Week8 monday

Proof: Suppose towards a contradiction that there is a Turing machine that decides A_{TM} . We call this presumed machine M_{ATM} .

By assumption, for every Turing machine M and every string w

- If $w \in L(M)$, then the computation of M_{ATM} on $\langle M, w \rangle$ _____
- If $w \notin L(M)$, then the computation of M_{ATM} on $\langle M, w \rangle$ _____

Define a **new** Turing machine using the high-level description:

D = "On input $\langle M \rangle$, where M is a Turing machine:

- 1. Run M_{ATM} on $\langle M, \langle M \rangle \rangle$.
- 2. If M_{ATM} accepts, reject; if M_{ATM} rejects, accept."

Is D a Turing machine?

Is D a decider?

What is the result of the computation of D on $\langle D \rangle$?

Theorem (Sipser Theorem 4.22): A language is Turing-decidable if and only if both it and its complement are Turing-recognizable.
Proof, first direction: Suppose language L is Turing-decidable. WTS that both it and its complement are Turing-recognizable.
Proof, second direction: Suppose language L is Turing-recognizable, and so is its complement. WTS that L is Turing-decidable.
Give an example of a decidable set:
Give an example of a recognizable undecidable set:
Give an example of an unrecognizable set:

