```
\begin{array}{l} \vdash = Def_{[(\iota \to o, \iota \to o) \to o]}(Ld_{1}{}_{[\iota \to o]}, \lambda X_{[\iota]} \blacksquare E_{[\iota \to o]}(F_{[(\iota, \iota) \to \iota]}(S_{[\iota \to \iota]}(N_{1}(\underline{L})caL))) + \\ \vdash \forall X_{[\iota]} \blacksquare [D_{[\iota \to o]}(X) \Rightarrow D(\mathbf{s}(X))] \end{array} \tag{Hyp}
Ld1.
                                        Ld1
A5.
                                        Α5
                                                                     \vdash \forall Dc - _{8215}{_{[\iota]}}\blacksquare [[\forall Dc - _{8244}{_{[\iota \to o]}}\blacksquare [[Dc - _{8244}(One_{[\iota]})
L10.
                                                                                                                                                                                            (Otter A5)
                                        \mathcal{H}_1
                                                                        \forall Dc -_{8248[\iota]} \bullet [Dc -_{8244}(Dc -_{8248})]
                                                                        Dc - 8244(s(Dc - 8248))]]
                                                                                                                                           Dc - 8244(Dc - 8215)]
                                                                                                                     [\forall Dc - _{8261}{}_{[\iota \rightarrow o]^{\blacksquare}}[[Dc - _{8261}(One)
                                                                        D(Dc - 8215)]
                                                                                                      \Rightarrow
                                                                                                                                                                                  Λ
                                                                        \forall Dc -_{8265[\iota]} [Dc -_{8261}(Dc -_{8265})]
                                                                        Dc - 8261(s(Dc - 8265))]]
                                                                                                                                       Dc-_{8261}(s(Dc-_{8215}))
                                                                                                                         \Rightarrow
                                                                        D(s(Dc-8215))]]
                                                                    \vdash \forall Y_{[\iota]^{\blacksquare}}[E(Y) \Rightarrow E(s(Y))]\vdash D(One)
L9.
                                                                                                                                                                                            (Defsi L10)
                                        \mathcal{H}_1
A4.
                                        Α4
                                                                                                                                                                                            (Hyp)
L8.
                                                                    \vdash [\forall Dc - 8208[\iota \to o]^{\bullet}[[Dc - 8208(One)]]
                                                                                                                                                                                            (Otter A4)
                                        \mathcal{H}_1
                                                                        \forall Dc - 8212[\iota] \bullet [Dc - 8208(Dc - 8212)]
                                                                        Dc-_{8208}(\dot{s}(Dc-_{8212}))]] \Rightarrow Dc-_{8208}(One)] \wedge D(One)]
L7.
                                                                    \vdash E(One)
                                                                                                                                                                                            (Defsi L8)
                                        \mathcal{H}_1
                                                                    \vdash E(s(One))
L11.
                                                                                                                                                                                            (Otter L7.L9)
                                        \mathcal{H}_1
                                                                    \vdash \forall N_{\lceil \iota \rceil} F(N, One) = \mathbf{s}(One)
A1.
                                                                                                                                                                                            (Hyp)
                                        A1
                                                                    \vdash E(F(s(N_1), One))
L28.
                                                                                                                                                                                            (Otter A1,L11)
                                        \mathcal{H}_2
L27.
                                                                    \vdash Ld_1(One)
                                                                                                                                                                                            (Defn-Contract-Local-Def
                                        \mathcal{H}_2
                                                                                                                                                                                            L28,Ld1)
                                                                    \vdash \forall Dc - _{8362[\iota]^{\blacksquare}}[N_{[\iota \rightarrow o]}(Dc - _{8362}) \Rightarrow E(F(N_1,Dc - _{8362}))]
L25.
                                                                                                                                                                                            (Hyp)
                                        L25
L31.
                                                                    \vdash Ld_1(N_{2[\iota]})
                                                                                                                                                                                            (Hyp)
                                        L31
                                                                    \vdash E(F(s(N_1), N_2))
L33.
                                        Ld1, L31
                                                                                                                                                                                            (Defn-Expand-Local-Def
                                                                                                                                                                                            L31,Ld1)
                                                                    \vdash [N(F(s(N_1), N_2)) \land D(F(s(N_1), N_2))]
L34.
                                        Ld1, L31
                                                                                                                                                                                            (Defne L33)
                                                                    \vdash \forall N, X_{\lceil \iota \rceil} F(\mathbf{s}(N), \mathbf{s}(X)) = F(N, F(\mathbf{s}(N), X))
                                                                                                                                                                                            (Hyp)
A3.
                                        А3
                                                                                                                                                                                            (Otter A3,L34,L25)
L35.
                                                                    \vdash E(F(s(N_1), s(N_2)))
                                        \mathcal{H}_3
                                                                                                                                                                                            (Defn-Contract-Local-Def
                                                                    \vdash Ld_1(s(N_2))
L32.
                                        \mathcal{H}_3
                                                                                                                                                                                            L35,Ld1)
L30.
                                                                    \vdash [Ld_1(N_2) \Rightarrow Ld_1(s(N_2))]
                                                                                                                                                                                            (⇒I L32)
                                        \mathcal{H}_2
                                                                    \vdash \forall X_{[\iota]} [Ld_1(X) \Rightarrow Ld_1(\mathbf{s}(X))]
L29.
                                        \mathcal{H}_2
                                                                                                                                                                                            (∀I L30)
                                                                    \vdash \forall Dc - _{8449} [\iota]^{\blacksquare} [\forall Dc - _{8460} [\iota \rightarrow o]^{\blacksquare} [[Dc - _{8460} (One)
                                                                                                                                                                                            (Otter L29,L27)
L42.
                                        \mathcal{H}_2
                                                                        \forall Dc - 8464[\iota] \cdot [Dc - 8460(Dc - 8464)]
                                                                                                                                                                                  \Rightarrow
                                                                                                                                      Dc-8460(Dc-8449)]
                                                                        Dc-_{8460}(s(Dc-_{8464}))]]
                                                                                                                                                                                  \Rightarrow
                                                                        Ld_1(Dc - 8449)]
                                                                    \vdash \forall X_{[\iota]} [N_{[\iota \to o]}(X) \Rightarrow Ld_1(X)]
L41.
                                        \mathcal{H}_2
                                                                                                                                                                                            (Defni L42)
                                                                    \vdash \forall Dc - _{8472\lceil\iota\rceil^{\blacksquare}}[N(Dc - _{8472}) \Rightarrow E(F(\mathbf{s}(N_1),Dc - _{8472}))]
L43.
                                                                                                                                                                                            (Defn-Expand-Local-Def
                                        \mathcal{H}_2
                                                                                                                                                                                            L41,Ld1)
L26.
                                        \mathcal{H}_2
                                                                    \vdash \forall Dc - 8366 [\iota] \bullet [N(Dc - 8366)) \Rightarrow E(F(s(N_1), Dc - 8366))]
                                                                                                                                                                                            (Otter L43)
                                                                    \vdash [\forall Dc -_{8362}[_{\iota}] \bullet [N(Dc -_{8362}) \quad \Rightarrow \quad
                                                                                                                                      E(F(N_1, Dc - 8362))
                                                                                                                                                                                            (⇒I L26)
L24.
                                        \mathcal{H}_1
                                                                        \forall Dc - 8366_{[l]} [N(Dc - 8366)) \Rightarrow E(F(s(N_1), Dc - 8366))]]
L23.
                                                                    \vdash \forall Dc - \mathbf{8350}_{\llbracket\iota\rrbracket} \blacksquare [\forall Dc - \mathbf{8362}_{\llbracket\iota\rrbracket} \blacksquare [N(Dc - \mathbf{8362})
                                                                                                                                                                                            (∀I L24)
                                        \mathcal{H}_1
                                                                        E(F(Dc-8350, Dc-8362))] \Rightarrow \forall Dc-8366[\iota] [N(Dc-8366)]
                                                                        E(F(s(Dc-8350), Dc-8366))]]
L22.
                                                                    \vdash \forall N_{\bullet}[M_{[\iota \to o]}(N) \Rightarrow M(\mathbf{s}(N))]
                                                                                                                                                                                            (Defni* L23)
                                        \mathcal{H}_1
L17.
                                                                    \vdash E(F(One, One))
                                                                                                                                                                                            (Otter A1,L11)
                                        \mathcal{H}_1
L16.
                                                                    \vdash Q_{[\iota \to o]}(One)
                                                                                                                                                                                            (Defni L17)
                                        \mathcal{H}_1
                                                                    \vdash \forall X_{[\iota]} \dot{\bar{F}}(One, \mathbf{s}(X)) = \mathbf{s}(\mathbf{s}(F(One, X)))
A 2.
                                        A2
                                                                                                                                                                                            (Hyp)
                                                                    \vdash \forall Dc - _{8293} [\iota]^{\blacksquare} [E(F(One,Dc - _{8293}))
                                                                                                                                                                                            (Otter A2,L9)
L19.
                                        \mathcal{H}_1
                                                                        E(F(One,s(Dc-8293)))]
L18.
                                                                    \vdash \forall X_{[\iota]} \cdot [Q(X) \Rightarrow Q(s(X))]
                                                                                                                                                                                            (Defni* L19)
                                        \mathcal{H}_1
                                                                    \vdash \forall Dc -_{8312\lceil \iota \rceil} [\forall Dc -_{8323\lceil \iota \to o \rceil} [[Dc -_{8323}(One)
L20.
                                                                                                                                                                                    Λ
                                                                                                                                                                                            (Otter L18,L16)
                                        \mathcal{H}_1
                                                                        \forall Dc - 8327[\iota] \bullet [Dc - 8323(Dc - 8327)]
                                                                                                                                                                                  \Rightarrow
                                                                        Dc - 8323(s(Dc - 8327))]]
                                                                                                                                      Dc - 8323(Dc - 8312)]
                                                                        Q(Dc - 8312)]
L15.
                                                                    \vdash \forall X_{[\iota]} [N_{[\iota \to o]}(X) \Rightarrow Q(X)]
                                                                                                                                                                                            (Defni L20)
                                        \mathcal{H}_1
                                                                    \vdash \forall Dc - _{8335}  {\scriptscriptstyle [\iota]}^{\bullet} [N(Dc - _{8335}) \Rightarrow E(F(One, Dc - _{8335}))]
L21.
                                        \mathcal{H}_1
                                                                                                                                                                                            (Defne L15)
                                                                    \vdash \forall Dc - 8282[\iota] \bullet [N(Dc - 8282) \Rightarrow E(F(One, Dc - 8282))]
                                                                                                                                                                                            (Otter L21)
L14.
                                        \mathcal{H}_1
                                                                    \vdash M(One)
L13.
                                                                                                                                                                                            (Defni L14)
                                        \mathcal{H}_1
L37.
                                                                    \vdash \forall Dc - 8397[\iota] \bullet [\forall Dc - 8408[\iota \to o] \bullet [[Dc - 8408(One)]]
                                                                                                                                                                                            (Otter L13,L22)
                                        \mathcal{H}_1
                                                                        \forall Dc - _{8412[\iota]} \cdot [Dc - _{8408}(Dc - _{8412})
                                                                                                                                                                                  \Rightarrow
                                                                                                                                      Dc-8408(Dc-8397)]
                                                                        Dc-_{8408}(s(Dc-_{8412}))]]
                                                                                                                                                                                  \Rightarrow
                                                                        M(Dc - 8397)]
L36.
                                                                    \vdash \forall N n_{[\iota]} \bullet [N(Nn) \Rightarrow M(Nn)]
                                                                                                                                                                                            (Defni L37)
                                        \mathcal{H}_1
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 \begin{array}{l} \vdash \forall Dc - _{8422[\iota]^{\blacksquare}}[N(Dc - _{8422}) \Rightarrow \forall Dc - _{8430[\iota]^{\blacksquare}}[N(Dc - _{8430}) \Rightarrow \\ E(F(Dc - _{8422}, Dc - _{8430}))]] \end{array} 
L38.
                                                                                                                                                                                                                                             (Defne L36)
                                                   \mathcal{H}_1
                                                                                      \vdash \forall N n_{[\iota]} \bullet [N(Nn) \Rightarrow \forall X_{[\iota]} \bullet [N(X) \Rightarrow E(F(Nn,X))]]
L12.
                                                   \mathcal{H}_1
                                                                                                                                                                                                                                             (Otter L38)
                                                                                      \begin{array}{l} + VM_{[l]} \bullet [V(M) \to VA_{[l]} \bullet [V(X) \to E(V(M,X))]] \\ + \forall Dc - 8189_{[l \to o]} \bullet [[Dc - 8189(One) \\ \forall Dc - 8193_{[l]} \bullet [Dc - 8189(Dc - 8193) \\ Dc - 8189(S(Dc - 8193))]] \to Dc - 8189(S(S(S(One)))))] \\ + V(S(S(S(S(One))))) \end{array} 
L6.
                                                                                                                                                                                                                                  Λ
                                                                                                                                                                                                                                             (Otter)
                                                   \mathcal{H}_1
                                                                                                                                                                                                                                \Rightarrow
                                                                                      \vdash N(s(s(s(One)))))
                                                                                                                                                                                                                                             (Defsi L6)
L5.
                                                   \mathcal{H}_1
                                                                                      \vdash E(F(s(s(s(s(One)))), s(s(s(s(One))))))
L39.
                                                                                                                                                                                                                                             (Otter L5,L12)
                                                   \mathcal{H}_1
                                                                                                                                                                                                                                             (Defne L39)
L40.
                                                                                      \vdash [N(F(\mathbf{s}(\mathbf{s}(\mathbf{s}(\mathbf{s}(One)))), \mathbf{s}(\mathbf{s}(\mathbf{s}(\mathbf{s}(One))))))
                                                   \mathcal{H}_1
                                                                                           D(F(s(s(s(One)))), s(s(s(s(One))))))]
Conc.
                                                                                      \vdash D(F(s(s(s(o(ne)))), s(s(s(s(one))))))
                                                                                                                                                                                                                                             (Otter L40)
                                                   \mathcal{H}_1
```

$$\begin{split} \mathcal{H}_1 &= \text{A1, A2, A3, A4, A5, Ld1} \\ \mathcal{H}_2 &= \text{A1, A2, A3, A4, A5, Ld1, L25} \\ \mathcal{H}_3 &= \text{A1, A2, A3, A4, A5, Ld1, L25, L31} \end{split}$$