[THEOREMS PROVED]

[T 1 1] [16.03 18 JULY 1973]

THEOREM TO BE PROVED:

LEQUAL CAPPEND A CAPPEND B C]] CAPPEND CAPPEND A B] C]]

MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

CAND

[EQUAL [APPEND NIL [APPEND B C]] [APPEND [APPEND NIL B] C]]

[IMPLIES

[EQUAL [APPEND A [APPEND B C]] [APPEND [APPEND A B] C]]

[EQUAL [APPEND [CONS A1 A] [APPEND B C]] [APPEND [APPEND [CONS A1 A] B] C]]]

WHICH IS EQUIVALENT TO:

FUNCTION DEFINITIONS:

EAPPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

EIMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

EAND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

PROFILE: [/ [A] , / E N R / E N R .]

TIME: 4.813 SECS.

[T 1 2] [16.03 18 JULY 1973]

THEOREM TO BE PROVED: .

CIMPLIES [EQUAL [APPFND A B] [APPEND A C]] [EQUAL B C]]

WHICH IS EQUIVALENT TO:

COND [EQUAL [APPEND A B] [APPEND A C]] [EQUAL B C] T]

MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

[AND [COND [EQUAL [APPEND NIL B] [APPEND NIL C]] [EQUAL B C] T] [IMPLIES [COND [EQUAL [APPEND A B] [APPEND A C]] [EQUAL B C] T] [COND [EQUAL [APPEND [CONS A1 A] B] [APPEND [CONS A1 A] C]] [EQUAL B C] T]]]

WHICH IS EQUIVALENT TO:

ľ

FUNCTION DEFINITIONS:

CAPPEND CLAMBDA CX Y] COND X CONS CCAR X] CAPPEND CCDR X] Y]] Y]]]

CIMPLIES CLAMBDA CX Y] COND X CCOND Y T NIL] T]]]

CAND CLAMBDA CX Y] COND X CCOND Y T NIL] NIL]]

PROFILE: [/ENR/ENR[A],/ENR/ENR.]

TIME: 4.188 SECS.

THEOREM TO BE PROVED:

CEQUAL [LENGTH [APPEND A B]] [LENGTH [APPEND B A]]]

MUST TRY INDUCTION.

INDUCT ON B.

THE THEOREM TO BE PROVED IS NOW:

EAND

CEQUAL CLENGTH CAPPEND A NIL]] CLENGTH CAPPEND NIL A]]] CIMPLIES

[EQUAL [LENGTH [APPEND A B]] [LENGTH [APPEND B A]]]
[EQUAL [LENGTH [APPEND A [CONS B1 B]]] [LENGTH [APPEND [CONS B1 B] A]]]]

WHICH IS EQUIVALENT TO:

ECOND

CEQUAL CLENGTH CAPPEND A NIL]] CLENGTH A]]
COOND CEQUAL CLENGTH CAPPEND A B]] CLENGTH CAPPEND B A]]]

• CEQUAL CLENGTH CAPPEND A CONS B1 B]]] CONS NIL CLENGTH CAPPEND B A]]]]

NILI

FERTILIZE WITH CEQUAL CLENGTH CAPPEND A BJ CLENGTH CAPPEND B AJJJ.

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THE THEOREM TO BE PROVED IS NOW:
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(WORK ON FIRST CONJUNCT ONLY)

MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

ECOND

CAND CEQUAL ELENGTH CAPPEND NIL NIL]] [LENGTH NIL]]

CIMPLIES CEQUAL CLENGTH CAPPEND A NIL]] CLENGTH A]]

- . [EQUAL [LENGTH [APPEND [CONS A1 A] NIL]] [LENGTH [CONS A1 A]]]]]
- . [EQUAL FLENGTH CAPPEND A2 [CONS B1 B]]] [CONS NIL [LENGTH CAPPEND A2 B]]]]
- . T
- . [*1]]

NILI

WHICH IS EQUIVALENT TO:

[CUND

EEQUAL ELENGTH [APPEND A2 ECONS B1 B]]] [CONS NIL ELENGTH [APPEND A2.B]]]
T
[*1]]

MUST TRY INDUCTION.

INDUCT ON A2.

THE THEOREM TO BE PROVED IS NOW:

```
[COND

. [EQUAL [LENGTH [APPEND NIL [CONS B1 B]]] [CONS NIL [LENGTH [APPEND NIL B]]]]

. T

. [*1]]

[IMPLIES

[COND

. [EQUAL [LENGTH [APPEND A2 [CONS B1 B]]] [CONS NIL [LENGTH [APPEND A2 B]]]]

. T

. [*1]]

[COND [EQUAL [LENGTH [APPEND [CONS A21 A2] [CONS B1 B]]]

. T

. [CONS NIL [LENGTH [APPEND [CONS A21 A2] B]]]]
```

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

CAPPEND CLAMBDA (X Y) COND X CONS (CAR X) CAPPEND (CDR X) Y)] Y)]]

CLENGTH CLAMBDA (X) COND X CONS NIL CLENGTH (CDR X)]) 0]]

LIMPLIES CLAMBDA (X Y) COND X COND Y T NIL] T)]]

CAND CLAMBDA (X Y) COND X COND Y T NIL] NIL]]

FERTILIZERS:

*1 = [COND [EQUAL [LENGTH [APPEND A B]] [LENGTH [APPEND B A]]] NIL T]

PROFILE: [/[B], /ENR/ENRX, / & [A], /ENR/ENR/ENR [A2], /ENR/ENR.]

TIME: 16.12 SECS.

[T 1 4] [16.04 18 JULY 1973]

THEOREM TO BE PROVED:

[EQUAL [REVERSE [APPEND A B]] [APPEND [REVERSE B] [REVERSE A]]]

MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

CAND [EQUAL [REVERSE [APPEND NIL B]] [APPEND [REVERSE B] [REVERSE NIL]]]
[IMPLIES [EQUAL [REVERSE [APPEND A B]] [APPEND [REVERSE B] [REVERSE A]]]
[EQUAL [REVERSE [APPEND [CONS A1 A] B]]
[APPEND [REVERSE B] [REVERSE [CONS A1 A]]]]]

WHICH IS EQUIVALENT TO:

LCUND [EQUAL [REVERSE B] [APPEND [REVERSE B] NIL]]

[COND [EQUAL [REVERSE [APPEND A B]] [CONS A1 NIL]]

[EQUAL [APPEND [REVERSE [APPEND A B]] [CONS A1 NIL]]

[APPEND [REVERSE B] [APPEND [REVERSE A] [CONS A1 NIL]]]

T]

NILI

FERTILIZE WITH [EQUAL [REVERSE [APPEND A 8]] [APPEND [REVERSE B] [REVERSE A]]].

THE THEOREM TO BE PROVED IS NOW:

COND [EQUAL [REVERSE B] [APPEND [REVERSE B] NIL]]

COND [EQUAL [APPEND [APPEND [REVERSE B] [REVERSE A]] [CONS A1 NIL]]

[APPEND [REVERSE B] [APPEND [REVERSE A] [CONS A1 NIL]]]

| [*1]]

NILI

(WORK ON FIRST CONJUNCT ONLY)

GENERALIZE COMMON SUBTERMS BY REPLACING [REVERSE B] BY GENRL1.

THE GENERALIZED TERM IS:

[EQUAL GENRL1 [APPEND GENRL1 NIL]]

MUST TRY INDUCTION.

INDUCT ON GENRL1.

THE THEOREM TO BE PROVED IS NOW:

LCOND

CAND

- . LEQUAL NIL CAPPEND NIL NIL]
- . CIMPLIES CEQUAL GENRL1 CAPPEND GENRL1 NIL]]
- . [EQUAL [CONS GENRL11 GENRL1] [APPEND [CONS GENRL11 GENRL1] NIL]]]]
 [COND [EQUAL [APPEND [APPEND [REVERSE B] [REVERSE A]] [CONS A1 NIL]]
- . [APPEND [REVERSE B] [APPEND [REVERSE A] [CONS A1 NIL]]]
- Т
- · [*1]]

NILJ

```
WHICH IS EQUIVALENT TO:
```

```
COND [EQUAL [APPEND [APPEND [REVERSE B] [REVERSE A]] [CONS A1 NIL]]

[APPEND [REVERSE B] [APPEND [REVERSE A] [CONS A1 NIL]]]

T
[*1]]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [REVERSE A] BY GENRL2 AND [REVERSE B] BY GENRL3.

THE GENERALIZED TERM IS:

[COND [EQUAL [APPEND [APPEND GENRL3 GENRL2] [CONS A1 NIL]]]

[APPEND GENRL3 [APPEND GENRL2 [CONS A1 NIL]]]]

T
[*1]]

MUST TRY INDUCTION.

INDUCT ON GENRL3.

THE THEOREM TO BE PROVED IS NOW:

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

CAPPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]

EREVERSE

[LAMBDA [X] [COND X [APPEND [REVERSE [CDR X]] [CONS [CAR X] NIL]] NIL]]

CIMPLIES FLAMBDA FX YI ECOND X ECOND Y T NILI TIII

EAND ELAMBDA [X Y] [COND X [COND Y T NIL] NIL]]

FERTILIZERS:

*1 = [COND [EQUAL [REVERSE [APPEND A B]] [APPEND [REVERSE B] [REVERSE A]]]

NIL
T]

GENERALIZATIONS:

GENRL1 = [REVERSE B]

GENRL3 = [REVERSE B]

GENRL2 = [REVERSE A]

PROFILE: [/ [A] , / E N R / E N R X , / & G [GENRL1] , / E N R

TIME: 21.38 SECS.

[T 1 5] [16,05 18 JULY 1973]

THEOREM TO BE PROVED:

[EQUAL [LENGTH [REVERSE D]] [LENGTH D]]

MUST TRY INDUCTION.

INDUCT ON D.

THE THEOREM TO BE PROVED IS NOW:

CAND [EQUAL [LENGTH [REVERSE NIL]] [LENGTH NIL]]

[IMPLIES [EQUAL [LENGTH [REVERSE D]] [LENGTH D]]

[EQUAL [LENGTH [REVERSE [CONS D1 D]]] [LENGTH [CONS D1 D]]]]

WHICH IS EQUIVALENT TO:

COND [EQUAL [LENGTH [REVERSE D]] [LENGTH D]]

[EQUAL [LENGTH [APPEND [REVERSE D] [CONS D1 NIL]]] [CONS NIL [LENGTH D]]]

T]

FERTILIZE WITH [EQUAL [LENGTH [REVERSE D]] [LENGTH D]].

THE THEOREM TO BE PROVED IS NOW:

[COND [EQUAL [LENGTH [APPEND [REVERSE D] [CONS D1 NIL]]] [CONS NIL [LENGTH [REVERSE D]]]]

[*1]]

GENERALIZE COMMON SUBTERMS BY REPLACING [REVERSE D] BY GENRL1.

THE GENERALIZED TERM IS:

COND [EQUAL [LENGTH [APPEND GENRL1 [CONS D1 NIL]]] [CONS NIL [LENGTH GENRL1]]]

[*1]

MUST TRY INDUCTION.

INDUCT ON GENRL1.

THE THEOREM TO BE PROVED IS NOW:

[COND [EQUAL [LENGTH [APPEND NIL [CONS D1 NIL]]] [CONS NIL [LENGTH NIL]]]

. T

. [*1]]
[IMPLIES
[COND

. [EQUAL [LENGTH [APPEND GENRL1 [CONS D1 NIL]]]] [CONS NIL [LENGTH GENRL1]]]

. T

. [*1]]
[COND [EQUAL [LENGTH [APPEND [CONS GENRL11 GENRL1]] [CONS D1 NIL]]]

[COND [EQUAL [LENGTH [APPEND [CONS GENRL11 GENRL1]]]]

WHICH IS EQUIVALENT TO:

[*1]]]

Τ

FUNCTION DEFINITIONS:

CREVERSE

CLAMBDA [X] [COND X [APPEND [REVERSE [CDR X]] [CONS [CAR X] NIL]] NIL]]]

CLENGTH [LAMBDA [X] [COND X [CONS NIL [LENGTH [CDR X]]]]]

CAPPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

CIMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

CAND [LAMBDA [X Y] [COND X [COND Y T NIL]]]

FERTILIZERS:

*1 = [COND [EQUAL [LENGTH [REVERSE D]] [LENGTH D]] NIL T]

GENERALIZATIONS:

GENRL1 = [REVERSE D]

PROFILE: [/ [D] , / E N R / E N R X , / G [GENRL1] , / E N R / E N R .]

TIME: 9.438 SECS.

[T 1 6] [16.05 18 JULY 1973]

THEOREM TO BE PROVED:

[EQUAL [REVERSE [REVERSE A]] A]

MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

[AND [EQUAL [REVERSE [REVERSE NIL]] NIL]

[IMPLIES [EQUAL [REVERSE [REVERSE A]] A]

[EQUAL [REVERSE [REVERSE [CONS A1 A]]] [CONS A1 A]]]

WHICH IS EQUIVALENT TO:

COND LEQUAL TREVERSE TREVERSE AT AT AT LEGUAL TREVERSE TAPPEND TREVERSE AT ECONS AT NILTER CONS. AT ATT.

FERTILIZE WITH [EQUAL [REVERSE [REVERSE A]] A].

THE THEOREM TO BE PROVED IS NOW:

[COND LEGUAL EREVERSE [APPEND FREVERSE A] [CONS A1 NIL]]]

[CONS A1 [REVERSE FREVERSE A]]]]

GENERALIZE COMMON SUBTERMS BY REPLACING CREVERSE AD BY GENRL1.

THE GENERALIZED TERM IS:

LCOND

[EQUAL [REVERSE [APPEND GENRL1 [CONS A1 NIL]]] [CONS A1 [REVERSE GENRL1]]]
T
[*1]]

MUST TRY INDUCTION.

INDUCT ON GENRL1.

THE THEOREM TO BE PROVED IS NOW:

CAND

[COND [EQUAL [REVERSE [APPEND NIL [CONS A1 NIL]]] [CONS A1 [REVERSE NIL]]]

CIMPLIES ECOND

. [EQUAL [REVERSE [APPEND GENRL1 [CONS A1 NIL]]] [CONS A1 [REVERSE GENRL1]]]

. T

• [*1]]

COND [EQUAL [REVERSE [APPEND [CONS GENRL11 GENRL1] [CONS A1 NIL]]]
[CONS A1 [REVERSE [CONS GENRL11 GENRL1]]]]

(*1)]]]

FERTILIZE WITH [EQUAL [REVERSE [APPEND GENRL1 [CONS A1 NIL]]] [CONS A1 [REVERSE GENRL1]]].

THE THEOREM TO BE PROVED IS NOW:

WHICH IS EQUIVALENT TO:

FUNCTION DEFINITIONS:

[REVERSE

CLAMBDA [X] [COND X CAPPEND [REVERSE [CDR X]] [CONS [CAR X] NIL]] NIL]]

CAPPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]]]

EAND CLAMBDA CX Y3 CCOND X CCOND Y T NIL3 NIL333

EIMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

FERTILIZERS:

*1 = [COND [EQUAL [REVERSE [REVERSE A]] A] NIL T]

*2 = [COND [EQUAL [REVERSE [APPEND GENRL1 [CONS A1 NIL]]] [CONS A1 [REVERSE GENRL1]]]

NIL T]

GENERALIZATIONS:

GENRL1 = [REVERSE A]

PROFILE: [/[A],/ENR/ENRX,/G [GENRL1],/ENR/ENR/ENRF,/ENR.]

TIME: 12.94 SECS.

[T 1 7] [16.05 18 JULY 1973]

THEOREM TO BE PROVED:

CIMPLIES A [EQUAL [LAST [REVERSE A]] [CAR A]]]

WHICH IS EQUIVALENT TO:

[COND A [EQUAL [LAST [REVERSE A]] [CAR A]] T]

MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

[AND [COND NIL [EQUAL [LAST [REVERSE NIL]] [CAR NIL]] T]
[IMPLIES
[COND A [EQUAL [LAST [REVERSE A]] [CAR A]] T]
[COND [CONS A1 A] [EQUAL [LAST [REVERSE [CONS A1 A]]] [CAR [CONS A1 A]]] T]]]

WHICH IS EQUIVALENT TO:

CCOND A

CCOND [EQUAL [LAST [REVERSE A]] [CAR A]]

[EQUAL [LAST [APPEND [REVERSE A] [CONS A1 NIL]]] A1]

T]

GENERALIZE COMMON SUBTERMS BY REPLACING CREVERSE AD BY GENRL1.

```
THE GENERALIZED TERM IS:
```

```
CCOND A

CCOND EEQUAL CLAST GENRL1] [CAR A]]

EQUAL CLAST CAPPEND GENRL1 [CONS A1 NIL]]] A1]

T]
```

MUST TRY INDUCTION.

INDUCT ON GENRL1.

THE THEOREM TO BE PROVED IS NOW:

```
LAND
ECOND. A
       [COND [EQUAL [LAST NIL] [CAR A]]
             [EQUAL [LAST [APPEND NIL [CONS A1 NIL]]] A1]
       T]
CIMPLIES
    ECOND A
           [COND [EQUAL [LAST GENRL1] [CAR A]]
                 [EQUAL [LAST [APPEND GENRL1 [CONS A1 NIL]]] A1]
                 T]
           T]
    ECOND A
           [COND [EQUAL [LAST [CONS GENRL11 GENRL1]] [CAR A]]
                 [EQUAL [LAST [APPEND [CONS GENRL11 GENRL1] [CONS A1 NIL]]] A1]
                 T]
           TJJJ
```

```
COND

CCOND

CEQUAL [LAST GENRL1] [CAR A]]

CCOND

CEQUAL [LAST [APPEND GENRL1 [CONS A1 NIL]]] A1]

CCOND GENRL1 [COND [APPEND GENRL1 [CONS A1 NIL]] T [EQUAL GENRL11 A1]] T]

T]

T]
```

FERTILIZE WITH [EQUAL [LAST [APPEND GENRL1 [CONS A1 NIL]]] A1].

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THE THEOREM TO BE PROVED IS NOW:
```

MUST TRY INDUCTION.

INDUCT ON GENRL1.

```
EAND
COND
 . A
 . ECOND
    [EQUAL [LAST NIL] [CAR A]]
    ECOND ECOND NIL
                [COND [APPEND NIL [CONS [LAST [APPEND NIL [CONS A1 NIL]]] NIL]]
                       [EQUAL GENRL11 [LAST [APPEND NIL [CONS A1 NIL]]]]
                T]
          Τ
          [*1]]
    T]
 .T]
[ [MPLIES
 ECOND
  . A
  . [COND
  • [EQUAL [LAST GENRL1] [CAR A]]
    ECOND
    . [COND
          GENRL1
          COND CAPPEND GENRL1 [CONS CLAST CAPPEND GENRL1 [CONS A1 NIL]] NIL]]
                CEQUAL GENEL11 CLAST CAPPEND GENEL1 CCONS A1 NIL]]]]
          T]
    • T
    .[*1]]
   T
  · T ]
 ECOND
   Α
   CCOND
   .[EQUAL [LAST [CONS GENRL12 GENRL1]] [CAR A]]
   . ECOND
    [COND
     . [CONS GENRL12 GENRL1]
     . ECOND
        [APPEND [CONS GENRL12 GENRL1]
                [CONS [LAST [APPEND [CONS GENRL12 GENRL1] [CONS A1 NIL]]] NIL]]
        [EQUAL GENRL11 [LAST [APPEND [CONS GENRL12 GENRL1] [CONS A1 NIL]]]]]
     .T]
    T
   . [*1]]
   . T]
   TIII
```

FUNCTION DEFINITIONS:

EREVERSE

[LAMBDA [X] [COND X [APPEND [REVERSE [CDR X]] [CONS [CAR X] NIL]] NIL]]]

LLAST [LAMBDA [X] [COND X [COND [CDR X] [LAST [CDR X]] [CAR X]] NIL]]]

EIMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[CARARG UNDEF]

CAPPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

CAND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]

FERTILIZERS:

*1 = [COND [EQUAL [LAST [APPEND GENRL1 [CONS A1 NIL]]] A1] NIL T]

GENERALIZATIONS:

GENRL1 = [REVERSE A]

PROFILE: [/ENR/ENR[A], /ENR/ENR/ENRG [GENRL1], /ENR/ENR/ENR/ENRG [GENRL1], /ENR/

TIME: 23.13 SECS.

[T 2 1] [16.06 18 JULY 1973]

THEOREM TO BE PROVED:

CIMPLIES [MEMBER A B] [MEMBER A [APPEND B C]]]

WHICH IS EQUIVALENT TO:

[COND [MEMBER A B] [MEMBER A [APPEND B C]] T]

MUST TRY INDUCTION.

INDUCT ON B.

THE THEOREM TO BE PROVED IS NOW:

EAND

[COND [MEMBER A NIL] [MEMBER A [APPEND NIL C]] T]
[IMPLIES [COND [MEMBER A B] [MEMBER A [APPEND B C]] T]
[COND [MEMBER A [CONS B1 B]] [MEMBER A [APPEND [CONS B1 B] C]] T]]]

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

EMEMBER.

CLAMBDA EX Y] COND Y COND CEQUAL X CCAR Y]] T EMEMBER X CCDR Y]]] NIL]]]

CAPPEND CLAMBDA [X Y] COND X CCONS (CAR X] CAPPEND (CDR X] Y]] Y]]

CIMPLIES CLAMBDA [X Y] COND X CCOND Y T NIL] T]]]

CAND CLAMBDA EX Y] COND X CCOND Y T NIL] NIL]]

PROFILE: [/ENR/ENR[B],/ENR/ENR.]

11ME: 5.063 SECS.

[T 2 2] [16.07 18 JULY 1973]

THEOREM TO BE PROVED:

CIMPLIES [MEMBER A B] [MEMBER A [APPEND C B]]]

WHICH IS EQUIVALENT TO:

CCOND [MEMBER A B] [MEMBER A [APPEND C B]] T]

MUST TRY INDUCTION.

INDUCT ON C.

THE THEOREM TO BE PROVED IS NOW:

[AND [COND [MEMBER A B] [MEMBER A [APPEND NIL B]] T]

[IMPLIES [COND [MEMBER A B] [MEMBER A [APPEND C B]] T]

[COND [MEMBER A B] [MEMBER A [APPEND [CONS C1 C] B]] T]]]

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

EMEMBER

[LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]

CAPPEND CLAMBDA CX Y3 CCOND X CCONS CCAR X3 CAPPEND CCDR X3 Y33 Y333 Y333 CIMPLIES CLAMBDA CX Y3 CCOND X CCOND Y T NIL3 T333 CAND CLAMBDA CX Y3 CCOND X CCOND Y T NIL3 NIL333

PROFILE: [/ENR/ENR[C],/ENR/ENR.]

TIME: 6.063 SECS.

[T 2 3] [16.07 18 JULY 1973]

THEOREM TO BE PROVED:

CIMPLIES [AND [NOT [EQUAL A [CAR B]]] [MEMBER A B]] [MEMBER A ECDR B]]]

WHICH IS EQUIVALENT TO:

COND [EQUAL A [CAR B]] T [COND [MEMBER A B] [MEMBER A [CDR B]] T]]

MUST TRY INDUCTION.

INDUCT ON B.

THE THEOREM TO BE PROVED IS NOW:

CAND

ľ

COND [EQUAL A [CAR NIL]] T [COND [MEMBER A NIL] [MEMBER A [CDR NIL]] T]]
[IMPLIES [COND [EQUAL A [CAR B]] T [COND [MEMBER A B] [MEMBER A [CDR B]] T]]
[COND [EQUAL A [CAR [CONS B1 B]]]

COND [MEMBER A [CONS B1 B]] [MEMBER A [CDR [CONS B1 B]]] T]]]]

WHICH IS EQUIVALENT TO:

FUNCTION DEFINITIONS:

ENOT [LAMBDA [X] [COND X NIL T]]]

EMEMBER

[LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

CIMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[CARARG UNDEF]

[CDRARG UNDEF]

PROFILE: [/ENR/ENR[B],/ENR/ENR.]

TIME: 5.938 SECS.

THEOREM TO BE PROVED:

CIMPLIES [OR [MEMBER A B] [MEMBER A C]] [MEMBER A [APPEND B C]]]

WHICH IS EQUIVALENT TO:

[COND [MEMBER A B]
[MEMBER A [APPEND B C]]
[COND [MEMBER A C] [MEMBER A [APPEND B C]] T]]

MUST TRY INDUCTION.

INDUCT ON B.

THE THEOREM TO BE PROVED IS NOW:

CAND [COND [MEMBER A NIL]

[MEMBER A [APPEND NIL C]]

[COND [MEMBER A C] [MEMBER A [APPEND NIL C]] T]]

[IMPLIES [COND [MEMBER A B]

[MEMBER A [APPEND B C]]

[COND [MEMBER A C] [MEMBER A [APPEND B C]] T]]

[COND [MEMBER A [CONS B1 B]]

[MEMBER A [APPEND [CONS B1 B] C]]

[COND [MEMBER A C] [MEMBER A [APPEND [CONS B1 B] C]] T]]]]

WHICH IS EQUIVALENT TO:

FUNCTION DEFINITIONS:

EMEMBER

[LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]

EOR [LAMBDA [X Y] [COND X T [COND Y T NIL]]]]

CAPPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]

CIMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

EAND ELAMBDA EX Y3 [COND X [COND Y T NIL] NIL]]

PROFILE: [/ENR/ENR[B],/ENR/ENR.]

11ME: 11.88 SECS.

[T 2 5] [16.07 18 JULY 1973]

THEOREM TO BE PROVED:

LIMPLIES [AND [MEMBER A B] [MEMBER A C]] [MEMBER A [INTERSEC B C]]]

WHICH IS EQUIVALENT TO:

COND [MEMBER A B] [COND [MEMBER A C] [MEMBER A [INTERSEC B C]] T] T]

MUST TRY INDUCTION.

INDUCT ON B.

THE THEOREM TO BE PROVED IS NOW:

CAND
[COND [MEMBER A NIL] [COND [MEMBER A C] [MEMBER A [INTERSEC NIL C]] T] T]
[IMPLIES [COND [MEMBER A B] [COND [MEMBER A C] [MEMBER A [INTERSEC B C]] T] T]
[COND [MEMBER A C] [MEMBER A [INTERSEC [CONS B1 B] C]] T]

[T]]]

WHICH IS EQUIVALENT TO:

[COND [MEMBER A B] T

[COND [EQUAL A B1]
[COND [MEMBER A C] [COND [MEMBER B1 C] T [MEMBER A [INTERSEC B C]]] T]
T]

```
FERTILIZE WITH [EQUAL A B1].
```

THE THEOREM TO BE PROVED IS NOW:

ECOND

[MEMBER A B]

COND COND [MEMBER A C] COND [MEMBER A C] T [MEMBER A [INTERSEC B C]]] T]

T
[*1]]

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

LMEMBER

[LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]

EAND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]

CINTERSEC [LAMBDA [X Y]

ECOND X

[COND [MEMBER [CAR X] Y]

[CONS [CAR X] [INTERSEC [CDR X] Y]]

[INTERSEC [CDR X] Y]]

NILJJJ

CIMPLIES CLAMBDA [X Y] COND X COND Y T NIL] T]]]

FERTILIZERS:

*1 = [COND [EQUAL A B1] NIL T]

PROFILE: [/ENR/ENR/ENR/ENR/ENR/ENRF,/R/ENR

[T 2 5]

TIME: 19.31 SECS.

[T 2 6] [16.08 18 JULY 1973]

THEOREM TO BE PROVED:

CIMPLIES FOR EMEMBER A B] EMEMBER A C]] EMEMBER A EUNION B C]]]

WHICH IS EQUIVALENT TO:

COND [MEMBER A B]

[MEMBER A CUNION B C]]

[COND [MEMBER A C] [MEMBER A [UNION B C]] T]]

MUST TRY INDUCTION.

INDUCT ON B.

THE THEOREM TO BE PROVED IS NOW:

CAND [COND [MEMBER A NIL]

[MEMBER A [UNION NIL C]]

COND [MEMBER A C] [MEMBER A [UNION NIL C]] T]]

[IMPLIES [COND [MEMBER A B]

[MEMBER A [UNION B C]]

[COND [MEMBER A C] [MEMBER A [UNION B C]] T]]

[COND [MEMBER A [CONS 31 8]]

[MEMBER A [UNION [CONS B1 B] C]]

[COND [MEMBER A C] [MEMBER A [UNION [CONS 81 B] C]] []]]]

WHICH IS EQUIVALENT TO:

ECOND

[MEMBER A B]

T

```
[CUND [MEMBER A C]
```

[COND [EQUAL A 81] [COND [MEMBER 81 C] [MEMBER A [UNION 8 C]] T] T]]]

FERTILIZE WITH [EQUAL A B1].

THE THEOREM TO BE PROVED IS NOW:

COND EMEMBER A CJ

CCOND COND [MEMBER A C] [MEMBER A [UNION B C]] T] T [*1]]]

WHICH IS EQUIVALENT TO:

1

FUNCTION DEFINITIONS:

EMEMBER

[LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]

EOR CLAMBDA EX YJ [COND X T [COND Y T NIL]]]]

LUNION

ELAMBDA

[X X]

ECOND

COND [MEMBER [CAR X] Y] [UNION [CDR X] Y] [CONS [CAR X] [UNION [CDR X] Y]]]

CIMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]

EAND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]

FERTILIZERS:

*1 = [COND [EQUAL A B1] NIL T]

PROFILE: [/ENR/ENR[B],/ENR/ENR/ENR/ENRF,/R/ENR

TIME: 27.06 SECS.

[T 2 7] [16.09 18 JULY 1973]

THEOREM TO BE PROVED:

[IMPLIES [SUBSET A B] [EQUAL [UNION A B] B]]

WHICH IS EQUIVALENT TO:

ECOND [SUBSET A B] [EQUAL [UNION A B] B] T]

MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

[AND [COND [SUBSET NIL B] [EQUAL [UNION NIL B] B] T]
[IMPLIES [COND [SUBSET A B] [EQUAL [UNION A B] B] T]
[COND [SUBSET [CONS A1 A] B] [EQUAL [UNION [CONS A1 A] B] B] T]]]

WHICH IS EQUIVALENT TO:

FUNCTION DEFINITIONS:

[SUBSET

CLÁMBDA [X Y] [COND X [COND [MEMBER [CAR X] Y] [SUBSET [CDR X] Y] NIL] T]]]

EMEMBER

CLAMBDA CX Y] COND Y COND CEQUAL X CCAR Y]] T CMEMBER X CCDR Y]]] NIL]]]

EUNION

CLAMBDA

[X X]

CCOND

χ

COND [MEMBER [CAR X] Y] [UNION [CDR X] Y] [CONS [CAR X] [UNION [CDR X] Y]]]

Y]]]

[[MPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

EAND ELAMBDA EX Y3 ECOND X ECOND Y T NIL3 NIL333

PROFILE: [/ENR/ENR[A],/ENR/ENR,]

TIME: 6.625 SECS.

[T 2 8] [16.09 18 JULY 1973]

THEOREM TO BE PROVED:

CIMPLIES [SUBSET A B] [EQUAL [INTERSEC A B] A]]

WHICH IS EQUIVALENT TO:

L'COND [SUBSET A B] [EQUAL [INTERSEC A B] A] T]

MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

[AND [COND [SUBSET NIL B] [EQUAL [INTERSEC NIL B] NIL] T]

[IMPLIES [COND [SUBSET A B] [EQUAL [INTERSEC A B] A] T]

[COND [SUBSET [CONS A1 A] B]

[EQUAL [INTERSEC [CONS A1 A] B] [CONS A1 A]]

T]]]

WHICH IS EQUIVALENT TO:

ſ

FUNCTION DEFINITIONS:

ESUBSET

CLAMBDA [X Y] [COND X [COND [MEMBER [CAR X] Y] [SUBSET [CDR X] Y] NIL] T]]]

EMEMBER

[LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]

CINTERSEC CLAMBDA (X Y)

ECOND X

[COND [MEMBER [CAR X] Y]
[CONS [CAR X] [INTERSEC [CDR X] Y]]
[INTERSEC [CDR X] Y]]

NILJJJ

EIMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

EAND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

PROFILE: [/ENR/ENR[A],/ENR/ENR/ENR,]

TIME: 8.563 SECS.

[T 2 9] [16.09 18 JULY 1973]

THEOREM TO BE PROVED:

[EQUAL [MEMBER A B] [NOT [EQUAL [ASSOC A [PAIRLIST B C]] NIL]]].

WHICH IS EQUIVALENT TO:

ECOND CASSOC A [PAIRLIST B C]] EMEMBER A B] [COND EMEMBER A B] NIL T]]

MUST TRY INDUCTION.

INDUCT ON C AND B.

THE THEOREM TO BE PROVED IS NOW:

EAND

.COND CASSOC A CPAIRLIST B NIL] CMEMBER A B] COND EMEMBER A B] NIL T]]

. ECOND [ASSOC A [PAIRLIST NIL C]] [MEMBER A NIL] [COND [MEMBER A NIL] NIL T]]]

[IMPLIES

COND [ASSOC A [PAIRLIST B C]] [MEMBER A R] [COND [MEMBER A B] NIL T]] [COND [ASSOC A [PAIRLIST [CONS B1 B] [CONS C1 C]]]

[MEMBER A [CONS B1 B]]
[COND [MEMBER A [CONS B1 B]] NIL T]]]

WHICH IS EQUIVALENT TO:

COUND CASSOC A [PAIRLIST B NIL]] [MEMBER A B] [COND [MEMBER A B] NIL T]]

```
MUST TRY INDUCTION.
```

INDUCT ON B.

THE THEOREM TO BE PROVED IS NOW:

CAND

[COND [ASSOC A [PAIRLIST NIL NIL]] [MEMBER A NIL] [COND [MEMBER A NIL] NIL T]]

[IMPLIES

[COND [ASSOC A [PAIRLIST B NIL]] [MEMBER A B] [COND [MEMBER A B] NIL T]]

[COND [ASSOC A [PAIRLIST [CONS B2 B] NIL]]

[MEMBER A [CONS B2 B]]

[COND [MEMBER A [CONS B2 B]] NIL T]]]

WHICH IS EQUIVALENT TO:

Τ

FUNCTION DEFINITIONS:

EMEMBER

CLAMBDA [X Y] [COND Y ECOND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]

[PAIRLIST

[LAMBDA [X Y]

ECOND X

ECOND Y

[CONS [CONS [CAR X] [CAR Y]] [PAIRLIST [CDR X] [CDR Y]]]

ECONS ECONS ECAR X3 NIL] [PAIRLIST ECDR X3 NIL]]

NILJJJ

LASSUC

ELAMBDA [X Y]

CCOND Y

ECOND ECAR YJ

[COND [EQUAL X [CAR [CAR Y]]] [CAR Y] [ASSOC X [CDR Y]]]

[ASSOC X [CDR Y]]]

NIL]]]

ENOT [LAMBDA [X] [COND X NIL T]]]

EAND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

EIMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

PROFILE: [/ENR/ENR[CB],/ENR/ENR/ENR[B],/ENR/ENR
.]

TIME: 15.0 SECS.

[T 3 1] [16.09 18 JULY 1973]

THEOREM TO BE PROVED:

[EQUAL [MAPLIST [APPEND A B] C] [APPEND [MAPLIST A C] [MAPLIST B C]]]

MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

EAND

[EQUAL [MAPLIST [APPEND NIL B] C] [APPEND [MAPLIST NIL C] [MAPLIST B C]]]
[IMPLIES [EQUAL [MAPLIST [APPEND A B] C] [APPEND [MAPLIST A C] [MAPLIST B C]]

[EQUAL [MAPLIST [APPEND [CONS A1 A] B] C]

[APPEND [MAPLIST [CONS A1 A] C] [MAPLIST B C]]]]

WHICH IS EQUIVALENT TO:

ſ

FUNCTION DEFINITIONS:

CAPPEND CLAMBDA [X Y] COND X CONS COAR X] CAPPEND CODR X] Y]]

EMAPLIST

ELAMBDA [X Y] [COND X [CONS [APPLY Y [CAR X]] [MAPLIST [CDR X] Y]] NIL]]]

[APPLY UNDEF]

EIMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

CAND CLAMBDA EX YJ CCOND X CCOND Y T NILJ NILJJJ

PROFILE: [/ [A] , / E N R / E N R .]

TIME: 6.25 SECS.

[T 3 2] [16.1 18 JULY 1973]

THEOREM TO BE PROVED:

CEQUAL CLENGTH [MAPLIST A B]] [LENGTH A]]

MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

CAND CEQUAL CLENGTH (MAPLIST NIL B]] [LENGTH NIL]]

CIMPLIES (EQUAL CLENGTH EMAPLIST A B]] (LENGTH A]]

CEQUAL CLENGTH (MAPLIST (CONS A1 A] B]] (LENGTH (CONS A1 A]]]]

WHICH IS EQUIVALENT TO:

Ŧ

FUNCTION DEFINITIONS:

EMAPLIST

CLAMBDA [X Y] [COND X [CONS [APPLY Y [CAR X]] [MAPLIST [CDR X] Y]] NIL]]]

[APPLY UNDEF]

[LENGTH [LAMBDA [X] [COND X [CONS NIL [LENGTH [CDR X]]] 0]]]

EIMPLIES CLAMBDA [X Y] COND X COND Y T NIL] T]]]

CAND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

PROFILE: [/ [A] , / E N R / E N R .]

TIME: 3.438 SECS.

[T 3 3] [16.13 18 JULY 1973]

THEOREM TO BE PROVED:

CEQUAL [REVERSE [MAPLIST A B]] [MAPLIST [REVERSE A] B]]

MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

WHICH IS EQUIVALENT TO:

COND [EQUAL [REVERSE [MAPLIST A B]] [MAPLIST [REVERSE A] B]]

LEQUAL [APPEND [REVERSE [MAPLIST A B]] [CONS [APPLY B A1] NIL]]

[MAPLIST [APPEND [REVERSE A] [CONS A1 NIL]] B]]

T]

FERTILIZE WITH [EQUAL [REVERSE [MAPLIST A B]] [MAPLIST [REVERSE A] B]].

THE THEOREM TO BE PROVED IS NOW:

```
[COND [EQUAL [APPEND [MAPLIST [REVERSE A] B] [CONS [APPLY B A1] NIL]]
[MAPLIST [APPEND [REVERSE A] [CONS A1 NIL]] B]]
[*1]]
```

GENERALIZE COMMON SUBTERMS BY REPLACING TREVERSE AT BY GENEL1.

THE GENERALIZED TERM IS:

COND CEQUAL CAPPEND (MAPLIST GENRL1 B) CONS CAPPLY B A1] NIL]]

EMAPLIST CAPPEND GENRL1 [CONS A1 NIL]] B]]

T
[*1]]

MUST TRY INDUCTION.

INDUCT ON GENRL1.

THE THEOREM TO BE PROVED IS NOW:

```
LAND

[COND [EQUAL [APPEND [MAPLIST NIL B] [CONS [APPLY B A1] NIL]]

. [MAPLIST [APPEND N]L [CONS A1 NIL]] B]]

. T

. [*1]]

[IMPLIES

[COND [EQUAL [APPEND [MAPLIST GENRL1 B] [CONS [APPLY B A1] NIL]]

. [MAPLIST [APPEND GENRL1 [CONS A1 NIL]] B]]

. T

. [*1]]

[COND

[EQUAL [APPEND [MAPLIST [CONS GENRL11 GENRL1] B] [CONS [APPLY B A1] NIL]]

. [MAPLIST [APPEND [CONS GENRL11 GENRL1] B] [CONS [APPLY B A1] NIL]]

. [MAPLIST [APPEND [CONS GENRL11 GENRL1] [CONS A1 NIL]] B]]

T'

[*1]]]]
```

WHICH IS EQUIVALENT TO:

Γ

FUNCTION DEFINITIONS:

EMAPLIST

ELAMBDA [X Y] [COND X [CONS [APPLY Y [CAR X]] [MAPLIST [CDR X] Y]] NIL]]]

[APPLY UNDEF]

CREVERSE

[LAMBDA [X] [COND X [APPEND [REVERSE [CDR X]] [CONS [CAR X] NIL]] NIL]]]

EAPPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

EIMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[[AND ELAMBDA [X Y] [COND X [COND Y T NIL] NIL]]

FERTILIZERS:

*1 = [COND [EQUAL [REVERSE [MAPLIST A B]] [MAPLIST [REVERSE A] B]] NIL T]

GENERALIZATIONS:

GENRL1 = [REVERSE A]

PROFILE: [/ [A] , / E N R / E N R X , / G [GENRL1] , / E N R / E N R .]

TIME: 12.19 SECS.

[T 4 1] [16.13 18 JULY 1973]

THEOREM TO BE PROVED:

[EQUAL [LIT [APPEND A B] C D] [LIT A [LIT B C D] D]]

MUST TRY INDUCTION.

INDÚCT ON A.

THE THEOREM TO BE PROVED IS NOW:

[AND
[EQUAL [LIT [APPEND NIL B] C D] [LIT NIL [LIT B C D] D]]
[IMPLIES
[EQUAL [LIT [APPEND A B] C D] [LIT A [LIT B C D] D]]
[EQUAL [LIT [APPEND [CONS A1 A] B] C D] [LIT [CONS A1 A] [LIT B C D] D]]]

WHICH IS EQUIVALENT TO:

ECOND CEQUAL CLIT CAPPEND A B] C D] CLIT A CLIT B C D] D]] CEQUAL CAPPLY D A1 CLIT CAPPEND A B] C D]] CAPPLY D A1 CLIT A CLIT B C D] D]]] T]

FERTILIZE WITH [EQUAL [LIT [APPEND A B] C D] [LIT A [LIT B C D] D]].

THE THEOREM TO BE PROVED IS NOW:

LCOND [EQUAL [APPLY D A1 [LIT [APPEND A B] C D]]

[APPLY D A1 [LIT [APPEND A B] C D]]

[
[*1]]

WHICH IS EQUIVALENT TO:

Т

FUNCTION DEFINITIONS:

EAPPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]]] Y]]]

ELlt [Lambda [X Y Z] [COND X [APPLY Z [CAR X] [LIT [CDR X] Y Z]] Y]]]

EAPPLY UNDEF]

EIMPLIES (LAMBDA (X Y) (COND X (COND Y T NIL) T))

EAND (LAMBDA (X Y) (COND X (COND Y T NIL) NIL))

FERTILIZERS:

*1 = [COND [EQUAL [LIT [APPEND A B] C D] [LIT A [LIT B C D] D]] NIL T]

PROFILE: [/[A],/ENR/ENRX,/ENR.]

fIME: 8.313 SECS.

[T 4 2] [16.14 18 JULY 1973]

THEOREM TO BE PROVED:

CIMPLIES [AND [BOOLEAN A] [BOOLEAN B]]

[EQUAL [AND [IMPLIES A B] [IMPLIES B A]] [EQUAL A B]]

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

EBUOLEAN [LAMBDA [X] [COND X [EQUAL X T] T]]]

EAND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

LIMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

PROFILE: [/ENR/ENR.]

TIME:, 4.313 SECS.

[T 4 3] [16.14 18 JULY 1973]

THEOREM TO BE PROVED:

CEQUAL CELEMENT B A] CELEMENT CAPPEND C B] CAPPEND C A]]]

MUST TRY INDUCTION.

INDUCT ON C.

THE THEOREM TO BE PROVED IS NOW:

CAND CEQUAL CELEMENT B A] CELEMENT CAPPEND NIL B] CAPPEND NIL A]]] CIMPLIES CEQUAL CELEMENT B AD CELEMENT CAPPEND C BD CAPPEND C ADD CEQUAL CELEMENT B A] [ELEMENT [APPEND [CONS C1 C] R] [APPEND [CONS C1 C] A]]]]

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

CELEMENT

CLAMBDA (X Y) [COND Y [COND X [ELEMENT [CDR X] [CDR Y]] [CAR Y]] NIL]]] LAPPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y] EIMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

PROFILE: [/ [C]; / E N R / E N R .]

TIME: 5.25 SECS.

[T 4 4] [16.14 18 JULY 1973]

THEOREM TO BE PROVED:

CIMPLIES CELEMENT B AD CMEMBER CELEMENT B AD ADD

WHICH IS EQUIVALENT TO:

CCUND CELEMENT B A] [MEMBER [ELEMENT B A] A] T]

MUST TRY INDUCTION.

INDUCT ON B AND A.

THE THEOREM TO BE PROVED IS NOW:

CAND CAND COND EELEMENT NIL A] [MEMBER CELEMENT NIL A] A] T]

COND CELEMENT B NIL] [MEMBER CELEMENT B NIL] NIL] T]]

CIMPLIES COND CELEMENT B A] [MEMBER CELEMENT B A] A] T]

COND CELEMENT CONS B1 B] CONS A1 A]]

[MEMBER CELEMENT CONS B1 B] CONS A1 A]]

T]]]

WHICH IS EQUIVALENT TO:

Ţ

FUNCTION DEFINITIONS:

CELEMENT

CLAMBDA (X Y) [COND Y [COND X [ELEMENT [CDR X] [CDR Y]] [CAR Y]] NIL]]

EMEMBER

[LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]

CIMPLIES CLAMBDA [X Y] [COND X [COND Y T NIL] T]]]

EAND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]

PROFILE: [/ENR/ENR[BA],/ENR/ENR.]

TIME: 6.188 SECS.

[T 4 5] [16,14 18 JULY 1973]

THEOREM TO BE PROVED:

LEQUAL CODRN C CAPPEND A BJJ CAPPEND CODRN C AJ CODRN CODRN A CJ BJJJ

MUST TRY INDUCTION.

INDUCT ON A AND C.

THE THEOREM TO BE PROVED IS NOW:

EAND

. [EQUAL [CDRN C [APPEND NIL B]] [APPEND [CDRN C NIL] [CDRN [CDRN NIL C] B]]]
. [EQUAL [CDRN NIL [APPEND A B]] [APPEND [CDRN NIL A] [CDRN [CDRN A NIL] B]]]
[IMPLIES [EQUAL [CDRN C [APPEND A B]] [APPEND [CDRN C A] [CDRN [CDRN A C] B]]]
[EQUAL [CDRN [CONS C1 C] [APPEND [CONS A1 A] B]]
[APPEND [CDRN [CONS C1 C] [CONS A1 A]]
[CDRN [CDRN [CONS A1 A] [CONS C1 C]] B]]]]]

WHICH IS EQUIVALENT TO:

FUNCTION DEFINITIONS:

CAPPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]

ECDRN CLAMBDA [X Y] [COND Y [COND X [CDRN [CDR X] [CDR Y]] Y] NIL]]]

EAND CLAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

PROFILE: [/ [A C] , / E N R / E N R .]

TIME: 6.563 SECS.

[T 4 6] [16.15 18 JULY 1973]

THEOREM TO BE PROVED:

CEQUAL [CDRN [APPEND B C] A] [CDRN C [CDRN B A]]]

MUST TRY INDUCTION.

INDUCT ON B AND A.

THE THEOREM TO BE PROVED IS NOW:

EAND EAND EEQUAL ECDRN EAPPEND NIL C] A] ECDRN C ECDRN NIL A]]]

EEQUAL ECDRN EAPPEND B C] NIL] ECDRN C ECDRN B NIL]]]]

EIMPLIES EEQUAL ECDRN EAPPEND B C] A] ECDRN C ECDRN B A]]]

ECDRN C ECDRN ECONS 31 B] C] ECONS A1 A]]

ECDRN C ECDRN ECONS B1 B] ECONS A1 A]]]]]

WHICH IS EQUIVALENT TO:

Т

FUNCTION DEFINITIONS:

CAPPEND CLAMBDA (X Y) COND X CONS (CAR X) CAPPEND (CDR X) Y) Y) Y) CORN CLAMBDA (X Y) COND Y COND X CCDRN (CDR X) CCDR Y) Y) NIL]]

CAND CLAMBDA (X Y) COND X CCOND Y T NIL] NIL]]

LIMPLIES CLAMBDA (X Y) COND X CCOND Y T NIL] T)]

PROFILE: E/ [B A] . / E N R / E N R . 3

TIME: 4.625 SECS.

THEOREM TO BE PROVED:

[EQUAL [EQUAL A B] [EQUAL B A]]

WHICH IS EQUIVALENT TO:

[COND [EQUAL A B] [EQUAL B A] [COND [EQUAL B A] NIL T]]

FERTILIZE WITH [EQUAL A B].

THE THEOREM TO BE PROVED IS NOW:

COND [COND [EQUAL A A] T [*1]]

COND [COND [EQUAL B A] NIL T] T [EQUAL A B]]

NIL]

WHICH IS EQUIVALENT TO:

LCOND [EQUAL B A] [EQUAL A B] T]

FERTILIZE WITH [EQUAL B A].

THE THEOREM TO BE PROVED IS NOW:

[COND [EQUAL B B] T [*2]]

WHICH IS EQUIVALENT TO:

T

FERTILIZERS:

*1 = [COND [EQUAL A B] NIL [COND [EQUAL B A] NIL T]]

*2 = [COND [EQUAL B A] NIL T]

PROFILE: [/NR/ENRF,/ENR/ENRF,/ENR.]

TIME: 2.063 SECS.

[T 4 8] [16.15 18 JULY 1973]

THEOREM TO BE PROVED:

EIMPLIES [AND [EQUAL A B] [EQUAL B C]] [EQUAL A C]]

WHICH IS EQUIVALENT TO:

CCUND [EQUAL A B] [COND [EQUAL B C] [EQUAL A C] T] T]

FERTILIZE WITH CEQUAL A BJ.

THE THEOREM TO BE PROVED IS NOW:

[COND [COND [EQUAL A C] [EQUAL A C] T] T [*1]]

WHICH IS EQUIVALENT TO:

FUNCTION DEFINITIONS:

CAND CLAMBDA (X Y) COND X COND Y T NIL] NIL]]

EIMPLIES CLAMBDA (X Y) COND X COND Y T NIL] T]]

FERTILIZERS:

*1 = ECOND REQUAL A BJ NIL TO

PROFILE: [/ENR/ENRF,/R/ENR.]

TIME: 1.938 SECS.

[T 4 9] [16.15 18 JULY 1973]

THEOREM TO BE PROVED:

CIMPLIES [AND [BOOLEAN A] [AND [BOOLEAN B] [BOOLEAN C]]]

CEQUAL [EQUAL A [EQUAL B C]] [EQUAL [EQUAL A B] C]]]

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

EAND CLAMBDA [X] COND X CCOND Y T NIL] NIL]]

EIMPLIES CLAMBDA [X Y] COND X CCOND Y T NIL] T]]

PROFILE: [/ENR/ENR.]

TIME: 13.69 SECS.

[T 5 1] [16.15 18 JULY 1973]

THEOREM TO BE PROVED:

[EQUAL [PLUS N M] [PLUS M N]]

MUST TRY INDUCTION.

INDUCT ON M.

THE THEOREM TO BE PROVED IS NOW:

[AND [EQUAL [PLUS N NIL] [PLUS NIL N]]
[IMPLIES [EQUAL [PLUS N M] [PLUS M N]]
[EQUAL [PLUS N [CONS NIL M]] [PLUS [CONS NIL M] N]]]]

WHICH IS EQUIVALENT TO:

COND [EQUAL [PLUS N NIL] N]

[COND [EQUAL [PLUS N M] [PLUS M N]]

[EQUAL [PLUS N [CONS NIL M]] [CONS NIL [PLUS M N]]]

T]

NIL]

FERTILIZE WITH [EQUAL [PLUS N M] [PLUS M N]].

THE THEOREM TO BE PROVED IS NOW:

[COND [EQUAL [PLUS N NIL] N]

[COND [EQUAL [PLUS N [CONS NIL M]] [CONS NIL [PLUS N M]]] T [*1]]
NIL]

(WORK ON FIRST CONJUNCT ONLY)

MUST TRY INDUCTION.

INDUCT ON N.

THE THEOREM TO BE PROVED IS NOW:

COND CAND CEQUAL CPLUS NIL NIL]

CIMPLIES (EQUAL CPLUS N NIL) N]

(EQUAL CPLUS CONS NIL N] NIL] CONS NIL N]]]

COND (EQUAL CPLUS N1 (CONS NIL M]] (CONS NIL CPLUS N1 M]]] T [*1]]

NIL]

WHICH IS EQUIVALENT TO:

[COND [EQUAL [PLUS N1 [CONS NIL M]] [CONS NIL [PLUS N1 M]]] T [*1]]

MUST TRY INDUCTION.

INDUCT ON N1.

THE THEOREM TO BE PROVED IS NOW:

[AND
[COND [EQUAL [PLUS NIL [CONS NIL M]] [CONS NIL [PLUS NIL M]]] T [*1]]
[IMPLIES
[COND [EQUAL [PLUS N1 [CONS NIL M]] [CONS NIL [PLUS N1 M]]] T [*1]]
[COND
[EQUAL [PLUS [CONS NIL N1] [CONS NIL M]] [CONS NIL [PLUS [CONS NIL N1] M]]]
T
[*1]]]]

WHICH IS EQUIVALENT TO:

FUNCTION DEFINITIONS:

EPLUS CLAMBDA (X Y.) [COND X [CONS NIL [PLUS [CDR X] Y]] Y]]]

LIMPLIES CLAMBDA (X Y) [COND X [COND Y T NIL] T]]]

LAND CLAMBDA (X Y) [COND X [COND Y T NIL] NIL]]

FERTILIZERS:

*1 = [COND [EQUAL [PLUS N M] [PLUS M N]] NIL T]

PROFILE: [/[M],/ENR/ENRX,/8[N],/ENR/ENR/ENR[N1],/ENR/ENR/ENR.]

TIME: 11.31 SECS.

[T 5 2] [16.16 18 JULY 1973]

THEOREM TO BE PROVED:

[EQUAL [PLUS N'[PLUS M K]] [PLUS [PLUS N M] K]]

MUST TRY INDUCTION.

INDUCT ON N.

THE THEOREM TO BE PROVED IS NOW:

[AND
[EQUAL [PLUS NIL [PLUS M K]] [PLUS [PLUS NIL M] K]]
[IMPLIES
[EQUAL [PLUS N [PLUS M K]] [PLUS [PLUS N M] K]]
[EQUAL [PLUS [CONS NIL N] [PLUS M K]] [PLUS [PLUS [CONS NIL N] M] K]]]]

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

CPLUS CLAMBDA (X Y) COOND X COONS NIL CPLUS CODR X) Y)) Y)))
CIMPLIES CLAMBDA (X Y) COOND X COOND Y T NIL) T)))
CAND CLAMBDA (X Y) COOND X COOND Y T NIL) NIL))

PROFILE: [/[N], /ENR/ENR.]

TIME: 4.688 SECS.

[T 5 3] [16.16 18 JULY 1973]

THEOREM TO BE PROVED:

CEQUAL CTIMES N MJ [TIMES M NJ]

MUST TRY INDUCTION.

INDUCT ON M.

THE THEOREM TO BE PROVED IS NOW:

[AND [EQUAL [TIMES N NIL] [TIMES NIL N]]
[ÎMPLIES [EQUAL [TIMES N M] [TIMES M N]]
[EQUAL [TIMES N [CONS NIL M]] [TIMES [CONS NIL M] N]]]

WHICH IS EQUIVALENT TO:

COND [TIMES N NIL]

NIL

COND [EQUAL [TIMES N M] [TIMES M N]]

[EQUAL [TIMES N [CONS NIL M]] [PLUS N [TIMES M N]]]

T]]

FERTILIZE WITH [EQUAL [TIMES N M] [TIMES M N]].

THE THEOREM TO BE PROVED IS NOW:

[COND [TIMES N NIL]

NIL
[COND [EQUAL [TIMES N [CONS NIL M]] [PLUS N [TIMES N M]]] T [*1]]

(WORK ON FIRST CONJUNCT ONLY)

MUST TRY INDUCTION.

INDUCT ON N.

THE THEOREM TO BE PROVED IS NOW:

COND [AND [NOT [TIMES NIL NIL]]

[IMPLIES [NOT [TIMES N NIL]] [NOT [TIMES [CONS NIL N] NIL]]]

[COND [EQUAL [TIMES N1 [CONS NIL M]] [PLUS N1 [TIMES N1 M]]] T [*1]]

NIL]

WHICH IS EQUIVALENT TO:

COND CEQUAL [TIMES N1 [CONS NIL M]] [PLUS N1 [TIMES N1 M]]] T [*1]]

MUST TRY INDUCTION.

INDUCT ON N1.

```
THE THEOREM TO BE PROVED IS NOW:
```

COND CEQUAL CTIMES NIL CCONS NIL M]] CPLUS NIL CTIMES NIL M]]] T [*1]]

LIMPLIES CCOND CEQUAL CTIMES N1 CCONS NIL M]] [PLUS N1 CTIMES N1 M]]] T [*1]]

COND CEQUAL CTIMES CCONS NIL N1] [CONS NIL M]]

CPLUS CCONS NIL N1] CTIMES CCONS NIL N1] M]]]

T

[*1]]]

WHICH IS EQUIVALENT TO:

[COND [EQUAL [TIMES N1 [CONS NIL M]] [PLUS N1 [TIMES N1 M]]] [COND [EQUAL [PLUS M [TIMES N1 [CONS NIL M]]] [PLUS N1 [PLUS M [TIMES N1 M]]]] . T . [*1]]

FERTILIZE WITH [EQUAL [TIMES N1 [CONS NIL M]] [PLUS N1 [TIMES N1 M]]].

THE THEOREM TO BE PROVED IS NOW:

COND CEQUAL CPLUS M CPLUS N1 CTIMES N1 M]]] CPLUS N1 CPLUS M CTIMES N1 M]]]

T

[*1]]

[*2]]

GENERALIZE COMMON SUBTERMS BY REPLACING CTIMES N1 MJ BY GENRL1.

THE GENERALIZED TERM IS:

ECOND [COND [EQUAL [PLUS M [PLUS N1 GENRL1]] [PLUS N1 [PLUS M GENRL1]]] T [*1]]

MUST TRY INDUCTION.

INDUCT ON N1.

```
THE THEOREM TO BE PROVED IS NOW:
```

WHICH IS EQUIVALENT TO:

CCUND [EQUAL [PLUS M [PLUS N1 GENRL1]] [PLUS N1 [PLUS M GENRL1]]]

ECOND [COND [EQUAL [PLUS M [CONS NIL [PLUS N1 GENRL1]]]]

T

[*1]]

T

[*2]]

T]

FERTILIZE WITH (EQUAL [PLUS M [PLUS N1 GENRL1]]] [PLUS N1 [PLUS M GENRL1]]].

THE THEOREM TO BE PROVED IS NOW:

ECOND ECOND [COND [EQUAL [PLUS M [CONS NIL [PLUS N1 GENRL1]]]]

ECONS NIL [PLUS M [PLUS N1 GENRL1]]]

[*1]]

T [*2]]

T [*3]]

GENERALIZE COMMON SUBTERMS BY REPLACING [PLUS N1 GENRL1] BY GENRL2.

THE GENERALIZED TERM IS:

L COND E COND

. [COND [EQUAL [PLUS M [CONS NIL GENRL2]] [CONS NIL [PLUS M GENRL2]]] T [*1]]

. T

. [*2]]

T

[*3]]

MUST TRY INDUCTION.

INDUCT ON M.

THE THEOREM TO BE PROVED IS NOW:

EAND

```
ECOND
.CCOND [COND [EQUAL [PLUS NIL [CONS NIL GENRL2]] [CONS NIL [PLUS NIL GENRL2]]]
             [*1]]
       T
       [*2]]
. T
.[*3]]
EIMPLIES
 ECOND
 . ECOND
  CCUND [EQUAL [PLUS M [CONS NIL GENRL2]] [CONS NIL [PLUS M GENRL2]]] T [*1]]
  [*2]]
 • T
 .[*3]]
 ECOND ECOND ECOND FEQUAL EPLUS ECONS NIL MD ECONS NIL GENRL233
                           [CONS NIL [PLUS [CONS NIL M] GENRL2]]]
                   [*1]]
             [*2]]
       [*3]]]
```

WHICH IS EQUIVALENT TO:

FUNCTION DEFINITIONS:

ETIMES CLAMBDA (X Y) COND X CPLUS Y CTIMES (CDR X) Y)] 0]]]

CPLUS CLAMBDA (X Y) COND X CONS NIL CPLUS (CDR X) Y)] Y)]]

CIMPLIES CLAMBDA (X Y) COND X COND Y T NIL] T)])

CAND CLAMBDA (X Y) COND X COND Y T NIL] NIL]]

ENOT CLAMBDA (X) COND X NIL T)]

FERTILIZERS:

- *1 = [COND [EQUAL [TIMES N M] [TIMES M N]] NIL T]
- *2 = [COND [EQUAL [TIMES N1 [CONS NIL M]] [PLUS N1 [TIMES N1 M]]] NIL T]

*3 = [COND [EQUAL [PLUS M [PLUS N1 GENRL1]] [PLUS N1 [PLUS M GENRL1]]] NIL T]

GENERALIZATIONS:

GENRL1 = [TIMES N1 M]

GENRL2 = [PLUS N1 GENRL1]

PROFILE: [/[M], /ENR/ENRX, /&[N], /ENR/ENR/ENR [N1], /ENR/ENR/ENRF, /G[M], /ENR/ENRF, /G[M], /ENR/ENRF, /G[M], /ENR/ENRF, /G[M], /ENR/ENR.]

TIME: 32.75 SECS.

[T 5 4] [16.17 18 JULY 1973]

THEOREM TO BE PROVED:

[EQUAL ETIMES N [PLUS M K]] [PLUS [TIMES N M] [TIMES N K]]]

MUST TRY INDUCTION.

INDUCT ON N.

THE THEOREM TO BE PROVED IS NOW:

[AND [EQUAL [TIMES NIL [PLUS M K]] [PLUS [TIMES NIL M] [TIMES NIL K]]]
[IMPLIES [EQUAL [TIMES N [PLUS M K]] [PLUS [TIMES N M] [TIMES N K]]

[EQUAL [TIMES [CONS NIL N] M] [TIMES [CONS NIL N] K]]]]

WHICH IS EQUIVALENT TO:

COND CEQUAL TIMES N CPLUS M K] CPLUS CTIMES N M] CTIMES N K]]]

CEQUAL CPLUS CPLUS M K] CTIMES N CPLUS M K]]]

CPLUS CPLUS M CTIMES N M]] CPLUS K CTIMES N K]]]]

T]

FERTILIZE WITH [EQUAL [TIMES N [PLUS M K]] [PLUS [TIMES N M] [TIMES N K]]].

THE THEOREM TO BE PROVED IS NOW:

LCOND DEQUAL CPLUS CPLUS M K) CPLUS CTIMES N M) CTIMES N K))]

[PLUS [PLUS M [TIMES N M]] [PLUS K [TIMES N K]]]]

T
[*1]]

GENERALIZE COMMON SUBTERMS BY REPLACING [TIMES N K] BY GENRL1 AND [TIMES N M] BY GENRL2.

THE GENERALIZED TERM IS:

[COND [EQUAL [PLUS [PLUS M K] [PLUS GENRL1]] [PLUS [PLUS M GENRL2] [PLUS K GENRL1]]]

T
[*1]]

MUST TRY INDUCTION.

INDUCT ON M.

THE THEOREM TO BE PROVED IS NOW:

[AND [COND [EQUAL [PLUS [PLUS NIL K] [PLUS GENRL2 GENRL1]] [PLUS [PLUS NIL GENRL2] [PLUS K GENRL1]]]

T

[*1]]
[1MPLIES [COND [EQUAL [PLUS [PLUS M K] [PLUS GENRL2 GENRL1]]
[PLUS [PLUS M GENRL2] [PLUS K GENRL1]]]

[*1]]
[COND [EQUAL [PLUS [PLUS [CONS NIL M] K] [PLUS GENRL2 GENRL1]]

[PLUS [PLUS [CONS NIL M] GENRL2] [PLUS K GENRL1]]]

T
[*1]]]]

```
WHICH IS EQUIVALENT TO:
```

COND [EQUAL [PLUS K [PLUS GENRL2 GENRL1]] [PLUS GENRL2 [PLUS K GENRL1]]]

T
[*1]]

MUST TRY INDUCTION.

INDUCT ON GENRL2.

THE THEOREM TO BE PROVED IS NOW:

COND [EQUAL [PLUS K [PLUS NIL GENRL1]] [PLUS NIL [PLUS K GENRL1]]] T [*1]]

[IMPLIES

[COND [EQUAL [PLUS K [PLUS GENRL2 GENRL1]]] [PLUS GENRL2 [PLUS K GENRL1]]]

. T

. [*1]]

[COND [EQUAL [PLUS K [PLUS [CONS GENRL2]]] [PLUS GENRL1]]

[PLUS [CONS GENRL2]] [PLUS K GENRL1]]

T

[*1]]]]

WHICH IS EQUIVALENT TO:

COND [EQUAL [PLUS K [PLUS GENRL2 GENRL1]] [PLUS GENRL2 [PLUS K GENRL1]]]

[COND [EQUAL [PLUS K [CONS NIL [PLUS GENRL2 GENRL1]]]]

[CONS NIL [PLUS GENRL2 [PLUS K GENRL1]]]]

[*1]]

T]

FERTILIZE WITH [EQUAL [PLUS K [PLUS GENRL2 GENRL1]] [PLUS GENRL2 [PLUS K GENRL1]]].

```
THE THEOREM TO BE PROVED IS NOW:

COND COND CEQUAL CPLUS K CONS NIL CPLUS GENRL2 GENRL1]]]

CONS NIL CPLUS K CPLUS GENRL2 GENRL1]]]

T
[*1]]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [PLUS GENRL2 GENRL1] BY GENRL3.

THE GENERALIZED TERM IS:

```
[COND [EQUAL [PLUS K [CONS NIL GENRL3]] [CONS NIL [PLUS K GENRL3]]] T [*1]]

T
[*2]]
```

MUST TRY INDUCTION.

INDUCT ON K.

THE THEOREM TO BE PROVED IS NOW:

```
EAND

[COND [COND [EQUAL [PLUS NIL [CONS NIL GENRL3]] [CONS NIL [PLUS NIL GENRL3]]]

. T

. [*1]]

. T

. [*2]]

[IMPLIES
[COND
```

- [COND [EQUAL [PLUS K [CONS NIL GENRL3]] [CONS NIL [PLUS K GENRL3]]] T [*1]]
- T[*2]]

COND [COND [EQUAL [PLUS [CONS NIL K] [CONS NIL GENRL3]]]

[CONS NIL [PLUS [CONS NIL K] GENRL3]]]

T [*1]] T [*2]]]]

WHICH IS EQUIVALENT TO:

Ī

FUNCTION DEFINITIONS:

EPLUS CLAMBDA (X Y) COND X CONS NIL EPLUS (CDR X) Y) Y) Y) CTIMES CLAMBDA (X Y) COND X EPLUS Y CTIMES (CDR X) Y) 0) 0) 0 CIMPLIES CLAMBDA (X Y) COND X COND Y T NIL) T) 1) CAND CLAMBDA (X Y) COND X COND Y T NIL) NIL)

FERTILIZERS:

- *1 = [COND [EQUAL [TIMES N [PLUS M K]] [PLUS [TIMES N M] [TIMES N K]]] NIL T]
- *2 = [COND [EQUAL [PLUS K [PLUS GENRL2 GENRL1]]] [PLUS GENRL2 [PLUS K GENRL1]]]
 NIL
 T]

GENERALIZATIONS:

GENRL2 = [TIMES N M]

GENRL1 = [TIMES N K]

GENRL3 = [PLUS GENRL2 GENRL1]

PROFILE: [/[N],/ENR/ENRX,/G[M],/ENR/ENR/ENR [GENRL2],/ENR/ENR/ENRF,/G[K],/ENR/ENR]

[T 5 4]

TIME: 31.19 SECS.

[T 5 5] [16.18 18 JULY 1973]

THEOREM TO BE PROVED:

CEQUAL [TIMES N [TIMES M K]] [TIMES [TIMES N M] K]]

MUST TRY INDUCTION.

INDUCT ON N.

THE THEOREM TO BE PROVED IS NOW:

[AND
[EQUAL [TIMES NIL [TIMES M K]] [TIMES [TIMES NIL M] K]]
[IMPLIES
[EQUAL [TIMES N [TIMES M K]] [TIMES [TIMES N.M] K]]
[EQUAL [TIMES [CONS NIL N] [TIMES M K]] [TIMES [TIMES [CONS NIL N] M] K]]]]

WHICH IS EQUIVALENT TO:

COND CEQUAL CTIMES N CTIMES M K] CTIMES CTIMES N M] K]

EQUAL CPLUS CTIMES M K] CTIMES N CTIMES M K]]

CTIMES CPLUS M CTIMES N M]] K]]

FERTILIZE WITH CEQUAL CTIMES N CTIMES M K]] CTIMES CTIMES N M] K]].

THE THEOREM TO BE PROVED IS NOW:

```
[COND [EQUAL [PLUS [TIMES M K] [TIMES [TIMES N M] K]]

[TIMES [PLUS M [TIMES N M]] K]]

[*1]
```

GENERALIZE COMMON SUBTERMS BY REPLACING [TIMES N M] BY GENRL1.

THE GENERALIZED TERM IS:

[COND [EQUAL [PLUS [TIMES M K] [TIMES GENRL1 K]] [TIMES [PLUS M GENRL1] K]]

T
[*1]]

MUST TRY INDUCTION.

INDUCT ON M.

THE THEOREM TO BE PROVED IS NOW:

[*1]]]

```
COND

COND

COND

COND

CEQUAL (PLUS (TIMES NIL K) (TIMES GENRL1 K)) (TIMES (PLUS NIL GENRL1) K))

Cincles

Cond (Equal (Plus (Times M K) (Times Genrl1 K)) (Times (Plus M GENRL1) K))

T

Cond (Equal (Plus (Times (Cons Nil M) K) (Times Genrl1 K))

Cond (Equal (Plus (Times (Cons Nil M) K) (Times Genrl1 K))

T
```

WHICH IS EQUIVALENT TO:

[COND [EQUAL [PLUS [TIMES M K] [TIMES GENRL1 K]] [TIMES [PLUS M GENRL1] K]]
[COND [EQUAL [PLUS [PLUS K [TIMES M K]] [TIMES GENRL1 K]]
[PLUS K [TIMES [PLUS M GENRL1] K]]]

T [*1]]

Tj

FERTILIZE WITH [EQUAL [PLUS [TIMES M K] [TIMES GENRL1 K]] [TIMES [PLUS M GENRL1] K]].

THE THEOREM TO BE PROVED IS NOW:

[COND [COND [EQUAL [PLUS [PLUS K [TIMES M K]] [TIMES GENRL1 K]] [PLUS K [PLUS [TIMES M K] [TIMES GENRL1 K]]]

T [*1]]

[*2]]

GENERALIZE COMMON SUBTERMS BY REPLACING [TIMES GENRL1 K] BY GENRL2 AND [TIMES M K] BY GENRL3.

THE GENERALIZED TERM IS:

COND COND CEQUAL CPLUS (PLUS K GENRL3] GENRL2] CPLUS K EPLUS GENRL3 GENRL2]]]
T
[*1]]

T [*2]]

MUST TRY INDUCTION.

INDUCT ON K.

```
THE THEOREM TO BE PROVED IS NOW:
CAND
 ECOND
 .[COND [EQUAL [PLUS [PLUS NIL GENRL3] GENRL2] [PLUS NIL [PLUS GENRL3 GENRL2]]]
        [*1]]
 . T
 .[*2]]
 [IMPLIES
  ECOND
      [COND [EQUAL [PLUS EPLUS K GENRL3] GENRL2] [PLUS K EPLUS GENRL3 GENRL2]]]
            [*1]]
      T
      [*2]]
  ECOND [COND [EQUAL [PLUS [PLUS [CONS NIL K] GENRL3] GENRL2]
                     [PLUS [CONS NIL K] [PLUS GENRL3 GENRL2]]]
              [*1]]
        [*2]]]]
```

WHICH IS EQUIVALENT TO:

FUNCTION DEFINITIONS:

ETIMES [LAMBDA [X Y] [COND X EPLUS Y [TIMES [CDR X] Y]] 0]]]

EPLUS ELAMBDA [X Y] [COND X [CONS NIL [PLUS [CDR X] Y]] Y]]]

EIMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

EAND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

*1 = CCOND (EQUAL CTIMES N CTIMES M K)] CTIMES (TIMES N M) K)] NIL T)

*2 = [COND]

CEQUAL CPLUS (TIMES M K) (TIMES GENRL1 K)) (TIMES CPLUS M GENRL1) K))
NIL
T]

GENERALIZATIONS:

GENRL1 = [TIMES N M]

GENRL3 = [TIMES M K]

GENRL2 = [TIMES GENRL1 K]

PROFILE: [/[N],/ENR/ENRX,/G[M],/ENR/ENR/ENRF,/G
[K],/ENR/ENR.]

TIME: 25.25 SECS.

[T 5 6] [16.18 18 JULY 1973]

THEOREM TO BE PROVED:

[EVEN1 [DOUBLE N]]

MUST TRY INDUCTION.

INDUCT ON N.

THE THEOREM TO BE PROVED IS NOW:

[AND [EVEN1 [DOUBLE NIL]] [EVEN1 [DOUBLE [CONS NIL N]]]]

WHICH IS EQUIVALENT TO:

FUNCTION DEFINITIONS:

EDUUBLE CLAMBDA [X] [COND X [CONS NIL [CONS NIL [DOUBLE [CDR X]]]] 0]]]

EEVEN1 [LAMBDA [X] [COND X [COND [EVEN1 [CDR X]] NIL T] T]]]

EIMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

EAND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

PROFILE: CV [N] , V E N R V E N R . 1

TIWE: S'SE SECS'

[9 5 1]

[T 5 7] [16.18 18 JULY 1973]

THEOREM TO BE PROVED:

[EQUAL [HALF [DOUBLE N]] N]

MUST TRY INDUCTION.

INDUCT ON N.

THE THEOREM TO BE PROVED IS NOW:

[AND [EQUAL [HALF [DOUBLE NIL]] NIL]

[IMPLIES [EQUAL [HALF [DOUBLE N]] N]

[EQUAL [HALF [DOUBLE [CONS NIL N]]] [CONS NIL N]]]

WHICH IS EQUIVALENT TO:

Ī

FUNCTION DEFINITIONS:

CDOUBLE CLAMBDA [X] [COND X [CONS NIL [CONS NIL [DOUBLE [CDR X]]]] 0]]]

CHALF CLAMBDA [X] [COND X [COND [CDR X] [CONS NIL CHALF [CDR ECDR X]]]] 0] 0]]]

CIMPLIES CLAMBDA [X Y] [COND X [COND Y T NIL] T]]]

CAND CLAMBDA [X Y] [COND X [COND Y T NIL] NIL]]

PROFILE: [/[N],/ENR/ENR.]

TIME: 2.625 SECS.

THEOREM TO BE PROVED:

CIMPLIES (EVEN1 N] [EQUAL [DOUBLE [HALF N]] N]]

WHICH IS EQUIVALENT TO:

[COND CEVEN1 N] [EQUAL [DOUBLE [HALF N]] N] T]

MUST TRY INDUCTION.

(SPECIAL CASE REQUIRED)

INDUCT ON N.

. THE THEOREM TO BE PROVED IS NOW:

CAND

COND CEVEN1 NIL] CEQUAL [DOUBLE [HALF NIL]] NIL] T]

CAND

CCOND [EVEN1 [CONS NIL NIL]]

[EQUAL [DOUBLE [HALF [CONS NIL NIL]]] [CONS NIL NIL]]

-T]

EIMPLIES

[COND [EVEN1 N] [EQUAL [DOUBLE [HALF N]] N] T]

ECOND

[EVEN1 [CONS NIL [CONS NIL N]]]

[EQUAL [DOUBLE [HALF [CONS NIL [CONS NIL N]]]] [CONS NIL [CONS NIL N]]]

T]]]]

WHICH IS EQUIVALENT TO:

FUNCTION DEFINITIONS:

CEVEN1 [LAMBDA [X] [COND X [COND [EVEN1 [CDR X]] NIL T] T]]

CHALF CLAMBDA [X] [COND X [COND [CDR X] [CONS NIL [HALF [CDR [CDR X]]]] 0] 0]]]

CDOUBLE CLAMBDA [X] [COND X [CONS NIL [CONS NIL [DOUBLE [CDR X]]]] 0]]]

CIMPLIES CLAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

PROFILE: [/ENR/ENRS2[N],/ENR/ENR.]

TIME: 4.75 SECS.

IT 5 9] [16.19 18 JULY 1973]

THEOREM TO BE PROVED:

[EQUAL [DOUBLE N] [TIMES 2 N]]

WHICH IS EQUIVALENT TO:

- CEQUAL CDOUBLE NJ [PLUS N [PLUS N 0]]]

MUST TRY INDUCTION.

INDUCT ON N.

THE THEOREM TO BE PROVED IS NOW:

EAND
[EQUAL [DOUBLE NIL] [PLUS NIL [PLUS NIL 0]]]
[IMPLIES
[FOUND FROUDLE NIL 5PLUS N 5PLUS N

CEQUAL [DOUBLE N] [PLUS N [PLUS N 0]]]

CEQUAL [DOUBLE [CONS NIL N]] [PLUS [CONS NIL N] [PLUS [CONS NIL N] 0]]]]]

WHICH IS EQUIVALENT TO:

[CUND [EQUAL [DOUBLE N] [PLUS N [PLUS N 0]]]

[EQUAL [CONS NIL [DOUBLE N]] [PLUS N [CONS NIL [PLUS N 0]]]]

T]

FERTILIZE WITH [EQUAL [DOUBLE N] [PLUS N [PLUS N 0]]].

```
THE THEOREM TO BE PROVED IS NOW:
```

[COND EEQUAL [CONS NIL [PLUS N [PLUS N 0]]] [PLUS N [CONS NIL [PLUS N 0]]]]

T

[*1]

GENERALIZE COMMON SUBTERMS BY REPLACING [PLUS N 0] BY GENRL1.

THE GENERALIZED TERM IS:

COUND [EQUAL CONS NIL [PLUS N GENRL1]] [PLUS N [CONS NIL GENRL1]]] T [*1]]

MUST TRY INDUCTION.

INDUCT ON N.

THE THEOREM TO BE PROVED IS NOW:

[*1]]]]

```
[AND
[COND [EQUAL [CONS NIL [PLUS NIL GENRL1]] [PLUS NIL [CONS NIL GENRL1]]]
. T
. [*1]]
[IMPLIES
[COND [EQUAL [CONS NIL [PLUS N GENRL1]] [PLUS N [CONS NIL GENRL1]]] T [*1]]
[COND [EQUAL [CONS NIL [PLUS [CONS NIL N] GENRL1]]
[PLUS [CONS NIL N] [CONS NIL GENRL1]]
```

WHICH IS EQUIVALENT TO:

FUNCTION DEFINITIONS:

EDOUBLE CLAMBDA [X] [COND X [CONS NIL [CONS NIL [DOUBLE [CDR X]]]] 0]]]

ETIMES CLAMBDA [X Y] [COND X [PLUS Y [TIMES [CDR X] Y]] 0]]]

EPLUS CLAMBDA [X Y] [COND X [CONS NIL [PLUS [CDR X] Y]] Y]]]

EIMPLIES CLAMBDA [X Y] [COND X [COND Y T NIL] T]]]

EAND CLAMBDA [X Y] [COND X [COND Y T NIL] NIL]]

FERTILIZERS:

*1 = [COND [EQUAL [DOUBLE N] [PLUS N [PLUS N 0]]] NIL T]

GENERALIZATIONS:

GENRL1 = [PLUS N 0]

PROFILE: [/ENR/ENR(N],/ENR/ENRX,/G[N],/ENR/ENR.

TIME: 9.188 SECS.

[T 5 10] [16.19 18 JULY 1973]

THEOREM TO BE PROVED:

[EQUAL [DOUBLE N] [TIMES N 2]]

MUST TRY INDUCTION.

INDUCT ON N.

THE THEOREM TO BE PROVED IS NOW:

CAND [EQUAL [DOUBLE NIL] [TIMES NIL 2]]

[IMPLIES [EQUAL [DOUBLE N] [TIMES N 2]]

[EQUAL [DOUBLE [CONS NIL N]] [TIMES [CONS NIL N] 2]]]

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

CDOUBLE CLAMBDA [X] [COND X [CONS NIL [CONS NIL [DOUBLE [CDR X]]]] 0]]]

ETIMES CLAMBDA [X Y] [COND X [PLUS Y [TIMES [CDR X] Y]] 0]]]

EPLUS CLAMBDA [X Y] [COND X [CONS NIL [PLUS [CDR X] Y]] Y]]]

EIMPLIES CLAMBDA [X Y] [COND X [COND Y T NIL] T]]]

EAND CLAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

PROFILE: C/ [N] , / E N R / E N R .]

THEOREM TO BE PROVED:

[EQUAL [EVEN1 N] [EVEN2 N]]

MUST TRY INDUCTION.

(SPECIAL CASE REQUIRED)

INDUCT ON N.

THE THEOREM TO BE PROVED IS NOW:

LAND

[EQUAL [EVEN1 NIL] [EVEN2 NIL]]

[AND

[EQUAL [EVEN1 [CONS NIL NIL]] [EVEN2 [CONS NIL NIL]]]

[IMPLIES

CEQUAL CEVEN1 N] CEVEN2 N]]
CEQUAL CEVEN1 CCONS NIL CCONS NIL N]]] CEVEN2 CCONS NIL CCONS NIL N]]]]]

WHICH IS EQUIVALENT TO:

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FUNCTION DEFINITIONS:

CEVEN1 [LAMBDA [X] [COND X [COND [EVEN1 [CDR X]] NIL T] T]]]

CEVEN2 [LAMBDA [X] [COND X [COND [CDR X] [EVEN2 [CDR [CDR X]]] NIL] T]]]

CIMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

CAND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]

PROFILE: [/ S2 [N] , / E N R / E N R .]

TIME: 2.938 SECS.

[T 6 1] [16.2 18 JULY 1973]

THEOREM TO BE PROVED:

EGT [LENGTH [CONS A B]] [LENGTH B]]

WHICH IS EQUIVALENT TO:

EGT ECONS NIL ELENGTH BIJ ELENGTH BIJ

MUST TRY INDUCTION.

INDUCT ON B.

THE THEOREM TO BE PROVED IS NOW:

CAND [GT [CONS NIL [LENGTH NIL]] [LENGTH NIL]]

EIMPLIES [GT [CONS NIL [LENGTH 8]] [LENGTH B]]

[GT [CONS NIL [LENGTH [CONS B1 B]]] [LENGTH [CONS B1 B]]]]

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

CLENGTH CLAMBDA CX3 COND X CONS NIL CLENGTH CODR X333 0333

EGT ELAMBDA (X Y) COND X (COND Y EGT (CDR X) ECDR Y) TO NIL)]

EIMPLIES CLAMBDA (X Y) COND X COND Y T NIL) T)

EAND ELAMBDA (X Y) COND X COND Y T NIL) NIL)

PROFILE: [/ENR/ENR[B],/ENR/ENR.]

TIME: 3.125 SECS.

THEOREM TO BE PROVED:

EIMPLIES [AND EGT A B] EGT B C]] EGT A C]]

WHICH IS EQUIVALENT TO:

ECOND EGT A BJ [COND EGT B C] [GT A C] T] T]

MUST TRY INDUCTION.

INDUCT ON B, A AND C.

THE THEOREM TO BE PROVED IS NOW:

CAND

EAND ECOND EGT A NIL] ECOND EGT NIL C3 EGT A C3 T3 T3

EAND ECOND EGT NIL B1 ECOND EGT B C3 EGT NIL C3 T3 T3

COND EGT A B3 ECOND EGT B NIL3 EGT A NIL3 T3 T333

CIMPLIES

COND EGT A BJ [COND [GT B C] [GT A C] T] T]

[COND [GT [CONS A1 A] [CONS B1 B]]

[COND [GT [CONS B1 B] [CONS C1 C]] [GT [CONS A1 A] [CONS C1 C]] T]

T]]]

WHICH IS EQUIVALENT TO:

[COND [GT A B] [COND B [COND A T NIL] T] T]

MUST TRY INDUCTION.

INDUCT ON A AND B.

THE THEOREM TO BE PROVED IS NOW:

[AND CAND COND EGT NIL B] COND B COND NIL T NIL] T] T]

[COND EGT A NIL] COND NIL [COND A T NIL] T] T]

[IMPLIES COND EGT A B] COND B COND A T NIL] T] T]

[COND EGT CONS A2 A] CONS B2 B]

[COND [CONS B2 B] COND [CONS A2 A] T NIL] T]

T]]]

WHICH IS EQUIVALENT TO:

FUNCTION DEFINITIONS:

EGT CLAMBDA (X Y) COND X COND Y CGT CCDR X3 CCDR Y) TO NIL) TO

PROFILE: [/ENR/ENR [BAC], /ENR/ENR/ENR [AB], /ENR,]

TIME: 9.375 SECS.

[T 6 3] [16.2 18 JULY 1973]

THEOREM TO BE PROVED:

CIMPLIES [GT A B] [NOT [GT B A]]]

WHICH IS EQUIVALENT TO:

[COND [GT A B] [COND [GT B A] NIL T] T]

MUST TRY INDUCTION.

INDUCT ON 8 AND A.

THE THEOREM TO BE PROVED IS NOW:

EAND CAND COND CGT A NIL] CCOND CGT NIL A] NIL T] T]

COND CGT NIL B] CCOND CGT B NIL] NIL T] T]

CIMPLIES CCOND CGT A B] CCOND CGT B A] NIL T] T]

COND CGT CONS A1 A] CCONS B1 B]

COND CGT CONS B1 B] CCONS A1 A] NIL T]

T]]]

WHICH IS EQUIVALENT TO:

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FUNCTION DEFINITIONS:

EGT ELAMBDA EX Y] [COND X [COND Y [GT [CDR X] [CDR Y]] T] NIL]]]

ENOT ELAMBDA EX] [COND X NIL T]]]

EIMPLIES ELAMBDA [X Y] [COND X [COND Y T NIL] T]]]

EAND ELAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

PROFILE: [/ENR/ENR[BA],/ENR/ENR.]

TIME: 4.375 SECS.

ET 6 4] [16.21 18 JULY 1973]

THEOREM TO BE PROVED:

ELTE A [APPEND B A]]

MUST TRY INDUCTION.

INDUCT ON B.

THE THEOREM TO BE PROVED IS NOW:

[AND [LTE A [APPEND NIL A]]
[IMPLIES [LTE A [APPEND B A]] [LTE A [APPEND [CONS B1 B] A]]]

WHICH IS EQUIVALENT TO:

[COND [LTE A A]
[COND [LTE A [APPEND B A]] [LTE A [CONS B1 [APPEND B A]]] T]
NIL]

(WORK ON FIRST CONJUNCT ONLY)

MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

COND [AND LITE NIL NIL] [IMPLIES LITE A A] LITE CONS A1 A] [CONS A1 A]]]

COND LITE A2 [APPEND B A2]] LITE A2 [CONS B1 [APPEND B A2]]] T]

NIL]

WHICH IS EQUIVALENT TO:

[COND [LTE A2 [APPEND 8 A2]] [LTE A2 [CONS B1 [APPEND B A2]]] T]

GENERALIZE COMMON SUBTERMS BY REPLACING [APPEND B A2] BY GENRL1.

THE GENERALIZED TERM IS:

[COND [LTE A2 GENRL1] [LTE A2 [CONS 31 GENRL1]] T]

MUST TRY INDUCTION.

INDUCT ON GENRL1 AND A2.

THE THEOREM TO BE PROVED IS NOW:

COND CLTE AS NIL CLTE AS CONS B1 NIL TO TO COND CLTE NIL GENRL1 CLTE NIL CONS B1 GENRL1 TTO

[IMPLIES [COND [LTE A2 GENRL1] [LTE A2 [CONS B1 GENRL1]] T]

| COND [LTE [CONS A21 A2] [CONS GENRL11 GENRL1]]

| [LTE [CONS A21 A2] [CONS B1 [CONS GENRL11 GENRL1]]]

WHICH IS EQUIVALENT TO:

[COND [LTE A2 GENRL1] [COND [LTE A2 [CONS GENRL11 GENRL1]] T]

T]

MUST TRY INDUCTION.

INDUCT ON GENRL1 AND A2.

THE THEOREM TO BE PROVED IS NOW:

CMAI

CAND COOND CLTE A2 NIL]

. [COND ELTE A2 [CONS B1 NIL]] ELTE A2 [CONS GENRL11 NIL]] []

T]

LCOND CLTE NIL GENRL1]

[COND [LTE NIL [CONS 31 GENRL1]] [LTE NIL [CONS GENRL11 GENRL1]] T]

T]]

EIMPLIES

[COND [LTE A2 GENRL1]

[COND [LTE A2 [CONS B1 GENRL1]] [LTE A2 [CONS GENRL11 GENRL1]] T]

T]

[COND [LTE [CONS A22 A2] [CONS GENRL12 GENRL1]]

[COND [LTE [CONS A22 A2] [CONS B1 [CONS GENRL12 GENRL1]]]

[LTE [CONS A22 A2] [CONS GENRL11 [CONS GENRL12 GENRL1]]]

T]

TIII

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FUNCTION DEFINITIONS:

CAPPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]]]]]

CLTE CLAMBDA [X Y] [COND X [COND Y [LTE [CDR X] [CDR Y]] NIL] T]]]

CIMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

EAND CLAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

GENERALIZATIONS:

GENRL1 = [APPEND B A2]

PROFILE: [/[B], / E N R / E N R & [A], / E N R / E N R G [GENRL1 A2], / E N R / E N R / E N R G [GENRL1 A2]

TIME: 17.75 SECS.

[T 6 5] [16.21 18 JULY 1973]

THEOREM TO BE PROVED:

COR CLTE A BJ CLTE B AJJ

WHICH IS EQUIVALENT TO:

CCOND [LTE A B] T [LTE B A]]

MUST TRY INDUCTION.

INDUCT ON B AND A.

THE THEOREM TO BE PROVED IS NOW:

CAND

CAND COOND (LTE A NIL) T (LTE NIL A)] (COND (LTE NIL B) T (LTE B NIL)]] [IMPLIES

[COND [LTE A B] T [LTE B A]]
[COND [LTE [CONS A1 A] [CONS B1 B]] T [LTE [CONS B1 B] [CONS A1 A]]]]]

WHICH IS EQUIVALENT TO:

FUNCTION DEFINITIONS:

ELTE (LAMBDA (X Y) [COND X [COND Y [LTE [CDR X] [CDR Y]] NIL] T]]]

EOR CLAMBDA (X Y) [COND X T [COND Y T NIL]]]]

EAND CLAMBDA (X Y) [COND X [COND Y T NIL] NIL]]]

EIMPLIES (LAMBDA (X Y) [COND X [COND Y T NIL] T]]

PROFILE: [/ENR/ENR[BA],/ENR/ENR.]

TIME: 3.625 SECS:

[T 6 6] [16.21 18 JULY 1973]

THEOREM TO BE PROVED: .

COR EGT A B] [OR [GT B A] [EQUAL [LENGTH A] [LENGTH B]]]]

WHICH IS EQUIVALENT TO:

COND [GT A B] T [COND [GT B A] T [EQUAL [LENGTH A] [LENGTH B]]]]

MUST TRY INDUCTION.

INDUCT ON A AND B.

THE THEOREM TO BE PROVED IS NOW:

CAND

LAND ECOND EGT NIL BJ T ECOND EGT B NILJ T EEQUAL ELENGTH NILJ ELENGTH BJJJJ

COND EGT A NILJ T ECOND EGT NIL AJ T EEQUAL ELENGTH AJ ELENGTH NILJJJJJ

EIMPLIES ECOND EGT A BJ T ECOND EGT B AJ T EEQUAL ELENGTH AJ ELENGTH BJJJJ

ECOND EGT ECONS A1 AJ ECONS B1 BJJ

COND EGT CONS 81 B] CONS A1 A]]

[EQUAL [LENGTH [CONS A1 A]] [LENGTH [CONS B1 B]]]]]]

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

EGT CLAMBDA (X Y) COND X (COND Y (GT (CDR X) (CDR Y)) T) NIL)]]

ELENGTH (LAMBDA (X) (COND X (CONS NIL (LENGTH (CDR X))) 0)))

EOR CLAMBDA (X Y) (COND X T (COND Y T NIL)))

EAND CLAMBDA (X Y) (COND X (COND Y T NIL) NIL)])

EIMPLIES (LAMBDA (X Y) (COND X (COND Y T NIL) T))

PROFILE: [/ENR/ENR[AB],/ENR/ENR.]

TIME: 7.875 SECS.

THEOREM TO BE PROVED:

CEQUAL EMONOT2P A] [MONOT1 A]]

WHICH IS EQUIVALENT TO:

LCOND A CEQUAL EMONOT2 TCAR AJ CCDR AJJ EMONOT1 AJJ TJ

MUST TRY INDUCTION.

(SPECIAL CASE REQUIRED)

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

LAND

[COND NIL [EQUAL [MONOT2 [CAR NIL] [CDR NIL]] [MONOT1 NIL]] T] EAND

ECOND [CONS A1 NIL]

- [EQUAL [MONOT2 [CAR [CONS A1 NIL]] [CDR [CONS A1 NIL]]]
- [MONOT1 [CONS A1 NIL]]]
- T]
- LIMPLIES

CCOND

- [CONS A2 A]
- [EQUAL [MONOT2 [CAR [CONS A2 A]] [CDR [CONS A2 A]]] [MONOT1 [CONS A2 A]]]
- [] ECOND

[CONS A1 [CONS A2 A]]

[EQUAL [MONOT2 [CAR [CONS A1 [CONS A2 A]]] [CDR [CONS A1 [CONS A2 A]]]]

. [MONOT1 [CONS A1 [CONS A2 A]]]]

WHICH IS EQUIVALENT TO:

CCOND

CCOND

CCOND

CCOND

CCOND (EQUAL A2 [CAR A])

CCOND (EQUAL [MONOT2 A2 A] [MONOT1 A])

CCOND (EQUAL A1 A2] (EQUAL [MONOT2 A1 A] [MONOT1 A]] T]

CCOND [MONOT2 A2 A] T [COND [EQUAL A1 A2] [COND [MONOT2 A1 A] NIL T] T]]]

FERTILIZE WITH [EQUAL A2 [CAR A]].

THE THEOREM TO BE PROVED IS NOW:

WHICH IS EQUIVALENT TO:

```
. [*1]]
. [COND EMONOT2 A2 A]
. T
. [COND [EQUAL A1 A2] [COND [MONOT2 A1 A] [EQUAL A2 [CAR A]] T] T]]
. NIL]
T]
```

FERTILIZE WITH [EQUAL [MONOT2 [CAR A] A] [MONOT1 A]].

THE THEOREM TO BE PROVED IS NOW:

```
ECUND
A
CCOND
.[COND
.[COND
COND [COND [EQUAL A1 [CAR A]] [EQUAL [MONOT2 A1 A] [MONOT2 [CAR A] A]] T]
. . T
. . [*2]]
. T
. [*1]]
.[CUND [MONOT2 A2 A]
. T
. [COND [EQUAL A1 A2] [COND [MONOT2 A1 A] [EQUAL A2 [CAR A]] T] T]]
.NIL]
T]
```

FERTILIZE WITH [EQUAL A1 [CAR A]].

THE THEOREM TO BE PROVED IS NOW:

```
ECOND
A
[COND
.COND
.COND [COND [COND [EQUAL [MONOT2 A1 A] [MONOT2 A1 A]] T [*3]] T [*2]] T [*1]]
.COND [MONOT2 A2 A]
. T
. [COND [EQUAL A1 A2] [COND [MONOT2 A1 A] [EQUAL A2 [CAR A]] T] T]]
.NIL]
T]
```

```
WHICH IS EQUIVALENT TO:
ECOND A
      ECOND EMONOT2 A2 A]
            COND [EQUAL A1 A2] [COND [MONOT2 A1 A] [EQUAL A2 [CAR A3] T] T]]
      T٦
FERTILIZE WITH LEQUAL A1 A2].
THE THEOREM TO BE PROVED IS NOW:
CCOAD
 [COND [MONOT2 A2 A] T [COND [COND [MONOT2 A1 A] [EQUAL A1 [CAR A]] T] T [*4]]]
MUST TRY INDUCTION.
INDUCT ON A.
THE THEOREM TO BE PROVED IS NOW:
LAND
 ECOND NIL
       ECOND EMONOT2 A2 NIL]
             [COND [COND [MONOT2 A1 NIL] [EQUAL A1 [CAR NIL]] T] T [*4]]]
       T]
 CIMPLIES
```

[COND [COND [MONOT2 A1 A] [EQUAL A1 [CAR A]] T] T [*4]]]

ECOND A

[COND [MONOT2 A2 A]

```
T]
  ECOND
      ECONS A3 A3
      [COND EMONOT2 A2 ECONS A3 A]]
            ECOND [COND [MONOT2 A1 [CONS A3 A]] [EQUAL A1 [CAR [CONS A3 A]]] T]
                  [*4]]]
      Tjjj
WHICH IS EQUIVALENT TO:
Γ
FUNCTION DEFINITIONS:
EMONOT2P [LAMBDA [X] [COND X EMONOT2 [CAR X] [CDR X]] T]]]
EMONOT2
     CLAMBDA EX Y] [COND Y [COND [EQUAL X [CAR Y]] [MONOT2 X [CDR Y]] NIL] T]]]
[MONOT1
 CLAMBDA
 [X]
  COND
     COND [CDR X] [COND [EQUAL [CAR X] [CAR [CDR X]]] [MONOT1 [CDR X]] NIL] T]
     Tlll
[CARARG UNDEF]
[CDRARG UNDEF]
LIMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]
FERTILIZERS:
*1 = [COND]
      [EQUAL A2 [CAR A]] -
      CCOND [MONOT2 A2 A] T [COND [EQUAL A1 A2] [COND [MONOT2 A1 A] NIL T] T]]]
*2 = [COND [EQUAL [MONOT2 [CAR A] A] [MONOT1 A]] NIL T]
```

*3 = [COND [EQUAL A1 [CAR A]] NIL T]

*4 = [COND [EQUAL A1 A2] NIL T]

PROFILE: [/ENR/ENR/ENRS1 [A],/ENR/ENR/ENRF,/NR/ENRF,/X,/ENR/ENRF,/[A],/ENR/ENR.]

TIME: 33.06 SECS.

THEOREM TO BE PROVED:

[ORDERED [SORT A]]

MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

[AND [URDERED [SORT NIL]]
[IMPLIES [ORDERED [SORT A]] [ORDERED [SORT [CONS A1 A]]]]]

WHICH IS EQUIVALENT TO:

[COND CORDERED [SORT A]] [ORDERED [ADDTOLIS A1 [SORT A]]] T]

GENERALIZE COMMON SUBTERMS BY REPLACING [SORT A] BY GENEL1.

THE GENERALIZED TERM IS:

[COND [ORDERED GENRL1] [ORDERED [ADDTOLIS A1 GENRL1]] T]

MUST TRY INDUCTION.

(SPECIAL CASE REQUIRED)

INDUCT ON GENRL1.

THE THEOREM TO BE PROVED IS NOW:

EAND [COND [ORDERED NIL] [ORDERED [ADDTOLIS A1 NIL]] T] EAND

[COND [ORDERED [CONS GENRL11 NIL]]

CORDERED CADDTOLIS A1 [CONS GENEL11 NIL]]

LIMPLIES [COND [ORDERED [CONS GENRL12 GENRL1]]

[ORDERED [ADDTOLIS A1 [CONS GENRL12 GENRL1]]]

[COND [ORDERED [CONS GENRL11 [CONS GENRL12 GENRL1]]]

[ORDERED [ADDTOLIS A1 [CONS GENRL11 [CONS GENRL12 GENRL1]]]]

Tjjj

WHICH IS EQUIVALENT TO:

COUND CLTE A1 GENRL113 T CLTE GENRL11 A133

MUST TRY INDUCTION.

INDUCT ON GENRL11 AND A1.

```
[AND [AND [COND [LTE A1 NIL] T [LTE NIL A1]]
          [COND [LTE NIL GENRL11] T [LTE GENRL11 NIL]]]
     [IMPLIES [COND [LTE A1 GENRL11] T [LTE GENRL11 A1]]
              [COND [LTE [CONS A11 A1] [CONS GENRL111 GENRL11]]
                    [LTE [CONS GENRL111 GENRL11] [CONS A11 A1]]]]
WHICH IS EQUIVALENT TO:
FUNCTION DEFINITIONS:
ESORT ELAMBDA [X] [COND X [ADDTOLIS [CAR X] [SORT [CDR X]]] NIL]]]
CORDERED
 ELAMBDA
  [X]
  ECOND
      LCOND [CDR X] [COND [LTE [CAR X] [CAR [CDR X]]] [ORDERED [CDR X]] NIL] T]
      Tjjj
ELTE CLAMBDA [X Y] [COND X [COND Y [LTE [CDR X] [CDR Y]] NIL] T]]]
EADDTOLIS
 ELAMBDA
    [X X]
    ECOND Y
          COND [LTE X [CAR Y]] [CONS X Y] [CONS [CAR Y] [ADDTOLIS X [CDR Y]]]]
          [CONS X NIL]]]
CIMPLIES CLAMBDA EX YD COND X COND Y T NILD TDD
[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]
```

GENERALIZATIONS:

THE THEOREM TO BE PROVED IS NOW:

GENRL1 = [SORT A]

TIME: 57.06 SECS.

THEOREM TO BE PROVED:

CIMPLIES [AND [MONOT1 A] [MEMBER B A]] [EQUAL [CAR A] B]]

WHICH IS EQUIVALENT TO:

ECOND [MONOT1 A] [COND [MEMBER B A] [EQUAL [CAR A] B] T] T]

MUST TRY INDUCTION.

(SPECIAL CASE REQUIRED)

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

[AND [MONOT1 NIL] [COND [MEMBER B NIL] [EQUAL [CAR NIL] B] T] T] [AND

[COND [MONOT1 [CONS A1 NIL]]

[COND [MEMBER B [CONS A1 NIL]] [EQUAL [CAR [CONS A1 NIL]] B] T]

· IJ Clmplies [COND [MONOT1 [CONS A2 A]]

[COND [MEMBER B [CONS A2 A]] [EQUAL [CAR [CONS A2 A]] B] T]

[COND [MONOT1 [CONS A1 [CONS A2 A]]]

[COND [MEMBER B [CONS A1 [CONS A2 A]]]

[EQUAL [CAR [CONS A1 [CONS A2 A]]] B]

T]]]

```
WHICH IS EQUIVALENT TO:
ECOND
 [COND [EQUAL B A1] [EQUAL A1 B] T]
 COND
 . A
 . ECOND
    CEQUAL A2 [CAR A]]
    ECUND EMONOT1 A]
          [COND [EQUAL B A2]
                 [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
                 [COND [MEMBER B A]
                       [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
                       TJ]
          T]
    11
 . ECOND [EQUAL B A2]
        [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
        T \supset \mathcal{I}
 NILI
FERTILIZE WITH [EQUAL B A1].
THE THEOREM TO BE PROVED IS NOW:
ECUND
 [COND [EQUAL B B] T [*1]]
 ECOND
 . A
 . ECUND
    [EQUAL A2 [CAR A]]
    ECOND [MONOT1 A]
          [COND [EQUAL B A2]
                 [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
                 CCOND EMEMBER 8 AJ
                       COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
                       T]]
          T]
    T]
 .[COND [EQUAL B A2]
        [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
        Tll
 NIL]
```


FERTILIZE WITH [EQUAL A2 [CAR A]].

THE THEOREM TO BE PROVED IS NOW:

```
ECOND
A
ECOND
```

```
.COND
.COND
.[MONOT1 A]
.[COND
. [EQUAL B [CAR A]]
. . [EQUAL B [CAR A] B] [COND [EQUAL A1 [CAR A]] [EQUAL A1 B] T] T]
. . [COND [MEMBER B A]
. . [COND [MEMBER B A]
. . [COND [EQUAL [CAR A] B] [COND [EQUAL A1 [CAR A]] [EQUAL A1 B] T] T]
. T]
. T]
.T
.[*2]]
[COND [EQUAL B A2]
```

[COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]

FERTILIZE WITH CEQUAL B [CAR AD].

T]]

```
ECUND
Α
ECOND
 . ECOND
 . [MONOT1 A]
 . rcond
 . . COND COND CEQUAL B B] COND CEQUAL A1 B] CEQUAL A1 B] T] T] T [*3]]
      [COND [MEMBER B A]
            [COND [EQUAL [CAR A] B] [COND [EQUAL A1 [CAR A]] [EQUAL A1 B] T] T]
            T]
     [EQUAL B [CAR A]]]
 . .NIL]
 . T]
 . T
 .[*2]]
 [COND [EQUAL B A2]
       COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
       T1]
WHICH IS EQUIVALENT TO:
ECUND
Α
 [COND
 . ECOND
 . [MONOT1 A]
 . rcond
      [MEMBER B A]
      [COND [EQUAL [CAR A] B]
            [COND [EQUAL A1 [CAR A]] [COND [EQUAL A1 B] T [EQUAL B [CAR A]]] T]
            T]
      []
  T D
 . T
 .[*2]]
 [COND [EQUAL B A2]
       CCOND EEQUAL A2 B] FCOND EEQUAL A1 A2] EEQUAL A1 B] T] T]
       Tll
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FERTILIZE WITH [EQUAL [CAR A] B].

THE THEOREM TO BE PROVED IS NOW:

```
THE THEOREM TO BE PROVED IS NOW:
ECOND
Α
 ECOND
 . ECOND
   [MUNOT1 A]
    [CUND [MEMBER B A]
          COND COND EQUAL A1 B] COND EQUAL A1 B] T EQUAL B B]] T] T [*4]]
    T]
 . T
 .[*2]]
 [COND [EQUAL B A2]
       ECOND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
       T]]
WHICH IS EQUIVALENT TO:
ECOND A
      [COND [EQUAL B A2]
            [COND [EQUAL A2 B] [COND [EQUAL A1 A2] [EQUAL A1 B] T] T]
FERTILIZE WITH [EQUAL B A2].
THE THEOREM TO BE PROVED IS NOW:
ECOND A
      [COND [COND [EQUAL B B] [COND [EQUAL A1 B] [EQUAL A1 B] T] T] T [*5]]]
WHICH IS EQUIVALENT TO:
```

Τ

FUNCTION DEFINITIONS:

EMONOT1 ELAMBDA EXJ ECOND X

COND [CDR X] [COND [EQUAL [CAR X] [CDR X]]] [MONOT1 [CDR X]] NIL] T

EMEMBER

CLAMBDA [X Y] [COND Y COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]

EAND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]

EIMPLIES CLAMBDA CX Y3 COND X COND Y T NIL3 T33

[CARARG UNDEF]

FERTILIZERS:

*1 = [COND [EQUAL B A1] NIL T]

*2 = [COND [EQUAL A2 [CAR A]] NIL T]

*3 = [COND]

[EQUAL B [CAR A]]

NIL

CCOND [MEMBER B A]

[COND [EQUAL [CAR A] B] [COND [EQUAL A1 [CAR A]] [EQUAL A1 B] T] T]

*4 = [COND [EQUAL [CAR A] B] NIL T]

*5 = [COND [EQUAL B A2] NIL T]

PROFILE: [/ENR/ENRS1 [A], /ENR/ENR/ENRF, /ENR/ENRF, / ENR/ENR/ENR.]

TIME: 49.63 SECS.

[T 6 10] [16.26 18 JULY 1973]

THEOREM TO BE PROVED:

[LTE [CDRN A B] B]

MUST TRY INDUCTION.

INDUCT ON A AND B.

THE THEOREM TO BE PROVED IS NOW:

CAND

[AND [LTE [CDRN NIL B] B] [LTE [CDRN A NIL] NIL]]
[IMPLIES [LTE [CDRN A B] B] [LTE [CDRN [CONS A1 A] [CONS B1 B]] [CONS B1 B]]]

WHICH IS EQUIVALENT TO:

CCOND CLTE B B3 [COND [LTE [CDRN A B] B] [LTE [CDRN A B] [CONS B1 B]] T3 NIL3

(WORK ON FIRST CONJUNCT ONLY)

MUST TRY INDUCTION.

INDUCT ON B.

THE THEOREM TO BE PROVED IS NOW:

COND CAND (LTE NIL NIL) [IMPLIES (LTE B 3) (LTE (CONS B2 B) (CONS B2 B)]]]

COND (LTE (CDRN A B3) B3) (LTE (CDRN A B3) (CONS B1 B3)) T)

NIL)

WHICH IS EQUIVALENT TO:

CCOND [LTE ECDRN A 83] B3] [LTE [CDRN A 83] [CONS 81 83]] T]

GENERALIZE COMMON SUBTERMS BY REPLACING [CDRN A B3] BY GENRL1.

THE GENERALIZED TERM IS:

CCUND LLTE GENRL1 B3] [LTE GENRL1 [CONS B1 B3]] T]

MUST TRY INDUCTION.

INDUCT ON B3 AND GENRL1.

THE THEOREM TO BE PROVED IS NOW:

[AND [AND [COND [LTE GENRL1 NIL] [LTE GENRL1 [CONS B1 NIL]] T]

[COND [LTE NIL B3] [LTE NIL [CONS B1 B3]] T]]

[IMPLIES [COND [LTE GENRL1 B3] [LTE GENRL1 [CONS B1 B3]] T]

[COND [LTE [CONS GENRL11 GENRL1] [CONS B31 B3]] [LTE [CONS GENRL11 GENRL1] [CONS B1 [CONS B31 B3]]]

WHICH IS EQUIVALENT TO:

[COND [LTE GENRL1 B3] CCOND [LTE GENRL1 [CONS B1 93]] [LTE GENRL1 [CONS B31 93]] T] T

MUST TRY INDUCTION.

INDUCT ON B3 AND GENRL1.

THE THEOREM TO BE PROVED IS NOW:

EAND CAND [COND [LTE GENRL1 NIL] [COND [LTE GENRL1 [CONS B1 NIL]] [LTE GENRL1 [CONS B31 NIL]] T] [COND [LTE NIL B3] [COND [LTE NIL [CONS B1 B3]] [LTE NIL [CONS B31 B3]] T] T]] [IMPLIES [COND [LTE GENEL1 B3] COND [LTE GENRL1 [CONS B1 B3]] [LTE GENRL1 [CONS R31 B3]] T] Т٦ [COND [LTE [CONS GENRL12 GENRL1] [CONS B32 B3]] [COND [LTE [CONS GENRL12 GENRL1] [CONS B1 [CONS B32 B3]]] [LTE [CONS GENRL12 GENRL1] [CONS B31 [CONS B32 B3]]]

Tjjj

TJ

WHICH IS EQUIVALENT TO:

FUNCTION DEFINITIONS:

CCDRN CLAMBDA (X Y) [COND Y [COND X [CDRN [CDR X] [CDR Y]] Y] NIL]]]

ELTE CLAMBDA [X Y] [COND X [COND Y [LTE [CDR X] [CDR Y]] NIL] T]]]

EAND CLAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

EIMPLIES CLAMBDA [X Y] [COND X [COND Y T NIL] T]]

GENERALIZATIONS:

GENRL1 = [CDRN A B3]

PROFILE: [/ [A B] , / E N R / E N R & [B] , / E N R / E N R / E N R G [B3 GENRL 1] , / E N R / E N R / E N R G [B3 GENRL 1] , / E N R / E N R .]

TIME: 18.06 SECS.

[T 6 12] [16.26 18 JULY 1973]

THEOREM TO BE PROVED:

[EQUAL [LENGTH A] [LENGTH [SORT A]]]

MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

[AND [EQUAL [LENGTH NIL] [LENGTH [SORT NIL]]]

[IMPLIES [EQUAL [LENGTH A] [LENGTH [SORT A]]]

[EQUAL [LENGTH [CONS A1 A]] [LENGTH [SORT [CONS A1 A]]]]]

WHICH IS EQUIVALENT TO:

COND DEQUAL CLENGTH AD CLENGTH [SORT ADDITIONS OF ADDITIO

FERTILIZE WITH [EQUAL [LENGTH A] [LENGTH [SORT A]]].

THE THEOREM TO BE PROVED IS NOW:

[CUND [EQUAL [CONS N]L [LENGTH [SORT A]]] [LENGTH [ADDTOLIS 41 [SORT A]]]]

T
[*1]]

GENERALIZE COMMON SUBTERMS BY REPLACING [SORT A] BY GENRL1.

THE GENERALIZED TERM IS:

[COND [EQUAL [CONS NIL [LENGTH GENRL1]] [LENGTH [ADDTOLIS A1 GENRL1]]] T [*1]]

MUST TRY INDUCTION.

INDUCT ON GENRL1.

THE THEOREM TO BE PROVED IS NOW:

CAND

[COND [EQUAL [CONS NIL [LENGTH NIL]] [LENGTH [ADDTOLIS A1 NIL]]] T [*1]]

[IMPLIES

[COND [EQUAL [CONS NIL [LENGTH GENRL1]] [LENGTH [ADDTOLIS A1 GENRL1]]]

. T

. [*1]]

[COND [EQUAL [CONS NIL [LENGTH [CONS GENRL11 GENRL1]]]

[LENGTH [ADDTOLIS A1 [CONS GENRL11 GENRL1]]]]

T

[*1]]]

WHICH IS EQUIVALENT TO:

ľ

FUNCTION DEFINITIONS:

[[ENGTH [LAMBDA [X] [COND X [CONS NIL [LENGTH [CDR X]]] 0]]]

ESORT ELAMBDA [X] [COND X [ADDTOLIS [CAR X] [SORT [CDR X]]] NIL]]]

[ADDTOLIS [LAMBDA [X Y]

ECOND Y

[COND [LTE X [CAR Y]] [CONS X Y] [CONS [CAR Y] [ADDTOLIS X [CDR Y]]]] [CONS X NIL]]]]

ELTE [LAMBDA [X Y] [COND X [COND Y [LTE [CDR X] [CDR Y]] NIL] T]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

EAND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]

FERTILIZERS:

*1 = ECOND [EQUAL [LENGTH A] [LENGTH [SORT A]]] NIL T]

GENERALIZATIONS:

GENRL1 = [SORT A]

PROFILE: [/ [A] , / E N R / E N R X , / G [GENRL1] , / E N R / E N R .]

TIME: 13.0 SECS.

[T 6 14] [16.26 18 JULY 1973]

THEOREM TO BE PROVED:

CIMPLIES [ORDERED A] [EQUAL A [SORT A]]]

WHICH IS EQUIVALENT TO:

[COND [ORDERED A] [EQUAL A [SORT A]] T]

MUST TRY INDUCTION.

(SPECIAL CASE REQUIRED)

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

EAND
[COND [ORDERED NIL] [EQUAL NIL [SORT NIL]] T]
[AND

COND [ORDERED [CONS A1 NIL]] [EQUAL [CONS A1 NIL] [SORT [CONS A1 NIL]]] TJ [IMPLIES

[COND [ORDERED [CONS A2 A]] [EQUAL [CONS A2 A] [SORT [CONS A2 A]]] T]
[COND [ORDERED [CONS A1 [CONS A2 A]]]
[EQUAL [CONS A1 [CONS A2 A]] [SORT [CONS A1 [CONS A2 A]]]]
T]]]]

FERTILIZE WITH [EQUAL [CONS A2 A] [ADDTOLIS A2 [SORT A]]].

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THE THEOREM TO BE PROVED IS NOW:
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WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

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CORDERED
CLAMBDA
CXJ
CCOND
```

ECOND [CDR X] [COND [LTE ECAR X] [CAR [CDR X]]] [ORDERED [CDR X]] NIL] T]
TJ]

ELTE ELAMBDA EX Y3 [COND X ECOND Y ELTE [CDR X3 [CDR Y3] NIL] T3]3

ESORT (LAMBDA (X) (COND X (ADDTOLIS (CAR X) (SORT (CDR X)) NIL))

CIMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]

COND Y

[COND [LTE X [CAR Y]] [CONS X Y] [CONS [CAR Y] [ADDTOLIS X [CDR Y]]]] [CONS X NIL]]]

EAND CLAMBDA EX Y3 COOND X COOND Y T NIL3 NIL333

FERTILIZERS:

*1 = [COND [EQUAL [CONS A2 A] [ADDTOLIS A2 [SORT A]]] NIL T]

TIME: 20.25 SECS.

[T 6 15] [16.27 18 JULY 1973]

THEOREM TO BE PROVED:

[IMPLIES [ORDERED [APPEND A B]] [ORDERED A]]

WHICH IS EQUIVALENT TO:

CCOND CORDERED [APPEND A B]] [ORDERED A] []

MUST TRY INDUCTION.

(SPECIAL CASE REQUIRED)

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

[AND COUND CORDERED CAPPEND NIL B]] [ORDERED NIL] T]

[AND COND CORDERED CAPPEND CONS A1 NIL] B]] [ORDERED CONS A1 NIL]] T]

[IMPLIES COND CORDERED CAPPEND CONS A2 A] B]] [ORDERED CONS A2 A]] T]

[COND CORDERED CAPPEND CONS A1 CONS A2 A]] B]]

[ORDERED CONS A1 CONS A2 A]]]

WHICH IS EQUIVALENT TO:

FUNCTION DEFINITIONS:

[APPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]]]

CORDERED CLAMBDA CXJ CCOND

[COND [CDR X] [COND [LTE [CAR X] [CAR [CDR X]]] [ORDERED [CDR X]] NIL] T]

ELTE CLAMBDA (X Y) COND X COND Y CLTE COR X) COR Y) NIL) T)))

CIMPLIES CLAMBDA (X Y) COND X COND Y T NIL) T))

CAND CLAMBDA (X Y) COND X COND Y T NIL) NIL))

PROFILE: [/ENR/ENRS1[A],/ENR/ENR.]

TIME: 14.69 SECS.

[T 6 16] [16.27 18 JULY 1973]

THEOREM TO BE PROVED:

CIMPLIES CORDERED [APPEND A B]] [ORDERED B]]

WHICH IS EQUIVALENT TO:

[COND CORDERED [APPEND A B]] [ORDERED B] T]

MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

[AND [COND [ORDERED [APPEND NIL B]] [ORDERED B] T]
[IMPLIES [COND [ORDERED [APPEND A B]] [ORDERED B] T]
[COND [ORDERED [APPEND [CONS A1 A] B]] [ORDERED B] T]]]

WHICH IS EQUIVALENT TO:

[COND [ORDERED [APPEND A B]] T [COND [APPEND A B] T [ORDERED B]]]

MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

CAND [COND [ORDERED [APPEND NIL B]] T [COND [APPEND NIL B] T [ORDERED B]]]

[IMPLIES [COND [ORDERED [APPEND A B]] T [COND [APPEND A B]] T [ORDERED B]]]

[COND [ORDERED [APPEND [CONS A2 A] B]]

[COND [APPEND [CONS A2 A] B] T [ORDERED B]]]]

WHICH IS EQUIVALENT TO:

CCOND CORDERED BJ T CCOND B T NILJJ

MUST TRY INDUCTION.

(SPECIAL CASE REQUIRED)

INDUCT ON B.

THE THEOREM TO BE PROVED IS NOW:

CAND COND CORDERED NIL] T COND NIL T NIL]]

[AND COND CORDERED CONS B1 NIL]] T COND CONS B1 NIL] T NIL]]

[IMPLIES COND CORDERED CONS B2 B]] T COND CONS B2 B] T NIL]]

[COND CORDERED CONS B1 CONS B2 B]]

T

[COND CONS B1 CONS B2 B]] T NIL]]]

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

CAPPEND (LAMBDA [X Y] (COND X (CONS (CAR X] (APPEND (CDR X] Y]] Y]]]

CORDERED CLAMBDA CXJ CCOND

COND [CDR X] [COND [LTE [CAR X] [CAR [CDR X]]] [ORDERED [CDR X]] NIL] T]

CLTE CLAMBDA CX Y] COND X COND Y CLTE COR X] COR Y] NIL] T]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

EAND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]

PROFILÉ: [/ENR/ENR[A],/ENR/ENR/ENR[A],/ENR/ENRS 1[8],/ENR.]

TIME: 17.0 SECS.

[T 6 18] [16.29 18 JULY 1973]

THEOREM TO BE PROVED:

[LTE [HALF A] A]

MUST TRY INDUCTION.

(SPECIAL CASE REQUIRED)

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

CAND [LTE CHALF NIL] NIL]

[AND [LTE CHALF [CONS A1 NIL]] [CONS A1 NIL]]

[IMPLIES [LTE [HALF A] A]

[LTE [HALF [CONS A1 [CONS A2 A]]] [CONS A1 [CONS A2 A]]]]]

WHICH IS EQUIVALENT TO:

[COND [LTE [HALF A] A] [LTE [HALF A] [CONS A2 A]] T]

GENERALIZE COMMON SUBTERMS BY REPLACING [HALF A] BY GENRL1.

THE GENERALIZED TERM IS:

[IMPLIES [NUMBERP GENRL1] [COND [LTE GENRL1 A] [LTE GENRL1 [CONS A2 A]] T]]

MUST TRY INDUCTION.

INDUCT ON A AND GENRL1.

THE THEOREM TO BE PROVED IS NOW:

EAND

CAND CIMPLIES [NUMBERP GENRL1]

[COND [LTE GENRL1 NIL] [LTE GENRL1 [CONS A2 NIL]] T]]

[IMPLIES [NUMBERP NIL] [COND [LTE NIL A] [LTE NIL [CONS A2 A]] T]]]

CIMPLIES

[IMPLIES [NUMBERP GENRL1] [COND [LTE GENRL1 A] [LTE GENRL1 [CONS A2 A]] T]]

CIMPLIES ENUMBERP ECONS GENRL11 GENRL1]]

[COND [LTE [CONS GENRL11 GENRL1] [CONS A3 A]]

[LTE [CONS GENRL11 GENRL1] [CONS A2 [CONS A3 A]]]

TIII

WHICH IS EQUIVALENT TO:

ECOND

[NUMBERP GENRL1]

FCOND

• [LTE GENRL1 A]

[COND [LTE GENRL1 [CONS A2 A]] [COND GENRL11 T [LTE GENRL1 [CONS A3 A]]] T]

· []

T]

MUST TRY INDUCTION.

INDUCT ON A AND GENRL1.

```
THE THEOREM TO BE PROVED IS NOW:
LEAND
 [AND
  . [COND [NUMBERP GENEL1]
         COND LLTE GENRL1 NIL]
               [COND [LTE GENRL1 [CONS A2 NIL]]
                     [COND GENRL11 T [LTE GENRL1 [CONS A3 NIL]]]
                     T]
               T]
         TJ
  . ECOND
     [NUMBERP NIL]
     ECOND [LTE NIL A]
           COND LITE NIL CONS A2 A]] COVD GENRL11 T LITE NIL CONS A3 A]]] T]
           T]
     T]]
 CIMPLIES
   ECOND
   . [NUMBERP GENRL1]
   . [COND
   . [L[E GENRL1 A]
   • ECUND ELTE GENRL1 [CONS A2 A]] [COND GENRL11 T ELTE GENRL1 [CONS A3 A]]] T]
   · T]
   . []
   ECOND
   ENUMBERP [CONS GENRL12 GENRL1]]
   ECOND
        [LTE [CONS GENRL12 GENRL1] [CONS A4 A]]
        [COND [LTE [CONS GENRL12 GENRL1] [CONS A2 [CONS A4 A]]]
              CCOND GENRL11 F DLTE [CONS GENRL12 GENRL1] [CONS A3 [CONS A4 A]]]]
              T]
        T]
   TJJJ .
WHICH IS EQUIVALENT TO:
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CHALF CLAMBDA CX3 CCOND X CCOND CCDR X3 CCONS NIL (HALF [CDR (CDR X]]]] 0] 0]]]

CLTE CLAMBDA CX Y3 CCOND X CCOND Y CLTE CCDR X3 [CDR Y]] NIL] T]]]

CIMPLIES CLAMBDA CX Y3 CCOND X CCOND Y T NIL] T]]]

CAND CLAMBDA [X Y3 CCOND X CCOND Y T NIL] NIL]]]

ENUMBERP CLAMBDA [X] CCOND X CCOND [CAR X] NIL [NUMBERP [CDR X]]] T]]]

GENERALIZATIONS:

GENRL1 = [HALF A]

PROFILE: [/ S2 [A] , / E N R / E N R G [A GENRL1] , / E N R / E N R / E N R [A GENRL1] , / E N R / E N R .]

TIME: 22.81 SECS.

[T 7 1] [16.3 18 JULY 1973]

THEOREM TO BE PROVED:

[EQUAL [COPY A] A]

MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

[AND [EQUAL [COPY NIL] NIL]

[IMPLIES [AND [EQUAL [COPY A1] A1] [EQUAL [COPY A] A3]

[EQUAL [COPY [CONS A1 A3] [CONS A1 A3]]

WHICH IS EQUIVALENT TO:

ſ

FUNCTION DEFINITIONS:

CCUPY CLAMBDA [X] [COND X [CONS [COPY [CAR X]] [COPY [CDR X]]] NIL]]]

EAND CLAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

EIMPLIES CLAMBDA [X Y] [COND X [COND Y T NIL] T]]

PROFILE: [/ [A] , / E N R / E N R]

TIME: 2.438 SECS.

[T 7 2] [16.3 18 JULY 1973]

THEOREM TO BE PROVED:

[EQUAL [EQUALP A B] [EQUAL A B]]

WHICH IS EQUIVALENT TO:

COND [EQUAL A B] [EQUALP A B] [COND [EQUALP A B] NIL T]]

FERTILIZE WITH CEQUAL A BJ.

THE THEOREM TO BE PROVED IS NOW:

WHICH IS EQUIVALENT TO:

LCOND LCOND [EQUALP A A] T [*1]] [COND [EQUALP A B] [EQUAL A B] T] NIL]

(WORK ON FIRST CONJUNCT ONLY)

MUST TRY INDUCTION.

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

ECOND

[AND [COND [EQUALP NIL NIL] T [*1]]

- · [IMPLIES CAND [COND [EQUALP A1 A1] T [*1]] [COND [EQUALP A A] T [*1]]]
- [COND [EQUALP [CONS A1 A] [CONS A1 A]] T [*1]]]

[COND [EQUALP A2 B] [EQUAL A2 B] T] NIL]

WHICH IS EQUIVALENT TO:

[COND [EQUALP A2 B] [EQUAL A2 B] T]

MUST TRY INDUCTION.

INDUCT ON A2 AND B.

THE THEOREM TO BE PROVED IS NOW:

[AND CAND COND [EQUALP NIL B] [EQUAL NIL B] T]

COND [EQUALP A2 NIL] [EQUAL A2 NIL] T]]

EIMPLIES [AND [COND [EQUALP A21 B1] [EQUAL A21 B1] T]

[COND [EQUALP A2 B] [EQUAL A2 B] T]]

[COND [EQUALP [CONS A21 A2] [CONS B1 B]]

[EQUAL [CONS A21 A2] [CONS B1 B]]

T]]]

WHICH IS EQUIVALENT TO:

FUNCTION DEFINITIONS:

CEQUALP CLAMBDA CX YJ CCOND

COND Y COND [EQUALP [CAR X] [CAR Y]] [EQUALP [CDR X] [CDR Y]] NIL] NIL] [COND Y NIL T]]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]]

EIMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]

FERTILIZERS:

*1 = [COND [EQUAL A B] NIL [COND [EQUALP A B] NIL T]]

PROFILÉ: [/NR/ENRF,/NR/ENR&[A],/ENR/ENR/ENR[A2B],/ENR/ENR.]

TIME: 14.87 SECS.

[T 7 3] [16.31 18 JULY 1973]

THEOREM TO BE PROVED:

[EQUAL [SUBST A A B] B]

MUST TRY INDUCTION.

INDUCT ON B.

THE THEOREM TO BE PROVED IS NOW:

[AND [EQUAL [SUBST A A NIL] NIL]
[IMPLIES [AND [EQUAL [SUBST A A B1] B1] [EQUAL [SUBST A A B] B]]
[EQUAL [SUBST A A [CONS B1 B]]]

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

[SUBST

ELAMBDA EX Y Z]

[COND [EQUAL Y Z]

COND Z CONS (SUBST X Y CCAR Z)] [SUBST X Y [CDR Z]]] NIL]]]

CAND CLAMBDA EX Y] [COND X ECOND Y T NIL] NIL]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

PROFILE: [/[B],/ENR/ENR/ENR.]

TIME: 6.125 SECS.

THEOREM TO BE PROVED:

[IMPLIES [MEMBER A B] [OCCUR A B]]

WHICH IS EQUIVALENT TO:

[COND [MEMBER A B] [OCCUR A B] T]

MUST TRY INDUCTION.

INDUCT ON B.

THE THEOREM TO BE PROVED IS NOW:

CAND

[COND [MEMBER A NIL] [OCCUR A NIL] T]

[IMPLIES

CAND [COND [MEMBER A B1] [OCCUR A B1] T] [COND [MEMBER A B] [OCCUR A B] T]] [COND [MEMBER A [CONS B1 B]] [OCCUR A [CONS B1 B]] T]]]

WHICH IS EQUIVALENT TO:

ECOND

[MEMBER A B1]

Τ

[COND [MEMBER A B]

[COND [EQUAL A B1]

[COND [EQUAL A [CONS 31 B]] T [COND [OCCUR A B1] T [OCCUR A B]]]

FERTILIZE WITH [EQUAL A 81].

THE THEOREM TO BE PROVED IS NOW:

CCOND

Τ

[MEMBER A B1]

[COND [MEMBER A B]

COND [COND [EQUAL A [CONS A B]] T [COND [OCCUR A A] T [OCCUR A B]]]

T
[*1]]]

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

EMEMBER

[LAMBDA [X Y] [COND Y [COND [EQUAL X [CAR Y]] T [MEMBER X [CDR Y]]] NIL]]]

COCCUR

[LAMBDA [X Y]

CCOND [EQUAL X Y]

ECOND Y ECOND EOCCUR X ECAR Y]] T EOCCUR X ECDR Y]]] NIL]]]

[IMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]

CAND CLAMBDA [X Y] [COND X [COND Y T NIL] NIL]]

FERTILIZERS:

*1 = [COND [EQUAL A B1] NIL T]

PROFILE: [/ENR/ENR[B],/ENR/ENRF,/ENR.]

TIME: 12.44 SECS.

[T 7 5] [16.31 18 JULY 1973]

THEOREM TO BE PROVED:

CIMPLIES [NOT [OCCUR A B]] [EQUAL [SUBST C A B] B]]

WHICH IS EQUIVALENT TO:

[COND EOCCUR A B] T [EQUAL [SUBST C A B] 3]]

MUST TRY INDUCTION.

INDUCT ON B.

THE THEOREM TO BE PROVED IS NOW:

CAND

[COND [OCCUR A NIL] T [EQUAL [SUBST C A NIL] NIL]]

[LMPLIES

[AND [COND [OCCUR A B1] T [EQUAL [SUBST C A B1] B1]]
. [COND [OCCUR A B] T [EQUAL [SUBST C A B] B]]]

[COND [OCCUR A [CONS B1 B]] T [EQUAL [SUBST C A [CONS B1 B]] [CONS B1 B]]]]

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

EOCCUR

ELAMBDA [X Y]

[COND [EQUAL X Y]

T

ECOND Y ECOND COCCUR X [CAR Y]] T [OCCUR X [CDR Y]]] NIL]]]

ENUT [LAMBDA [X] [COND X NIL T]]]

ESUBST

CLAMBDA [X Y Z]

ECOND EEQUAL Y Z]

χ

[COND Z [CONS [SUBST X Y [CAR Z]] [SUBST X Y [CDR Z]]] NIL]]]

EIMPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]

EAND CLAMBDA EX YD COND X COND Y T NILD NILDD

PROFILE: [/ENR/ENR[B],/ENR/ENR/ENR.]

TIME: 14.19 SECS.

THEOREM TO BE PROVED:

[EQUAL [EQUALP A B] [EQUALP B A]]

MUST TRY INDUCTION.

INDUCT ON B AND A.

THE THEOREM TO BE PROVED IS NOW:

CNAD

. CEQUAL CEQUALP A NIL] (EQUALP NIL A)]
. EQUAL CEQUALP NIL 3) (EQUALP B NIL)]]

CIMPLIES

CAND CEQUAL CEQUALP A1 B1] CEQUALP B1 A1]] CEQUAL CEQUALP A B] CEQUALP B A]]]
CEQUAL CEQUALP CONS A1 A] CCONS B1 B] CEQUALP CONS B1 B] CCONS A1 A]]]]

WHICH IS EQUIVALENT TO:

COND [EQUAL [EQUALP A1 B1] [EQUALP B1 A1]]

[COND [EQUAL EQUALP A B] [EQUALP B A]]

[COND [EQUALP A1 B1]]

[COND [EQUALP B1 A1] T [COND [EQUALP A B] NIL T]]

[COND [EQUALP B1 A1] [COND [EQUALP B A] NIL T]]

T]

TJ

FERTILIZE WITH [EQUAL [EQJALP A1 B1] [EQUALP B1 A1]].

```
THE THEOREM TO BE PROVED IS NOW:
[COND [COND [EQUAL [EQUALP A B] [EQUALP B A]]
            [COND [EQUALP A1 B1]
                  [COND [EQUALP A1 B1] T [COND [EQUALP A B] NIL T]]
                  [COND [EQUALP A1 B1] [COND [EQUALP B A] NIL T] T]]
            Τ٦
      T
      [*1]]
WHICH IS EQUIVALENT TO:
FUNCTION DEFINITIONS:
LEQUALP
CLAMBUA
 LX XJ
  COND
      COND Y COND TEQUALP [CAR X] [CAR Y]] TEQUALP CODR X] [CDR Y]] NIL] NIL]
     [COND Y NIL T]]]
EAND (LAMBDA EX Y) [COND X ECOND Y T NIL] NIL]]
[[MPLIES [LAMBDA [X Y] [COND X [COND Y T NIL] T]]]
FERTILIZERS:
*1 = [COND [EQUAL [EQUALP A1 B1] [EQUALP B1 A1]] NIL T]
PROFILE: [/ [B A] , / E N R / E N R F , / R / E N R .]
```

10.69 SECS.

TIME:

THEOREM TO BE PROVED:

CIMPLIES [AND EEQUALP A B] [EQUALP B C]] [EQUALP A C]]

WHICH IS EQUIVALENT TO:

COND LEGUALP A B] [COND [EQUALP B C] [EQUALP A C] T] T]

MUST TRY INDUCTION.

INDUCT ON B, A AND C.

THE THEOREM TO BE PROVED IS NOW:

EAND

EAND COOND EEQUALP A NIL] COOND EEQUALP NIL C] EEQUALP A C] T] T]
. CAND COOND EEQUALP NIL B] COOND EEQUALP B C] EEQUALP NIL C] T] T]
. COOND EEQUALP A B] COOND EEQUALP B NIL] EEQUALP A NIL] T] T]]]

[IMPLIES
 [AND [COND [EQUALP A1 B1] [COND [EQUALP B1 C1] [EQUALP A1 C1] T] T]
 [COND [EQUALP A B] [COND [EQUALP B C] [EQUALP A C] T] T]]

ECOND

EEQUALP [CONS A1 A] [CONS B1 B]]
ECOND [EQUALP [CONS B1 B] [CONS C1 C]] [EQUALP [CONS A1 A] [CONS C1 C]] T]
T]]]

WHICH IS EQUIVALENT TO:

ECOND [EQUALP A B] [COND B T [COND A NIL T]] T]

MUST TRY INDUCTION.

INDUCT ON A AND B.

THE THEOREM TO BE PROVED IS NOW:

CAND CAND COOND EQUALP NIL B3 COOND B T COOND NIL NIL T33 T3

COOND EQUALP A NIL3 COOND NIL T COOND A NIL T33 T33

CIMPLIES CAND COOND EEQUALP A2 323 COOND B2 T COOND A2 NIL T33 T33

COOND EEQUALP A B3 COOND B T COOND A NIL T33 T33

COOND EQUALP COONS A2 A3 COONS B2 B33

COOND COONS B2 B3 T COOND COONS A2 A3 NIL T33

T333

WHICH IS EQUIVALENT TO:

FUNCTION DEFINITIONS:

CEQUALP CLAMBDA CX YI CCOND

1

COND Y [COND [EQUALP [CAR X] [CAR Y]] [EQUALP [CDR X] [CDR Y]] NIL] NIL]
COND Y NIL T]]]]

EAND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]

CIMPLIES CLAMBDA (X Y) [COND X [COND Y T NIL] T]]

PROFILE: [/ E N R / E N R EB A C] , / E N R / E N R / E N R / E N R [A B] , / E

TIME: 29.88 SECS.

[T 7 8] [16.33 18 JULY 1973]

THEOREM TO BE PROVED:

[EQUAL [SWAPTREE [SWAPTREE A]] A]

MUST TRY INDUCTION.

(SPECIAL CASE REQUIRED)

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW: .

[AND [EQUAL [SWAPTREE [SWAPTREE NIL]] NIL] [AND [EQUAL [SWAPTREE [SWAPTREE [CONS A1 NIL]]] [CONS A1 NIL]]

[AND [EQUAL [SWAPTREE [SWAPTREE A2]] A2] [EQUAL [SWAPTREE [SWAPTREE A3] A]] [EQUAL [SWAPTREE [SWAPTREE [CONS A1 [CONS A2 A]]]] [CONS A1 [CONS A2 A]]]]]

WHICH IS EQUIVALENT TO:

T

FUNCTION DEFINITIONS:

LAND CLAMBDA [X Y] [COND X [COND Y T NIL] NIL]]

EIMPLIES CLAMBDA [X Y] [COND X [COND Y T NIL] T]]

PROFILE: [/ S2 [A] , / E N R / E N R / E N R .]

TIME: 7.875 SECS.

THEOREM TO BE PROVED:

CEQUAL CFLATTEN (SWAPTREE A)] (REVERSE (FLATTEN A)]]

MUST TRY INDUCTION.

(SPECIAL CASE REQUIRED)

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

CAND

[EQUAL [FLATTEN [SWAPTREE NIL]] [REVERSE [FLATTEN NIL]]]

EAND

requal [flatten [swaptree [cons a1 NIL]]] [reverse [flatten [cons a1 NIL]]] rimplies [and [equal [flatten [swaptree a2]] [reverse [flatten a2]]] [equal [flatten [swaptree a]] [reverse [flatten a]]]] [equal [flatten [swaptree [cons a1 [cons a2 a]]]]

[REVERSE [FLATTEN [CONS A1 [CONS A2 A]]]]]]]]

WHICH IS EQUIVALENT TO:

[COND]

[EQUAL [FLATTEN [SWAPTREE A2]] [REVERSE [FLATTEN A2]]]
[COND [EQUAL [FLATTEN [SWAPTREE A]] [REVERSE [FLATTEN A]]]

- . ECOND A1
- . Т
- . [EQUAL CAPPEND CFLATTEN CSWAPTREE A]] [FLATTEN CSWAPTREE A2]]]
- . [REVERSE [APPEND [FLATTEN A2] [FLATTEN A]]]]]

T]

FERTILIZE WITH [EQUAL [FLATTEN [SWAPTREE A2]] [REVERSE [FLATTEN A2]]].

THE THEOREM TO BE PROVED IS NOW:

COND COND EQUAL [FLATTEN [SWAPTREE A]] [REVERSE [FLATTEN A]]]

[COND A1

T

[EQUAL CAPPEND [FLATTEN [SWAPTREE A]] [REVERSE [FLATTEN A2]]]

[REVERSE [APPEND [FLATTEN A2] [FLATTEN A]]]]

T]

[**1]]

FERTILIZE WITH REQUAL RELATTEN ESWAPTREE ADD REVERSE RELATTEN ADDD.

THE THEOREM TO BE PROVED IS NOW:

COND LCOND COND A1
T
CEQUAL CAPPEND (REVERSE (FLATTEN A)) [REVERSE (FLATTEN A2]]]
T
T
[*2]]
T

GENERALIZE COMMON SUBTERMS BY REPLACING EFLATTEN ALBY GENRL1 AND EFLATTEN ALB BY GENRL1.

THE GENERALIZED TERM IS:

LCOND ECOND ECOND A1

[*1]]

[EQUAL [APPEND [REVERSE GENRL1] [REVERSE GENRL2]]
[REVERSE [APPEND GENRL2 GENRL1]]]

```
T
[*2]]
T
[*1]]
```

MUST TRY INDUCTION.

INDUCT ON GENRL2.

THE THEOREM TO BE PROVED IS NOW:

```
CAND
ECOND ECOND ECOND A1
                   [EQUAL [APPEND [REVERSE GENRL1] [REVERSE NIL]]
                          [REVERSE [APPEND NIL GENRL1]]]
             [*2]]
       [*1]]
[IMPLIES
  ECOND ECOND ECOND A1
                    requal cappend creverse general reeverse general.
                           [REVERSE [APPEND GENRL2 GENRL1]]]]
              T
              [*2]]
        [*1]]
  LCOND
  ECOND ECOND A1
               [EQUAL [APPEND [REVERSE GENRL1] [REVERSE [CONS GENRL21 GENRL2]]]
                      [REVERSE [APPEND [CONS GENRL21 GENRL2] GENRL1]]]]
         Τ
         [*2]]
  [*1]]]]
```

```
WHICH IS EQUIVALENT TO:
COND
[COND [COND [COND A1 T [EQUAL [APPEND [REVERSE GENRL1] NIL] [REVERSE GENRL1]]]
             [*2]]
       Τ
       [*1]]
 ECOND
 . A1
 . T
 . ECOND
 . [EQUAL [APPEND [REVERSE GENRL1] [REVERSE GENRL2]]
         CREVERSE CAPPEND GENRL2 GENRL1333
 . [COND
 . . ECUND
  - [EnUAL
         CAPPEND [REVERSE GENRL1] [APPEND [REVERSE GENRL2] [CONS GENRL21 NIL]]]
         [APPEND [REVERSE [APPEND GENRL2 GENRL1]] [CONS GENRL21 NIL]]]
  . T
  . [*2]]
  . T
 . .[*1]]
 . T]]
NILI
FERTILIZE WITH [EQUAL [APPEND [REVERSE GENRL1] [REVERSE GENRL2]]
                      [REVERSE [APPEND GENRL2 GENRL1]]].
THE THEOREM TO BE PROVED IS NOW:
ECOND
 [COND [COND [COND A1 T [EQUAL [APPEND [REVERSE GENRL1] NIL] [REVERSE GENRL1]]]
             [*2]]
       T
       [ *1]]
 ECOND
 . A 1
 . T
 · [COND
 . [COND
  . ECOND
  . [⊨ŋUAL
        [APPEND [REVERSE GENRL1] [APPEND [REVERSE GENRL2] [CONS GENRL21 NIL]]]
        [APPEND [APPEND [REVERSE GENRL1] [REVERSE GENRL2]] [CONS GENRL21 NIL]]]
   . T
```

. [*2]]

```
. . T
```

[T 7 9]

```
. .[*1]]
. T
. [*3]]] ·
```

(WORK ON FIRST CONJUNCT ONLY)

GENERALIZE COMMON SUBTERMS BY REPLACING CREVERSE GENRL1] BY GENRL3.

THE GENERALIZED TERM IS:

ECOND [COND [COND A1 T [EQUAL [APPEND GENRL3 NIL] GENRL3]] T [*2]] T [*1]]

MUST TRY INDUCTION.

INDUCT ON GENRL3.

THE THEOREM TO BE PROVED IS NOW:

```
ECUND
```

- .COND COND COND A1 T CEQUAL [APPEND NIL NIL] NIL] T [*2]] T [*1]]
- . LIMPLIES
- . [COND [COND [COND A1 T [EQUAL [APPEND GENRL3 NIL] GENRL3]] T [*2]] T [*1]]
- . COND
- . ECOND
- · · COND A1
- . . . T
- · · · [EQUAL [APPEND [CONS GENRL31 GENRL3] NIL] [CONS GENRL31 GENRL3]]]
- . . T
- . . [*2]]

```
T
```

```
[T 7 9]
```

```
. [*1]]]]
[COND
.A1
.T
.LCOND
. [COND
. [COND
. [COND
. [EQUAL
. . [APPEND [REVERSE GENRL1] [APPEND [REVERSE GENRL2] [CONS GENRL21 NIL]]]
. . [APPEND [APPEND [REVERSE GENRL1] [REVERSE GENRL2]] [CONS GENRL21 NIL]]]
. . T
. [*2]]
. . T
. [*3]]
. . T
. [*3]]
NIL]
```

WHICH IS EQUIVALENT TO:

```
COND

COND

COND

COND

COND

CEQUAL

CAPPEND (REVERSE GENRL1) (APPEND (REVERSE GENRL2) (CONS GENRL21 NIL))]

CAPPEND (APPEND (REVERSE GENRL1) (REVERSE GENRL2)] (CONS GENRL21 NIL)]]

CEQUAL

CAPPEND (APPEND (REVERSE GENRL1) (REVERSE GENRL2)] (CONS GENRL21 NIL)]]

CEXTINATION OF THE PROPERT OF THE PROPERT
```

GENERALIZE COMMON SUBTERMS BY REPLACING [REVERSE GENRL2] BY GENRL4 AND [REVERSE GENRL1] BY GENRL5.

```
THE GENERALIZED TERM IS:
```

```
ECOND
A1
T
```

COND COND COND LEGUAL CAPPEND GENRLS CAPPEND GENRL4 CONS GENRL21 NIL]]]

[T 7 9]

```
[*2]]
       T
       [*1]]
[*3]]]
```

MUST TRY INDUCTION.

INDUCT ON GENRL5.

```
THE THEOREM TO BE PROVED IS NOW:
EAND
CCOND
      A1
      COND COND COND [EQUAL [APPEND NIL [APPEND GENRL4 [CONS GENRL21 NIL]]]
                                [APPEND [APPEND NIL GENRL4] [CONS GENRL21 NIL]]
                         Τ
                         [*2]]
                  Τ
                  [*1]]
            Τ
            [*3]]]
 CIMPLIES
  ECOMD
  . A1
  . [COND [COND [COND [EQUAL [APPEND GENRL5 [APPEND GENRL4 [CONS GENRL21 NIL]]]
                             [APPEND [APPEND GENRL5 GENRL4] [CONS. GENRL21 NIL]]]
                     [*2]]
               [*1]]
         Τ
         [*3]]]
  LCOND
   Α1
   T
   CCOND
    ECOND
    .[COND
```

```
[APPEND [APPEND [CONS GENRL51 GENRL5] GENRL4] [CONS GENRL21 NIL]]]
                                                             [T 7 9]
    T
    · [*2]]
    • T
    •[*1]]
    [*3]]]]
WHICH IS EQUIVALENT TO:
FUNCTION DEFINITIONS:
ESWAPTREE
 ELAMBDA
  [X]
  ECOND
   Χ
   CCOND
     [CAR X]
      [COND [CDR X]
            CONS NIL CONS (SWAPTREE COR COR XIII ESWAPTREE CCAR CODR XIIIII
            X]]
   NILJJJ
CFLATTEN
 LLAMBDA
     [X]
     ECOND X
           [COND [CAR X]
                 [CONS X NIL]
                 ECOND ECDR X3
                       CAPPEND CFLATTEN [CAR [CDR X]]] [FLATTEN [CDR [CDR X]]]]
                       [CONS X NIL]]
           [CONS NIL NIL]]]
EREVERSE
       CLAMBDA [X] [COND X [APPEND [REVERSE [CDR X]] [CONS [CAR X] NIL]] NIL]]]
CAPPEND CLAMBDA EX Y] COND X CONS CCAR X] CAPPEND CCDR XJ Y]] Y]]
CAND CLAMBDA EX Y] ECOND X ECOND Y T NIL] NIL]]
EIMPLIES [LAMBDA [X Y] [COND X [COND Y T VIL] T]]]
```

T

• CEQUAL CAPPEND CONS GENRL51 GENRL5] CAPPEND GENRL4 CONS GENRL21 NIL]]]

FERTILIZERS:

- *1 = [COND FEQUAL EFLATTEN ESWAPTREE A2]] [REVERSE EFLATTEN A2]]] NIL T]
- *2 = [COND [EQUAL [FLATTEN [SWAPTREE A]] [REVERSE [FLATTEN A]]] NIL T]
- *3 = [COND [EQUAL [APPEND [REVERSE GENRL1] [REVERSE GENRL2]]

 [REVERSE [APPEND GENRL2 GENRL1]]]

 NIL

[T 7 10] [16.36 18 JULY 1973]

THEOREM TO BE PROVED:

CEQUAL [LENGTH [FLATTEN A]] [TIPCOUNT A]]

MUST TRY INDUCTION.

(SPECIAL CASE REQUIRED)

INDUCT ON A.

THE THEOREM TO BE PROVED IS NOW:

[AND [EQUAL [LENGTH [FLATTEN NIL]] [TIPCOUNT NIL]]

[AND [EQUAL [LENGTH [FLATTEN [CONS A1 NIL]]] [TIPCOUNT [CONS A1 NIL]]]

[EQUAL [LENGTH [FLATTEN A2]] [TIPCOUNT A2]]

[EQUAL [LENGTH [FLATTEN A]] [TIPCOUNT A]]]

[EQUAL [LENGTH [FLATTEN [CONS A1 [CONS A2 A]]]]]

WHICH IS EQUIVALENT TO:

COND CEQUAL [LENGTH [FLATTEN A2]] [TIPCOUNT A2]]

[COND [EQUAL [LENGTH [FLATTEN A]] [TIPCOUNT A]]

[COND A1

T

[EQUAL [LENGTH [APPEND [FLATTEN A2] [FLATTEN A]]]
[PLUS [TIPCOUNT A2] [TIPCOUNT A]]]]

T]

FERTILIZE WITH [EQUAL [LENGTH [FLATTEN A2]] [TIPCOUNT A2]].

THE THEOREM TO BE PROVED IS NOW:

COND COND (EQUAL [LENGTH EFLATTEN A]) [TIPCOUNT A]]

[COND A1

T

[EQUAL ELENGTH [APPEND [FLATTEN A2] [FLATTEN A]]]

[PLUS ELENGTH [FLATTEN A2]] [TIPCOUNT A]]]]

T]

[*1]

FERTILIZE WITH CEQUAL CLENGTH (FLATTEN A]] CTIPCOUNT A]].

THE THEOREM TO BE PROVED IS NOW:

COND COND COND A1

T

CEQUAL CLENGTH CAPPEND (FLATTEN A2) (FLATTEN A)]]

CPLUS CLENGTH (FLATTEN A2)] (LENGTH (FLATTEN A)]]]

T

[*2]]

T

[*1]]

GENERALIZE COMMON SUBTERMS BY REPLACING [FLATTEN A] BY GENRL1 AND [FLATTEN A2] BY GENRL2.

THE GENERALIZED TERM IS:

COND COND COND A1

T

CEQUAL CLENGTH [APPEND GENRL2 GENRL1]]

CPLUS CLENGTH GENRL2] [LENGTH GENRL1]]]

```
[*2]]
T
[*1]]
```

MUST TRY INDUCTION.

INDUCT ON GENRL2.

THE THEOREM TO BE PROVED IS NOW:

```
EAND
COND
 . ECOND
 . [COND
      A1
     [EQUAL [LENGTH [APPEND NIL GENRL1]] [PLUS [LENGTH NIL] [LENGTH GENRL1]]]
  [*2]]
 .[*1]]
 CIMPLIES
  ECOND ECOND ECOND A1
                    T
                    [EQUAL [LENGTH [APPEND GENRL2 GENRL1]]
                           [PLUS [LENGTH GENRL2] [LENGTH GENRL1]]]]
              [*2]]
        [*1]]
  ECOND
     ECOND ECOND A1
                 Т
                 [EQUAL [LENGTH [APPEND [CONS GENRL21 GENRL2] GENRL1]]
                        [PLUS [LENGTH [CONS GENRL21 GENRL2]] [LENGTH GENRL1]]]]
           [*2]]
     [*1]]]
```

```
WHICH IS EQUIVALENT TO:
T .
FUNCTION DEFINITIONS:
CFLATTEN
 ELAMBDA
     [X]
     [COND X
           [COND [CAR X]
                 [CONS X NIL]
                 ECOND ECDR XI
                       [APPEND [FLATTEN [CAR [CDR X]]] [FLATTEN [CDR [CDR X]]]]
                       [CONS X NIL]]]
           [CONS NIL NIL]]]
CLENGTH (LAMBDA [X] [COND X [CONS NIL [LENGTH [CDR X]]] 0]]]
ETIPCOUNT
 LLAMBDA
  [X]
  CCOND
   Χ
   ECOND
   . [CAR X]
   . [COND [CDR X] [PLUS ETIPCOUNT [CAR [CDR X]]] ETIPCOUNT [CDR [CDR X]]] 1]]
   1]]]
EAND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]
CAPPEND [LAMBDA [X Y] [COND X [CONS [CAR X] [APPEND [CDR X] Y]] Y]]]
EPLUS CLAMBDA [X Y] [COND X [CONS NIL [PLUS [CDR X] Y]] Y]]]
CIMPLIÉS [LAMBDA EX Y] [COND X [COND Y T VIL] T]]]
```

FERTILIZERS:

*1 = [COND [EQUAL [LENGTH [FLATTEN A2]] [TIPCOUNT A2]] NIL T]

*2 = [COND [EQUAL [LENGTH [FLATTEN A]] [TIPCOUNT A]] NIL T]

GENERALIZATIONS:

GENRL1 = [FLATTEN A]

PROFILE: [/ S2 [A] , / E N R / E N R F , / F , / G [GENRL2] , / E N R / E N R .]

TIME: 25.0 SECS.

[T 8 2] [16.37 18 JULY 1973]

THEOREM TO BE PROVED:

[EQUAL [LINEAR [BINARYOF N]] N] "

MUST TRY INDUCTION.

INDUCT ON N.

THE THEOREM TO BE PROVED IS NOW:

CAND [EQUAL [LINEAR [BINARYOF NIL]] NIL]

CIMPLIES [EQUAL [LINEAR [BINARYOF N]] N]

CEQUAL [LINEAR [BINARYOF [CONS NIL N]]]]

WHICH IS EQUIVALENT TO:

COND CEQUAL CLINEAR [BINARYOF N]] N]

EQUAL CLINEAR CRINADD CONS 1 NIL] [BINARYOF N]]] CONS NIL N]]

T]

FERTILIZE WITH [EQUAL [LINEAR [BINARYOF N]] N].

THE THEOREM TO BE PROVED IS NOW:

COND [EQUAL [LINEAR [BINADD CONS 1 NIL] [BINARYOF N]]]

[CONS NIL [LINEAR [BINARYOF N]]]

T

GENERALIZE COMMON SUBTERMS BY REPLACING [BINARYOF N] BY GENRL1.

THE GENERALIZED TERM IS:

COND [EQUAL [LINEAR [BINADD [CONS 1 NIL] GENRL1]] [CONS NIL [LINEAR GENRL1]]]

T
[*1]]

MUST TRY INDUCTION.

INDUCT ON GENRL1.

THE THEOREM TO BE PROVED IS NOW:

EAND

[COND [EQUAL [LINEAR [BINADD [CONS 1 NIL] NIL]] [CONS NIL [LINEAR NIL]]]

Ţ

[*1]]

CIMPLIES

ECOND

[EQUAL [LINEAR [BINADD [CONS 1 NIL]] GENRL1]] [CONS NIL [LINEAR GENRL1]]]

1

[*1]]

[COND [EQUAL [LINEAR [BINADD [CONS 1 NIL] [CONS GENRL11 GENRL1]]]]
[CONS NIL ELINEAR [CONS GENRL11 GENRL1]]]

[*1]]]

WHICH IS EQUIVALENT TO:

```
ECUND [EQUAL [LINEAR [BINADD [CONS 1 NIL] GENRL1]] [CONS NIL [LINEAR GENRL1]]]
      ECOND [COND GENRL11
                  EEQUAL [DOUBLE [LINEAR [BINADD [CONS 1 NIL] GENRL1]]]
                         [CONS NIL [CONS VIL [DOUBLE [LINEAR GENRL1]]]]
                  T]
            Т
            [*1]]
      T]
FERTILIZE WITH [EQUAL [LINEAR [BINADD [CONS 1 NIL] GENRL1]]
                      [CONS NIL [LINEAR GENRL1]]].
THE THEOREM TO BE PROVED IS NOW:
ECOND ECOND ECOND GENRL11
                  [EQUAL [DOUBLE [CONS NIL [LINEAR GENRL1]]]
                         CONS NIL [CONS NIL [DOUBLE [LINEAR GENRL1]]]]
                  T]
            Т
            [*1]]
      Т
      [*2]]
WHICH IS EQUIVALENT TO:
T
FUNCTION DEFINITIONS:
[BINARYOF [LAMBDA [X] [COND X [BINADD [CONS 1 NIL] [BINARYOF [CDR X]]] NIL]]]
CLINEAR CLAMBDA [X]
                ECOND X
                      [COND [CAR X]
                            [CONS NIL [DOJBLE [LINEAR [CDR X]]]]
```

[DOUBLE [LINEAR [CDR X]]]]

NILJJJ

CX YJ AGGMAJJ CY YJ

```
ECOND Y

COND CAR X;

COND CAR Y;

COND CAR Y;

CONS 0 [BINADD CONS 1 NIL] [BINADD [CDR X] [CDR Y]]]]

CONS 1 [BINADD CCDR X] [CDR Y]]]]

CONS [CAR Y] [BINADD [CDR X] [CDR Y]]]]

X]

Y]]]
```

EIMPLIES CLAMBDA (X Y) COND X COND Y T NIL) T)]]

[AND [LAMBDA [X Y] [COND X [COND Y T NIL] NIL]]

EDOUBLE [LAMBDA [X] [COND X [CONS NIL [CONS NIL [DOUBLE [CDR X]]]] 0]]]

FERTILIZERS:

*1 = [COND [EQUAL [LINEAR [BINARYOF N]] N] NIL T]

*2 = [COND [EQUAL [LINEAR [BINADD [CONS 1 NIL] GENRL1]] [CONS NIL [LINEAR GENRL1]]]

NIL

T1

GENERALIZATIONS:

GENRL1 = [BINARYOF N]

PROFILE: [/[N],/ENR/ENRX,/G[GENRL1],/ENR/ENRF,/ENR.]

TIME: 19.81 SECS.

[]