Week 2 at a glance

Textbook reading: Sections 1.1, 1.2

Before Monday, read pages 41-43 (Figures 1.18, 1.19, 1.20) for examples of automata and languages.

Before Wednesday, read pages 48-50 (Figures 1.27, 1.29) which introduces nondeterminism.

Before Friday, read pages 60-61 Theorem 1.47 and Theorem 1.48 that we'll refer to as "closure proofs".

For Week 3 Monday: Theorem 1.39 "Proof Idea", Example 1.41, Example 1.56, Example 1.58.

We will be learning and practicing to:

- Clearly and unambiguously communicate computational ideas using appropriate formalism. Translate across levels of abstraction.
  - Give examples of sets that are regular (and prove that they are).
    - **State the definition of the class of regular languages**
    - **Give examples of regular languages, using each of the three equivalent models of computation for proving regularity.**
  - Describe and use models of computation that don't involve state machines.
    - **Given a DFA or NFA, find a regular expression that describes its language.**
    - **Given a regular expression, find a DFA or NFA that recognizes its language.**
  - Use precise notation to formally define the state diagram of finite automata.
  - Use clear English to describe computations of finite automata TM informally.
    - **Design an automaton that recognizes a given language**
    - **Specify a general construction for DFA based on parameters**
    - **Design general constructions for DFA**
    - **Motivate the use of nondeterminism**
    - **State the formal definition of NFA**
    - **Trace the computation(s) of a NFA on a given string using its state diagram**
    - **Determine if a given string is in the language recognized by a NFA**
    - **Translate between a state diagram and a formal definition of a NFA**
- Understand, guide, shape impact of computing on society/the world. Connect the role of Theory CS classes to other applications (in undergraduate CS curriculum and beyond). Model problems using appropriate mathematical concepts.
  - Explain nondeterminism and describe tools for simulating it with deterministic computation.
    - **Given a nondeterministic finite automaton, find a deterministic finite automaton that recognizes its language.**

TODO:

#FinAid Assignment on Canvas (complete as soon as possible) and read syllabus on Canvas

Schedule your Test 1 Attempt 1, Test 2 Attempt 1, Test 1 Attempt 2, and Test 2 Attempt 2 times at PrairieTest (http://us.prairietest.com)

Homework 1 submitted via Gradescope (https://www.gradescope.com/), due Tuesday 10/8/2024

Review Quiz on PrairieLearn (http://us.prairielearn.com), complete by Sunday 10/13/2024

In Computer Science, we operationalize "hardest" as "requires most resources", where resources might be memory, time, parallelism, randomness, power, etc. To be able to compare "hardness" of problems, we use a consistent description of problems

Input: String

Output: Yes/ No, where Yes means that the input string matches the pattern or property described by the problem.

So far: we saw that regular expressions are convenient ways of describring patterns in strings. **Finite automata** give a model of computation for processing strings and and classifying them into Yes (accepted) or No (rejected). We will see that each set of strings is described by a regular expression if and only if there is a FA that recognizes it. Another way of thinking about it: properties described by regular expressions require exactly the computational power of these finite automata.

Monday: Finite automaton constructions

Review: Formal definition of DFA: $M = (Q, \Sigma, \delta, q_0, F)$

- Finite set of states Q
- Alphabet Σ
- Transition function δ
- Start state q_0
- Accept (final) states F

In the state diagram of M, how many outgoing arrows are there from each state?

$M = (\{q, r, s\}, \{a, b\}, \delta, q, \{s\})$ where δ is (rows labelled by states and columns labelled by symbols):

δ	a	b
q	r	q
r	r	s
s	s	s

The state diagram for M is

Give two examples of strings that are accepted by M and two examples of strings that are rejected by M:

Add "labels" for states in the state diagram, e.g. "have not seen any of desired pattern yet" or "sink state".

We can use the analysis of the roles of the states in the state diagram to describe the language recognized by the DFA.

$L(M) =$

A regular expression describing $L(M)$ is

Let the alphabet be $\Sigma_1 = \{0, 1\}$.

A state diagram for a DFA that recognizes $\{w \mid w \text{ contains at most two 1's}\}$ is

A state diagram for a DFA that recognizes $\{w \mid w \text{ contains more than two 1's}\}$ is

Extra example: A state diagram for DFA recognizing

$$\{w \mid w \text{ is a string over } \{0,1\} \text{ whose length is not a multiple of } 3\}$$

Let n be an arbitrary positive integer. What is a formal definition for a DFA recognizing

$$\{w \mid w \text{ is a string over } \{0,1\} \text{ whose length is not a multiple of } n\}?$$

Note: On Wednesday, we'll see a new kind of finite automaton. It will be helpful to distinguish it from the machines we've been talking about so we'll use **Deterministic Finite Automaton** (DFA) to refer to the machines from Section 1.1.

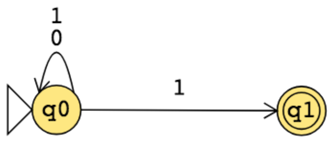
Wednesday: Nondeterministic automata

Nondeterministic finite automaton (Sipser Page 53) Given as $M = (Q, \Sigma, \delta, q_0, F)$

Finite set of states Q	Can be labelled by any collection of distinct names. Default: $q0, q1, \ldots$
Alphabet Σ	Each input to the automaton is a string over Σ.
Arrow labels Σ_ε	$\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$.
	Arrows in the state diagram are labelled either by symbols from Σ or by ε
Transition function δ	$\delta : Q \times \Sigma_\varepsilon \to \mathcal{P}(Q)$ gives the **set of possible next states** for a transition from the current state upon reading a symbol or spontaneously moving.
Start state q_0	Element of Q. Each computation of the machine starts at the start state.
Accept (final) states F	$F \subseteq Q$.

M accepts the input string $w \in \Sigma^*$ if and only if **there is** a computation of M on w that processes the whole string and ends in an accept state.

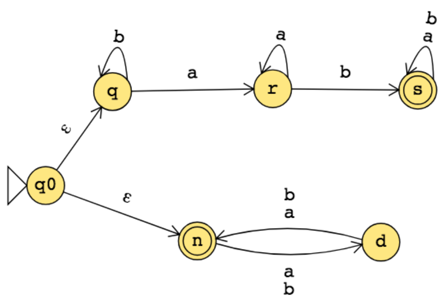
The formal definition of the NFA over $\{0, 1\}$ given by this state diagram is:



The language over $\{0, 1\}$ recognized by this NFA is:

Change the transition function to get a different NFA which accepts the empty string (and potentially other strings too).

The state diagram of an NFA over $\{a, b\}$ is below. The formal definition of this NFA is:



The language recognized by this NFA is:

Friday: Automata constructions

Warmup: Design a DFA (deterministic finite automaton) and an NFA (nondeterministic finite automaton) that each recognize each of the following languages over $\{a, b\}$

$$\{w \mid w \text{ has an } a \text{ and ends in } b\}$$

$$\{w \mid w \text{ has an } a \text{ or ends in } b\}$$

Strategy: To design DFA or NFA for a given language, identify patterns that can be built up as we process strings and create states for intermediate stages. Or: decompose the language to a simpler one that we already know how to recognize with a DFA or NFA.

Recall (from Wednesday of last week, and in textbook Exercise 1.14): if there is a DFA M such that $L(M) = A$ then there is another DFA, let's call it M', such that $L(M') = \overline{A}$, the complement of A, defined as $\{w \in \Sigma^* \mid w \notin A\}$.

Let's practice defining automata constructions by coming up with other ways to get new automata from old.

Suppose A_1, A_2 are languages over an alphabet Σ. **Claim:** if there is a NFA N_1 such that $L(N_1) = A_1$ and NFA N_2 such that $L(N_2) = A_2$, then there is another NFA, let's call it N, such that $L(N) = A_1 \cup A_2$.

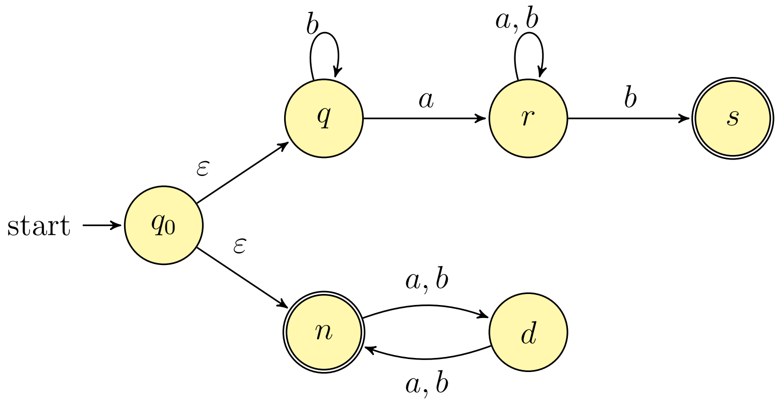
Proof idea: Use nondeterminism to choose which of N_1, N_2 to run.

Formal construction: Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and $N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ and assume $Q_1 \cap Q_2 = \emptyset$ and that $q_0 \notin Q_1 \cup Q_2$. Construct $N = (Q, \Sigma, \delta, q_0, F_1 \cup F_2)$ where

- $Q =$
- $\delta : Q \times \Sigma_\varepsilon \to \mathcal{P}(Q)$ is defined by, for $q \in Q$ and $x \in \Sigma_\varepsilon$:

Proof of correctness would prove that $L(N) = A_1 \cup A_2$ by considering an arbitrary string accepted by N, tracing an accepting computation of N on it, and using that trace to prove the string is in at least one of A_1, A_2; then, taking an arbitrary string in $A_1 \cup A_2$ and proving that it is accepted by N. Details left for extra practice.

Example: The language recognized by the NFA over $\{a, b\}$ with state diagram



is:

Could we do the same construction with DFA?

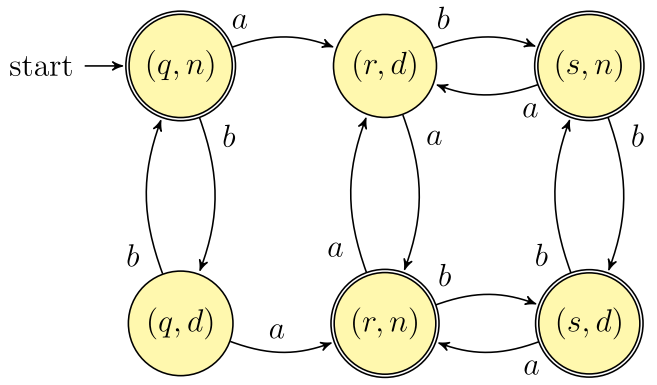
Happily, though, an analogous claim is true!

Suppose A_1, A_2 are languages over an alphabet Σ. **Claim:** if there is a DFA M_1 such that $L(M_1) = A_1$ and DFA M_2 such that $L(M_2) = A_2$, then there is another DFA, let's call it M, such that $L(M) = A_1 \cup A_2$. *Theorem 1.25 in Sipser, page 45*

Proof idea:

Formal construction:

Example: When $A_1 = \{w \mid w \text{ has an } a \text{ and ends in } b\}$ and $A_2 = \{w \mid w \text{ is of even length}\}$.



Suppose A_1, A_2 are languages over an alphabet Σ. **Claim:** if there is a DFA M_1 such that $L(M_1) = A_1$ and DFA M_2 such that $L(M_2) = A_2$, then there is another DFA, let's call it M, such that $L(M) = A_1 \cap A_2$. *Sipser Theorem 1.25, page 45*

Proof idea:

Formal construction: