

Week4 friday

Big picture: PDAs were motivated by wanting to add some memory of unbounded size to NFA. How do we accomplish a similar enhancement of regular expressions to get a syntactic model that is more expressive?

DFA, NFA, PDA: Machines process one input string at a time; the computation of a machine on its input string reads the input from left to right.

Regular expressions: Syntactic descriptions of all strings that match a particular pattern; the language described by a regular expression is built up recursively according to the expression’s syntax

Context-free grammars: Rules to produce one string at a time, adding characters from the middle, beginning, or end of the final string as the derivation proceeds.

Definitions below are on pages 101-102.

| Term | Typical symbol or Notation | Meaning |
|---|--------------------------------------|--|
| Context-free grammar (CFG) | G | $G = (V, \Sigma, R, S)$ |
| The set of variables | V | Finite set of symbols that represent phases in production pattern |
| The set of terminals | Σ | Alphabet of symbols of strings generated by CFG $V \cap \Sigma = \emptyset$ |
| The set of rules | R | Each rule is $A \rightarrow u$ with $A \in V$ and $u \in (V \cup \Sigma)^*$ |
| The start variable | S | Usually on left-hand-side of first/ topmost rule |
| Derivation | $S \Rightarrow \cdots \Rightarrow w$ | Sequence of substitutions in a CFG (also written $S \Rightarrow^* w$). At each step, we can apply one rule to one occurrence of a variable in the current string by substituting that occurrence of the variable with the right-hand-side of the rule. The derivation must end when the current string has only terminals (no variables) because then there are no instances of variables to apply a rule to. |
| Language generated by the context-free grammar G | $L(G)$ | The set of strings for which there is a derivation in G . Symbolically: $\{w \in \Sigma^* \mid S \Rightarrow^* w\}$ i.e. $\{w \in \Sigma^* \mid \text{there is derivation in } G \text{ that ends in } w\}$ |
| Context-free language | | A language that is the language generated by some context-free grammar |

Examples of context-free grammars, derivations in those grammars, and the languages generated by those grammars

$G_1 = (\{S\}, \{0\}, R, S)$ with rules

$$S \rightarrow 0S$$

$$S \rightarrow 0$$

In $L(G_1)$...

Not in $L(G_1)$...

$$G_2 = (\{S\}, \{0, 1\}, R, S)$$

$$S \rightarrow 0S \mid 1S \mid \varepsilon$$

In $L(G_2) \dots$

Not in $L(G_2) \dots$

$(\{S, T\}, \{0, 1\}, R, S)$ with rules

$$S \rightarrow T1T1T1T$$

$$T \rightarrow 0T \mid 1T \mid \varepsilon$$

In $L(G_3) \dots$

Not in $L(G_3) \dots$

$G_4 = (\{A, B\}, \{0, 1\}, R, A)$ with rules

$$A \rightarrow 0A0 \mid 0A1 \mid 1A0 \mid 1A1 \mid 1$$

In $L(G_4)$...

Not in $L(G_4)$...

Design a CFG to generate the language $\{a^n b^n \mid n \geq 0\}$

Sample derivation: