How ‘Super’ is a Super Moon?
A Physics Exploration Using an Online Simulation

Introduction

Leading up to this investigation, I came across a series of posts on my Facebook feed: ‘SUPER MOON November 16, 2016 Spread this message so everyone can enjoy!’ Often these posts were captions for images of a moon so large that the details on individual craters were visible. I have gone stargazing with my family throughout my life and I have never seen a moon remotely close to the images all over the Internet.

This investigation explores how the apparent size of the lunar disk varies based on its distance from Earth. As a result of the moon’s elliptical orbit and a variety of other factors, the moon’s distance from the Earth varies considerably throughout the years in fact never travelling the same path. My research question, therefore, is: how does the moon’s apparent size, measured based on its distance from Earth, vary with time?

In order to investigate lunar distance I used an online simulation written by Jurgen Giesen based on Jean Meeus’s algorithm for lunar position to develop data points for distance at various times of the year. By comparing this data to dates for the full moon, the dates for ‘super’ moons can be calculated. Through this investigation, it will also be of interest to measure the significance of the moon’s changing distance on its apparent size and the point in the near future at which the moon would appear largest.

Background Information

'Super' Moons

In actuality a 'super' moon is not an official astronomical term but a concept coined by the astrologer Richard Nolle. Nolle defined a 'super' moon as a full moon that occurred when the moon was within 90% of its closest approach to Earth's orbit[1]. Due to the lack of clarity as to the origin of the 90% cut-off, the modern definition is a full moon that occurs when the center of the moon is less than 360,000km from the center of the Earth (the origin of this number will be discussed later in the investigation). In order to investigate the phenomenon, it is necessary to explore why the moon's distance from Earth varies and how it waxes (approaches a full moon) and wanes (approaches a new moon).

Kepler's 1st Law

Kepler's first law of planetary motion states that all planets move in elliptical orbits with the sun placed at one focus[2]. The law can similarly be extended to any other orbit of celestial bodies including the moon about the Earth[3].

The paths orbiting bodies travel depend on their initial velocity when formed. Orbits result from the force of gravity pulling in a direction contrary to this velocity. If the velocity of the orbiting body is too large it will arc past in the shape of a hyperbola rather than being captured by the planet (Fig. 1a). Smaller velocities could lead to parabolic motion where the orbiting body would arc around due to gravity but still not remain captured (Fig. 1b). At low enough velocity, the orbiting body can be captured by gravity. Like in parabolic motion, the orbiting body will arc around. However, rather travelling into space, the body will continue to bend around forming an ellipse (Fig. 1c). For this motion to occur, the orbiting body's velocity would vary based on its distance from the

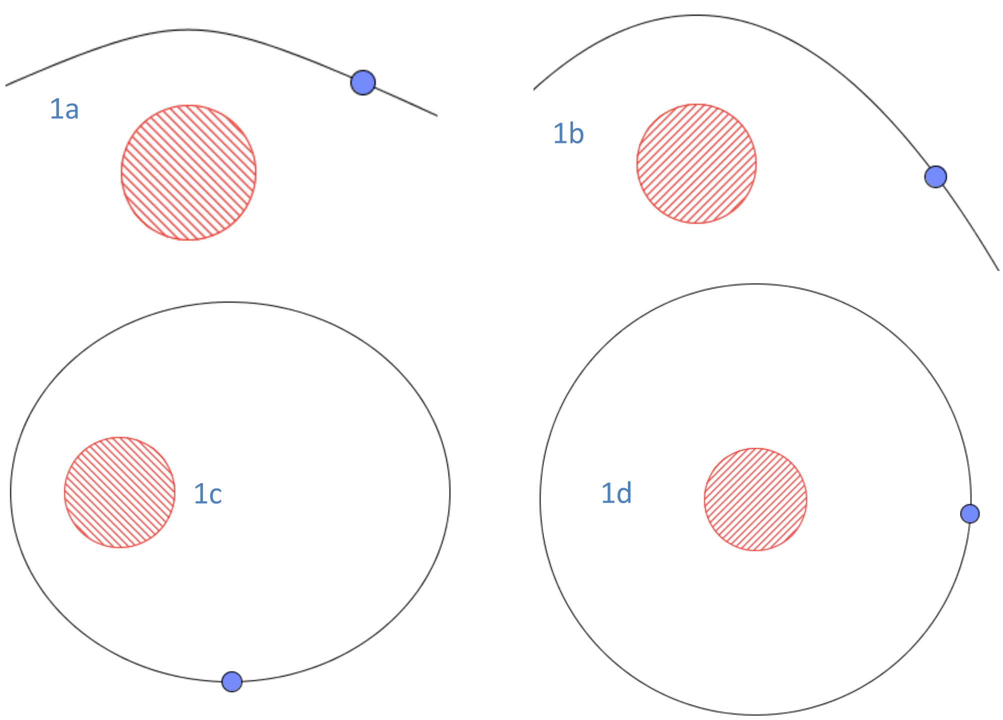


Figure 1
1a represents a hyperbolic path of an orbiting body
1b represents a parabolic path of an orbiting body
1c represents an elliptical path of an orbiting body
1d represents a circular path of an orbiting body

[1] "What Is a Supermoon and When Is the Next One?" Supermoon / Super Moon - Why and When? Accessed November 25, 2016. https://www.timeanddate.com/astronomy/moon/super-full-moon.html.
[2] Nave, R. "Kepler's Laws." HyperPhysics. Accessed November 25, 2016. http://hyperphysics.phy-astr.gsu.edu/hbase/kepler.html.
[3] "Kepler's Laws of Orbital Motion." How Things Fly. Accessed January 19, 2017. http://howthingsfly.si.edu/flight-dynamics/kepler%E2%80%99s-laws-orbital-motion.

object it is orbiting[4]. The case of a circular orbit is a special form of an ellipse (Fig. 1d). For a circular orbit to exist, the force of gravity on the orbiting body must be equivalent to a centripetal force, which can be expressed using Newton's Law of Gravitation and the equation for circular motion.

$$F_{grav} = \frac{GMm}{r^2}$$

$$F_{cent} = \frac{mv^2}{r}$$

$$F_{grav} = F_{cent} \rightarrow \frac{GMm}{r^2} = \frac{mv^2}{r} \leftrightarrow v = \sqrt{\frac{GM}{r}}$$

Only when the orbiting body has a velocity in accordance with the above equation will the orbit be a circle, any other will be an ellipse. Since the moon's velocity does not follow the equation it has an elliptical orbit. As a result, the distance between the Earth and moon varies over time. In figure 2, the minimum distance occurs when the moon is at point P, the perigee, and the maximum distance occurs at point A, the apogee. However, if this model were exact the perigee and apogee would always be at the same distance but in reality they vary. The average distance of perigee is approximately 360,000km leading to its use as the cut-off for defining a super moon[5].

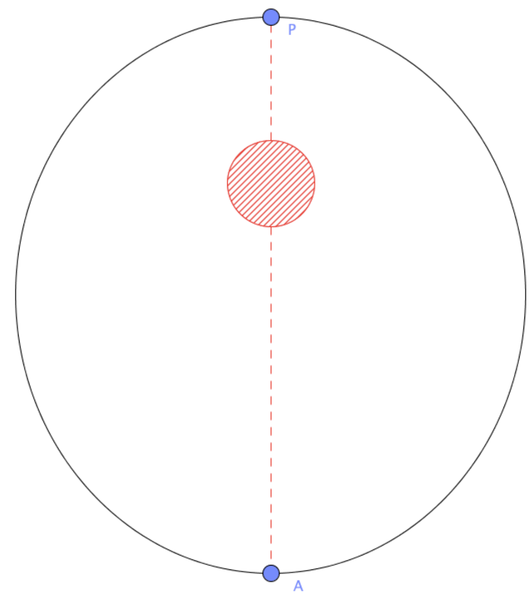


Figure 2
Point P is the position of lunar perigee and point A is the position of lunar apogee

Newton's Law of Gravitation

Newton's law of gravitation states that any two objects will exert a gravitational force of attraction of one another and this is true for every object in the universe[6]. Due to this further consideration, the moon is attracted to the other celestial bodies in the solar system to a degree that warps its elliptical orbit of the Earth. For example, during Northern Hemisphere winter, the Earth reaches its perigee for its orbit of the sun. As a result, the moon is also nearer to the sun and is influenced to a larger degree by its gravitational field. Since the moon has a smaller mass than the Earth, the gravitational force exerted on it by the sun will be larger pulling it closer to the Earth. Due to such gravitational effects from the sun and other bodies the distance to perigee varies through time rather than being a constant value[7].

[4] Slater, Sara. "Why Do Planets Have Elliptical Orbits? (Beginner)." Ask an Astronomer. January 31, 2016. Accessed November 25, 2016. http://curious.astro.cornell.edu/about-us/57-our-solar-system/planets-and-dwarf-planets/orbits/244-why-do-planets-have-elliptical-orbits-beginner.

[5] "What Is a Supermoon and When Is the Next One?" Supermoon / Super Moon - Why and When? Accessed November 25, 2016. https://www.timeanddate.com/astronomy/moon/super-full-moon.html.

[6] Drakos, Nikos. "Newton's Law of Gravitation." IUN/FYDE Introductory Physics Notes. September 10, 1997. Accessed November 27, 2016. http://theory.uwinnipeg.ca/physics/circ/node7.html.

[7] "What Is a Supermoon and When Is the Next One?" Supermoon / Super Moon - Why and When? Accessed November 25, 2016. https://www.timeanddate.com/astronomy/moon/super-full-moon.html.

Lunar Phase

The moon's phases result from the positioning of the moon relative to the Earth and Sun. The Sun always illuminates half of the moon (except in the event of a lunar eclipse); however, the visibility of the illuminated portion from Earth varies based on the moon's position. As the moon orbits the Earth different hemispheres are illuminated. The four main stages are displayed in figure 3. At point NM (a new moon) the moon is between the Sun and Earth leading the illuminated portion to be beyond view[8]. At points FQ (a first quarter) and LQ (last quarter), through similar reasoning the moon appears half lit. Since the moon's elliptical orbit about the Earth is tilted[9], when the Earth is between the moon and sun, the moon remains outside the Earth's shadow and is fully lit which is shown at point FM (full moon). The moon will return to the same position at an average of every 29.53 days (a synodic month), which would be a complete lunar cycle[10].

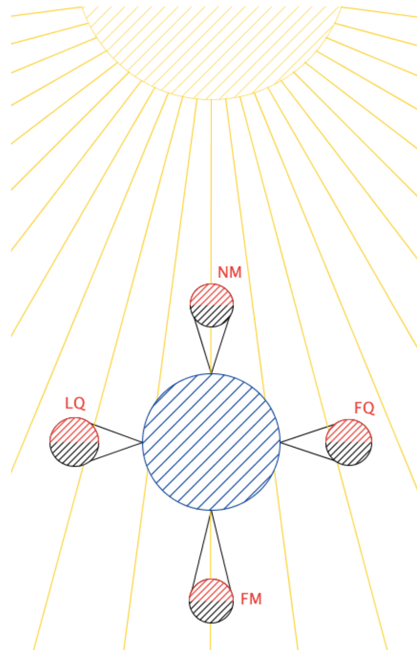


Figure 3
The sun is shown shining on the different lunar positions where NM is the new moon, FQ the first quarter moon, FM the full moon, and LQ the last quarter moon.

Apparent Size of Objects

The apparent size of an object depends on the angle of the field of view it subtends. As shown in figure 4 smaller objects at a closer distance will be bounded by the same angle as larger objects at a farther distance leading them to appear the same size. Therefore, the relative size of objects at a distance can be calculated using a ratio. In the case of the moon, its actual size never changes. However, as it moves closer or farther from the Earth, it is bounded by a smaller or larger angle leading its apparent size to an observer to vary.

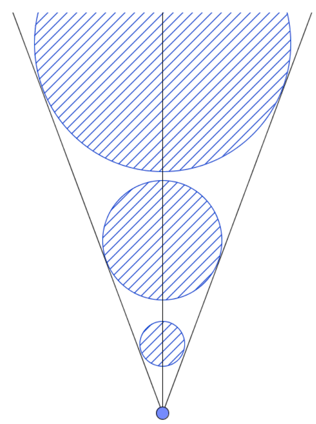


Figure 4
Three objects with the same relative size due to being bound by the same angle

[8] Palermo, Elizabeth. "Why Does the Moon Have Phases?" LiveScience. June 30, 2014. Accessed November 27, 2016. http://www.livescience.com/46606-why-does-the-moon-have-phases.html.
[9] Williams, David R. "Moon Fact Sheet." Planetary Factsheet-Metric. Accessed November 27, 2016. http://nssdc.gsfc.nasa.gov/planetary/factsheet/moonfact.html.
[10] Ibid.

Methodology: Lunar Distance Simulation[11]

Independent variable:

Point in time in days. For the investigation, time values were either days in a month or year.

Dependent variable:

Apparent lunar size calculated as the reciprocal of lunar topocentric distance (distance measured from the Earth's surface to the moon's surface).

Controlled variables:

The time of day at which distance was calculated via the simulation was kept at a constant 19:00 since lunar distance can vary by a significant quantity over a day.

The co-ordinate from which lunar distance was calculated was kept constantly at a latitude of 52° 31' and a longitude of 13° 25' (Berlin, Germany). This is necessary, as the rotation of the Earth would affect the topocentric distance from an observer to the moon at a certain point in time.

For my exploration, I gathered lunar distance data from an online applet written by Jurgen Giesen.
The image in figure 5 displays the simulation. For a given date and time, the simulation calculates the topocentric and geocentric distances (distance measured from the Earth's center to the moon's center), the lunar phase, tidal acceleration, and a variety of other quantities.
For this investigation, topocentric distance was used, as it would represent the distance to the moon from an observer on the Earth's surface. The distance values for a series of dates were developed from the table generated by the simulation displayed in figure 6.

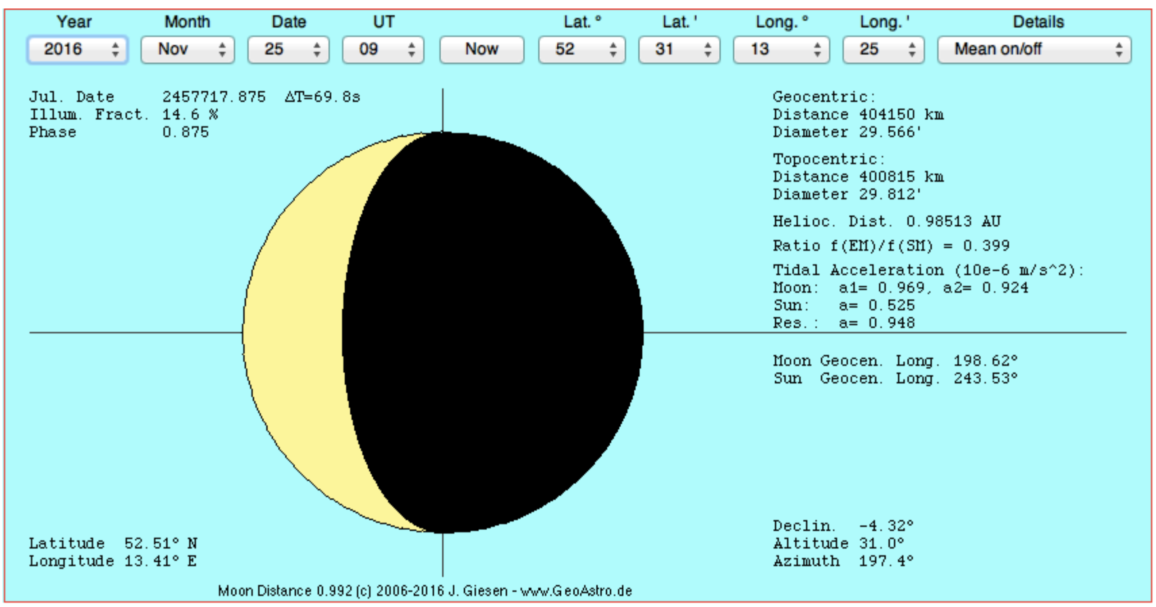


Figure 5
The figure displays the user interface for the lunar distance simulation. Dropdowns are available for to vary year, month, and time as well as the co-ordinate from, which the moon is being observed. The details drop provides access to a table of values and a series of graphs of different lunar data.

[11] Giesen, Jurgen. Moon Distance. Computer software. Version 0.98. J. G.'s GeoAstro Applet Collection. Accessed November 27, 2016. http://www.jgiesen.de/moondistance/index.htm.

Moon Distance 0.992
Lat.= 52.51° Long.= 13.41°
(c) 2006-2016 J. Giesen - www.GeoAstro.de

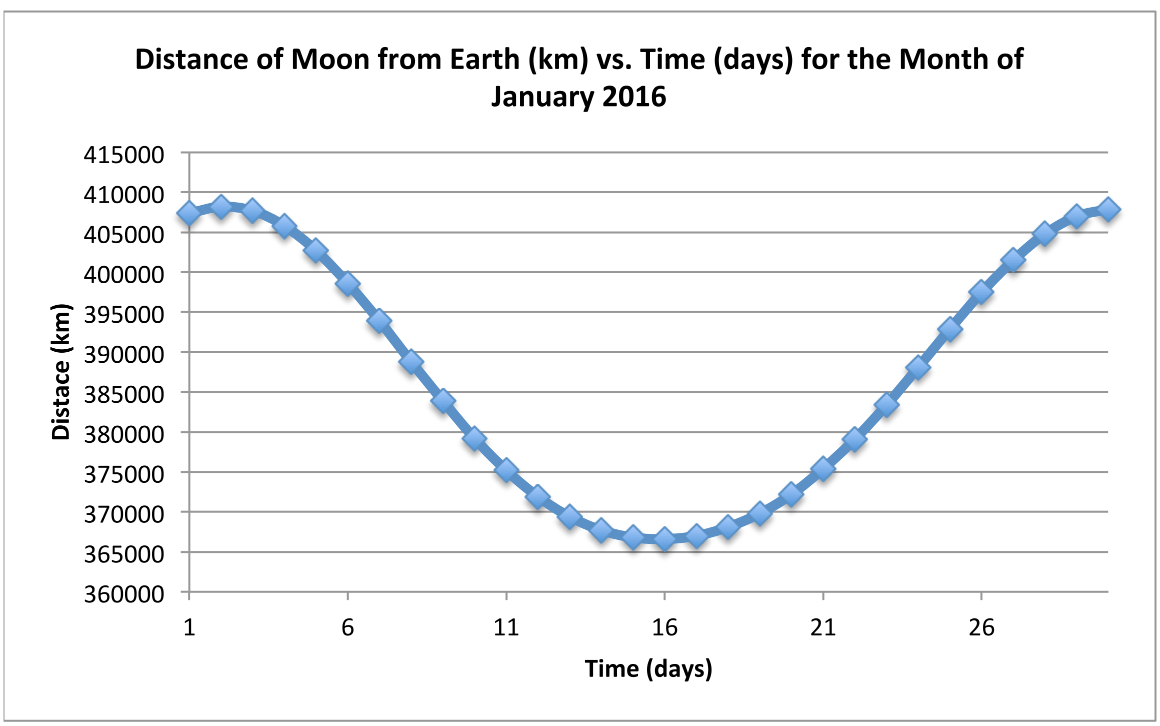
	Geocentric Dist./km	Geocentric arcmin	Phase	Topocentric Dist./km	Topocentric arcmin	Tidal Accel. a*10e6 m/(s*s)	Heliocent. Dist./AU	F(EM)/F(SM)
at 9 UT								
D E M O								
Jan 02	404268	29.557	0.754	402540	29.684	0.415	0.98324	0.39700
Jan 03	403767	29.594	0.784	401816	29.738	0.467	0.98274	0.39800
D E M O								
Jan 05	399128	29.938	0.845	396954	30.102	0.789	0.98182	0.40700
Jan 06	395382	30.222	0.877	393215	30.388	1.022	0.98145	0.41400
D E M O								
Jan 08	386527	30.914	0.942	384617	31.068	1.479	0.98096	0.43300
Jan 09	382107	31.272	0.976	380440	31.409	1.628	0.98087	0.44300
D E M O								
Jan 11	374798	31.881	0.046	373803	31.966	1.630	0.98106	0.46000
Jan 12	372291	32.096	0.082	371699	32.147	1.482	0.98133	0.46700
D E M O								
Jan 14	369789	32.313	0.155	370052	32.290	1.039	0.98220	0.47400
Jan 15	369645	32.326	0.192	370322	32.267	0.842	0.98275	0.47500
D E M O								
Jan 17	371006	32.207	0.264	372396	32.087	0.702	0.98398	0.47300
Jan 18	372341	32.092	0.301	374001	31.949	0.784	0.98459	0.47000
D E M O								
Jan 20	376205	31.762	0.372	378182	31.596	1.155	0.98570	0.46100
Jan 21	378767	31.547	0.407	380784	31.380	1.360	0.98614	0.45500
D E M O								
Jan 23	385152	31.024	0.476	387013	30.875	1.601	0.98675	0.44100
Jan 24	388828	30.731	0.510	390504	30.599	1.589	0.98689	0.43300
D E M O								
Jan 26	396289	30.153	0.575	397429	30.066	1.317	0.98688	0.41700

Figure 6
The figure displays the lunar distance for a series of dates. By varying, values in the simulation, the table can show distance data for a month, year, or apogees and perigees.

Data and Analysis

Lunar Distance in km ±100km	Day in month of January 2016									
	1	2	3	4	5	6	7	8	9	10
	407400	408200	407700	405800	402700	398600	393900	388800	383900	379200
	11	12	13	14	15	16	17	18	19	20
	375200	371900	369400	367700	366800	366600	367000	368100	369800	372200
	21	22	23	24	25	26	27	28	29	30
	375400	379100	383400	388100	392900	397500	401500	404800	407000	407900

Table 1
The table displays the lunar distance data for each day in the month of January 2016

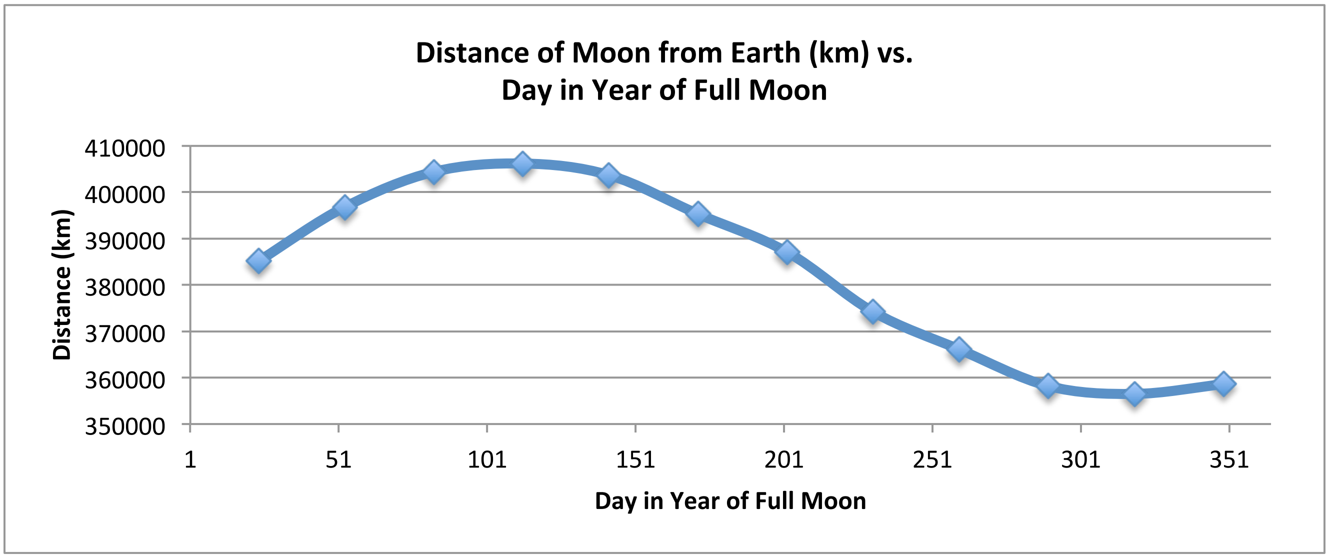


Graph 1
The graph displays the variation in lunar distance over the month of January 2016. The effect of the uncertainty is small and, as a result, the error bars are not visible on the graph.

As can be seen from the graph and table, lunar perigee occurs on the 16th and apogee on the 2nd. Between these two dates the lunar distance has had a maximum change of over 40,000km. Over the period of 30 days, the moon has returned to approximately the same position, which is close to the average time for a synodic month. By examining the distances for days at which a full moon occurs, the effect of this change in distance on the apparent size of lunar disk can be investigated.

Lunar Distance in km ±100km	Day in Year of Full Moon					
	24	53	83	113	142	172
	385200	396800	404400	406200	403700	395300
	202	231	260	290	319	349
	387100	374200	366100	358200	356500	358600

Table 2
The table displays the lunar distance data for the day in the year of each full moon in 2016



Graph 2
The graph displays the variation in lunar distance for the full moons in 2016. The effect of the uncertainty is small and, as a result, the error bars are not visible on the graph.

As can be seen from graph 2, the distance of full moons from Earth varies also approximately periodically throughout the year with its nearest points being in Northern Hemisphere winter as mentioned earlier. According to the aforementioned definition of a 'super' moon as being less than 360,000km from the Earth, the full moons in October, November, and December (days 290, 319, and 349) would all qualify as such.
Apparent, relative size of the lunar disk was calculated using the following formula:

$$Apparent\ Relative\ Size = \frac{1}{Lunar\ Distance}$$

$$Absolute\ Uncertainty\ in\ Relative\ Size = Relative\ Size \times \frac{100}{Lunar\ Distance}$$

$$Apparent\ Relative\ Size\ for\ 1st\ Day\ in\ Year = \frac{1}{403625} \approx 2.478 \times 10^{-6} km^{-1}$$

$$Absolute\ Uncertainty\ for\ 1st\ Day\ of\ Year = 2.478 \times 10^{-6} \times \frac{100}{403625} \approx \pm 6.138 \times 10^{-10} km^{-1}$$

Variation in apparent, relative size at perigee and apogee throughout 2016 was then compared to dates of full moons using data from table 3 and the intersections of the dashed lines with the curve in graph 3 below.

Relative Size of Lunar Disk (10^{-6} km^{-1})	Day of Lunar Perigee or Apogee in 2016								
	1	15	30	42	58	70	85	98	112
	2.478	2.706	2.472	2.744	2.467	2.782	2.462	2.799	2.461
	127	139	155	167	183	195	208	224	235
	2.795	2.464	2.769	2.469	2.732	2.473	2.703	2.481	2.724
	250	262	278	291	305	319	332	348	360
	2.469	2.762	2.462	2.794	2.459	2.805	2.459	2.773	2.464

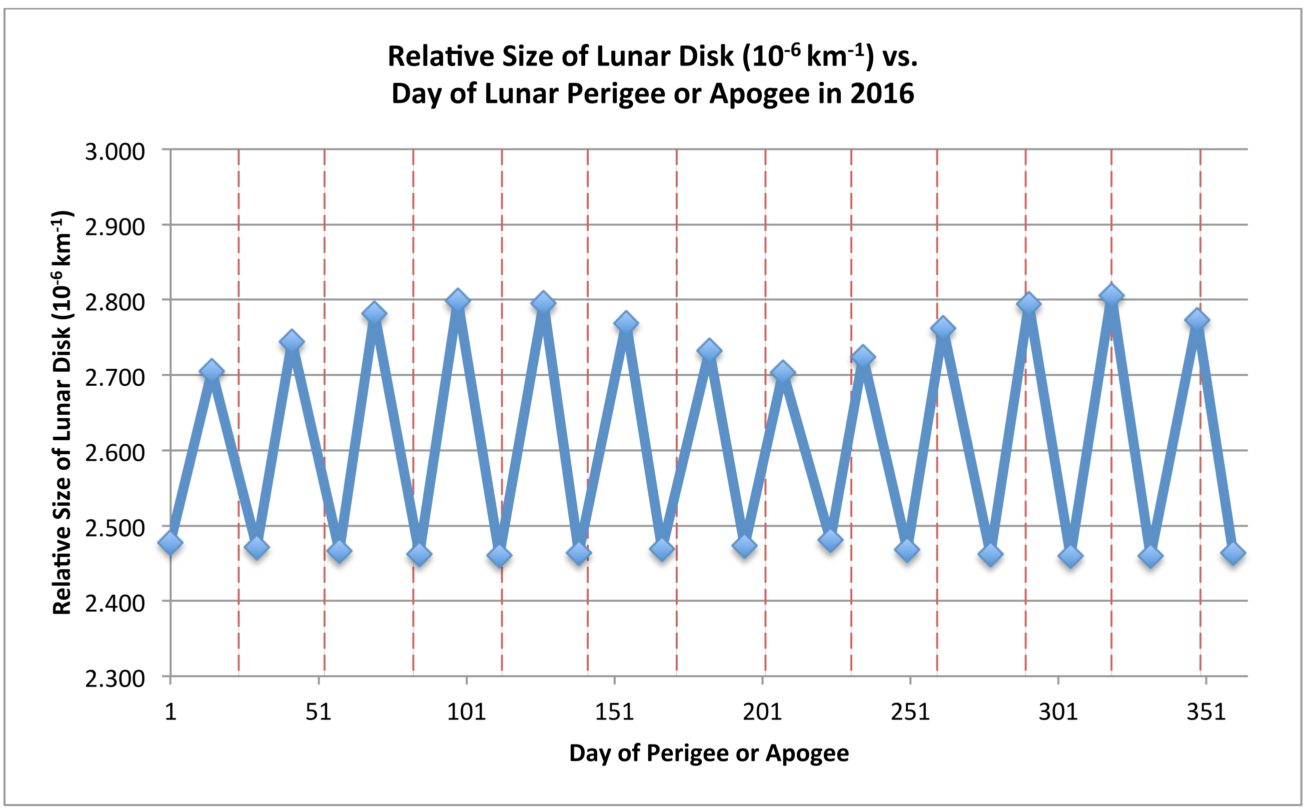
Table 3

The table displays the apparent, relative size for the day in the year of each perigee and apogee in 2016

Absolute Uncertainty in Relative Size of Lunar Disk ($\pm 10^{-10}$ km^{-1})	Day of Lunar Perigee or Apogee in 2016								
	1	15	30	42	58	70	85	98	112
	6.138	7.319	6.110	7.532	6.086	7.737	6.063	7.833	6.056
	127	139	155	167	183	195	208	224	235
	7.810	6.071	7.667	6.096	7.466	6.119	7.308	6.156	7.422
	250	262	278	291	305	319	332	348	360
	6.095	7.632	6.064	7.809	6.049	7.868	6.050	7.690	6.071

Table 4

The table displays the absolute uncertainty in the apparent, relative size of each perigee and apogee in 2016



Graph 3

The graph displays the variation in apparent, relative size for the perigees and apogees in 2016. Dashed lines are used to represent the days in the year of full moons. The effect of the uncertainty is small and, as a result, the error bars are not visible on the graph.

Full Moon	Furthest	Average	Closest
Relative Size 10^{-6} km^{-1}	2.459	2.601	2.805

Table 5
The table displays the values for the apparent, relative size of the furthest, average, and closest moons in 2016

As can be seen from graph 3, the relatively closest perigees in 2016 coincided roughly with the dates for full moons, which led to an abundance of 'super' moons during the year. One full moon coincided exactly with a perigee on November 14th leading to the closest and apparently largest moon of the year. Relative size of the apparently smallest and largest full moons during 2016 (respectively day 332 and 319) was then compared to the average lunar size.

$$\% \, diff. perigee \text{ and } apogee = 100 - 100 \times \frac{2.495}{2.805} \approx 11\%$$

$$\% \, diff. perigee \text{ and } average = 100 - 100 \times \frac{2.601}{2.805} \approx 7\%$$

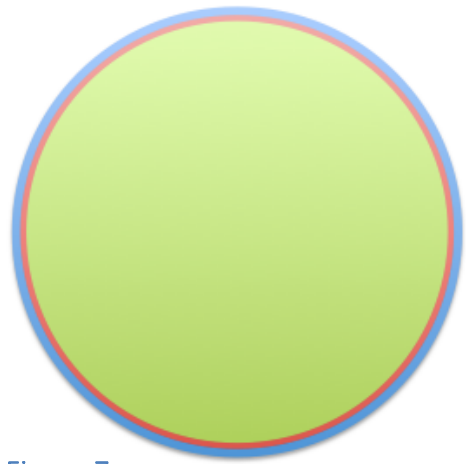


Figure 7
The figure displays an overlay of the apparent lunar disk size of the closest (blue), average (red), and furthest (green) moons.

From figure 7 it is evident that the change in lunar distance in reality does very little to effect the perceived size of the lunar disk. There is only an 11% difference between the full moon at the smallest distance and the one at the largest distance and an even smaller difference of 7% when compared to average lunar size. Especially considering how this change occurs gradually over many days rather than from one to the next, it is barely noticeable. To put this difference in perspective it will be compared to the size of a Euro coin placed at a distance given that a Euro coin has a radius of 11.675mm and the moon a radius of 1,737km.

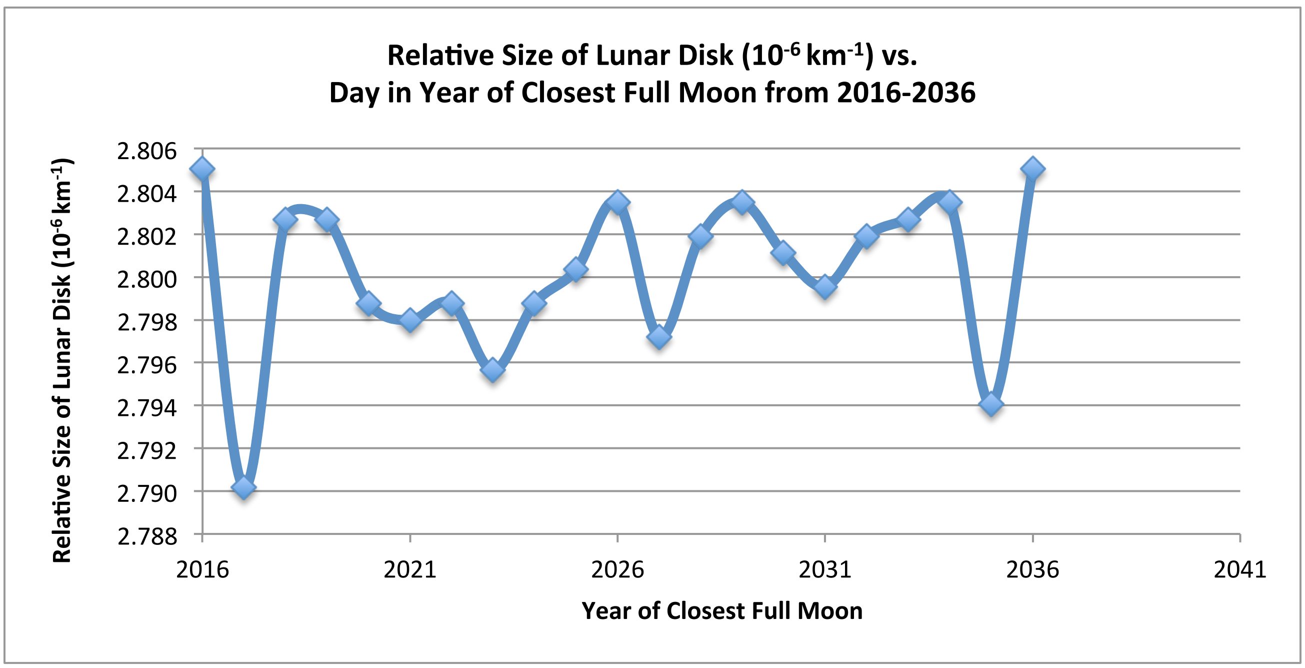
$$\frac{11.675}{d_1} = \frac{1737}{356500} \rightarrow d_1 \approx 2.396 \, meters$$

$$\frac{11.675}{d_2} = \frac{1737}{406200} \rightarrow d_2 \approx 2.730 \, meters$$

This change in position of roughly 30cm would not even be noticeable considering the coin's small size and the distance the coin is placed from the observer. Thus in relation the change in the size of the moon cannot be observed either. The next perceived, largest lunar disk was then extrapolated by charting the closest full moons from 2016 onward.

Year	Lunar Distance in km ±100km	Relative Size (10^{-6} km^{-1})	Uncertainty in Relative Size (10^{-10} km^{-1})
2016	356500	2.805	7.868
2017	357200	2.800	7.786
2018	356600	2.804	7.854
2019	356800	2.803	7.856
2020	356900	2.802	7.832
2021	356800	2.803	7.828
2022	357300	2.799	7.834
2023	357200	2.800	7.815
2024	357200	2.800	7.833
2025	356800	2.803	7.842
2026	356700	2.803	7.860
2027	357300	2.799	7.823
2028	356700	2.803	7.852
2029	356700	2.803	7.860
2030	357000	2.801	7.845
2031	357000	2.801	7.845
2032	356900	2.802	7.850
2033	356800	2.803	7.854
2034	356400	2.806	7.859
2035	357400	2.798	7.807
2036	356500	2.805	7.868

Table 6
The table displays the values for the lunar distance and relative size of the closest full moons from 2016-2036.



Graph 4
The graph displays the variation in apparent, relative size of the closest full moon for each year from 2016-2036. The effect of the uncertainty is small and, as a result, the error bars are not visible on the graph.

As can be seen from table 5 and graph 4, following the 'super' moon of November 14 2016, the next time a 'super' moon will be at a smaller distance and thus apparent, relative size will be on December 24, 2026.

Conclusion

From graph 1, it is apparent that the moon's distance from the Earth will vary periodically with each synodic month due to its elliptical orbit. Additionally, from graph 2, it is evident that the moon's distance will also vary on a larger period of a year due to the effect of the sun's gravity. Since apparent, relative size is inversely proportional to distance it will also vary on the same periods. Therefore, the moon will appear largest at perigee and in Northern Hemisphere winter. For this year in November, the super moon in the news was 356500 from Earth and coincided with a perigee during Northern Hemisphere winter supporting the conclusion that moons at these moments in time appear largest.

But is a super moon really 'super'? Not particularly. A 7% difference from the average, or relatively, the change in position of 2.7m to 2.4m of a Euro coin is barely noticeable. Furthermore the super moon this year in November was among the largest generated by the simulation foreseeably only being surpassed in 2026, which shows the difference is typically even smaller.

It is possible that the distances used for data were not the absolute minimums for each cycle of the full moon due to the time being kept at a constant 19:00 in the simulation. The point in time at which the moon was actually at its closest position to an observer on Earth's surface could have varied; however, this would have little effect on the overall conclusions but could change the dates of apparently larger super moons.

After finishing this investigation, it occurred to me that I could simply use an online distance calculator to find the absolute minimum rather than tracking changes using the simulation; however, this would have defeated the purpose of the investigation.

Evaluation

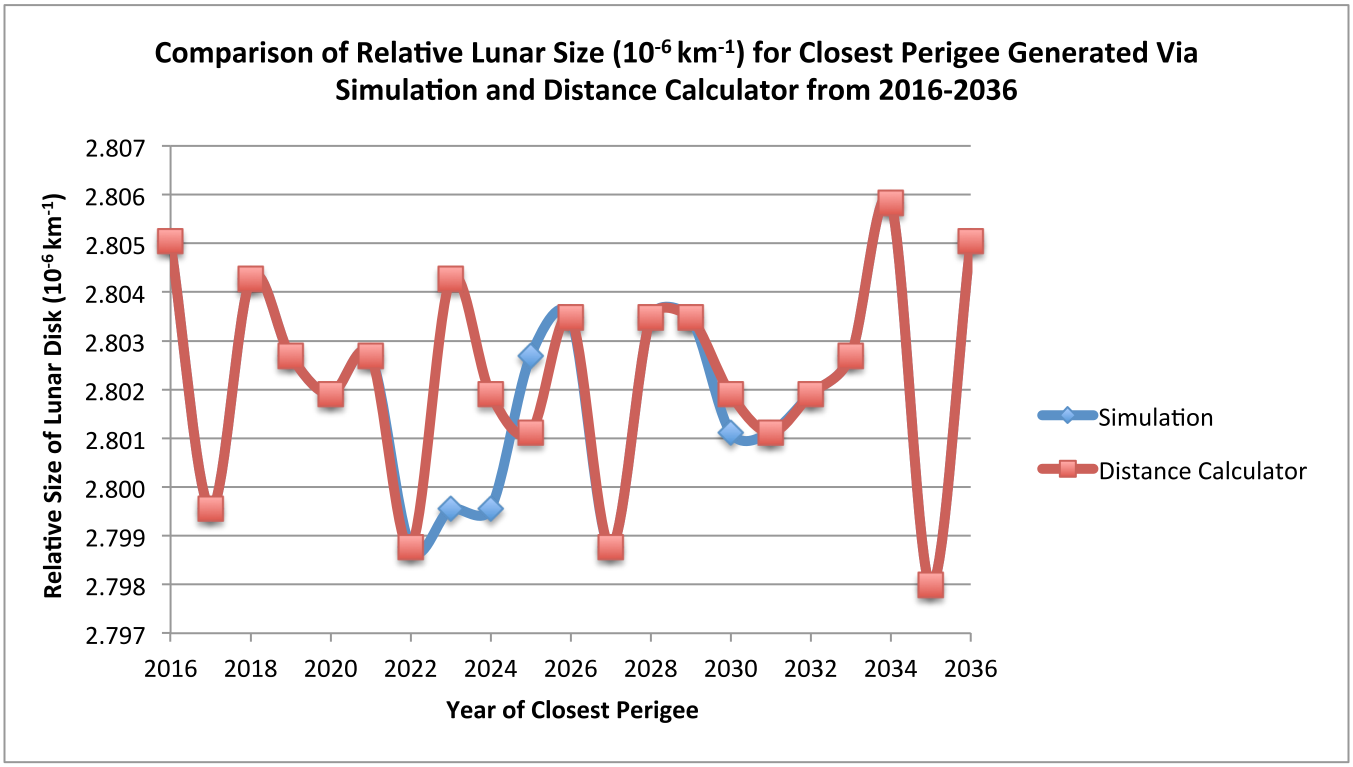
For this investigation, topocentric distance was used since it would provide the distance from an observer to the moon, which would determine its apparent size. However, the actual point at which the moon was closest to the Earth would be given by geocentric distance since it would not depend on the position of an observer.

In order to verify the values from the simulation, the data for minimum geocentric distance from 2016-2036 was compared to values from the lunar distance calculator hosted by timanddate.com.

Year	Simulation Distance ±100km	Distance Calculator ±100km	Simulation Relative Size (10^{-6} km^{-1})	Uncertainty in Simulation Relative Size (±10^{-10} km^{-1})	Distance Calculator Relative Size (10^{-6} km^{-1})	Uncertainty in Distance Calculator Relative Size (±10^{-10} km^{-1})
2016	356500	356500	2.805	7.868	2.805	7.868
2017	357200	357200	2.800	7.786	2.800	7.786
2018	356600	356600	2.804	7.854	2.804	7.854
2019	356800	356800	2.803	7.856	2.803	7.856
2020	356900	356900	2.802	7.832	2.802	7.832
2021	356800	356800	2.803	7.828	2.803	7.828
2022	357300	357300	2.799	7.834	2.799	7.834
2023	357200	356600	2.800	7.815	2.804	7.815
2024	357200	356900	2.800	7.833	2.802	7.833
2025	356800	357000	2.803	7.842	2.801	7.842
2026	356700	356700	2.803	7.860	2.803	7.860
2027	357300	357300	2.799	7.823	2.799	7.823
2028	356700	356700	2.803	7.852	2.803	7.852
2029	356700	356700	2.803	7.860	2.803	7.860
2030	357000	356900	2.801	7.845	2.802	7.845
2031	357000	357000	2.801	7.837	2.801	7.837
2032	356900	356900	2.802	7.850	2.802	7.850
2033	356800	356800	2.803	7.854	2.803	7.854
2034	356400	356400	2.806	7.859	2.806	7.859
2035	357400	357400	2.798	7.807	2.798	7.807
2036	356500	356500	2.805	7.868	2.805	7.868

Table 7

The table displays the values for the lunar distance and relative size of the closest perigees from 2016-2036 generated by the simulation and lunar distance calculator.



Graph 5

The graph compares the variation in apparent, relative size of the closest perigee for each year from 2016-2036 developed through the simulation and distance calculator. The effect of the uncertainty is small and, as a result, the error bars are not visible on the graph.

When compared to the distance calculator (graph 5), the values from the simulation differed from 0 to 600km, which is visually insignificant but shows that the simulation data is generally verified by another source. The majority of the points were even within a few kilometers of one another aside from the large differences from 2024-2026. Using the value for geocentric distance, it is noticeable that the next date at which the moon will be absolutely closest and thus apparently largest will be in 2036, specifically the 13th of January.

Aside from this, the method was straightforward and the only means by which the method could be improved would be through directly measuring distances using reflected light however even this would be prone to uncertainty possibly to an even greater degree than the simulation where the effect of uncertainty was not even noticeable. Unfortunately, there are no up to date collections of lunar distance measurements to use for comparison.

Further Studies

While I was researching super moons I found some sources, which suggested that super moons could lead to dangerous, and possibly damaging tides. It would be interesting to follow up the investigation by investigating the degree of the effect a super moon could have on tides. Another additional comparison would be to look at a super moon's effect on lunar brightness since some sources said it could be increased by as much as 30%.

Furthermore, it would be interesting for me to make calculations myself using the algorithm rather than relying on simulations or online calculators. If I understood the algorithm for lunar distance, I could extend it to other orbiting bodies to investigate the significance of their apparent size change due to perigee.

Bibliography

"What Is a Supermoon and When Is the Next One?" Supermoon / Super Moon - Why and When? Accessed November 25, 2016. https://www.timeanddate.com/astronomy/moon/super-full-moon.html.

Slater, Sara. "Why Do Planets Have Elliptical Orbits? (Beginner)." Ask an Astronomer. January 31, 2016. Accessed November 25, 2016. http://curious.astro.cornell.edu/about-us/57-our-solar-system/planets-and-dwarf-planets/orbits/244-why-do-planets-have-elliptical-orbits-beginner.

Nave, R. "Kepler's Laws." HyperPhysics. Accessed November 25, 2016. http://hyperphysics.phy-astr.gsu.edu/hbase/kepler.html.

Drakos, Nikos. "Newton's Law of Gravitation." IUN/FYDE Introductory Physics Notes. September 10, 1997. Accessed November 27, 2016. http://theory.uwinnipeg.ca/physics/circ/node7.html.

Palermo, Elizabeth. "Why Does the Moon Have Phases?" LiveScience. June 30, 2014. Accessed November 27, 2016. http://www.livescience.com/46606-why-does-the-moon-have-phases.html.

Williams, David R. "Moon Fact Sheet." Planetary Factsheet-Metric. Accessed November 27, 2016. http://nssdc.gsfc.nasa.gov/planetary/factsheet/moonfact.html.

Giesen, Jurgen. Moon Distance. Computer software. Version 0.98. J. G.'s GeoAstro Applet Collection. Accessed November 27, 2016. http://www.jgiesen.de/moondistance/index.htm.

"Kepler's Laws of Orbital Motion." How Things Fly. Accessed January 19, 2017. http://howthingsfly.si.edu/flight-dynamics/kepler%E2%80%99s-laws-orbital-motion.