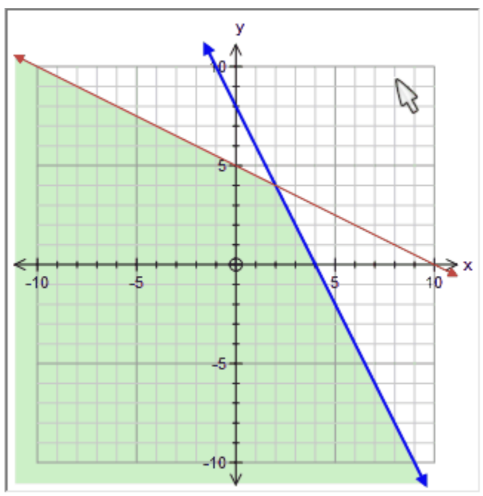
Lesson 6 – Anatomy of an Optimization Problem

Why Optimization Problems?

Optimization shows up in everyday life: choosing the fastest route, planning a study schedule, or buying items under a budget. To solve these problems clearly, we break them into:

- **Decision Variables:** what we control (our choices).
- **Objective Function:** what we want to *maximize* or *minimize* (The goal that we have).
- **Constraints:** the rules/limits we must respect.
- **Feasible Region:** all solutions that satisfy the constraints.



Big Idea

Optimization = **pick values for the decision variables** that satisfy all **constraints** and make the **objective function** as good as possible.

The Parts of an Optimization Problem

1) Decision Variables

Variables represent our choices.

- Example: let x = number of apples, y = number of bananas.
- Types: can be whole numbers (integers), decimals, or even yes/no (binary: 0 or 1).

2) Objective Function

What are we trying to optimize?

- **Maximize** fun, profit, score, etc.
- **Minimize** cost, time, distance, etc.

3) Constraints

Rules and limits, usually written as inequalities or equalities.

- Limited example
- Time/capacity limits, resource availability, at least/at most requirements
- Nonnegativity: $x \geq 0, \ y \geq 0$

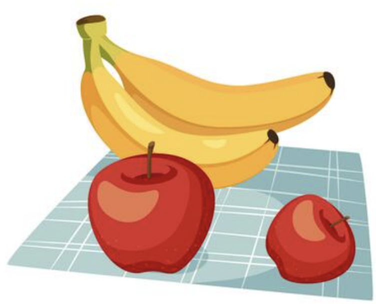
4) Feasible Region

All variable values that satisfy all the constraints.

Example 1: Snacks on a Budget (Formulation Only)

You have \$20 to spend. Apples cost \$3 and each apple has 2 fun points, bananas cost \$2 and each banana has 3 fun points. You want to maximize your "fun points."

- **Decision variables:** $x =$ number of apples, $y =$ number of bananas.
- **Objective (maximize fun):** $\max\ 2x + 3y$
- **Constraint (budget):** $3x + 2y \leq 20$
- **Nonnegativity:** $x \geq 0, \ y \geq 0$



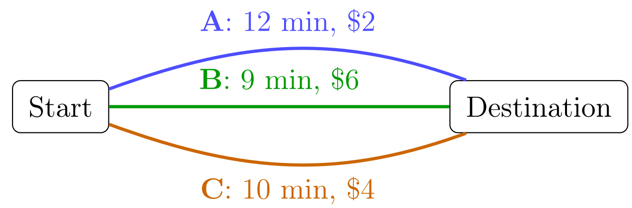
Note

We are *not* solving by hand here. In later lessons, we will use Python to solve it automatically with `linprog`.

Example 2: Travel Routes (Feasibility Focus)

You want the **fastest** route, but you cannot spend more than **10 minutes** *and* your budget is at most **$5**.

- **Decision variable:** choose a route (A, B, or C).
- **Objective:** minimize travel time.
- **Constraints:** time $\leq$ 10 minutes, cost $\leq$ \$5.



Route	Time (min)	Cost ($)	Feasible under time ≤ 10 & cost ≤ 5?
A	12	2	No (time)
B	9	6	No (cost)
C	10	4	Yes

Feasibility First

Even before optimizing, we must check what is **allowed**. With time ≤ 10 and cost ≤ 5, routes A and B are *infeasible* for different reasons (time vs. cost). Only route C is feasible, so it is the only candidate for optimization.

From Words to Math (Workflow)

When translating a word problem into math:

1. **Define variables clearly.** (What do x, y, etc. represent?)
2. **Write the objective.** (Maximize or minimize . . .)
3. **List the constraints.** (Budgets, limits, at-least/at-most conditions)
4. **Include nonnegativity, specify if variables are integer or binary.**

Lesson 6 Practice: From Words to Math

1. **Clothes shopping:** Shirts cost \$15, pants cost \$25. Your budget is \$100. Each shirt gives 2 style points, each pant gives 4 style points. *Task:* Define variables, write the objective to **maximize** style points, and write the budget and nonnegativity constraints.
2. **Shipping boxes:** You can ship boxes by truck or train. Truck costs \$5 per box, train costs \$3 per box. You must ship at least 40 boxes. *Task:* Define variables, write the objective to

minimize cost, and write constraints (including "at least 40 boxes").

3. **Study planning:** You have at most 10 hours this week. Each hour of math study gives 3 points, each hour of science gives 2 points. You want to **maximize** points. *Task:* Define variables, write the objective, the time constraint, and nonnegativity.

Looking Ahead

Next, we will turn these math formulations into arrays (like c, A, b) and then solve them in Python with `linprog` from `scipy.optimize`.