2T\frac{d^2T}{dl^2} + 2\left(\frac{dT}{dl}\right)^2 = \frac{4\pi^2}{g}\,\frac{2k^2}{l^3}$$

$$\Rightarrow \frac{d^2T}{dl^2} = \frac{4\pi^2}{gkT} > 0 \text{ for } l = k \text{ and } dT/dl = 0$$

This shows that $\frac{d^2T}{dl^2}$ is positive.

Therefore, T is minimum when $l = k$

and minimum time period, $T_{min} = 2\pi\sqrt{\frac{k^2/k + k}{g}} = 2\pi\sqrt{\frac{2k}{g}}$

and the time period is maximum when $l = 0$

For $l = 0$, $T = \infty$ or a maximum.

Here, the distance of point of suspension from c.g = l

and The distance of point of oscillation from c.g = $k^2/l = l^2/l = l$

Therefore, time period of compound pendulum is minimum when the distance of point of suspension from c.g = distance of point of oscillation from c.g

Moment of Inertia

Inertia

The inability of a body to change its state of rest or of uniform linear motion by itself is called inertia. In linear motion mass is the measures of its inertia.

Moment of Inertia

The quantity which plays the same role in rotational motion as mass does in linear motion is called moment of inertia.

The moment of inertia of a rigid body about a given axis of rotation is the sum of products of the masses of various particles of the body and the square of their respective distances from the axis, that is,

$$I = \sum mr^2 = Mk^2$$

Where $M = m_1 + m_2 + \ldots\ldots$ is the total mass of the body.

k is the effective distance of its particle from axis called it's radius of gyration.

Radius of Gyration

It is defined as the distance from axis of rotation at which if all the masses of the body were supposed to be concentrated the moment of inertia of the body would same as the moment of inertia of the body with actual distribution of the particle.

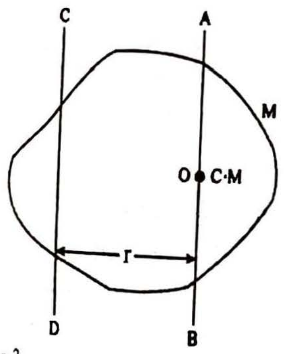
$$I = Mk^2 = \sum mr^2 = M\left(\frac{r_1^2 + r_2^2 + r_3^2 + \ldots\ldots + r_n^2}{n}\right)$$

$$k = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + \ldots\ldots + r_n^2}{n}}$$

Thus it is also defined as the root mean square of the distances of the particles of the body from the axis of rotation.

Theorem of Parallel Axis

It states that the moment of inertia of a body about any axis is equal to its moment of inertia about a parallel axis passing through it's centre of mass, plus the product of the mass of the body and the square of the distance between the two axis.

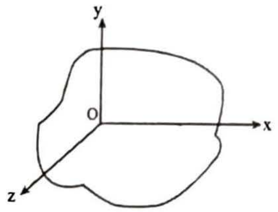


$$I = I_{cm} + Mr^2 = Mk^2 + Mr^2$$

Theorem of Perpendicular Axis

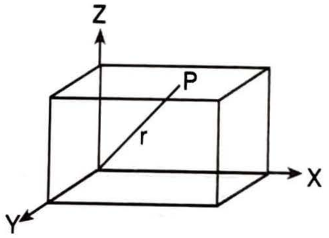
1. **For plane laminar body**

   The moment of inertia of a plane lamina about an axis perpendicular to the plane is equal to the sum of the moments of the inertia about two mutually perpendicular axis in the plane of lamina which intersect that axis. i.e. $I_z = I_x + I_y$.



2. **For three dimensional body**

   It states that the sum of moments of inertia of a three dimensional body about its three mutually perpendicular axis is equal to twice the total moment of inertia of the body about the origin.



$$r^2 = x^2 + y^2 + z^2$$

i.e. $I_x + I_y + I_z = 2\sum mr^2$

Bar Pendulum

A bar pendulum is the simplest form of a compound pendulum, consist of uniform metal bar having equally spaced holes drilled along its length on either side of c.g.

The time period of bar pendulum is

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \ldots\ldots(1)$$

Where $L = k^2/l + l$, is the length of equivalent simple pendulum.

Therefore, $T = 2\pi\sqrt{\dfrac{k^2/l + l}{g}}$

Squaring both sides, $T^2 = 4\pi^2\left(\dfrac{k^2 + l^2}{lg}\right)$

or, $lT^2 = \dfrac{4\pi^2}{g}l^2 + \dfrac{4\pi^2}{g}k^2 \ldots\ldots(2)$

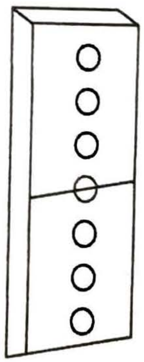


Fig. 1, bar pendulum

or, $\frac{4\pi^2}{g} l^2 - T^2 . l + \frac{4\pi^2}{g} . k^2 = 0$

Which is the quadratic equation in l so it possesses two roots.

If l_1 and l_2 are two roots

The sum of roots

[Since, for quadratic equation $ax^2 + bx + c$, the sum of roots = $-b/a$ and the product of roots = c/a]

$$l_1 + l_2 = -\frac{(-T^2)}{(4\pi^2/g)} = \frac{gT^2}{4\pi^2}$$

or, $$g = \frac{4\pi^2}{T^2}(l_1 + l_2) = \frac{4\pi^2}{T^2} L \text{ ....(3)}$$

And the product of roots $l_1 . l_2 = \left(\frac{4\pi^2}{g} k^2\right) / \frac{4\pi^2}{g} = k^2$

$$k^2 = l_1 . l_2 \Rightarrow k = \pm\sqrt{l_1 l_2} \text{ .....(4)}$$

Here l_1 and l_2 are two values of l for one side for which the time period will be same. [or $l_1 = l$, $l_2 = l$ such that $l_1 + l_2 = L$, total length of pendulum (equivalent length of simple pendulum)

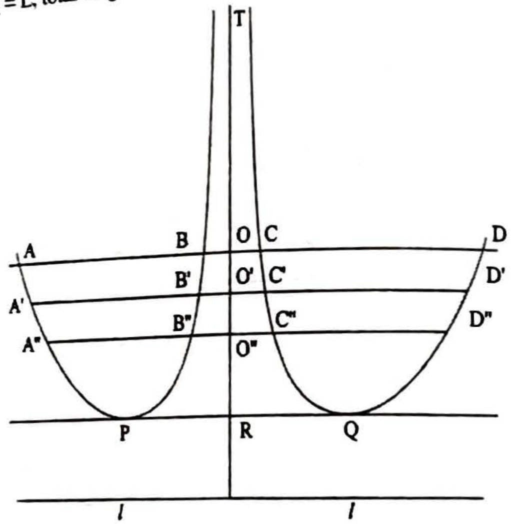


Figure (2)

First the time periods of bar pendulum at different value of it's length are determined by setting it into small oscillation. A graph is then plotted between the lengths of pendulum along the x-axis and the time periods (T) along y - axis.

The experiment is repeated with the holes on the other side of the c.g. of the pendulum and similar graph is drawn along side the first on the same graph paper.

A horizontal line ABOCD be drawn parallel to the x - axis so as to cut the curve in points A,B,C and D. These four points are collinear points having same time period.

Then AC = BD = L represents the length of equivalent simple pendulum. Now, the acceleration due to gravity can be determined by using equation (3) [Here. AC = AO + OC = $l + \frac{k^2}{l}$ = L and BD = BO + OD = $\frac{k^2}{l} + l$ = L]

Also in figure (2), AO = l_1 & OC = l_2 or BO = l_2 and OD = l_1

Therefore the radius of gyration can be calculated using equation (4).

Ferguson Method: (to find g and k)

Again plotting a graph between lT^2 and l^2 in accordance with equation (2), it will be a straight line as shown in figure (3)

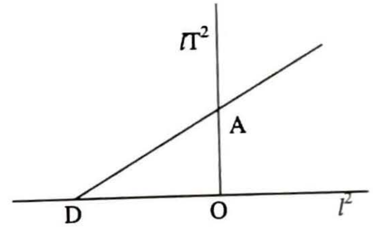


Figure (3)

Here, slope = $\frac{4\pi^2}{g} \Rightarrow g = \frac{4\pi^2}{\text{slope}}$

and intercept = $\frac{4\pi^2}{g} k^2$

or intercept = slope. $k^2 \Rightarrow k = \sqrt{\frac{\text{intercept}}{\text{slope}}} = \sqrt{OA . \frac{OD}{OA}} = \sqrt{OD}$

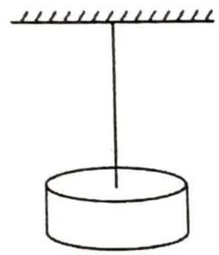
4. Torsion Pendulum

A rigid body, like a cylinder or a disc, suspended at its mid point by a long and thin wire to a rigid support, constitutes a torsoinal pendulum.

It is so called because when it is twisted and then released it executes torsional vibrations or oscillations about the wire as axis.

If the disc is turned through an angle θ, the wire too gets twisted through angle θ. This twisted wire exert a restoring torque on the disc given by.

$$\tau = -C\theta \quad \text{.......(1)}$$



Where, C is called torsoin constant, which depends upon the property of wire.

The value of C is, $C = \frac{\pi\eta r^4}{2l}$; where, r = radius of wire, l = length of wire, η = modulus of rigidity of wire.

But $\tau = I\alpha$ ....(2)

From (1) and (2) $I\alpha = -C\theta$ or $I \cdot \frac{d^2\theta}{dt^2} = -C\theta \Rightarrow \frac{d^2\theta}{dt^2} + \frac{C}{I}\theta = 0$

$$\text{or, } \frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \qquad \text{......(2)}$$

$$\text{Here } \omega^2 = \frac{C}{I}$$

$$\Rightarrow \omega = \sqrt{\frac{C}{I}} \Rightarrow f = \frac{1}{2\pi} \cdot \sqrt{\frac{C}{I}}$$

Therefore, time period, $T = \frac{1}{f} = 2\pi\sqrt{\frac{I}{C}}$ ...(3)

This is the time period of torsoinal pendulum.

The solution of equation (2) is $\theta = \theta_m \sin(\omega t + \phi)$.

The time period is

$$T = 2\pi\sqrt{\frac{I}{C}} \Rightarrow T^2 = 4\pi^2\frac{I}{C}$$

$$\text{or,} \quad T^2 = 4\pi^2 I \times \frac{2l}{\pi\eta r^4} = \frac{8\pi I l}{\eta r^4}$$

$$\Rightarrow \text{Modulus of rigidity, } \eta = \frac{8\pi I\, l}{T^2 r^4}$$

Here I is the moment of inertia of disc about the wire as axis. [For disc, $I = \frac{1}{2}MR^2$]

Note: Here to derive the relation for T no approximation is used like in the case of compound or simple pendulum. [Where we used $\sin\theta \sim \theta$ for small amplitude] The time period of a torsional pendulum therefore remains un effected even if the amplitude be large].

Difference between Compound and Torsion Pendulum

Compound Pendulum	Torsion Pendulum
1. Definition: A compound pendulum is a rigid body of whatever shape capable of oscillating about a horizontal axis passing through it.	1. Definition: A torsion pendulum is a rigid body capable of executing torsional vibration about an axis passing through it's c.g.
2. It oscillates about any axis passing through it other than c.g.	2. It oscillates about an axis along the wire passing through c.g.
3. It executes angular vibration.	3. It executes torsional vibration.
4. c.g. moves in an arc.	4. c.g remains fixed.
5. Time period $T = 2\pi\sqrt{\frac{k^2/l + l}{g}}$	5. Time period, $T = 2\pi\sqrt{I/C}$ where, $C = \frac{\pi\eta r^4}{2l}$
6. Time period is dependent of 'g'.	6. Time period is independent of g.

7. The approximation, for small θ, $\sin\theta \approx \theta$ is used to derive the time period. If θ is large the time period of compound pendulum is affected and the motion remain no longer simple harmonic. Therefore θ should be small.	7. No approximation is used to derive the time period. Therefore the time period of torsion pendulum remains unaffected even if the amplitude be large, provided the elastic limit of the suspension wire is not exceeded.

Solved Examples

1. A wave of frequency 500 cycles/sec. has a phase velocity of 350 m/sec. How for apart are two points 60° out of phase.

Solution:

Frequency f = 500 cycles/sec

Phase velocity v = 350 m/sec

Phase difference = 60°

Path difference = ?

Since $v = f\lambda \Rightarrow \lambda = \frac{v}{f} = \frac{350}{500}$

Since phase difference 2π = path difference λ

Phase difference 1 = path difference $\frac{\lambda}{2\pi}$

Phase difference 60° = path difference $\frac{\lambda}{2\pi} \times 60°$

Therefore, path difference $= \frac{\lambda}{2\pi} \times 60° = \frac{\lambda}{6} = \frac{350}{500 \times 6} = 0.117\text{m}$

2. The equation of transverse wave travelling in a rope is given by $y = 10 \sin \pi(0.01x - 2t)$ centimeters. Find the amplitude, frequency, velocity and wave length of the wave.

Solution:

The given equation is $y = 10 \sin \pi (0.01x - 2t)$

Comparing this equation with the general displacement equation for a transverse wave,

$y = A \sin (kx - \omega t)$

a. $A = 10$ cm

b. $k = 0.01\,\pi \Rightarrow \frac{2\pi}{\lambda} = 0.01\,\pi \Rightarrow \lambda = \frac{2}{0.01} = 200\text{ cm} = 2\text{m}$

c. $\omega = 2\pi \Rightarrow 2\pi f = 2\pi \Rightarrow f = 1\text{ Hz}$

d. The wave speed is $v = f\lambda = 2\text{m/s}$

3. A particle is moving with simple harmonic motion in a straight line. If it has a speed v_1 when the displacement is x_1 and speed v_2 when the displacement is x_2 then show that the amplitude of the motion is

$$a = \left[\frac{v_2^2 x_1^2 - v_1^2 x_2^2}{v_2^2 - v_1^2}\right]^{1/2}$$

Solution:

We have the displacement equation $x = a \sin \omega t$

The speed is v_1 when the displacement is $x_1 = a \sin \omega t$

Therefore, $v_1 = \frac{dx_1}{dt} = \omega a \cos \omega t = \omega a \sqrt{1 - \sin^2 \omega t} = \omega a \sqrt{1 - \frac{x_1^2}{a^2}}$

$v_1 = \omega \sqrt{a^2 - x_1^2}$

The speed is v_2 when the displacement is $x_2 = a \sin\omega t$

Therefore, $v_2 = \frac{dx_2}{dt} = \omega a \cos \omega t = \omega a \sqrt{1 - \sin^2 \omega t} = \omega a \sqrt{1 - \frac{x_2^2}{a^2}}$

$v_2 = \omega \sqrt{a^2 - x_2^2}$

Now, $\frac{v_1^2}{v_2^2} = \frac{a^2 - x_1^2}{a^2 - x_2^2} \Rightarrow v_1^2 a^2 - v_1^2 x_2^2 = v_2^2 a^2 - v_2^2 x_1^2$

$\Rightarrow v_1^2 a^2 - v_2^2 a^2 = v_1^2 x_2^2 - v_2^2 x_1^2$

$$a^2 = \frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2} = \frac{v_2^2 x_1^2 - v_1^2 x_2^2}{v_2^2 - v_1^2}$$

$$a = \left[\frac{v_2^2 x_1^2 - v_1^2 x_2^2}{v_2^2 - v_1^2}\right]^{1/2}$$

4. **An oscillatory motion of a body is represented by $y = ae^{i\omega t}$ where y is displacement in time t, a is its amplitude and ω is angular frequency. Show that the motion is simple harmonic.**

Solution:

Here, $y = ae^{i\omega t}$

$$\frac{dy}{dt} = (i\omega)\, ae^{i\omega t}$$

$$\frac{d^2y}{dt^2} = (i\omega)^2\, ae^{i\omega t} = -\omega^2 y$$

$$\frac{d^2y}{dt^2} + \omega^2 y = 0$$

This is the equation of simple harmonic motion. Therefore, the motion is simple harmonic.

5. **A linear spring whose force constant is 0.2 N/m hangs vertically supporting a 1kg mass at rest. The mass is pulled down a distance 0.2m and then released. What will be its maximum velocity? Also find the frequency of vibration.**

Solution:

Spring constant (k) = 0.2 N/m

Mass (m) = 1kg

amplitude (A) = 0.2m

1. $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{0.2}{1}} = 0.447$

   Maximum velocity $V_{max} = A\omega = 0.2 \times 0.447 = 0.089$ m/sec.

2. Frequency (f) $= \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 0.071$Hz.

6. **A small body of mass 0.1 kg is undergoing a SHM of amplitude 0.1m and period 2sec. (1) What is the maximum force on the body? (2) If the oscillations are produced in the spring, what should be the force constant?**

Solution:

Mass of the body, m = 0.1kg

Amplitude, A = 0.1m

Time period, T = 2sec

i. F_{max} = ?   ii. k = ?

We have, $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow T^2 = \frac{4\pi^2 m}{k}$

$k = \frac{4\pi^2 m}{T^2} = 0.986$ N/m

$F_{max} = kA = 0.0986$ N.

7. **A light spring is suspended from a rigid support and its free end carries a mass of 0.4kg, which produces an extension of 6cm in the string. The mass is then pulled down further 4cm and then released so that the mass oscillate with SHM calculate the kinetic energy generated as it passes through mean position.**

Solution:

Mass of the body, m = 0.4kg

Extension on the string, x = 6cm = 0.06m

Amplitude of oscillation, A = 4 cm = 0.04m

We have, maximum kinetic energy K.E. $= \frac{1}{2}kA^2$

Also, F = mg = kx

$$\Rightarrow k = \frac{mg}{x} = \frac{0.4 \times 9.8}{0.06} = 65.33 \text{ N/m}$$

$K.E_{max} = \frac{1}{2}kA^2 = 0.0523$J

8. **The balance wheel of watch oscillates with an angular amplitude of π rad and a period of 0.5sec. Find (a) maximum angular speed of wheel (b) The angular sped of wheel when the displacement is $\frac{\pi}{2}$ rad, and (c) the magnitude of angular acceleration of wheel when its displacement is $\frac{\pi}{4}$ rad.**

Solution:

Here, angular amplitude, $\theta_{max} = \pi$ rad.

Time period T = 0.5 sec

a. Since, the maximum linear speed $v_{max} = \omega A$

The maximum angular speed, $v_{max} = \omega\theta_{max}$

$$= \frac{2\pi}{T}.\pi = 39.44 \text{ rad/sec}$$

b. Here, $\theta = \frac{\pi}{2}$ rad

We have, $v = \omega\sqrt{a^2 - x^2}$

$$= \omega\sqrt{\theta_{max}{}^2 - \theta^2}$$

$$= \frac{2\pi}{T}\sqrt{\pi^2 - \left(\frac{\pi}{2}\right)^2}$$

$$= \frac{2\pi}{0.5}\cdot\sqrt{\frac{3\pi^2}{4}} = 34.15 \text{ rad/sec}$$

c. Since in linear motion, $a = \omega^2 x$

in angular motion $\alpha = \omega^2\theta$

$$= \left(\frac{2\pi}{T}\right)^2 \cdot \frac{\pi}{4} = 123.84 \text{ rad/sec}^2$$

9. In simple harmonic motion, when the displacement is one half the amplitude, wh fraction of total energy is the kinetic energy and what fraction is potential energy? what displacement is the energy half K.E. and half P.E.?

Solution:

We have in SHM,

$$\text{P.E.} = \frac{1}{2}kx^2 = \frac{1}{2}m\,\omega^2.x^2 = \frac{1}{2}m\,\omega^2A^2\sin^2(\omega t + \phi)$$

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 = \frac{1}{2}m\left[\frac{d}{dt}\{A\sin(\omega t + \phi)\right]^2$$

$$\text{K.E.} = \frac{1}{2}m\omega^2A^2\cos^2(\omega t + \phi)$$

$$\text{and T.E.} = \text{K.E.} + \text{P.E.} = \frac{1}{2}m\omega^2A^2$$

The general displacement equation is $x = A\sin(\omega t + \phi)$

From question, $x = \frac{A}{2}$

Therefore, $\frac{A}{2} = A\sin(\omega t + \phi) \Rightarrow \sin(\omega t + \phi) = \frac{1}{2}$

And $\cos(\omega t + \phi) = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{2}}$

Now, $\text{K.E.} = \frac{1}{2}m\omega^2A^2\cos^2(\omega t + \phi) = \left(\frac{\sqrt{3}}{2}\right)^2 \text{ T.E.} = \frac{3}{4}\text{ T.E.}$

and $\text{P.E.} = \frac{1}{2}m\omega^2A^2\sin^2(\omega t + \phi) = \left(\frac{1}{2}\right)^2, \text{ T.E.} = \frac{1}{4}\text{ T.E.}$

When P.E. is half of T.E. i.e. P.E. $= \frac{1}{2}$ T.E.

$$\frac{1}{2}m\omega^2x^2 = \frac{1}{2}\left(\frac{1}{2}m\omega^2A^2\right)$$

$$x^2 = \frac{A^2}{2}$$

$$x = \frac{A}{\sqrt{2}}$$

Therefore, at $x = \frac{A}{\sqrt{2}}$, half the energy is K.E. and half the energy is P.E.

10. In an electric shaver, the blades moves back and forth over a distance of 2mm in simple harmonic motion with a frequency of 120 Hz. Find the maximum blade speed and the magnitude of the maximum acceleration.

Solution:

Here, $A = 2\text{mm} = 2\times10^{-3}$m

f = 120 Hz

Maximum speed, $V_{max} = \omega A = 2\pi f.A = 1.5$ m/sec

and maximum acceleration, $a_{max} = \omega^2 A = (2\pi f)^2.\ A = 1135.83\text{m/sec}^2$

11. An oscillating block spring system has mechanical energy of 1.18 J, an amplitude of 9.84cm and maximum speed of 1.22m/sec. Find i) The force constant of spring ii) The mass of the block and iii) the frequency of oscillation.

Solution:

Here, Energy = 1.18J

Amplitude (A) = 9.84cm = 9.84×10^{-2}m

Maximum speed, V_{max} = 1.22m/sec

We have, $v_{max} = \omega A$

$$\Rightarrow \omega = \frac{v_{max}}{A} = \frac{1.22}{9.84\times10^{-2}} = 12.4 \text{ rad/sec}$$

Also, Total energy $= \frac{1}{2}m\,\omega^2A^2$

$$\Rightarrow \frac{1}{2}m\omega^2A^2 = 1.18$$

$\therefore m = \frac{1.18 \times 2}{(12.4)^2 \times (9.84 \times 10^{-2})^2} = 1.59$ kg

1. Since $\omega = \sqrt{\frac{k}{m}} \Rightarrow k = m\omega^2 = 1.59 \times 12.4^2 = 243.74$N/m
2. Mass of the block (m) = 1.59 kg
3. Since, $\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} = 1.97$ Hz.

12. Show that, if a spring of spring constant 'k' is cut into 'n' equal parts i) the spring constant of each part becomes nk. ii) The time period of each part will be $T = 2\pi\sqrt{\frac{m}{nk}}$, where m is the mass of load in each part.

Solution:

Let a mass less spring with spring constant k and x be the extension produced when a force F is applied to the spring, then,

$F = kx$

When the spring is cut into n equal ports, let $x_1, x_2 \ldots x_n$ are extension produced in 1st, 2nd ... n^{th} part.

Then for first spring, $F = k_1x_1$

For second spring $F = k_2x_2$

For nth spring $F = k_nx_n$

Now the total extension is given by

$x = x_1 + x_2 + x_3 + \ldots + x_n$

$\frac{F}{k} = \frac{F}{k_1} + \frac{F}{k_2} + \frac{F}{k_3} + \ldots + \frac{F}{k_n}$

$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \ldots + \frac{1}{k_n}$

Since all the parts are of equal length and of same material

$k_1 = k_2 = \ldots = k_n = k'$ (say)

$\therefore \frac{1}{k} = \frac{n}{k'}$

$\therefore k' = nk$

ii. Time period $T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{nk}}$

13. The displacement of an oscillating particle is given by $x = a \sin \omega t + b \cos \omega t$. Show that the amplitude 'A' is given by $A = (a^2 + b^2)^{1/2}$, if the motion is simple harmonic.

Solution:

Here, $x = a \sin \omega t + b \cos \omega t$ ..... (1)

Since the motion is simple harmonic we may also write the displacement as $x = A \sin(\omega t + \phi)$ ..... (2) where A is the amplitude.

Comparing equations (1) and (2)

$A \sin \omega t \cos \phi + A \cos \omega t \sin \phi = a \sin \omega t + b \cos \omega t$

$(A \cos \phi - a) \sin \omega t + (A \sin \phi - b) \cos \omega t = 0$

This equation will hold for all values of t if

$A \cos \phi - a = 0$ and $A \sin \phi - b = 0$

or, $A \cos \phi = a$ and $A \sin \phi = b$

Squaring and adding these two equations.

$A^2 = a^2 + b^2$

$A = (a^2 + b^2)^{1/2}$

14. Show that if the displacement of a moving point at any time is given by an equation of the form $x = a \cos \omega t + b \sin \omega t$ the motion is simple harmonic. If a = 3cm, b = 4cm and ω = 2 rad/sec determine the period, amplitude, maximum velocity and maximum acceleration of the motion.

Solution:

Here, the displacement is given by $x = a \cos\omega t + b \sin\omega t$

velocity, $v = \frac{dx}{dt} = -\omega a \sin \omega t + \omega b \cos \omega t$

acceleration, $a = \frac{d^2x}{at^2} = -\omega^2 a \cos \omega t - \omega^2 b \sin \omega t$

$= -\omega^2 (a \cos \omega t + b \sin \omega t) = -\omega^2 x$.

The particle, therefore, execute simple harmonic motion,

The amplitude is given by $A = \sqrt{a^2 + b^2} = \sqrt{3^2 + 4^2} = 5$cm

Time period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi = 3.14$ sec.

Maximum velocity of particle, $v_{max} = \omega A = 2 \times 5 = 10$cm/sec

Maximum acceleration of particle, $a_{max} = \omega^2 A = 2^2 \times 5 = 20$ cm/sec^2

15. If a particle moves in a potential energy field $U = U_0 - ax + bx^2$, where a and b are positive constants, a) obtain an expression for the force acting on it as a function of position b) At what point does the force vanish? c) Is this a point of stable equilibrium d) calculate the force constant, time period and frequency.

Solution:

a) The force acting on the particle is given by

$F = -\frac{dU}{dx} = -\frac{d}{dx}(U_0 - ax + bx^2) = a - 2bx$

b) The force vanishes at the point where $\frac{dU}{dx} = 0$

i.e. $a - 2bx = 0$

$x = \frac{a}{2b}$

c) Here, $\frac{d^2U}{dx^2} = \frac{d}{dx}\left(\frac{dU}{dx}\right) = \frac{d}{dx}(-a + 2bx) = 2b$, which is positive. The point $x = \frac{a}{2b}$ represents the point of minimum potential energy. It is therefore, a point of stable equilibrium.

d) We have $F = a - 2bx$, from this relation if is clear that F is a linear restoring force with force constant $k = 2b$.

Therefore, time period, $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{2b}}$

and frequency, $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{2b}{m}}$

16. A particle of mass 10 gm moves under a potential $V_x = 8\times10^5x^2$ ergs/gm, where x is in cm. Deduce the time displacement relation when the total energy is 8×10^5 ergs.

Solution:

Here potential $V_x = 8\times10^5\ x^2$ ergs/gm

Potential energy acquired by particle,

$U = m\ V_x = 10\times8\times10^5x^2$

$= 8\times10^6\ x^2$ ergs

Since P.E. is maximum at extreme position, that is when x = A (maximum displacement or amplitude). Where total energy is only P.E.

$\therefore\ 8\times10^6\ A^2 = 8\times10^5$

$A^2 = \frac{1}{10}$

$A = \frac{1}{\sqrt{10}}$ cm

Now, force acting on particle, $F = -\frac{dU}{dx} = -16\times10^6x$

$\Rightarrow m\frac{d^2x}{dt^2} = -16\times10^6x$

$m\frac{d^2x}{dt^2} + (16\times10^6)x = 0$

$\frac{d^2x}{dt^2} + \left(\frac{16\times10^6}{m}\right)x = 0$

Therefore, $\omega^2 = \frac{16\times10^6}{m} = \frac{16\times10^6}{10} = 16\times10^5$

$\omega = \sqrt{16\times10^5} = 400\sqrt{10}$

The time displacement relation is given by

$x = \frac{1}{\sqrt{10}}\sin(400\sqrt{10}\,t + \phi)$

17. Show that the time period of oscillation of loaded spring is $T = 2\pi\sqrt{\frac{x}{g}}$.

Solution:

If k is the force constant, we have

$mg = kx$

$\frac{m}{k} = \frac{x}{g}$

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{x}{g}}$

18. Show that if a uniform stick of length 'l' is mounted so as to rotate about a horizontal axis perpendicular to the stick and at a distance 'd' from the centre of gravity the period has minimum value when d = 0.289l.

Solution:

Here, Total length of stick = l

We have for a rod, $I = mk^2 = \frac{ml^2}{12} \Rightarrow k = \frac{l}{\sqrt{12}} = 0.289l$.

The time period of compound pendulum is minimum when length of pendulum (d) = radius of gyration (k)

Therefore, d = k = 0.289l.

19. A uniform circular disc of radius 'R' oscillates in a vertical plane about horizontal axis. Find the distance of the axis of rotation from the centre for which the time period is minimum. Find the value of time period also.

Solution:

For a circular disc, $I = mk^2 = \frac{mR^2}{2}$

$\Rightarrow k = \frac{R}{\sqrt{2}}$

Time period is minimum when, $l = k = \frac{R}{\sqrt{2}}$

Since, $T = 2\pi\sqrt{\frac{\frac{k^2}{l} + l}{g}}$

Therefore, $T_{min} = 2\pi\sqrt{\frac{k + k}{g}}$

$= 2\pi\sqrt{\frac{2k}{g}} = 2\pi\sqrt{\frac{2}{g}\cdot\frac{R}{\sqrt{2}}} = 2\pi\sqrt{\frac{1.41R}{g}}$

20. A metal disc of radius 0.5m oscillates in its own plane about an axis passing through a point on its edge. Find the length of equivalent simple pendulum.

Solution:

Radius of disc (R) = 0.5m

Therefore, length of pendulum l = R = 0.5m

For disc we have,

$I = mk^2 = \frac{mR^2}{2} \Rightarrow k = \frac{R}{\sqrt{2}}$

Therefore, Length of equivalent simple pendulum $L = \frac{k^2}{l} + l$

$$L = \frac{R^2}{2} \cdot \frac{1}{R} + R = \frac{R}{2} + R = \frac{3R}{2}$$

$L = 0.75m$

21. A thin straight uniform rod of length l = 1m and mass m = 160 gm hangs from a pivot at one end. (a) What is it's time period for small amplitude oscillation? (b) What is the length of a simple pendulum that will have the same time period?

Solution:

Here length of rod, $l' = 1m$

Therefore, length of pendulum $l = \frac{l'}{2} = 0.5m$

Mass, m = 160 gm = 0.16kg

Since, the moment of inerfra rod about an axis passing through one end is $I = \frac{1}{3} ml'^2 = \frac{1}{3} \times$

$0.16 \times 1 = 0.053$ kgm^2

a. $T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{0.053}{0.16 \times 9.8 \times 0.5}} = 1.63$ sec.

b. $T = 2\pi\sqrt{\frac{k^2/l + l}{g}} = 2\pi\sqrt{\frac{L}{g}}$

$\Rightarrow L = \frac{gT^2}{4\pi^2} = \frac{9.8 \times 1.63^2}{4 \times \pi^2} = 0.66m$

22. A wire has torsional constant of 2 Nm/rad. A disc of radius 5cm and mass 100 gm is suspended at it's centre. What is the frequency?

Solution:

Here,

C = 2Nm/rad, R = 5cm = 0.05m, m = 100 gm = 0.1kg

We have, $I = \frac{1}{2} mR^2 = \frac{1}{2} \times 0.1 \times 0.05^2 = 1.25 \times 10^{-4}$ kgm^2

and, $T = 2\pi\sqrt{\frac{I}{C}} = 2\pi\sqrt{\frac{1.25 \times 10^{-4}}{2}}$

$= 0.04965$ Sec.

Therefore, frequency, $f = \frac{1}{T} = 20.14$ Hz.

23. A flat uniform circular disc has mass of 3kg and radius of 70cm. It is suspended in a horizontal plane by a wire attached to it's centre. If the disc is rotated through 2.5 radian about the wire, a torque of 0.6Nm is required to maintain that orientation. Calculate i) The rotational inertia of the disc about the wire ii) The torsion constant iii) The angular frequency of the torsion pendulum when it is set oscillating.

Solution:

Here, m = 3kg, R = 70cm = 0.7m, θ = 2.5 radian and τ = 0.6 Nm.

i. $I = \frac{1}{2} mR^2 = \frac{1}{2} \times 3 \times 0.7^2 = 0.735$ kgm^2

ii. $\tau = C\theta \Rightarrow C = \frac{\tau}{\theta} = \frac{0.6}{2.5} = 0.24$ Nm/rad

iii. $\omega = \sqrt{\frac{C}{I}} = \sqrt{\frac{0.24}{0.735}} = 0.571$ rad/sec.

24. A solid sphere of radius 0.3m executes torsional oscillation of time period $2\pi\sqrt{12}$ sec at the end of suspension wire where upper end is fixed in a rigid support. If the torque constant of the wire be 6×10^{-3} Nm/rad, calculate the mass of the sphere.

Solution:

Here, R = 0.3m, $T = 2\pi\sqrt{12}$ sec, $C = 6 \times 10^{-3}$ Nm/rad, m = ?

We have, $T = 2\pi\sqrt{\frac{I}{C}} \Rightarrow I = \frac{CT^2}{4\pi^2} = \frac{6 \times 10^{-3} \times 4\pi^2 \times 12}{4\pi^2}$

$I = 7.2 \times 10^{-2}$ kg m^2

Since, for sphere, $I = \frac{2}{5} mR^2$

$m = \frac{5I}{2R^2} = 2kg.$

25. A block is in SHM on the end of a spring with position given by $x = x_m \sin(\omega t + \phi)$. If $\phi = \frac{\pi}{5}$ rad, then at t = 0 what percentage of total mechanical energy is potential energy?

Solution:

The total mechanical energy T.E. $= \frac{1}{2} kA^2$

The potential energy, P.E. $= \frac{1}{2} kx^2$

$= \frac{1}{2} kA^2 \sin^2(\omega t + \phi)$

When, t = 0 and $\phi = \frac{\pi}{5}$

P.E. $= \frac{1}{2} kA^2 \sin^2 \frac{\pi}{5}$

The percentage of P.E. $= \frac{P.E.}{T.E} \times 100\%$

$= \sin^2 \frac{\pi}{5} \times 100\%$

$= 34.54\%$

26. A sinusoidal wave travels along a string. The time for a particular point to move from maximum displacement to zero is 0.17 S. What are the

(i) Period and frequency?

(ii) The wavelength is 1.40 m; what is the wave speed?

Solution.

Let T be the time period of oscillation.

The time interval between two extreme (maximum) position $= \frac{T}{2}$

Then, the time interval between mean position and one maximum position $= \frac{T}{4}$

(i) Time period (T) = 4 × (Time for maximum displacement to zero)

= 4 × 0.17 S

= 0.68 sec.

and Frequency $(f) = \frac{1}{T} = \frac{1}{0.68} = 1.47$ Hz

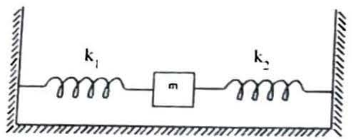
(ii) $\lambda = 1.40$ m, wave speed $(v) = f\lambda = 1.47 \times 1.40 = 2.05$ m/S

Exercise

1. Define simple harmonic motion. Show that the gravitational force has no effect on it's the vertical oscillation of mass.
2. Explain phase and phase constant in SHM. Show that total energy of mass spring system which is oscillating in SHM is conserved.
3. Define SHM. Derive an expression for the time period of a physical pendulum and establish the interchangeability of centre of oscillation and suspension.
4. Distinguish between simple and physical pendulum. Deduce the time period of a compound pendulum and show that it is minimum when the length of the pendulum is equal to radius of gyration.
5. Define angular harmonic motion. Write down it's differential equation.
6. Derive an expression for the time period and radius of gyration of compound pendulum.
7. What are the limitation of a simple pendulum?
8. Derive time period of torsion pendulum and find an expression for modulus of rigidity of the suspension wire.
9. Show that particle velocity at any instant is the product of wave velocity and the slope of displacement curve at that instant.
10. Write down the characteristics of simple harmonic motion.
11. In which condition time period of compound pendulum is maximum and minimum.
12. Define simple harmonic motion. Why is really S.H.M. rare? Describe with necessary theory how you will determine the value of the modulus of rigidity of a metal in laboratory using torsional pendulum.

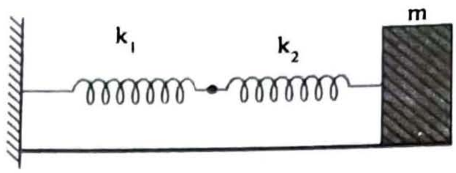
13. Define restoring force and state Hook's law.
14. Differentiate between SHM and periodic motion.
15. For a compound pendulum prove that the minimum time period is obtained if the point of suspension and point of oscillation are equidistance from C.G.
16. Point out similarities and dissimilarities between the oscillations of bar pendulum and torsional pendulum.
17. Prove that if a transverse wave is travelling along a string, then the slope at any point of the string is numerically equal to the ratio of the particle speed to the wave speed at that point.
18. Derive a relation to determine the radius of gyration of a compound pendulum. Why determination of the acceleration due to gravity is more accurate from a compound pendulum than a simple pendulum.
19. List the common pendulum in practice. Which of them is a physical pendulum and why.
20. What is compound pendulum? Deduce the expression for the time period of a compound pendulum and compare this period with torsional pendulum.
21. Differentiate between linear and angular harmonic motion. Show that the motion of torsion pendulum is angular harmonic motion. Also find it's time period.
22. Obtain an expression for the time period of a compound pendulum and show that it's time period is unaffected by the fixing of a small additional mass to it's centre of suspension.
23. What are draw backs of simple pendulum? Show that the period of torsion pendulum remains unaffected even if the amplitude be large, provided that the elastic limit of the wire is not exceeded.
24. Show that there are four collinear points with in compound pendulum having same time period. Give their physical significance.
25. Deduce the expression for the time period of a compound pendulum and formulate the equivalent length of the simple pendulum.
26. What is SHM? Discuss the theory of a simple spring mass system and derive an expression for its time period and frequency.
27. Define the terms frequency, amplitude of S.H.M.
28. Two springs of force constant k_1 and k_2 are attached to mass 'm' and joined to fixed supports as shown in figure. Show that the frequency of oscillation of 'm' is

$$f = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$$



29. Two springs A and B having force constant k_1 and k_2 are attached to a block of mass 'm' as shown in figure. Show that the frequency of oscillation on the frictionless surface is given by,

$$f = \frac{1}{2\pi}\sqrt{\frac{k_1 . k_2}{k_1 + k_2}}$$



30. A body of mass 100 gm is suspended from a spring of negligible mass and is found to be stretch the spring 1cm 1) what is the force constant of spring 2) What is the period of oscillation of the body pulled down and released.
31. A body of mass 0.01 kg is attached to a light spring of force constant 5N/m. The motion starts from rest by displacing the body 0.01m to the right and releasing it. Calculate the frequency and maximum velocity.
32. A mass 'm' is suspended from a light vertical spring of force constant K. The period of oscillation is 0.50 second. The spring is cut into four equal parts. The same mass 'm' is suspended from one of the parts and set into oscillation. Calculate the new period of oscillation.
33. A meter stick swings about pivot at one end, at distance 'l' from the stick's centre of mass. Calculate the period of oscillation using parallel axis theorem. What would be the length of the simple pendulum that would have the same period?
34. An iron ball of mass 4kg falls from a height of 200 cm and hits a light plat form on a helical spring of spring constant 400 N/m. What will be time period of vertical oscillation?
35. A meter stick swings about a pivot point at its one end. What is the time period of oscillation? What is the distance of the pivot from the centre of oscillation of the stick?
36. A thin rod of mass 0.1kg and length 0.1m is suspended by a wire, which passes through center and is perpendicular to its length. The wire is twisted and the rod is set into oscillation. The period is found to be 2 sec. When a flat body in the shape of an equilateral triangle is suspended similarly through its center of mass, the period is found to be 6 sec. Find the moment of inertia of triangle about this axis.
37. A uniform circular disc of a diameter 20cm vibrates about a horizontal axis perpendicular to its plane and at a distance of 5cm from the center. Calculate, the period of oscillation and the equivalent length of simple pendulum.
38. A metallic disc of mass 0.5kg and radius of 0.1m is suspended by wire of length 40cm and radius 1mm. The torsion oscillation of the disc is found to have a period of 2.5sec. Find the modulus of rigidity of wire.
39. A uniform circular disc whose radius R is 12.6cm is suspended as a physical pendulum from a pivot point on it's rim a) What is its period? b) At what radial distance $r < R$, is there a pivot point that gives the same period.
40. A solid cylinder of mass 500 gm is suspended by a thin wire in such a way that its axis remains vertical. If the period of torsion oscillation be 2.5 seconds what will be the period if the cylinder is replaced by a solid sphere of 6 cm radius and mass 400 gm?
41. What is SHM? Derive an expression for the period and radius of gyration of a compound pendulum and show that centre of oscillation and centre of suspension are interchangeable.
42. Calculate the frequency and maximum particle velocity due to wave represented by $y(x, t) = 0.03 \sin(60\pi t - 0.03\pi x)$. The values of x and y are in centimeters.

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