Week10 friday

NP-Complete Problems

3SAT: A literal is a Boolean variable (e.g. x) or a negated Boolean variable (e.g. \bar{x}). A Boolean formula is a **3cnf-formula** if it is a Boolean formula in conjunctive normal form (a conjunction of disjunctive clauses of literals) and each clause has three literals.

$$3SAT = \{ \langle \phi \rangle \mid \phi \text{ is a satisfiable 3cnf-formula} \}$$

Example string in 3SAT

$$\langle (x \vee \bar{y} \vee \bar{z}) \wedge (\bar{x} \vee y \vee z) \wedge (x \vee y \vee z) \rangle$$

Example string not in 3SAT

$$\langle (x \vee y \vee z) \wedge (x \vee y \vee \bar{z}) \wedge (x \vee \bar{y} \vee z) \wedge (x \vee \bar{y} \vee \bar{z}) \wedge (\bar{x} \vee y \vee z) \wedge (\bar{x} \vee y \vee \bar{z}) \wedge (\bar{x} \vee \bar{y} \vee z) \wedge (\bar{x} \vee \bar{y} \vee \bar{z}) \rangle$$

Cook-Levin Theorem: 3SAT is NP-complete.

Are there other NP-complete problems? To prove that X is NP-complete

- From scratch: prove X is in NP and that all NP problems are polynomial-time reducible to X.
- Using reduction: prove X is in NP and that a known-to-be NP-complete problem is polynomial-time reducible to X.



$$CLIQUE = \{\langle G, k \rangle \mid G \text{ is an undirected graph with a } k\text{-clique}\}$$

Example string in CLIQUE

Example string not in CLIQUE

Theorem (Sipser 7.32):

$$3SAT <_{P} CLIQUE$$

Given a Boolean formula in conjunctive normal form with k clauses and three literals per clause, we will map it to a graph so that the graph has a clique if the original formula is satisfiable and the graph does not have a clique if the original formula is not satisfiable.

The graph has 3k vertices (one for each literal in each clause) and an edge between all vertices except

- vertices for two literals in the same clause
- vertices for literals that are negations of one another

Example: $(x \vee \bar{y} \vee \bar{z}) \wedge (\bar{x} \vee y \vee z) \wedge (x \vee y \vee z)$

Model of Computation	Class of Languages
Deterministic finite automata: formal definition, how to design for a given language, how to describe language of a machine? Nondeterministic finite automata: formal definition, how to design for a given language, how to describe language of a machine? Regular expressions: formal definition, how to design for a given language, how to describe language of expression? Also: converting between different models.	Class of regular languages: what are the closure properties of this class? which languages are not in the class? using pumping lemma to prove nonregularity.
Push-down automata: formal definition, how to design for a given language, how to describe language of a machine? Context-free grammars: formal definition, how to design for a given language, how to describe language of a grammar?	Class of context-free languages: what are the closure properties of this class? which languages are not in the class?
Turing machines that always halt in polynomial time	P
Nondeterministic Turing machines that always halt in polynomial time	NP
Deciders (Turing machines that always halt): formal definition, how to design for a given language, how to describe language of a machine?	Class of decidable languages: what are the closure properties of this class? which languages are not in the class? using diagonalization and mapping reduction to show undecidability
Turing machines formal definition, how to design for a given language, how to describe language of a machine?	Class of recognizable languages: what are the closure properties of this class? which languages are not in the class? using closure and mapping reduction to show unrecognizability

Given	a	language,	prove	it	is	regui	ar
Given	а	language,	prove	ւլ	\mathbf{IS}	regu	aı

Strategy 1: construct DFA recognizing the language and prove it works.

Strategy 2: construct NFA recognizing the language and prove it works.

Strategy 3: construct regular expression recognizing the language and prove it works.

"Prove it works" means . . .

Example: $L = \{w \in \{0,1\}^* \mid w \text{ has odd number of 1s or starts with } 0\}$

Using NFA

Using regular expressions $\,$

Example: Select all and only the options that result in a true statement: "To show a language A is not regular, we can..."

- a. Show A is finite
- b. Show there is a CFG generating A
- c. Show A has no pumping length
- d. Show A is undecidable

Example: What is the language generated by the CFG with rules

$$S \rightarrow aSb \mid bY \mid Ya$$

$$Y \rightarrow bY \mid Ya \mid \varepsilon$$



Example:	Prove t	that th	e class o	of decidal	ble langı	ages is o	closed un	der conc	atenation	