DSA Mini Textbook

Theo Park

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Preface

Chapter 1

Runtime Analysis

Algorithms are any well-defined computational procedures that take some value(s) as input and produce more value(s) as output. They are **effective**, **precise**, and **finite**. There are several ways to analyze the runtime of an algorithm.

1.1 Power Law

1. For the algorithm, get a table for the input size n and the runtime $T(n)$.

n	$T(n)$
250	0.0
500	0.012
1000	0.0954
2000	0.7727
4000	6.1664

2. Make sure that the data plots:

   - **have enough data plots.** For instance, if there are only two data plots, you should not make the power law conjecture.
   - **fits the power law.** You can verify this by finding the ratio between data plots.

n	$T(n)$	ratio
250	0.0	–
500	0.012	–
1000	0.0954	0.0954 / 0.012 = 7.95
2000	0.7727	0.7727 / 0.0954 = 8.10
4000	6.1664	6.1664 / 0.7727 = 7.98

   For the ratios we found, //TODO

1.2 Runtime Expressions

1.3 Asymptotic Runtime Analysis

1.4 Recursive Relationship

Chapter 2

Intro to Data Structures

Data structures are collections of data values, the relationships among them, and the functions or operations that can be applied to the data. All three characteristics need to be present.

2.1 Array

Array is a linear container of items.

Array length 6	250	251	252	253	254	255
	0	1	2	3	4	5

- Access time: $\Theta(1)$
- Inserting n items in the *tail* for array size n: $\Theta(1)$ per item, $n \times \Theta(1) \in \Theta(1)$
- Inserting n items in the *tail* for array size *unknown*: $\Theta(n)$ per item, $n \times \Theta(n) \in \Theta(n)$

Lesson? **Keep track of the tail!**

2.2 Linked List

2.3 Stack

2.4 Queue

2.5 Binary Heap

2.5.1 Building a Heap – Top-down v.s. Bottom-up

2.6 Tree

Chapter 3

Sorting Algorithms

Once you store all the items in a data structure, you might want to organize them for the future use (such as selecting nth largest element). For this, you have to *sort* the data structure (in this book, array will be assumed). *Sorting* is deciding how to permute the array elements until they are sorted.

There are couple aspects of sorting algorithms you need to consider:

- Runtime: When analyzing a runtime of a sorting algorithm, both number of compares and number of swaps are considered. **Most sorting algorithms make more comparisons than swaps**, but if a sorting algorithm makes more swaps, it must be used for the asymptotic runtime analysis
- Stability: An algorithm is stable if it preserves the input ordering of equal items For example: //TODO
- In-place: An algorithm is in-place if it can directly sorts the items without making a copy or extra array(s)

3.1 Bubble Sort

BUBBLE-SORT goes through the array and swap elements that are out of place, and if such element is found, it repeats from the beginning.

```
function BUBBLE-SORT(A)                          ▷ A is an array size n
    repeat ← True
    while repeat is True do
        repeat ← False
        for i = 0 to n − 2 do
            if A[i] > A[i + 1] then
                SWAP(A, i, i + 1)                ▷ Assume SWAP(A, i, j) swaps A[i] and A[j]
                repeat ← True
            end if
        end for
    end while
    return A
end function
```

In-place?	Stable?
True	True

-	NumCompares	NumSwaps
Already Sorted	$n-1$	0
Worst Case	$n^2 - n$	$\frac{1}{2}n^2 - \frac{1}{2}n$

3.2 Selection Sort

SELECTION-SORT is a sorting algorithm closest to our "natural" thought of sorting an array. It makes the same number of comparisons no matter what.

```
function SELECTION-SORT(A)                          ▷ A is an array size n
    for i = 0 to n − 2 do
        index ← i
        for i = i + 1 to n − 1 do
            if [j] < A[index] then
                index ← j
            end if
        end for
        if i ≠ index then
            SWAP(A, i, index)              ▷ Assume SWAP(A, i, j) swaps A[i] and A[j]
        end if
    end for
    return A
end function
```

In-place?	Stable?
True	False

-	NumCompares	NumSwaps
Already Sorted	$\frac{1}{2}n^2 - \frac{1}{2}n$	0
Worst Case	$\frac{1}{2}n^2 - \frac{1}{2}n$	$\lfloor \frac{1}{2}n \rfloor$

3.3 Insertion Sort

```
function INSERTION-SORT(A)                          ▷ A is an array size n
    for i = 1 to n − 1 do
        j ← i − 1
        while j ≥ 0 and A[j] > A[j + 1] do
            SWAP(A, j, j + 1)              ▷ Assume SWAP(A, i, j) swaps A[i] and A[j]
            j ← j − 1
        end while
    end for
    return A
end function
```

In-place?	Stable?
True	True

-	NumCompares	NumSwaps
Already Sorted	$n - 1$	0
Worst Case	$\frac{1}{2}n^2 - \frac{1}{2}n$	$\frac{1}{2}n^2 - \frac{1}{2}n$

3.4 Shell Sort

3.5 Heap Sort

HEAP-SORT uses binary max-heap to sort an array. While it's the first sorting algorithm to utilize a data structure, it's not preferred in real life due to cache issue.

```
function HEAP-SORT(A)                         ▷ A is an array size n
    A ← BUILD-HEAP(A)
    for i = n − 1 down to 0 do
        SORT-DOWN(A, i)
    end for
    return A
end function
```

The algorithm first builds the heap from the array elements (refer to section 2.5 for methods for building a heap). BOTTOM-UP is used for its runtime. Then the algorithm calls SORT-DOWN from the last heap elements down to the first.

3.5.1 Sort Down Algorithm

3.6 Merge Sort

MERGE-SORT is an algorithm //TODO

```
function MERGE-SORT(A, l, r)                  ▷ A is an array size n
    if l < r then
        m ← (l + r)/2
        MERGE-SORT(A, l, m)
        MERGE-SORT(A, m + 1, r)
        MERGE(A, l, m, r)
    end if
    return A
end function
```

3.6.1 Merge Algorithm

```
function MERGE(A, l, m, r)                    ▷ A is an array size n
    n1 ← m − l + 1
    n2 ← r − m
    L ← array size of (n1 + 1)
    R ← array size of (n2 + 1)
    ▷ Assign elements to each array
    for i = 0 to n1 − 1 do
        L[i] ← A[l + i]
    end for
    for i = 0 to n2 − 1 do
        R[i] ← A[m + j + 1]
    end for
```

```
    L[n1], R[n2] ← ∞
    i, j ← 0

    for k = l to r do
        if L[i] ≤ R[j] then
            A[k] ← L[i]
            i ← i + 1
        else
            A[k] ← R[i]
            j ← j + 1
        end if
    end for
    return A
end function
```

3.7 Quick Sort

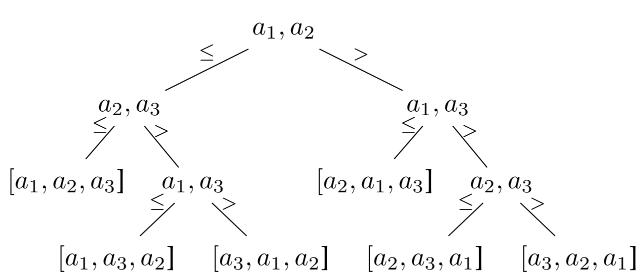
QUICK-SORT is another divide-and-conquer sorting algorithm.

```
function QUICK-SORT(A, l, r)                                  ▷ A is an array size n
    if l < r then
        m ← PARTITION(A, l, r)
        QUICK-SORT(A, l, m − 1)
        QUICK-SORT(A, m + 1, r)
    end if
    return A
end function
```

3.7.1 Partition and Pivot

3.8 Decision Tree and the Lower Bound for Comparison Sorting Algorithm

So far, we have been discussing *comparison sorting algorithms* (sorting algorithms that only reads array elements through $>, =, <$ comparison). We can draw a decision tree with all the permutations of how a comparison sorting algorithms would compare and sort the array $[a_0, a_1, a_2]$.



The decision tree for array size of 3 created $3! = 6$ leaves, and it is trivial that a decision tree for an **array with** n **elements will have** $n!$ **leaves**. The height of the decision tree represents the worst-case number of comparisons the algorithm has to make in order to sort the array.

A tree with the height h has at most 2^h leaves. Using this property, we can have the lower Big-omega bound for height of the decision tree for comparison sorting algorithms.

$$
\begin{aligned}
2^h \geq n! \therefore h \geq \log_2(n!) & \qquad \text{(h is the height of the decision tree)} \\
h \geq \log_2(n!) = \log_2(1 \cdot 2 \cdot \dots \cdot (n-1) \cdot n) & \\
= \log_2(1) + \log_2(2) + \dots + \log_2(n-1) + \log_2(n) & \\
= \sum_{i=1}^{n} \log_2(i) = \sum_{i=1}^{\frac{n}{2}-1} \log_2(i) + \sum_{i=\frac{n}{2}}^{n} \log_2(i) & \\
\geq 0 + \sum_{i=\frac{n}{2}}^{n} \log_2(i) \geq 0 + \sum_{i=\frac{n}{2}}^{n} \log_2(\frac{n}{2}) & \qquad \text{(i from the prev. expression starts at $\frac{n}{2}$)} \\
= \frac{n}{2} \log_2(\frac{n}{2}) & \qquad \text{($i = \frac{n}{2}$ to n is exactly $\frac{n}{2}$ iterations)} \\
\in \Theta(n \log_2(n)) &
\end{aligned}
$$

Because $h \geq \log_2(n!) \in \Theta(n \log_2(n))$, $h \in \Omega(n \log_2(n))$. Therefore, we can conclude that any **comparison sorting algorithms cannot run faster than $O(n \log_2(n))$ time** in the worst-case scenario.

3.9 Bucket Sort

BUCKET-SORT is a sorting algorithm where elements are divided into each "bucket" and a different sorting algorithm is called for each bucket. While BUCKET-SORT is a comparison sorting algorithm, it's an attempt to reduce the runtime by reducing the number of elements that the sorting algorithm has to sort.

3.10 Counting Sort

COUNTING-SORT is a *non-comparison* sorting algorithm. It uses the extra array *count*, where its index initially represents the value of each element in A (e.g., if there are three 5's in A, $count[5] = 3$ before the "accumulation" step to determine the final index), to sort the array.

```
function COUNTING-SORT(A, k)                    ▷ A is an array size n, k is the max element of A
    count ← array size k + 1 filled with 0
    for i = 0 to n − 1 do                        ▷ Num occurrence in each element in A, O(n)
        count[A[i]] ← count[A[i]] + 1
    end for

    for i = 1 to k do                            ▷ Accumulate the values in count from left to right, O(k)
        count[i] ← count[i] + count[i − 1]
    end for

    out ← array size n
    for i = n − 1 down to 0 do    ▷ Use count values to determine the index for the elements in A, O(n)
        out[count[A[i]] − 1] ← A[i]
        count[A[i]] ← count[A[i]] − 1
    end for

    return out
end function
```

1. Suppose we have an array $A = [2, 5, 3, 0, 2, 3, 0, 3]$. $k = \text{MAX}(A) = 5$.
2. Initialize $count$, the array size $5 + 1$, with 0's. $count = [0, 0, 0, 0, 0, 0]$.
3. Count number of occurrence. $count = [2, 0, 2, 3, 0, 1]$ (e.g., 2 occurred 2 times)
4. Accumulate values of $count$ from left to right. $count = [2, 2, 4, 7, 7, 8]$ (e.g., $count[1] = 2 + 0, count[2] = 2 + 0 + 2, \ldots$)
5. Initialize out, the array size $n = 8$, with nil's. $out = [nil, nil, nil, nil, nil, nil, nil, nil]$.
6. Place each element to the out array using $count$ array
   (a) When $i = n - 1 = 7$: $A[7] = 3$ and $count[3] = 7 \Rightarrow out[7 - 1] := A[7] = 3$ and $count[3] := 7 - 1$

   out = [nil, nil, nil, nil, nil, nil, 3, nil]
   count = [2, 2, 4, 6, 7, 8]

   (b) When $i = n - 2 = 6$: $A[6] = 0$ and $count[0] = 2 \Rightarrow out[2 - 1] := A[6] = 0$ and $count[0] := 2 - 1$

   out = [nil, 0, nil, nil, nil, nil, 3, nil]
   count = [1, 2, 4, 6, 7, 8]

   (c) When $i = n - 3 = 5$: $A[5] = 3$ and $count[3] = 6 \Rightarrow out[6 - 1] := A[5] = 3$ and $count[3] := 6 - 1$

   out = [nil, 0, nil, nil, nil, 3, 3, nil]
   count = [1, 2, 4, 5, 7, 8]

   (d) ...

   out = [0, 0, 2, 2, 3, 3, 3, 5]
   count = [0, 2, 2, 4, 7, 7]

In-place?	Stable?
False	True

Because of its use for RADIX-SORT, COUNTING-SORT must be stable, and it indeed is. If there are items with the same value, it will be moved to the out array in order in the last (third) for loop.

Runtime	Space Usage
$O(n + k)$	$O(n + k)$

As the algorithm iterates both the size of the array n and the maximum element in the array k, **the algorithm runs in $O(n + k)$ time and uses $O(n + k)$ space**.

3.11 Radix Sort

RADIX-SORT is a non-comparative sorting algorithm for elements with more than one significant digits. It utilizes a stable sorting algorithm such as COUNTING-SORT to sort elements lexicographically.

```
function RADIX-SORT(A, k)        ▷ A is an array where the maximum dimension of an element is d
    for i = d down to 1 do
        Call a stable sorting algorithm at dimension i
    end for
    return A
end function
```

3.11.1 Lexicographic Order

$$(x_1, x_2, \ldots, x_d) < (y_1, y_2, \ldots, y_d) \Leftrightarrow (x_i < y_i) \vee (x_1 = y_1 \wedge (x_2, \ldots, x_d) < (y_2, \ldots, y_d))$$

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