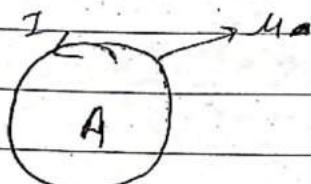


Magnetic Materials

*

Magnetic dipole moment.

Magnetic dipole moment is simply a current carrying loop represented by μ_m .



$$\mu_m = IA$$

If N = no. of turns on,

$$\mu_m = NIA$$

*

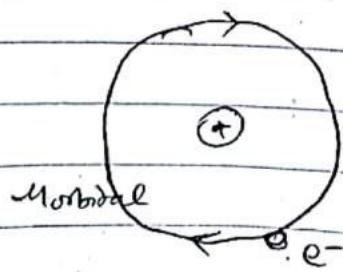
Atomic magnetic moment

An orbiting electron in an atom behaves much like a current carrying loop and has a magnetic dipole moment associated with it called orbital magnetic moment (μ_{orbital}). Also the e⁻ has spin with intrinsic angular momentum called spin magnetic moment (μ_{spin}).

The overall magnetic moment of e⁻ consists of both μ_{orbital} and μ_{spin} .

∴ Overall magnetic moment is given by

$$\vec{\mu}_{\text{atom}} = \vec{\mu}_{\text{orbital}} + \vec{\mu}_{\text{spin}}$$



* Magnetization (M)

The magnetization of a magnetic material is defined as total magnetic dipole moments per unit volume.

$$\text{ie. } M = \frac{\sum_{i=1}^n m_i}{V}$$

m_i = magnetic dipole moment
 V = volume (cm³)

* Magnetic permeability and Susceptibility

$$M = \frac{B}{H}$$

B ← magnetic field ~~intensity~~
 H ← magnetic field intensity, magnetizing field

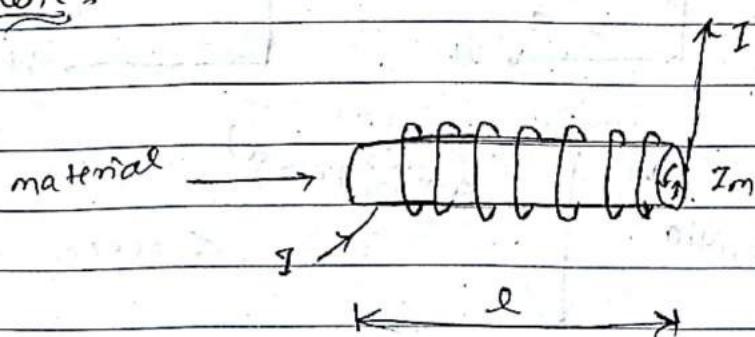
$$\Rightarrow M = 4\pi \cdot 10^{-7} \text{ Vs/A}$$

Relative permeability of a medium is the fractional increase in the magnetic field with respect to the magnetic field in free space after the introduction of the medium. And is given by

$$\Rightarrow \mu_r = \frac{B}{B_0} = \frac{B}{\mu_0 H}$$

$$B_p = \mu_0 \mu_r H$$

Note:



Before introducing material $B_0 = \mu_0 H$

After introducing material $B = B_0 + \mu_0 M$

$$= \mu_0 H + \mu_0 M$$

\uparrow magnetization

where $\chi_m = \frac{M}{H} \Rightarrow B = \mu_0 (H + \chi_m H)$

susceptibility $\Rightarrow B = \mu_0 (H + \chi_m H)$
 $= \mu_0 (1 + \chi_m) H$

$$\boxed{B = \mu_0 \mu_r H}$$

where,

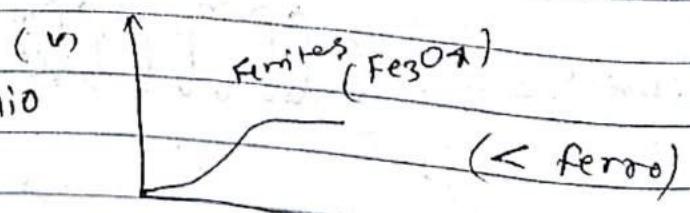
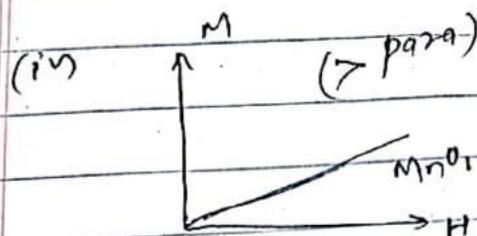
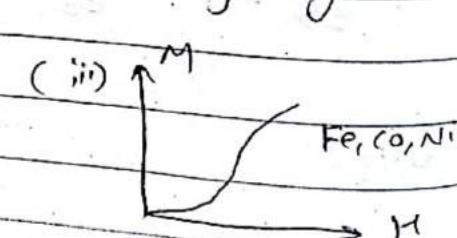
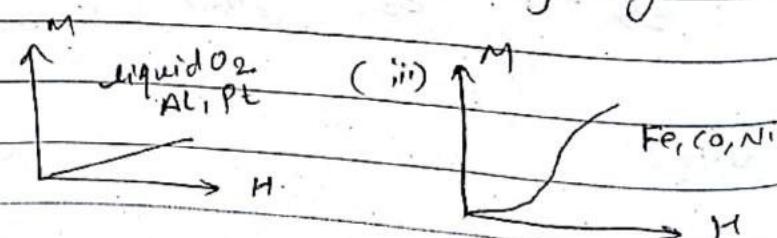
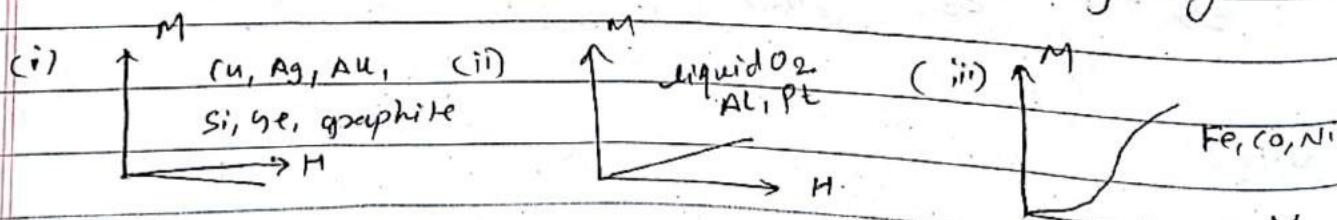
$$\boxed{\mu_r = 1 + \chi_m}$$

* Classification

Permanent mag. dipoles

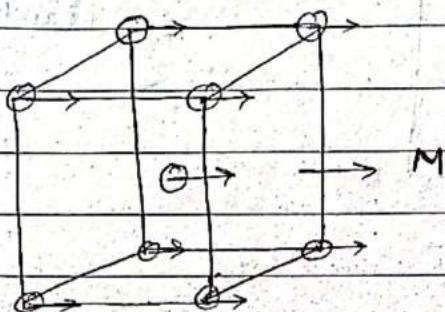
χ_m

i)	Diamagnetic	Absent	none and small ($\approx 10^{-6}$)
ii)	paramagnetic	Present	+ve and small (10^{-5} to 10^{-4})
iii)	Ferro-magnetic	present	+ve and very large.
iv)	Anti-ferro	present	+ve and small [\gg iii)]
v)	Ferr	present	+ve and very large.



(*) Ferromagnetism

Fe

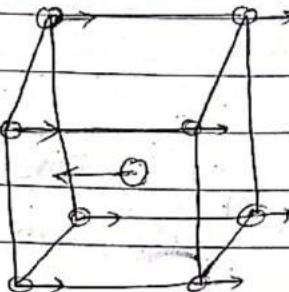
 $B \rightarrow$ developed $H \rightarrow$ applied

Ferromagnetic crystal
with aligned magnetic
moments.

Ferromagnets have positive magnetic susceptibility (χ_m) and possess large permanent magnetization even in the absence of applied external magnetic field. For ferromagnetic materials, the relationship betⁿ magnetization (M) and H ^(ext.) magnetizing field is highly non linear and saturates at sufficiently large external field.

In ferromagnetic crystals magnetic moments are aligned in particular direction, which gives rise to one net magnetization. Below the critical temp. or curie temperature, ferromagnetism occurs but at temp. greater than T_c , ferromagnetism is lost and becomes paramagnetic.

(ii)

Anti-ferro

BCC structure.

Antiferro crystal of chromium (r).

Antiferro magnetic materials have positive susceptibility, but the net magnetization of the material is zero in the absence of external magnetic field. Such materials possess magnetic ordering in which alternate atoms in crystal aligned in opposite direction.

Antiferro magnetism occurs below a critical temp. called Neel temperature and above this temp. the material is paramagnetic. These materials when subjected to external mag. field gets feebly magnetized. Ex. MnO, FeO, Mn₂O₃, Cr etc.

(iii) Ferri magnetic

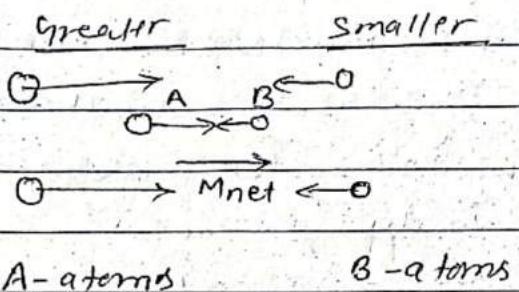


Fig. Magnetic ordering in ferri-magnetic crystals like Fe_3O_4 .

These materials exhibit magnetic behaviors similar to ferromagnetics below a critical or curie temperature. Above T_c they are paramagnetic. As shown in the figure, the mag. moment of atom A is greater than that of atom B. Thus the net magnetization will be in the direction of atom A even in the absence of external magnetic field. These are used for high frequency electronic applications and also they are non-conducting. So, doesn't suffers from eddy current losses.

These materials are generally called ferrides and is represented by XFe_3O_4 . Where $\text{X} = \text{Mg, Cu, Mn, Ni, Zn, Cd}$ etc.

(iv)

Paramagnetism

Paramagnetic materials have small $\mu_0 M$ mag. susceptibility. In the absence of mag. field the average magnetic dipole moment μ_{av} and net magnetization both are zero. But in the presence of ext. mag. field μ_{av} is not equals to zero and depends upon external magnetizing field (H).

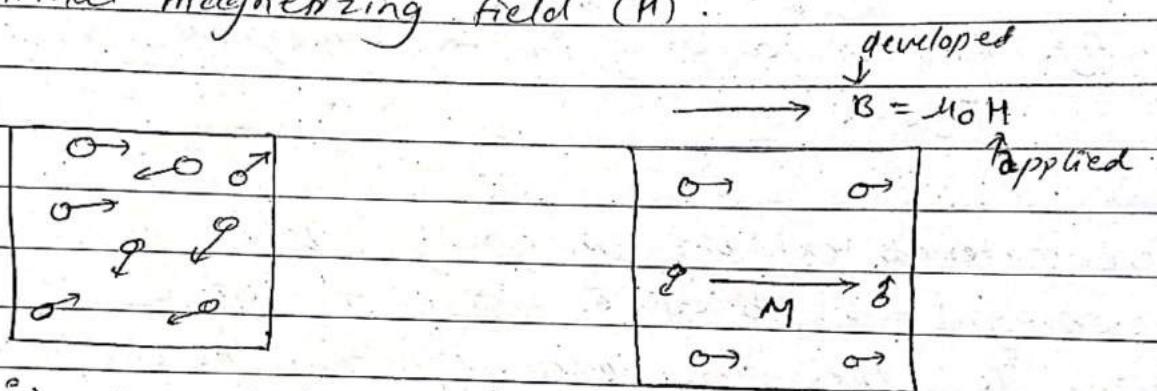
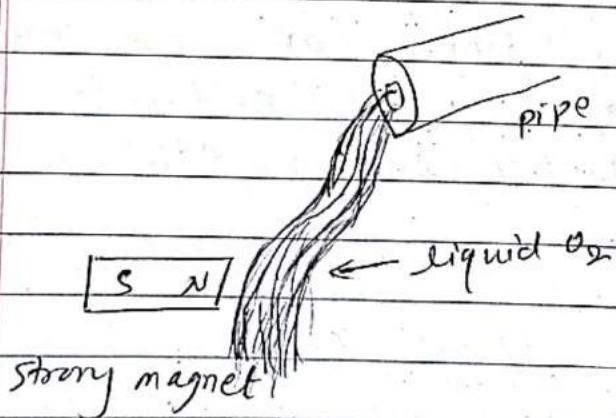


fig.i) $\mu_{av} = 0, M = 0$

fig.ii) $\mu_{av} \neq 0, M \neq 0$

$$(M = x\mu H)$$



Liquid oxygen is paramagnetic in nature. So, it will get attracted to ~~the~~ towards the magnet, when posed from the pipe.

At higher temperature, even in the presence of external mag. field, the molecular collision destroys the molecular magnetic moments. It destroy ~~or~~ decrease the magnetization value.

- Q. Calculate the current necessary to produce magnetic flux of 5×10^{-4} wb. in a soft iron ring of cross sectional diameter of 8 cm, mean circumference 200 cm with 700 turns of wire and relative permeability of 1800.

^{SOL}

$$\phi = 5 \times 10^{-4} \text{ wb.}$$

$$d = 8 \text{ cm}$$

$$A = \frac{\pi d^2}{4}$$

$$l = 200 \text{ cm}$$

$$B = \frac{\phi}{A}$$

$$\mu_r = 1800$$

$$\phi = \mu_0 B A$$

$$N = 700$$

$$= 5.02 \times 10^{-3} \text{ m}^2$$

$$\text{since, } B = \frac{\phi}{A} = \frac{5 \times 10^{-4}}{5.02 \times 10^{-3}} = 0.0996 \text{ wb/m}^2.$$

Also, $B = \mu_0 \mu_r H$

0.22 A

$$\mu_0 \mu_r n I = \mu_0 \mu_r H$$

$$H = \frac{B}{\mu_0 \mu_r}$$

$$H = n I$$

$$\phi H = \frac{N}{l} I$$

$$I = \frac{H l}{N} = \frac{0.0996 \times 200 \times 10^{-2}}{700 \times 4 \pi \times 10^{-7} \times 1800} = 0.22 \text{ Amp.}$$

- Q. Calculate the permeability & susceptibility of iron bar of cross-sectional area 0.2 cm^2 . When a magnetizing field of 1200 A/m produces a magnetic flux of $2.9 \mu\text{wb}$.

$$\cancel{\mu_0} = ?$$

$$\chi_m = ?$$

$$A = 0.2 \times 10^{-4} \text{ m}^2$$

$$H = 1200 \text{ A/m}$$

$$\phi = 2.9 \times 10^{-6} \text{ web.}$$

$$B = \frac{\phi}{A} = \frac{2.9 \times 10^{-6}}{0.2 \times 10^{-4}} = 1.2 \text{ web/m}^2$$

$$B = \mu_0 \mu_r H$$

$$\mu_r = \frac{B}{\mu_0 H} = \frac{1.2}{4\pi \times 10^{-7} \times 1200} = 795.77$$

$$\mu_m = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 795.77 = 1 \times 10^{-3}$$

~~$\mu_r = 1 + \chi_m$~~

$$\chi_m = 794.77$$

* Hard and soft magnetic materials.

B ← developed magnetic field

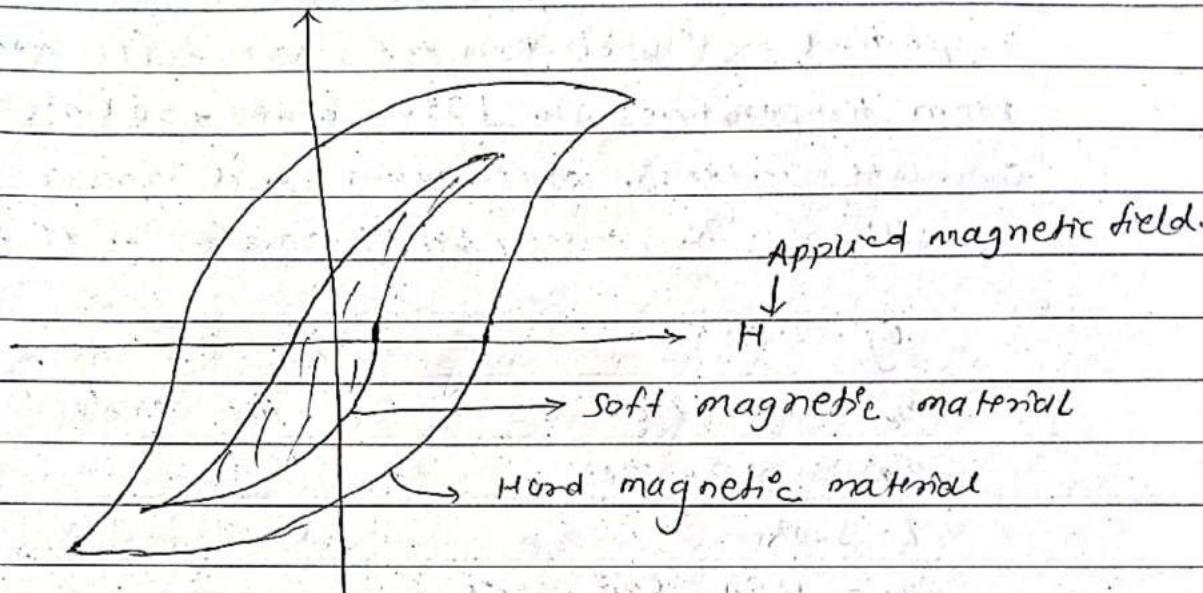


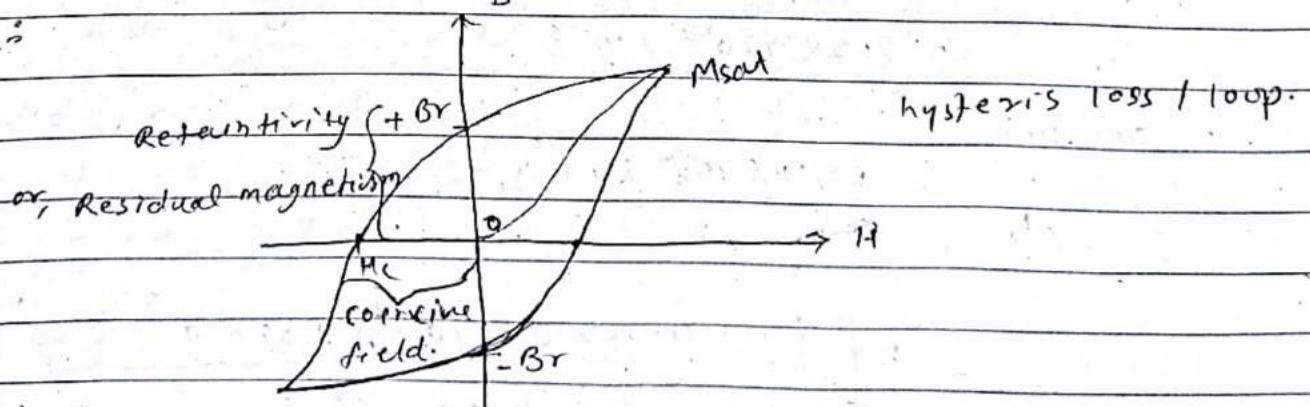
Fig. Hysteresis loop or B-H curve.

soft :- Easy to magnetize and demagnetize, used to make temporary magnets,

- small hysteresis loop / loss or B-H curve. e.g. Si-Fe, supermalloy etc

Hard :- Difficult to magnetize & demagnetize, used to make permanent magnets, large hysteresis loop / loss or B-H curve.
e.g. Alnico, Sm₂Co₁₇ etc.

Note:



Numericals

1. The density of states related effective masses of e⁻ and h⁺ in Si are approximately 1.08 me and 0.56 me respectively. The electron and hole drifts mobilities at room temperature are 1350 and 450 cm²V⁻¹s⁻¹ resp. Calculate intrinsic concentration and intrinsic resistivity of silicon. The energy band gap for Si is 1.1 eV.

S.Q.

$$E_g = 1.1 \text{ eV}$$

$$m_e^* = 1.08 \text{ me}$$

$$m_h^* = 0.56 \text{ me}$$

$$T = 300 \text{ K}$$

$$\mu_e = 1350 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$$

$$\mu_h = 450 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$$

$$n_i = ?$$

$$S = ?$$

we have,

$$N_c = 2 \left(\frac{2\pi m_e^* k T}{h^2} \right)^{3/2}$$

$$= 2 \left(\frac{2\pi \times 1.08 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(6.624 \times 10^{-34})^2} \right)^{3/2}$$

$$= 2.54 \times 10^{25} \text{ m}^{-3}$$

$$= 2.54 \times 10^{19} \text{ cm}^{-3}$$

similarly,

$$N_v = 2 \left(\frac{2\pi m_h^* k T}{h^2} \right)^{3/2}$$

$$= 2 \left[\frac{2\pi \times 0.56 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(6.624 \times 10^{-34})^2} \right]^{3/2}$$

$$= 1.05 \times 10^{25} \text{ cm}^{-3}$$

$$= 1.05 \times 10^{19} \text{ cm}^{-3}$$

Intensive concentration, $n_i = (\lambda e N_A)^{1/2} \left(-\frac{E_g}{kT} \right)$

$$= (2.5 \times 10^{25} \times 1.05 \times 10^{25})^{1/2} \left[\frac{-9.1 \times 1.6 \times 10^{19}}{1.38 \times 10^{-23} \times 300} \right]$$

$$= 9.58 \times 10^{15} \text{ m}^{-3}$$

and resistivity

$$\sigma = \frac{1}{S}$$

$$= \frac{1}{ne\mu + p\mu_h}$$

for intensive,

$$n = p = n_i$$

so

$$\sigma_i = e n_i (ne + n_h)$$

$$= 1.6 \times 10^{-31} \times 9.58 \times 10^{15} \times (1350 + 950)$$

$$= 3.62 \times 10^5 \text{ S.cm.}$$

A.D.

2.

Calculate the resistance of pure silicon cubic crystal of 1 cm^3 at room temperature. What will be the resistance of the cube when it is doped with 1 arsenic in 10^9 silicon atoms and 1 boron atom per billion silicon atoms? Atomic concentration of Si is $5 \times 10^{22} \text{ cm}^{-3}$, $n_i = 1.45 \times 10^{10} \text{ cm}^{-3}$. Assume the mobility as in the previous example.

$$V = 1\text{ cm}^3$$

$$N_{\text{Si}} = 5 \times 10^{22} \text{ cm}^{-3}$$

$$n_i = 1.45 \times 10^{10} \text{ cm}^{-3}$$

case I. 1 As \rightarrow in 10^9 Si

$$\text{i.e. } 10^9 \text{ Si} = 1 \text{ As}$$

$$5 \times 10^{22} \text{ Si} = \frac{1}{10^9} \times 5 \times 10^{22} \text{ As}$$

$$= 5 \times 10^{13} \text{ cm}^{-3} \text{ As.}$$

Hence As is donor so

$$\text{concn. of donor } N_d \approx n = 5 \times 10^{13} \text{ cm}^{-3}$$

and hole concentration,

$$\mu = \frac{n_i^2}{N_d} = \frac{(1.45 \times 10^{10})^2}{5 \times 10^{13}}$$

$$= 4.2 \times 10^6 \text{ cm}^{-3}$$

Initially for pure Si,

$$n = p = n_i$$

$$R = \frac{8l}{A} = \frac{l}{8A}$$

$$= \frac{l}{e n_i (m_e + m_h)}$$

$$= \frac{l}{1.6 \times 10^{-19} \times 1.95 \times 10^{10} (1350 + 450)} \\ = 2.39 \times 10^5 \Omega$$

After addition of Si,

$$R = \frac{l}{6A} = \frac{l}{e n_i m_e + e p_i m_h}$$

$$= \frac{l}{1.6 \times 10^{-19} [5 \times 10^{13} \times 1350 + 4.2 \times 10^6 \times 450]}$$

$$= 92.6 \Omega.$$

Case II.

Similarly, sample is doped with 1 B per billion Si atoms.

$$\text{donor acceptor and, } p \approx n_d = \frac{5 \times 10^{22}}{10^9} = 5 \times 10^{13} \text{ cm}^{-3}$$

$$\text{and } n = 4.2 \times 10^6 \text{ cm}^{-3}$$

and resistance,

$$R = \frac{l}{1.6 \times 10^{-19} [4.2 \times 10^6 \times 1350 + 5 \times 10^{13} \times 450]}$$

$$= 277.7 \Omega.$$

A

3. An n-type SiC doped with 10^{16} cm^{-3} phosphorous atoms has been doped with 10^{17} cm^{-3} boron atoms. Calculate the e-s and h-s conc' in the SiC.

Sol: Here, n-type SiC doped with 10^{16} cm^{-3} ph atoms
so no of donors $N_d \approx n = 10^{16} \text{ cm}^{-3}$

which is further doped with 10^{17} cm^{-3} boron
ie. $N_a = 10^{17} \text{ cm}^{-3}$.

so, resulting conc' = $n_a' = n_a - N_d = 10^{17} - 10^{16}$
 $= 9 \times 10^{16} \text{ cm}^{-3}$

Hence,

$$h^+ \text{ conc}' , p = N_a' = 9 \times 10^{16} \text{ cm}^{-3}$$

$$\text{and } e^- \text{ conc}' \\ \eta = \frac{n}{p}$$

$$= \frac{(1.45 \times 10^{10})^2}{9 \times 10^{16}}$$

$$= 2.34 \times 10^8 \text{ cm}^{-3}.$$

4. An n-type Si wafer is uniformly doped with 10^{16} antimony atoms per cm^3 . Where will be the Fermi level compared to its intrinsic Fermi level?

Sol: Here, Si is doped with 10^{16} antimony atoms
 $\eta = N_d = 10^{16} \text{ cm}^{-3}$.

and we have

$$n = n_i \exp \left[-\frac{E_c - E_{F_0}}{kT} \right] \quad \textcircled{1}$$

Before doping, intrinsic concentration.

$$n_i = n_i \exp \left(-\frac{E_c - E_{F_i}}{kT_i} \right) \quad \textcircled{2}$$

solving \textcircled{1} & \textcircled{2}

$$\frac{n}{n_i} = \frac{n_i}{n_i} \exp \left[-\frac{E_c - E_{F_0}}{kT_i} + \frac{E_c - E_{F_i}}{kT_i} \right]$$

$$\text{or } \ln \left(\frac{n}{n_i} \right) = \exp \left(\frac{E_{F_0} - E_{F_i}}{kT_i} \right)$$

$$E_{F_0} - E_{F_i} = \ln \left(\frac{n}{n_i} \right) kT_i$$

$$= \ln \left(\frac{10^{16}}{1.95 \times 10^{10}} \right) \times 1.38 \times 10^{-23} \times 300$$

$$= 5.56 \times 10^{-20} \text{ J}$$

Continue second part.

Where will the Fermi level be shifted if the sample is further doped with 2×10^{17} boron atoms per cm^{-3} ?

Hence

$$N_A = 2 \times 10^{17} \text{ cm}^{-3}$$

Acceptor concentration is high, so

$$P = N_A - N_D = 2 \times 10^{17} - 10^{16} = 19 \times 10^{16} \text{ cm}^{-3}$$

we have for holes concentration

$$P = Nv \exp \left[-\frac{E_F - E_V}{kT} \right] \quad \text{--- (3)}$$

for intensity,

$$n_i^p = Nv \exp \left[-\frac{E_{F_i} - E_V}{kT} \right] \quad \text{--- (4)}$$

solving (3) & (4)

$$\frac{P}{n_i} = \exp \left(-\frac{E_F - E_V}{kT} + \frac{E_{F_i}}{kT} + \frac{E_F - E_{F_i}}{kT} + \frac{E_V}{kT} \right)$$

$$\text{or, } \frac{P}{n_i} = \exp \left(-\frac{E_F - E_{F_i}}{kT} \right)$$

$$\text{or, } E_F - E_{F_i} = -kT \ln \left(\frac{P}{n_i} \right)$$

$$= -1.38 \times 10^{-23} \times 300 \times \ln \left(\frac{1.9 \times 10^{16}}{1.95 \times 10^{10}} \right)$$

$$= -6.78 \times 10^{-20} \text{ J}$$

5. A silicon sample has been doped with 10^{17} arsenic atoms per cm^3 . calculate the conductivity of the sample at 300 K and 400 K.

$$n_e \text{ at } 300\text{K} = 800 \text{ cm}^2 \text{ V}^{-1} \text{s}^{-1}$$

$$n_e \text{ at } 400\text{K} = 420 \text{ cm}^2 \text{ V}^{-1} \text{s}^{-1}$$

Here,

$$n_d \approx n = 10^{17} \text{ cm}^{-3}$$

thus

$$\sigma_{300\text{K}} = 10^{17} \times 1.6 \times 10^{-19} \times 800 \quad \sigma = \eta e u e \\ = 12.8 \text{ S/cm}$$

$$\text{and} \quad \sigma_{400\text{K}} = 10^{17} \times 1.6 \times 10^{-19} \times 420 \\ = 6.72 \text{ S/cm.}$$

6. What happens to conductivity if the sample in (5) is further doped with 9×10^{16} boron atoms cm^{-3} ?

initially, sample is n-type with $n = 10^{17} \text{ cm}^{-3}$

now, 9×10^{16} acceptor are added

so,

$$\text{new concn of n-type sc. is} \Rightarrow 10^{17} - 9 \times 10^{16} \\ = 10^{16} \text{ cm}^{-3}.$$

and similarly,

$$\sigma_{300\text{K}} = 10^{16} \times 1.6 \times 10^{-19} \times 800 \quad \left[\because \text{still using same mobility as in (5)} \right] \\ = 1.28 \text{ S/cm}$$

$$\sigma_{400\text{K}} = 10^{16} \times 1.6 \times 10^{-19} \times 420 \\ = 0.672 \text{ S/cm}$$

7. Calculate the diffusion coefficient of electron at 300 K in n-type silicon SiC doped with 10^{15} arsenic atoms per cm^3 ?

^{So} we have form Einstein's relationship

$$\frac{D_e}{n_e} = \frac{kT}{e}$$

$$\text{Here, } T = 300 \text{ K}$$

$$n_d = 10^{15} \text{ cm}^{-3}$$

$$\text{we find drift mobility } \mu_e = 1300 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$$

So,

$$D_e = \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \times 1300$$

$$= 33.67 \text{ cm}^2 \text{ s}^{-1} \text{ - diffusion coefficient.}$$

8. consider a Schottky junction diode between Al and n-type Si, doped with 10^{16} donors per cm^3 . The cross sectional area of Si is 1 mm^2 and electron affinity of silicon is 0.901 eV , work function of Al is 4.28 eV . What will be the theoretical potential barrier height from metal to Si? Using the Richardson's eqⁿ for thermionic emission $J_0 = A_0 T^2 \exp(-\Phi_B / kT)$ calculate the reverse saturation current and forward biased current if 0.1 V is applied. $A_0 = 3000 \text{ A K}^{-2} \text{ m}^{-2}$ - J_0 represents the saturation current density.

50

Hence,

$$N_d = 10^6 \text{ cm}^{-3}$$

$$A_{Si} = 1 \text{ mm}^2 = (1 \times 10^{-3})^2 = 10^{-6} \text{ m}^2$$

$$\chi_{Si} = 4.01 \text{ eV}$$

$$\phi_{AL} = 4.28 \text{ eV}$$

$$\begin{aligned} \text{Then, Theoretical p. bammr, } \phi_B &= \phi_{AL} - \chi_{Si} \\ &= 4.28 - 4.01 \\ &= 0.27 \text{ eV.} \end{aligned}$$

Reverse current density

$$J_0 = A_0 T^2 \exp\left(-\frac{\phi_B}{kT}\right)$$

current

$$I_0 = A_{Si} A_0 T^2 \exp\left(-\frac{\phi_B}{kT}\right)$$

$$= 10^{-6} \times 3000 \times (300)^2 \times \exp\left(\frac{-0.27}{1.38 \times 10^{-23} \times 300}\right)$$

$$= 7.99 \times 10^{-3} \text{ A.}$$

Forward current.

$$I_F = I_0 \left[\exp\left(\frac{eV}{kT}\right) - 1 \right]$$

$$= 7.99 \times 10^{-3} \times \left[\exp\left(\frac{1.6 \times 10^{-19} \times 0.2}{1.38 \times 10^{-23} \times 300}\right) - 1 \right]$$

$$= 0.37 \text{ A.}$$

Page - 112 .

- (3) calculate the magnetic moment of a bar of iron having length 0.5 m and cross-sectional area 2 mm^2 . When placed in a long solenoid of 25 turns/cm carrying a current 2 A. Take the relative permeability of material 400.

S.N Here, $l = 0.5 \text{ m}$

$$A = 2 \text{ mm}^2 = 2 \times 10^{-6} \text{ m}^2$$

$$n = 25 \text{ turn/cm} = 25 \times 10^2 \text{ turns/m}$$

$$I = 2 \text{ A};$$

magnetic moment = ?

$$\mu_r = 400.$$

since

$$B = \mu_0 \mu_r n I$$

$$\Rightarrow 4\pi \times 10^{-7} \times 400 \times 25 \times 10^2 \times 2$$

$$= 2.51 \text{ tesla}.$$

now magnetization = $\frac{B}{\mu_0}$

$$= \frac{2.51}{4\pi \times 10^{-7}}$$

$$= 1.99 \times 10^6 \text{ Am}^{-1}$$

Magnetic moment = $M \times V$

$$= 1.99 \times 10^6 \times 2 \times 10^{-6} \times 0.5$$

$$= 1.99 \text{ Am}^2$$

8

8) Note:-

Hard magnetic materials can be used in magnetic storage of digital data, where $+B_r$ and $-B_r$ represents 1 and 0 respectively.

* Application of soft magnetic materials.

- (1) Si-Fe (2 to 4%) \rightarrow These are used in power transformer and electric machineries.
- (2) Ni-Fe (77% Ni, 23% Fe) \rightarrow Used in low magnetic field applications and are typically found in high frequency work in electronics. e.g. Audio and wide-band transformers, CT, magnetic recording heads etc.
 ↗ Current transformer (used to read current)
- (3) Supermalloy (79% Ni, 16% Fe, 5% Co) \rightarrow They have high relative permeability & can be used in low loss electric devices. e.g. special transformers, magnetic amplifiers etc.
- (4) Permalloy (78.5% Ni, 21.5% Fe) \rightarrow These are used in low loss electric devices, audio transformers, high freq. transformers, filters, recording heads etc.
- (5) Glassy metals (Fe-Si-B) \rightarrow Used in low loss transformer cores.
- (6) Mn-Zn ferrites \rightarrow Used in high freq. low loss application. They have low conductivity hence eddy current losses are negligible

These are used in inductors, recording heads, high freq. transformers etc.

* Application of

Hard magnetic materials (Application)

① Ceramic magnets (powdered solid permanent magnet)

→ These are made by compacting Barium-Ferrite ($\text{BaFe}_{12}\text{O}_9$) and Strontium ferrite ($\text{SrFe}_{12}\text{O}_9$)

These are used in low cost magnetic applications like telephone receivers, toys, loud speakers, dc motors etc.

② Samarium Cobalt ($\text{Sm}_2\text{-Co}_7$) alloys → These are used in stepper and servo motors, quality audio headphones etc.

③ Neodymium-iron-boron (NdFeB) → These are used in small motors for hand tools, MRI body scanners, computer applications, headphones and toys etc.

alloy of 2

④ Alnico (Al, Ni, Co, Fe) → These are horse shoe shape magnets and have wide range of permanent magnet applications.

⑤ Hard particles ($\text{Y}-\text{Fe}_2\text{O}_3$) → These are used in audio and video tapes, floppy disk etc.

$\text{Y}_3\text{Fe}_5\text{O}_{12} \rightarrow$ Yttrium Iron garnet

Note:

Hysteresis loss is given by Steinmetz formula,

$$P_h = K_f B_m^k \text{ watt/m}^3 \rightarrow K_f B_m^k$$

$k \rightarrow$ hysteresis coefficient J/m^3

$f \rightarrow$ freq. Hz.

$B_m \Rightarrow$ max. flux density in web/ m^2 or Tesla.

$K \rightarrow$ Steinmetz constant

* Eddy current loss.

When an applied magnetic field is changed, current is induced in the magnetic material, this current is called eddy current and produces loss in power resulting in heating of materials called eddy current loss. To reduce this loss, resistance has to be made large such that small current is allowed and hence the eddy current loss is reduced.

2018 Spring Q.(b) Find the percentage increase in the magnetic field B , when one space within the current carrying coil is fixed with Al. The susceptibility of Al is 2.132×10^{-5} .

Soln

$$B_0 = \mu_0 H$$

$$B = \mu_0 \mu_r H = \mu_r B_0$$

$$\mu_m = \mu_r - 1$$

$$\begin{aligned} \mu_r &= \mu_m + 1 = 1 + 2.132 \times 10^{-5} \\ &= 1.00002132 \end{aligned}$$

$$\% \text{ change, } \rightarrow \frac{B - B_0}{B_0} \times 100\%$$

$$= \frac{\mu_r B_0 - B_0}{B_0} \times 100\%$$

$$= (\mu_r - 1) 100\%$$

$$= 2.132 \times 10^{-3} \%$$

2019 Q. The magnetic field strength in a piece of Fe_2O_3 is 10^6 Am^{-1} . Given that the susceptibility at room temp is 1.9×10^{-3} . Find the flux density and magnetization in the material.

~~2022-23~~ Here,

$$H = 10^6 \text{ Am}^{-1}$$

$$x_m = 1.4 \times 10^{-3}$$

$$B = ?$$

$$M = 2$$

$$\text{Since, } x_m = \frac{M}{H}$$

$$B = 4\pi(H + M)$$

$$M = 1.4 \times 10^{-3} \times 10^6 \text{ Am}^{-1}$$

$$= 1400 \text{ Am}^{-1}$$

$$\begin{aligned} \text{Now, } B &= 4\pi(H + M) \\ &= 4\pi \times 10^{-7} (10^6 + 1400) \\ &= 3.25 \times 10^{-7} \text{ Weber/m}^2 \end{aligned}$$

#

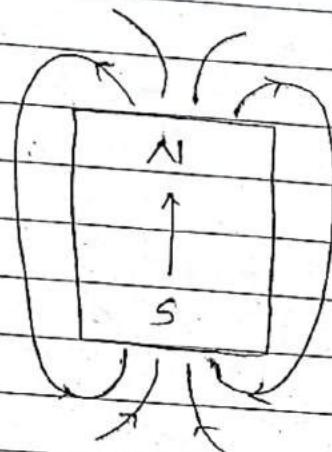
Magnetic Domains

Fig (a) single domain of Fe crystal without magnetization.

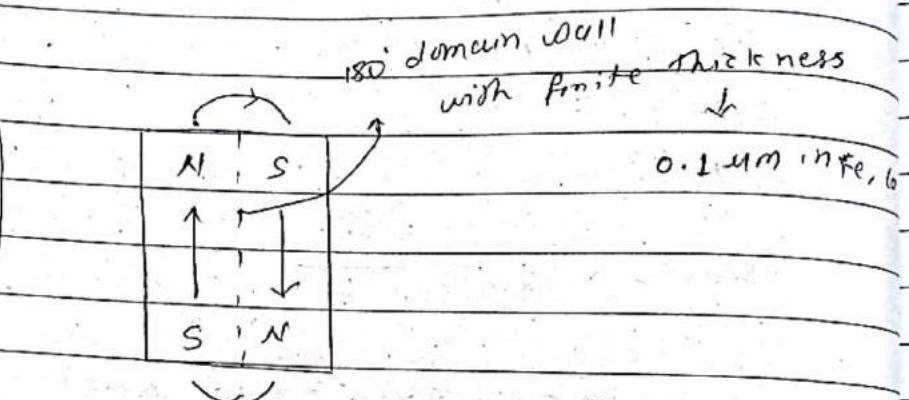


Fig (b) Two domains with domain or Bloch wall.

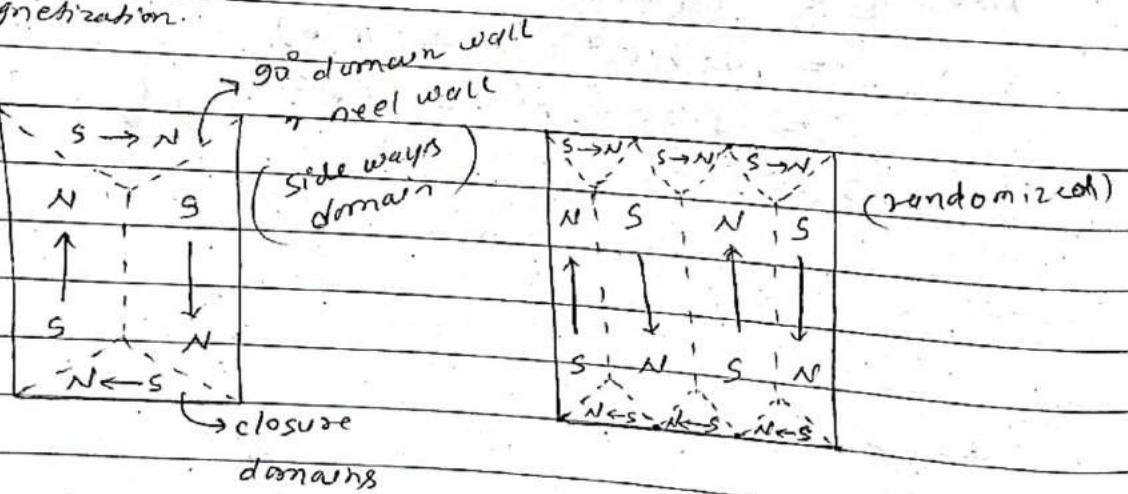


Fig (c) Fe crystal with closure domains & unmagnetised (ie. $B = 0$)

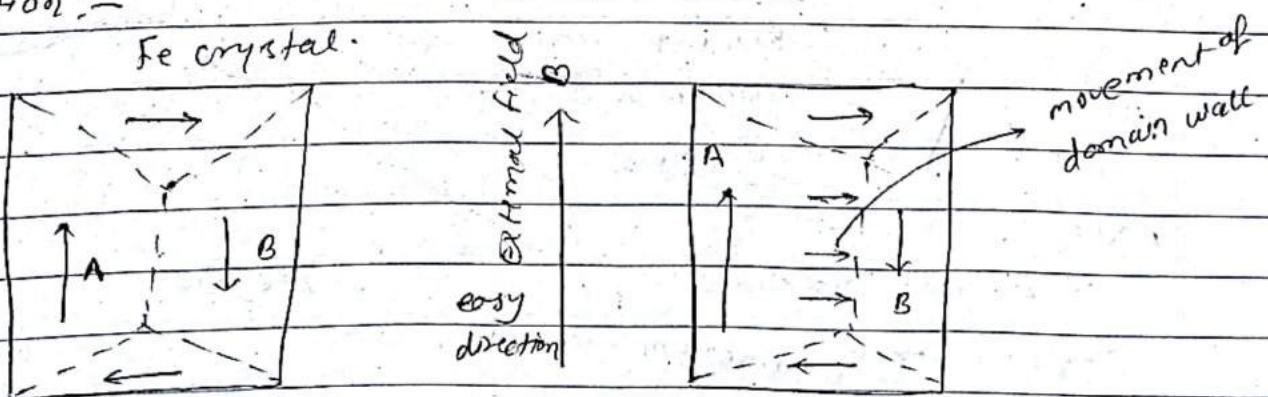
Fig (d) max closure domains in Fe crystal.

Demagnetization

magnetization

- A region of the crystal or solid in which all the spin magnetic moments are aligned to produce a magnetic moment in only one direction is called magnetic domains.
- Domain or Block wall has finite thickness (for Fe 0.1 μm) and usually the thickness is of several hundred atomic spacings.
- The creation of magnetic domains will continue until the decrease in magneto static potential energy in creating an additional domain is same as the increase in magneto static potential energy in creating an addition wall. This is equilibrium condition with minimum potential energy and no net magnetization.
- Magnetization of the Fe crystal along an externally applied field occurs by growth of domains with magnetization along the direction of the field.

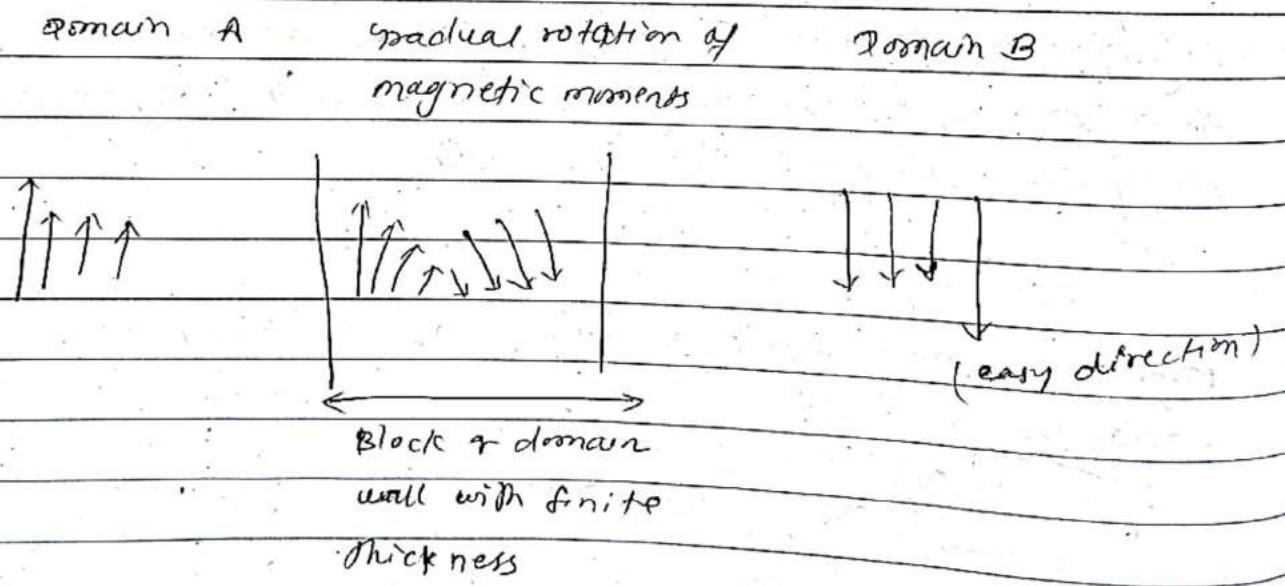
* Migration / movement or growth of domains / domain wall motion :-



fig(a) Domain A & B of equal & opposite size so net magnetization is zero.

fig (b) Domain A grows & Domain B shrinks on applying field.
(net M in the dir of ↑)

- The migration of the block or domain wall is caused by the gradual rotation of the spin magnetic moments of the wall and adjacent to the wall in the direction of externally applied magnetic field. Thus the magnetization process involves the motion of domain wall in the crystal of Fe.
- As shown in figure B, on applying external magnetic field, domain A grows and shrinks domain B due to applied external field or easy direction, all the spin magnetic moments in the wall and adjacent to the wall in region B will experience torque along the easy direction.



Note:

square loop materials:

$\text{Sm}_2\text{Co}_{17}$ → } samarium-cobalt

SmCo_5 → }

AlnFeB → neodymium iron boron

$\text{YFe}_5\text{O}_{13}$

Q) The saturation induction of Ni is 0.65 Weber/m^2 , density is 8906 kg/m^3 & its atomic weight is 58.7 gm/mol^{-1} . Calculate the magnetic moment of Ni atom.

$\frac{\text{No. of}}{\text{Volume}}$

$$\text{No. of atoms per unit volume (N)} = \frac{g \times N_A}{M_{\text{atom}}} \text{ m}^{-3}$$

$$M = \frac{\Sigma M}{V} = \frac{\text{no. of atoms} \times M}{V} = \frac{8906 \times 6.023 \times 10^{23}}{58.7 \times 10^{-3}}$$

$$\mu = \frac{M}{\text{no. of atoms/volume}} = \frac{M}{n} = 9.138 \times 10^{28} \text{ m}^3$$

$$B = \mu_0 \mu_r H \Rightarrow B = \mu_0 M$$

$$\frac{M}{H} \quad M = \frac{B}{\mu_0}$$

$$\text{So, } \mu = \frac{B}{\mu_0 n} = \frac{0.65}{4\pi \times 10^{-7} \times 9.138 \times 10^{28}}$$

$$= 5.66 \times 10^{-29} \text{ Am}^{-2}$$

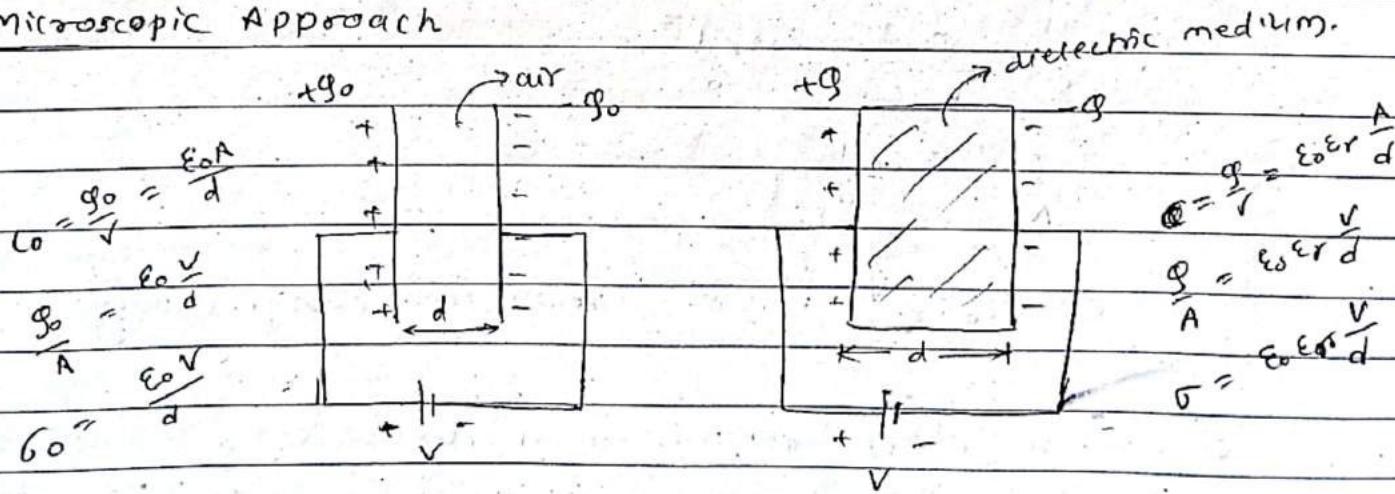
A good dielectric medium is

- (i) Has high dielectric breakdown strength
- (ii) Is thermally stable and chemically inert
- (iii) Is non-flammable and non-toxic with less cost
- (iv) A good dielectric material serves as insulating material for current carrying conductor which are at different voltage levels.
- (v) It also helps to quench electric discharge and serves as coolant.

Q. Why cannot be simply used air as dielectric medium between two very high voltage conductors? ($> 33 \text{ kV/cm}$) air breakdown.

$$\frac{\text{dipole moment}}{V} \rightarrow \text{polarization.}$$

* Microscopic Approach



$$P = \sigma - \sigma_0 \quad (\text{polarization})$$

$$= \frac{\epsilon_0 V}{d} (\epsilon_r - 1) = \epsilon_0 E x \quad \alpha = \epsilon_0 - 1 \quad \text{electric susceptibility}$$

When a voltage V is applied across the parallel plate capacitor charges $+g_0$ and $-g_0$ will be induced on the surface of the plates by electrostatic induction. The capacitance of the parallel plate capacitor is then given by

$$C_0 = \frac{g_0}{V} = \frac{\epsilon_0 A}{d}$$

$$\frac{g_0}{A} = \frac{\epsilon_0 V}{d}$$

$$g_0 = \epsilon_0 V/d ; g_0 = \text{surface charge density}$$

When a dielectric medium is inserted between these plates, then the charges on the surface of the plates gets increases due to appearance of the charge on the surface of the dielectric near the plates. such that new capacitance,

$$C = \frac{q}{V} = \frac{\epsilon_0 \epsilon_r A}{d}$$

$$\frac{q}{A} = \frac{\epsilon_0 \epsilon_r V}{d}$$

$$\sigma = \epsilon_0 \epsilon_r \frac{V}{d} ; \sigma = \text{new surface charge density}$$

Increase in charge density when dielectric medium is inserted compared with that without dielectric medium is called polarization

$$\text{i.e. } P = \sigma - g_0$$

$$= \frac{\epsilon_0 V (\epsilon_r - 1)}{d}$$

$$P = \epsilon_0 E \chi ; \quad \chi = \text{dielectric susceptibility} = \epsilon_r - 1$$

$E = \frac{V}{d}$ electric field.

Polarization can also be defined as induced charge within dielectric per unit area.

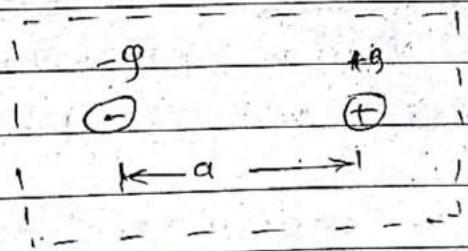
i.e. $P = \frac{Q}{A}$

Note: The relative permittivity can be defined as increase in charge stored on the capacitor plates due to insertion of the dielectric medium compared to the charge without medium.

i.e. $\epsilon_r = \frac{Q}{Q_0}$

$$= \frac{C}{C_0}$$

* Electric dipole moment



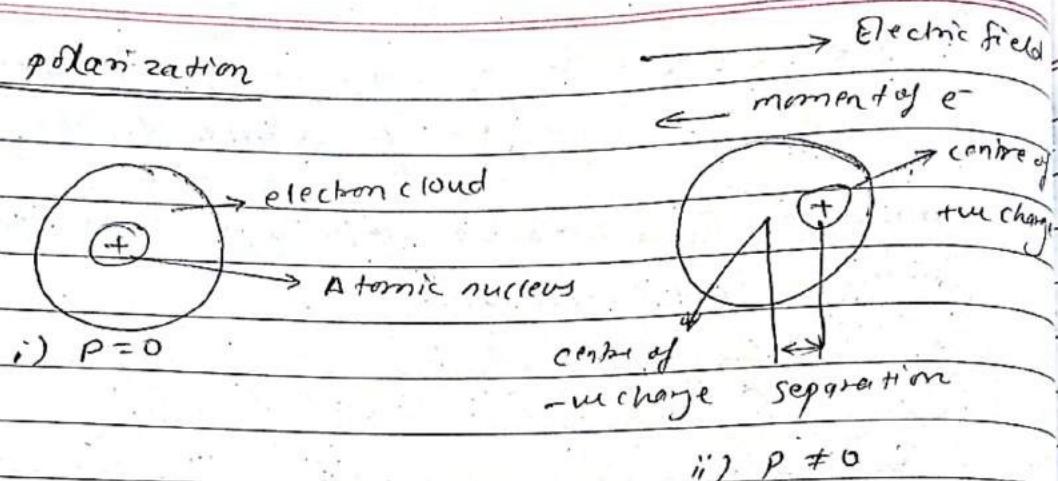
$$q_{\text{net}} = 0$$

$$\mathbf{p} = \mathbf{a} \cdot \mathbf{q}$$

An electric dipole moment is simply a separation between +ve & -ve charge of equal magnitude q . If a is the vector from -ve to +ve charge, the electric dipole moment is defined by the vector given by

$$\mathbf{p} = \mathbf{a} \cdot \mathbf{q} \quad \text{--- (1)}$$

* Electronic polarization



In an atom when there is no applied field, the centre of mass of atom & the electron charge centre coincide with each other, so the net dipole within the atom is zero. When an external field is applied, the centre of +ve charge doesn't change but the -ve charge centre due to e⁻'s will be shifted in the direction opposite to the applied field. Hence there will be an induced electric dipole moment, called as electronic polarization.

This induced dipole moment depends primarily on applied electric field causing it and is given by

$$\text{p}_{\text{induced}} = \alpha E$$

; α is the coefficient of electronic polarizability.

Note :-

Electronic polarizability α can be calculated as

$$\alpha_e = \frac{Z e^2}{m_e \cdot \omega_0^2}$$

Z = atomic number

ω_0 = oscillation frequency of centre of mass of e⁻ about nucleus.

m_e = mass of e⁻

e = charge of electron

g. calculate the electronic polarization resonant frequency of the Ar atom. whose atomic no. is 18 and the coefficient of electronic polarizability is $1.7 \times 10^{-70} \text{ fm}^2$.

$$\omega_0 = 2\pi f_0$$

$$\alpha_e = \frac{2e^2}{m_e \cdot \omega_0^2}$$

$$\begin{aligned}\omega_0 &= \left(\frac{2e^2}{m_e \cdot \alpha_e} \right)^{1/2} \\ &= \left(\frac{18 \times (1.6 \times 10^{-19})^2}{9.1 \times 10^{-31} \times 1.7 \times 10^{-70}} \right)^{1/2}\end{aligned}$$

$$= 5.46 \times 10^{16} \text{ rad/sec.}$$

now,

$$\omega_0 = 2\pi f_0$$

$$f_0 = \frac{\omega_0}{2\pi}$$

$$= 8.686 \times 10^{15} \text{ Hz.}$$

* polarization vector (\mathbf{P}) & polarization

polarization of dielectric medium is represented by a polarization vector, and is defined as total dipole moment per unit volume.

$$\text{I.e. } \mathbf{P} = \frac{\mathbf{p}_1 + \mathbf{p}_2 + \dots + \mathbf{p}_N}{V} = \frac{\sum_{i=1}^N \mathbf{p}_i}{V}$$

where $\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_N$ are dipole moment induced for N molecules.

If p_{avg} is average dipole moment per molecule
then polarization vector,

$$\mathbf{P} = N \cdot p_{avg} \quad \text{--- (1)}$$

N = no of molecules per unit volume (cm^{-3})

since, polarization

$$\mathbf{P} = \epsilon_0 \chi_e \mathbf{E} \quad \text{--- (2)}$$

and $\mathbf{P}_{\text{induced}} = \chi_e \mathbf{E}$

so, from (1)

$$\mathbf{P} = N \cdot \mathbf{P}_{\text{induced}}$$

$$\mathbf{P} = N \chi_e \mathbf{E} \quad \text{--- (3)}$$

from (2) & (3)

we can write.

$$N \chi_e \mathbf{E} = \epsilon_0 \chi_e \mathbf{E}$$

$$\Rightarrow \chi_e = \frac{N \chi_e}{\epsilon_0}$$

This is the relation showing relationship between dielectric susceptibility and electronic polarizability coefficient such that

$$\epsilon_r = 1 + \chi_e$$

$$\Rightarrow \epsilon_r = 1 + \frac{N \chi_e}{\epsilon_0}$$

~~App~~

Local electric field. (E_{loc})

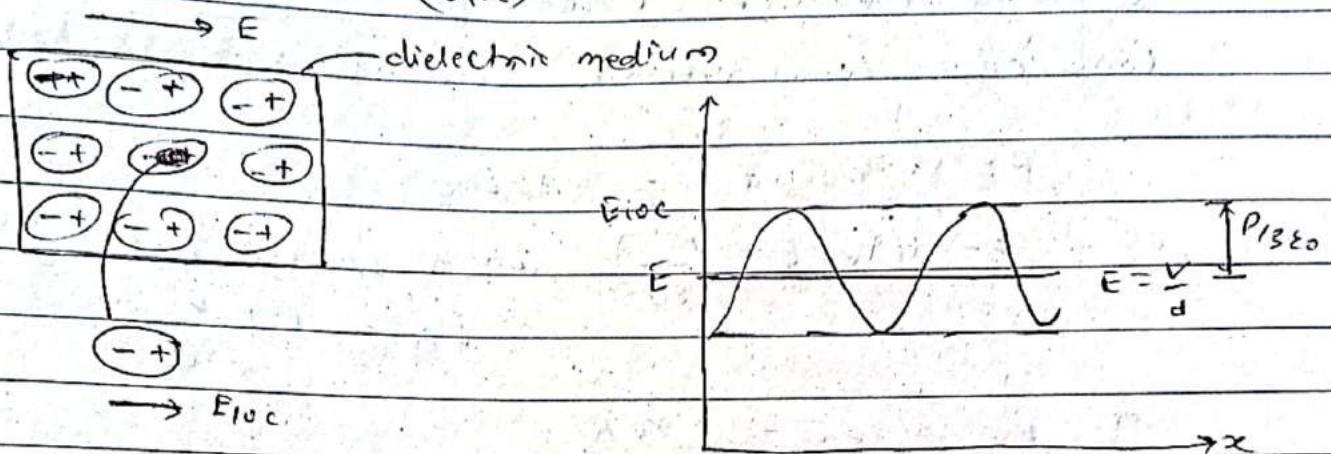


fig. Non-uniform electric field (E_{loc})

with a polarized dielectric medium

In atomic scale, as we move through the dielectric, the electric field acting on atom or molecule is not uniform. Hence as the dielectric becomes polarized, the field at some arbitrary point inside the dielectric depends not only on charges on the plates but also on arrangements/ orientations of all other dipoles around this point in the dielectric medium. This actual field, experience by a molecule or atom in dielectric is called local electric field (E_{loc}). Greater the polarization, greater will be the dipoles around the arbitrary point in the dielectric medium and hence greater will be the local field.

E_{loc} is the actual field that acts on a molecule and for a simple cubic crystal structure or a liquid, it is given by Lorentz formula or field as

$$E_{loc} = E + \frac{P}{3\epsilon_0}$$

II Clausius-Mosotti Equation (C-M eqn)

Considering local electric field as E_{loc} , we have

$$\cdot P = N \cdot \text{Produced} = N \alpha_e E_{loc}$$

$$\text{also, } \text{or } P = N \alpha_e \left(E + \frac{P}{3\epsilon_0} \right) \text{ where, Lorentz field} \\ \left(E_{loc} = E + \frac{P}{3\epsilon_0} \right)$$

$$\text{or, } P = N \alpha_e E + \frac{N \alpha_e P}{3\epsilon_0}$$

$$\rightarrow P \left(1 - \frac{N \alpha_e}{3\epsilon_0} \right) = N \alpha_e E$$

$$\Rightarrow P = \frac{N \alpha_e E}{\left(1 - \frac{N \alpha_e}{3\epsilon_0} \right)} \quad \text{--- (1)}$$

since,

$$P = \epsilon_0 \chi_e E \quad \text{--- (2)}$$

from (1) and (2) we get.

$$\epsilon_0 \chi_e E = \frac{N \alpha_e E}{\left(1 - \frac{N \alpha_e}{3\epsilon_0} \right)}$$

$$\Rightarrow \epsilon_0 \chi_e - \frac{\epsilon_0 \chi_e N \alpha_e}{3\epsilon_0} = N \alpha_e \quad \text{--- (3)}$$

$$\text{or, } \epsilon_0 \chi_e = N \alpha_e + \frac{N \alpha_e \chi_e}{3}$$

$$\Rightarrow \epsilon_0 \chi_e = N \alpha_e \left(1 + \frac{\chi_e}{3} \right)$$

$$\Rightarrow \epsilon_0 \chi_e = N \alpha_e \left(\frac{3 + \chi_e}{3} \right)$$

$$\Rightarrow \boxed{\frac{N \alpha_e}{3\epsilon_0} = \frac{\chi_e}{\chi_e + 3} = \frac{\epsilon_r - 1}{\epsilon_r + 2}}$$

⇒ $\propto \epsilon_0$

This is called Clausius-Mossotti equation, which shows relationship between polarizability $\propto (\epsilon_0)$, dielectric susceptibility (χ_e) and relative permittivity (ϵ_r) .

- Q. A solid contains 5×10^{28} identical atoms m^{-3} each with a polarizability of $2 \times 10^{-40} Fm^2$. Assuming that the internal field is given by the Lorentz formula calculate the ratio of internal field to the applied field.

$N = 5 \times 10^{28} \text{ atom } m^{-3}$

$\propto_e = 2 \times 10^{-40} Fm^2$

$E_{loc} = ?$

Lorentz formula, $E_{loc} = E + \frac{P}{3\epsilon_0}$

$\therefore E_{loc} = E + \frac{\propto_e E_{loc}}{3\epsilon_0}$

$\therefore E_{loc} \left(1 - \frac{\propto_e}{3\epsilon_0} \right) = E$

$\therefore \frac{E_{loc}}{E} = \frac{1}{1 - \frac{\propto_e}{3\epsilon_0}}$

$$= \frac{1}{1 - \frac{5 \times 10^{28} \times 2 \times 10^{-40}}{3 \times 8.85 \times 10^{-12}}}$$

= 1.607

Ans.

dielec. constnt = ϵ_r \leftarrow same.

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- Q. The electronic polarizability of Ar atom is $1.7 \times 10^{-28} \text{ Fm}^2$. What is the static dielectric constant of Ar, if its density is 1.8 gm/cm^3 . Given $N_A = 6.023 \times 10^{23} \text{ mol}^{-1}$, atomic wt. of Ar = 39.95 gm/mol .

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N \epsilon_e}{3 \epsilon_0}$$

$$N = \frac{g \times N_A}{M_A} \text{ cm}^{-3}$$

$$g = \frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{2.71 \times 10^{28} \times 1.7 \times 10^{-28}}{3 \times 8.85 \times 10^{-12}} = 2.71 \times 10^{22} \text{ cm}^{-3}$$

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = 0.1735 = \frac{2.71 \times 10^{22}}{(10^{-2})^3} \text{ m}^{-3}$$

$$w. \quad \epsilon_r - 1 = 0.1735 \epsilon_r + 0.3477 = 2.71 \times 10^{28} \text{ m}^{-3}$$

$$\epsilon_r = \frac{0.3477 + 1}{1 - 0.1735}$$

$$\epsilon_r = 1.63$$

↑

Dielectric constant.

Different polarization mechanism

- ① Electronic polarization
- ② Ionic polarization
- ③ Dipolar/orientational polarization
- ④ Interfacial polarization

(2) Ionic polarization

fig. a)

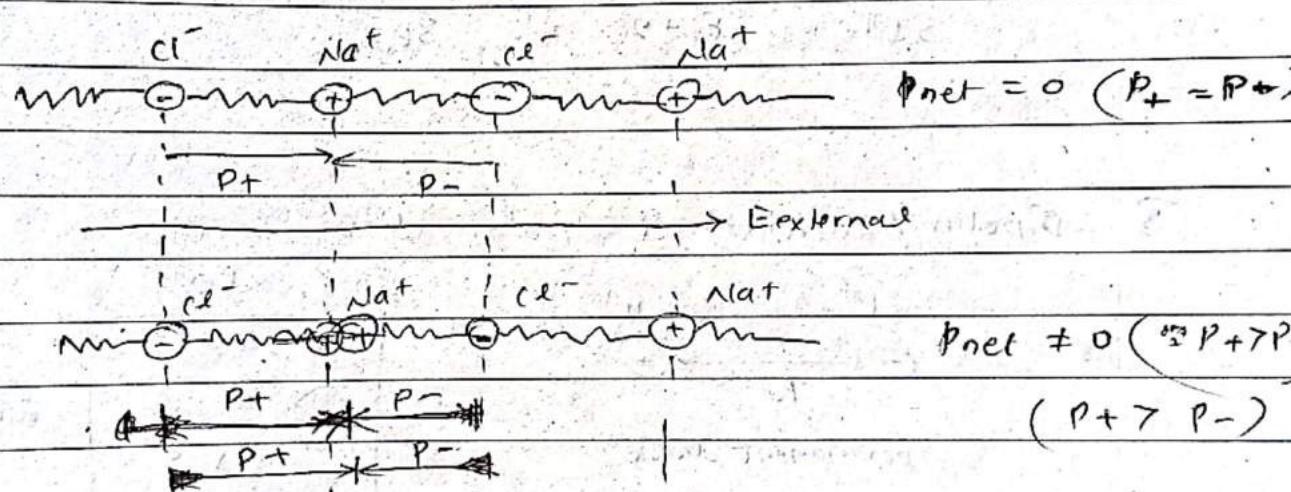


fig. NaCl chain with and without external electric field.

Ionic crystals like NaCl , KCl , LiBr etc. shows ionic polarization as shown in the NaCl chain. In the absence of electric field, the dipole moment have same magnitude but are in opposite in direction such that $P_{\text{net}} = 0$. But in presence of external electric field, Cl^- ion is pushed opposite to the electric field direction and Na^+ to the same direction of applied electric field about their equilibrium position such that $P_+ > P_-$. Hence $P_{\text{net}} \neq 0$.

The induced average dipole moment per ion pair is,

$$P_{\text{induced}} = \alpha_i E_{\text{loc}}$$

such that $P = N_i P_{\text{induced}}$ $N_i = \text{no. of ion pairs}$

$$P = N_i \alpha_i E_{\text{loc}}$$

Hence the C.M eqn becomes,

$$\frac{\pi e}{3 + \pi e} = \frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N_i \alpha_i}{3 \epsilon_0}$$

③ Dipolar / orientational polarization

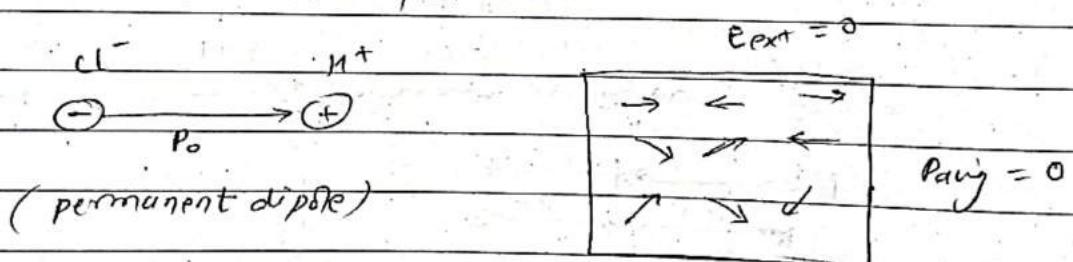


fig. a) absence of E_{ext}

fig. b) liquid or gas state

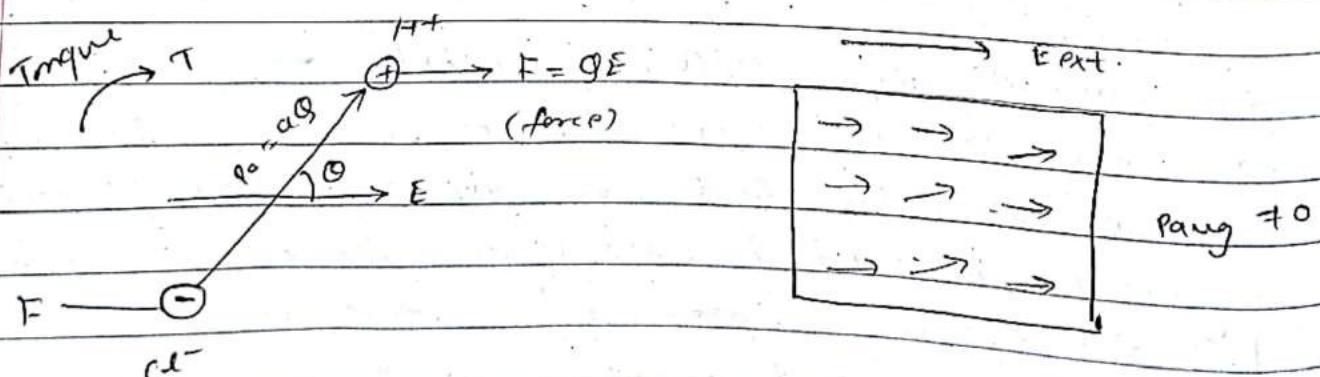


fig. c) presence of E_{ext}

fig. d) polarized medium

Polar gases like HCl and steam, water, alcohol, electrolytes, acetone etc have permanent dipole moments associated with them. Such motion in presence of external field exhibits a ~~polymer~~ polarization called dipolar orientation polarization.

In the absence of external field, all the dipoles are randomly oriented due to thermal agitation. Hence $\langle \cos\theta \rangle = 0$. But in presence of external field, dipoles will try to align its direction along external field, experiencing some torque and also some dipoles will be lost due to molecular collision.

The torque acts on the permanent dipole P_0 and tries to rotate it with applied field E given by

$$\begin{aligned} \tau &= (F \sin\theta) a \\ &= Q \cdot E \sin\theta a \\ &= Q a \cdot E \sin\theta \end{aligned}$$

$$\tau = P_0 E \sin\theta$$

when $\theta = 0^\circ$, PE is minimum

$\theta = 180^\circ$, PE is maximum

$$\begin{aligned} \text{at } 0^\circ, \quad 180^\circ \\ (PE)_{\max} &= \int_0^{180^\circ} P_0 E \sin\theta d\theta \\ &= 2P_0 E \end{aligned}$$

so the average dipole potential energy will be then

$$\frac{1}{2}(PE)_{\max} = P_0 E$$

At Hence the average dipole per molecule is directly proportional to permanent dipole ρ_0 and also directly proportional to ratio of average dipole energy to the avg average thermal energy.

$$\text{Pavg} \propto \rho_0$$

$$\text{Pavg} \propto \frac{\rho_0 E}{3kT} \quad \leftarrow \text{Thermal energy (avg)}$$

$$\frac{\rho_0}{3kT} \times \frac{e^2}{\epsilon_0}$$

$$\therefore \text{Pavg} \propto \frac{\rho_0^2 E}{3kT}$$

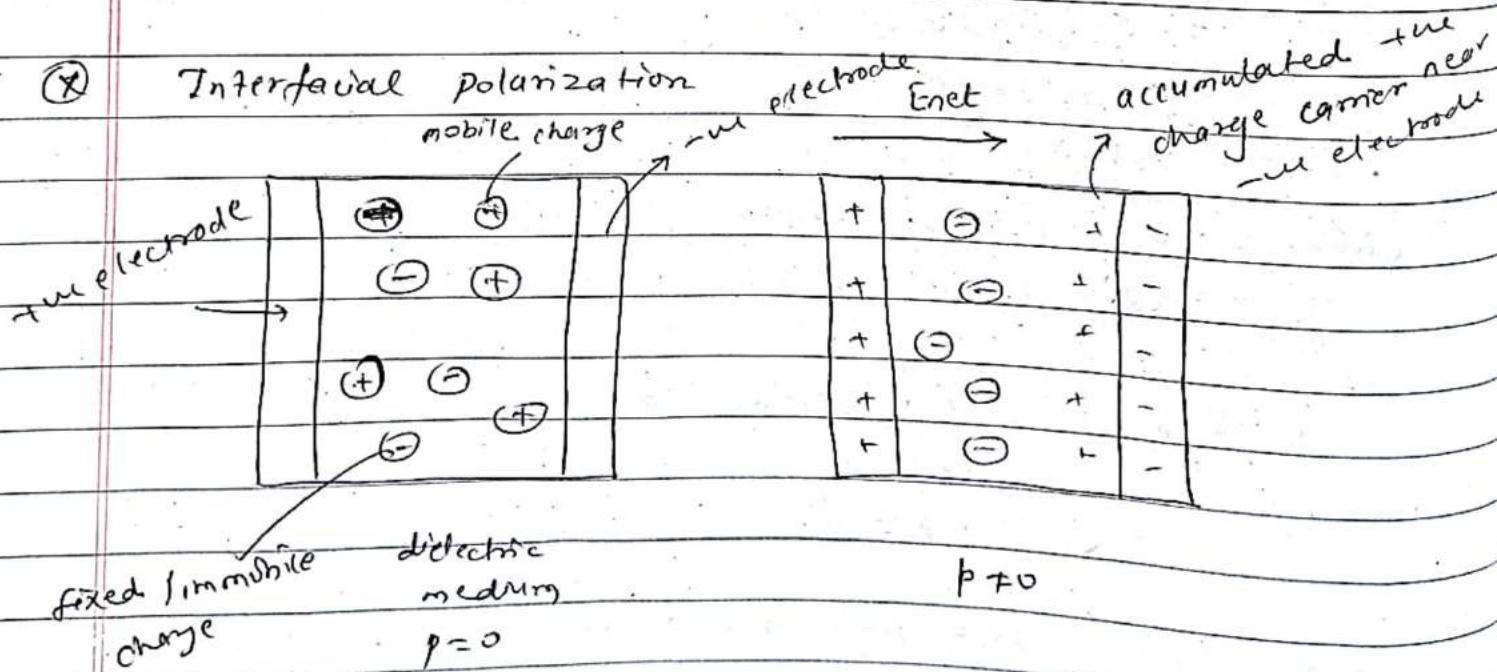
$$\frac{e^2}{\epsilon_0} = n e^{+3}$$

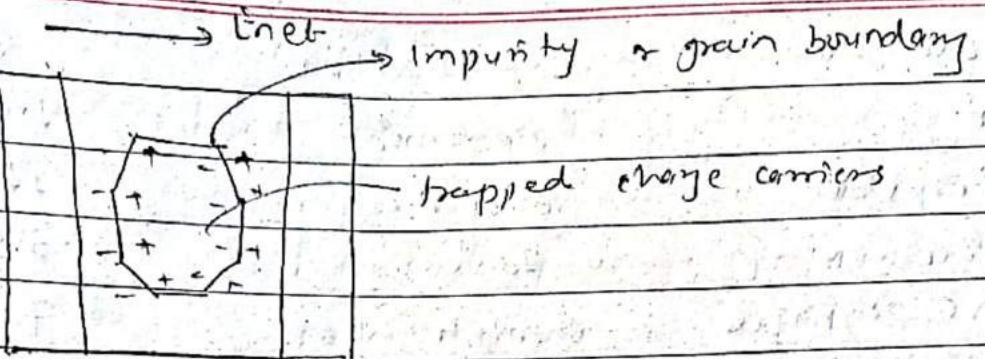
Hence,

$$\text{Pavg} = \alpha_d E$$

where, α_d = dipolar polarizability

$$\Rightarrow \alpha_d = \frac{\rho_0^2}{3kT}$$





The accumulation of charge at an interface bet' two materials or two regions within a material causes interfacial polarization. Under the applied field, the ions migrate near the electrode and this accumulation of +ve charge near interface of one electrode attracts more e-s to one electrode. That appears as increase in dielectric constant. Thus the formation of more dipoles will occur at the interface giving raise the polarization vector P called interfacial polarization and the average value of the dipole is given by

$$P_{avg} = \alpha_{if} \cdot E_{loc}$$

α_{if} = interfacial polarization coefficient.

Note: The average value of dipole due to all the polarization mechanism is given by

$$\Rightarrow P_{avg} = P_{\text{induced}} = \alpha_e \cdot E_{loc} + \alpha_i \cdot E_{loc} + \alpha_d \cdot E_{loc} + \alpha_{if} \cdot E_{loc}$$

Example	polarization	ϵ_r
→ Ar gas	electronic	1.005
→ Si crystal	electronic	11.9
→ NaCl	Ionic	5.9
→ PVC (polyvinyl chloride)	orientational / dipolar	7
→ water	dipolar	80

Ferroelectricity & Piezoelectricity

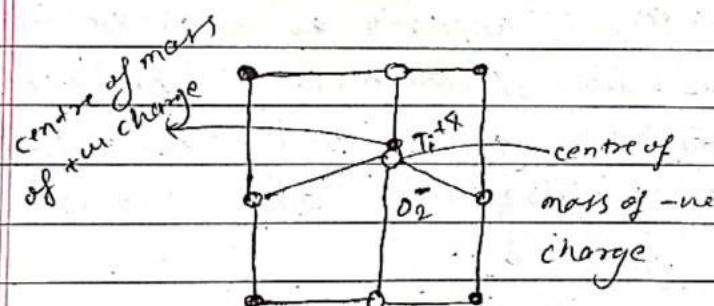


fig. a) BaTiO_3 at temp $< 130^\circ$ has tetragonal structure (ferroelectric)

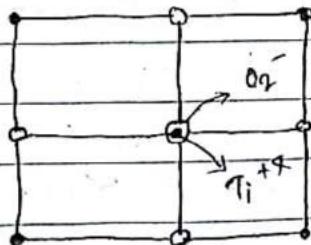
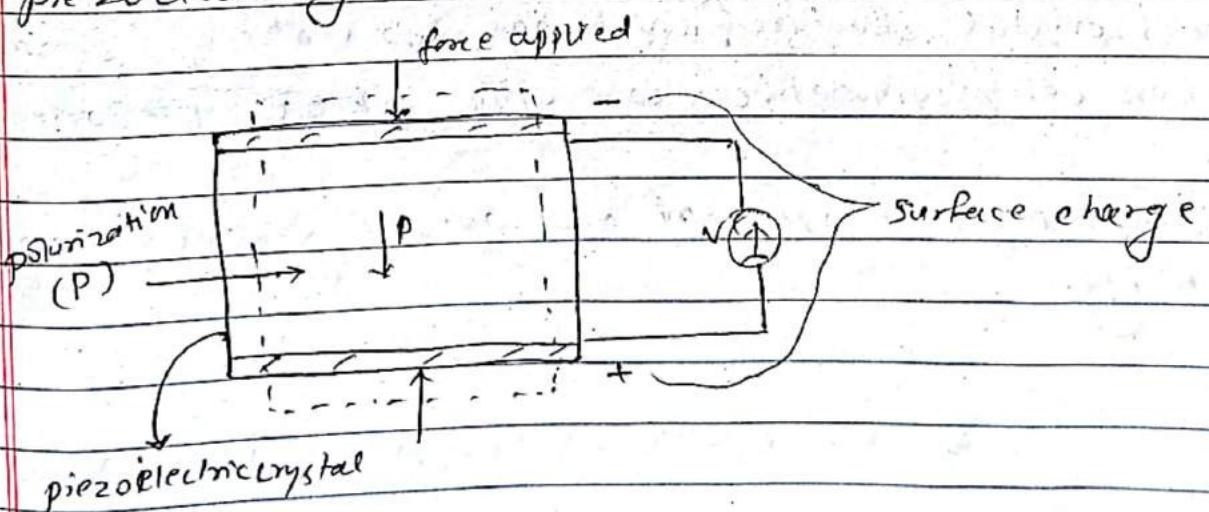
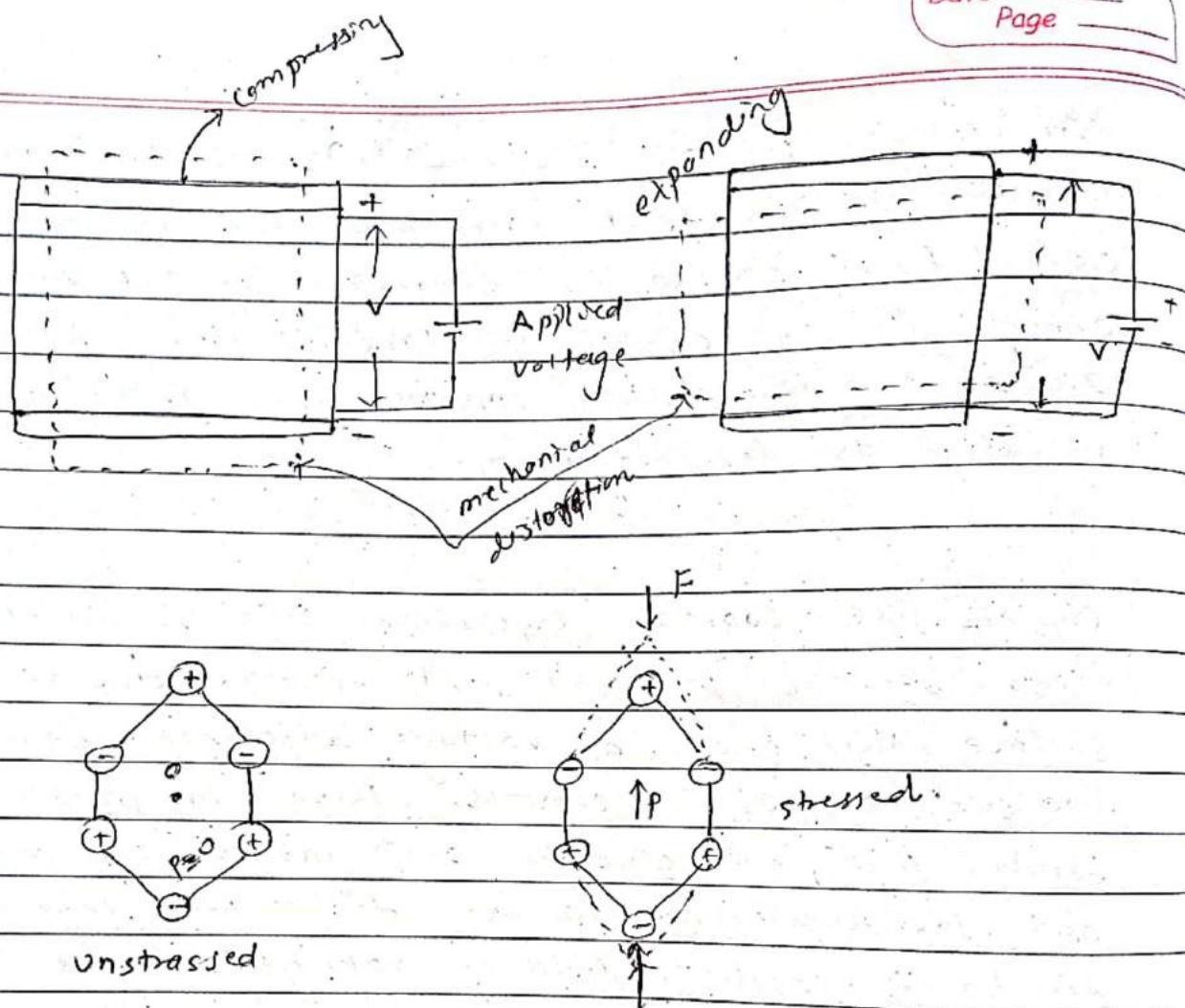


fig. b) BaTiO_3 at temp $> 130^\circ$ has cubic structure (not ferroelectric)

In certain crystal like BaTiO_3 (Barium Titanate), there exists permanent polarization even in the absence of applied field due to the separation of +ve and -ve charges in the crystal structure. Hence such crystals are called ferroelectric crystals and this phenomenon is called as ferroelectricity.

Crystals like quartz (crystalline SiO_2) under mechanical stress becomes polarized and charges appear on its surface that creates a voltage difference between the two surfaces of the crystals. Also in presence of an electric field, such crystals show mechanical distortion and the direction mechanical deformation depends on direction of applied field or voltage. These two phenomena are complementary and is defined as piezoelectricity.





like quartz

Special crystal structure, which has no centre of symmetry or non centrosymmetric can only exhibit piezoelectricity.

Null has centre of symmetry and doesn't exhibit piezoelectricity.

SiC Materials

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Page _____

9. Calculate the built-in potential across the junction formed by joining silicon doped with 10^{17} donor atoms per cm^{-3} with silicon doped with 10^{16} acceptor atoms per cm^{-3} at room temperature. Take the intrinsic concentration from previous example.

SiC, built-in potential, $V_0 = \frac{kT}{e} \ln \left(\frac{N_{\text{A}} N_{\text{D}}}{n_i^2} \right)$

$$= \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \ln \left[\frac{10^{16} \times 10^{17}}{(1.95 \times 10^{10})^2} \right]$$

$$= 0.755 \text{ V.}$$

10. A heavily doped p-side with acceptor concentration of 10^{18} cm^{-3} is connected to n-side with donor concentration of 10^{16} cm^{-3} . Calculate the built-in potential, depletion width in n-side and p-side, and overall depletion width. The intrinsic concentration is as in previous case and $T = 300 \text{ K}$.

Built-in potential, $V_0 = \frac{kT}{e} \ln \left(\frac{N_{\text{A}} N_{\text{D}}}{n_i^2} \right)$

$$= \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \ln \left[\frac{10^{18} \times 10^{16}}{(1.95 \times 10^{10})^2} \right]$$

$$\Rightarrow V_0 = 0.815 \text{ V.}$$

Now

$$E_r = 11.9$$

Depletion layer, $w_0 = \sqrt{\frac{2EV_0}{e} \left(\frac{1}{N_A} + \frac{1}{N_D} \right)}$

$$= \sqrt{\frac{2 \times 11.9 \times 8.85 \times 10^{12}}{1.6 \times 10^{-19}} \left(\frac{1}{10^{27}} + \frac{1}{10^{22}} \right)}$$

$$= 3.67 \times 10^{-7} \text{ m.}$$

depletion width in n-side

$$w_n = \frac{w_0 \cdot N_A}{N_A + N_D}$$

$$= \frac{3.67 \times 10^{-7} \times 10^{27}}{10^{27} + 10^{22}}$$

$$= 3.6 \times 10^{-7} \text{ m}$$

depletion width in p-side, $w_p = w_0 - w_n$

$$= 3.67 \times 10^{-7} - 3.6 \times 10^{-7}$$

$$= 0.07 \times 10^{-7} \text{ m}$$

$$= 7 \times 10^{-9} \text{ m.}$$

S

11) An abrupt p+n junction diode has a cross sectional area of 1 mm^2 , an acceptor concentration of $5 \times 10^{18} \text{ cm}^{-3}$ in p-side and a donor concentration of 10^{16} cm^{-3} on n-side. The lifetime of holes in n-region is 417 ns whereas that of electrons in p-region is 5 ns. Mean thermal generation lifetime is about 1 ms. The lengths of p and n sides are 5 micron and 100 micron resp. calculate.

(a) Diffusion coefficient and the minority carrier diffusion length,

since from Einstein relationship

$$\frac{D_e}{n_e} = \frac{kT}{e}$$

$$n_e = 120 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1}$$

$$n_h = 800 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1}$$

$$D_e = n_e \frac{kT}{e}$$

$$kT \rightarrow \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} = 0.026 \text{ eV}$$

$$\text{then, } D_e = 120 \times 0.026 \cdot \text{cm}^2 \text{V}^{-1} \text{s}^{-1} \cdot \text{eV} \\ = 3.12 \text{ cm}^2 \text{s}^{-1}$$

$$\text{Similarly, } D_h = n_h \times \frac{kT}{e}$$

$$= 800 \times 0.026 \\ = 10.4 \text{ cm}^2 \text{s}^{-1}$$

For minority carrier diffusion length

$$L_d = \sqrt{D_e \tau_e}$$

$$= \sqrt{3.12 \times 5 \times 10^{-9}}$$

$$= 1.25 \times 10^{-4} \text{ cm}$$

$$L_h = \sqrt{10.4 \times 4.17 \times 10^{-9}}$$

$$= 2.08 \times 10^{-3} \text{ cm.}$$

(b) Built-in potential across the Junction,

$$V_0 = \frac{kT}{e} \ln \left(\frac{N_A N_D}{n n_i^2} \right)$$

$$= 0.026 \times \ln \left(\frac{5 \times 10^{18} \times 10^{16}}{(1.75 \times 10^{10})^2} \right)$$

$$= 0.86 \text{ V.}$$

Magnetic Materials

Date _____

Page _____

1. Calculate the permeability and susceptibility of an iron bar of cross sectional area 0.2 cm^2 . when a magnetizing field of 1200 Am^{-1} produces magnetic field of $2 \times 10^{-6} \text{ Weber}$.

Ans,

$$A = 0.2 \text{ cm}^2 = 0.2 \times 10^{-4} \text{ m}^2$$

$$H = 1200 \text{ Am}^{-1}$$

$$\Phi = \cancel{2 \times} 2 \times 10^{-6} \text{ web. } \chi_m = ?$$

$$\mu = ?$$

$$B = \frac{\Phi}{A} = \frac{2 \times 10^{-6}}{0.2 \times 10^{-4}}$$

$$= 1.2 \text{ Tesla}$$

$$\text{permeability, } \mu = \frac{B}{H} = \frac{1.2 \text{ Tesla}}{1200 \text{ Am}^{-1}} \Rightarrow 10^{-3} \text{ Vs} \cdot \text{Am}^{-1}$$

$$\mu_r = \frac{\mu}{\mu_0} = \frac{10^{-3}}{4\pi \times 10^{-7}} = 795.77$$

$$\text{now, } \chi_m + 1 = \mu_r$$

$$\chi_m = 795.77$$

susceptibility.

2. calculate the magnetization and flux density in a diamagnetic sample having susceptibility -0.3×10^{-5} and magnetic field strength 1000 A/m .

$$\text{H.e.r. } H = 1000 \text{ A/m}$$

$$\chi = -0.3 \times 10^{-5}$$

$$M = ?$$

$$B = ?$$

$$\text{since } B = \mu_0 M$$

$$= 4\pi \times 10^{-7} \times 1000$$

$$> 1.25 \times 10^{-3} \text{ T.}$$

$$\text{also, } \chi = \frac{M}{B}$$

$$M = n I I$$

$$= -0.3 \times 10^{-5} \times 1000$$

$$= -0.3 \times 10^{-2} \text{ A/m.}$$

3. calculate the magnetic moment of a bar of iron having length 0.5 m and cross sectional area 2 mm^2 when placed in a long solenoid of 25 turns/cm carrying a current 2 A . Take the relative permeability of the material 400 .

$$\text{mag. moment} = M \times V \quad \rightarrow (1)$$

$$\text{we know } B = \mu_0 M$$

$$M = \frac{B}{\mu_0} \quad \rightarrow (2)$$

For soft iron,

$$B = \mu_0 I$$

$$= 4\pi \times 10^{-7} \times 400$$

$$= 4\pi \times 10^{-7} \times 400 \times 25 \times 10^2 \times 2$$

$$= 2.51 \text{ T}$$

$$M = \frac{2.51}{4\pi \times 10^{-7}} = 2 \times 10^6 \text{ Am}^{-1}$$

$$\text{magnetic moment} = 2 \times 10^6 \times 0.5 \times 2 \times (10^{-3})^2$$

$$= 2 \text{ Am}^2$$

4. The saturation induction of nickel is 0.65 wb m^{-2} . If the density is 8906 kg m^{-3} and its atomic weight is 58.7 ; calculate the magnetic moment of nickel atom.

HFM,

$$B = 0.65 \text{ wb m}^{-2}$$

$$\rho = 8906 \text{ kg m}^{-3}$$

$$\text{Awt} = 58.7 \text{ g}$$

$$M$$

$$n = \frac{\text{no of atm}}{\text{volume}}$$

$$\text{since, } \mu = \frac{\text{no. of atoms}}{V}$$

$$= \frac{N_A \times 3}{\text{Awt}}$$

$$\mu = \frac{M}{n}$$

$$M = \frac{B}{\mu_0}$$

$$= \frac{6.023 \times 10^{23} \times 8906}{58.7 \times 10^{-3}}$$

$$\mu = \frac{B}{\mu_0 n}$$

$$= \frac{0.65}{4\pi \times 10^{-7} \times 9.138 \times 10^{23}}$$

$$= 9.138 \times 10^{28} \text{ Am}^2$$

$$= 5.66 \times 10^{-27} \text{ Am}^{-2}$$

THEORY OF METALS

Physical behaviours of metals are described in terms of quantum mechanics. Quantum mechanics is a generalized form of mechanics applicable to very small objects like e⁻s in the atom. Large systems can also be described by quantum mechanics but mostly a simpler form of mechanics i.e. classical mechanics is used to describe them.

Various processes & phenomenon that takes place within a material can not be explain without having good understanding of quantum mechanics. Although classical mechanics gives us the information about various behaviours of different materials but fails to explain the material behaviours where it comes to microscopic level just as the wave-particle duality behaviour of an electron.

(short notes)

* wave-particle duality.

It was discovered that the particles of atomic dimensions sometimes behave more like waves than discrete classical particles. It was also observed that the electromagnetic waves e.g. X-rays, gamma rays, visible light etc. sometimes exhibit the properties similar to the properties of discrete particle of materials.

Experiment shows that the particles like e⁻s, protons and neutrons exhibit an equivalent wavelength given as

$$\lambda = \frac{h}{P}$$

(1) De-Broglie wavelength.

$$\lambda = \frac{h}{p} \quad \text{--- (i)}$$

where, λ = equivalent wavelength exhibit by particle

p = momentum of the particle ($m \times v$)

h = Planck's universal constant.

$$= 6.629 \times 10^{-34} \text{ Js}$$

$$\rightarrow \frac{h}{\text{charge of } e^-} \text{ in eV}$$

Also we can write,

$$\lambda = h/k \quad \text{--- (ii)} \quad (k = \frac{h}{2\pi}) \quad \text{--- (iii)}$$

$$k = \frac{2\pi}{\lambda} \quad \text{wave number}$$

$$\left. \begin{array}{l} E = mc^2 \\ E = hf \end{array} \right\} \Rightarrow mc^2 = hf \quad \left. \begin{array}{l} p = \frac{h}{\lambda} \\ \lambda = \frac{h}{p} \end{array} \right\}$$

pico $\rightarrow 10^{-12} \text{ m}$
nano $\rightarrow 10^{-9} \text{ m}$
micro $\rightarrow 10^{-6} \text{ m}$
mili $\rightarrow 10^{-3} \text{ m}$

Q.L calculate the wavelength of an electron travelling at 2200 m/s.

$$\text{since, wavelength, } \lambda = \frac{h}{p} = \frac{h}{m_e \times v}$$

$$= \frac{6.629 \times 10^{-34}}{9.1 \times 10^{-31} \times 2200}$$

$$= 3.3 \times 10^{-7} \text{ m.}$$

Ques Another experiment shows that the electro magnetic radiation energy is sometimes contained in packet known as photons ie.

$$E = hf \quad \text{--- (v)}$$

The above eq? (5) is called planck's equation.

Frequency of electromagnetic radiation f is related to the radian frequency (angular frequency) ω by

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi} \quad \text{--- (vi)}$$

Now from eq? (v) & (vi)

$$E = h \frac{\omega}{2\pi}$$

$$\left(\text{since } h = \frac{h}{2\pi} \right)$$

$$\boxed{E = h \omega} \quad \text{--- (vii)}$$

Q.2. What is the energy of photon with a wavelength of 450 nm.

$$\text{Ans. wavelength } \lambda = 450 \text{ nm} = 450 \times 10^{-9} \text{ m}$$

$$\text{since, } E = hf = h \frac{c}{\lambda}$$

$$= \frac{6.624 \times 10^{-34} \times 3 \times 10^8}{450 \times 10^{-9}}$$

$$= 4.416 \times 10^{-19} \text{ J.}$$

Q.3. Calculate the wavelength of a 50 gm bullet travelling at a velocity of 20 ms^{-1} ?

$$\therefore m = 50\text{ gm} = 50 \times 10^{-3}\text{ kg.}$$

$$v = 20\text{ ms}^{-1}$$

$$\lambda = ?$$

$$\text{since, } p = h/\lambda$$

$$m \cdot v = h/\lambda$$

$$\lambda = \frac{h}{m \cdot v} = \frac{6.629 \times 10^{-38}}{50 \times 10^{-3} \times 20}$$

$$= 6.629 \times 10^{-38}\text{ m.}$$

Q.4. Calculate the De-Broglie wavelength associated with a proton moving with a velocity equal to $\frac{1}{90}$ the velocity of light.

$$\therefore \lambda = ?$$

$$v = \frac{1}{90} \times 3 \times 10^8 \text{ m/s}$$

$$m_p = 1.67 \times 10^{-27}\text{ kg.}$$

$$\text{since, } p = \frac{\lambda}{\lambda} \Rightarrow \lambda = \frac{h}{p} = \frac{h}{m_p v}$$

$$= \frac{6.629 \times 10^{-38}}{1.67 \times 10^{-27} \times \frac{1}{90} \times 3 \times 10^8}$$

$$= 2.679 \times 10^{-17}\text{ m.}$$

np (short note)

at Heisenberg's Uncertainty principle.

The wave function of a free electron corresponds to a travelling wave with a single wavelength; this means the travelling wave expands over all space with one same amplitude. So the probability distribution function is uniform throughout the whole space. The e^- can be found anywhere. This means the uncertainty in position of e^- is infinite. At the same time the e^- has fixed well defined wavelength which gives the exact value of momentum of a particle. This corresponds to the uncertainty in momentum being zero.

Hence we can define Heisenberg's uncertainty principle in one sentence as, "It is impossible to measure both the position and momentum of the microscopic particles at the same time".

Mathematically we can write:

$$\Delta x \cdot \Delta p_x \geq \frac{h}{2\pi} \quad \text{or} \quad \Delta x \cdot \Delta p_x \geq \hbar$$

The required value either Δx or Δp_x can be calculated by the boundary condition

$$\text{i.e. } \Delta x \cdot \Delta p_x = \frac{h}{2\pi} \quad \text{where } \Delta p_x = m \cdot \Delta v_x.$$

Q.1. For an electron confined to an infinite potential well of width 0.1 nm. Determine the uncertainty in momentum and kinetic energy.

~~Given~~ $\Delta x = 0.1 \text{ nm}$
 $= 0.1 \times 10^{-9} \text{ m}$

Since, $\Delta x \cdot \Delta p_x = \frac{\hbar}{2\pi c}$

$$\Delta p_x = \frac{6.624 \times 10^{-38}}{2\pi \times 0.1 \times 10^{-9}}$$

$$= 1.054 \times 10^{-28} \text{ kg m/s}$$

Now Kinetic energy

$$KE = \frac{1}{2} mv^2$$

$$= \frac{1}{2m} (mv)^2$$

$$= \frac{(\Delta p_x)^2}{2me}$$

$$= \frac{(1.054 \times 10^{-28})^2}{2 \times 9.1 \times 10^{-31}}$$

$$= 6.107 \times 10^{-19} \text{ J.}$$

Q.2 Estimate the minimum velocity of an apple of mass 100 gm confined to a crate of size 1 m.

$$m = 100 \text{ gm} = 0.1 \text{ kg}$$

$$\Delta x = 1 \text{ m}$$

since,

$$\Delta x \cdot \Delta p_x = \frac{\hbar}{2\pi}$$

$$\Delta p_x = \frac{6.629 \times 10^{-34}}{2\pi \times 1} = 1.059 \times 10^{-34} \text{ kg m/s}$$

$$m \times \Delta v_x = \Delta p_x$$

$$\cancel{\Delta v_x} = \frac{1.059 \times 10^{-34}}{0.1} = 1.059 \times 10^{-33} \text{ m/s.}$$

Need for quantum mechanics

there were two famous experiments conducted early in the 20th century that were unexplained by the classical mechanics and so warranted the need for quantum mechanical ideas.

Experiment - I

It was observed that the beams of particles such as electrons were diffracted from a crystal in a manner similar to the diffraction of x-rays which are electromagnetic waves and the energy released is given by

$$E = mv^2 = \frac{p^2}{2m}$$

$$\text{or, } p = (2mE)^{1/2} \quad \text{--- (1)}$$

Experiment - 2

When the electromagnetic radiation interacts with electrons in a solid or gas, a certain maximum wavelength of radiation is required to eject the electrons from the surface of the material. It is found that for each material, a characteristic energy is required to remove the electrons from the surface of the material. The minimum frequency (and hence the maximum wavelength) of this electromagnetic radiation was found to be

$$f = \frac{\phi}{h} \quad \text{--- (2)}$$

where ϕ = characteristic energy known as work function of the material.

Now comparing the above equations, we can conclude that, the electromagnetic radiation behaves like particles with packet of energy when it collides with microscopic particles like electrons in the material.

From the above discussion, we can clearly see the duality of wave and particle. Particles like electrons sometime behave like wave and wave like electromagnetic radiation sometimes behave like particles. (photons with packet of energy).

In quantum mechanics, there are no clear distinctions between wave and particle. so we can called this phenomenon as wave particle duality or waveicle.

imp for short notes.

Free Electron Theory :-

According to this theory, the outermost electrons known as valence electrons of an atom in metals are very loosely attached to the centre of the atom; so they are not bound to the parent atom. These e⁻s are free to move throughout the whole metal. These e⁻s are attached to different atoms at different instants. These freely roaming e⁻s within the metals form an electron gas. When an atom loses its valence e⁻s, it becomes fully charged ions. So the metallic bond is an unsaturated covalent bond due to the electrostatic attraction between the fully charged ion and -vely charged e⁻s cloud.

If the e⁻s has kinetic energy $E = \frac{1}{2}mv^2$ ——— (i)

$$\text{and } p = \frac{h}{\lambda} = \frac{h}{mv} \quad \text{——— (ii)}$$

$$K = \frac{2\pi}{\lambda} \quad \text{and } V = \frac{\hbar K}{m}$$

we get eq? (i) as

$$E = \frac{m}{2} \cdot \frac{(\hbar K)^2}{m^2}$$

$$\Rightarrow E = \frac{\hbar^2 K^2}{2m} \quad \text{——— (iii)}$$

The eq? (iii) represents the kinetic energy of the free e-s within the material. The relationship between energy released (E) and wave number (K) given by the above equation (iii) is parabolic in nature as shown in the figure below.

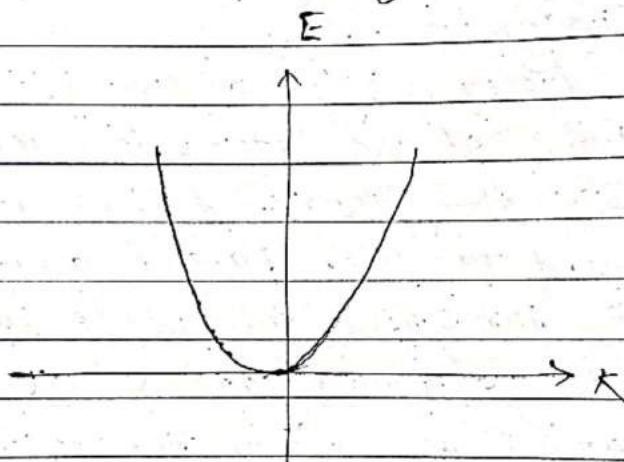
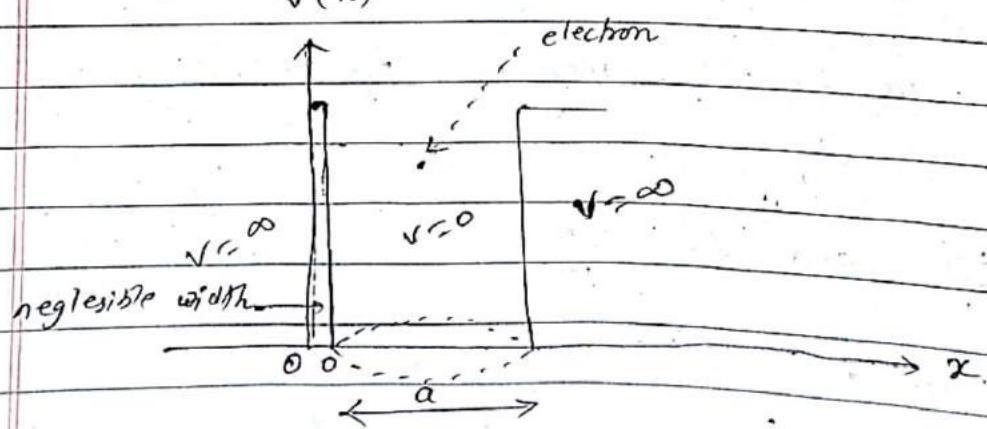


Fig. E-K diagram for the free electron within the metal

~~v. imp~~

Energy Well model of an Electron



consider the behaviour of an electron when it is confined to a certain region between 0 and a . So its potential energy is zero inside the region and infinite outside the region as shown in the above figure. Since the probability of finding of e^- outside the well is zero so,

$$\psi = 0 \text{ when } x \leq 0$$

$$\& x > a$$

Schrodinger's wave equation describes the behaviour of e^- 's nature which is independent of time and is given by

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} (E - V) \psi = 0 \quad (1)$$

where, $\hbar = \frac{h}{2\pi}$, m = mass of e^-

E = total energy of the e^-

V = potential energy of the e^-

since the potential energy inside the well is zero i.e. $V=0$ so eqn (1) can be written as

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{\hbar^2} E \psi = 0 \quad (2) \quad \hbar = \frac{h}{2\pi}$$

This is the second order differential equation and its general solution is

$$\psi(x) = A e^{jkx} + B e^{-jkx} \quad (3)$$

Where the values of A, B & k are calculated by boundary conditions.

i.e. $\Psi(0) = 0$ [when $x=0, \psi=0$]

we get eq 1 (3) as,

$$\Psi(0) = Ae^0 + Be^0$$

$$A + B = 0$$

$$\therefore A = -B \Rightarrow B = -A \quad \text{--- (4)}$$

$$\begin{aligned} \therefore \text{We can write } \Psi(x) &= Ae^{jkx} - Ae^{-jkx} \\ &= A(e^{jkx} - e^{-jkx}) \end{aligned}$$

$$\Psi(x) = A(\cos kx + j \sin kx - \cos kx + j \sin kx)$$

$$\therefore \Psi(x) = 2A j \sin kx \quad \text{--- (5)}$$

We have another boundary condition as

$$\Psi(a) = 0$$

\therefore we can write eq 1 (5) as,

$$2A j \sin ka = 0$$

$$\sin ka = 0$$

$$\text{or } \sin ka = \sin n\pi \quad \text{when } n = 0, 1, 2, \dots, n$$

$$\therefore ka = n\pi$$

$$k = \frac{n\pi}{a} \quad \text{--- (6)}$$

Now substituting the value of $\Psi(x)$ in (2). we get

$$\frac{d^2(2A j \sin ka)}{dx^2} + \frac{8\pi^2 m}{h^2} E (2A j \sin ka) = 0$$

$$\therefore -2A j k^2 \sin ka + \frac{8\pi^2 m}{h^2} E (2A j \sin ka) = 0$$

$$2Ajsinkar \left(-\frac{k^2}{h^2} + \frac{8\pi^2 m}{h^2} E \right) = 0$$

$$-\frac{k^2}{h^2} + \frac{8\pi^2 m}{h^2} E = 0$$

Now putting the value of $k = \frac{2\pi}{a}$ from $g^n(6)$, we get

$$\frac{n^2 \pi^2}{a^2} = \frac{8\pi^2 m}{h^2} E$$

$$E = \frac{n^2 h^2}{8ma^2}$$

Numericals

(1) Consider an electron in an infinite potential well of width 0.1 nm . What is the ground level energy of the electron?

What is the energy required to put the electron from ground energy level to third energy level? Also find the wavelength required to put the electron from second energy level to fourth energy level.

(2) An electron is bounded by potential which closely approaches an infinite square well of width $2.5 \times 10^{-10} \text{ m}$. Calculate the lowest three permissible quantum energies the electron can have. Also calculate the wavelength required to put the electron from first to third energy level.

(1) ∞

$$a = 0.1 \text{ nm} = 0.1 \times 10^{-9} \text{ m}$$

$$\text{since, } E = \frac{n^2 h^2}{8ma^2}$$

for ground level $n=1$,

$$E_1 = \frac{1 \times (6.626 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (0.1 \times 10^{-9})^2}$$

$$= 6.027 \times 10^{-18} \text{ J.}$$

$$E_3 = 3^2 E_1$$

$$= 9 E_1$$

$$= 5.4243 \times 10^{-17} \text{ J.}$$

again,

$$E_3 - E_2 = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{(E_3 - E_2)}$$

$$= \frac{hc}{(16-9) E_1}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{12 \times 6.027 \times 10^{-18}}$$

$$= 2.797 \times 10^{-9} \text{ m.}$$

(8) 509

$$q = 2.5 \times 10^{-20} \text{ m}$$

$$E = \frac{ne^2}{8\pi a^2}$$

$$E_1 = \frac{1^2 \times (6.626 \times 10^{-38})^2}{8 \times 9.1 \times 10^{-31} \times (2.5 \times 10^{-10})^2}$$

$$= 9.63 \times 10^{-19} \text{ J}$$

$$E_2 = 4E_1 = 4 \times 9.63 \times 10^{-19} \text{ J}$$

$$E_3 = 9E_1 = 8.67 \times 10^{-19} \text{ J}$$

$$\text{Now, } E_3 - E_1 = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{9E_1 - E_1}$$

$$= \frac{hc}{8E_1}$$

$$= 2.57 \times 10^{-8} \text{ m}$$

Schrödinger's Wave Equation

Since there is no clear distinction between wave and particle in quantum mechanics, there should be some clear representation (maybe in the form of mathematical expression) of what we called as wave-particle duality.

In quantum mechanics there exists an expression in the form of equation that is used to represent the wave-particle duality.

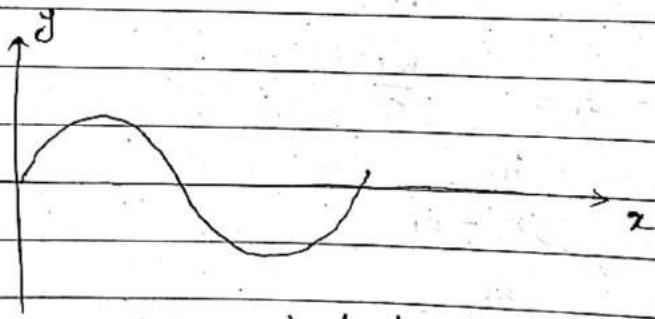


Fig. ordinary wave represented by the microscopic particle.

Similar to the ordinary wave, the displacement about its mean position is represented by wave function as

$$\Psi = A e^{-i(\omega t - \frac{2\pi}{\lambda} x)} \quad \text{(i)}$$

Now by using

$$E = hf = 2\pi\hbar f = 2\pi\hbar c/\lambda$$

$$\& \lambda = \frac{\hbar}{p} = \frac{2\pi\hbar}{P}$$

we get eqn (i) as

$$\psi = Ae^{-\frac{i}{\hbar}(Et - px)} \quad (ii)$$

where

A = maximum displacement from the mean position

λ = wavelength of wave

x = displacement from the origin

f = frequency of vibration

ω = angular radian frequency

$2\pi/\lambda = k$ = wave number

The above equation (ii) is a mathematical description of the wave equivalent of an unrestricted particles with total energy E and momentum p moving in the +ve x -direction. and is correct only for freely moving particles.

Schrodinger's equation is the fundamental equation of quantum mechanics and is also a wave equation in the variable ψ .

Now differentiating the above equation (i) twice w.r.t x we get,

$$\frac{d\psi}{dx} = A j \frac{2\pi}{\lambda} e^{-i(\omega t - \frac{2\pi}{\lambda} px)}$$

$$\therefore \frac{d^2\psi}{dx^2} = A j^2 \frac{4\pi^2}{\lambda^2} e^{-i(\omega t - \frac{2\pi}{\lambda} px)}$$

$$\therefore \psi j^2 \frac{4\pi^2}{\lambda^2}$$

$$\frac{d^2\psi}{dx^2} = -\frac{p^2}{\hbar^2} \psi \quad k^2 = \left(\frac{p}{\hbar}\right)^2$$

..... (ii)

Now differentiating the above eqn (ii) w.r.t. t .

we get

$$\frac{d\psi}{dt} = -\frac{iE}{\hbar} A e^{\frac{it}{\hbar}} \quad -\frac{i}{\hbar} [Et - px] \quad \text{..... (iii)}$$

$$\frac{d\psi}{dt} = -\frac{iE}{\hbar} \psi \quad \text{..... (iv)}$$

As ~~space~~ speeds small compared to that of ~~light~~, the total energy (E) of a particle is the sum of its K.E (1/2 $m v^2$) and P.E (V). where V is in general a function of position (x) and time (t).

$$\text{i.e. } E = \text{K.E.} + \text{P.E.}$$

$$E = \frac{p^2}{2m} + V \quad \text{..... (v)}$$

now multiplying both side of the above eqn (v) by wave function ψ . we get,

$$E\psi = \frac{p^2}{2m} \psi + V\psi \quad \text{..... (vi)}$$

now from eqn (ii), (iv) & (vi) we can write.

$$i\hbar \frac{d\psi}{dt} = -\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V\psi \quad \text{..... (vii)}$$

which is the time dependent form of schrodinger's wave eqn.