Answers to Practice Problems — Lesson 6

Answer 1: Clothes shopping (maximize style)

Prompt recap: Shirts cost \$15, pants \$25, budget \$100. Each shirt gives 2 style points, each pant 4 points. Write variables, objective (maximize), constraints.

Decision variables:

$$s = \text{number of shirts}, \qquad p = \text{number of pants}.$$

Objective (maximize style):

$$\max \; Z = 2s + 4p.$$

Constraints (budget & nonnegativity):

$$15s + 25p \leq 100, \qquad s \geq 0,\; p \geq 0.$$

Optional matrix form (for later use with `linprog`)

Maximization with `linprog` will later be handled by minimizing $-Z$. For now, matrix form of the inequality:

$$A_{\text{ub}} = \begin{bmatrix} 15 & 25 \end{bmatrix}, \quad b_{\text{ub}} = \begin{bmatrix} 100 \end{bmatrix}, \quad \text{nonnegativity: } s \geq 0,\; p \geq 0.$$

Objective coefficient (maximize): $c_{\max} = \begin{bmatrix} 2 & 4 \end{bmatrix}$.

Answer 2: Shipping boxes (minimize cost)

Prompt recap: Truck costs \$5/box, train \$3/box. Must ship at least 40 boxes. Write variables, objective (minimize), constraints.

Decision variables:

$$t = \text{boxes shipped by truck}, \qquad r = \text{boxes shipped by train}.$$

Objective (minimize cost):

$$\min \; C = 5t + 3r.$$

Constraints (demand & nonnegativity):

$$t + r \geq 40, \qquad t \geq 0,\; r \geq 0.$$

Optional matrix form (convert "≥" to "≤")

To fit the standard $A_{\text{ub}}x \leq b_{\text{ub}}$ form, rewrite $t + r \geq 40$ as

$$-(t + r) \leq -40.$$

Thus,

$$A_{\text{ub}} = \begin{bmatrix} -1 & -1 \end{bmatrix}, \quad b_{\text{ub}} = \begin{bmatrix} -40 \end{bmatrix}, \quad \text{nonnegativity: } t \geq 0,\ r \geq 0.$$

Objective coefficient (minimize): $c_{\min} = \begin{bmatrix} 5 & 3 \end{bmatrix}$.

Answer 3: Study planning (maximize points)

Prompt recap: At most 10 hours. Math hour gives 3 points, science hour 2 points. Write variables, objective (maximize), constraints.

Decision variables:

$$m = \text{hours of math study}, \qquad s = \text{hours of science study}.$$

Objective (maximize points):

$$\max\ P = 3m + 2s.$$

Constraints (time & nonnegativity):

$$m + s \leq 10, \qquad m \geq 0,\ s \geq 0.$$

Optional matrix form

$$A_{\text{ub}} = \begin{bmatrix} 1 & 1 \end{bmatrix}, \quad b_{\text{ub}} = \begin{bmatrix} 10 \end{bmatrix}, \quad \text{nonnegativity: } m \geq 0,\ s \geq 0.$$

Objective coefficient (maximize): $c_{\max} = \begin{bmatrix} 3 & 2 \end{bmatrix}$.

Notes

- In later lessons, we will pass the inequality matrices/vectors to `linprog`. Since `linprog` minimizes by default, we handle maximization by minimizing the *negative* of the objective.
- If a problem requires whole numbers (e.g., shirts/pants), that is an *integer* optimization. We start with continuous versions; integrality comes later as an extension.