NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2014-2015

EE2008 / IM1001 – DATA STRUCTURES AND ALGORITHMS

April / May 2015 Time Allowed: 2 hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 4 pages.
2. Answer ALL questions.
3. All questions carry equal marks.
4. This is a closed-book examination.
5. Unless specifically stated, all symbols have their usual meanings.

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1. (a) Consider the following recurrence equation, defining a function $T(n)$:

$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ T(n-1) + n & \text{otherwise} \end{cases}$$

Show, by induction, that $T(n) = \dfrac{n(n+1)}{2}$.

(7 Marks)

(b) Determine the asymptotic upper bound for the number of times the statement "$r = r + 1$" is executed in each of the following algorithms.

(i)
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for i = 1 to n do
    for j = 1 to i do
        r = r + 1
```

Note: Question No. 1 continues on page 2.

(ii) **for** $i = 1$ to $n/2$ **do**
for $j = i$ to $n - i$ **do**
for $k = 1$ to j **do**
$r = r + 1$

(10 Marks)

(c) Determine whether the following statement is true or false. If the statement is true, prove it. If the statement is false, give a counterexample.

If $f_1(n) = \Omega(g_1(n))$ and $f_2(n) = \Omega(g_2(n))$, then
$f_1(n) + f_2(n) = \Omega(g_1(n) + g_2(n))$.

(8 Marks)

2. (a) Given the binary tree in Figure 1, write the tree traversal orderings in *inorder* and *postorder*, respectively.

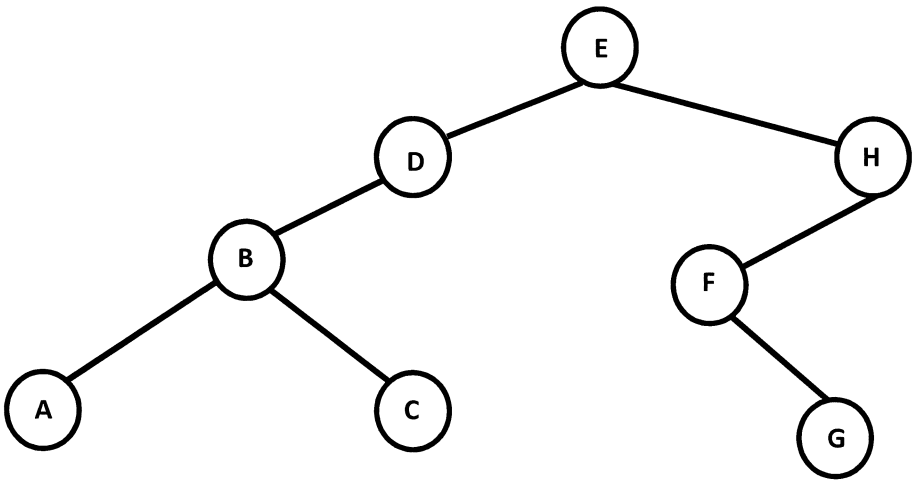

Figure 1: A Binary Tree

(8 Marks)

Note: Question No. 2 continues on page 3.

(b) (i) A pointer *start* points to the first element of a singly-linked list L. All the elements in L are integers. Write a recursive algorithm that computes the number of elements that have value 0 in L.

(ii) Write a recursive algorithm that computes the number of nodes in a binary tree.

(11 Marks)

(c) A doubly-linked list is used to implement the abstract data type *Queue* with the variables f and r pointing to the first and last elements of *Queue,* respectively. Write the pseudo-code for the function *dequeue*() for *Queue*.

(6 Marks)

3. (a) Explain clearly each step of the partition algorithm on the following array:

60	47	90	12	58	70

(10 Marks)

(b) Continuing from your answer in part (a), show how the select algorithm finds the third smallest element in the array in part (a).

(5 Marks)

(c) Given a sorted array consisting of n integers, not necessarily distinct, write in pseudo-code an algorithm with $O(n)$ time complexity to determine if any integer occurs more than $\lceil n/2 \rceil$ times in the array.

(10 Marks)

4. (a) Use Dijkstra's algorithm to find the shortest path from vertex A to vertex D in the following weighted graph. Show each step clearly.

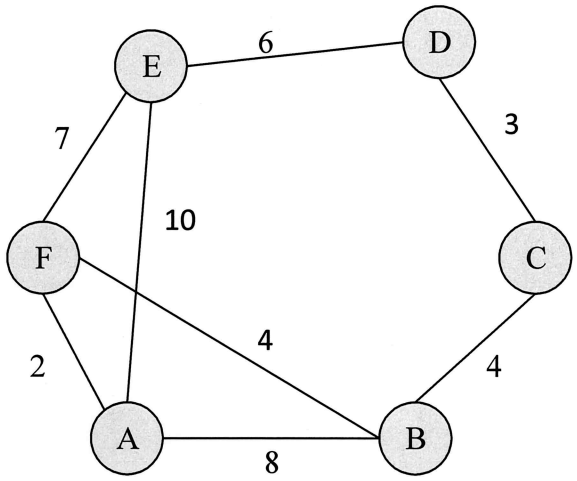

Figure 2: A Weighted Graph

(10 Marks)

(b) Suppose that $G = (V, E)$ is a tree represented by an adjacency list. Write in pseudo-code an algorithm that constructs the adjacency list for a new graph $G' = (V, E')$ with the same set of vertices V as G, and with edges between any two vertices if and only if they are 2 hops away in G, i.e., G' contains the edge (u, v) if and only if there is a path of length 2 in G connecting u and v.

(10 Marks)

(c) What is the time complexity of your algorithm in part (b)? Justify your answer.

(5 Marks)

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.