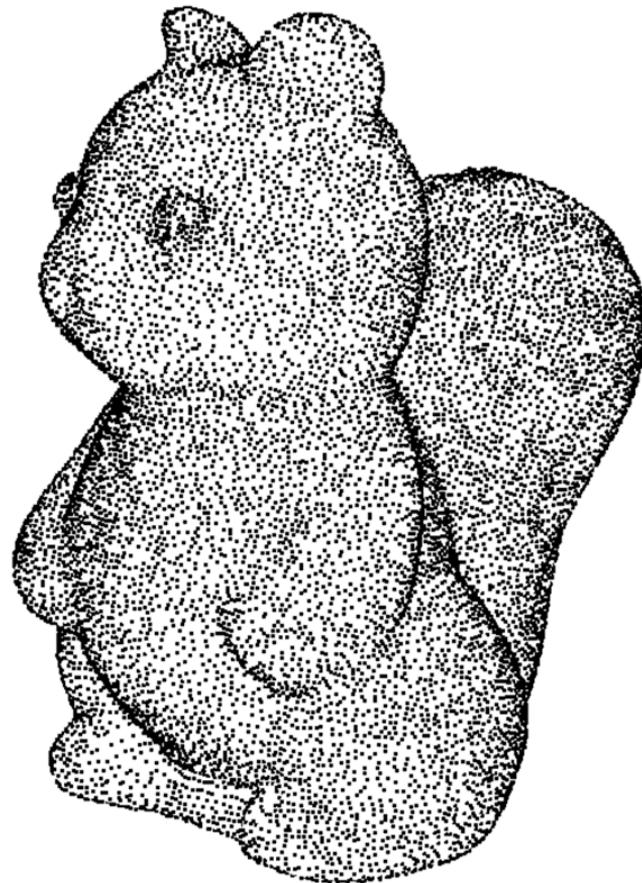


# Funções de Base Radial (RBF)

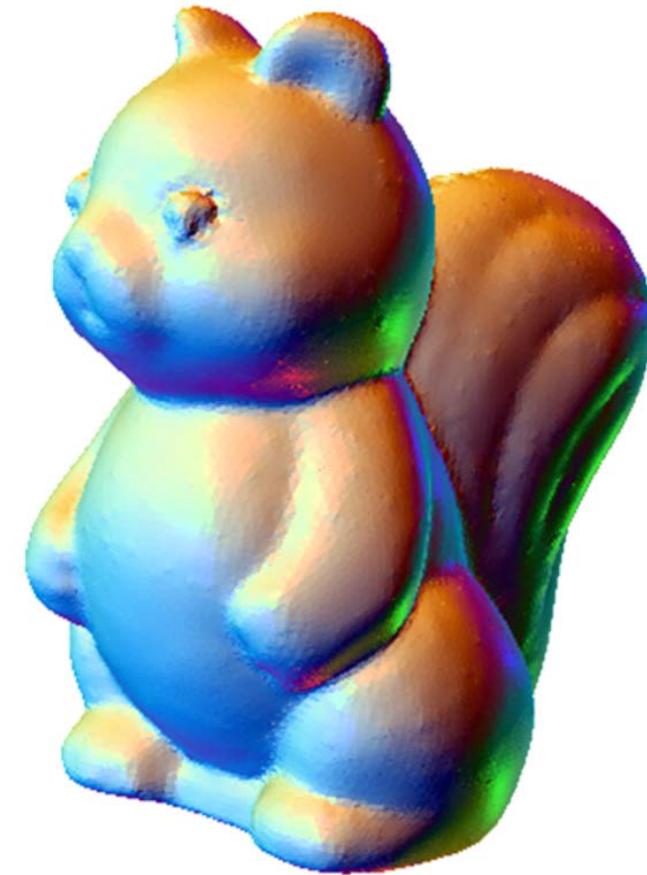
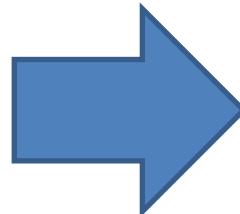
Afonso Paiva

ICMC-USP

# Motivação

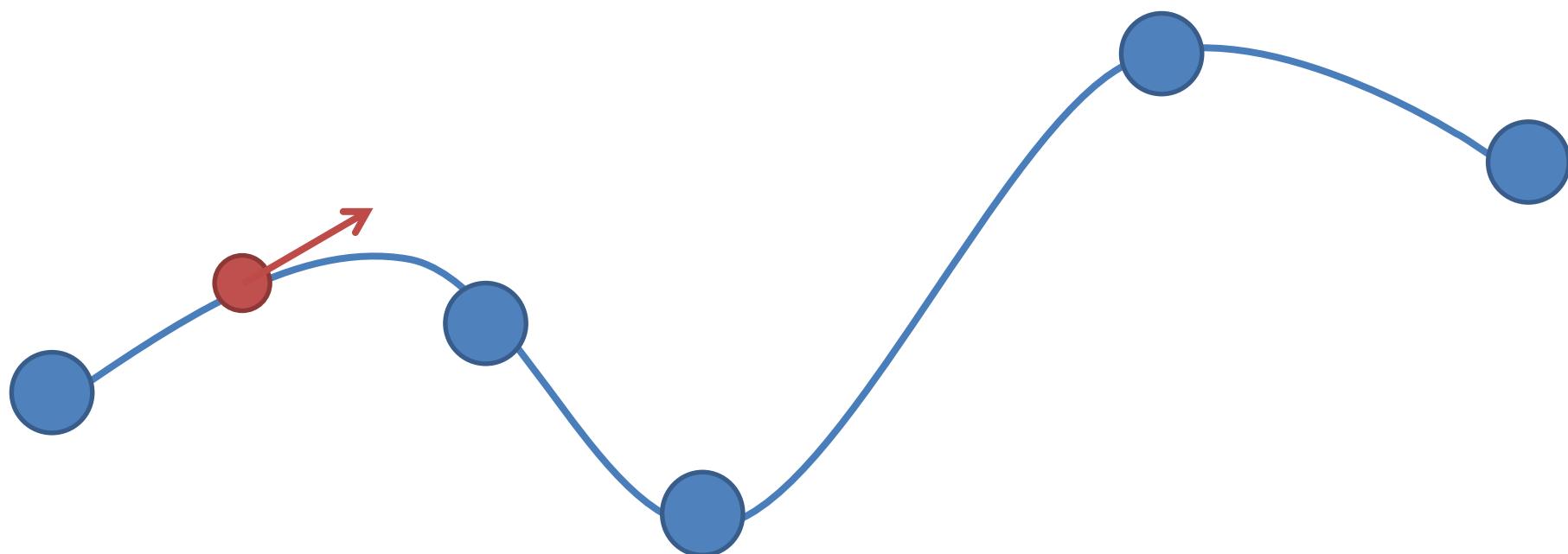


Nuvem de pontos



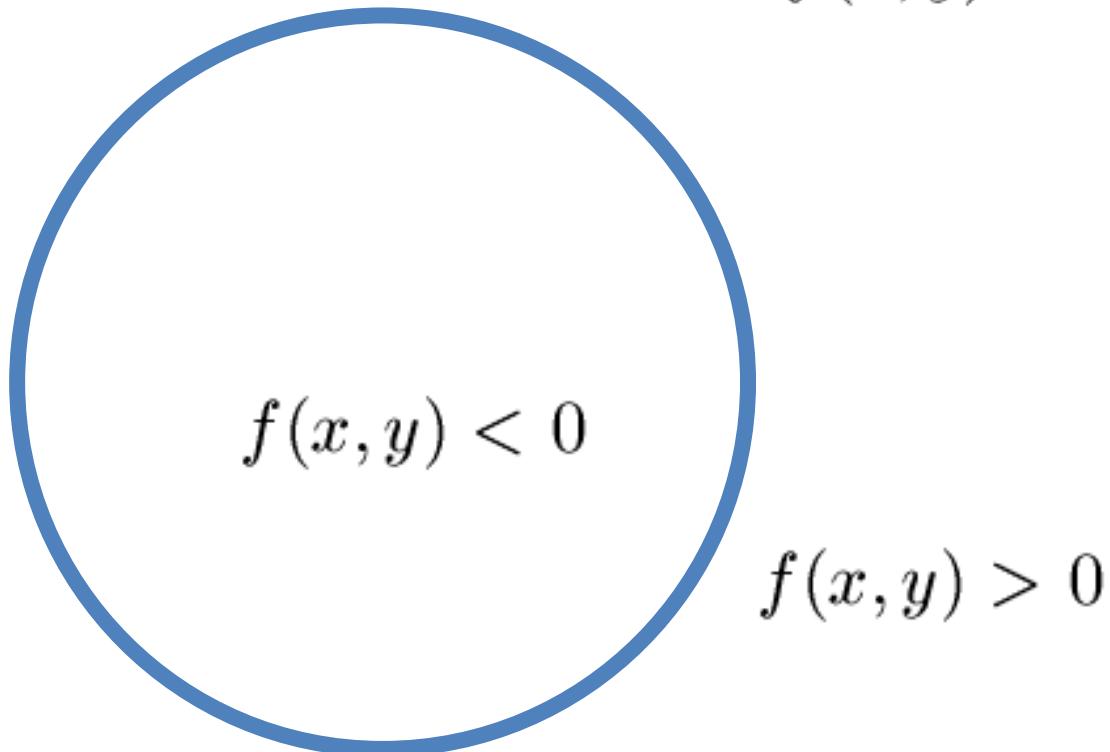
Encontrar uma superfície implícita  
que interpola os pontos

# Lembra de Interpolação?



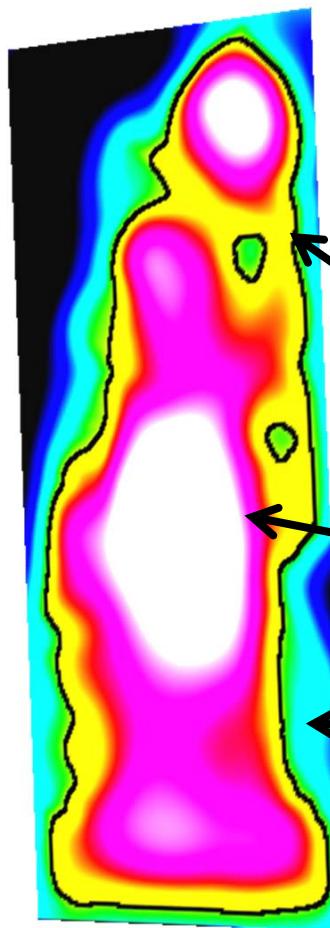
# Representação Implícita

$$f(x, y) = x^2 + y^2 - 1$$



$$C = f^{-1}(0) = \{(x, y) \in \mathbb{R}^2, f(x, y) = 0\}$$

# Representação Implícita



Uma seção de  $S$



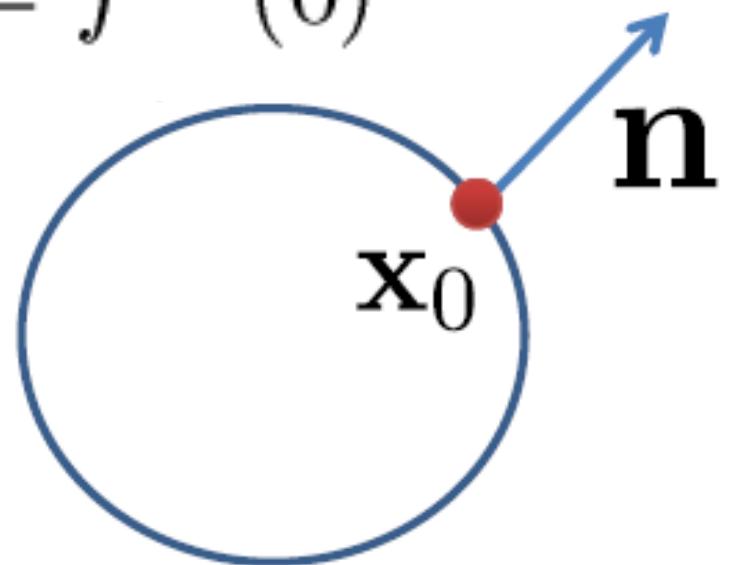
$$S = f^{-1}(0)$$

# Cálculo do vetor normal

$$S = f^{-1}(0)$$

**Teorema:**

$$\mathbf{n} = \nabla f(\mathbf{x}_0)$$



**Prova:**

$$f(\mathbf{x}_0) = f(\gamma(0)) = 0 \Rightarrow \nabla f(\mathbf{x}_0) \cdot \gamma'(0) = 0$$

# Superfícies Implícitas em CG



*Star Trek VI*  
- Paramount, 1991

*Flubber*

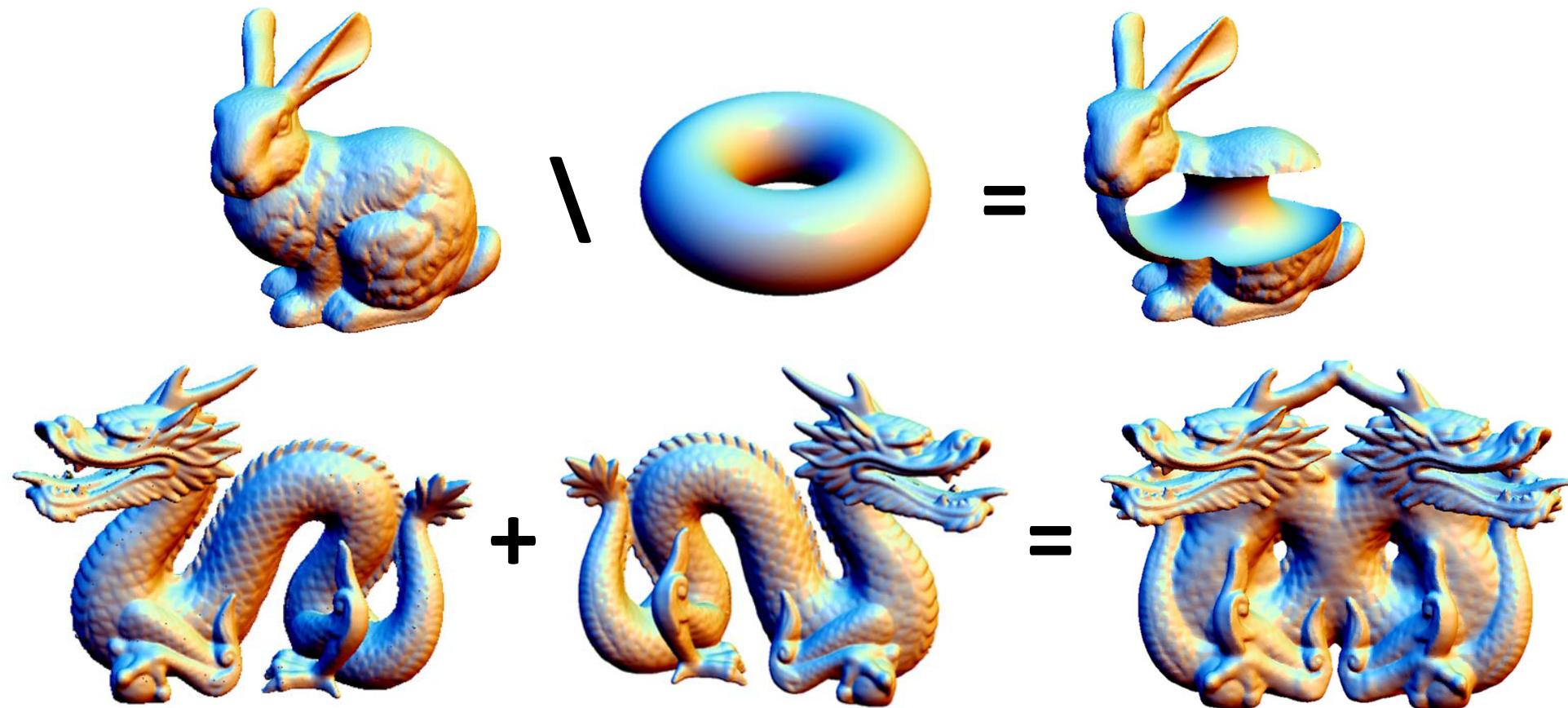
- Disney, 1997



# Por que usar superfícies implícitas?

Operações Booleanas

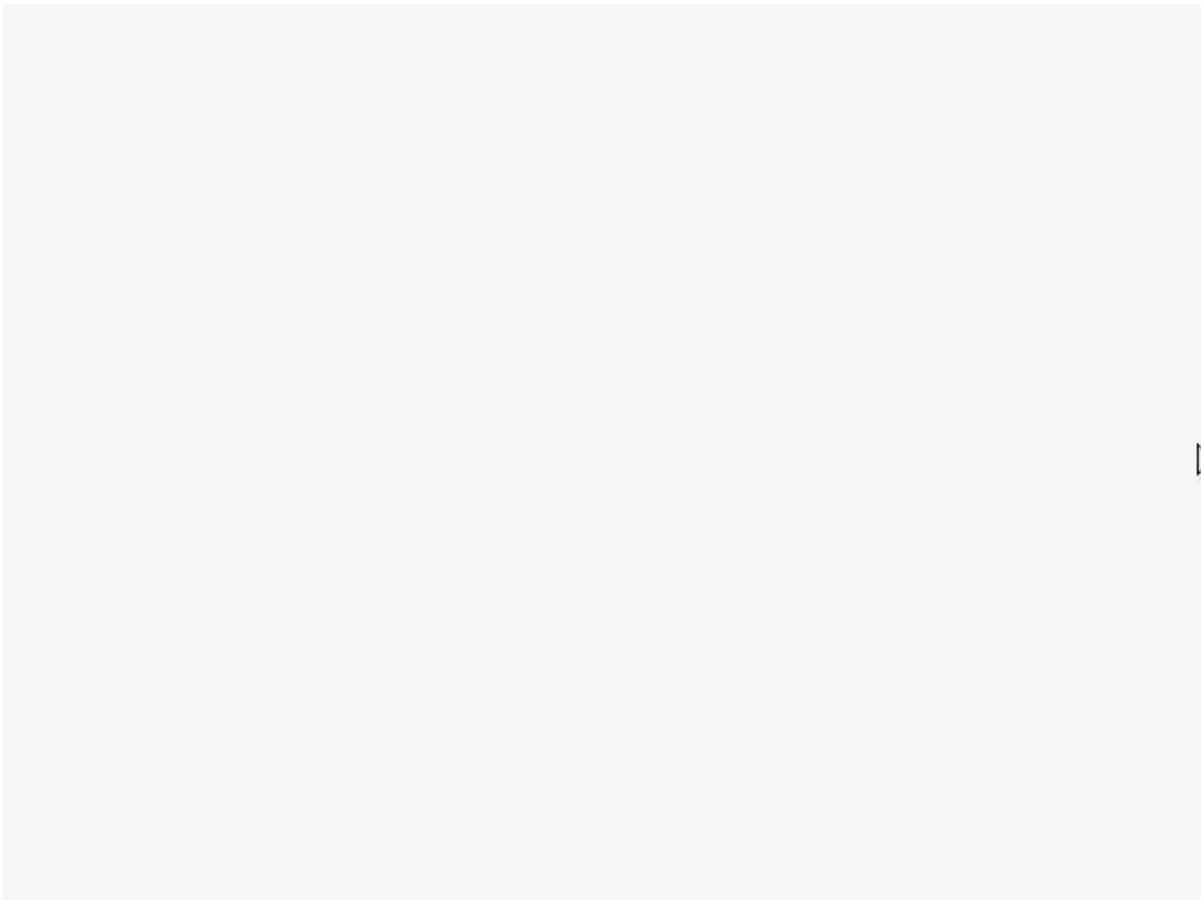
- Otake *et al.*, 2003.



# Por que usar superfícies implícitas?

Edição Interativa (*sketching*)

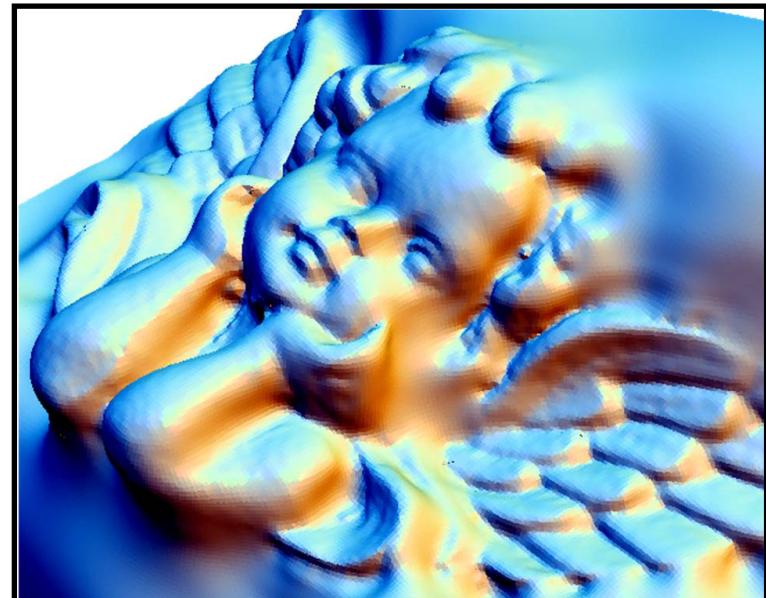
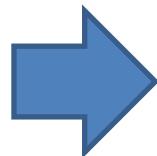
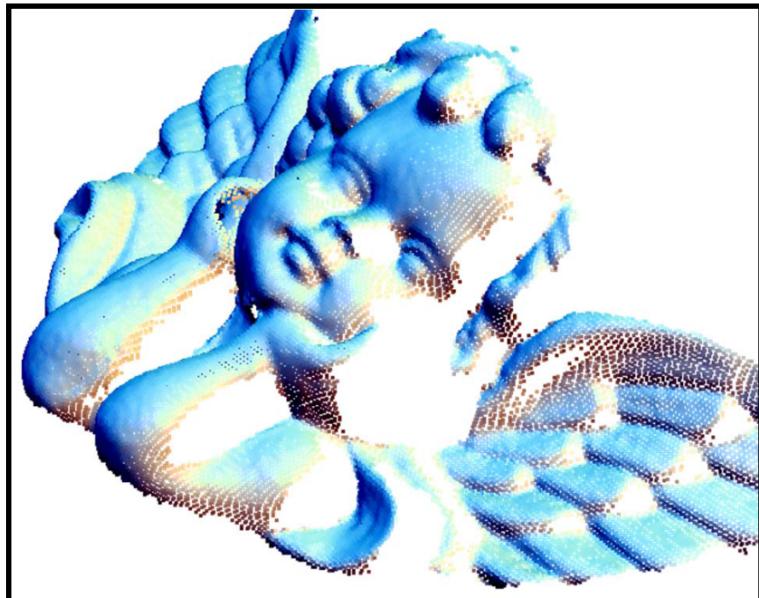
- Schmidt *et al.*, 2005.



# Por que usar superfícies implícitas?

## Restauração de Superfícies

- Otake *et al.*, 2003.



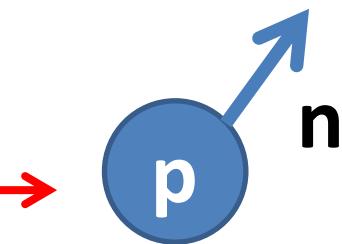
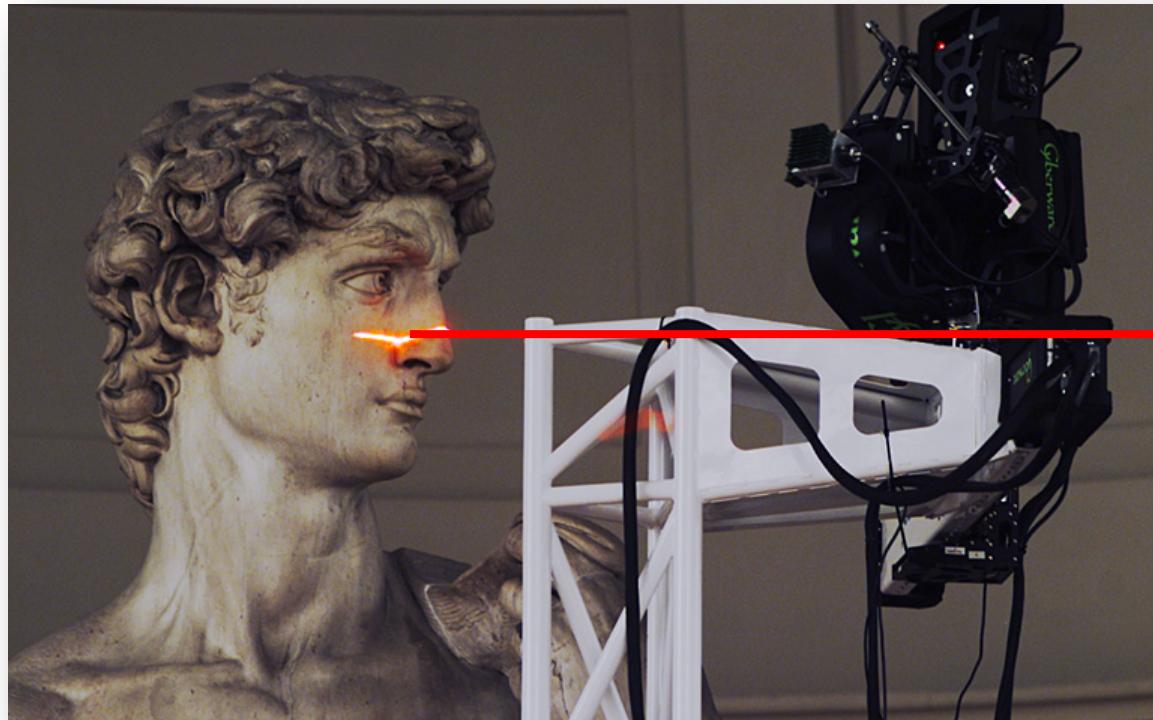
# Por que usar superfícies implícitas?

Superfície Livre de Fluido

- Losasso *et al.*, 2007.



# Scanner 3D



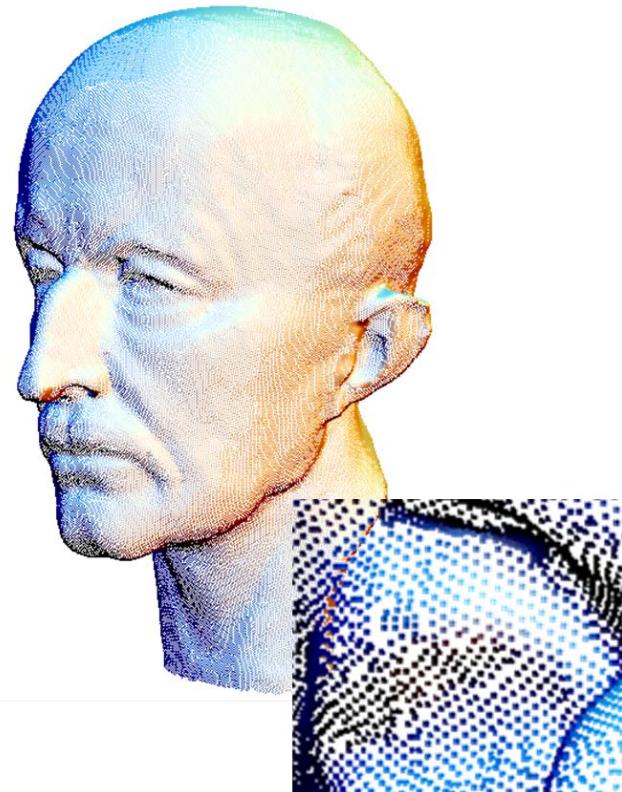
Dados de Hermite  
(posição + normal)

The Digital Michelangelo Project

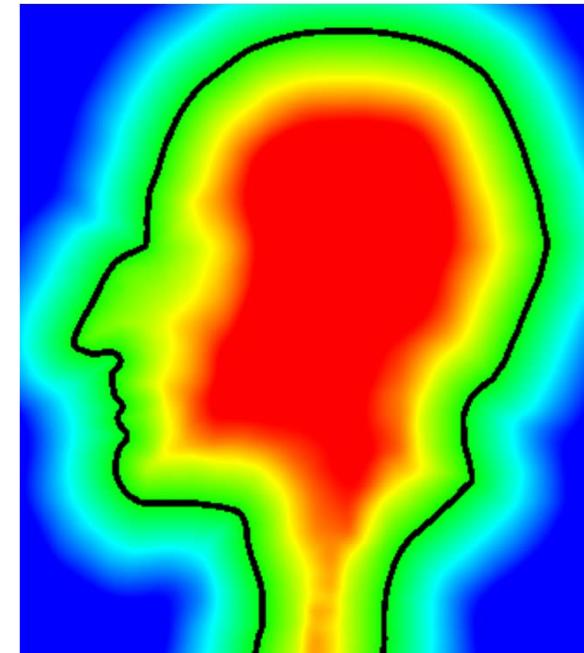
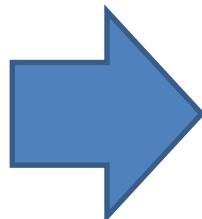


KINECT™  
for XBOX 360.

# Reconstrução via Superfícies Implícitas



Dados de Hermite



Função distância com sinal  
 $f(x,y,z) < 0 \dots \text{dentro}$   
 $f(x,y,z) > 0 \dots \text{fora}$   
 $f(x,y,z) = 0 \dots \text{pontos aproximados}$

# Funções de Base Radial (RBF)

Função real cujo valor depende apenas da distância de um ponto ao *centro p*.

$$\phi(\mathbf{x}, \mathbf{p}) = \varphi(\|\mathbf{x} - \mathbf{p}\|)$$

$$\phi : \mathbb{R}^3 \xrightarrow{\|\cdot\|} \mathbb{R} \xrightarrow{\varphi} \mathbb{R}$$

*Reconstruction and representation of 3D objects  
with radial basis functions*

- Carr *et al.*, 2001.

# Exemplos de RBF

**Gaussiana:**

$$\varphi(r) = \exp(-\beta r^2), \text{ com } \beta > 0$$

**Multi-quádrica:**

$$\varphi(r) = \sqrt{r^2 + \beta^2}, \text{ com } \beta > 0$$

**Spline poli-harmônica:**

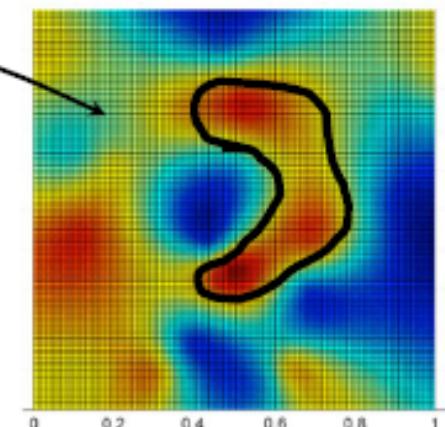
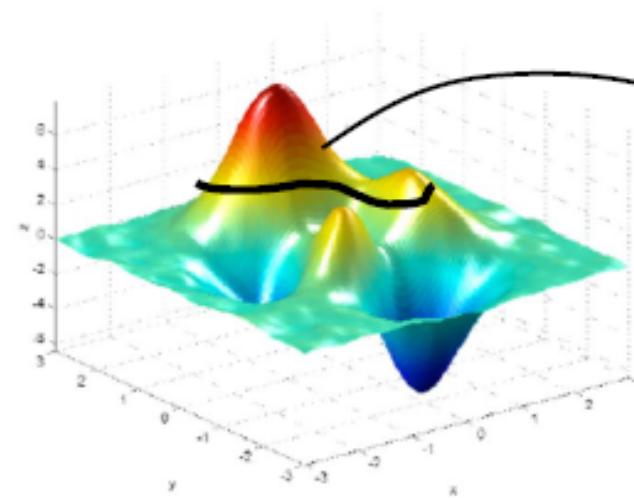
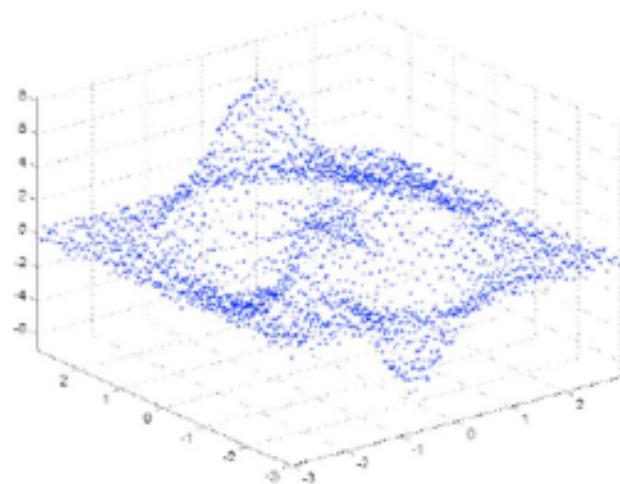
$$\varphi(r) = r^k, \quad k = 1, 3, 5 \dots$$

**Thin-plate Spline:**

$$\varphi(r) = r^2 \ln(r)$$

# Idéia Chave em RBF

Interpolar dados de Hermite em  $\mathbb{R}^{n+1}$



Depois tomar uma superfície de nível dessa função.

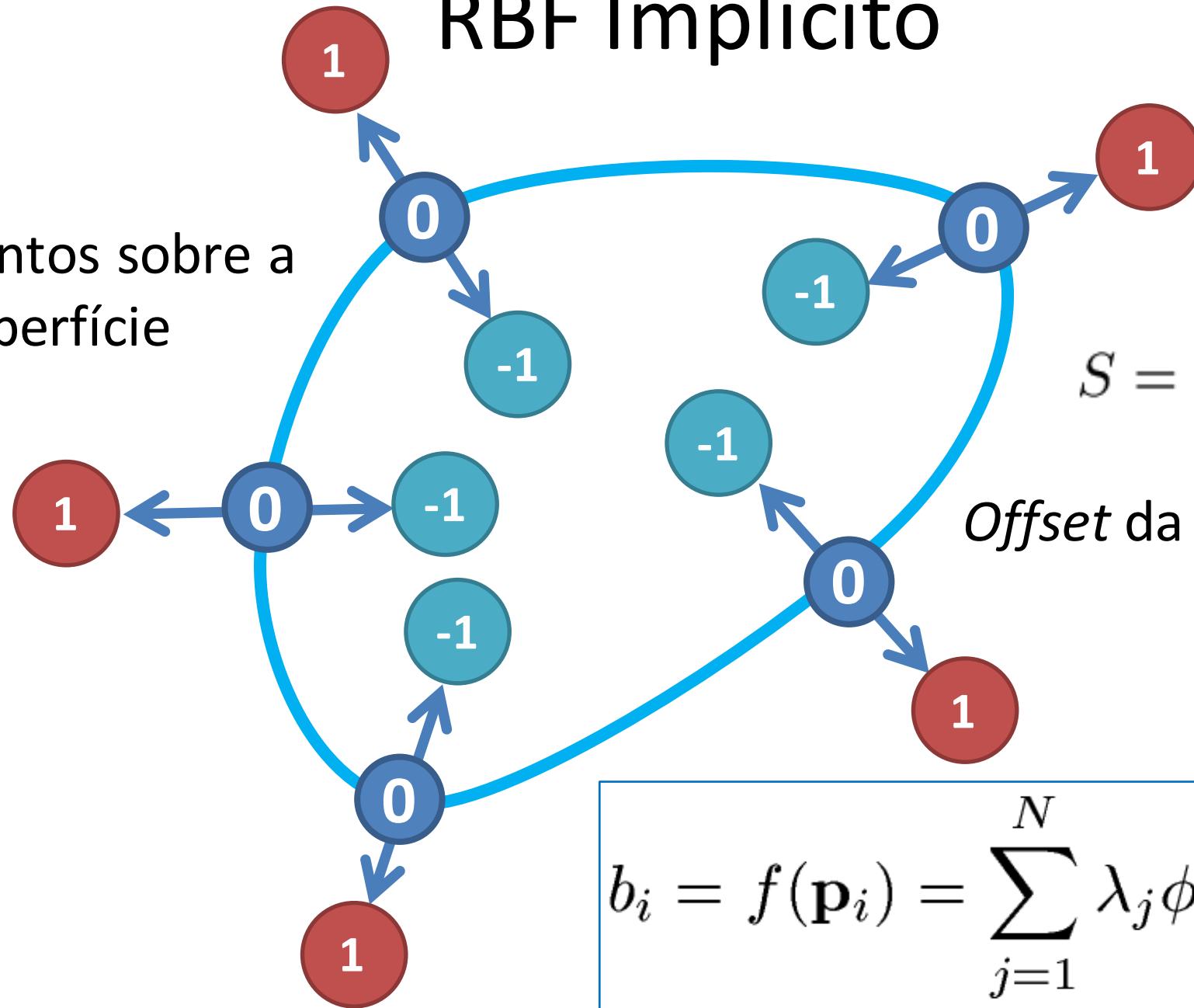
# Interpolação via RBF

$$f(\mathbf{x}) = \sum_{j=1}^N \lambda_j \phi(\mathbf{x}, \mathbf{p}_j)$$

Para obter os pesos  $\lambda$  precisamos resolver um sistema linear.  
Mas como é esse sistema linear???

# RBF Implícito

Pontos sobre a superfície



$$S = f^{-1}(0)$$

*Offset da superfície*

# Problema de Interpolação RBF

Precisamos resolver o sistema linear:

$$b_i = f(\mathbf{p}_i) = \sum_{j=1}^N \lambda_j \phi(\mathbf{p}_i, \mathbf{p}_j), \quad i = 1, \dots, N$$



$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{12} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1N} & a_{2N} & \cdots & a_{NN} \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_N \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{pmatrix}$$

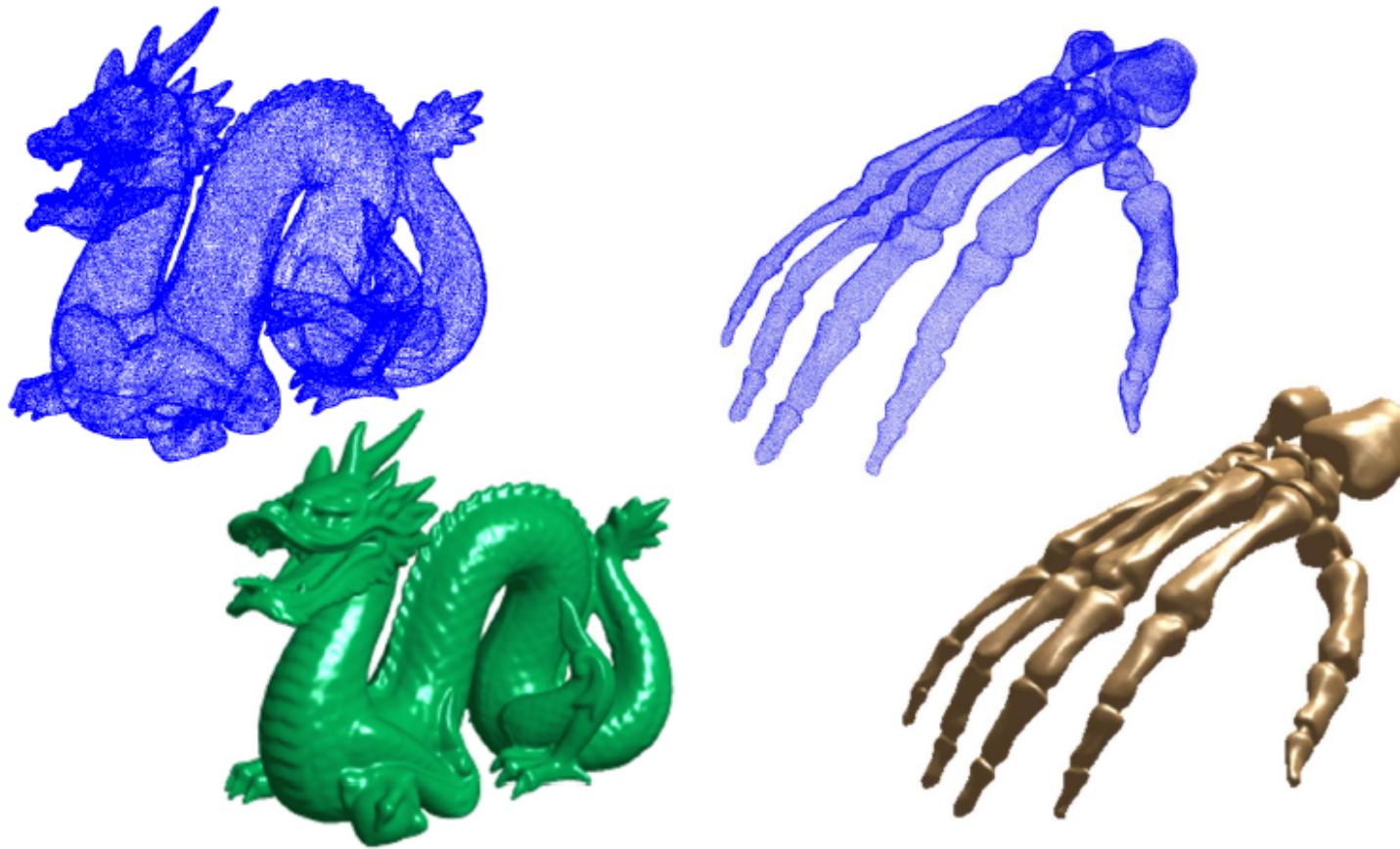
com  $a_{ij} = \phi(\mathbf{p}_i, \mathbf{p}_j) = \varphi(\|\mathbf{p}_i - \mathbf{p}_j\|)$

# Problema de Interpolação RBF

- A matriz  $(a_{ij})$  é simétrica positiva definida
  - Método de Cholesky
  - Método do Gradiente Conjugado
- Grandes problemas em RBF:
  - número de restrições aumenta com o número de pontos;
  - problemas de consumo de memória e de tempo computacional;
  - comprimento do offset;



# Resultados



*Reconstruction and representation of 3D objects  
with radial basis functions*

- Carr *et al.*, 2001.

# Resultados



*Reconstruction and representation of 3D objects  
with radial basis functions*

- Carr *et al.*, 2001.

# Partição da Unidade (PU)

- Família de funções de suporte compacto que satisfaz:

$$\sum_k \Phi_k = 1$$

- Uma função pode ser aproximada por PU da seguinte forma:

$$f(\mathbf{x}) \approx \sum_{k=1}^N \Phi_k(\mathbf{x}) p_k(\mathbf{x})$$

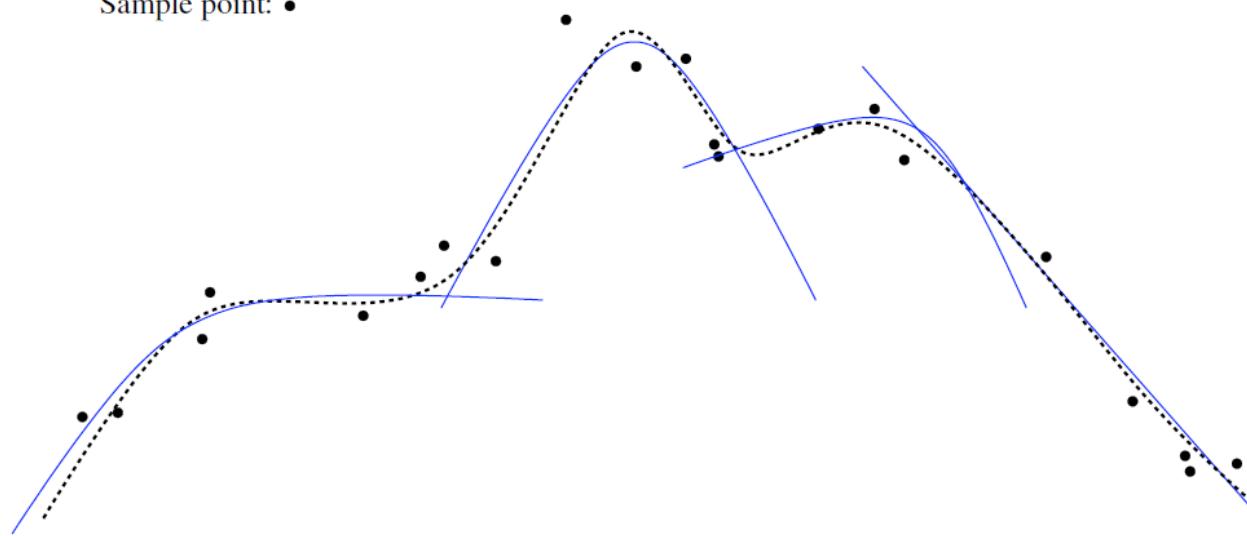
$$\Phi_k(\mathbf{x}) = \frac{\phi_k(\mathbf{x})}{\sum_j \phi_j(\mathbf{x})} \quad \text{com} \quad \phi_k(\mathbf{x}) = \exp(-\|\mathbf{x} - \mathbf{c}_k\|^2/r^2)$$

# RBF + PU

Local reconstruction: —————

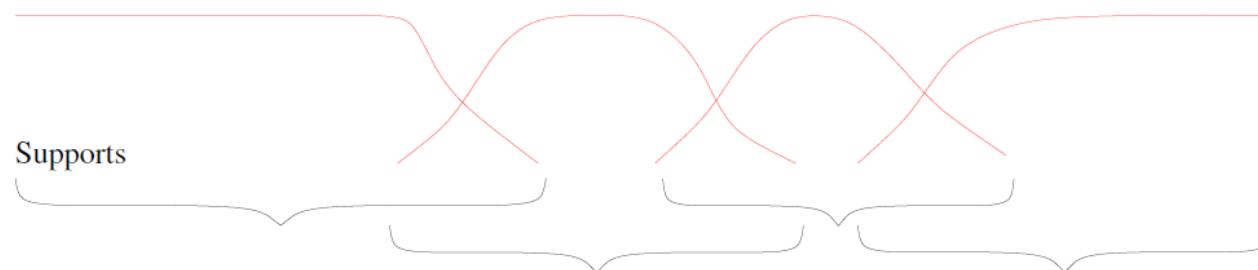
Global reconstruction: - - - - -

Sample point: •

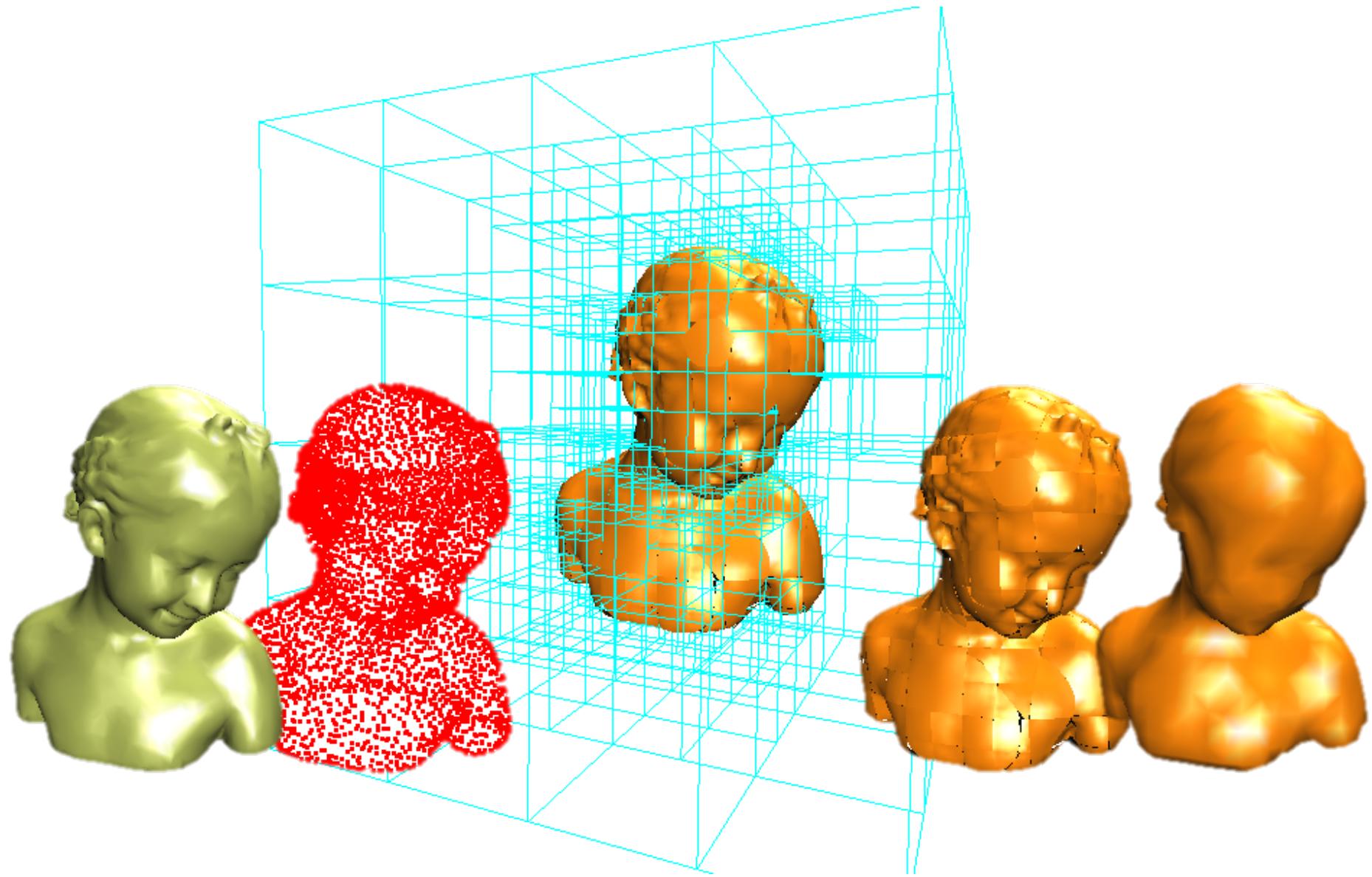


Partition of unity

Supports



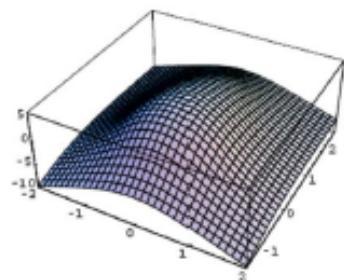
# RBF +PU



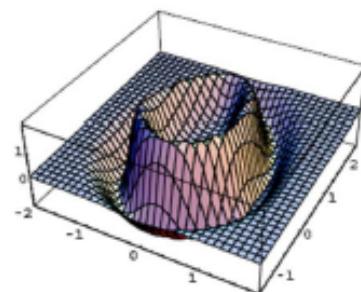
# Estado da Arte

## RBF com suporte compacto

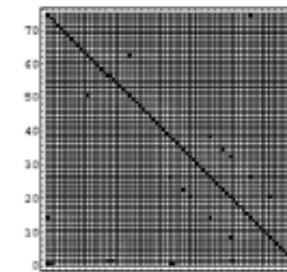
- Otake *et al.*, 2004.



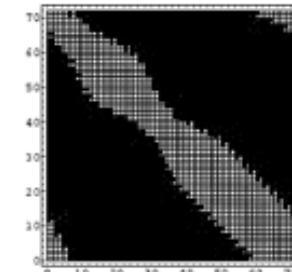
Thin-plate



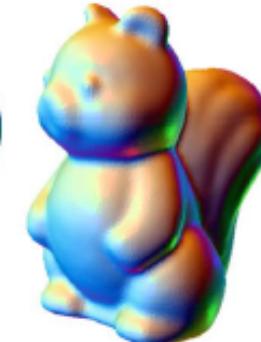
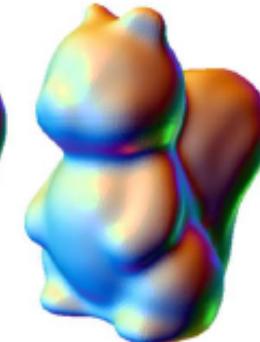
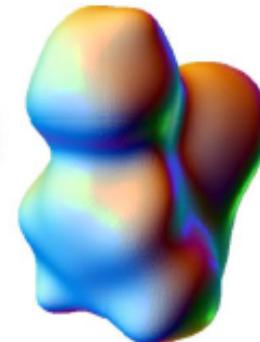
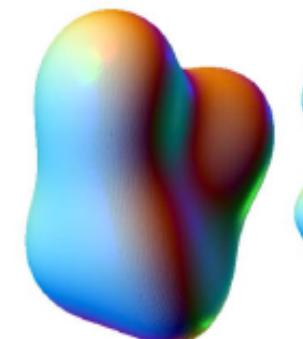
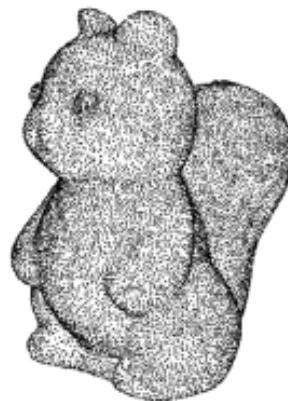
Compactly-supported



Thin-plate



Compactly-supported



# Estado da Arte

## Hermite RBF

- Macedo *et al.*, 2010.

