Modern Problems in Nuclear Physics: neutron-star EoS and high-mass twins

1 Solution of the TOV equation

Write a program for solving the TOV equation

$$\frac{dP(r)}{dr} = -\frac{[E(r)+P(r)][M(r)+4\pi r^3 P(r)]}{r(r-2M(r))}, \tag{1}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 E(r) \tag{2}$$

for a given set of central densities or pressures. Here the units are $G = c = 1$; you may find the conversion factors and detailed discussion on how to solve the equations in [1]. Use the EoS in the file "MKVOR.dat" to obtain the solution. The following plots are expected as a result:

- Neutron star gravitational mass M in units of $M_\odot = 1.4766$ km as a function of the central density n_c in units of $n_0 = 0.16$ fm^{-3}. The NS with a maximum mass should be clearly seen from the plot.
- Neutron star mass $M(n_c)$ vs. NS radius $R(n_c)$ for the same set of the central densities.

2 Third branch of compact stars

The piecewise-polytropic EoS is defined as:

$$P(n) = \begin{cases} \kappa_1 n^{\Gamma_1}, & n < n_{12} \\ P_c, & n_{12} < n < n_{23} \\ \kappa_3 n^{\Gamma_3}, & n_{23} < n. \end{cases} \tag{3}$$

$$E(n) = \begin{cases} \dfrac{\kappa_1 n^{\Gamma_1}}{(\Gamma_1 - 1)} + m_1 n, & n < n_{12} \\ P_c, & n_{12} < n < n_{23} \\ \dfrac{\kappa_3 n^{\Gamma_3}}{(\Gamma_3 - 1)} + m_3 n, & n_{23} < n. \end{cases} \tag{4}$$

We will employ this assumption about the EoS shape for studying the possible consequences of the first-order phase transition from hadronic to quark matter on the mass-radius diagram. The task in composed of the following steps:

1. Implement and test these formulas in the code.
2. Perform a fit of the hadronic EoS given for the Task 1 using this formula with $n_{12} \to \infty$

   Output:

   - Obtain the parameters κ_1, Γ_1.
   - Plot the fitting curve compared with the original.
3. Partially reproduce the results of [2] for the values $\Gamma_1 = 4.92$ and $\kappa_1 = 17906.60\,\mathrm{MeV} \cdot \mathrm{fm}^{3(\Gamma_1 - 1)}$, and $n_{12} = 0.32\,\mathrm{fm}^{-3}$, $n_{23} = 0.53\,\mathrm{fm}^{-3}$. The mass parameter $m_1 = 938\,\mathrm{MeV}$ is the nucleon mass.

   Output

   - $M(n_c)$, MR diagram
   - Pressure and density profile inside a star with the maximum mass as a function of the radial coordinate.
4. Write a code for finding n_{23} for given $\kappa_1, \Gamma_1, \kappa_3, \Gamma_3, n_{12}$. For a fixed n_{12} and a fixed "hadronic" EoS vary κ_3, Γ_3 to change n_{23}. in order to check the validity of the Seidov's criterion [3] for the existence of the third branch:

$$\frac{\Delta E}{E_{12}} = \frac{1}{2} + \frac{3}{2}\frac{P_c}{E_{12}}, \tag{5}$$

where $E_{12} = E(n_{12})$, $E_{23} = E(n_{23})$, and $\Delta E = E_{23} - E_{12}$.

Output:

- MR plot for several values of n_{23} like Fig. 2 in [4].
- Comparison of $\Delta E/E_{12}$ with the formula for the critical value of n_{12}.

References

[1] Glendenning, N.K. 2000, Compact Stars, Nuclear Physics, Particle Physics, and General Relativity, 2nd ed.

[2] D. E. Alvarez-Castillo and D. B. Blaschke, arXiv:1703.02681 [nucl-th].

[3] Z. F. Seidov, Soviet Astronomy, Vol. 15, p.347

[4] L. Lindblom, Phys. Rev. D **58**, 024008 (1998) doi:10.1103/PhysRevD.58.024008 [gr-qc/9802072].