Day9

Definition and Theorem: For an alphabet Σ , a language L over Σ is called **regular** exactly when L is recognized by some DFA, which happens exactly when L is recognized by some NFA, and happens exactly when L is described by some regular expression

We saw that: The class of regular languages is closed under complementation, union, intersection, set-wise concatenation, and Kleene star.

Extra practice:

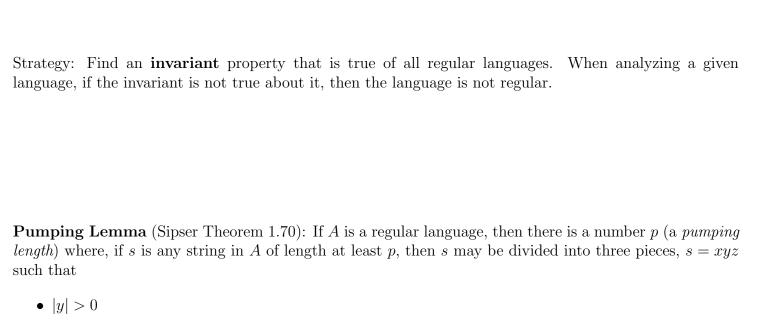
Disprove: There is some alphabet Σ for which there is some language recognized by an NFA but not by any DFA.

Disprove: There is some alphabet Σ for which there is some finite language not described by any regular expression over Σ .

Disprove: If a language is recognized by an NFA then the complement of this language is not recognized by any DFA.

Fix alphabet Σ . Is every language L over Σ regular?

| Set | Cardinality |
|---|-------------|
| $\{0,1\}$ | |
| $\{0,1\}^*$ | |
| $\mathcal{P}(\{0,1\})$ | |
| The set of all languages over $\{0,1\}$ | |
| The set of all regular expressions over $\{0,1\}$ | |
| The set of all regular languages over $\{0,1\}$ | |
| | |



- for each $i \ge 0$, $xy^i z \in A$
- $|xy| \leq p$.

Proof idea: In DFA, the only memory available is in the states. Automata can only "remember" finitely far in the past and finitely much information, because they can have only finitely many states. If a computation path of a DFA visits the same state more than once, the machine can't tell the difference between the first time and future times it visits this state. Thus, if a DFA accepts one long string, then it must accept (infinitely) many similar strings.

Proof illustration

True or False: A pumping length for $A = \{0, 1\}^*$ is p = 5.

True or False: A pumping length for $A = \{0, 1\}^*$ is p = 2.

True or False: A pumping length for $A = \{0, 1\}^*$ is p = 105.

Restating **Pumping Lemma**: If L is a regular language, then it has a pumping length.

Contrapositive: If L has no pumping length, then it is nonregular.

The Pumping Lemma cannot be used to prove that a language is regular.

The Pumping Lemma can be used to prove that a language is not regular.

Extra practice: Exercise 1.49 in the book.

Proof strategy: To prove that a language L is **not** regular,

- ullet Consider an arbitrary positive integer p
- ullet Prove that p is not a pumping length for L
- \bullet Conclude that L does not have any pumping length, and therefore it is not regular.

Negation: A positive integer p is **not a pumping length** of a language L over Σ iff

$$\exists s \ \big(\ |s| \geq p \land s \in L \land \forall x \forall y \forall z \ \big(\ (s = xyz \land |y| > 0 \land |xy| \leq p \) \rightarrow \exists i (i \geq 0 \land xy^iz \not\in L) \big) \ \big)$$

Day10

Proof strategy: To prove that a language L is **not** regular,

- \bullet Consider an arbitrary positive integer p
- Prove that p is not a pumping length for L. A positive integer p is **not a pumping length** of a language L over Σ iff

$$\exists s \ (\ |s| \ge p \land s \in L \land \forall x \forall y \forall z \ (\ (s = xyz \land |y| > 0 \land |xy| \le p \) \rightarrow \exists i (i \ge 0 \land xy^i z \notin L)) \)$$

Informally:

• Conclude that L does not have any pumping length, and therefore it is not regular.

Example: $\Sigma = \{0, 1\}, L = \{0^n 1^n \mid n \ge 0\}.$

Fix p an arbitrary positive integer. List strings that are in L and have length greater than or equal to p:

Pick s =

Suppose s = xyz with $|xy| \le p$ and |y| > 0.

Then when i =, $xy^iz =$

Example: $\Sigma = \{0, 1\}$, $L = \{ww^{\mathcal{R}} \mid w \in \{0, 1\}^*\}$. Remember that the reverse of a string w is denoted $w^{\mathcal{R}}$ and means to write w in the opposite order, if $w = w_1 \cdots w_n$ then $w^{\mathcal{R}} = w_n \cdots w_1$. Note: $\varepsilon^{\mathcal{R}} = \varepsilon$. Fix p an arbitrary positive integer. List strings that are in L and have length greater than or equal to p: Pick s =Suppose s = xyz with $|xy| \le p$ and |y| > 0. $, xy^iz =$ Then when i =**Example**: $\Sigma = \{0, 1\}, L = \{0^j 1^k \mid j \ge k \ge 0\}.$ Fix p an arbitrary positive integer. List strings that are in L and have length greater than or equal to p: Pick s =Suppose s = xyz with $|xy| \le p$ and |y| > 0. $, xy^iz =$ Then when i =**Example**: $\Sigma = \{0, 1\}, L = \{0^n 1^m 0^n \mid m, n \ge 0\}.$ Fix p an arbitrary positive integer. List strings that are in L and have length greater than or equal to p: Pick s =Suppose s = xyz with $|xy| \le p$ and |y| > 0.

 $, xy^iz =$

Then when i =

Extra practice:

| Language | $s \in L$ | $s \notin L$ | Is the language regular or nonregular? |
|---|-----------|--------------|--|
| $\{a^nb^n\mid 0\leq n\leq 5\}$ | | | |
| $\{b^na^n\mid n\geq 2\}$ | | | |
| $\{a^mb^n\mid 0\leq m\leq n\}$ | | | |
| $\{a^mb^n\mid m\geq n+3, n\geq 0\}$ | | | |
| $\{b^ma^n\mid m\geq 1, n\geq 3\}$ | | | |
| $\{w \in \{a, b\}^* \mid w = w^{\mathcal{R}}\}$ | | | |
| $\{ww^{\mathcal{R}} \mid w \in \{a, b\}^*\}$ | | | |
| | | | |