ATTENTION: The Singapore Copyright Act applies to the use of this document. Nanyang Technological University Library

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2015-2016

EE2008 / IM1001 – DATA STRUCTURES AND ALGORITHMS

April / May 2016 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 4 pages.
2. Answer ALL questions.
3. All questions carry equal marks.
4. This is a closed-book examination.
5. Unless specifically stated, all symbols have their usual meanings.

---

1. (a) Determine the asymptotic upper bound for the number of times the statement "$y = y + 2$" is executed in each of the following algorithms.

(i)
```
for i = 1 to n
    for j = i + 1 to n
        for k = j + 1 to n
            y = y + 2
```

(ii)
```
for i = 1 to n
    for j = i to 2i
        y = y + 2
```

(10 Marks)

Note: Question No. 1 continues on page 2.

ATTENTION: The Singapore Copyright Act applies to the use of this document. Nanyang Technological University Library

(b) (i) Solve the following recurrence relation: $b_n = \frac{b_{n-1}}{1+b_{n-1}}$, $b_0 = 1$, where n is a non-negative integer.

(ii) Determine whether the following statement is true or false. Justify your answer.

$$\text{If } f(n) = \Theta(g(n)) \text{ then } g(n) = \Theta(f(n)).$$

(8 Marks)

(c) A pointer *start* points to the first element of a doubly-linked list L. Write an algorithm that reverses the elements of L. You are not allowed to use any additional data structure in your solution.

(7 Marks)

2. (a) (i) Let Q be a non-empty queue and S be an empty stack. Using the stack and queue ADT functions and the stack S, write an algorithm to reverse the order of the elements in Q.

(ii) Draw the 7-item hash table resulting from hashing the keys 19, 26, 13, 48, and 17 using the hash function $h(x) = x \bmod 7$. Assume that collisions are handled by double hashing using a second hash function $h'(x) = 5 - (x \bmod 5)$.

(9 Marks)

(b) Assume that the LIST ADT is implemented using a doubly linked list. Using pseudo-code, describe the implementation of the method *insertBefore*(p,e) of the LIST ADT.

(6 Marks)

(c) Suppose that the data stored at each node in a binary search tree is a positive integer. Write a recursive algorithm that finds the sum of all values, which are less than a given value x in the binary search tree.

(10 Marks)

ATTENTION: The Singapore Copyright Act applies to the use of this document. Nanyang Technological University Library

3. (a) Show clearly what the following array looks like in each step of the siftdown algorithm when applied at the index $i=2$, assuming that we are restoring a maxheap and the index of the array starts at 1.

102	20	69	67	33	58	65	23	15

(5 Marks)

(b) Continuing from your answer obtained in part (a), show each step in the heapsort algorithm.

(12 Marks)

(c) Write an algorithm using counting sort to sort an array of integers in the range $[-k,k]$. You may make use of the functions discussed in lecture to construct your algorithm.

(8 Marks)

4. (a) Use the depth first search (dfs) algorithm starting at vertex 1 to perform topological sorting of the directed acyclic graph shown in Figure 1. Explain each step clearly by drawing the dfs trees generated and the output array at each step.

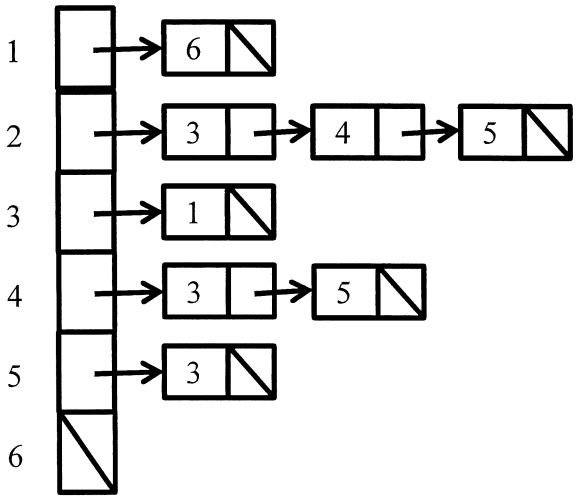

Figure 1

(10 Marks)

Note: Question No. 4 continues on page 4.

ATTENTION: The Singapore Copyright Act applies to the use of this document. Nanyang Technological University Library

(b) Write an algorithm that finds the sum of the in-degrees of all the vertices in a directed graph. Assume that the directed graph is represented by an adjacency list.

(10 Marks)

(c) What is the time complexity of your algorithm in part (b) in terms of the number of vertices and edges? Justify your answer.

(5 Marks)

END OF PAPER

ATTENTION: The Singapore Copyright Act applies to the use of this document. Nanyang Technological University Library

ATTENTION: The Singapore Copyright Act applies to the use of this document. Nanyang Technological University Library

ATTENTION: The Singapore Copyright Act applies to the use of this document. Nanyang Technological University Library

ATTENTION: The Singapore Copyright Act applies to the use of this document. Nanyang Technological University Library

EE2008 DATA STRUCTURES & ALGORITHMS
IM1001 DATA STRUCTURES & ALGORITHMS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.