

## Week5 monday

Warmup: Design a CFG to generate the language  $\{a^i b^j \mid j \geq i \geq 0\}$

*Sample derivation:*

Design a PDA to recognize the language  $\{a^i b^j \mid j \geq i \geq 0\}$

**Theorem 2.20:** A language is generated by some context-free grammar if and only if it is recognized by some push-down automaton.

Definition: a language is called **context-free** if it is the language generated by a context-free grammar. The class of all context-free language over a given alphabet  $\Sigma$  is called **CFL**.

Consequences:

- Quick proof that every regular language is context free
- To prove closure of the class of context-free languages under a given operation, we can choose either of two modes of proof (via CFGs or PDAs) depending on which is easier
- To fully specify a PDA we could give its 6-tuple formal definition or we could give its input alphabet, stack alphabet, and state diagram. An informal description of a PDA is a step-by-step description of how its computations would process input strings; the reader should be able to reconstruct the state diagram or formal definition precisely from such a description. The informal description of a PDA can refer to some common modules or subroutines that are computable by PDAs:
  - PDAs can “test for emptiness of stack” without providing details. *How?* We can always push a special end-of-stack symbol, \$, at the start, before processing any input, and then use this symbol as a flag.
  - PDAs can “test for end of input” without providing details. *How?* We can transform a PDA to one where accepting states are only those reachable when there are no more input symbols.

Suppose  $L_1$  and  $L_2$  are context-free languages over  $\Sigma$ . **Goal:**  $L_1 \cup L_2$  is also context-free.

*Approach 1: with PDAs*

Let  $M_1 = (Q_1, \Sigma, \Gamma_1, \delta_1, q_1, F_1)$  and  $M_2 = (Q_2, \Sigma, \Gamma_2, \delta_2, q_2, F_2)$  be PDAs with  $L(M_1) = L_1$  and  $L(M_2) = L_2$ .

Define  $M =$

*Approach 2: with CFGs*

Let  $G_1 = (V_1, \Sigma, R_1, S_1)$  and  $G_2 = (V_2, \Sigma, R_2, S_2)$  be CFGs with  $L(G_1) = L_1$  and  $L(G_2) = L_2$ .

Define  $G =$

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*Approach 1: with PDAs*

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*Approach 2: with CFGs*

Let  $G_1 = (V_1, \Sigma, R_1, S_1)$  and  $G_2 = (V_2, \Sigma, R_2, S_2)$  be CFGs with  $L(G_1) = L_1$  and  $L(G_2) = L_2$ .

Define  $G =$

# Week4 friday

*Big picture:* PDAs were motivated by wanting to add some memory of unbounded size to NFA. How do we accomplish a similar enhancement of regular expressions to get a syntactic model that is more expressive?

DFA, NFA, PDA: Machines process one input string at a time; the computation of a machine on its input string reads the input from left to right.

Regular expressions: Syntactic descriptions of all strings that match a particular pattern; the language described by a regular expression is built up recursively according to the expression's syntax

**Context-free grammars:** Rules to produce one string at a time, adding characters from the middle, beginning, or end of the final string as the derivation proceeds.

Definitions below are on pages 101-102.

Term	Typical symbol or Notation	Meaning
<b>Context-free grammar</b> (CFG)	$G$	$G = (V, \Sigma, R, S)$
The set of <b>variables</b>	$V$	Finite set of symbols that represent phases in production pattern
The set of <b>terminals</b>	$\Sigma$	Alphabet of symbols of strings generated by CFG $V \cap \Sigma = \emptyset$
The set of <b>rules</b>	$R$	Each rule is $A \rightarrow u$ with $A \in V$ and $u \in (V \cup \Sigma)^*$
The <b>start</b> variable	$S$	Usually on left-hand-side of first/ topmost rule
<b>Derivation</b>	$S \Rightarrow \dots \Rightarrow w$	Sequence of substitutions in a CFG (also written $S \Rightarrow^* w$ ). At each step, we can apply one rule to one occurrence of a variable in the current string by substituting that occurrence of the variable with the right-hand-side of the rule. The derivation must end when the current string has only terminals (no variables) because then there are no instances of variables to apply a rule to.
Language <b>generated</b> by the context-free grammar $G$	$L(G)$	The set of strings for which there is a derivation in $G$ . Symbolically: $\{w \in \Sigma^* \mid S \Rightarrow^* w\}$ i.e.  $\{w \in \Sigma^* \mid \text{there is derivation in } G \text{ that ends in } w\}$
<b>Context-free language</b>		A language that is the language generated by some context-free grammar

Examples of context-free grammars, derivations in those grammars, and the languages generated by those grammars

$G_1 = (\{S\}, \{0\}, R, S)$  with rules

$$S \rightarrow 0S$$

$$S \rightarrow 0$$

In  $L(G_1)$  ...

Not in  $L(G_1)$  ...

$$G_2 = (\{S\}, \{0, 1\}, R, S)$$

$$S \rightarrow 0S \mid 1S \mid \varepsilon$$

In  $L(G_2) \dots$

Not in  $L(G_2) \dots$

$(\{S, T\}, \{0, 1\}, R, S)$  with rules

$$S \rightarrow T1T1T1T$$

$$T \rightarrow 0T \mid 1T \mid \varepsilon$$

In  $L(G_3) \dots$

Not in  $L(G_3) \dots$

$G_4 = (\{A, B\}, \{0, 1\}, R, A)$  with rules

$$A \rightarrow 0A0 \mid 0A1 \mid 1A0 \mid 1A1 \mid 1$$

In  $L(G_4)$  ...

Not in  $L(G_4)$  ...



Design a CFG to generate the language  $\{a^n b^n \mid n \geq 0\}$

*Sample derivation:*