

set - 1      2018 fall

1(a) What do you mean by wave function? Derive the time independent form of Schrodinger's wave equation.

→ In quantum mechanics there is a variable quantity that mathematically describe the wave characteristics of a particle, called wave function. Denoted by  $\psi$  (4).

we have the wave function

$$\psi = A e^{-j(\omega t - \frac{2\pi}{\lambda} x)} \quad \text{--- (1)}$$

Now using  $E = hf = 2\pi\hbar f = 2\pi\hbar c/s$

$$\lambda = \frac{\hbar}{P} = \frac{2\pi\hbar}{P}$$

we get (1) as

$$\psi = A e^{-j(Et - Px)} \quad \text{--- (2)}$$

Now differentiating above eq (1) twice w.r.t.  $x$

$$\frac{d\psi}{dx} = A j \frac{2\pi}{\lambda} e^{-j(\omega t - \frac{2\pi}{\lambda} x)}$$

$$\frac{d^2\psi}{dx^2} = A j^2 \frac{4\pi^2}{\lambda^2} e^{-j(\omega t - \frac{2\pi}{\lambda} x)}$$

$$\begin{aligned} \frac{d^2\psi}{dx^2} &= \psi j^2 k^2 \\ &= -\frac{P^2}{\hbar^2} \psi \quad k^2 = \left(\frac{P}{\hbar}\right)^2 \end{aligned} \quad \text{--- (3)}$$

~~Note~~ The total energy of one particle is the sum of its K.E ( $\frac{p^2}{2m}$ ) and P.E (V).

$$\text{i.e. } E = \frac{p^2}{2m} + V$$

Multiply both sides by  $\psi$  we get

$$E\psi = \frac{p^2}{2m}\psi + V\psi \quad \text{--- (4)}$$

from (3) and (4)

$$E\psi = -\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V\psi$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} (E-V)\psi = 0 \quad \text{--- (5)}$$

which is the time independent form of Schrodinger's wave equation.

In 3-D it can be written as

$$\nabla^2\psi + \frac{2m}{\hbar^2} (E-V)\psi = 0$$

(b) Find the temperature at which there is 98% probability that a state 0.3 eV below the Fermi energy level will be occupied by an electron.

By Fermi-Dirac distribution function probability that occupied by electron is

$$F(E) = \frac{1}{1 + e^{\frac{(E - E_F)}{kT}}}$$

$$0.98 = \frac{1}{1 + e^{(-0.3/kT)}}$$

$$e^{-0.3/kT} = \frac{1}{0.98} - 1$$

$$\frac{(-0.3/kT)}{e} = \frac{1}{99}$$

$$\frac{-0.3}{kT} = \ln(\frac{1}{99})$$

$$kT = \frac{-0.3}{\ln(\frac{1}{99})} \quad k = 1.38 \times 10^{-23} \text{ J/K}$$

$$1J = \frac{1}{1.6 \times 10^{-19} \text{ eV}}$$

$$T = \frac{0.07708}{k}$$

$$1.38 \times 10^{-23} \text{ J/K} = \frac{1.38 \times 10^{-23}}{1.6 \times 10^{-19}}$$

$$T = \frac{0.07708 \text{ eV}}{1.38 \times 10^{-23} \text{ J/K}}$$

$$= 5.625 \times 10^{-5} \text{ eV/K}$$

$$T = \frac{0.07708 \text{ eV}}{8.625 \times 10^{-5} \text{ eV/K}}$$

$$T = 893.68 \text{ K} \quad \text{Ans}$$

Q.2-(a) Define the term electrons drift velocity and mobility. How are they related? Show that the conductivity of electrons with in metallic conductor is product of charge density and mobility.

- (b) The average velocity attained by charged particles in a material due to an electric field is called drift velocity and denoted by  $v_d$ .

Electron mobility is the movement of electron and is the ratio of drift velocity to the strength of the electric field.

$$\text{i.e. } \mu = \frac{v_d}{E} \Rightarrow E = \mu v_d.$$

Electrical conductivity is the quantity of electricity that flow in unit time per unit area of cross section of the metallic conductor per unit potential gradient. The quantity of electricity flowing through conductor is

$$q = \sigma AET$$

$$\sigma = \frac{q}{AE} \quad T = 1 \text{ sec}$$

$$J = \sigma E \quad \text{--- (1)}$$

Now from Ohm's law

$$\sigma = \frac{n e^2 \lambda r}{6 k_B T} \quad \text{--- (2)}$$

Now,  $\sigma = \frac{ne^2 \lambda v}{6mv^2} \times 3 \quad k_B T = \frac{mv^2}{3}$

$$= \frac{ne^2 \lambda}{2mv}$$

$$= \frac{ne^2 \lambda}{2m} t$$

$$\sigma = \frac{ne^2 \lambda}{m} \quad \lambda \text{ is mean free path.}$$

$$\boxed{\sigma = ne \mu} \quad \mu = \frac{e \lambda}{m}$$

which is required equation.

(b) Molybdenum has the BCC crystal structure, a density of  $10.22 \text{ gm cm}^{-3}$  and an atomic mass of  $95.97 \text{ g mol}^{-1}$ . What is atomic concentration, lattice parameter and atomic radius of molybdenum?

$$\rightarrow \text{density } (\rho) = 10.22 \text{ gm cm}^{-3}$$

$$\text{at mass (M.M.)} = 95.97 \text{ g mol}^{-1}$$

$$\text{at conc (C)} = ?$$

$$\text{lattice parameter} = ?$$

$$\text{atomic radius} = ?$$

In BCC crystal there are  $1/8$  part of an atom in each corner of the cube  $1/8 \times 8 = 1$  atom. and 1 central atom in the central position of the cube

$$n = 2 \text{ atoms per unit cell}$$

$$\text{at. conc} = 2 \text{ atoms / unit cell.}$$

volume of the cube

$$a^3 = \frac{\text{at. mass} \times \text{at. one}^n}{\text{density} \times \text{A-number.}}$$

$$a^3 = \frac{95.94 \times 2}{10.22 \times 6.023 \times 10^{23}}$$

$$a^3 = 3.12 \times 10^{-23} \text{ m}^3$$

$$a = 3.22 \times 10^{-10} \text{ m.}$$

Now

$$a = \frac{4r}{\sqrt{3}}$$

$$r = \frac{a\sqrt{3}}{4}$$

$$= \frac{3.22 \times 10^{-10} \times \sqrt{3}}{4}$$

$$= 1.39 \times 10^{-10} \text{ m}$$

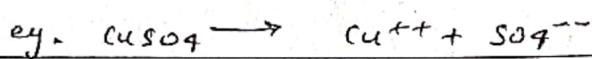
∴ atomic concentration = 2 units/cell atoms / unit cell

$$\text{lattice parameter} = 3.22 \times 10^{-10} \text{ m}$$

$$\text{atomic radius} = 1.39 \times 10^{-10} \text{ m.}$$

Q. 3. (a) Show that ionic conductivity depends inversely with temperature.

→ Ionic conductivity can be visualized from purposely done electrolysis, electroplating etc. In electrolysis two metals anode and cathode are placed in aqueous medium. When external potential is applied, cation and anion are formed and cation is attracted towards anode while the anion goes towards cathode.



The main conduction is caused due to movement of such positively or negatively ion in particular direction on application of electric field and it is given by

$$\sigma_{\text{ionic}} = e N_{\text{ion}} u_{\text{ion}} \quad \text{--- (1)}$$

From Einstein diffusion relation we have

$$u_{\text{ion}} = \frac{e}{kT} D \quad \text{--- (2)}$$

The diffusion coefficient varies with temperature as

$$D = D_0 \exp\left(-\frac{q}{kT}\right) \quad \text{--- (3)}$$

Now the ionic conductivity can be expressed as

$$\sigma_{\text{ion}} = e N_{\text{ion}} u_{\text{ion}}$$

$$= e N_{\text{ion}} \frac{e D}{kT}$$

$$= \frac{e^2 N_{\text{ion}} D_0}{kT} \exp\left(-\frac{q}{kT}\right)$$

$$\sigma_{\text{ion}} = \frac{e^2}{kT} N_{\text{ion}} D_0 \exp\left(\frac{-q}{kT}\right)$$

$$\sigma_{\text{ion}} = \sigma_0 \exp\left(\frac{-q}{kT}\right) \quad \text{--- (4)}$$

where  $\sigma_0 = \frac{e^2}{kT} N_{\text{ion}} D_0$   $\text{--- (5)}$

Taking ln on both side of (4) we get

$$\ln \sigma_{\text{ion}} = \ln \sigma_0 - \frac{q}{kT}$$

$$\ln \sigma_{\text{ion}} = \ln \sigma_0 - \left(\frac{q}{k}\right) \frac{1}{T} \quad \text{--- (6)}$$

From the above equation (6), it is concluded that the ionic conductivity is inversely proportional to the temperature.

$\ln \sigma_{\text{ion}}$

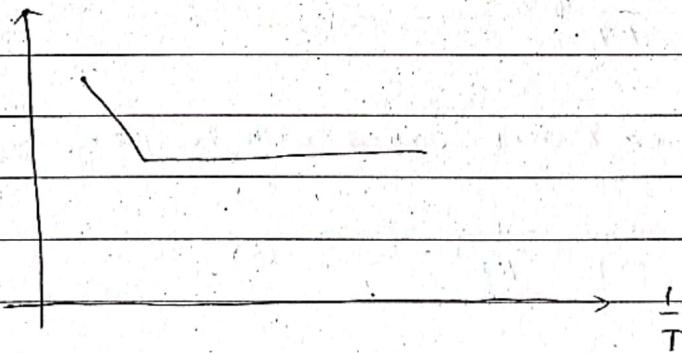


fig. dependences of ionic conductivity to the temperature.

(b) Write the types of polarization! Derive the Clausius-Mossotti equation for electronic polarization, relating to polarizability with the permittivity.

→ Types of polarization:

1. Ionic polarization
2. Dipolar / orientational polarization
3. Interfacial polarization
4. Electronic polarization

Consider local electric field as  $E_{loc}$ , we have

$$P = N \cdot P_{\text{produced}} = N \alpha_e E_{loc}$$

$$\Rightarrow P = N \alpha_e \left( E + \frac{P}{3 \epsilon_0} \right) \quad \text{Lorentz field } E_{loc} = E + \frac{P}{3 \epsilon_0}$$

$$P = N \alpha_e E + \frac{N \alpha_e P}{3 \epsilon_0}$$

$$\Rightarrow P \left( 1 - \frac{N \alpha_e}{3 \epsilon_0} \right) = N \alpha_e E$$

$$P = \frac{N \alpha_e E}{\left( 1 - \frac{N \alpha_e}{3 \epsilon_0} \right)} \quad \text{--- (1)}$$

$$\text{Since, } P = \epsilon_0 \chi_e E \quad \text{--- (2)}$$

from (1) and (2) we get

$$\epsilon_0 \chi_e E = \frac{N \alpha_e E}{1 - \frac{N \alpha_e}{3 \epsilon_0}}$$

$$\Rightarrow \epsilon_0 \chi_e - \frac{\epsilon_0 \pi e N \chi_e}{3 \epsilon_0} = N \chi_e$$

$$\text{or, } \epsilon_0 \chi_e = N \chi_e + \frac{N \chi_e \pi e}{3}$$

$$\text{or } \epsilon_0 \chi_e = N \chi_e \left( 1 + \frac{\pi e}{3} \right)$$

$$\therefore \epsilon_0 \chi_e = N \chi_e \left( \frac{3 + \pi e}{3} \right)$$

$$\boxed{\frac{N \chi_e}{3 \epsilon_0} = \frac{\chi_e}{\chi_e + 3} = \frac{\epsilon_r - 1}{\epsilon_r + 2}}$$

This is called Clausius-Mossotti equation, which shows relationship between polarizability ( $\chi_e$ ) and susceptibility ( $\chi_e$ ) and relative permittivity ( $\epsilon_r$ ).

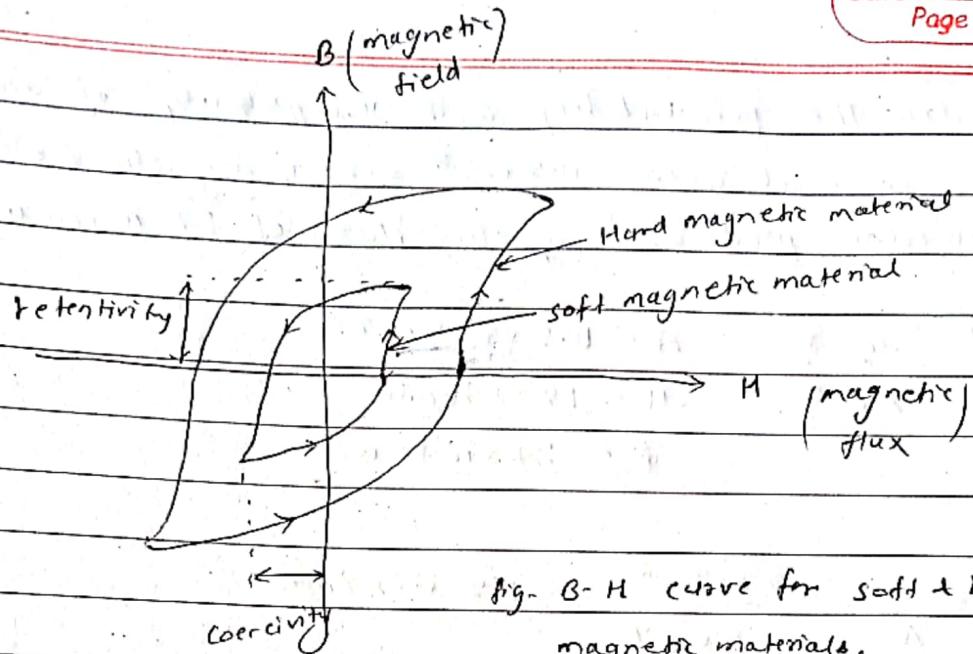
Q.4-(a) Explain the eddy current loss and hysteresis loss in magnetic materials?

$\Rightarrow$  Eddy current loss:-

When an applied magnetic field is changed, current is induced in magnetic material. This current is called eddy current and produces loss in power resulting in heating of materials called eddy current loss. To reduce this loss, resistance has to be made large such that small current is allowed and hence the eddy current loss is reduced.

Hysteresis loss:-

When a magnetic material is magnetize in a certain unit and then demagnetized, energy loss take place that is given by one area of the hysteresis loop.



The size of loop is different for different materials, these are materials which are easily magnetized and then demagnetized, called soft magnetic material and other materials, hard to both magnetized and demagnetized are hard magnetic materials.

The value of retentivity is large enough in hard magnetic materials than soft magnetic materials. The value of coercivity also large in hard magnetic material.

Hence large hysteresis loop means large loss so hard magnetic materials have large hysteresis loss than soft.

(b) Calculate the permeability and susceptibility of an iron bar of cross-sectional area  $0.2 \text{ cm}^2$  when a magnetic field of strength of  $1200 \text{ A/m}$  produces magnetic flux of  $24 \text{ micro weber}$ .

$$\Rightarrow \text{Here, } \mu_r = ? \quad A = 0.2 \times 10^{-4} \text{ m}^2 \\ \chi_m = ? \quad H = 1200 \text{ A/m} \\ \phi = 24 \times 10^{-6} \text{ weber}$$

$$B = \frac{\phi}{A} = \frac{24 \times 10^{-6}}{0.2 \times 10^{-4}} = 1.2 \text{ weber/m}^2$$

Also,

$$B = \mu_0 \mu_r H$$

$$\mu_r = \frac{B}{\mu_0 H} = \frac{1.2}{4\pi \times 10^{-7} \times 1200} = 795.77$$

$$\mu_m = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 795.77 = 1 \times 10^{-3}$$

Also,

$$\mu_r = 1 + \chi_m$$

$$\chi_m = \mu_r - 1$$

$$= 794.77$$

Q.5 (a) Show that the Fermi level of intrinsic SiC lies midway between the valence band and conduction band. ( $E_F = E_g/2$ ).

$\Rightarrow$  Since from mass law action

$$p = n = n_i \quad \text{for intrinsic}$$

Taking  $p = n_i$  we can write

$$N_V \exp\left(-\frac{(E_F - E_V)}{kT}\right) = (N_c N_V)^{1/2} \exp\left(\frac{-E_g}{2kT}\right)$$

$$\text{or } \exp\left(-\frac{E_{F_i}}{kT} + \frac{E_V}{kT} + \frac{E_g}{2kT}\right) = \left(\frac{N_c N_V}{N_V^2}\right)^{1/2} = \left(\frac{N_c}{N_V}\right)^{1/2}$$

Taking ln on both sides

$$-\frac{E_{F_i}}{kT} + \frac{E_V}{kT} + \frac{E_g}{2kT} = \ln\left(\frac{N_c}{N_V}\right)^{1/2}$$

$$\text{or } -\frac{E_{F_i}}{kT} = -\frac{E_V}{kT} + \ln\left(\frac{N_c}{N_V}\right)^{1/2} - \frac{E_g}{2kT}$$

$$\Rightarrow E_{F_i} = E_V + \frac{E_g}{2} - \frac{1}{2} kT \ln\left(\frac{N_c}{N_V}\right) \quad \text{--- ①}$$

we have

$$\frac{N_c}{N_V} = \frac{2}{n^2} \left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$$

$$\frac{N_c}{N_V} = \frac{2}{n^2} \left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$$

$$\frac{N_c}{N_V} = \left(\frac{m_e^*}{m_h^*}\right)^{3/2} \quad \text{--- ②}$$

From ① and ②

$$E_{Fi} = Ev + \frac{Eg}{2} - \frac{1}{2} kT \left( \frac{m^*}{m_h^*} \right)^{3/2}$$

$$E_{Fi} = Ev + \frac{Eg}{2} - \frac{3}{4} kT \ln \left( \frac{m^*}{m_h^*} \right)$$

if  $N_c = N_v$  or  $m^* = m_h^*$ , then

$$E_{Fi} = Ev + \frac{Eg}{2}$$

⇒ The intrinsic fermi level lies midway between valence and conduction band. which is required expression.

(b) A silicon sample is doped with  $10^{15}$  Antimony atoms/cm<sup>3</sup>.

Find carrier concentration, its resistance and the shift in fermi level from its intrinsic fermi level at 27°C. If it is further doped with  $2 \times 10^{17}$  boron atoms/cm<sup>3</sup>, what is new resistance?

⇒ since Si is doped with  $10^{15}$  antimony so

$$n \approx N_d = 10^{15} \text{ cm}^{-3}$$

intrinsic concentration

$$n_i = N_c \exp \left( \frac{-(E_c - E_{Fi})}{kT} \right) \quad \text{--- (1)}$$

doped concentration

$$n = N_d = N_c \exp \left( \frac{-(E_c - E_{Fn})}{kT} \right) \quad \text{--- (2)}$$

dividing ② by ①

$$\frac{N_d}{n_i} = \exp \left( -\frac{E_c + E_{F_n} + E_c - E_{F_i}}{kT} \right)$$

$$\frac{E_{F_n} - E_{F_i}}{kT} = \ln \left( \frac{N_d}{n_i} \right)$$

$$E_{F_n} - E_{F_i} = \ln \left( \frac{10^{15}}{1.95 \times 10^{10}} \right) \times 300 \times 1.38 \times 10^{-23}$$

$$E_{F_n} - E_{F_i} = 4.612 \times 10^{-20} \text{ eV}$$

Carrier concentration

$$n = 10^{15} \text{ cm}^{-3}$$

$$n_i = 1.45 \times 10^{10} \text{ cm}^{-3}$$

$$n_p = n_i^2$$

$$p = \left( \frac{1.45 \times 10^{10}}{10^{15}} \right)^2 = 2.1 \times 10^5 \text{ cm}^{-3}$$

$$n_e = 1350 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$$

$$\begin{aligned} \sigma &= n_e n_i e + n_p \mu_h \\ &= 1.6 \times 10^{19} (10^{15} \times 1350 + 2.1 \times 10^5 \times 450) \\ &= 0.216 \text{ mho} \cdot \text{cm}^{-1} \end{aligned}$$

$$\text{resistance } R = \frac{1}{\sigma} = 4.6296 \text{ ohm} \cdot \text{cm}^2$$

$$R = 4.6296 \text{ ohm}$$

Now sample is further doped with  $2 \times 10^{17}$  boron so

$$N_a = 2 \times 10^{17}$$

$$N_a > N_d$$

$$\begin{aligned} p &= N_a - N_d = 2 \times 10^{17} - 10^{15} \\ &= 1.99 \times 10^{17} \end{aligned}$$

$$n = \frac{h^2}{P}$$

$$\frac{(1.45 \times 10^{10})^2}{1.99 \times 10^{17}}$$

$$= 1056.53$$

$$\text{Now } \sigma = 1.6 \times 10^{-19} (1056.53 \times 1350 + 1.99 \times 10^{17} \times 450)$$

$$= 14.328 \text{ } \Omega^{-1} \text{ cm}^{-1}$$

$$g = \frac{1}{\sigma} = 0.06979 \text{ } \Omega \text{ cm}$$

$$g = \frac{RA}{l}$$

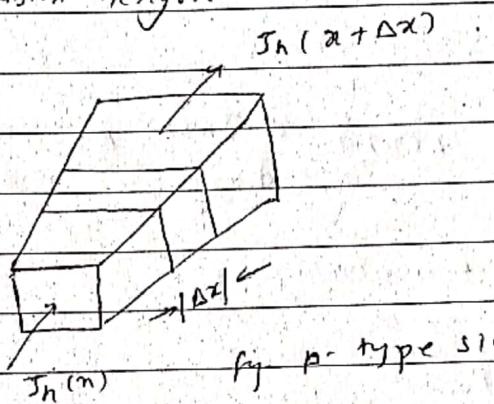
$R = g \text{ for } A = 1 \text{ cm}^2, l = 1 \text{ cm}$

$$R = 0.06979 \text{ } \Omega \text{ g}$$

6. (a) Define diffusion length and obtain the continuity equation for holes.



Diffusion length is defined as the length, a hole diffuse on average before it recombine with an  $e^-$ . In n-type it is also called minority carrier diffusion length.



Here the current density of  $h^+$  entering the sample can be smaller or larger than the current density of holes leaving the sample.

Depending on the generation and recombination taking place within the sample, the net increase in  $h^+$  concentration per unit time is the difference between  $h^+$  flow per unit volume entering and leaving.

rate of combination is given by

$$\frac{dAP}{dt} = g_{ph} - \frac{\Delta P}{T_h} \quad \text{--- (1)}$$

rate of increase in excess  $h^+$  concn = rate of photogeneration - rate of recombination.

This can be written as

$$\frac{dAP}{dt} = \frac{1}{c} \left[ \frac{J_h(m) - J_h(x+\Delta x)}{\Delta x} \right] - \frac{\Delta P}{T_h} \quad \text{--- (2)}$$

$\dot{G}_{px}$  is defined as increase in hole concn per unit volume ( $A \Delta x$ ) per unit time

$$\text{i.e. } \frac{p}{t} = \frac{p_x}{t_x} + \frac{e p_x}{e t_x} - \frac{e p v}{e x} = \frac{J}{e x}$$

$$J = e p v$$

using Taylor series

$$J_h(n+\Delta x) = J_h(n) + \frac{\partial J_h(n)}{\partial n} \Delta x$$

now excess hole concentration

$$\Delta p = p - p_0$$

$$\frac{\partial \Delta p}{\partial t} = \frac{\partial p}{\partial t}$$

from eq? (2) we can write

$$\frac{\partial \Delta p}{\partial t} = \frac{1}{e} \left[ J_h(n) - J_h(n) - \frac{\partial J_h(n)}{\partial x} \Delta x \right] - \frac{\Delta p}{T_h}$$

$$\Rightarrow \frac{\partial \Delta p}{\partial t} = - \frac{1}{e} \frac{\partial J_h(n)}{\partial x} - \frac{\Delta p}{T_h}$$

This is continuity equation for holes.

(b) With neat sketch, explain the Czochralski crystal growth technique to obtain the single crystal of semiconductor.

⇒

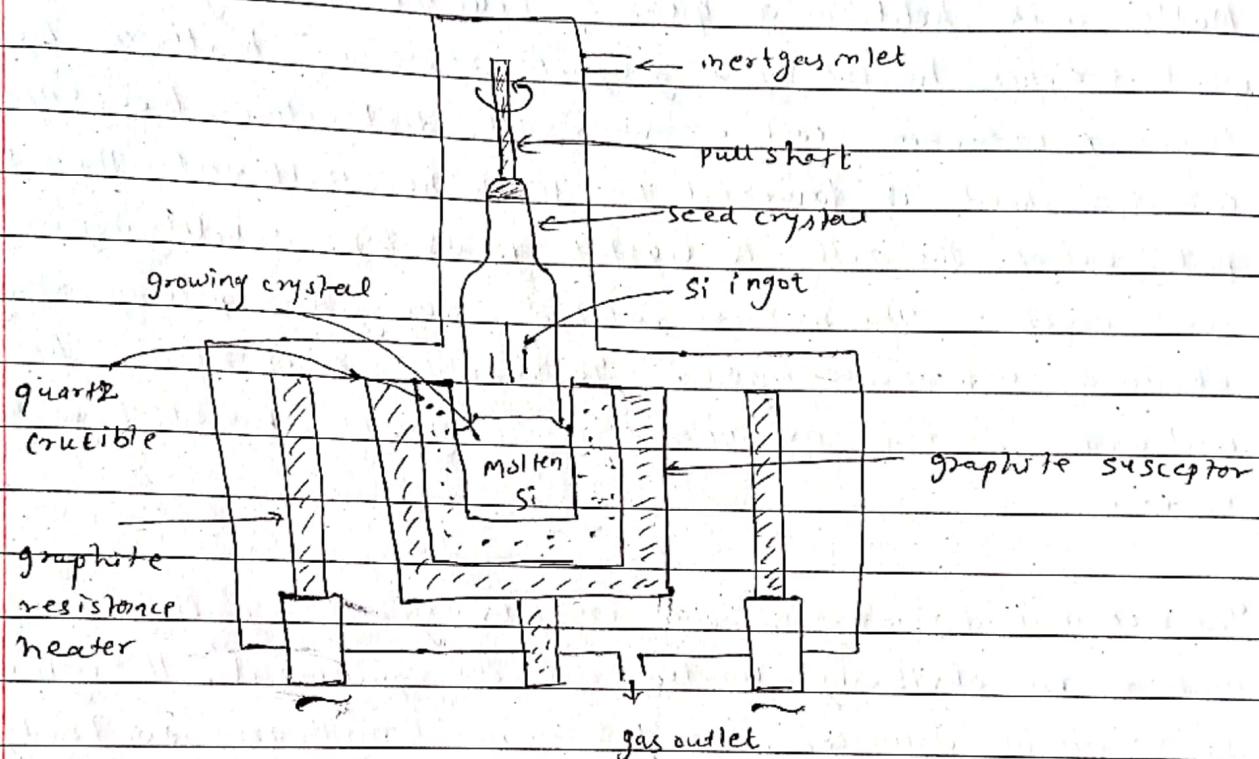


fig. growth of single crystal si ingot by the czochralski technique.

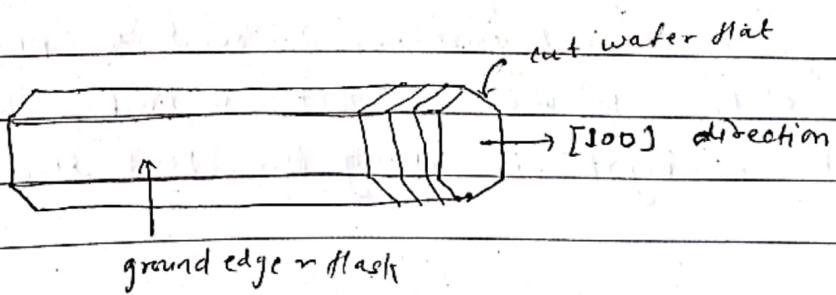


Fig. crystallographic orientation of the si ingot into marked by grounding a flat.

This method involves growing a single-crystal ingot from the melt using solidification on a seed crystal as shown above.

Molten Si is held in a quartz crucible in a graphite susceptor, which is either heated by a graphite resistance heater or by radio frequency induction coil. A small dislocation free crystal, called a seed is lowered to touch the melt and then slowly pulled out of the melt. A crystal grows by solidifying on the seed crystal. The seed is rotated during the pulling stage, to obtain a cylindrical ingot. To suppress evaporation from the melt and prevent oxidation, argon gas is passed through the system.

The size and diameters of the crystals grown by the Czochralski method are obviously limited by the equipment, though crystals 20-30 cm in diameter and 1-2 m in length are routinely grown for the IC fabrication industry. Also the crystal orientation of the seed and its flatness with melt surface are important engineering requirements.

For very large scale integration, the seed is placed with its (100) plane flats to the melt, so that the axis of the cylindrical ingot is along the [100] direction.

Q7. Write short notes.

(a) ion implantation

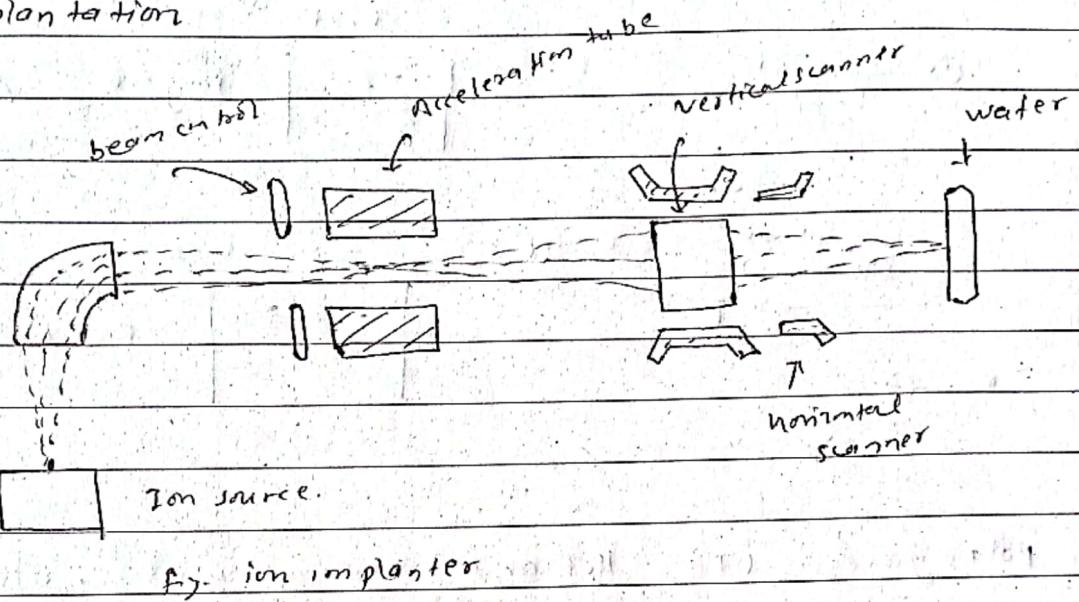
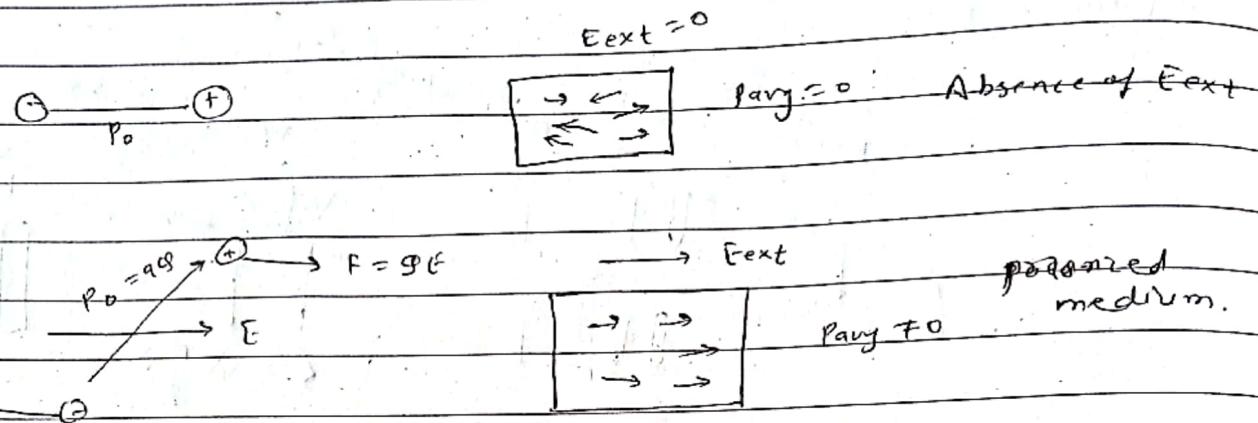


Fig. ion implanter

Ion implantation process has many advantages over diffusion for the production of impurity atoms into the Si wafer, so it is most widely used technology in modern IC fabrication.

An ion implanter is a high-voltage particle accelerator producing a high velocity beam of impurity ions, which can penetrate the surface of Si wafer.

## (b) orientational (or dipolar) polarization



polar gases like HCl and steam, water, alcohol, electrolytes, acetone etc have permanent dipole moments associated with them. Such medium in presence of external field exhibits a polarization called dipolar orientational polarization.

In absence of external field, all the dipoles are randomly oriented due to thermal agitation. Hence  $p_{avg} = 0$ .

But in presence of external field, dipoles will try to align its direction along external field experiencing some torque and also some dipoles will be lost due to molecular collision.

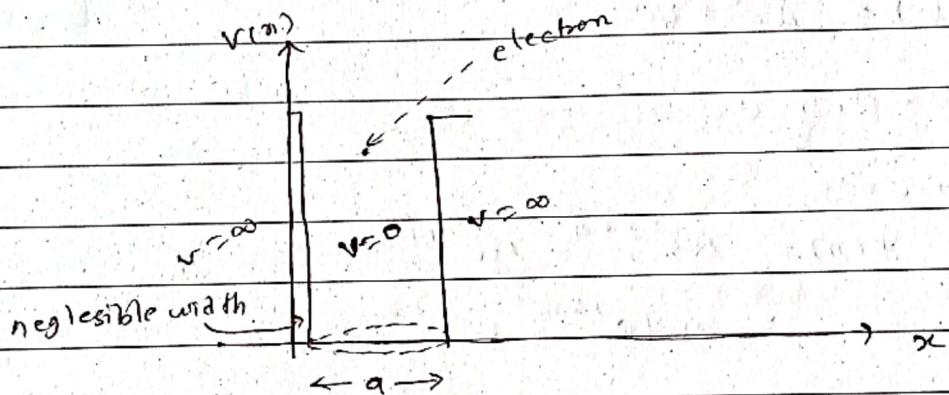
Set - 2. 2028 Spring

- 1.(a) Define Heisenberg uncertainty principle. Derive the expression for the energy of an electron confined to an infinite potential well.

Heisenberg uncertainty principle states that, "it is impossible to measure both the position and momentum of the microscopic particles at the same time."

Mathematically,

$$\Delta x \cdot \Delta p_x \geq \frac{\hbar}{2\pi}$$



Consider the behaviour of an  $e^-$  when it is confined to a certain region between 0 and  $a$ . Its potential energy is zero inside the region and infinite outside the region as shown in fig above. since the probability of finding of  $e^-$  outside the well is zero.

$$\psi = 0 \text{ when } x \leq 0 \text{ & } x \geq a$$

Schrodinger's wave equation describes the behaviour of  $e^-$ 's nature in independent of time and is given by

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} (E - V) \psi = 0 \quad \text{--- (1)}$$

Since the potential energy inside the well is zero i.e.  $V=0$   
so.

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{\hbar^2} E\psi = 0 \quad \text{--- (2)}$$

Its general soln is

$$\psi(x) = Ae^{jknx} + Be^{-jknx} \quad \text{--- (3)}$$

$$\psi(0) = 0$$

we get eq (3) as

$$\psi(0) = Ae^0 + Be^0$$

$$A = -B \text{ or } B = -A \quad \text{--- (4)}$$

we can write

$$\begin{aligned}\psi(x) &= Ae^{jknx} - Ae^{-jknx}, \\ &= A(e^{jknx} - e^{-jknx})\end{aligned}$$

$$\psi(x) = A(\cos knx + j \sin knx, - \cos knx + j \sin knx)$$

$$\psi(x) = 2Aj \sin knx \quad \text{--- (5)}$$

we have another boundary condition

$$\psi(a) = 0$$

(5) can be written as

$$0 = 2Aj \sin kna$$

$$\sin kna = 0$$

$$\sin kna = \sin n\pi$$

$$k = \frac{n\pi}{a} \quad \text{--- (6)}$$

now from (2)

$$\frac{d^2}{dn^2} (2A_j \sin kx) + \frac{8\pi^2 m}{h^2} E (2A_j \sin kx) = 0$$

$$⇒ -2A_j k^2 \sin kx + \frac{8\pi^2 m}{h^2} E (2A_j \sin kx) = 0$$

$$⇒ 2A_j \sin kx (-k^2 + \frac{8\pi^2 m}{h^2} E) = 0$$

$$-k^2 + \frac{8\pi^2 m}{h^2} E = 0$$

$$\frac{n^2 \pi^2}{a^2} = \frac{8\pi^2 m}{h^2} E$$

$$\boxed{E = \frac{n^2 h^2}{8 m a^2}} \quad \text{which is required expression.}$$

(b) Already done Set-1 2(a).

2. (a) Calculate the drift mobility and the mean scattering time of conduction electrons in copper at room temperature, given that the conductivity of copper is  $5.9 \times 10^5 \text{ S}^{-1} \text{ cm}^{-1}$ . The density of copper is  $8.96 \text{ gm cm}^{-3}$  and its atomic mass is  $63.5 \text{ gm mol}^{-1}$ . (Assume one free electron per atom).

Here,  $\sigma = 5.9 \times 10^5 \text{ S}^{-1} \text{ cm}^{-1}$

$\rho = 8.96 \text{ gm cm}^{-3}$   $n = \text{no. of e}^- \text{ per unit volume}$

$M_{at} = 63.5 \text{ gm mol}^{-1}$

$a = ?$

$T = ?$

$$= \frac{\sigma \times N_A}{M_{at}}$$

$$= \frac{8.96 \times 6.023 \times 10^{23}}{63.5}$$

$$= 8.498 \times 10^{22} \text{ cm}^{-3}$$

now when

$$\sigma = neu$$

$$u = \frac{\sigma}{ne} = \frac{5.9 \times 10^5}{8.998 \times 10^{22} + 1.6 \times 10^{-19}} = 43.3925 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1}$$

now,

$$u = \frac{ne^2 \tau}{m} \quad \text{for 1 free } e^- \text{ at mass} = m$$

$$\tau = \frac{om}{ne^2}$$

$$= \frac{m u}{e}$$

$$= \frac{63.5 \times 43.39}{1.6 \times 10^{-19}}$$

$$= 1.7 \times 10^{22}$$

(b) What do you mean by electrical breakdown in liquid?

Electric breakdown occurs when the gap pressure is close to atmospheric pressure. It is the condition when two terminals in liquid are conduct each other. We use Ar discharge in electrical breakdown to avoid breakdown.

Already done 2018 Fall B-(a).

Q3. (a) Define local field in relation to polarization.

- (i) Local field is the actual field that acts on a molecule and for a simple cube crystal structure or a liquid. It is given by Lorentz formula as

$$E_{loc} = E + \frac{P}{3\epsilon_0}$$

Done 2028 Fall 3 (b)

- (b) An electronic polarizability of Ar atom is  $1.7 \times 10^{-40} Fm^2$ .

Calculate the static dielectric constants of solid Ar given its density is  $1.8 \text{ g cm}^{-3}$ ,  $M_{at} = 39.5 \text{ g mol}^{-1}$  and  $N_A = 6.023 \times 10^{23} \text{ mol}^{-1}$

$$\alpha_e = 1.7 \times 10^{-40} F m^2$$

$$\delta = 1.8 \text{ g cm}^{-3} = \frac{1.8 \times 10^{-3}}{(10^{-2})^3} = \frac{1.8 \times 10^{-3}}{10^{-6}} = 1.8 \times 10^3 \text{ kg m}^{-3}$$

$$M_{at} = 39.5 \text{ g mol}^{-1}$$

$$= 39.5 \times 10^3 \text{ kg mol}^{-1}$$

$$N_A = 6.023 \times 10^{23} \text{ mol}^{-1}$$

$$k = ? \text{ or } \epsilon_r = ?$$

$$N = \frac{\delta \times N_A}{M_{at}}$$

$$= \frac{1.8 \times 6.023 \times 10^{23}}{39.5}$$

$$= 2.71 \times 10^{22} \text{ cm}^{-3}$$

$$= 2.71 \times 10^{-28} \text{ m}^{-3}$$

Now using Clausius - Massotti eqn

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N \Phi_e}{3 \epsilon_0}$$

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{2.71 \times 10^{28} \times 1.7 \times 10^{-90}}{3 \times 8.85 \times 10^{-12}}$$

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = 1.735 \times 10^{-30}$$

$$\epsilon_r - 1 = 1.735 \times 10^{-30} \epsilon_r + 2 \times 1.735 \times 10^{-30}$$

$$\epsilon_r - 0.1735 \epsilon_r = 0.347 + 1$$

$$\epsilon_r = \frac{0.347 + 1}{1 - 0.1735}$$

$$\epsilon_r = 1.63$$

$$\kappa = \epsilon_r = 1.63$$

Q. 9 (a) What do you mean by magnetic dipole moments? Explain the different types of magnetic materials according to the magnetic behavior.

→ A magnetic dipole moment is a quantity that represents the magnetic strength and orientation of magnet or any other object that produces a magnetic field.  
simply it is a current carrying loop represented by  $\mu_m$ .

$$\mu_m = NIA$$



Different types of magnetic materials:-

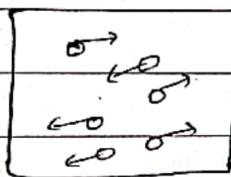
### 1. Diamagnetic materials:

Materials with negative susceptibility are termed as diamagnetic substance. Their relative permeability is less than unity. When diamagnetic materials are placed in magnetic field, the magnetization vector  $m$  is in opposite direction to the applied field. This causes magnetic field within the material to be less than the applied field.

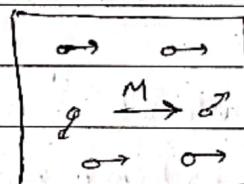
e.g. air,  $H_2O$ ,  $Bi$  etc.

### 2. Paramagnetic materials:-

It has small positive magnetic susceptibility. In absence of mag. field the average mag. dipole moment and net magnetization both are zero.



$$\mu_{av} = 0, M = 0$$

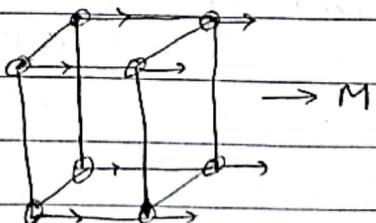


$$\mu_{av} \neq 0, M \neq 0$$

e.g. liquid  $O_2$ ,  $Al$ ,  $Pt$  etc.

### 3. Ferromagnetic materials.

It have positive magnetic susceptibility ( $\chi_m$ ) and posses large permanent magnetization even in the absence of applied external magnetic field.



For ferromagnetic materials, the relationship between magnetization ( $M$ ) and external magnetizing field ( $H$ ) is highly non linear and saturates at sufficiently large external field.  
e.g. Fe, Co, Ni etc.

#### 7. Anti-ferro magnetic material

It have positive susceptibility but the net magnetization of the material is zero in the absence of external magnetic field. Such materials posses magnetic ordering in which alternate atoms in crystal aligned in opposite direction.

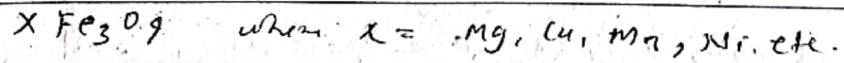
Antiferromagnetism occurs below a critical temp. called Neel temp and above this temp. material is paramagnetism.

e.g. MnO, FeO, MnO<sub>2</sub>, Cr etc.

#### 5. Ferrimagnetic material

These materials exhibit magnetic behavior similar to ferromagnets below a critical or curie temp. Above it they are paramagnetic. These are used for high frequency electronic applications and also they are non-conducting so doesn't suffer from eddy current losses.

These materials are generally called ferrites and are represented by



(b) Find the percentage increase in the magnetic field  $B$ , when the space within current carrying coil is fixed with aluminium. The susceptibility of aluminium is  $2.132 \times 10^{-5}$ .

$\Rightarrow H_m \chi_m = 2.132 \times 10^{-5}$

Initial mag. field ( $B_0$ ) =  $\mu_0 H$

After filled with aluminium ( $B$ ) =  $\mu_0(H + M)$

$$= \mu_0(1 + \chi_m) H$$

change in magnetic field =  $B - B_0$

$$= \mu_0(1 + \chi_m) H - \mu_0 H$$

$$= \mu_0 H (1 + \chi_m - 1)$$

$$= \mu_0 H \chi_m$$

Percent change in magnetic field =  $\frac{B - B_0}{B_0} \times 100\%$

$$= \frac{\mu_0 H \chi_m}{\mu_0 H} \times 100\%$$

$$= \chi_m \times 100\%$$

$$= 2.132 \times 10^{-5} \times 100\%$$

$$= 2.132 \times 10^{-3}\%$$

HZ

Q.5 (a) what are the types of semiconductor? prove that the position of fermi level is near the middle of band gap in pure silicon sic.

$\Rightarrow$  Types of sic.

1. p-type sic.

It has holes as majority charge carriers. when pure sic is doped with trivalent impurities like B, Al it become p type sic.

2. n-type sic.

It has electron as majority charge carriers. when pure sic is doped with pentavalent impurities like As, Sb it become n-type sic.

Done Already 2018 Fall 5(9)

(b) A heavily doped p-type side with acceptor concentration of  $10^{18} \text{ cm}^{-3}$  is connected to n-side with donor concentration  $10^{16} \text{ cm}^{-3}$ . calculate the built in potential, depletion width in n-side and p-side and overall depletion width. the intrinsic concentration is  $1.95 \times 10^{10} \text{ cm}^{-3}$  and  $T = 300 \text{ K}$ .

$\Rightarrow$  Here,

$$N_A = 10^{18} \text{ cm}^{-3}$$

$$N_D = 10^{16} \text{ cm}^{-3}$$

$$n_i = 1.95 \times 10^{10} \text{ cm}^{-3}$$

$$T = 300 \text{ K}$$

Built in potential,  $V_0 = \frac{kT}{e} \ln \left( \frac{N_a N_d}{n_i^2} \right)$

$$= \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \ln \left[ \frac{10^{18} \cdot 10^{16}}{(1.95 \times 10^{10})^2} \right]$$

$$\Rightarrow V_0 = 0.815 \text{ V}$$

Depletion width (overall)  $w_0 = \sqrt{\frac{2 \epsilon V_0}{e} \left( \frac{1}{N_a} + \frac{1}{N_d} \right)}$

$$= \sqrt{\frac{2 \times 11.9 \times 8.85 \times 10^{-12}}{1.6 \times 10^{-19}} \left( \frac{1}{10^{24}} + \frac{1}{10^{22}} \right)}$$

$$= 3.64 \times 10^{-7} \text{ m.}$$

Depletion width in n-side  $w_n = \frac{w_0 N_a}{N_a + N_d}$

$$\frac{3.64 \times 10^{-7} \times 10^{29}}{10^{29} + 10^{22}}$$

$$= 3.6 \times 10^{-7} \text{ m}$$

Depletion width in p-side  $w_p = \frac{w_0 N_d}{N_a + N_d}$

$$\therefore w_0 - w_n$$

$$w_p = 3.64 \times 10^{-7} - 3.6 \times 10^{-7}$$

$$= 4 \times 10^{-9} \text{ m}$$

Ans

Q.6 (a) what do you mean by diffusion coefficient? Define the expression for the Einstein's relationship between mobility and diffusion coefficient.

Diffusion coefficient is the measure of how rapidly the particle diffused in the medium. For most medium diffusion coefficient is found to be vary with temp in an activated manner and given by

$$D = D_0 \exp\left(-\frac{\phi}{kT}\right).$$

Einstein's relationship:-

The diffusion coefficient for free electron in one dimension and along  $x$ -direction is given by

$$D_e = \frac{d^2}{t} \quad \text{--- (1)}$$

Let  $d$  be the mean free path and  $\tau$  be mean free time then

$$d = v_n \tau \quad \text{--- (2)}$$

(1) becomes

$$D_e = v_n^2 \tau \quad \text{--- (3)}$$

so Energy of free  $e^-$  in conduction band is  $\frac{1}{2} kT$  and this energy is equal to KE of the  $e^-$   
so we can write

$$\frac{1}{2} m v_n^2 = \frac{1}{2} kT \quad \text{--- (4)}$$

$$v_n^2 = \frac{kT}{m_e}$$

Now (3) becomes

$$D_e = \frac{kT}{me} \tau$$

$$= \frac{kT \cdot e\tau}{eme}$$

$$= \frac{kT}{e} m_e$$

$$D_e = \frac{kT}{e} m_e \quad \text{--- (3)} \quad m_e = \frac{e\tau}{me}$$

which is required expression for Einstein's relationship between diffusion coefficient and mobility.

- (b) Why silicon is preferred for semiconducting materials than Germanium? Explain the floating zone process of crystal growth with necessary figure.

→ Silicon is abundant on the earth's surface and therefore cheaper than germanium. The peak inverse voltage rating of silicon is much higher than germanium and therefore it can withstand much higher temperatures than germanium. Hence we preferred silicon for manufacturing semiconducting materials.

Floating zone process of crystal growth:

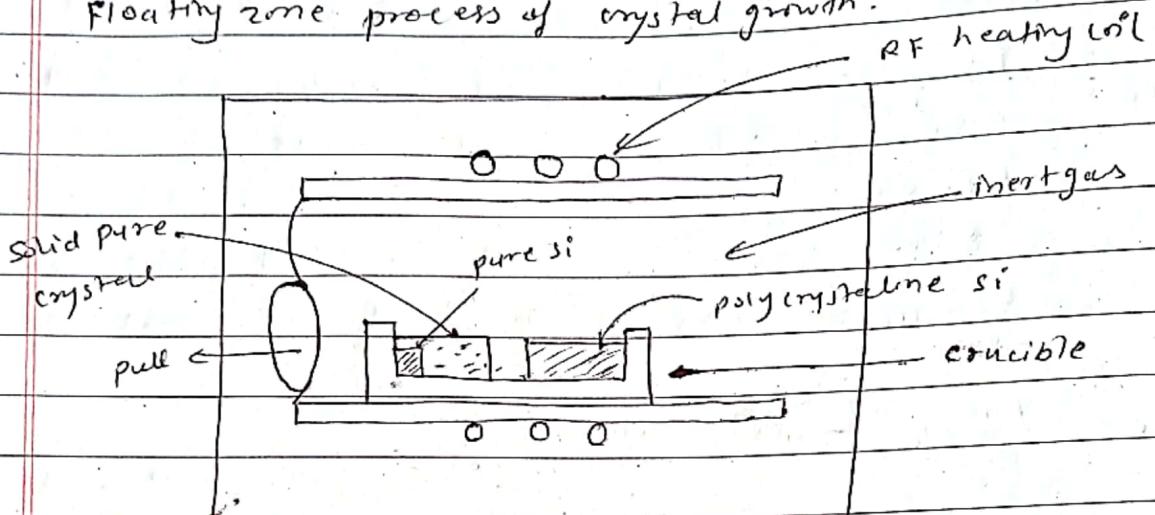


fig. Floating zone method for crystal growth.

This method,

in addition to being capable of producing single crystals and also removing removing the extra impurities contained in the SiC material. The impurities can be removed very easily as they prefer to go to the liquid phase.

This type of crystal growth is rarely used because of the high temp requirement of Si. There is always possibility of Si crystals adhering to their crucible wall which causes deviation from the perfect lattice structure required for Si crystal.

The RF heating coil heats the crucible in which the polycrystalline Si is placed. As the Si is heated, it melts at high temp. and by solidification by slowly pulling it to the left where there is seed crystal, single pure crystal for IC fabrication is obtained. The inert gas is used to suppress the evaporation and prevent the oxidation.

Q.7. Write short notes on:

(a) Free electron theory of metal

According to this theory, the outermost electrons, known as valence  $e^-$  of an atom in metals are very loosely attached to the centre of the atom, so they are not bound to the parent atom. These  $e^-$ s are free to move throughout the whole metal. These  $e^-$ s are attached to different atoms at different instants.

These freely roaming  $e^-$ s within the metal forms an electron gas. When an atom loses its valence  $e^-$ , it becomes a positively charged ion. So the metallic bond is an unsaturated covalent bond due to the electrostatic attraction between the positively charged ions and -vely charged  $e^-$ s cloud.

The electron has kinetic energy.

$$E = \frac{1}{2} mv^2$$

$$E = \frac{\hbar^2 k^2}{2m}$$

### (b) photolithography

photolithography is a process used in micro fabrication to selectively remove parts of a thin film or the bulk of a substrate.

It uses light to transfer a geometric pattern from a photomask to a light sensitive chemical photoresists or simply resist on substrate.

photolithography shares same fundamental processes with photography

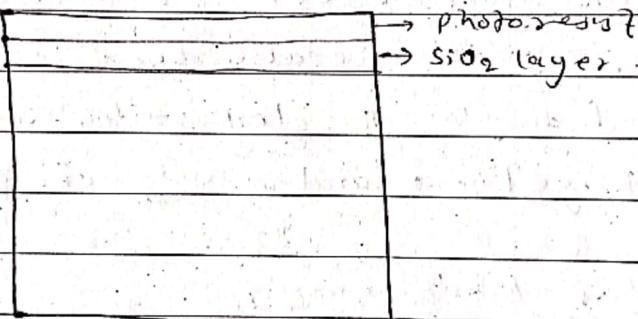


fig. A single crystal of Si