# **First-Order Theorem Prover PRV**

**Wolfgang Bibel**

The following SNOBOL4 program for proving formulas in first-order logic was written and run by the author end of 1970 at Wayne State University, Detroit, USA. The original out-print of a run of the program with eight different formulas as input is contained in the private collection of the author’s publications under the label 3-E136. The subsequent code was typed-off this copy in 2016.

The program is the subject of several publications including the following one:

Wolfgang Bibel, An approach to a systematic theorem proving procedure in first-order logic. Computing *12*, 43-55 (1974).

SNOBOL4 (VERSION 3.4, JULY 10, 1970)

(MTS IMPLEMENTATION AUG. 15, 1970)

\*1 ENTH FORM PT1 :S(RETURN)

\*2 ENTH = 1 :(RETURN)

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\*3 APR APR PT2

\*4 IDENT(Q, ‘(’ ) :F(MA1)

\*5 APR ‘E’ V = NE(ENTH(APR,V)) :(RETURN)

\*6 MA1 APR PT3

\*7 E1 = ENTH(Q,V)

\*8 E2 = ENTH(R,V)

\*9 EQ(E1 + E2, 2) :S(MA7)

\*10 P ’N‘ :S(MA2)

\*11 EQ(E1 + E2) :S(MA3)

\*12 EQ(E1) :S(MA4)

\*13 APR = P ‘D(‘ Q ‘,‘ APR(‘E‘ V R) ‘)‘ :(RETURN)

\*14 MA7 APR ’E’ V = :(RETURN)

\*15 MA3 APR = P ‘D(‘ APR(‘E‘ V Q) ‘,‘ APR(‘E‘ V R) ‘)‘ :(RETURN)

\*16 MA4 APR = P ‘D(‘ APR(‘E‘ V Q) ‘,‘ R ‘)‘ :(RETURN)

\*17 MA2 P ‘N‘ FENCE RPOS(0) :F(RETURN)

\*18 EQ(E1 + E2) :S(RETURN)

\*19 EQ(E1) :S(MA6)

\*20 R POS(0) ‘N’ = :F(MA8)

\*21 APR = P ‘D(‘ Q ‘,N‘ APR(‘E‘ V R) ‘)‘ :(RETURN)

\*22 MA8 APR = APR(‘E‘ V ‘N‘ R) ‘,‘ APR(‘E‘ V R) ‘)‘

\*23 APR POS(0) ‘N‘ = :F(MA9)

\*24 APR = P ‘D(‘ Q ‘,‘ APR ‘)‘ :(RETURN)

\*25 MA9 APR = P ‘D(‘ Q ‘,N‘ APR ‘)‘ :(RETURN)

\*26 MA6 Q POS(0) ’N’ = :F(MA10)

\*27 APR = P ‘D(N‘ APR(‘E‘ V Q) ‘,‘ R ‘)‘ :(RETURN)

\*28 MA10 APR = APR(‘E‘ V ‘N‘ Q)

\*29 APR POS(0) ‘N‘ = :F(MA11)

\*30 APR = P ‘D(‘ APR ‘,‘ R ‘)‘ :(RETURN)

\*31 APR = P ‘D(N‘ APR ‘,‘ R ‘)‘ :(RETURN)

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\*32 AEQ AEQ PT4 :S(RETURN)

\*33 AEQ PT5 :F(MAE1)

\*34 AEQ = IDENT(Q, ‘B’) ‘ND(ND(’ AEQ(‘N’ P) ‘,’ AEQ(R)

\*34 . ‘),ND)’ AEQ(‘N’ R) ‘,’ AEQ(P) ‘))’ :S(RETURN)

\*35 AEQ = IDENT(Q, ‘C’) ‘ND(’ AEQ(‘N’ P) ‘,’ AEQ(‘N’ R) ‘)’ :S(RETURN)

\*36 AEQ = IDENT(Q, ‘D’) ‘D(’ AEQ(P) ‘,’ AEQ(R) ‘)’ :S(RETURN)

\*37 AEQ = ‘D(’ AEQ(‘N’ P) ‘,’ AEQ(R) ‘)’ :(RETURN)

\*38 MAE1 AEQ POS(0) ‘N’ = :F(MAE2)

\*39 AEQ = AEQ(AEQ)

\*40 AEQ POS(0) ‘N’ = :S(RETURN)

\*41 AEQ = ‘N’ AEQ :(RETURN)

\*42 MAE2 AEQ PT6 = :F(MAE3)

\*43 AEQ = APR(P AEQ(AEQ)) :(RETURN)

\*44 MAE3 AEQ PT7 = AEQ(‘NE’ P ‘N’ Q) :(RETURN)

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\*45 ERS FRM PT8 = Q :F(ME)

\*46 ERS = ERS P V2 :(ERS)

\*47 ME ERS = ERS FRM :(RETURN)

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\*48 ALZ I = LT(I, NC) I + 1 :F(RETURN)

\*49 H = $(‘B’ I)

\*50 DIFFER(H) :F(ALZ)

\*51 MAL8 H PT09 =

\*52 MAL7 H PT9 :F(ALZ)

\*53 IDENT(P, ‘(’ ) :S(MAL2)

\*54 IDENT(P, ‘D’ ) :S(MAL3)

\*55 IDENT(P, ‘E’ ) :S(MAL4)

\*56 IDENT(Q, ‘D’ ) :S(MAL5)

\*57 IDENT(Q, ‘E’ ) :S(MAL6)

\*58 IDENT(Q, ‘(’ ) :S(MAL2)

\*59 H LEN(2) = :(MAL7)

\*60 MAL2 H PT10 =

\*61 $(‘C’ I) = $(‘C’ I) P :(MAL8)

\*62 MAL3 H PT11 = P G ‘)’ Q :(MAL7)

\*63 MAL4 H PT12 =

\*64 ND = ND + 1

\*65 G = ‘D’ ND G

\*66 $(‘BC’ I) = $(‘BC’ I) G ‘)E’ P Q

\*67 H = ERS(Q, P, ‘D’ ND) H :(MAL7)

\*68 MAL5 NC = NC + 1

\*69 H PT13 = ‘N’ P R

\*70 $(‘C’ NC) = $(‘C’ I)

\*71 $(‘BC’ NC) = $(‘BC’ I)

\*72 $(‘B’ NC) = G ‘)N’ Q R :(MAL7)

\*73 MAL6 NV = NV + 1

\*74 $(‘V’ NV) = G

\*75 G = ‘V’ NV

\*76 H PT14 = ‘N’ ERS(Q, P, G) :(MAL7)

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\*77 AX I = 1

\*78 AX4 &FULLSCAN = 0

\*79 $(‘C’ I) PT16 :(AX1)

\*80 $(‘K’ I) =

\*81 $(‘L’ I) =

\*82 I = NE(I, 1) I – 1 :F(FRETURN)

\*83 $(‘C’ I) PT17

\*84 AX1 &FULLSCAN = 1

\*85 $(‘C’ I) PT18 :F(AX4)

\*86 &FULLSCAN = 0

\*87 I = LT(I, NC) I + 1 :S(AX4)F(RETURN)

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\*88 VRTR L = LT(L, ND) L + 1 :F(VR1)

\*89 $(‘H’ L) = $(L ‘,’ I – 1) :(VRTR)

\*90 VR1 IDENT(Q, ‘)’ ) :S(VR2)

\*91 Q PT19 =

\*92 R PT20 =

\*93 IDENT(W, U) :S(VR3)

\*94 IDENT(W, ‘D’) :S(VR4)

\*95 H = J

\*96 J = K

\*97 K = H

\*98 VR4 DV(J, K) :S(VR1)F(FRETURN)

\*99 VR3 IDENT(W, ‘V’) :S(VR22)

\*100 $(‘H’ J) PT21 :F(VR11)

\*101 $(‘H’ K) PT22 :F(VR12)

\*102 IDENT(J1, M) :S(VR13)F(FRETURN)

\*103 VR22 EQ(J, K) :S(VR1)F(FRETURN)

\*104 VR11 $(‘H’ K) PT22 :F(VR13)

\*105 DV(J, M) :S(VR13)F(FRETURN)

\*106 VR12 DV(K, J1) :F(FRETURN)

\*107 VR13 V = $(‘H’ J)

\*108 W = $(‘H’ K)

\*109 V ‘D’ K PT24 :S(VR14)

\*110 X = ‘D’ K

\*111 VR14 W ‘D’ J PT24 :S(VR15)

\*112 W1 = ‘D’ J

\*113 VR15 V PT23 :F(VR16)

\*114 VR18 W U PT24 :S(VR17)

\*115 W1 = W1 U

\*116 VR17 V POS(L) PT23 :S(VR18)

\*117 VR16 W PT23 :F(VR19)

\*118 VR21 V U PT24 :S(VR20)

\*119 X = X U

\*120 VR20 W POS(L) PT23 :S(VR21)

\*121 VR19 $(‘H’ J) = $(‘H’ J) X

\*122 $(‘H’ K) = $(‘H’ K) W1 :(VR1)

\*123 VR2 L = 0

\*124 VR10 L = LT(L, ND) L + 1 :F(RETURN)

\*125 $(L ‘,’ I) = $(‘H’ L) :(VR10)

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\*126 DV $(‘H’ J) PT21 :F(VR5)

\*127 C = EQ(J1, K) 1 :F(FRETURN)

\*128 VR5 J1 = ‘D’ J

\*129 $(‘H’ J) PT23 :F(VR6)

\*130 VR7 J1 = J1 | U

\*131 $(‘H’ J) POS(L) PT23 :S(VR7)

\*132 VR6 W = $(‘V’ K)

\*133 VR9 IDENT(W) :S(VR8)

\*134 W J1 :S(FRETURN)

\*135 W (‘V’ REM) . W :F(VR8)

\*136 W = $W :(VR9)

\*137 VR8 $(‘H’ J) = EQ(C) ‘V’ K $(‘H’ J) :(RETURN)

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\*138 START

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\*139 DEFINE( ‘ ENTH(FORM, VAR) ’ )

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\*140 DEFINE( ‘ APR(APR) E1, E2, P, Q, R, V ’ )

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\*141 DEFINE( ‘ AEQ(AEQ) P, Q, R ’ )

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\*142 DEFINE( ‘ ERS(FRM, V1, V2) P, Q ’ )

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\*143 DEFINE( ‘ ALZ( ) G, H, I, P, Q, R ’ )

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\*144 DEFINE( ‘ AX( ) I, P, Q, R ’ )

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\*145 DEFINE( ‘ VRTR(Q, R) H, J, J1, K, L, M, U, V, W, W1, X ’ )

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\*146 DEFINE( ‘ DV(J, K) L, C, J1 ’ )

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\*147 PT1 = ‘,’ \*VAR ANY( ‘,)’ )

\*148 PT2 = POS(1) BREAK( ‘DEN(’ ) . V BREAK( ‘D(’ ) . P LEN(1) . Q

\*149 PT3 = ‘(’ BAL . Q ‘,’ BAL . R ‘)’

\*150 PT4 = POS(0) ‘(’ BREAK( ‘)’ ) RPOS(1)

\*151 PT5 = POS(0) ‘(’ BAL . P ANY(‘BCDS’) . Q BAL . R ‘)’

\*152 PT6 = POS(0) (‘E’ BREAK( ‘AEN(’ ) . P

\*153 PT7 = ‘A’ BREAK( ‘AEN(’ ) . P RTAB(0) . Q

\*154 PT8 = (ARB ‘,’ ) . P \*V1 ANY( ‘,)’ ) . Q

\*155 PT9 = LEN(1) . P LEN(1) . Q

\*156 PT09 = BREAK( ‘)’ ) . G ‘)’

\*157 PT10 = (BREAK( ‘(’ ) BAL) . P

\*158 PT11 = TAB(2) BAL . P ‘,’ BAL . Q ‘)’

\*159 PT12 = LEN(1) BREAK( ‘DEN(’ ) . P (BREAK( ‘(’ BAL) . Q

\*160 PT13 = TAB(3) BAL . P ‘,’ BAL . Q ‘)’ REM . R

\*161 PT14 = TAB(1) PT12

\*162 PT16 = \*EQ($(‘K’ I)) POS(0) ‘(’ @\*$(‘K’ I) BREAK( ‘,)’ ) . P

. (BREAK( ‘)’ ) ‘)’ ) . Q | POS(\*$(‘K’ I) ARB ‘)(’ @\*$(‘K’ I)

. BREAK( ‘,)’ ) . P ( BREAK( ‘)’ ) ‘)’ ) . Q

\*163 PT17 = POS( \*$(‘K’ I) ) BREAK( ‘,)’ ) . P ( BREAK( ‘)’ ) ‘)’ ) . Q

\*164 PT18 = POS( \*$(‘L’ I) ) (NULL | BAL) ‘N(’ \*P ( BREAK( ‘)’ ) ‘)’ )

. $ R \*VRTR(Q,R) @\*$(‘L’ I)

\*165 PT19 = LEN(1) LEN(1) . W BREAK( ‘,)’ ) . J

\*166 PT20 = LEN(1) LEN(1) . U BREAK( ‘,)’ ) . K

\*167 PT21 = POS(0) ‘V’ (BREAK(‘D’) | REM) . J1

\*168 PT22 = POS(0) ‘V’ (BREAK(‘D’) | REM) . M

\*169 PT23 = (‘D’ (BREAK(‘D’) | REM)) . U @L

\*170 PT24 = ‘D’ | RPOS(0)

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\*171 &TRIM = 1

\*172 M2 F = INPUT :F(END)

\*173 OUTPUT =

\*174 OUTPUT = ‘INPUT FORMULA:’

\*175 OUTPUT =

\*176 F BREAK( ‘AEN(’ ) . MST REM . OUTPUT

\*177 F = AEQ(OUTPUT)

\*178 OUTPUT =

\*179 OUTPUT = ‘EQUIVALENT TRANSFORMATION:’

\*180 OUTPUT =

\*181 OUTPUT = F

\*182 STN = 1

\*183 B1 = ‘)’ F

\*184 NC = 1

\*185 M9 OUTPUT =

\*186 OUTPUT = ‘LIST OF CLAUSES:’

\*187 OUTPUT = ALZ( )

\*188 I =

\*189 M3 I = I + 1

\*190 OUTPUT = LE(I, NC) $(‘C’ I) :S(M3)

\*191 OUTPUT =

\*192 OUTPUT = ‘VARIABLES WITH INCOMPATIBLE DUMMIES:’

\*193 OUTPUT =

\*194 M4 J = J + 1

\*195 OUTPUT = LE(J, NV) ‘V’ J ‘ ’ $(‘V’ J) :S(M4)

\*196 OUTPUT =

\*197 OUTPUT = AX( ) ‘IN ’ STN ‘TH STEP OF PROCEDURE CURRENT ’

\*197 . ‘FORMULA TURNS OUT TO BE VALID’ :S(M6)

\*198 OUTPUT = LT(STN, MST) ‘AFTER ’ STN ‘TH STEP OF ’

\*198 . ‘PROCEDURE NO DECISION ABOUT ’

\*198 . ‘CURRENT FORMULA’ :S(M7)

\*199 OUTPUT = ‘PROCEDURE TERMINATES AFTER ’ STN

\*199 . ‘TH STEP. NO DECISION ABOUT ’

\*199 . ‘CURRENT FORMULA’ :(M6)

\*200 M7 STN = STN + 1

\*201 J = J – 1

\*202 I =

\*203 M8 I = LT(I, NC) I + 1 :(M9)

\*204 $(‘B’ I) = $(‘BC’ I)

\*205 $(‘BC’ I) = :(M8)

\*206 M6 I =

\*207 M11 I = LT(I, NC) I + 1 :(M10)

\*208 $(‘C’ I) =

\*209 $(‘BC’ I) =

\*210 $(‘K’ I) =

\*211 $(‘L’ I) =

\*212 K = 0

\*213 M12 K = LT(K, ND) K + 1 :F(M11)

\*214 $(K ‘,’ I) = :(M12)

\*215 M10 NV =

\*216 J =

\*217 F =

\*218 ND = :(M2)

\*219 END START

0 SYNTACTIC ERROR(S) IN SOURCE PROGRAM

Comments added in 2016

In the program A stands for all-quantification, B for equivalence, C for conjunction, D for disjunction, E for existential quantification, N for negation, and S for implication. As input the program expects any first-order formula written in a relatively usual way such as (Ax((F,x)CNEy(G,x,y)) representing x(F(x)yG(x,y)).

The symbols used in the program are derived from German expressions. For instance, ALZ derives from “Abbau logischer Zeichen” (handling of logical symbols), APR from “Antipränexierung” (antiprenexing), ENTH from “enthalten” (contained), AEQ from “Äquivalenzumformung” (equivalence transformation), ERS from “Ersetzung” (substitution), and so forth. The proof for any formula is attempted in a stepwise fashion whereby the limit of steps is given along with the input formula (taken to be 1 by default if none is given).

The statistics of the run in 1970 mention among several other values 2.96 ms average time per statement executed, 9264 ms compilation and 48479 ms exection time for the proof attempts of the eight input formulas.

While great care was taken in copying the code from the paper copy, mistyping can of course never be excluded a hundred percent. In the original there is no difference between the symbol ‘ and the symbol ’ while WORD makes a difference; I am not aware whether this might effect a possible run of the program. Blanks were added for readability and probably have no influence on a possible run.

Lindenau, 27 Nov. 2016