

6

Chapter

Semiconductor Materials

6.1 Band Structure of Group IV Materials, Energy Gap

Semiconductors in the present day world are one of the most important materials owing to their widespread use. Almost all the modern sophisticated engineering creations employ semiconductor devices for their effective functioning. XX century has witnessed phenomenal growth and advancement in semiconductor technology.

The group IV materials in Mendeleev's Periodic Table such as Si and Ge have four electrons in the outermost orbit. These electrons are called valence electrons. To fulfill the outermost orbit each atom shares an electron with other atom and the result is the covalent bond. Not all the group IV elements are semiconductor in nature. Si and Ge in particular are semiconductors by their own in-built characteristics. They are called intrinsic semiconductors.

6.1.1 Energy Band in Silicon Crystal

The electronic configuration of Si is $[Ne] 3s^2 3p^2$. However in the vicinity of other atoms, the 3s and 3p energy levels are so close that their interactions result in four new hybrid orbitals ψ_{hyb} that are symmetrically directed away from each other. Each hybrid orbital of one Si atom has one electron and half empty. So, hybrid orbitals of neighbouring atoms overlap and covalent bond with spin-paired electron is formed. A simplified two-dimensional illustration of bonding in Si is shown in figure 6.1.

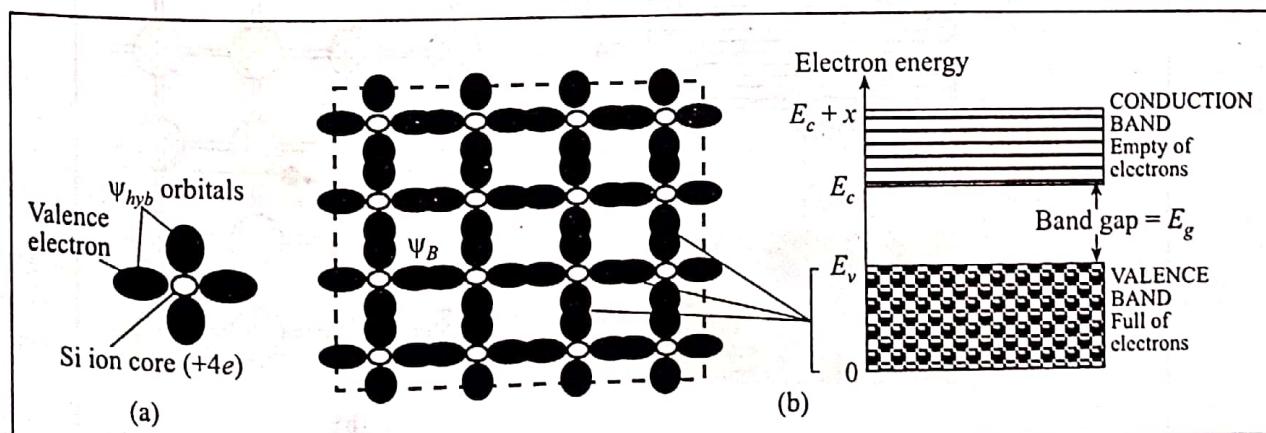


Fig. 6.1 Two dimensional sketch of bonding in Silicon crystal (a); Energy band diagram at 0K (b).

The conduction band contains electronic states that are at higher energies whereas the valence band contains electronic states that correspond to lower energies. The conduction band is separated by valence band by an energy gap E_g , called band gap. The width of the conduction band is called electron affinity and denoted by χ . The energy level E_c represents the bottom of the conduction band and E_v represents top of the valence band. An electron in the valence band like one in conduction band is not localized to an atomic site but extends throughout the whole solid. In the crystals, electrons can always interchange places by tunneling from one bond to the other. The wavefunction of a valence electron extends throughout the solid. So, the electrons in covalent bonds are indistinguishable. The silicon crystal can be crudely represented in as shown in figure 6.2.

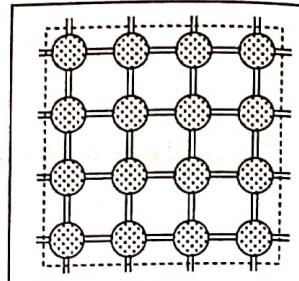


Fig. 6.2 Two-dimensional representation of Silicon crystal showing covalent bonds where each line represents a valence electron.

6.1.2 Electron and Hole

The covalent bond is saturated and there are no empty states available at valence band at absolute zero temperature. The conduction band is completely empty. An electron excited to conduction band is free to move around the crystal and can also respond to an applied field because of the vacant energy levels around. At absolute zero, the valence electron cannot be excited to the conduction band. In order to overcome the energy gap between the two energy bands, electron at top of the valence band must gain an energy at least equal to the band gap, E_g . When a photon with energy greater than or equal to E_g is incident on the electron, it will be able to surmount the energy gap and find itself in the conduction band. The electron in the conduction band with its ability to move freely throughout the solid is termed as **free electron**. The space left behind in the valence band by electron excited to the conduction band is called **hole**. So, in intrinsic semiconductor (the materials are semiconductors on their own), only a free electron or

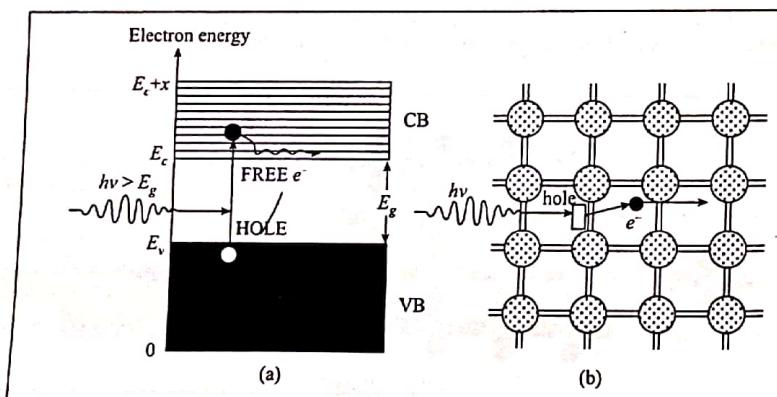


Fig. 6.3 Photon excites electron to conduction band (a). Electron-hole pair generation by incident photon (b).

conduction electron can not be generated but the generation of a conduction electron is inevitably accompanied by the generation of a valence hole. Such generated conduction electron and valence hole are together called **electron-hole pair**.

Electron-hole pair can be generated not only by incident photons, but also by thermal means. Thermal energy causes lattice vibration in crystal, which will periodically deform the Si-Si bond, and in certain regions, the atoms may be moving in such a way that the bond is overstretched and rupturing of the overstretched bond takes place. As the bond ruptures, the electron corresponding to that bond becomes free and is excited to conduction band. The empty electronic state with missing electron becomes positively charged, as there is shortage of one electron. This empty space with positive charge is denoted by h^+ and can also move throughout the whole valence band in the solid in a manner similar to the conduction electron. As we see, in figure 6.4, the neighbouring electron can tunnel through to the vacant place and the hole or vacant place is moved in the direction opposite to the direction of movement of electron in valence band. So, the hole moves in the crystal in the manner if it were a free positively charged particle. When an electric field is applied, the holes will drift in the direction of applied field and contribute to the electric current in the manner similar to the current due to conduction electrons. The current due to both the conduction electron and valence hole is in the same direction. Hole is an empty electronic state in valence band, which behaves as if it were a positively charged particle that is free to respond to an applied electric field.

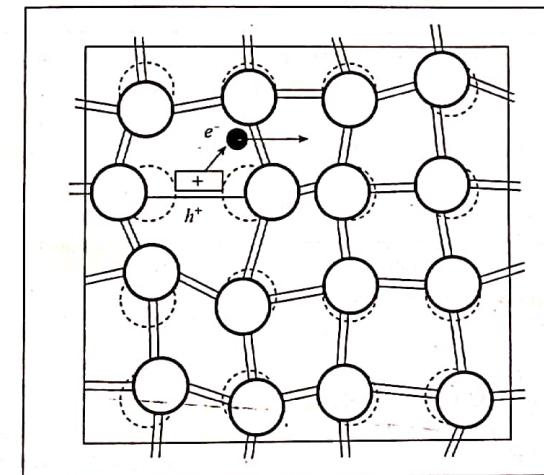


Fig. 6.4 Thermal vibrations of atoms can break bonds and create electron-hole pair.

When a wandering electron in conduction band meets a hole in valence band, the electron will occupy the empty state, which is at lower energy than the present energy state of electron. The electron falls from conduction band to valence band and fills the hole. This process is called recombination (figure 6.5e).

6.1.3 Electrical Conduction in Semiconductor

When an electric field is applied to the semiconductor, both the electrons in the conduction band and holes in the valence band start drifting. Valence holes drift in the direction of applied field whereas

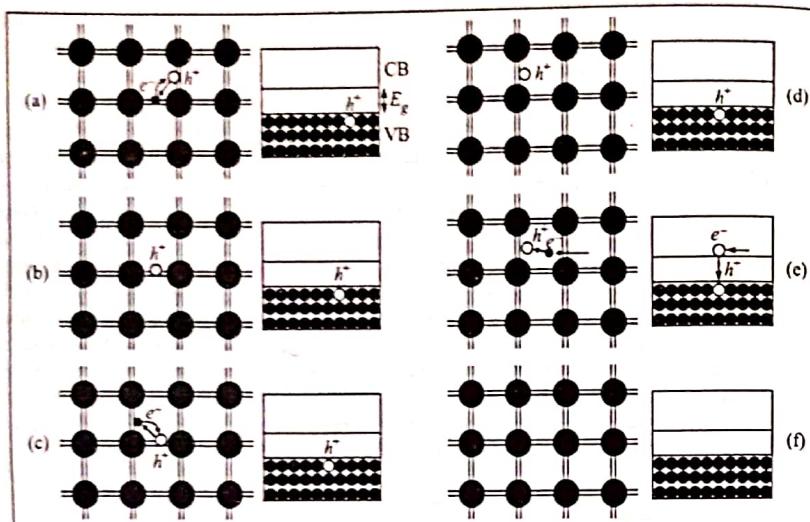


Fig. 6.5 A pictorial view of hole in valence band wandering around the crystal due to tunneling of electrons from neighboring bonds.

conduction electrons drift in the direction opposite to the applied field. Both the valence holes and conduction electrons contribute to current in one direction – in the direction of applied field. From the expression of current density in metals, we can make an analogy and write

$$J = env_{de} + ev_{dh} \quad (6.1)$$

where n and p are respective concentrations of electrons and holes in conduction band and valence band; v_{de} and v_{dh} are electron and hole drift velocities.

The drift velocities are (in x -direction)

$$\begin{aligned} v_{de} &= \mu_e E_x \\ v_{dh} &= \mu_h E_x \end{aligned} \quad (6.2)$$

where μ_e and μ_h are electron and hole drift mobilities.

The drift mobilities can be calculated in analogy with that for metals in chapter II. The only difference is in that case we derived drift mobility for free electron in metal, whereas, here these are the mobilities for conduction band electron and valence band holes where these bands are separated by considerable energy gap. In case of metal we used mass of a free electron to calculate mobility, but here we have to use the effective mass of electron in crystal to account for the internal forces that hinders its movement. For hole also we will use its effective mass, which is different from effective mass of electron in conduction band.

$$\mu_e = \frac{e\tau_e}{m_e^*} \quad (6.3)$$

$$\text{And, } \mu_h = \frac{e\tau_h}{m_h^*} \quad (6.4)$$

The conductivity of the semiconductor is given by

$$\sigma = en\mu_e + en\mu_h \quad (6.5)$$

6.2 Electron and Hole Concentrations in Intrinsic Semiconductor

The conductivity of semiconductor is given by equation (6.5) and mainly depends up on the electron and hole concentrations in conduction and valence bands respectively. By using the Fermi-Dirac function, we will calculate the number of electrons and holes per unit volume in semiconductor.

If n_E is the number of electrons per unit energy per unit volume, $Z(E)$ be the density of states in conduction band, $F(E)$ is the probability of occupation, then the number of electrons in the energy interval E to $E + dE$ is given by

$$n_E dE = Z(E) F(E) dE \quad (6.6)$$

Integrating the above expression from bottom of the conduction band, E_c to top of conduction band, $E_c + \chi$, we will find the electron concentration or number of electrons per unit volume in conduction band.

$$n_i = \int_{E_c}^{E_c + \chi} n_E dE = \int_{E_c}^{E_c + \chi} Z(E) F(E) dE \quad (6.7)$$

Assuming that $(E_c - E_F) \gg kT$, we can write

$$F(E) = \exp\left[-\frac{E - E_F}{kT}\right] \quad (6.8)$$

which is Boltzmann's statistics.

The upper limit in the integral will be $E = \infty$ rather than $E_c + \chi$ as $F(E)$ decays rapidly with energy so that $Z(E) F(E)$ tends to zero near the top of the band. $Z(E)$ is significant only close to the bottom of conduction band.

$$Z(E) = \frac{8\sqrt{2}\pi(m_e^*)^{3/2}}{h^3} (E - E_c)^{1/2} \quad (6.9)$$

Putting from equation (6.9) into equation (6.7), we get

$$n_i = \frac{8\sqrt{2}\pi(m_e^*)^{3/2}}{h^3} \int_{E_c}^{\infty} (E - E_c)^{1/2} \exp\left[-\frac{E - E_F}{kT}\right] dE$$

$$\text{Let us make a substitution } \frac{E - E_c}{kT} = x.$$

So that

$$dE = (kT) dx$$

And the above expression becomes

$$n_i = 4\pi \left(\frac{2kTm_e^*}{h^2}\right)^{3/2} \exp\left(-\frac{E_c - E_F}{kT}\right) \int_{E_0}^{\infty} (x)^{1/2} \exp(-x) dx \quad (6.10)$$

But, we have from table of integrals

$$\int_0^{\infty} (x)^{1/2} \exp(-x) dx = \frac{\sqrt{\pi}}{2}$$

And the expression (6.10) becomes

$$n = 2 \left(\frac{2\pi k T m_e^*}{h^2} \right)^{1/2} \exp \left[-\frac{E_c - E_F}{kT} \right]$$

or

$$n = N_c \exp \left[-\frac{E_c - E_F}{kT} \right] \quad (6.11)$$

where $N_c = 2 \left(\frac{2\pi k T m_e^*}{h^2} \right)^{1/2}$ is called effective density of states at the conduction band edge.

So, by multiplying effective density of states at conduction band edge by Boltzmann's statistics, we can calculate the electron concentration at E_c .

In the similar manner, we can obtain the concentration of holes at the valence band. The probability of occupancy of hole can be found by subtracting the probability of occupation function from unity, indicating the absence of electron. Integrating over valence band, we obtain

$$p = \int_0^{E_v} p_E dE = \int_0^{E_v} Z_{VB}(E) [1 - F(E)] dE \quad (6.12)$$

As in the previous case, assuming that E_v is few kT above the top of valence band, and performing the integration, we will find

$$p = N_v \exp \left[-\frac{E_F - E_v}{kT} \right] \quad (6.13)$$

where N_v is called effective density of states at the valence band edge and equal to

$$N_v = 2 \frac{(2\pi m_h^* k T)^{3/2}}{h^3} \quad (6.14)$$

Equations (6.10) and (6.13) give the electron concentration in conduction band and hole concentration in valence band.

Multiplying the hole concentration and electron concentration, we will have

$$np = N_c \exp \left[-\frac{E_c - E_F}{kT} \right] N_v \exp \left[-\frac{E_F - E_v}{kT} \right]$$

$$np = N_c N_v \exp \left[-\frac{E_c - E_v}{kT} \right] \quad (6.15)$$

Equation (6.15) is commonly termed as "Mass action law".

But

$$E_c - E_v = E_g$$

So

$$np = N_c N_v \exp \left[-\frac{E_g}{kT} \right] \quad (6.16)$$

For intrinsic semiconductors, the electron and hole are generated simultaneously. This means their concentration must be same. The product of np is constant for a given temperature.

$$n = p = n_i$$

where n_i represents intrinsic concentration of the semiconductor.

The intrinsic concentration can be found as

$$n_i = (N_c N_v)^{1/2} \exp \left[-\frac{E_g}{2kT} \right] \quad (6.17)$$

Since

$$n = p = n_i$$

$$\text{or, } N_v \exp \left[-\frac{E_F - E_v}{kT} \right] = (N_c N_v)^{1/2} \exp \left[-\frac{E_g}{2kT} \right] \quad (6.18)$$

For intrinsic semiconductor, the Fermi energy level is denoted by E_{Fi} , which can be found from the above relationship.

$$\exp \left[-\frac{E_{Fi} - E_v}{kT} \right] = \frac{1}{N_v} (N_c N_v)^{1/2} \exp \left[-\frac{E_g}{2kT} \right]$$

Taking natural logarithm on both the sides, we will have

$$\text{or, } -\frac{E_{Fi} - E_v}{kT} = \ln(N_c/N_v)^{1/2} + \left[-\frac{E_g}{2kT} \right]$$

$$\text{or, } -E_{Fi} + E_v = kT \ln(N_c/N_v)^{1/2} + kT \left[-\frac{E_g}{2kT} \right]$$

$$\text{or, } E_{Fi} = E_v - \frac{1}{2} kT \ln(N_c/N_v) + \frac{E_g}{2}$$

$$\text{or, } E_{Fi} = E_v - \frac{1}{2} kT \ln \frac{(8\pi m_e^* k T / h^2)^{3/2}}{(8\pi m_h^* k T / h^2)^{3/2}} + \frac{E_g}{2}$$

$$\text{or, } E_{Fi} = E_v + \frac{E_g}{2} - \frac{3}{4} kT \ln \frac{m_e^*}{m_h^*} \quad (6.19)$$

In the above equation (6.19), if effective masses of electron in conduction band and of hole in valence band are equal to each other, the intrinsic Fermi level in semiconductor lies in the middle of the energy gap.

$$E_{Fi} = E_v + \frac{E_g}{2}$$

$$E_{Fi} = E_v + \frac{E_c - E_v}{2} = E_c - \frac{E_c - E_v}{2} = E_c - \frac{E_g}{2}$$

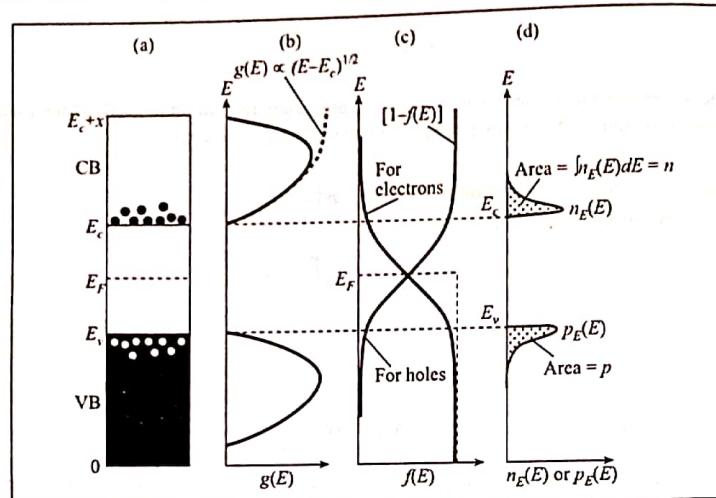


Fig. 6.6 Energy band (a), Density of states (b), Probability of occupation (c), electron or hole concentration (d).

or,

$$E_{F1} = E_c - \frac{E_g}{2} \quad (6.20)$$

Normally, effective masses of conduction electron and valence hole differ slightly, which means the intrinsic Fermi level does not lay precisely at the center of the energy band gap, but is shifted a little bit. In Si, the hole effective mass is slightly greater than that of electron, so the intrinsic Fermi level is slightly above the mid-gap.

Some important properties of the important semiconductors are presented in table 6.1.

6.3 Extrinsic Semiconductors

Pure semiconductors do not always serve our purpose or give operating characteristics of semiconductor material according to our requirement. For this, pure semiconductors are added with materials belonging to either group III or group V in Mendeleev's periodic table. These group III or group V elements (materials)

Table 6.1 Different parameters of different semiconductors

Semi-conductor	E_g , eV	χ , eV	$N_{c'}$, cm^{-3}	$N_{v'}$, cm^{-3}	n_i , cm^{-3}	μ_e , $cm^2 V^{-1} s^{-1}$	μ_h , $cm^2 V^{-1} s^{-1}$	m_e/m_e	m_h/m_e	ϵ_r
Ge	0.66	4.13	1.04 10^{19}	6 10^{18}	2.4 10^{13}	3900	1900	.12a	0.23a	16
Si	1.1	4.01	2.8 10^{19}	1.04 10^{19}	1.45 10^{10}	1350	450	0.56b	0.4b	11.9
GaAs	1.42	4.07	4.7 10^{17}	7 10^{18}	1.8 10^6	8500	400	0.067a,b	0.4a 0.5b	13.1

Note: Effective masses related to conductivity(a) and density of states (b).

added to the pure semiconductor (mostly the silicon) are called **impurities**. And the resulting semiconductor is known as **extrinsic semiconductor**. We can obtain different types of extrinsic semiconductors by adding different types of impurities to the pure semiconductor. There are four valence electrons in silicon. If pentavalent impurities like arsenic and phosphorous are added, four electrons will participate to form covalent bond with neighbouring silicon atoms, whereas the fifth electron cannot participate in bonding since all the bonds are saturated. This extra electron can be contributed to the conduction band. The impurity atom is donating an extra electron to the conduction band of the semiconductor. Such semiconductors are called **n-type** (remember **n** from word **donor**). Similarly, we can get **p-type** (remember **p** from word **acceptor**) semiconductor by adding trivalent impurities like boron, aluminum, gallium, etc.

6.3.1 n-type Semiconductor

By adding small amount of pentavalent impurities into the silicon, we can obtain wide range of characteristics of the resultant semiconductor. When As is added to Si crystal as shown in figure 6.7, it is forced to bond with neighbouring Si atoms in the same diamond crystal structure as Si as it is surrounded by millions of these silicon atoms. Only four of the five valence electrons in the arsenic will participate in bonding with neighbouring silicon atoms. The fifth has nothing to do in bond formation. This extra electron is continuously orbiting the As^+ ion core as shown in figure 6.7 similar to the hydrogen atom in silicon environment. The energy required to free the electron from As site and consequently ionizing As atom can be calculated in analogy with ionization energy of hydrogen, which means to free its electron from the ground state corresponding to $n = 1$. The binding energy of the electron in hydrogen atom is

$$E_b = -E_1 = \frac{e^4 m_e}{8 \epsilon_0^2 h^2} = 13.6 \text{ eV}$$

For electron around As^+ ion core, we have to use instead of absolute permittivity of air, permittivity of silicon and effective mass of electron in Si.

$$E_b^{Si} = \frac{e^4 m_e^*}{8 \epsilon_0^2 \epsilon_r^2 h^2} = 0.032 \text{ eV}$$

where $m_e^* = m_e/3$ and $\epsilon_r = 11.9$.

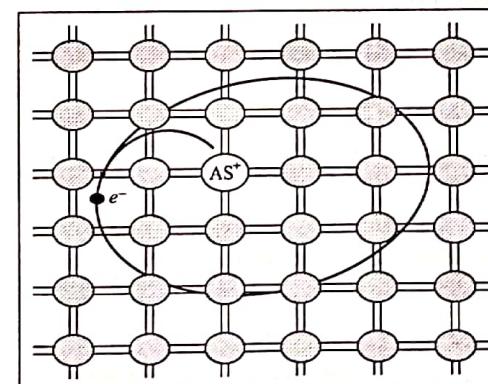


Fig. 6.7 Simplified two-dimensional pictorial view of As-doped Si crystal.

Average thermal energy of atomic vibration is $-3kT(-0.07\text{eV})$. Thus the free fifth valence electron can readily be freed by thermal vibrations in the silicon lattice and excited to the conduction band. The introduction of pentavalent impurity like arsenic creates localized electronic state with localized hydrogenic wavefunction around As^+ . In the energy band diagram, these donor sites are represented by E_d as shown above in figure 6.8.

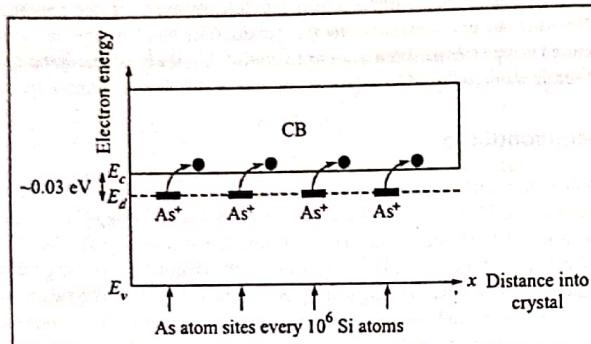


Fig. 6.8 Energy band diagram for an n-type semiconductor doped with arsenic.

In n-type semiconductor, the conduction electrons, which outnumber valence holes, are called **majority charge carrier** and the holes called **minority charge carrier**. In equilibrium condition, a semiconductor is charge neutral, i.e., it possesses equal negative and positive charges. All the donor sites are easily ionized. If n and p represent the extrinsic electron and hole concentration of semiconductor, N_d the donor concentration, then supposing all donor sites are ionized (all donor atoms contribute one electron to CB), electron concentration is given by

$$n = N_d + p \quad (6.21)$$

But

$$np = n_i^2$$

$$p = n_i^2/n$$

Substituting for p in equation (6.21), we have

$$n = N_d + \frac{n_i^2}{n}$$

which gives

$$n = \frac{N_d}{2} \pm \left(\frac{N_d^2}{4} + n_i^2 \right)^{\frac{1}{2}} \quad (6.22)$$

Ignoring the negative sign and for $\frac{N_d^2}{4} \gg n_i^2$, we will have

$$n \approx \frac{N_d}{2} + \frac{N_d}{2} = N_d \quad (6.23)$$

Similarly, we can find the minority carrier concentration as

$$p = \frac{n_i^2}{N_d} \quad (6.24)$$

The hole concentration in n-type semiconductor given by equation (6.24) is less than the intrinsic hole concentration. This is because some of the electrons in conduction band recombine with holes in valence band so as to maintain $np = n_i^2$. As we go on increasing concentration of donor atoms thereby increasing the majority carrier concentration, the minority carrier concentration will be decreased simultaneously. This is called **minority carrier suppression**. It is similar to if the minority carriers were suppressed by the majority carriers.

The conductivity of semiconductor is then

$$\sigma = en\mu_e + ep\mu_h = eN_d\mu_e + e\mu_h \left(\frac{n_i^2}{N_d} \right) \quad (6.25)$$

or,

So, the conductivity is mainly due to the mobility of electrons in the n-type semiconductors.

6.3.2 p-type Semiconductor

As we already know, by adding trivalent impurities to the silicon, we get extrinsic semiconductor of p-type. When very few trivalent impurities (1 impurity atom in 1 mln Si atoms) are added to silicon, they are forced to occupy the same diamond crystalline structure as that of silicon – there is no alternative for the impurity atoms. But when doing so, the bond formed between impurity atom and surrounding four silicon atoms is incomplete because there are only three valence electrons in impurity atom, which can participate in bonding. The unfilled electronic state is the hole. By doping silicon with trivalent impurity like boron, we can create hole in the valence band without creating a conduction electron at the same time. So, we can create only one of the free charge carriers in semiconductor by adding suitable impurities, which is not the case in intrinsic semiconductor (electron and hole are simultaneously created).

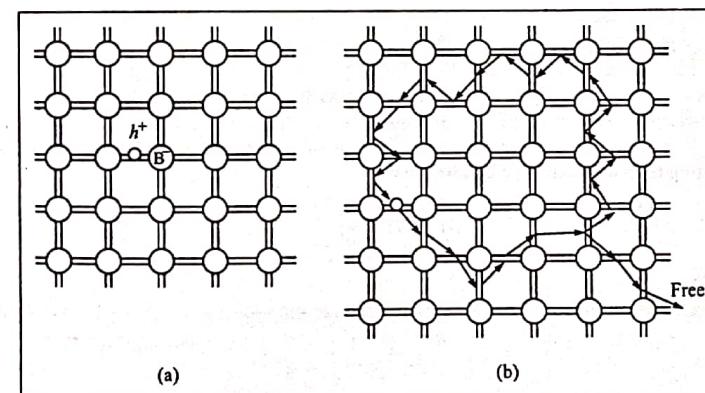


Fig. 6.9 Adding B to Si creates a hole (a).

The hole orbits around the B^- site by tunneling of electrons from neighboring Si atoms (b). Thermally vibrating Si atoms provide enough energy to free the hole from B^- site into the valence band.

As shown in the figure, as the hole moves away from negatively charged ion core, it is attracted by it. Consequently the hole orbits around the ion core. The binding energy is very small ~ 0.05 eV, so at room temperature all the acceptor sites are ionized by thermal vibrations in the lattice. When boron accepts electron from neighboring Si-Si bond, the hole is shifted away from the boron to Si-Si bond in valence band. The boron atom doped to silicon thus accepts an electron from Si-Si bond, and the resulting semiconductor is called p-type (remember p from acceptor).

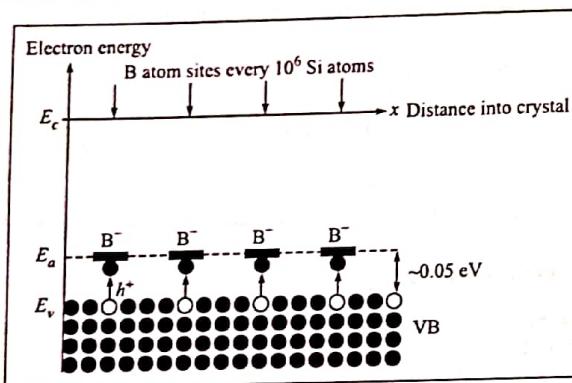


Fig. 6.10 Energy band diagram for p-type semiconductor doped with B showing the ionized acceptor sites and creation of hole in valence band.

Adding trivalent impurities will increase the hole concentration in valence band but at the same time does not increase the electron concentration in conduction band. If n and p be the electron and hole concentration in the semiconductor and N_a the acceptor concentration, then hole concentration is given by

$$p = N_a + n \quad (6.26)$$

But

$$np = n_i^2$$

$$n = \frac{n_i^2}{p}$$

Substituting for p in equation (6.26), we have

$$p = N_a + \frac{n_i^2}{N_a + n}$$

which gives

$$p = \frac{N_a}{2} \pm \left(\frac{N_a}{2} + n_i^2 \right)^{1/2} \quad (6.27)$$

Ignoring the negative sign and for $\frac{N_a}{2} \gg n_i^2$, we will have

$$p = \frac{N_a}{2} + \frac{N_a}{2} = N_a \quad (6.28)$$

The conductivity in p-type semiconductor is mainly due to the hole concentration which can be calculated as

$$\sigma = en\mu_e + ep\mu_h = e\mu_e \left(\frac{n_i^2}{N_a} \right) + eN_a\mu_h \quad (6.29)$$

or,

$$\sigma = eN_a\mu_h$$

Table 6.2 Examples of donor and acceptor ionization energies (eV) in silicon.

Phosphorous	Donors Arsenic	Antimony	Boron	Acceptors Aluminum	Gallium
0.045	0.054	0.039	0.045	0.057	0.072

In trivalent doping of silicon, the electron concentration in hole is decreased as a result of some of the large number of holes created by doping recombine with electrons in conduction band thereby decreasing the electron concentration, which is in minority in p-type semiconductor. This means the minority charge carriers are suppressed by majority charge carriers. This phenomenon is called *minority carrier suppression*.

6.3.3 Compensation Doping

As we have observed in previous discussions, the properties of semiconductors are changed by doping with either trivalent or pentavalent impurities. The level of doping can be different so as to obtain different properties and operating characteristics of the resulting semiconductor device. It is interesting to see what happens when both the trivalent and pentavalent impurities are added to silicon. Adding trivalent impurities will increase the hole concentration in the semiconductor whereas by adding pentavalent impurities to the same will compensate for the increase in hole concentration and decrease in electron concentration. Assuming all donor impurities and acceptor impurities ionized at room temperature, the effective hole concentration and electron concentration are calculated based on which of the impurity concentrations is greater. If N_d and N_a are donor concentration and acceptor concentration respectively, and $N_d > N_a$, the holes due to acceptor doping will recombine with the electrons due to donor doping. So, the electron concentration is

$$n = N_d - N_a$$

$$p = \frac{n_i^2}{n} = \frac{n_i^2}{N_d - N_a} \quad (6.30)$$

Similarly, when $N_a > N_d$, the electrons due to donor doping recombine with holes due to acceptor doping and the resultant concentrations are

$$p = N_a - N_d$$

$$n = \frac{n_i^2}{p} = \frac{n_i^2}{N_a - N_d} \quad (6.31)$$

where n_i is intrinsic concentration in the semiconductor.

6.4 Temperature Dependence of Carrier Concentration

Rewriting the expression (6.17) for intrinsic concentration of semiconductor

$$n_i = \sqrt{N_c N_v} \exp\left[-\frac{E_g}{2kT}\right]$$

where $n_i = n = p$ is valid for intrinsic semiconductors. Equation (6.17) implies that electron and hole concentrations in intrinsic semiconductors are the functions of energy band gap and temperature of the sample. Both the N_c and N_v have $T^{3/2}$ in their expressions, so they are also the functions of temperature. Since there is negative sign in exponential term, it signifies that the increase in band gap will reduce the intrinsic concentration and vice versa. Both from the exponential term and N_c and N_v , it can be observed that the increase in temperature will cause increase in concentration as well. The energy band gap is the built-in characteristic of all materials, which cannot be altered, whereas the temperature of the sample can be controlled to suit our purpose. The intrinsic concentrations of some of the important semiconductors as function of temperature are presented in figure 6.11.

In extrinsic semiconductor, the carrier concentration depends on different parameters at different temperatures. For an n-type semiconductor, at very low temperature, the donor sites will not be ionized and carrier concentration will be very low and determined only by the intrinsic concentration at these temperatures. As temperature increases, thermal energy of lattice vibrations become more and more, and

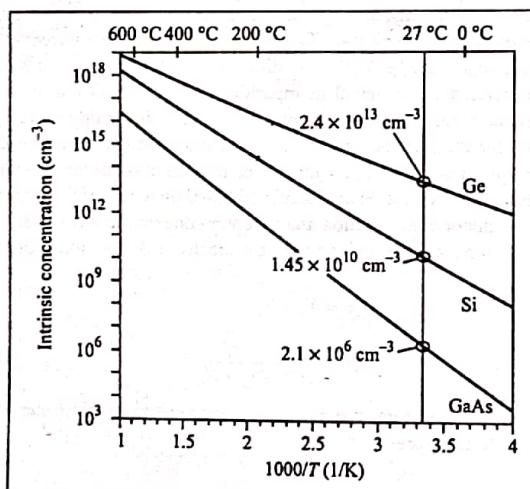


Fig. 6.11 Temperature dependence of carrier concentration of intrinsic semiconductors

most of the donor sites are ionized thereby defining the carrier concentration. In the high temperature range again, the generation of electron-hole pair in semiconductor is more than the impurity concentration, which means even though all impurity sites are ionized their contribution to carrier concentration is surpassed by the electron-hole concentration from within the semiconductor itself. So at high temperatures, doped semiconductor again behaves like intrinsic semiconductor.

In extrinsic semiconductor, the electron concentration at low temperature when thermal vibrations are unable to ionize donor sites and excite the electrons from donor site E_d to conduction band E_c can be expressed as

$$n = \sqrt{\frac{1}{2} N_c N_v} \exp\left[-\frac{E_c - E_d}{2kT}\right] \quad (6.32)$$

$$n = \sqrt{\frac{1}{2} N_c N_v} \exp\left[-\frac{\Delta E}{2kT}\right]$$

The ionization of donor atoms and electron concentration in conduction band at different temperatures in arsenic doped silicon semiconductor is shown in figure 6.12.

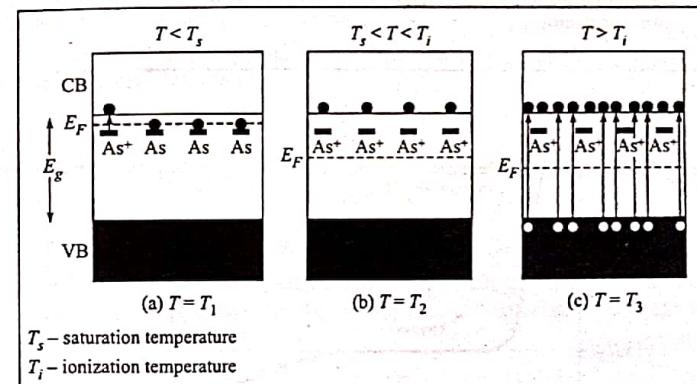


Fig. 6.12 Energy band diagram showing ionized donors at different temperatures

In the log scale, temperature dependence of electron concentration in n-type semiconductor will be as presented in figure 6.13 below.

6.5 Drift and Diffusion of Charge Carriers

6.5.1 Drift of Charge Carriers

The drifting of charge carriers under the influence of applied electric field is similar to that of electrons in metal. In semiconductor, in addition to drift of conduction electrons, there is drift of valence holes as well. The drift velocities of electron and hole are expressed in x -direction as (in analogy with drift velocity of electron in metal)

$$v_{de} = \mu_e E_x \quad (6.33)$$

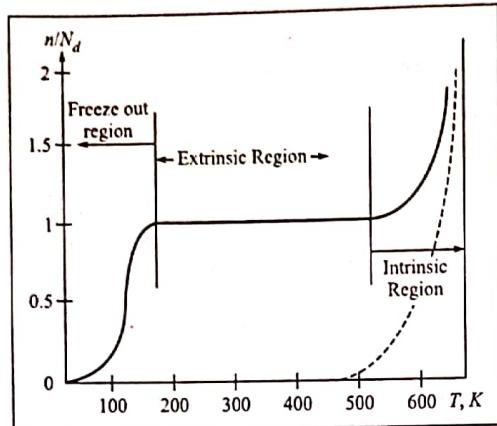


Fig. 6.13 Temperature dependence of extrinsic semiconductor in n-type silicon semiconductor

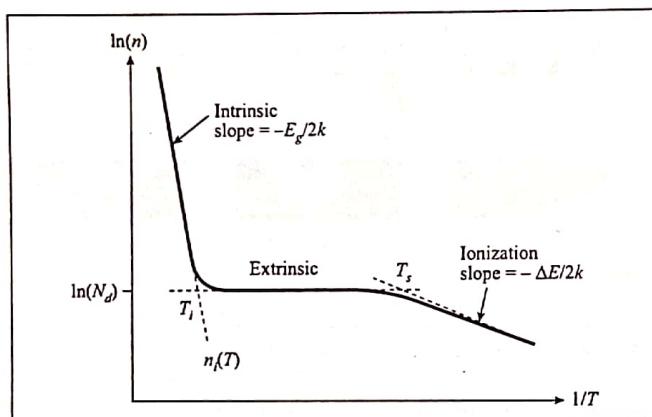


Fig. 6.14 Temperature dependence of electron concentration in n-type semiconductor

$$v_{dk} = \mu_h E_x \quad (6.34)$$

where E_x is the applied electric field in x -direction; μ_e and μ_h are electron and hole mobilities respectively.

The current density due to the drift of conduction electrons and valence holes is given by

$$J_{drift} = \sigma E_x \quad (6.35)$$

where σ is the conductivity of semiconductor.

$$J_{drift} = (en\mu_e + ep\mu_h) E_x \quad (6.36)$$

Conductivity and hence drift current density both depend on the mobility of charge carrier. Mobility is the measure of ease with which the charge carrier can move within the semiconductor crystal. The charge

carriers when drifting in the crystal constantly collide with lattice atoms and with each other, which reduces their mobility. This is the reason why in highly doped semiconductors, the mobility is considerably reduced than in lightly doped one. Similarly, increase in temperature causes increase in lattice vibrations, which in turn will reduce the mobility of the carriers.

6.5.2 Temperature Dependence of Mobility

Recalling from the previous chapter, electron drift mobility in metal is given by

$$\mu = \frac{e\tau}{m_e}$$

where τ is the mean free time between scattering, which itself is given by

$$\tau = \frac{1}{S v_{th} N_s} \quad (6.37)$$

where S – cross-sectional area of scatterer

v_{th} – mean speed of electrons, called thermal velocity;

N_s – number of scatterers per unit volume.

After series of mathematical manipulations, we will arrive at a relationship between mobility limited by lattice vibration scattering in the form

$$\mu_L \propto T^{-\frac{3}{2}} \quad (6.38)$$

And similarly, ionization scattering limited mobility will be

$$\mu_I \propto \frac{T^{\frac{3}{2}}}{N_I} \quad (6.39)$$

where N_I – is the concentration of ionized impurities. Expression (6.39) shows that with increase in temperature as all the impurities are ionized, the mobility limited by ionization scattering also decreases.

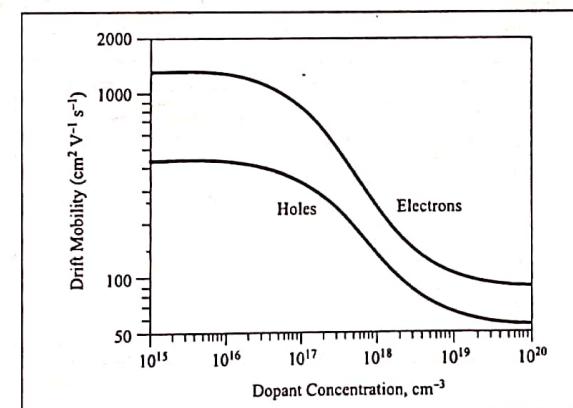


Fig. 6.15 Log-log plot of experimental values of drift mobility versus temperature for n-type semiconductors. The concentrations are in per cm^{-3} .

Mobility of electrons and holes in n-type semiconductor at a doping concentration of $\sim 10^{14}$ per cm^{-3} can be expressed as function of temperatures as $\mu_e \propto T^{-2.3}$ and $\mu_h \propto T^{-2.3}$

Mobility in extrinsic semiconductor heavily depends on the doping concentration. Assuming all impurities ionized at room temperature, the variation in drift mobility of charge carriers in extrinsic semiconductors will be as shown in figure 6.16.

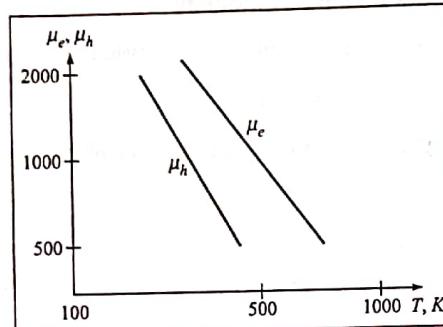


Fig. 6.16 Temperature dependence of mobility in extrinsic semiconductor

The conductivity of semiconductor as given by $\sigma = en\mu_e + ep\mu_h$ depends on carrier concentration and drift mobility. For n-type semiconductor (in extrinsic range), it will depend on donor concentration and electron drift mobility whereas in p-type – on acceptor concentration and hole drift mobility. In n-type semiconductor, at lowest temperatures in the ionization range, the electron concentration depends exponentially on the temperature.

$$n = \left(\frac{1}{2} N_c N_v \right)^{1/2} \exp \left[-\frac{E_c - E_d}{2kT} \right]$$

which dominates the temperature dependence of conductivity. In the intrinsic range at highest temperatures, the conductivity is dominated by temperature dependence of n_i since $\sigma = en_i(\mu_e + \mu_h)$ and n_i is exponential

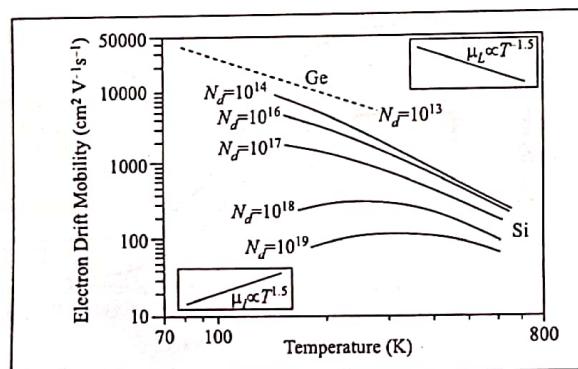


Fig. 6.17 Drift mobility variations according to doping concentration in silicon.

function of temperature in contrast to $\mu \propto T^{-1.5}$. In the extrinsic range $n = N_d$ and is constant, so that the conductivity follows the temperature dependence of the drift mobility.

6.5.3 Diffusion in Semiconductor

Diffusion is a process in which particles tend to spread out, and redistribute themselves as a result of their random thermal vibrations, migrating from region of high concentration to the region of low concentration. This process, if allowed to continue, will end only when there is uniform distribution of particles (same concentration) throughout the material. For diffusion to take place, the particle need not be charged, as thermal motion is responsible for this.

We will examine what happens to the particles confined in compartments shown in figure 6.18. Only the far left compartment has particle in it. The left wall of the compartment is so high that these particles cannot jump out of the compartment in that direction (left wall of the far left compartment). If τ is the mean free time of the particles, there is 50% probability that these particles will jump out of the compartment in either direction. Only in right hand side compartment, there will be half of the particle after τ seconds. After 2τ seconds, again the same process will repeat and there would be particles in 3rd compartment from the left. This process continues till there are equal particles in the entire compartments as the particles cannot jump out of the far right hand side wall as well.

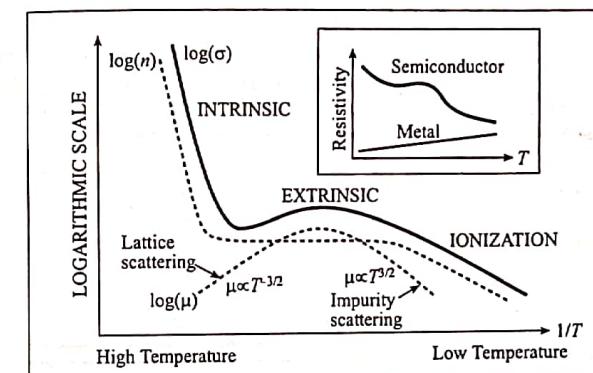


Fig. 6.18 Temperature dependence of conductivity in a n-type semiconductor.

In semiconductors, there is abundance of one of the charge carriers in doped semiconductors (fully uncompensated semiconductors). So, we can visualize a macroscopic diffusion from the region of high concentration of charge carriers to the region where their concentration is low. In semiconductor, the particles are charged, which means their diffusion will give rise to electrical current called the diffusion current in semiconductor.

For diffusion to occur in semiconductor and give rise to diffusion current, there should be abundance of charge carriers in one region and scarcity in the other region. This means, there must be a concentration gradient along the axis along which diffusion is to take place.

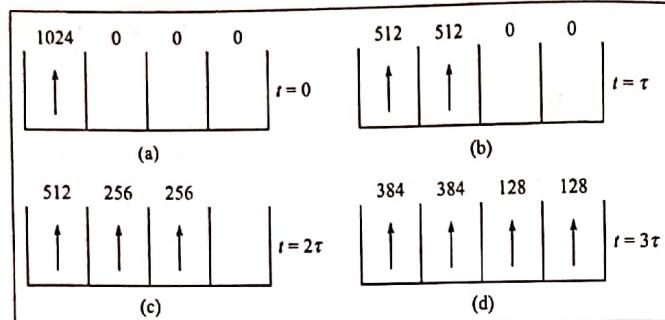


Fig. 6.19 Diffusion of particles in 1-dimensional compartments.

In single-dimension, for diffusion to take place the following condition must be met

$$\frac{dn}{dx} \neq 0, \text{ and } \frac{dp}{dx} \neq 0$$

And, for three-dimension, for diffusion to take place

$$\nabla p \neq 0, \text{ and } \nabla n \neq 0$$

There must be carrier concentration gradient for diffusion to take place. The larger the concentration gradient, the greater is the diffusion of carriers and hence the greater will be flow of current. Diffusion of either of the charge carriers will give rise to current in one direction only since the carriers will always diffuse from region of high concentration to the region of low concentration.

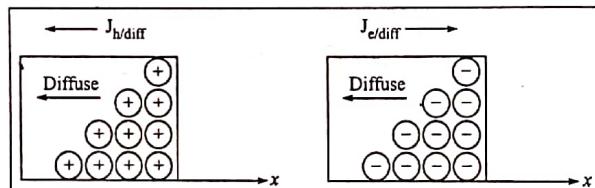


Fig. 6.20 Diffusion of charge carriers giving diffusion currents in doped semiconductors.

The diffusion currents are directly proportional to the carrier concentration gradients as given by

$$J_{e/diff} = eD_e \frac{dn}{dx} \quad (6.40)$$

$$J_{h/diff} = eD_h \frac{dp}{dx} \quad (6.41)$$

is also called Fick's first law of diffusion in one dimension, where D_e and D_h are electron and hole diffusion coefficients respectively measured in cm^2/sec . The negative sign in the expression for diffusion current density complies with the fact that for positive concentration gradient in x -direction (i.e., $dn/dx > 0$ and $dp/dx > 0$), the diffusion will take place in negative x -direction for both the holes and electrons. So, hole diffusion current will flow in negative x -direction (in the direction of diffusion) and electron diffusion current will flow in positive x -direction (in the direction opposite to the diffusion of electron).

For most materials, the diffusion coefficient is found to vary with temperature in an activated manner. Thus $D = D_0 e^{-\Phi/kT}$, where Φ is the activation energy and D_0 is pre-factor.

The total current density in the semiconductor due to electrons (both by drift and diffusion) is given by

$$J_e = J_{e/drift} + J_{e/diff} = en\mu_e E + eD_e \frac{dn}{dx} \quad (6.42)$$

Similarly, total current density in the semiconductor due to holes is given by

$$J_h = J_{h/drift} + J_{h/diff} = ep\mu_h E - eD_h \frac{dp}{dx} \quad (6.43)$$

Now, total current in semiconductor, which is the algebraic sum of contributions by both the electrons and holes, is given by

$$J = J_{drift} + J_{diff} = (J_{e/drift} + J_{h/drift}) + (J_{e/diff} + J_{h/diff}) \quad (6.44)$$

$$J = (J_{e/drift} + J_{e/diff}) + (J_{h/drift} + J_{h/diff}) = J_e + J_h \quad (6.45)$$

6.5.4 Degenerate and Non-degenerate Semiconductors

The general exponential expression for the concentration of electron in conduction band

$$n \approx N_c \exp\left[-\frac{E_c - E_F}{kT}\right] \quad (6.46)$$

is based on replacing Fermi-Dirac statistics with Boltzmann statistics, which is only valid when E_c is several kT above E_F . We assumed that the number of states in the conduction band far exceeds the number of electrons there, so likelihood of two electrons trying to occupy the same state is almost none. So we can neglect Pauli's exclusion principle and can apply Boltzmann statistics for the electron concentration. N_c is the density of states in conduction band. So, the Boltzmann statistics is valid up to $n \ll N_c$. Semiconductors with $n \ll N_c$ and $p \ll N_v$ are termed non-degenerate semiconductors.

When the semiconductors have been excessively doped with, say, donors, electron concentration may reach a very large value of the order of $10^{19} - 10^{20}$ per cm^3 . This electron concentration is comparable to density of states N_c in the conduction band edge. In this case, Pauli's exclusion principle can no longer be neglected and Fermi-Dirac statistics have to be used. Such semiconductors exhibit properties, which are more metal-like than semiconductor-like. For example, resistivity is now directly proportional to temperature. So, semiconductors with $n \gg N_c$ and $p \gg N_v$ are called degenerate semiconductors. In n-type semiconductor, when donor concentration is sufficiently high, the donor atoms become so close to each other that their orbitals overlap to form a narrow energy band that overlaps and becomes part of the conduction band thereby slightly shifting E_c downward. The valence electrons from the donor fill the conduction band right from E_c . The Fermi level in n-type degenerate semiconductor is within the conduction band. In other words, the Fermi level in degenerate n-type semiconductor lies above E_c and in degenerate p-type semiconductor it is below E_v .

6.6 Einstein Relationship

In a non-degenerate semiconductor, the Boltzmann statistics is obeyed and the Fermi level is at least $\sim 3kT$ away from either conduction band-edge or valence band-edge. In such semiconductors, as doping level changes along the semiconductor, there will be corresponding change in the Fermi energy level.

For a non-degenerate semiconductor, the doping profile along its length is given by figure 6.21.

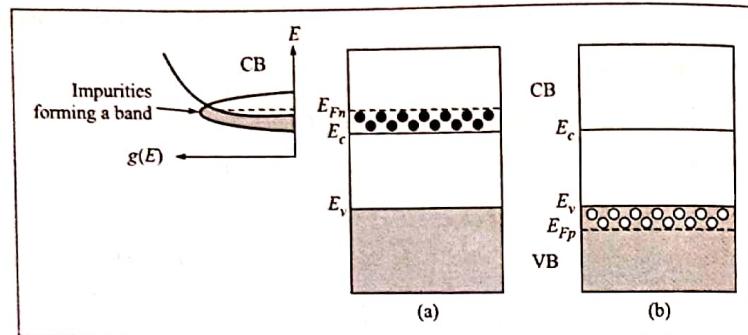


Fig. 6.21 Degenerate n-type semiconductor (a), and p-type semiconductor (b).

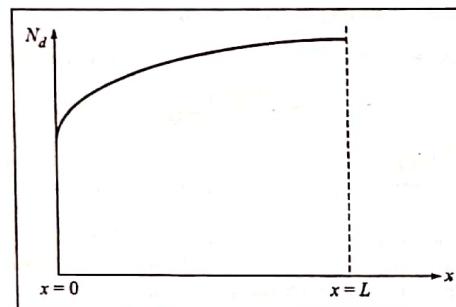


Fig. 6.22 Doping profile of an n-type non-degenerate semiconductor

But the energy band diagram shown in figure (6.23) is not accurate because it shows the variation in Fermi energy level within the semiconductor along its length. In equilibrium condition, Fermi level inside a material or in two materials in contact cannot vary with respect to position. Fermi energy level inside a material must be same. Mathematically

$$\frac{dE_F}{dx} = \frac{dE_F}{dy} = \frac{dE_F}{dz} = 0 \quad (6.47)$$

To keep the Fermi level same throughout the material, other energy bands- conduction band, valence band and intrinsic Fermi level have to bend, which is commonly referred as band bending in non-uniformly doped semiconductors.

Under equilibrium condition, the current in the semiconductor must be zero. The current due to drift and diffusion of electrons is given by equation 6.45. Recalling this equation, we have

$$J_e = (J_{e\text{drift}} + J_{e\text{diff}}) \\ en\mu_e E + eD_e \frac{dn}{dx} = 0 \quad (6.48)$$

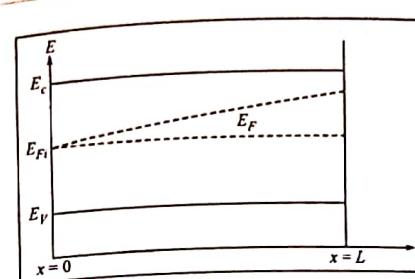


Fig. 6.23 Energy band diagram corresponding to doping profile in figure 6.22.

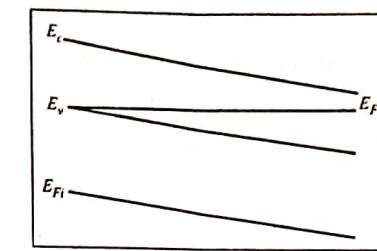


Fig. 6.24 Band bending in non-uniformly donor-doped semiconductor

But the potential energy in electron-volt is related to electrostatic potential V at a point by

$$PE = -eV \quad (6.49)$$

Potential energy can also be expressed with respect to some reference as

$$PE = E_c - E_{ref} \quad (6.50)$$

From 6.49 and 6.50, we have

$$V = -\frac{1}{e} (E_c - E_{ref}) \quad (6.51)$$

The electric field is related to electrostatic potential as

$$\vec{E} = -\nabla V \text{ in 3-dimension} \\ \vec{E} = -\frac{dV}{dx} \text{ in 1-dimension} \quad (6.52)$$

From equation 6.51 and 6.52 for 1-dimension, we have

$$\vec{E} = \frac{1}{e} \frac{dE_c}{dx} = \frac{1}{e} \frac{dE_v}{dx} = \frac{1}{e} \frac{dE_{Fi}}{dx} \quad (6.53)$$

The electron concentration in n-type semiconductor is given by

$$n = N_c \exp\left[-\frac{E_c - E_F}{kT}\right]$$

and, the intrinsic concentration is

$$n_i = N_c \exp\left[-\frac{E_c - E_{Fi}}{kT}\right]$$

So

$$\frac{n}{n_i} = \frac{\exp\left[-\frac{E_c - E_F}{kT}\right]}{\exp\left[-\frac{E_c - E_{Fi}}{kT}\right]} = \exp\left[\frac{E_F - E_{Fi}}{kT}\right]$$

or,

$$n = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right] \quad (6.54)$$

Since Fermi level throughout the material is same, $\frac{dE_F}{dx} = 0$ under equilibrium condition.

Differentiating equation 6.54 with respect to x , we have

$$\begin{aligned}\frac{dn}{dx} &= n_i \exp\left[\frac{E_F - E_{F_i}}{kT}\right] \left[\frac{1}{kT}\right] \left[-\frac{dE_{F_i}}{dx}\right] \\ \frac{dn}{dx} &= -\frac{n_i}{kT} \exp\left[\frac{E_F - E_{F_i}}{kT}\right] \frac{dE_{F_i}}{dx} = -\frac{n}{kT} \frac{dE_{F_i}}{dx}\end{aligned}\quad (6.55)$$

But we have, from equation (6.53)

$$\frac{1}{e} \frac{dE_{F_i}}{dx} = \bar{E}$$

or,

$$\frac{dE_{F_i}}{dx} = e\bar{E}$$

Putting this in equation (6.55) yields

$$\frac{dn}{dx} = -\frac{n}{kT} e\bar{E} \quad (6.56)$$

Substituting from equation (6.56) in equation 6.48, we get

$$en\mu_e \bar{E} + eD_e \left(-\frac{n}{kT} e\bar{E}\right) = 0$$

or,

$$en \left(\mu_e - \frac{D_e}{kT} e\right) \bar{E} = 0$$

or,

$$\mu_e = \frac{D_e}{kT} e$$

or,

$$\frac{kT}{e} = \frac{D_e}{\mu_e} \quad (6.57)$$

Equation (6.57) relating electron diffusion coefficient in n-type semiconductor to electron mobility is called Einstein relationship. For a given n-type semiconductor material at a constant temperature, the ratio of diffusion coefficient to mobility of electron is constant and given by kT/e .

Similarly, for p-type semiconductor, diffusion coefficient and mobility of hole are related to each other in the following manner

$$\frac{kT}{e} = \frac{D_h}{\mu_h} \quad (6.58)$$

The equation (6.57) derived above and known as Einstein's equation can also be derived in the following manner:

Recalling the expression (6.48), we have

$$en\mu_e E + eD_e \frac{dn}{dx} = 0$$

or,

$$en\mu_e E = -eD_e \frac{dn}{dx}$$

$$\frac{dn}{n} = -\mu_e E dx$$

or,
Integrating both sides, we get

$$\ln(n) = -\frac{\mu_e E x}{D_e} + A$$

or,

$$n = A \exp\left(-\frac{\mu_e x E}{D_e}\right) \quad (6.59)$$

According to Boltzmann's statistics, for an electric field we have

$$n = A \exp\left(-\frac{e x E}{kT}\right) \quad (6.60)$$

Hence from equations (5.59) and (5.60), we get

$$-\frac{\mu_e x E}{D_e} = \frac{e x E}{kT}$$

i.e.

$$\frac{\mu_e}{D_e} = \frac{e}{kT} \quad (6.61)$$

The above equation is extensively used in ionic system.

6.7 Recombination and Generation

6.7.1 Direct Recombination and Generation

Above absolute zero temperature, the thermal excitation of electrons from the valence band to the conduction band continuously generates electron-hole pairs, i.e. conduction electron and valence hole pairs. In equilibrium condition, there is an annihilation mechanism by which conduction electron returns to valence hole and fills this empty state. So, when a conduction electron wandering freely in that band falls down to the low energy empty electronic state and meets the hole there, the process is called **recombination**. By this process free electron in conduction band and free hole in valence band are annihilated or destroyed. In figure 6.25 direct recombination mechanism in GaAs is shown. The excess energy of electron is lost as a photon of energy $h\nu = E_g$. Such recombination process results in the emitted light from light emitting diodes (LEDs).

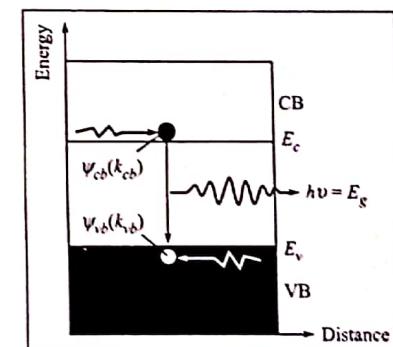


Fig. 6.25 Direct recombination in GaAs

6.7.2 Indirect Thermal Generation and Recombination

In elemental semiconductor crystals like Si and Ge, electrons and holes usually recombine through recombination centers. A recombination center can take up any momentum difference between a hole and

electron, so increases the probability of recombination. This process essentially involves a third party, which may be an impurity atom or crystal defect. The electron is captured at the recombination center and localized at this site. The electron is held or trapped at this center until a hole arrives and recombines with it. The recombination center provides localized electronic state below E_c in the band gap. In figure 6.26 recombination center is shown. When an electron approaches the center, it is captured, and is now bound to the local state. The localized electron waits the arrival of hole and recombines during which some energy is lost lattice vibrations via third party. Emited lattice vibrations are called **phonons**. A phonon is a quantum energy associated with atomic vibration in crystal analogous to photon. Typical recombination centers might be metallic impurities and crystal defects.

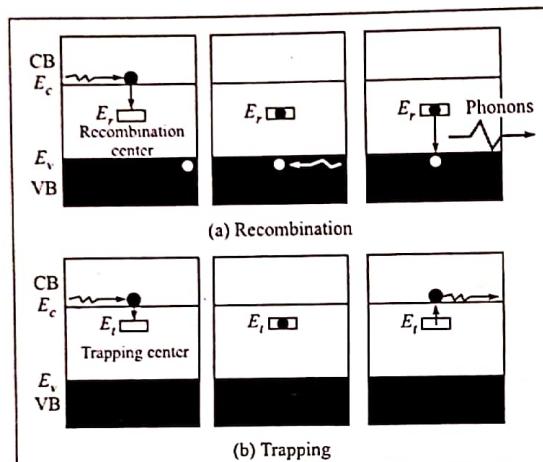


Fig. 6.26 Indirect recombination and generation. Recombination in Si via recombination centre (a); trapping and detrapping of electrons by trapping centres (b).

6.7.3 Photogeneration

By illuminating the surface of the semiconductor, a steady state non-equilibrium carrier concentration can be established. It is apparent that by photon of sufficient energy we can promote electrons from valence band to conduction band thereby creating electron-hole pair. Let the thermal equilibrium concentration of electrons and holes be n_0 and p_0 respectively, and the excess concentrations are Δn and Δp .

Now the total electron and hole concentrations are

$$n = n_0 + \Delta n \quad (6.62)$$

$$p = p_0 + \Delta p \quad (6.63)$$

Photogeneration creates equal number of electron and holes.

$$\Delta n = \Delta p$$

The excess concentrations for example reached at a time $t = 0$, can be denoted by $\Delta n(0)$ and $\Delta p(0)$ are equal to each other and are the values of excess concentrations at $t = 0$ for steady state condition.

If steady state has been reached before $t = 0$ and the source of illumination is removed at $t = 0$, then the excess carrier concentration will decrease to an equilibrium value with a time constant τ_h called excess carrier lifetime or minority carrier lifetime or mean recombination time. This is the average time for which a hole exists in valence band from its generation to recombination, that is, the mean time the hole is free before recombining with an electron. Although the excess majority charge carriers also die along with excess minority carrier, the time during which excess carriers die is referred as minority carrier lifetime because there is still very large number of majority carriers whereas the number of minority carriers is drastically reduced

Rate of increase in hole concentration is equal to the difference of rate of photogeneration and rate of recombination of excess holes with electrons.

$$\frac{d\Delta p}{dt} = G_{ph} - \frac{\Delta p}{\tau_h} \quad (6.64)$$

where G_{ph} is the rate of photogeneration of holes,

$\Delta p/\tau_h$ represents rate of recombination of excess holes.

The recombination time τ_h depends on semiconductor material, impurities, crystal defects, temperature and so forth, and therefore can not be quoted by a typical value but is somewhere from nanoseconds to seconds.

Consider an n-type semiconductor where illumination is switched on at $t = 0$ and switched off at $t = t_{off}$. Using the fact that $G_{ph} = \text{const}$ in $0 < t < t_{off}$, integrating equation (6.64), we have

$$\ln [G_{ph} \tau_h - \Delta p] = -\frac{t}{\tau_h} + C$$

where C is the constant of integration. At $t = 0$, $\Delta p = 0$, so that $C = G_{ph}$.

The solution is

$$\Delta p(t) = \tau_h G_{ph} [1 - \exp(-t/\tau_h)] \text{ in the region } 0 < t < t_{off}$$

As soon as the illumination is switched on, the minority carrier concentration rises exponentially toward its steady-state value $\Delta p(\infty) = \tau_h G_{ph}$ which is reached after $t > \tau_h$.

When illumination is switched off at $t_{off} \gg \tau_h$, $\Delta p(t_{off}) = \tau_h G_{ph}$

Let t' be the time measured from $t = t_{off}$, then $t' = t - t_{off}$.

$$\Delta p(t' = 0) = \tau_h G_{ph}$$

Solving equation 6.61 with $G_{ph} = 0$ in $t > t_{off}$ or $t' > 0$, we have

$$\Delta p(t') = \Delta p(0) \exp(-t'/\tau_h) \quad (6.65)$$

Differentiating equation (6.65) with respect to t , we get

$$\frac{d\Delta p(t')}{dt} = -\frac{1}{\tau_h} \Delta p(0) \exp\left[-\frac{t}{\tau_h}\right]$$

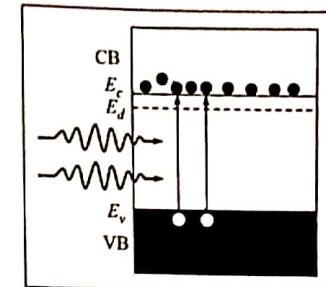


Fig. 6.27 Low level photo-injection into an n-type semiconductor where $D_n < n_0$

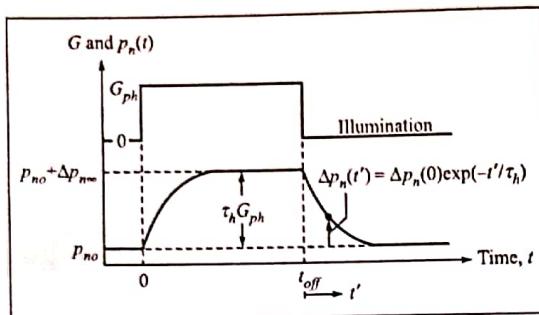


Fig. 6.28 Hole concentration profile when illumination is switched on at $t = 0$ and switched off at $t = t_{off}$. Excess minority carrier concentration (of holes) both rises and decays exponentially.

or,

$$\frac{d\Delta p}{dt} = \frac{\Delta p(t')}{\tau_h} \quad (6.66)$$

which also gives the rate of recombination of excess minority carriers.

6.7.4 Continuity Equations for Carriers

As shown in the figure the hole current density entering the sample $J_h(x)$ can be greater or smaller than the current density leaving the sample $J_h(x + \Delta x)$ depending on the generation or recombination taking place within the sample volume. The net increase in hole concentration per unit time is the difference between the hole flow per unit volume entering and leaving minus the rate of recombination as given by equation (6.64).

The hole current density entering the area A can be converted to hole flow by dividing by electron charge, i.e. $J_h(x)/e$. And, by dividing hole flow entering the area A by Δx , we will get number of carriers per unit volume entering the sample $J_h(x)/(e\Delta x)$. Similarly, the hole flow leaving the volume per unit time is $J_h(x + \Delta x)/(e\Delta x)$ and number of holes per unit volume per unit time leaving the sample is $J_h(x + \Delta x)/(e\Delta x)$.

$$\frac{\partial \Delta p}{\partial t} \Big|_{x \rightarrow \Delta x} = \frac{1}{e} \frac{J_h(x) - J_h(x + \Delta x)}{\Delta x} - \frac{\Delta p}{\tau_h} \quad (6.67)$$

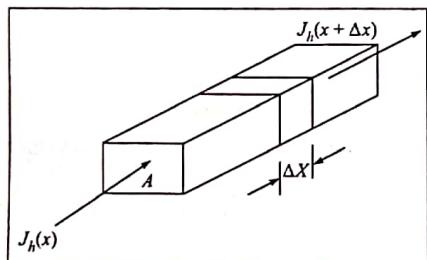


Fig. 6.29 Particle current entering and leaving the sample.

The term in left hand side gives the rate of hole build-up where as the first term in right hand side represents increase in hole concentration in volume $A\Delta x$ per unit time, and the last term is recombination rate.

But, limit $\Delta x \rightarrow 0$

$$\frac{\partial p(x,t)}{\partial t} = -\frac{1}{e} \frac{\partial J_h}{\partial x} - \frac{\Delta p}{\tau_h} \quad (6.68)$$

The equilibrium concentration is independent of time. $P_0 = \text{constant}$. So,

$$\frac{\partial p}{\partial t} = \frac{\partial \Delta p}{\partial t} \quad (6.69)$$

Now from the above two expressions, we have

$$\frac{\partial \Delta p}{\partial t} = -\frac{1}{e} \frac{\partial J_h}{\partial x} - \frac{\Delta p}{\tau_h} \quad (6.70)$$

Equation (6.70) is called the *continuity equation for holes*.

Similarly, for electrons we have

$$\frac{\partial n}{\partial t} = -\frac{\Delta n}{\tau_e} + \frac{1}{e} \frac{\partial J_e}{\partial x} \quad (6.71)$$

is the *continuity equation for electrons*. The positive sign before the last term in right hand side of the expression signifies the opposite nature of flow of electron and resulting current density.

Recalling the expression for hole current density from equation (6.43), we have

$$J_h = J_{h\text{drift}} + J_{h\text{diff}} = ep\mu_e E - eD_h \frac{dp}{dx}$$

Now substituting for J_h in equation (6.67), we will have

$$\frac{\partial \Delta p}{\partial t} = -\frac{\Delta p}{\tau_h} - \frac{1}{e} \frac{\partial}{\partial x} \left(ep\mu_h \bar{E} - eD_h \frac{dp}{dx} \right) \quad (6.72)$$

In the above expression for uniformly doped semiconductor, \bar{E} is small. So the term consisting \bar{E} can be neglected from the expression. So,

$$\frac{\partial \Delta p}{\partial t} = -\frac{\Delta p}{\tau_h} + D_h \frac{\partial^2 \Delta p}{\partial x^2} \quad (6.73)$$

Equation (6.73) is called Fick's 2nd law of diffusion (or *diffusion equation for holes*).

For metals, $\frac{\Delta p}{\tau_h}$ and $p = n$

Under steady-state condition, i.e. $\frac{\partial \Delta p}{\partial t} = 0$, equation (6.73) becomes

$$\frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{\tau_h D_h} = 0 \quad (6.74)$$

$$\frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{L_h^2} = 0 \quad (6.75)$$

where $L_h = \sqrt{\tau_h D_h}$ is called diffusion length for the holes in n-type semiconductor. L_h is the length a hole diffuses, in average before it recombines with an electron in n-type semiconductor, that is why, L_h is also called the minority carrier diffusion length.

Solution of the equation (6.75) is in the form

$$\Delta p = A \exp\left(\frac{x}{L_h}\right) + B \exp\left(-\frac{x}{L_h}\right) \quad (6.76)$$

Similarly, for electrons in p-type semiconductor, we can write second order differential equation in the form

$$\frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{L_e^2} = 0 \quad (6.77)$$

where $L_e = (\tau_e D_e)^{1/2}$ is called diffusion length of electrons in p-type semiconductor. The solution of expression (6.77) is

$$\Delta n = A \exp\left(\frac{x}{L_e}\right) + B \exp\left(-\frac{x}{L_e}\right) \quad (6.78)$$

The diffusion length of injected carriers is important in determining certain dimensions in semiconductor devices such as diodes, transistors, etc. For example, for efficient operation of transistor, the width of base region must be small compared to the diffusion length of injected carriers. Otherwise a large portion of them would be lost by recombination before reaching the collector-base junction.

6.8 PN Junction

6.8.1 Ideal Abrupt Junction

In an n-type silicon semiconductor electrons are majority charge carriers and holes are minority charge carriers. Similarly, in a p-type semiconductor, holes are majority carriers and electrons are minority carriers.

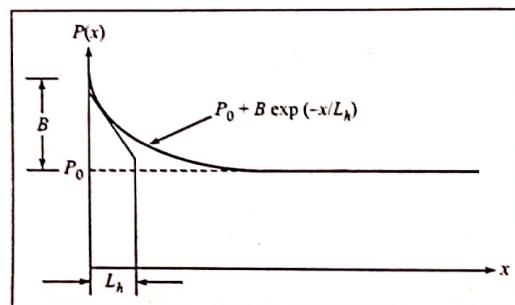


Fig. 6.30 Steady-state minority carrier concentration profile in an n-type semiconductor illuminated at one end.

Let us see what happens when these two semiconductors are brought together to form an abrupt junction (there is abrupt discontinuity between p and n regions). The abrupt junction is called metallurgical junction.

As shown in figure 6.31.a an n-type and a p-type semiconductor are brought together. The n-region has excess electrons in conduction band and immobile positive ions whereas the p-region has excess free holes and immobile negative ions. The electron concentration and hole concentrations in n-region are n_{n0} and p_{n0} respectively. Similarly, the hole and electron concentrations in p-region are p_{p0} and n_{p0} respectively. Due to concentration gradient of holes from p region to n region, they will diffuse toward n-region and electrons will diffuse toward p region thanks to the concentration gradient from n region to p region. These holes and electrons diffusing toward each other meet and recombine around the junction. The junction region consequently becomes depleted of free charge carriers in comparison to the bulk n and p regions away from the junction. The region around junction is thus depleted of free charge carriers, so it is called depletion layer. The free charge carriers diffusing to p and n regions leave behind exposed positively charged impurity ions at or around the junction. So, this region is also called space charge region or layer

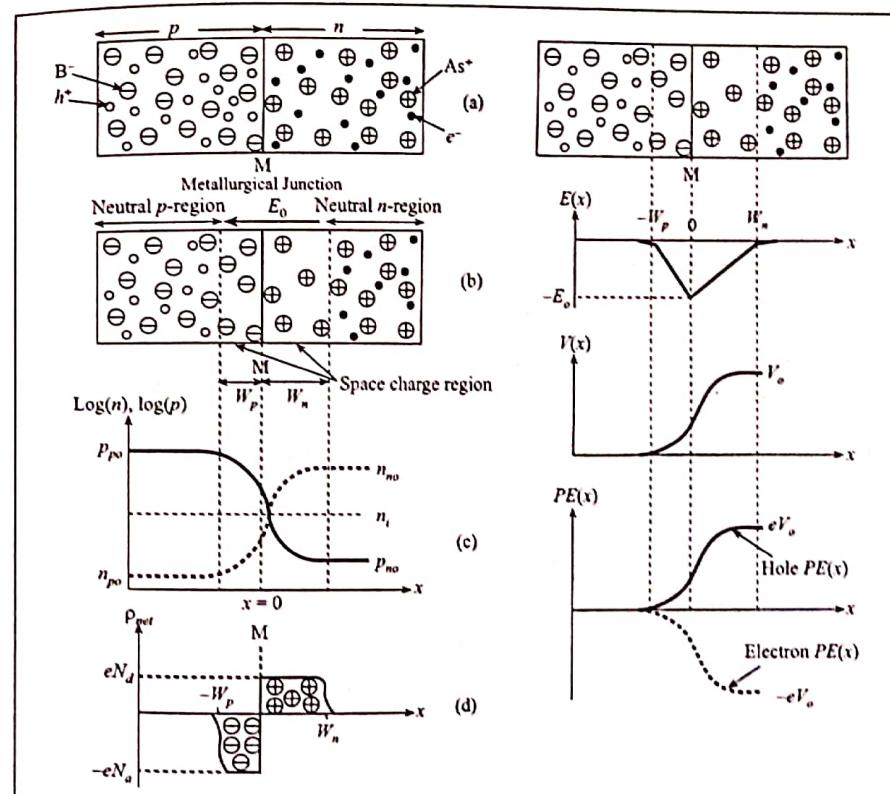


Fig. 6.31 Abrupt idealized pn junction

(SCL). Through this region, the concentration of electron and hole changes in such a way that they transform from majority to minority carriers and vice versa. So, this region is also called transition region. Figure 6.31.b shows the space charge region and figure 6.31.d shows the carrier concentration profile in logarithmic scale. Here the acceptor concentration is greater than the donor concentration.

Even without applied field, there is an internal field E_0 directed from positive ions to negative ions. This internal field tries to drift holes back to the p-region and electrons back to the n-region. In other words, the internal field tries to drive the carriers in direction opposite to their diffusion. The strength of the field will be determined by the equilibrium condition at which the number of carriers diffusing is compensated by the number of carriers drifting in opposite direction. For uniformly doped p and n regions, the net space charge density is as shown in figure 6.31.d. The exposed charged in the left side of the junction must be equal to the exposed charge in the right side of the junction to maintain the charge neutrality.

$$N_a W_p = N_d W_n \quad (6.79)$$

Since

$$N_a > N_d, W_p < W_n.$$

The electric field and net space charge density are related by

$$\frac{dE}{dx} = \frac{\rho_{net}}{\epsilon} \quad (6.80)$$

where ρ_{net} is the net space charge density and ϵ is the permittivity of the medium.

The electric field can be found as

$$\bar{E}(x) = \frac{1}{\epsilon} \int_{-W_p}^x \rho_{net}(x) dx \quad (6.81)$$

which will give the maximum electric field at the junction and is given by

$$\bar{E}_0 = -\frac{eN_d W_n}{\epsilon} = \frac{eN_a W_d}{\epsilon} \quad (6.82)$$

The potential at any point can be calculated from the electric field.

$$\bar{E} = -\frac{dV}{dx}$$

Net current due to electron and due to hole are separately equal to zero. So,

$$J_e = J_{edrift} + J_{ediff} = kn\mu_e \bar{E} + eD_e \frac{dn}{dx} = 0$$

$$\bar{E} = -\frac{D_e}{n\mu_e} \frac{dn}{dx}$$

$$-\frac{dV}{dx} = -\frac{kT}{en} \frac{dn}{dx}$$

Integrating from $-W_p$ to W_n , we have

$$\int_{-W_p}^{W_n} dV = \frac{kT}{e} \int_{-W_p}^{W_n} \frac{dn}{n}$$

or,

$$V(W_n) - V(-W_p) = \frac{kT}{e} [\ln n(W_n) - \ln n(-W_p)] \quad (6.83)$$

$$V_0 = V(W_n) - V(W_p) = \text{Built-in potential}$$

The electron concentration at $x = -W_p$ is $n_{p0} = \frac{n_i^2}{N_a}$ and at $x = W_n$, $n_{n0} = N_d$

Putting these values in equation (6.80), we will find the built-in potential.

$$V_0 = \frac{kT}{e} \left[\ln N_d - \ln \frac{n_i^2}{N_a} \right]$$

or,

$$V_0 = \frac{kT}{e} \ln \frac{N_a N_d}{n_i^2} \quad (6.84)$$

At $x = -W_p$, $n_{p0} = n_i^2/N_a$ and at $x = W_n$, $n_{n0} = N_d$. Putting these values in equation (6.84) yields

$$V_0 = \frac{kT}{e} \ln \left(\frac{n_{n0}}{n_{p0}} \right) \quad (6.85)$$

From which we can find

$$\frac{n_{n0}}{n_{p0}} = \exp \left(\frac{eV_0}{kT} \right) \quad (6.86)$$

Similarly we can write

$$\underbrace{V_0}_{\sim} = \frac{kT}{e} \ln \left(\frac{p_{p0}}{p_{n0}} \right) \quad (6.87)$$

And

$$\frac{p_{p0}}{p_{n0}} = \exp \left(\frac{eV_0}{kT} \right) \quad (6.88)$$

In the above expressions, n_{n0} and n_{p0} represent electron concentrations in bulk of n-region and p-region respectively. Similarly, p_{p0} and p_{n0} represent hole concentration in bulk p and n-region respectively.

The built in potential

$$V_0 = -\frac{\bar{E}_0 W_0}{2} = \frac{eN_d W_n}{2\epsilon} W_0 \quad (6.89)$$

Where $W_0 = W_p + W_n$ is the depletion width. So, $W_p = W_0 - W_n$

We also have,

$$W_n N_d = W_p N_a$$

$$W_n N_d = (W_0 - W_n) N_a = W_0 N_a - W_n N_a$$

or,

$$W_n = \frac{W_0 N_a}{N_a + N_d} \quad (6.90)$$

Now substituting W_n from equation (6.90) in to equation (6.89), we have

$$V_0 = \frac{eN_a N_d W_0^2}{2\epsilon(N_a + N_d)} \quad (6.91)$$

Depletion width can be expressed as

$$W_0 = \sqrt{\frac{2eV_0}{\epsilon} \left(\frac{N_a + N_d}{N_a N_d} \right)} = \sqrt{\frac{2eV_0}{\epsilon} \left(\frac{1}{N_a} + \frac{1}{N_d} \right)} \quad (6.92)$$

Substituting for V_0 from equation (6.84) in to equation (6.92), we have

$$W_0 = \sqrt{\frac{2ekT}{e^2} \ln \frac{N_a N_d}{n_i^2} \left(\frac{1}{N_a} + \frac{1}{N_d} \right)} \quad (6.93)$$

If one of the impurity concentrations is very small compared to the other, let $N_a \ll N_d$, then built-in potential and depletion width can be found as

$$V_0 = \frac{eN_a W_0^2}{2\epsilon} \quad (6.94)$$

$$W_0 = \sqrt{\frac{2ekT}{e^2} \ln \frac{N_d}{n_i^2}} \quad (6.95)$$

In this case, almost all of the depletion region will fall in p-region as doping concentration of n-region is much heavier than that of p-region, which requires charge from very small range in n-region to be balanced by charge from relatively larger range in p-region to balance the former.

6.8.2 Forward Biased pn Junction

When there is no external voltage applied to the pn junction, there will be no current through it. When an external voltage source (battery) is connected to the pn junction in such a manner that the positive terminal of the battery is connected to far-end of p-region and negative terminal is connected to far end of n-region, then the pn junction is called forward biased. The internal electric field and built-in potential both will be reduced as shown in figure 6.32. The new electric field is still directed from right of the junction (depletion region in n-side) to left of the junction (to the depletion region in p-side) because $E_o > E$. The applied voltage will mostly drop in the depletion region as it has high resistivity compared to the bulk because there is far higher number of free charge carriers in bulk compared to immobile ions in the depletion region. So, applied voltage directly opposes built-in potential. Consequently, built-in potential is reduced from V_0

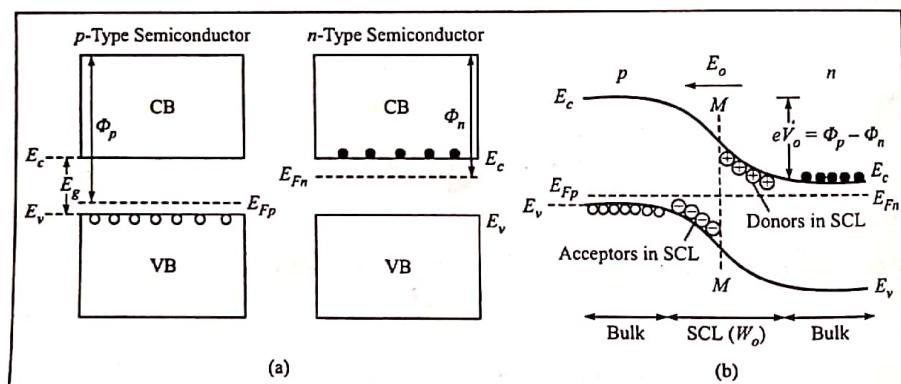


Fig. 6.32 Energy band diagram of two isolated p and n-type semiconductors (a). Energy band diagram when p and n-type semiconductors are in contact (b).

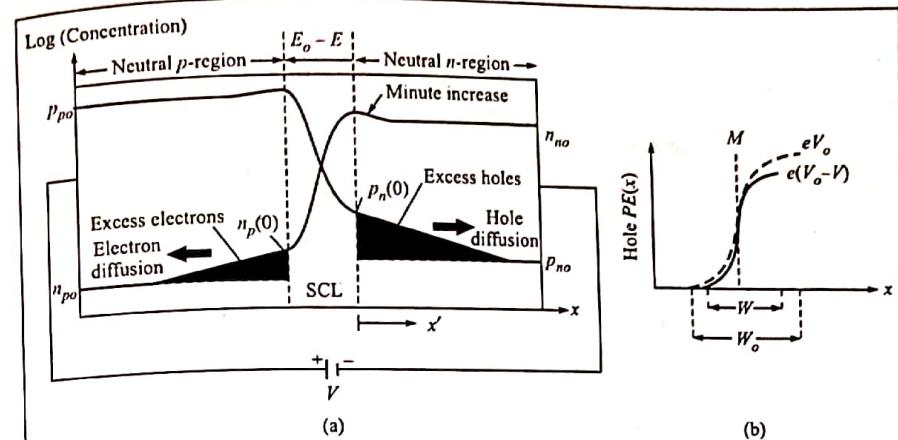


Fig. 6.33 Forward biased pn junction. Carrier concentration profile across the device (a); hole potential energy with or without bias.

to new value ($V_0 - V$). The potential barrier to diffusion is then reduced which has drastic consequences. The probability that a hole will surmount this height and diffuse to n-region is now proportional to $\exp[-e(V_0 - V)/kT]$. Many holes will diffuse across depletion region and enter the n-side. This is like injection of excess minority carriers, holes, into the n-region. In the similar manner electrons are injected into p-region.

The hole concentration in n-region is $p_n(0) = p_n(x' = 0)$ just outside the depletion region at $x' = 0$ is due to excess of holes diffusing as a result of reduction in- built-in potential barrier. This concentration $p_n(0)$ is found from probability of surmounting the potential energy barrier $e(V_0 - V)$ and given by

$$p_n(0) = p_{po} \exp\left[-\frac{e(V_0 - V)}{kT}\right] \quad (6.96)$$

But, we have, recalling equation (6.88)

$$\frac{p_{po}}{p_{no}} = \exp\left(\frac{eV_0}{kT}\right)$$

or,

$$p_{po} = p_{no} \exp\left(\frac{eV_0}{kT}\right)$$

Substituting this in equation (6.96), we get

$$p_n(0) = p_{no} \exp\left(\frac{eV_0}{k}\right) \exp\left[-\frac{e(V_0 - V)}{kT}\right] = p_{no} \exp\left(\frac{eV}{kT}\right) \quad (6.97)$$

The expression (6.97) represents what is called "Law of junction".

Injected holes diffuse in n-region and eventually recombine with electrons in this region, as there are plenty of electrons there. These electrons lost by recombination with holes are readily replenished by negative terminal of the battery connected to n-side. As the p-region can supply more and more holes to

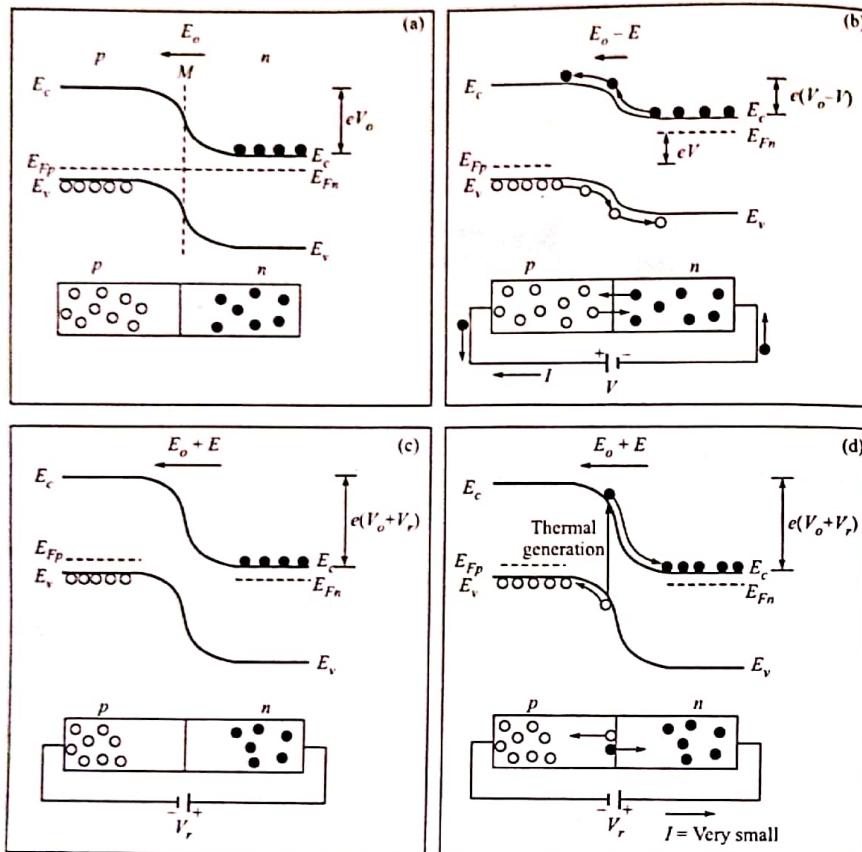


Fig. 6.34 Energy band diagram of *pn* junction. Open circuit (a), forward bias (b), reverse bias(c), reverse bias including thermal generation of electron-hole pair (d).

n-side, current due to diffusion of holes can be sustained. The positive terminal connected to p-side will supply the holes to this side. Similarly, electrons are injected from n-side to p-side and electron concentration just outside the depletion region in p-side is given by

$$n_p(0) = n_{p0} \exp\left(\frac{eV_0}{kT}\right) \exp\left[-\frac{e(V_0 - V)}{kT}\right] = n_{p0} \exp\left(\frac{eV}{kT}\right) \quad (6.98)$$

By forward biasing a *pn* junction, a current through the junction can be maintained. Holes diffusion in n-side and electrons diffusing in p-side will give rise to current in the same direction. The current flow seems to be surprisingly due to the diffusion of minority charge carriers, there is, however, current due to drift of majority carriers as well.

If the length of p-side and n-side are longer than minority carrier diffusion lengths, then the hole concentration $p_n(x')$ on the n-side will fall exponentially toward thermal equilibrium value p_{n0} . So,

$$\Delta p(x') = \Delta p_n(0) \exp\left(-\frac{x'}{L_h}\right) \quad (6.99)$$

where $\Delta p_n(x') = p_n(x') - p_{n0}$ is the excess carrier distribution and L_h is the hole diffusion length. The hole current density is

$$J_{h/diff} = -eD_h \frac{dp_n(x')}{dx'} = -eD_h \frac{d\Delta p_n(x')}{dx'} \quad (6.100)$$

or,

$$J_{h/diff} = \frac{eD_h}{L_h} \Delta p_n(0) \exp\left(-\frac{x'}{L_h}\right) \quad (6.100)$$

At $x' = 0$ just outside the depletion region, the hole diffusion current is

$$J_{h/diff} = \frac{eD_h}{L_h} \Delta p_n(0)$$

But

$$\Delta p_n(0) = p_n(0) - p_{n0} = p_{n0} \exp\left(\frac{eV}{kT}\right) - p_{n0} = p_{n0} \left[\exp\left(\frac{eV}{kT}\right) - 1 \right]$$

Putting this in to above equation will give

$$J_{h/diff} = \frac{eD_h}{L_h} p_{n0} \left[\exp\left(\frac{eV}{kT}\right) - 1 \right] \quad (6.101)$$

But

$$p_{n0} = \frac{n_i^2}{n_{n0}} = \frac{n_i^2}{N_d}$$

$$J_{h/diff} = \frac{eD_h n_i^2}{L_h N_d} \left[\exp\left(\frac{eV}{kT}\right) - 1 \right] \quad (6.102)$$

Similarly, the diffusion current density for electrons is given by

$$J_{e/diff} = \frac{eD_e n_i^2}{L_e N_a} \left[\exp\left(\frac{eV}{kT}\right) - 1 \right] \quad (6.103)$$

The total diffusion current density is

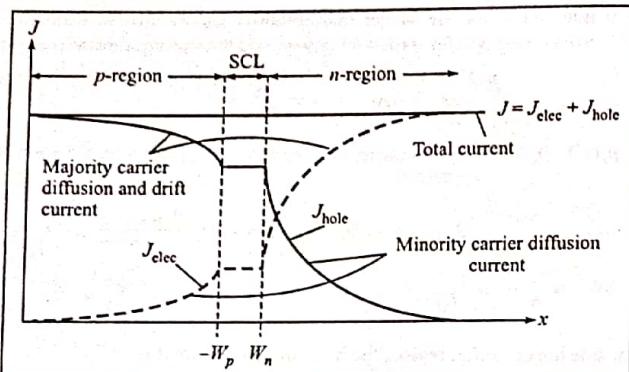
$$J_{diff} = J_{e/diff} + J_{h/diff} = e n_i^2 \left(\frac{D_e}{L_e N_a} + \frac{D_h}{L_h N_d} \right) \left[\exp\left(\frac{eV}{kT}\right) - 1 \right] \quad (6.104)$$

or,

$$J_{diff} = J_{s0} \left[\exp\left(\frac{eV}{kT}\right) - 1 \right] \quad (6.104)$$

Equation (6.104) is called *ideal diode equation*. In terms of current, this can be expressed as

$$I_{diff} = J_{diff} A = A J_{s0} \left[\exp\left(\frac{eV}{kT}\right) - 1 \right] = I_{s0} \left[\exp\left(\frac{eV}{kT}\right) - 1 \right] \quad (6.105)$$

Fig. 6.35 Current through the *pn* junction during forward bias condition.

Where

$$J_{s0} = \left(\frac{D_e}{L_e N_a} + \frac{D_h}{L_h N_d} \right) e n_i^2 \quad (6.106)$$

 J_{s0} is called reverse saturation current density. And I_{s0} is reverse saturation current.

6.8.3 Reverse Biased *pn* Junction

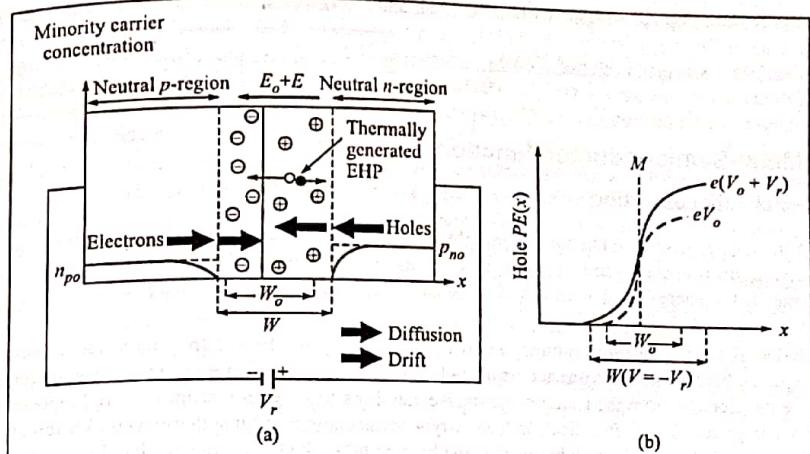
When *pn* junction is connected to a voltage source with p-region connected to negative terminal of voltage source and n-region to positive terminal, then the junction is said to be reverse biased.

The negative terminal of the battery will attract holes in p-region to move away from the depletion layer resulting in more exposed negative acceptor ions and thus a wider space charge layer. In the same way, the positive terminal will attract electrons away from the depletion region exposing more positively charged donors and consequently widening the depletion region in this side as well. The movement of electrons in the n-region toward the positive battery terminal cannot be sustained, as there is no electron supply to this side. And, p-side cannot supply electrons to n-side, as it has almost none. The electrons from the battery cannot enter the p-side and transit across to n-side because before they get a chance to do so, they will be recombined with holes. So, a current cannot be sustained through *pn* junction in reverse biased condition. The applied voltage adds to the built-in potential. Similarly, the electric field in the depletion layer is the sum of internal field and applied field. The holes in the n-side near the depletion layer can be extracted by the field and driven across depletion layer top the p-region. These holes can exit the p-region and enter the battery, giving rise to a very small current. Similarly, the electrons in p-side near the depletion region are extracted and driven away through the depletion layer onto n-side and again give rise to a small current. There will be small diffusion current through the junction due to holes diffusing from n-region to depletion layer and electrons diffusing to depletion layer from p-region.

Recalling the expression (6.106) the reverse saturation current density is given by

$$J_{s0} = \left(\frac{D_e}{L_e N_a} + \frac{D_h}{L_h N_d} \right) e n_i^2 \quad (6.106)$$

is called *Shockley equation*.

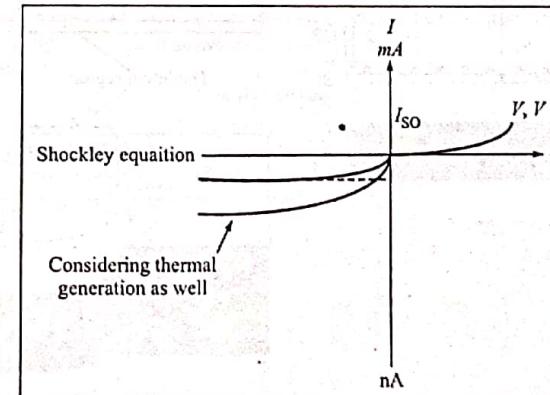
Fig. 6.36 Reversed biased *pn* junction. Minority carrier profile (a); Hole PE across the junction (b).

Considering the effect of thermal generation the equation for reverse current will be modified to

$$J_{rev} = \left(\frac{D_e}{L_e N_a} + \frac{D_h}{L_h N_d} \right) e n_i^2 + \frac{e n_i W}{\tau_g} \quad (6.107)$$

where τ_g is the mean thermal generation time to generate electron-hole pair. And, thermal generation current is

$$J_{gen} = \frac{e n_i W}{\tau_g}$$

Fig. 6.37 Reverse I-V characteristics of a *pn* junction.

The width of the depletion region will be increased and can be given by

$$W = \sqrt{\frac{2\epsilon(V_0 + V)}{e} \frac{(N_a + N_d)}{N_a N_d}} = \sqrt{\frac{2\epsilon}{e} \left(\frac{1}{N_a} + \frac{1}{N_d} \right) (V_0 + V)} \quad (6.108)$$

6.9 Metal-Semiconductor Junction

6.9.1 Schottky Junction

A metal and an n-type semiconductor are brought into contact. The work function of the metal is greater than the work function of the semiconductor, that is, the Fermi level in semiconductor is higher than Fermi level in metal. The energy band diagrams of metal and n-type semiconductor when placed separately and brought in to contact are shown in figure 6.38.

In metal work function is the minimum energy required to free the electron from its surface, whereas in semiconductor there are no electrons at Fermi level. Still the work function represents the energy required to remove the electron from the semiconductor. We can think that electron affinity should represent the minimum energy required to free electron from n-type semiconductor. But at thermal equilibrium, not all electrons are located in conduction band, so some have to be excited from valence band to maintain the equilibrium. That is why work function gives the minimum energy required for freeing electron from the semiconductor.

The more energetic electrons in the conduction band of semiconductor can easily tunnel into the metal in search of lower energy levels just above the Fermi level in metal. These electrons from semiconductor pile up on the surface of the metal leaving behind exposed positively charged region with immobile ions, which is depleted of free charge carriers. The depletion region extends into the n-region only. An internal electric field from depleted n-region toward the metal surface near junction where electrons have accumulated

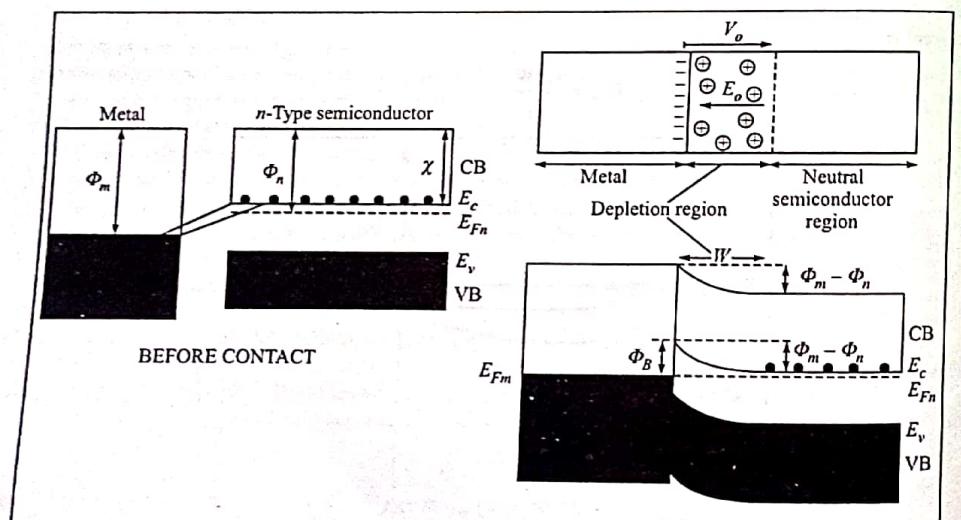


Fig. 6.38 Formation of Schottky junction between metal and n-type semiconductor.

is set up. There is charge difference between the depleted n-region and the metal surface with access electrons, consequently a potential difference known as built-in potential develops between the metal and the semiconductor. The internal electric field will cause drift of electrons from metal surface back to semiconductor. The increase in built-in potential and internal electric field continues till an equilibrium is reached when equal number of electrons cross in both directions. The maximum value of internal field is at the metal-semiconductor junction.

The Fermi level throughout the whole solid (both the metal and semiconductor) must be same. In depletion layer which is depleted of electrons, $E_c - E_{Fn}$ must increase for n to decrease. So, the band bends near the junction to increase $E_c - E_{Fn}$. The bending is just enough for vacuum level to be continuous and changing by $\Phi_m - \Phi_n$ from semiconductor to the metal, as this much energy is required to take an electron from the semiconductor to the metal. The potential energy barrier for electrons moving from metal to the semiconductor is given by $\Phi_B = \Phi_m - \chi = eV_0 + (E_c - E_{Fn})$ is greater than eV_0 . Such junction is called Schottky junction and the barrier potential energy as Schottky barrier height.

Under open circuit condition, the number of electrons thermally emitted from metal to semiconductor over potential barrier is equal to the number of electrons thermally emitted over eV_0 from semiconductor to metal. Therefore, no net current flows through the junction. The current density due to electrons thermally emitted from metal to semiconductor is

$$J_1 = A_1 \exp(-\Phi_B/kT) \quad (6.109)$$

where A_1 is a constant with same dimension as the J_1 .

Similarly, current due to electrons emitted from conduction band of semiconductor to the metal is given by

$$J_2 = A_2 \exp(-eV_0/kT) \quad (6.110)$$

where A_2 is another constant different from A_1 .

During open circuit condition, there will be no current through the junction.

$$J_{oc} = J_2 - J_1 = 0 \quad (6.111)$$

Under forward biased condition, the semiconductor is connected to negative terminal of the battery and the metal is connected to the positive terminal. The whole applied voltage drops in the depletion region owing to its greater resistance compared to the rest. The built-in potential is reduced but the barrier height Φ_B is unchanged. The current due to emission of electron from metal to conduction band of semiconductor over the barrier is unchanged, whereas the current due to emission of electrons from conduction band of semiconductor to the metal surface is given by

$$J_2^{for} = A_2 \exp\left[-\frac{e(V_0 - V)}{kT}\right] \quad (6.112)$$

So, the current from semiconductor to metal in forward bias condition considerably increases, whereas current from metal to semiconductor remains the same as in open circuit condition. It is apparent that there will be sustained current from semiconductor to metal given by

$$J = J_2^{for} - J_1 = \exp\left[-\frac{e(V_0 - V)}{kT}\right] - A_2 \exp\left(-\frac{eV_0}{kT}\right)$$

or,

$$J = A_2 \exp\left(-\frac{eV_0}{kT}\right) \left[\exp\left(\frac{eV}{kT}\right) - 1 \right]$$

$$J = J_0 \left[\exp\left(\frac{eV}{kT}\right) - 1 \right] \quad (6.113)$$

where J_0 is a constant that depends upon material and surface properties of the two solids.

When the Schottky junction is reversed biased, i.e. the positive terminal of battery connected to n-type semiconductor and negative terminal connected to metal, the depletion region will expand as in case of pn junction as the applied field will simply add up to the internal field. The built-in potential will increase from V_0 to $(V_0 + V_r)$. The semiconductor energy band diagram will be shifted down with respect to the metal side because the charge is negative and the voltage is positive. Potential energy barrier for thermal emission of electrons from conduction band of semiconductor to metal increases to $e(V_0 + V_r)$ and the corresponding current will be

$$J_2^{rev} = A_2 \exp \left[-\frac{e(V_0 + V_r)}{kT} \right] \quad (6.114)$$

which will be less than the current due to emission of electrons from metal to semiconductor over potential barrier ϕ_B , which is still unchanged from its value at open circuit. So

$$J_2^{rev} = A_2 \exp \left[-\frac{e(V_0 + V_r)}{kT} \right] \ll J_1 \quad (6.115)$$

Since generally, V_0 is a fraction of volts and the reverse bias is more than a few volts, the reverse bias current is essentially limited by emission current from metal to the conduction band of semiconductor and is very small. The I-V characteristics exhibit rectifying properties and the device is called *Schottky diode*.

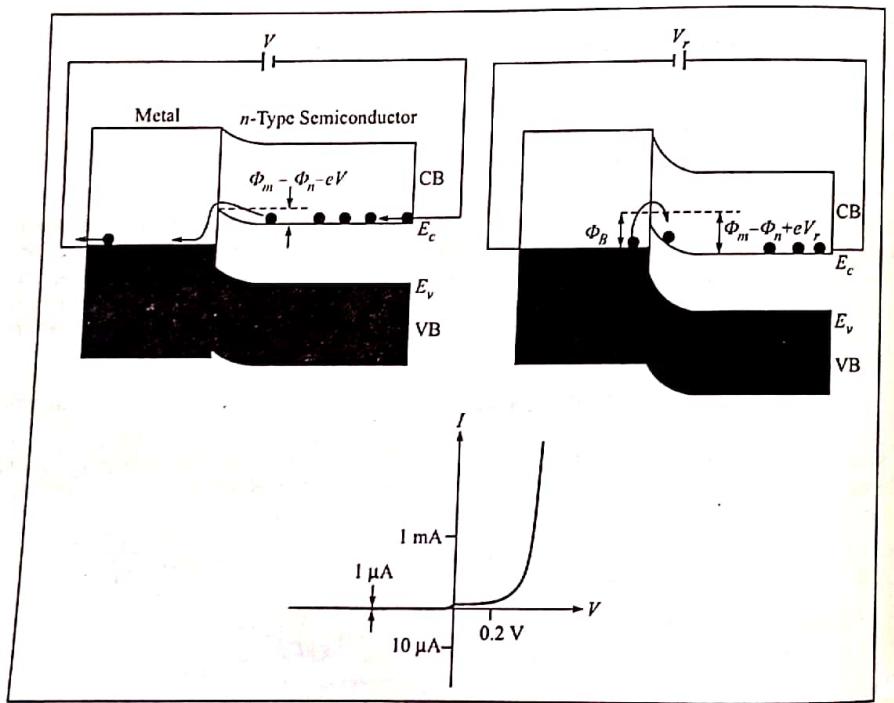


Fig. 6.39 The Schottky junction. Forward biased (a); reverse biased (b); I-V characteristics (c)

6.9.2 Ohmic Contact

An Ohmic contact is a junction between metal and semiconductor that does not limit the current flow. The current is limited by the resistance of the semiconductor outside the contact region. When we say Ohmic contact here it does not mean that the I-V characteristics is linear.

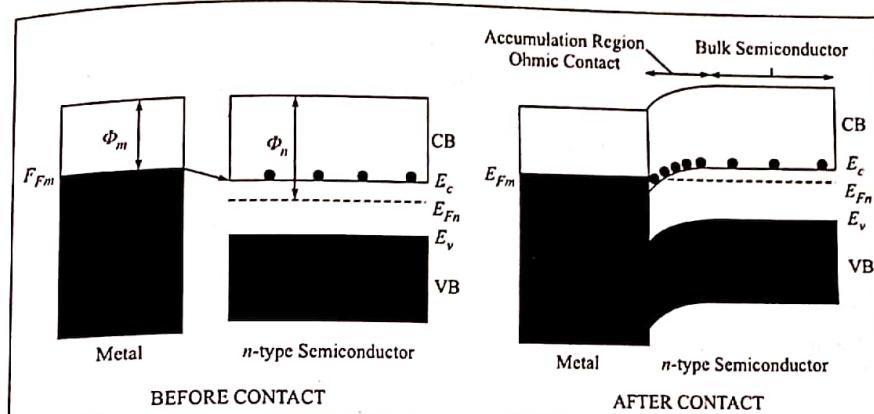


Fig. 6.40 Metal-semiconductor junction with $\phi_m < \phi_n$

When a metal and n-type semiconductor are brought together, the electrons in metal near the junction tunnel through to the conduction band of semiconductor because of lower energy level there. Many electrons pile up in conduction band near the junction and equilibrium is reached when further electron cannot tunnel to the conduction band. The energy band is bent downward to decrease $E_c - E_{Fn}$ and increase the electron concentration in conduction band. The electrons in metal and n-region just outside the junction have same energy, so they can cross in any direction under applied field. There is no depletion layer as there are electrons from far left side of metal to far right side of semiconductor. Both the metal and the semiconductor region with electron accumulation have high concentration of electron compared to the bulk of semiconductor. So the current flow is limited by the resistance of the bulk.

6.10 Solved Numerical Examples

- The density of states related effective masses of electrons and holes in silicon are approximately $1.08 m_e$ and $0.56 m_e$, respectively. The electron and hole drift mobilities at room temperature are 1350 and $450 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ respectively. Calculate intrinsic concentration and intrinsic resistivity of silicon. The energy band gap for silicon is 1.1 eV .

Solution:

Effective densities of states at conduction band edge and valence band edge are given by

$$N_e = \left[\frac{8\pi m_e kT}{h^2} \right]^{3/2} \text{ and } N_v = \left[\frac{8\pi m_h kT}{h^2} \right]^{3/2}$$

So,

$$N_c = \left[\frac{8\pi(1.08 \times 9.1 \times 10^{-31} \text{ kg})(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})}{(6.624 \times 10^{-34} \text{ Js})^2} \right]^{3/2}$$

$$= 2.8 \times 10^{25} \text{ m}^{-3} = 2.8 \times 10^{19} \text{ cm}^{-3}$$

Similarly,

$$N_v = \left[\frac{8\pi(0.56 \times 9.1 \times 10^{-31} \text{ kg})(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})}{(6.624 \times 10^{-34} \text{ Js})^2} \right]^{3/2}$$

$$= 1.05 \times 10^{25} \text{ m}^{-3} = 1.05 \times 10^{19} \text{ cm}^{-3}$$

The intrinsic concentration is given by

$$n_i = \sqrt{N_c N_v} \exp \left[-\frac{E_g}{2kT} \right]$$

$$n_i = \sqrt{(2.8 \times 10^{25} \text{ m}^{-3})(1.05 \times 10^{25} \text{ m}^{-3})} \exp \left[-\frac{1.1 \text{ eV} \times 1.6 \times 10^{-19} \text{ J/eV}}{2(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})} \right]$$

$$= 1.0 \times 10^{16} \text{ m}^{-3}$$

The resistivity is given by

$$\rho = \frac{1}{en_i(\mu_e + \mu_h)} = \frac{1}{(1.6 \times 10^{-19} \text{ C})(10^{16} \text{ m}^{-3})(1350 + 450) \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}}$$

$$= 3.5 \times 10^3 \Omega \cdot \text{m} = 3.5 \times 10^5 \Omega \cdot \text{cm}$$

2. Calculate the resistance of pure silicon cubic crystal of 1 cm^3 at room temperature. What will be the resistance of the cube when it is doped with 1 arsenic in 10^9 silicon atoms and 1 boron atom per billion silicon atoms? Atomic concentration of silicon is $5 \times 10^{22} \text{ cm}^{-3}$, $n_i = 1.45 \times 10^{10} \text{ cm}^{-3}$. Assume the mobilities as in the previous example (1).

Solution:

Resistance of the cube is

$$R = \rho \frac{l}{A} = \rho = \frac{l}{\sigma} = \frac{1}{en_i(\mu_e + \mu_h)}$$

$$R = \frac{1}{(1.6 \times 10^{-19} \text{ C})(1.45 \times 10^{10} \text{ cm}^{-3})(1350 + 450) \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}}$$

$$= 2.39 \times 10^5 \Omega$$

When the silicon is doped with 1 arsenic atom per 10^9 silicon atoms, then doping concentration is

$$N_d = \frac{N_{Si}}{10^9} = \frac{5 \times 10^{22} \text{ cm}^{-3}}{10^9} = 5 \times 10^{13} \text{ cm}^{-3}$$

At room temperature, all the donors are ionized, so the electron concentration is given by the donor concentration.

$$n \approx N_d = 5 \times 10^{13} \text{ cm}^{-3}$$

The hole concentration then becomes

$$p = \frac{n_i^2}{N_d} = \frac{(1.45 \times 10^{10} \text{ cm}^{-3})^2}{5 \times 10^{13} \text{ cm}^{-3}} = 4.2 \times 10^6 \text{ cm}^{-3}$$

Since $p \ll n$, we can neglect the hole concentration for calculation of resistance, which is given by

$$R = \frac{1}{(1.6 \times 10^{-19} \text{ C})(5 \times 10^{13} \text{ cm}^{-3})(1350 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1})} = 92.6 \Omega$$

So, there is drastic fall in resistance of the cubic crystal when it is doped with 1 arsenic atom in 10^9 silicon atoms.Similarly, when the sample is doped with 1 boron atom per billion silicon atoms, the acceptor concentration is again $5 \times 10^{13} \text{ cm}^{-3}$ and the resistance of the sample is calculated as, by neglecting the contribution from electrons,

$$R = \frac{1}{(1.6 \times 10^{-19} \text{ C})(5 \times 10^{13} \text{ cm}^{-3})(450 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1})} = 278 \Omega$$

From this example, we can see the significance of donor doping and acceptor doping, as for the same doping concentrations the resistance of the resulting sample is different.

3. An n-type semiconductor doped with 10^{16} cm^{-3} phosphorous atoms has been doped with 10^{17} cm^{-3} boron atoms. Calculate the electron and hole concentrations in the semiconductor.

Solution:

A semiconductor doped with both the donors and acceptors is called compensated semiconductor. Here pentavalent phosphorous is donor and trivalent boron is acceptor. The resultant acceptor concentration is $N_a - N_d = 10^{17} - 10^{16} = 9 \times 10^{16} \text{ cm}^{-3}$. The hole concentration is simply given by the resultant acceptor concentration as the intrinsic concentration is very small compared to acceptor concentration.

$$p = N_a - N_d = 10^{17} - 10^{16} \text{ cm}^{-3} = 9 \times 10^{16} \text{ cm}^{-3}$$

And, the electron concentration is given by

$$n = \frac{n_i^2}{p} = \frac{(1.45 \times 10^{10} \text{ cm}^{-3})^2}{9 \times 10^{16} \text{ cm}^{-3}} = 2.34 \times 10^3 \text{ cm}^{-3}$$

This semiconductor, though having much higher acceptor concentration than donors, is still called compensation-doped semiconductor.

4. An n-type silicon wafer is uniformly doped with 10^{16} antimony atoms per cm^3 . Where will be the Fermi level compared to its intrinsic Fermi level? Where will the Fermi level be shifted if the sample is further doped with 2×10^{17} boron atoms per cm^3 ?

Solution:

An antimony-doped silicon is n-type semiconductor. The intrinsic electron concentration is very small compared to donor concentration. Therefore $n \approx N_d = 10^{16} \text{ cm}^{-3}$. The intrinsic concentration and doped concentrations are given by

$$n_i = N_c \exp \left[-\frac{E_c - E_F}{kT} \right]$$

$$n = N_c \exp\left[-\frac{E_c - E_F}{kT}\right] = N_d$$

So,

$$\frac{N_d}{n_i} = \exp\left[-\frac{E_{F_n} - E_{F_i}}{kT}\right]$$

Solving the above, we have

$$E_{F_n} - E_{F_i} = kT \ln \frac{N_d}{n_i}$$

$$E_{F_n} - E_{F_i} = (1.38 \times 10^{-23} \text{ JK}^{-1})(300 \text{ K}) \ln \left(\frac{10^{16} \text{ cm}^{-3}}{1.45 \times 10^{10} \text{ cm}^{-3}} \right) = 0.348 \text{ eV}$$

When the sample is further doped with boron by acceptor concentration which is greater than already doped donor concentration, the silicon now becomes compensated p-type with resultant hole concentration given by

$$p = N_a - N_d = 2 \times 10^{17} - 10^{16} = 1.9 \times 10^{16} \text{ cm}^{-3}$$

For intrinsic semiconductor the carrier concentration is, as before, given by

$$n_i = N_v \exp\left[-\frac{E_{F_i} - E_v}{kT}\right]$$

And for doped silicon resulting in p-type semiconductor, the hole concentration is

$$p = N_a \exp\left[-\frac{E_{F_p} - E_v}{kT}\right]$$

So,

$$\frac{p}{n_i} = \exp\left[-\frac{E_{F_p} - E_{F_i}}{kT}\right]$$

After some series of mathematical manipulations we get

$$E_{F_p} - E_{F_i} = -kT \ln \frac{p}{n_i}$$

$$E_{F_i} - E_{F_p} = kT \ln \frac{p}{n_i} = (0.0259 \text{ eV}) \ln \left(\frac{1.9 \times 10^{17}}{1.45 \times 10^{10}} \right) = 0.42 \text{ eV}$$

5. A silicon sample has been doped with 10^{17} arsenic atoms per cm^3 . Calculate the conductivity of the sample at 300 K and 400 K. Use the mobility vs impurity concentration graph to determine mobility.

Solution:

To calculate conductivity we can use the donor concentration only since $N_d \ll n_i$, and $p \ll n$. The electron drift mobility at $N_d = 10^{17} \text{ cm}^{-3}$ is $\sim 800 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ at 300 K. So, the required conductivity at 300 K is

$$\begin{aligned} \sigma &= en\mu_e = (1.6 \times 10^{-19} \text{ C})(10^{17} \text{ cm}^{-3})(800 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}) \\ &= 12.8 \text{ Sm/cm} \end{aligned}$$

Similarly at 400 K, the drift mobility of electron is $420 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ at the given donor concentration. So, the conductivity of the sample is

$$\sigma = en\mu_e = (1.6 \times 10^{-19} \text{ C})(10^{17} \text{ cm}^{-3})(420 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}) = 6.72 \text{ Sm/cm}$$

6. What happens to the conductivity if the sample in example 5 is further doped with 9×10^{16} boron atoms cm^{-3} ?

Solution:

Still the donor concentration is greater than the acceptor concentration in the sample. So, the sample is n-type. The electron concentration is now reduced to

$$n = N_d - N_a = 10^{17} - 9 \times 10^{16} = 10^{16} \text{ cm}^{-3}$$

There is one interesting change associated with the electron drift mobility both at 300 K and 400 K. Due to the crowding of atoms by addition of more impurities (boron atoms), the carriers will face more hindrance to their mobility. So, addition of more impurities although reduces the carrier concentration, but to find mobility we should rather take the sum of the impurity concentration ($1.9 \times 10^{17} \text{ cm}^{-3}$ in this case). The electron mobility at 300 K and 400 K corresponding to this concentration are $\sim 600 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ and $400 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ respectively. So the conductivities at these temperatures are given by

$$\sigma_{300K} = (1.6 \times 10^{-19} \text{ C})(10^{17} \text{ cm}^{-3})(600 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}) = 0.96 \text{ Sm/cm}$$

$$\sigma_{400K} = (1.6 \times 10^{-19} \text{ C})(10^{17} \text{ cm}^{-3})(400 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}) = 0.63 \text{ Sm/cm}$$

For both the temperatures, we neglected the conductivity associated with hole as their concentration is very small compared to electron concentration and will not affect the result obtained.

7. Calculate the diffusion coefficient of electrons at 300 K in n-type silicon semiconductor doped with 10^{15} arsenic atoms per cm^3 ?

Solution:

Using mobility versus dopant concentration graph, we find the electron drift mobility for the given donor concentration is $\sim 1300 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$. And using Einstein relation, we have

$$\frac{D_e}{\mu_e} = \frac{kT}{e}$$

$$D_e = \mu_e \frac{kT}{e} = (1300 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1})(0.0259 \text{ V}) = 33.7 \text{ cm}^2 \text{ s}^{-1}$$

8. Consider a Schottky junction diode between Al and n-type Si, doped with 10^{16} donors per cm^3 . The cross-sectional area of Si is 1 m^2 and electron affinity of silicon is 4.01 eV, work function of Al is 4.28 eV. What will be the theoretical potential barrier height from metal to semiconductor? Using the Richardson's equation for thermionic emission $J_0 = A_0 T^2 \exp(-\Phi_B/kT)$ calculate the reverse saturation current and forward bias current if 0.1V is applied. $A_0 = 3000 \text{ AK}^{-2} \text{ m}^{-2}$. J_0 represents the saturation current density.

Solution:

The theoretical barrier height is

$$\Phi_B = \Phi_m - \chi = 4.28 - 4.01 = 0.27 \text{ eV}$$

The typical experimental value is -0.5 eV due to various reasons such as defects, impurities, dislocations, etc at metal-semiconductor interface. The reverse saturation current is

$$\begin{aligned}I_0 &= A_{J0} A_{A0} T^2 \exp(-\Phi_B/kT) \\I_0 &= (0.01 \text{ cm}^2)(30 \text{ A K}^{-2} \text{ cm}^{-2})(300 \text{ K})^2 \exp(-0.5 \text{ eV}/0.0259 \text{ eV}) \\&= 1.2 \times 10^{-4} \text{ A}\end{aligned}$$

And the forward current is

$$\begin{aligned}I_f &= I_0 \left[\exp\left(\frac{eV_f}{kT}\right) - 1 \right] = 1.2 \times 10^{-4} \text{ A} \left[\exp\left(\frac{0.1 \text{ eV}}{0.0259 \text{ eV}}\right) - 1 \right] \\&= 5.5 \times 10^{-3} \text{ A}\end{aligned}$$

9. Calculate the built-in potential across the junction formed by joining silicon doped with 10^{17} donor atoms per cm^3 with silicon doped with 10^{16} acceptor atoms per cm^3 at room temperature. Take the intrinsic concentration from previous examples.

Solution:

$$kT/e = 0.0259 \text{ V at } 300 \text{ K.}$$

$$\begin{aligned}V_0 &= \frac{kT}{e} \ln \frac{N_a N_d}{n_i^2} \\&= (0.0259 \text{ V}) \ln \left[\frac{10^{17} \times 10^{16} \text{ cm}^{-6}}{(1.45 \times 10^{10} \text{ cm}^{-3})^2} \right] = 0.756 \text{ V}\end{aligned}$$

10. A heavily doped p-side with acceptor concentration of 10^{18} cm^{-3} is connected to n-side with donor concentration of 10^{16} cm^{-3} . Calculate the built-in potential, depletion width in n-side and p-side, and overall depletion width. The intrinsic concentration is as in previous case and $T = 300 \text{ K}$.

Solution:

The built-in potential in the junction is calculated as

$$\begin{aligned}V_0 &= \frac{kT}{e} \ln \frac{N_a N_d}{n_i^2} \\&= (0.0259 \text{ V}) \ln \frac{(10^{16} \times 10^{18})}{(1.45 \times 10^{10})^2} = 0.816 \text{ V}\end{aligned}$$

The depletion width can be calculated by using the following expression

$$W_0 = \sqrt{\frac{2eV_0}{e} \left(\frac{1}{N_a} + \frac{1}{N_d} \right)}$$

But here since $N_a \gg N_d$, we can simply the formula as

$$\begin{aligned}W_0 &= \sqrt{\frac{2eV_0}{e} \frac{1}{N_d}} = \frac{2 \times (11.9 \times 8.85 \times 10^{12} \text{ Fm}^{-1})(0.816 \text{ V})}{(1.6 \times 10^{-19} \text{ C})(10^{22} \text{ m}^{-3})} \\&= 3.3 \times 10^{-7} \text{ m}\end{aligned}$$

Now, depletion width in n-region is

$$W_n = \frac{W_0 N_a}{N_a + N_d} = \frac{(3.3 \times 10^{-7} \text{ m})(10^{24} \text{ m}^{-3})}{(10^{22} \text{ m}^{-3} + 10^{24} \text{ m}^{-3})} = 3.3 \times 10^{-9} \text{ m}$$

Similarly, the depletion width in p-region is

$$W_p = W_0 - W_n = 3.3 \times 10^{-7} \text{ m} - 3.267 \times 10^{-7} \text{ m} = 3.3 \times 10^{-9} \text{ m}$$

So almost 99% of depletion layer falls in n-region due to heavily doped p-region where only around 1% depletion layer falls.

11. An abrupt p^+n junction diode has a cross-sectional area of 1 mm^2 , an acceptor concentration of $5 \times 10^{18} \text{ cm}^{-3}$ on p-side and a donor concentration of 10^{16} cm^{-3} on n-side. The lifetime of holes in n-region is 417 ns , whereas that of electrons in p-region is 5 ns . Mean thermal generation lifetime is about 1 ms . The lengths of p and n-sides are 5 micron and 100 micron respectively. Calculate:

- (a) Diffusion coefficients and the minority carrier diffusion lengths
- (b) Built-in potential across the junction
- (c) Current when a forward bias of 0.6 V is applied assuming the current is by the minority carrier diffusion only at 300 K and 373 K and also at 373 K temperature dependence of intrinsic concentration dominates over mobility, diffusion length and diffusion coefficient.
- (d) Reverse current when a reverse bias of 5 V is applied.

Solution:

Since we are interested only to the diffusion of minority charge carriers, for an acceptor concentration of $5 \times 10^{18} \text{ cm}^{-3}$, the electron mobility (which is minority carrier in p-region) is $120 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ and for a donor concentration of 10^{16} cm^{-3} , the hole mobility (which is minority charge carrier in n-region) is $440 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ is obtained from the graph of mobility vs doping concentration. From Einstein relation, we have

$$\frac{D_e}{\mu_e} = \frac{kT}{e} \quad \text{and} \quad \frac{D_h}{\mu_h} = \frac{kT}{e}$$

$$D_e = \mu_e \frac{kT}{e} = (120 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1})(0.0259 \text{ eV}) = 3.1 \text{ cm}^2 \text{ s}^{-1}$$

$$D_h = \mu_h \frac{kT}{e} = (440 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1})(0.0259 \text{ eV}) = 11.39 \text{ cm}^2 \text{ s}^{-1}$$

Diffusion lengths for electrons and holes are

$$L_e = \sqrt{D_e \tau_e} = \sqrt{(3.1 \text{ cm}^2 \text{ s}^{-1})(5 \times 10^{-9} \text{ s})} = 1.2 \times 10^{-6} \text{ m}$$

$$L_h = \sqrt{D_h \tau_h} = \sqrt{(11.39 \text{ cm}^2 \text{ s}^{-1})(417 \times 10^{-9} \text{ s})} = 21.8 \times 10^{-6} \text{ m}$$

The built-in potential across the junction is

$$\begin{aligned}V_0 &= \frac{kT}{e} \ln \frac{N_a N_d}{n_i^2} \\&= (0.0259 \text{ V}) \ln \frac{5 \times 10^{18} \text{ cm}^{-3} \times 10^{16} \text{ cm}^{-3}}{(1.45 \times 10^{10} \text{ cm}^{-3})^{1/2}} \\&= 0.856 \text{ V}\end{aligned}$$

Assuming the forward current is due to diffusion of minority carriers in neutral regions,

$$I = I_{s0} \left[\exp\left(\frac{eV}{kT}\right) - 1 \right] = I_{s0} \exp\left(\frac{eV}{kT}\right) \text{ for } V \gg \frac{kT}{e}$$

The saturation current I_{s0} is calculated as

$$I_{s0} = AJ_{s0} = A \left[\frac{D_e}{L_e N_a} + \frac{D_h}{L_h N_d} \right] e n_i^2 = A \frac{D_h}{L_h N_d} e n_i^2 \cdot \text{As } \frac{1}{N_a} \ll \frac{1}{N_d}$$

$$I_{s0} = \frac{(0.01 \text{ cm}^2)(11.39 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1})(1.6 \times 10^{-19} \text{ C})(1.45 \times 10^{10} \text{ cm}^{-3})^2}{(21.8 \times 10^{-4} \text{ cm})(10^{16} \text{ cm}^{-3})} \\ = 1.76 \times 10^{-13} \text{ A}$$

So, the diode current in forward bias is

$$I = I_{s0} \exp\left(\frac{eV}{kT}\right) \\ = 1.76 \times 10^{-13} \ln\left(\frac{0.6 \text{ V}}{0.0259 \text{ V}}\right)$$

$$2 \times 10^{-3} \text{ A} = 2 \text{ mA}$$

At 373 K, the intrinsic concentration is, from graph, equal to $1.2 \times 10^{12} \text{ cm}^{-3}$. Reverse saturation current at 373 K can be calculated as

$$I_{s0}(373 \text{ K}) \approx I_{s0}(300 \text{ K}) \left[\frac{n_i(373 \text{ K})}{n_i(300 \text{ K})} \right]^2 \\ = (1.76 \times 10^{-13} \text{ A}) \left[\frac{1.2 \times 10^{12} \text{ cm}^{-3}}{1.45 \times 10^{10} \text{ cm}^{-3}} \right]^2 \\ = 1.21 \times 10^{-9} \text{ A}$$

And the forward current at this temperature is

$$I(373 \text{ K}) = I_{s0}(373 \text{ K}) \exp\left(\frac{eV}{kT}\right) \\ I(373 \text{ K}) = (1.2 \times 10^{-9} \text{ A}) \exp\left[\frac{(1.6 \times 10^{-19} \text{ C}) \times (0.6 \text{ V})}{(1.38 \times 10^{-23} \text{ J/K})(373 \text{ K})}\right] \\ = 0.15 \text{ A}$$

When a reverse bias is applied, the depletion width is given by

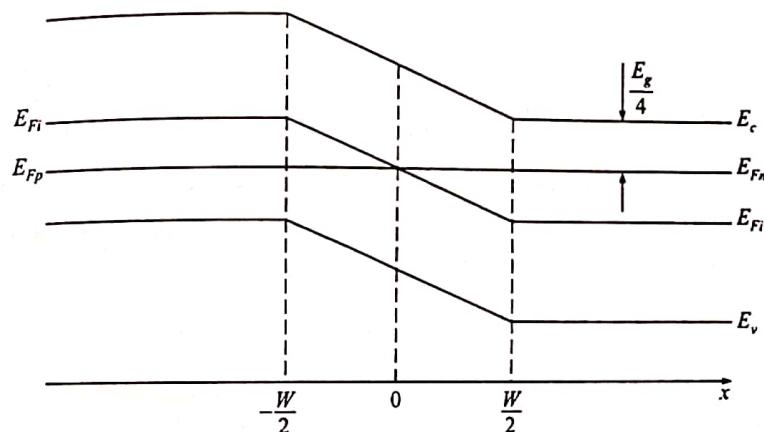
$$W = \sqrt{\frac{2\varepsilon(V_0 + V_r)}{N_d e}} = \frac{2 \times (11.9 \times 8.85 \times 10^{-12} \text{ F m}^{-1})(0.856 + 5 \text{ V})}{(1.6 \times 10^{-19} \text{ C})(10^{22} \text{ m}^{-3})} \\ = 0.88 \times 10^{-6} \text{ m} = 0.88 \text{ micron}$$

The thermal generation current with reverse bias of 5 V is

$$I_{gen} = \frac{en_i A W}{\tau_g} \\ = \frac{(1.6 \times 10^{-19} \text{ C})(1.45 \times 10^{-10} \text{ cm}^{-3})(0.01 \text{ cm}^2)(0.88 \times 10^{-4} \text{ cm})}{10^{-6} \text{ s}} \\ = 2 \times 10^{-9} \text{ A}$$

This current is greater than the reverse saturation current of $1.76 \times 10^{-13} \text{ A}$ at 300 K, so the reverse current is dominated by thermal generation current and is 2 nA at this condition.

12. A semiconductor is characterized by the energy band diagram given below:



- (a) If the material is Si at 300 K, determine the resistivity of the region $x > -W/2$;
- (b) An electron located at $x = W/2$ attempts to move to the $x < -W/2$ region of the semiconductor without changing its total energy. What is the minimum kinetic energy required for the electron to do this?
- (c) Determine whether or not the semiconductor is in equilibrium.
- (d) Determine the electron and hole current density at $x = 0$.
- (e) Is there drift current at $x = 0$? If yes, find its magnitude and direction.
- (f) Is there diffusion current at $x = 0$? If yes, find its magnitude and direction. The intrinsic concentration at 300 K is $1.188 \times 10^{10} \text{ cm}^{-3}$ and the $E_g = 1.12 \text{ eV}$.

Solution:

- (a) In the region $x > W/2$, the semiconductor is n-type as the Fermi level is between intrinsic Fermi level and conduction band edge. The electron concentration, and hence donor concentration, in n-region is determined in terms of intrinsic concentration as

$$N_d \approx n = n_i \exp\left[-\frac{E_{Fn} - E_{Fi}}{kT}\right]$$

$$N_d = (1.18 \times 10^{10} \text{ cm}^{-3}) \exp\left[-\frac{1.12/4}{0.0259}\right] = 5.58 \times 10^{14} \text{ cm}^{-3}$$

For dopant concentration of $5.85 \times 10^{14} \text{ cm}^{-3}$, from the graph mobility versus dopant concentration, electron mobility is found to be $1350 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ which will determine the resistivity of the region.

$$\rho_n = \frac{1}{en\mu_e} = \frac{1}{(1.6 \times 10^{-19} \text{ C})(5.85 \times 10^{14} \text{ cm}^{-3})(1350 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1})} = 7.91 \Omega \cdot \text{cm}$$

- (b) To be able to jump the p region from n region, an electron must possess a kinetic energy at least equal to potential energy barrier between n and p-regions.

$$KE_{\min} = E_c \left(-\frac{w}{2}\right) - E_c \left(\frac{w}{2}\right) = \frac{E_s}{2}$$

- (c) The material is in equilibrium as Fermi level does not change with position.
(d) Under equilibrium condition, the current densities due to electrons and holes are zero everywhere.
(e) There is a drift current at $x=0$ and is given by $J_{e,drift} = en\mu_e \bar{E}$, where n and E both are non zero. The drift current will flow in the direction of electric field which is from n-side to p-side in the depletion layer.
(f) There is current due to diffusion of electrons as well at $x=0$. The electron concentration is given by

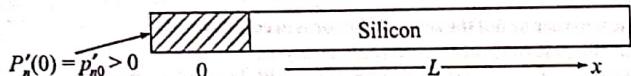
$n = n_i \exp\left[\frac{E_{Fn} - E_{Fi}}{kT}\right]$ which increase ingoing from $x=-w/2$ to $x=w/2$. Thus $\frac{dn}{dx} \neq 0$, and electron diffusion current is non-zero at $x=0$.

$$J_{e,diff} = eD_e \frac{dn}{dx} \neq 0$$

which flows in positive x-direction.

The electron drift current, which is in negative x-direction, must exactly be equal to the electron diffusion current, which is in positive x-direction under equilibrium condition.

13. A semi-infinite bar shown below is subject to a stimulus such that $p'_n(0) = p'_{n0} > 0$. The bar is at room temperature at steady-state condition and $N_d = 10^{14} \text{ cm}^{-3}$ for all x . There are no recombination-generation centers in $0 < x < L$ region. There are no other processes taking place in the bar.



Determine the equilibrium electron concentration. Will low level injection exist at $p'_{n0} = 10^{14} \text{ cm}^{-3}$?

Solution:

All the donor atoms are ionized at room temperature. So the equilibrium electron concentration is given by the donor concentration as

$$n_0 = N_d$$

or,

$$n_0 = 10^{14} \text{ cm}^{-3}$$

Since $p'_{n0} = n_0$, low level injection will not exist.

14. Evaluate the change in conductivity of Silicon specimen when its temperature changes from 300 K to 400 K. Assume usual relationship for temperature dependence of mobility. The effective mass of electron and holes are $1.08m_e$ and $0.56m_e$ respectively. The electron drift mobility and hole drift mobility at 300 K is $1350 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ and $450 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ respectively for the given Si sample. $E_g = 1.1 \text{ eV}$ for Si.

Solution:

We know that the conductivity of Si sample is given by

$$\sigma_{300} = en_{i(300)} (\mu_{e(300)} + \mu_{h(300)}) \text{ and} \\ \sigma_{400} = en_{i(400)} (\mu_{e(400)} + \mu_{h(400)})$$

where $n_{i(300)}$ and $n_{i(400)}$ are the intrinsic concentrations of Si at 300 K and 400 K respectively. Intrinsic concentration in general is given by

$$n_i = \sqrt{N_c N_v} \exp\left[-\frac{E_g}{2kT}\right]$$

where

$$N_c = \left[\frac{8\pi m_e^* kT}{h^2}\right]^{3/2} \text{ and } N_v = \left[\frac{8\pi m_h^* kT}{h^2}\right]^{3/2}$$

We know that the mobilities are related to the temperature in intrinsic semiconductor by

$$\mu_e \propto T^{-2.3} \text{ and } \mu_h \propto T^{-2.2}$$

Now, we can find the ratio of conductivities at 300 K and 400 K as

$$\frac{\sigma_{i(300)}}{\sigma_{i(400)}} = \frac{\left(\frac{8\pi m_e^* kT_{300}}{h^2}\right)^{3/2} \left(\frac{8\pi m_h^* kT_{300}}{h^2}\right)^{3/2} \exp\left(-\frac{E_g}{2kT_{300}}\right) [\mu_{e(300)} + \mu_{h(300)}]}{\left(\frac{8\pi m_e^* kT_{400}}{h^2}\right)^{3/2} \left(\frac{8\pi m_h^* kT_{400}}{h^2}\right)^{3/2} \exp\left(-\frac{E_g}{2kT_{400}}\right) [\mu_{e(400)} + \mu_{h(400)}]}$$

$$\frac{\sigma_{i(300)}}{\sigma_{i(400)}} = \frac{T_{300}^3 \exp\frac{E_g}{2K} \left(\frac{1}{T_{400}} - \frac{1}{T_{300}}\right) [\mu_{e(300)} + \mu_{h(300)}]}{T_{400}^3 [\mu_{e(400)} + \mu_{h(400)}]}$$

$$\frac{\sigma_{e(400)}}{\sigma_{e(300)}} = \left[\frac{T_{400}}{T_{300}}\right]^{-2.3}$$

$$\text{So, } \mu_{e(400)} = \mu_{e(300)} \left[\frac{T_{400}}{T_{300}}\right]^{-2.3} = 1350 \times \left[\frac{400}{300}\right]^{-2.3} = 696.6 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$$

Similarly, the hole mobility at 400 K can be calculated as

$$\mu_{h(400)} = \mu_{h(300)} \left[\frac{T_{400}}{T_{300}}\right]^{-2.2} = 450 \times \left[\frac{400}{300}\right]^{-2.2} = 239 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$$

$$\text{Now, } \frac{\sigma_{i(300)}}{\sigma_{i(400)}} = \frac{300^3 \exp\left[\left(\frac{1.1 \times 1.6 \times 10^{-19}}{2 \times 1.38 \times 10^{-23}}\right)\left(\frac{1}{400} - \frac{1}{300}\right)\right][1350 + 450] \times 10^{-4}}{400^3 [696.6 + 239] \times 10^{-4}}$$

$$\sigma_{i(300)} = 3.995 \times 10^{-3} \times \sigma_{i(400)}$$

Then we can write,

$$\sigma_{i(400)} = 250.31 \times \sigma_{i(300)}$$

The intrinsic conductivity of the given Si sample increases by around 250 times compared to the conductivity at 300 K when temperature reaches 400 K.

15. How the conductivity of the donor doped Si changes as temperature is reduced from 250 K to 200 K. The doping concentration is 10^{25} m^{-3} .

Solution:

Conductivity of donor-doped Si is given by

$$\mu_n \approx en\mu_e$$

$$\text{where } n \approx N_d = 10^{25} \text{ m}^{-3}$$

Now, the electron drift mobility for the doping concentration of 10^{25} m^{-3} in the given temperature changes positively with the change in temperature, which is due to impurity scattering domination.

From the experimental plot of drift mobility vs. temperature for the given doping concentration, we have

$$\mu_{200} = 93 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$$

$$\mu_{250} = 100 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$$

Now, the conductivities can be evaluated as

$$\frac{\sigma_{250}}{\mu_{200}} = \frac{100}{93} = 1.075$$

Significant to note is that, a change of temperature by almost a quarter causes change in conductivity by merely 7.5%. This is mainly due to saturation of the ionization in the doped semiconductor.

From the same graph of electron drift mobility vs. temperature for n-type semiconductor, one might conclude that as mobility is seen decreasing in most of the cases with increase in temperature, the conductivity might be actually decreasing. This is true but for only a narrow temperature range only. For all other cases, the conductivity of doped semiconductor will be increasing with the rise in temperature as the semiconductor will start behaving like intrinsic above a certain temperature and any reduction in mobility will be compensated by increase in carrier concentration both of which affect the conductivity. The same is verified in problem no. (6) above.