

Week3 wednesday

Definition and Theorem: For an alphabet Σ , a language L over Σ is called **regular** exactly when L is recognized by some DFA, which happens exactly when L is recognized by some NFA, and happens exactly when L is described by some regular expression

We saw that: The class of regular languages is closed under complementation, union, intersection, set-wise concatenation, and Kleene star.

Prove or Disprove: There is some alphabet Σ for which there is some language recognized by an NFA but not by any DFA.

Prove or Disprove: There is some alphabet Σ for which there is some finite language not described by any regular expression over Σ .

Prove or Disprove: If a language is recognized by an NFA then the complement of this language is not recognized by any DFA.

Fix alphabet Σ . Is every language L over Σ regular?

| Set | Cardinality |
|--|-------------|
| $\{0, 1\}$ | |
| $\{0, 1\}^*$ | |
| $\mathcal{P}(\{0, 1\})$ | |
| The set of all languages over $\{0, 1\}$ | |
| The set of all regular expressions over $\{0, 1\}$ | |
| The set of all regular languages over $\{0, 1\}$ | |

Strategy: Find an **invariant** property that is true of all regular languages. When analyzing a given language, if the invariant is not true about it, then the language is not regular.

Pumping Lemma (Sipser Theorem 1.70): If A is a regular language, then there is a number p (a *pumping length*) where, if s is any string in A of length at least p , then s may be divided into three pieces, $s = xyz$ such that

- $|y| > 0$
- for each $i \geq 0$, $xy^iz \in A$
- $|xy| \leq p$.

Proof illustration

True or False: A pumping length for $A = \{0, 1\}^*$ is $p = 5$.

Week3 friday

Recap so far: In DFA, the only memory available is in the states. Automata can only “remember” finitely far in the past and finitely much information, because they can have only finitely many states. If a computation path of a DFA visits the same state more than once, the machine can’t tell the difference between the first time and future times it visits this state. Thus, if a DFA accepts one long string, then it must accept (infinitely) many similar strings.

Definition A positive integer p is a **pumping length** of a language L over Σ means that, for each string $s \in \Sigma^*$, if $|s| \geq p$ and $s \in L$, then there are strings x, y, z such that

$$s = xyz$$

and

$$|y| > 0, \quad \text{for each } i \geq 0, xy^iz \in L, \quad \text{and} \quad |xy| \leq p.$$

Negation: A positive integer p is **not a pumping length** of a language L over Σ iff

$$\exists s (|s| \geq p \wedge s \in L \wedge \forall x \forall y \forall z ((s = xyz \wedge |y| > 0 \wedge |xy| \leq p) \rightarrow \exists i (i \geq 0 \wedge xy^iz \notin L)))$$

Informally:

Restating **Pumping Lemma**: If L is a regular language, then it has a pumping length.

Contrapositive: If L has no pumping length, then it is nonregular.

The Pumping Lemma *cannot* be used to prove that a language *is* regular.

The Pumping Lemma **can** be used to prove that a language *is not* regular.

Extra practice: Exercise 1.49 in the book.

Proof strategy: To prove that a language L is **not** regular,

- Consider an arbitrary positive integer p
- Prove that p is not a pumping length for L
- Conclude that L does not have *any* pumping length, and therefore it is not regular.

Example: $\Sigma = \{0, 1\}$, $L = \{0^n 1^n \mid n \geq 0\}$.

Fix p an arbitrary positive integer. List strings that are in L and have length greater than or equal to p :

Pick $s =$

Suppose $s = xyz$ with $|xy| \leq p$ and $|y| > 0$.

Then when $i =$, $xy^iz =$

Example: $\Sigma = \{0, 1\}$, $L = \{ww^{\mathcal{R}} \mid w \in \{0, 1\}^*\}$. Remember that the reverse of a string w is denoted $w^{\mathcal{R}}$ and means to write w in the opposite order, if $w = w_1 \cdots w_n$ then $w^{\mathcal{R}} = w_n \cdots w_1$. Note: $\varepsilon^{\mathcal{R}} = \varepsilon$.

Fix p an arbitrary positive integer. List strings that are in L and have length greater than or equal to p :

Pick $s =$

Suppose $s = xyz$ with $|xy| \leq p$ and $|y| > 0$.

Then when $i =$, $xy^iz =$

Example: $\Sigma = \{0, 1\}$, $L = \{0^j1^k \mid j \geq k \geq 0\}$.

Fix p an arbitrary positive integer. List strings that are in L and have length greater than or equal to p :

Pick $s =$

Suppose $s = xyz$ with $|xy| \leq p$ and $|y| > 0$.

Then when $i =$, $xy^iz =$

Example: $\Sigma = \{0, 1\}$, $L = \{0^n1^m0^n \mid m, n \geq 0\}$.

Fix p an arbitrary positive integer. List strings that are in L and have length greater than or equal to p :

Pick $s =$

Suppose $s = xyz$ with $|xy| \leq p$ and $|y| > 0$.

Then when $i =$, $xy^iz =$

Extra practice:

| Language | $s \in L$ | $s \notin L$ | Is the language regular or nonregular? |
|---|-----------|--------------|--|
| $\{a^n b^n \mid 0 \leq n \leq 5\}$ | | | |
| $\{b^n a^n \mid n \geq 2\}$ | | | |
| $\{a^m b^n \mid 0 \leq m \leq n\}$ | | | |
| $\{a^m b^n \mid m \geq n + 3, n \geq 0\}$ | | | |
| $\{b^m a^n \mid m \geq 1, n \geq 3\}$ | | | |
| $\{w \in \{a, b\}^* \mid w = w^R\}$ | | | |
| $\{ww^R \mid w \in \{a, b\}^*\}$ | | | |