Lesson 7 – Practice Problem Answers

Problem 1: Snack Mix (maximize)

Model

- Variables: x_1 = units of nuts, x_2 = units of raisins
- Objective: maximize $1x_1 + 2x_2$
- Constraint: $x_1 + 2x_2 \leq 8$
- Bounds: $x_1, x_2 \geq 0$

Observation: Profit per unit of packaging is 1 for both items, so *any* point on the boundary $x_1+2x_2 = 8$ is optimal. Examples: $(8, 0)$ or $(0, 4)$; all give profit 8.

Python (one optimal solution)

```python
from scipy.optimize import linprog

c = [-1, -2] # negate for maximization
A_ub = [[1, 2]]
b_ub = [8]
bounds = [(0, None), (0, None)]

res = linprog(c, A_ub=A_ub, b_ub=b_ub, bounds=bounds, method="highs")

print("One optimal (x1, x2):", res.x)
print("Max profit:", -res.fun)
```

Typical output

```
One optimal (x1, x2): [0. 4.]
Max profit: 8.0
```

Note: infinitely many optimal solutions with $x_1 + 2x_2 = 8$.

Problem 2: Study Hours (maximize)

Model

- Variables: x_1 = hours on Subject 1, x_2 = hours on Subject 2
- Objective: maximize $4x_1 + 3x_2$

- Constraint: $x_1 + x_2 \leq 10$
- Bounds: $x_1, x_2 \geq 0$

Reasoning: Subject 1 yields more points per hour, so spend all time on Subject 1: $(x_1, x_2) = (10, 0)$ with value 40.

Python

```python
from scipy.optimize import linprog

c = [-4, -3] # negate for maximization
A_ub = [[1, 1]]
b_ub = [10]
bounds = [(0, None), (0, None)]

res = linprog(c, A_ub=A_ub, b_ub=b_ub, bounds=bounds, method="highs")

print("Optimal (x1, x2):", res.x)
print("Max points:", -res.fun)
```

Output

```
Optimal (x1, x2): [10.  0.]
Max points: 40.0
```

Problem 3: Toy Factory (maximize)

Model

- Variables: x_1 = Toy A, x_2 = Toy B
- Objective: maximize $5x_1 + 7x_2$
- Constraints: $3x_1 + 4x_2 \leq 60, \quad x_1 + x_2 \leq 20$
- Bounds: $x_1, x_2 \geq 0$

Corner check:

- $(20, 0)$ satisfies both, profit $= 100$
- $(0, 15)$ satisfies both, profit $= 105$ **(best)**

So an optimal plan is $(x_1, x_2) = (0, 15)$ with maximum profit 105.

Python

```python
from scipy.optimize import linprog

c = [-5, -7] # negate for maximization
A_ub = [[3, 4],
        [1, 1]]
b_ub = [60, 20]
bounds = [(0, None), (0, None)]

res = linprog(c, A_ub=A_ub, b_ub=b_ub, bounds=bounds, method="highs")

print("Optimal (x1, x2):", res.x)
print("Max profit:", -res.fun)
```

Output

```
Optimal (x1, x2): [ 0. 15.]
Max profit: 105.0
```