DSA Mini Textbook

Theo Park

Contents

Preface **3**

1 Runtime Analysis **4**
1.1 Power Law . . . 4
1.2 Runtime Expressions . . . 4
1.3 Asymptotic Runtime Analysis . . . 4
1.4 Recursive Relationship . . . 4

2 Intro to Data Structures **5**
2.1 Array . . . 5
2.2 Linked List . . . 5
2.3 Stack . . . 5
2.4 Queue . . . 5
2.5 Binary Heap . . . 5
2.5.1 Building a Heap – Top-down v.s. Bottom-up . . . 5
2.6 Tree . . . 5

3 Sorting Algorithms **6**
3.1 Bubble Sort . . . 6
3.2 Selection Sort . . . 7
3.3 Insertion Sort . . . 7
3.4 Shell Sort . . . 7
3.5 Heap Sort . . . 7
3.5.1 Sort Down Algorithm . . . 7
3.6 Merge Sort . . . 7
3.6.1 Merge Algorithm . . . 8
3.7 Quick Sort . . . 8
3.7.1 Pivot and Partition . . . 9
3.8 Decision Tree, Limit for Comparison Algo . . . 9
3.9 Counting Sort . . . 9
3.10 Radix Sort . . . 10
3.11 Chapter 3 Review . . . 10

4 Hash Tables **11**
4.1 Division Method . . . 11
4.2 Multiplication Method . . . 11
4.3 Collision . . . 11
4.3.1 Chaining . . . 11
4.3.2 Open Addressing . . . 11

5 Search Tree **12**
5.1 Binary Search Tree and Its Limit . . . 12
5.2 2-3 Tree . . . 12
5.3 Red-Black Tree . . . 12
5.4 Left-Leaning Red-Black Tree . . . 12
5.4.1 Deletion in LLRBT . . . 12

6 Graph Traversal **13**
6.1 Adjacency Matrix and List . . . 13
6.2 DFS . . . 13
6.3 BFS . . . 13

7 Directed Graphs **14**
7.1 Strong Connectivity . . . 14
7.1.1 Brute-force Strong Connectivity Algorithm . . . 14
7.1.2 Brute-force using Stack . . . 14
7.1.3 Strongly Connected Components and Kosaraju's Algorithm . . . 14
7.2 Directed Acyclic Graphs . . . 14
7.2.1 Topological Sort . . . 14

8 Weighted Graphs **15**
8.1 Shortest Path . . . 15
8.1.1 Dijkstra's Algorithm . . . 15
8.1.2 Bellman-Ford Algorithm . . . 15
8.2 Articulation Points . . . 15
8.3 Minimum Spanning Tree . . . 15
8.3.1 Cycle and Cut Properties . . . 15
8.3.2 Prim's Algorithm . . . 15
8.4 Union-Find . . . 15
8.4.1 Kruskal MST Algorithm . . . 15

9 Strings **16**
9.1 Brute-force String Pattern Matching . . . 16
9.2 KMP Algorithm . . . 16
9.3 Trie . . . 16
9.4 PATRICIA . . . 16
9.5 Huffman Coding . . . 16

Preface

Chapter 1

Runtime Analysis

Algorithms are any well-defined computational procedures that take some value(s) as input and produce more value(s) as output. They are **effective**, **precise**, and **finite**. There are several ways to analyze the runtime of an algorithm.

1.1 Power Law

1. For the algorithm, get a table for the input size n and the runtime $T(n)$.

n	$T(n)$
250	0.0
500	0.012
1000	0.0954
2000	0.7727
4000	6.1664

2. Make sure that the data plots:
   - **have enough data plots.** For instance, if there are only two data plots, you should not make the power law conjecture.
   - **fits the power law.** You can verify this by finding the ratio between data plots.

n	$T(n)$	ratio
250	0.0	–
500	0.012	–
1000	0.0954	0.0954 / 0.012 = 7.95
2000	0.7727	0.7727 / 0.0954 = 8.10
4000	6.1664	6.1664 / 0.7727 = 7.98

   For the ratios we found, //TODO

1.2 Runtime Expressions

1.3 Asymptotic Runtime Analysis

1.4 Recursive Relationship

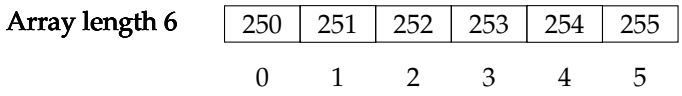
Chapter 2

Intro to Data Structures

Data structures are collections of data values, the relationships among them, and the functions or operations that can be applied to the data. All three characteristics need to be present.

2.1 Array

Array is a linear container of items.



- Access time: $\Theta(1)$
- Inserting n items in the *tail* for array size n: $\Theta(1)$ per item, $n \times \Theta(1) \in \Theta(1)$
- Inserting n items in the *tail* for array size *unknown*: $\Theta(n)$ per item, $n \times \Theta(n) \in \Theta(n)$

Lesson? **Keep track of the tail!**

2.2 Linked List

2.3 Stack

2.4 Queue

2.5 Binary Heap

2.5.1 Building a Heap – Top-down v.s. Bottom-up

2.6 Tree

Chapter 3

Sorting Algorithms

Once you store all the items in a data structure, you might want to organize them for the future use (such as selecting nth largest element). For this, you have to *sort* the data structure (in this book, array will be assumed). *Sorting* is deciding how to permute the array elements until they are sorted.

There are couple aspects of sorting algorithms you need to consider:

- Runtime: When analyzing a runtime of a sorting algorithm, both number of compares and number of swaps are considered. **Most sorting algorithms make more comparisons than swaps**, but if a sorting algorithm makes more swaps, it must be used for the asymptotic runtime analysis
- Stability: An algorithm is stable if it preserves the input ordering of equal items For example: //TODO
- In-place: An algorithm is in-place if it can directly sorts the items without making a copy or extra array(s)

3.1 Bubble Sort

BUBBLE-SORT goes through the array and swap elements that are out of place, and if such element is found, it repeats from the beginning.

```
procedure BUBBLE-SORT(A)                          ▷ A is an array size n
    repeat ← True
    while repeat is True do
        repeat ← False
        for i = 0 to n − 2 do
            if A[i] > A[i + 1] then
                SWAP(i, i + 1)                    ▷ Assume SWAP(i, j) is externally defined
                repeat ← True
            end if
        end for
    end while
end procedure
```

In-place?	Stable?
True	True

-	NumCompares	NumSwaps
Already Sorted	$n-1$	0
Worst Case	n^2-n	$\frac{1}{2}n^2-\frac{1}{2}n$

3.2 Selection Sort

SELECTION-SORT is a sorting algorithm closest to our "natural" thought of sorting an array. It makes the same number of comparisons no matter what.

```
procedure SELECTION-SORT(A)                                  ▷ A is an array size n
    for i = 0 to n − 2 do
        index ← i
        for i = i + 1 to n − 1 do
            if [j] <A[index] then
                index ← j
            end if
        end for
        if i ≠ index then SWAP(i, index)
        end if
    end for
end procedure
```

In-place?	Stable?
True	False

-	NumCompares	NumSwaps
Already Sorted	$\frac{1}{2}n^2 - \frac{1}{2}n$	0
Worst Case	$\frac{1}{2}n^2 - \frac{1}{2}n$	$\lfloor \frac{1}{2}n \rfloor$

3.3 Insertion Sort

3.4 Shell Sort

3.5 Heap Sort

HEAP-SORT uses binary max-heap to sort an array. While it's the first sorting algorithm to utilize a data structure, it's not preferred in real life due to cache issue.

```
function HEAP-SORT(A)                                        ▷ A is an array size n
    A ← BUILD-HEAP(A)
    for i = n − 1 down to 0 do
        SORT-DOWN(A, i)
    end for
    return A
end function
```

The algorithm first builds the heap from the array elements (refer to section 2.5 for methods for building a heap). BOTTOM-UP is used for its runtime. Then the algorithm calls SORT-DOWN from the last heap elements down to the first.

3.5.1 Sort Down Algorithm

3.6 Merge Sort

MERGE-SORT is an algorithm //TODO

```
function MERGE-SORT(A, l, r)                                   ▷ A is an array size n
    if l < r then
        m ← (l + r)/2
        MERGE-SORT(A, l, m)
        MERGE-SORT(A, m + 1, r)
        MERGE(A, l, m, r)
    end if
    return A
end function
```

3.6.1 Merge Algorithm

```
function MERGE(A, l, m, r)                                     ▷ A is an array size n
    n1 ← m − l + 1
    n2 ← r − m
    L ← array size of (n1 + 1)
    R ← array size of (n2 + 1)
    ▷ Assign elements to each array
    for i = 0 to n1 − 1 do
        L[i] ← A[l + i]
    end for
    for i = 0 to n2 − 1 do
        R[i] ← A[m + j + 1]
    end for

    L[n1], R[n2] ← ∞
    i, j ← 0

    for k = l to r do
        if L[i] ≤ R[j] then
            A[k] ← L[i]
            i ← i + 1
        else
            A[k] ← R[i]
            j ← j + 1
        end if
    end for
end function
```

3.7 Quick Sort

QUICK-SORT is another divide-and-conquer sorting algorithm.

```
function QUICK-SORT(A, l, r)                                   ▷ A is an array size n
    if l < r then
        m ← PARTITION(A, l, r)
        QUICK-SORT(A, l, m − 1)
        QUICK-SORT(A, m + 1, r)
    end if
    return A
end function
```

3.7.1 Pivot and Partition

3.8 Decision Tree and $\Omega(n \log n)$ Limit for Comparison Sorting Algorithms

3.9 Counting Sort

COUNTING-SORT is *not* a comparison based sorting algorithm. It uses the extra array $count$, where its index initially represents the value of each element in A (e.g., if there are three 5's in A, $count[5] = 3$ before the "accumulation" step to determine the final index), to sort the array.

```
function COUNTING-SORT(A, k)                    ▷ A is an array size n, k is the max element of A
    count ← array size k + 1 filled with 0
    for i = 0 to n − 1 do                       ▷ Num occurrence in each element in A, O(n)
        count[A[i]] ← count[A[i]] + 1
    end for

    for i = 1 to k do                           ▷ Accumulate the values in count from left to right, O(k)
        count[i] ← count[i] + count[i − 1]
    end for

    out ← array size n
    for i = n − 1 down to 0 do      ▷ Use count values to determine the index for the elements in A, O(n)
        out[count[A[i]] − 1] ← A[i]
        count[A[i]] ← count[A[i]] − 1
    end for

    return out
end function
```

1. Suppose we have an array $A = [2, 5, 3, 0, 2, 3, 0, 3]$. $k = \text{MAX}(A) = 5$.
2. Initialize $count$, the array size $5 + 1$, with 0's. $count = [0, 0, 0, 0, 0, 0]$.
3. Count number of occurrence. $count = [2, 0, 2, 3, 0, 1]$ (e.g., 2 occurred 2 times)
4. Accumulate values of $count$ from left to right. $count = [2, 2, 4, 7, 7, 8]$ (e.g., $count[1] = 2 + 0, count[2] = 2 + 0 + 2, \ldots$)
5. Initialize out, the array size $n = 8$, with nil's. $out = [nil, nil, nil, nil, nil, nil, nil, nil]$.
6. Place each element to the out array using $count$ array
   (a) When $i = n - 1 = 7$: $A[7] = 3$ and $count[3] = 7 \Rightarrow out[7 - 1] := A[7] = 3$ and $count[3] := 7 - 1$

out = [nil, nil, nil, nil, nil, nil, 3, nil]
count = [2, 2, 4, 6, 7, 8]

(b) When $i = n - 2 = 6$: $A[6] = 0$ and $count[0] = 2 \Rightarrow out[2 - 1] := A[6] = 0$ and $count[0] := 2 - 1$

out = [nil, 0, nil, nil, nil, nil, 3, nil]
count = [1, 2, 4, 6, 7, 8]

(c) When $i = n - 3 = 5$: $A[5] = 3$ and $count[3] = 6 \Rightarrow out[6 - 1] := A[5] = 3$ and $count[3] := 6 - 1$

out = [nil, 0, nil, nil, nil, 3, 3, nil]
count = [1, 2, 4, 5, 7, 8]

(d) ...

out = [0, 0, 2, 2, 3, 3, 3, 5]
count = [0, 2, 2, 4, 7, 7]

In-place?	Stable?
False	True

Because of its use for RADIX-SORT, COUNTING-SORT must be stable, and it indeed is. If there are items with the same value, it will be moved to the out array in order in the last (third) for loop.

Runtime	Space Usage
$O(n + k)$	$O(n + k)$

As the algorithm iterates both the size of the array n and the maximum element in the array k, **the algorithm runs in $O(n + k)$ time and uses $O(n + k)$ space.

3.10 Radix Sort

3.11 Chapter 3 Review

Chapter 4

Hash Tables

4.1 Division Method

4.2 Multiplication Method

4.3 Collision

4.3.1 Chaining

4.3.2 Open Addressing

Chapter 5

Search Tree

5.1 Binary Search Tree and Its Limit

5.2 2-3 Tree

5.3 Red-Black Tree

5.4 Left-Leaning Red-Black Tree

5.4.1 Deletion in LLRBT

Chapter 6

Graph Traversal

6.1 Adjacency Matrix and List

6.2 DFS

6.3 BFS

Chapter 7

Directed Graphs

7.1 Strong Connectivity

7.1.1 Brute-force Strong Connectivity Algorithm

7.1.2 Brute-force using Stack

7.1.3 Strongly Connected Components and Kosaraju's Algorithm

7.2 Directed Acyclic Graphs

7.2.1 Topological Sort

Chapter 8

Weighted Graphs

8.1 Shortest Path

8.1.1 Dijkstra's Algorithm

8.1.2 Bellman-Ford Algorithm

8.2 Articulation Points

8.3 Minimum Spanning Tree

8.3.1 Cycle and Cut Properties

8.3.2 Prim's Algorithm

8.4 Union-Find

8.4.1 Kruskal MST Algorithm

Chapter 9

Strings

9.1 Brute-force String Pattern Matching

9.2 KMP Algorithm

9.3 Trie

9.4 PATRICIA

9.5 Huffman Coding