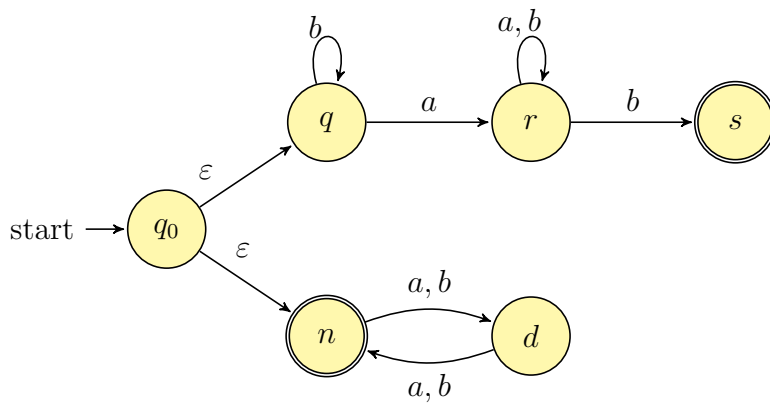


## Week2 friday

**Review:** The language recognized by the NFA over  $\{a, b\}$  with state diagram



is:

So far, we know:

- The collection of languages that are each recognizable by a DFA is **closed** under complementation.  
*Could we do the same construction with NFA?*
- The collection of languages that are each recognizable by a NFA is **closed** under complementation.  
*Could we do the same construction with DFA?*

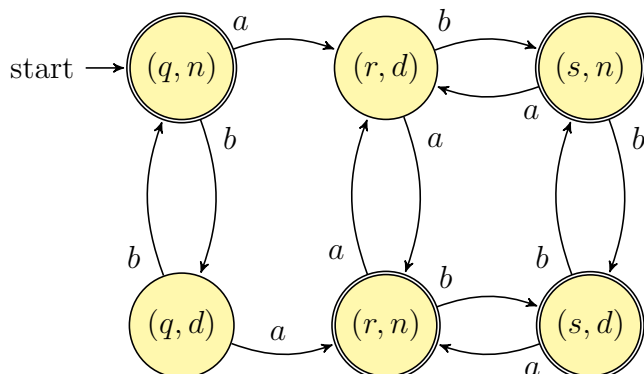
Happily, though, an analogous claim is true!

Suppose  $A_1, A_2$  are languages over an alphabet  $\Sigma$ . **Claim:** if there is a DFA  $M_1$  such that  $L(M_1) = A_1$  and DFA  $M_2$  such that  $L(M_2) = A_2$ , then there is another DFA, let's call it  $M$ , such that  $L(M) = A_1 \cup A_2$ .  
*Theorem 1.25 in Sipser, page 45*

**Proof idea:**

**Formal construction:**

**Example:** When  $A_1 = \{w \mid w \text{ has an } a \text{ and ends in } b\}$  and  $A_2 = \{w \mid w \text{ is of even length}\}$ .



Suppose  $A_1, A_2$  are languages over an alphabet  $\Sigma$ . **Claim:** if there is a DFA  $M_1$  such that  $L(M_1) = A_1$  and DFA  $M_2$  such that  $L(M_2) = A_2$ , then there is another DFA, let's call it  $M$ , such that  $L(M) = A_1 \cap A_2$ .  
*Footnote to Sipser Theorem 1.25, page 46*

**Proof idea:**

**Formal construction:**