HW1CSE105W24: Homework assignment 1

CSE105W24

Due: January 18th at 5pm (no penalty late submission until 8am next morning), via Gradescope

In this assignment,

You will practice reading and applying the definitions of alphabets, strings, languages, Kleene star, and regular expressions. You will use regular expressions and relate them to languages and finite automata. You will use precise notation to formally define the state diagram of finite automata, and you will use clear English to describe computations of finite automata informally.

Resources: To review the topics for this assignment, see the class material from Week 1. We will post frequently asked questions and our answers to them in a pinned Piazza post.

Reading and extra practice problems: Sipser Section 0, 1.3, 1.1. Chapter 1 exercises 1.1, 1.2, 1.3, 1.18, 1.23.

For all HW assignments: Weekly homework may be done individually or in groups of up to 3 students. You may switch HW partners for different HW assignments. Please ensure your name(s) and PID(s) are clearly visible on the first page of your homework submission and then upload the PDF to Gradescope. If working in a group, submit only one submission per group: one partner uploads the submission through their Gradescope account and then adds the other group member(s) to the Gradescope submission by selecting their name(s) in the "Add Group Members" dialog box. You will need to re-add your group member(s) every time you resubmit a new version of your assignment. Each homework question will be graded either for correctness (including clear and precise explanations and justifications of all answers) or fair effort completeness. For "graded for correctness" questions: collaboration is allowed only with CSE 105 students in your group; if your group has questions about a problem, you may ask in drop-in help hours or post a private post (visible only to the Instructors) on Piazza. For "graded for completeness" questions: collaboration is allowed with any CSE 105 students this quarter; if your group has questions about a problem, you may ask in drop-in help hours or post a public post on Piazza.

All submitted homework for this class must be typed. You can use a word processing editor if you like (Microsoft Word, Open Office, Notepad, Vim, Google Docs, etc.) but you might find

it useful to take this opportunity to learn LaTeX. LaTeX is a markup language used widely in computer science and mathematics. The homework assignments are typed using LaTeX and you can use the source files as templates for typesetting your solutions. To generate state diagrams of machines, we recommend using Flap.js or JFLAP. Photographs of clearly hand-drawn diagrams may also be used. We recommend that you submit early drafts to Gradescope so that in case of any technical difficulties, at least some of your work is present. You may update your submission as many times as you'd like up to the deadline.

Integrity reminders

- Problems should be solved together, not divided up between the partners. The homework is designed to give you practice with the main concepts and techniques of the course, while getting to know and learn from your classmates.

- You may not collaborate on homework questions graded for correctness with anyone other than your group members. You may ask questions about the homework in office hours (of the instructor, TAs, and/or tutors) and on Piazza (as private notes viewable only to the Instructors). You *cannot* use any online resources about the course content other than the class material from this quarter – this is primarily to ensure that we all use consistent notation and definitions (aligned with the textbook) and also to protect the learning experience you will have when the 'aha' moments of solving the problem authentically happen.

- Do not share written solutions or partial solutions for homework with other students in the class who are not in your group. Doing so would dilute their learning experience and detract from their success in the class.

You will submit this assignment via Gradescope (https://www.gradescope.com) in the assignment called "hw1CSE105W24".

Assigned questions

1. (*Graded for completeness*) [1] **Finding examples and edge cases** (12 points):

With $\Sigma_1 = \{0, 1\}$ and $\Sigma_2 = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$ and $\Gamma = \{0, 1, x, y, z\}$

(a) Give an example of the shortest string over Σ_1 that is meaningful to you in some way, and explain why it's meaningful to you.

(b) List all examples of strings of length 1 over Σ_2 and explain why your list is exhaustive.

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[1]This means you will get full credit so long as your submission demonstrates honest effort to answer the question. You will not be penalized for incorrect answers. To demonstrate your honest effort in answering the question, we expect you to include your attempt to answer *each* part of the question. If you get stuck with your attempt, you can still demonstrate your effort by explaining where you got stuck and what you did to try to get unstuck.

(c) Calculate the number of distinct strings of length 3 over Γ and explain your calculation.

(d) With the ordering $x < y < z < 0 < 1$, list the first ten strings over Γ in string order.

(e) Give an example of a finite set that is a language over Σ_1 and over Σ_2 and over Γ, or explain why there is no such set.

(f) Give an example of an infinite set that is a language over Σ_1 and over Γ, or explain why there is no such set.

2. (*Graded for completeness*) **Regular expressions** (10 points):

(a) Give three regular expressions that all describe the set of all strings over $\{a, b\}$ that have even length. Ungraded bonus challenge: Make the expressions as different as possible!

(b) A friend tells you that each regular expression that has a Kleene star (*) describes an infinite language. Are they right? Either help them justify their claim or give a counterexample to disprove it and then fix the formula.

3. (*Graded for correctness*) [2] **Functions over languages** (15 points):

For languages L_1, L_2 over the alphabet $\Sigma_1 = \{0, 1\}$, we have the associated sets of strings

$$SUBSTRING(L_1) = \{w \in \Sigma_1^* \mid \text{there exist } a, b \in \Sigma_1^* \text{ such that } awb \in L_1\}$$

and

$$L_1 \circ L_2 = \{w \in \Sigma_1^* \mid w = uv \text{ for some strings } u \in L_1 \text{ and } v \in L_2\}$$

(a) Specify an example language A over Σ_1 such that $A \neq \emptyset$ and yet $\text{SUBSTRING}(A) = \emptyset$, or explain why there is no such example. A complete solution will include either (1) a precise and clear description of your example language A and a precise and clear description of the result of computing $\text{SUBSTRING}(A)$ using relevant definitions to justify this description and to justify the set equality with $\emptyset$, or (2) a sufficiently general and correct argument why there is no such example, referring back to the relevant definitions.

(b) Specify example languages B, C over Σ_1 such that $B \neq \Sigma_1^*$ and $C \neq \Sigma_1^*$ and yet $B \circ C = \Sigma_1^*$, or explain why there are no such examples. A complete solution will include either (1) a precise and clear description of your example languages B, C and a precise and clear description of the result of computing $B \circ C$ using relevant definitions to justify this description and to justify the set equality with Σ_1^*, or (2) a sufficiently general and correct argument why there is no such example, referring back to the relevant definitions.

(c) Specify example **finite** languages L_1, L_2 over Σ_1 such that $L_1 \circ L_2 \neq L_1$ but $|L_1 \circ L_2| = |L_1|$, or explain why there are no such examples. A complete solution will include either (1) a precise and clear description of your example languages L_1, L_2 and a precise and clear description

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[2]This means your solution will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

of the result of computing $L_1 \circ L_2$ using relevant definitions to justify this description and to justify the cardinality claims and set (in)equality claims, or (2) a sufficiently general and correct argument why there is no such example, referring back to the relevant definitions.

4. (*Graded for correctness*) **Finite automata** (13 points):

Consider the finite automaton $(Q, \Sigma, \delta, q_0, F)$ whose state diagram is depicted below

where $Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{0, 1\}$, and $F = \{q_0\}$, and $\delta : Q \times \Sigma \to Q$ is specified by the look-up table

	0	1
q_0	q_1	q_0
q_1	q_2	q_3
q_2	q_0	q_3
q_3	q_3	q_3

(a) A friend tries to summarize the transition function with the formula

$$\delta(q_i, x) = \begin{cases} q_0 & \text{when } i = 0 \text{ and } x = 1 \\ q_3 & \text{when } 0 < i \leq 3 \text{ and } x = 1 \\ q_j & \text{when } j = (i+1) \mod 3 \text{ and } x = 0 \end{cases}$$

Are they right? Either help them justify their claim or give a counterexample to disprove it and then fix their formula.

(b) Give a regular expression R so that $L(R)$ is the language recognized by this finite automaton. Justify your answer by referring to the definition of the semantics of regular expressions and computations of finite automata. Include an explanation for why each string in $L(R)$ is accepted by the finite automaton *and* for why each string not in $L(R)$ is rejected by the finite automaton.

(c) Keeping the same set of states $Q = \{q_0, q_1, q_2, q_3\}$, alphabet $\Sigma = \{0, 1\}$, same start state q_0, and same transition function δ, choose a new set of accepting states F_{new} so that the new finite automaton that results accepts at least one string that the original one rejected **and** rejects at least one string that the original one accepted, or explain why there is no such choice of F_{new}. A complete solution will include either (1) a precise and clear description of your choice of F_{new} and a precise and clear the two example strings using relevant definitions to justify them, or (2) a sufficiently general and correct argument why there is no such example, referring back to the relevant definitions.

HW2CSE105W24: Homework assignment 2 Due: February 1st at 5pm (no penalty late submission until 8am next morning), via Gradescope

In this assignment,

You will practice designing multiple representations of regular languages and working with general constructions of automata to demonstrate the richness of the class of regular languages. You will also distinguish between regular and nonregular languages using both closure arguments and the pumping lemma.

Resources: To review the topics for this assignment, see the class material from Weeks 2-4. We will post frequently asked questions and our answers to them in a pinned Piazza post.

Reading and extra practice problems: Sipser Chapter 1, Section 2.2. Chapter 1 exercises 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 1.10, 1.11, 1.12, 1.14, 1.15, 1.16, 1.17, 1.19, 1.20, 1.21, 1.22, 1.29, 1.30. Chapter 1 problems 1.49, 1.50, 1.51.

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Assigned questions

1. **Number representations** (12 points): Integers can be represented using base b expansions, for a convenient choice of base b: for b an integer greater than 1 and n a positive integer, the **base b expansion of** n is defined to be

$$(a_{k-1} \cdots a_1 a_0)_b$$

where k is a positive integer, $a_0, a_1, \ldots, a_{k-1}$ are nonnegative integers less than b, $a_{k-1} \neq 0$, and

$$n = \sum_{i=0}^{k-1} a_i b^i$$

Notice: *The base b expansion of a positive integer n is a string over the alphabet $\{x \in \mathbb{Z} \mid 0 \leq x < b\}$ whose leftmost character is nonzero.*

An important property of base b expansions of integers is that, for each integer b greater than 1, each positive integer $n = (a_{k-1} \cdots a_1 a_0)_b$, and each nonnegative integer a less than b,

$$bn + a = (a_{k-1} \cdots a_1 a_0 a)_b$$

In other words, shifting the base b expansion to the left results in multiplying the integer value by the base. In this question we'll explore building deterministic finite automata that recognize languages that correspond to useful sets of integers.

(a) (*Graded for correctness*) [3] Design a DFA that recognizes the set of binary (base 2) expansions of positive integers that are powers of 2. A complete solution will include the state diagram of your DFA and a brief justification of your construction by explaining the role each state plays in the machine, as well as a brief justification about how the strings accepted and rejected by the machine connect to the specified language.

Hints: (1) A power of 2 is an integer x that can be written as 2^y for some nonnegative integer y, (2) the DFA should accept the strings 100, 10 and 100000 and should reject the strings 010, 1101, and ε (can you see why?).

(b) (*Graded for completeness*) [4] Consider arbitrary positive integer m. Design a DFA that recognizes the set of binary (base 2) expansions of positive integers that are multiples of m. A complete solution will include the formal definition of your DFA (paramterized by m) and a brief justification of your construction by explaining the role each state plays in the machine, as well as a brief justification about how the strings accepted and rejected by the machine connect to the specified language.

Hints: (1) Consider having a state for each possible remainder upon division by m. (2) To determine transitions, notice that reading a new character will shift what we already read over by one slot.

(c) (*Graded for correctness*) Choose a positive integer m_0 between 4 and 8 (inclusive) and draw the state diagram of a DFA recognizing the language over $\{0, 1, 2\}$

$$\{w \in \{0, 1, 2\}^* \mid w \text{ is a base 3 expansion of a positive integer that is a multiple of } m_0\}$$

A complete solution will include the state diagram of your DFA and a brief justification of your construction by explaining the role each state plays in the machine, as well as a brief justification about how the strings accepted and rejected by the machine connect to the specified language.

Bonus extension to think about (ungraded): Which other languages related to sets of integers can be proved to be regular using a similar strategy?

2. **Multiple representations** (10 points): For any language $L \subseteq \Sigma^*$, recall that we define its *complement* as

$$\overline{L} := \Sigma^* - L = \{w \in \Sigma^* \mid w \notin L\}$$

That is, the complement of L contains all and only those strings which are not in L. Our notation for regular expressions does not include the complement symbol. However, it turns out that the

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complement of a language described by a regular expression is guaranteed to also be describable by a (different) regular expression. For example, over the alphabet $\Sigma = \{0, 1\}$, the complement of the language described by the regular expression Σ^*0 is described by the regular expression $\varepsilon \cup \Sigma^*1$ because any string that does not end in 0 must either be the empty string or end in 1.

For each of the regular expressions R over the alphabet $\Sigma = \{a, b\}$ below, write the regular expression for $\overline{L(R)}$. Your regular expressions may use the symbols $\varnothing$, ε, a, b, and the following operations to combine them: union, concatenation, and Kleene star.

Briefly justify why your solution for each part works by giving plain English descriptions of the language described by the regular expression and of its complement and connecting them to the regular expression via relevant definitions. An English description that is more detailed than simply negating the description in the original language will likely be helpful in the justification.

Alternatively, you can justify your solution by first designing a DFA that recognizes $L(R)$, using the construction from class and the book to modify this DFA to get a new DFA that recognizes $\overline{L(R)}$, and then applying the constructions from class and the book to convert this new DFA to a regular expression.

For each part of the question, clearly state which approach you're taking and include enough intermediate steps to illustrate your work.

(a) (*Graded for correctness*) a^*b^*

(b) (*Graded for correctness*) $(a \cup b)ab^*$

3. **Applying general constructions** (12 points): In this question, you'll practice working with formal general constructions for NFAs and translating between state diagrams and formal definitions. Consider the following general construction: Let $N_1 = (Q, \Sigma, \delta_1, q_1, F_1)$ be a NFA and assume that $q_0 \notin Q$. Define the new NFA $N_2 = (Q \cup \{q_0\}, \Sigma, \delta_2, q_0, \{q_1\})$ where

$$\delta_2 : (Q \cup \{q_0\}) \times \Sigma_\varepsilon \to \mathcal{P}(Q \cup \{q_0\})$$

is defined by

$$\delta_2(q, a) = \begin{cases} \{q' \in Q \mid q \in \delta_1(q', a)\} & \text{if } q \in Q, \, a \in \Sigma_\varepsilon \\ F_1 & \text{if } q = q_0, \, a = \varepsilon \\ \emptyset & \text{if } q = q_0, \, a \in \Sigma \end{cases}$$

(a) (*Graded for correctness*) Illustrate this construction by defining a specific example NFA N and applying the construction above to create a new NFA. Your example NFA should

- Have exactly three states (all reachable from the start state),
- Have at least one spontaneous move (arrow labelled ε),
- Accept at least one string and reject at least one string, and
- Not have any states labelled q_0.

Apply the construction above to create the new NFA. A complete submission will include the state diagram of your example NFA N and the state diagram of the NFA resulting from this construction.

(b) (*Graded for correctness*) Use Theorem 1.39 on page 55 of the book (see also page 7 in Week 3 notes) to construct a DFA equivalent to your example NFA N from part (a). A complete submission will include the state diagram of your example NFA N and the state diagram of the DFA resulting from this construction, with the correct state labels for this DFA. You may prune the DFA so that only the "macro-states" reachable from the start state are included.

(c) (*Graded for completeness*) Explain the relationship between N_1 and N_2 in the general construction. Give an example string that is accepted by your example NFA N and is rejected by the NFA that results from applying the general construction that illustrates this relationship, or explain why there is no such example string.

4. **Pumping** (8 points):

(a) (*Graded for correctness*) Give an example of a language over the alphabet $\{0, 1\}$ that has cardinality 3 and for which 5 is a pumping length and 4 is not a pumping length. A complete solution will give a clear and precise description of the language, a justification for why 5 is a pumping length, and a justification for why 4 is not a pumping length. Is this language regular?

(b) (*Graded for completeness*) Consider the following attempted "proof" that the set

$$X = \{uw \mid u \text{ and } w \text{ are strings over } \{0, 1\} \text{ and have the same length}\}$$

is nonregular.

> "Proof" that X is not regular using the Pumping Lemma: Let p be an arbitrary positive integer. We will show that p is not a pumping length for X.
> Choose s to be the string $1^p 0^p$, which is in X because we can choose $u = 1^p$ and $w = 0^p$ which each have length p. Since s is in X and has length greater than or equal to p, if p were to be a pumping length for X, s ought to be pump'able. That is, there should be a way of dividing s into parts x, y, z where $s = xyz$, $|y| > 0$, $|xy| \leq p$, and for each $i \geq 0$, $xy^i z \in X$. Suppose x, y, z are such that $s = xyz$, $|y| > 0$ and $|xy| \leq p$. Since the first p letters of s are all 1 and $|xy| \leq p$, we know that x and y are made up of all 1s. If we let $i = 2$, we get a string $xy^i z$ that is not in X because repeating y twice adds 1s to u but not to w, and strings in X are required to have u and w be the same length. Thus, s is not pumpable (even though it should have been if p were to be a pumping length) and so p is not a pumping length for X. Since p was arbitrary, we have demonstrated that X has no pumping length. By the Pumping Lemma, this implies that X is nonregular.

Find the (first and/or most significant) logical error in the "proof" and describe why it's wrong. Then, either prove that the set is actually regular (by finding a regular expression that describes it or a DFA/NFA that recognizes it, and justifying why) **or** fix the proof so that it is logically sound.

(c) (*Graded for completeness*) In class and in the reading so far, we've seen the following examples of nonregular sets:

$$\{0^n1^n \mid n \geq 0\} \qquad \{0^n1^m \mid 0 \leq m \leq n\} \qquad \{0^n1^m0^n \mid n, m \geq 0\}$$

$$\{0^n1^n \mid n \geq 2\} \qquad \{0^i1^{2i} \mid 0 \leq i\} \qquad \{w \in \{0,1\}^* \mid w = w^R\}$$

$$\{0^n1^m \mid 0 \leq n \leq m\} \qquad \{0^i1^{i+1} \mid 0 \leq i\} \qquad \{ww^R \mid w \in \{0,1\}^*\}$$

Modify one of these sets in some way and use the Pumping Lemma to prove that the resulting set is still nonregular.

5. **Regular and nonregular languages** (8 points): In Week 2's review quiz, we saw the definition that a set X is said to be **closed under an operation** if, for any elements in X, applying to them gives an element in X. For example, the set of integers is closed under multiplication because if we take any two integers, their product is also an integer .

Prove or disprove each closure claim statement below about the class of regular languages and the class of nonregular languages. Your arguments may refer to theorems proved in the textbook and class, and if they do, should include specific page numbers and references (i.e. write out the claim that was proved in the book and/or class).

Recall the definitions we have:

For languages L_1, L_2 over the alphabet $\Sigma_1 = \{0, 1\}$, we have the associated sets of strings

$$SUBSTRING(L_1) = \{w \in \Sigma_1^* \mid \text{there exist } a, b \in \Sigma_1^* \text{ such that } awb \in L_1\}$$

and

$$L_1 \circ L_2 = \{w \in \Sigma_1^* \mid w = uv \text{ for some strings } u \in L_1 \text{ and } v \in L_2\}$$

(a) (*Graded for completeness*) The set of regular languages over $\{0, 1\}$ is closed under set-wise concatenation.

(b) (*Graded for completeness*) The set of nonregular languages over $\{0, 1\}$ is closed under set-wise concatenation.

(c) (*Graded for completeness*) The set of regular languages over $\{0, 1\}$ is closed under the $SUBSTRING$ operation.

(d) (*Graded for completeness*) The set of nonregular languages over $\{0, 1\}$ is closed under the $SUBSTRING$ operation.

HW3CSE105W24: Homework assignment 3 Due: February 15th at 5pm (no penalty late submission until 8am next morning), via Gradescope

In this assignment,

You will practice with the definition of pushdown automata and context-free grammars and reason about regular and context-free languages. You will also practice analyzing and designing Turing machines.

Resources: To review the topics for this assignment, see the class material from Weeks 4-6. We will post frequently asked questions and our answers to them in a pinned Piazza post.

Reading and extra practice problems: Sipser Chapter 2, Section 3.1 Chapter 2 exercises 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.9, 2.10, 2.11, 2.12, 2.13, 2.16, 2.17. Chapter 3 exercises 3.1, 3.2.

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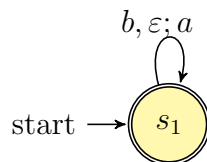
You will submit this assignment via Gradescope (https://www.gradescope.com) in the assignment called "hw3CSE105W24".
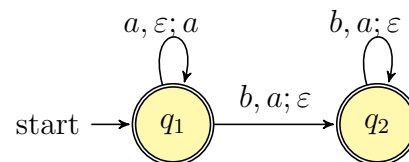
Assigned questions

1. **Constructions** (18 points):

Consider the push-down automata M_1 and M_2 over $\{a, b\}$ with stack alphabet $\{a, b\}$ whose state diagrams are

State diagram for M_1

$$b, \varepsilon; a$$

start $\longrightarrow$ s_1

State diagram for M_2

$$a, \varepsilon; a \qquad b, a; \varepsilon$$

start $\longrightarrow$ q_1 $\xrightarrow{b, a; \varepsilon}$ q_2

(a) (*Graded for completeness*) [5] What is the language A_1 recognized by M_1 and what is the language A_2 recognized by M_2? Include a sample string that is accepted and one that is rejected for each of these PDA. Justify these examples with sample accepting computations or with an explanation why there is no accepting computation.

(b) (*Graded for correctness*) [6] Design CFGs G_1 and G_2 over $\{a, b\}$ so that $L(G_1) = A_1$ and $L(G_2) = A_2$. A complete solution will include precise definitions for each of the parameters

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[5]This means you will get full credit so long as your submission demonstrates honest effort to answer the question. You will not be penalized for incorrect answers. To demonstrate your honest effort in answering the question, we expect you to include your attempt to answer *each* part of the question. If you get stuck with your attempt, you can still demonstrate your effort by explaining where you got stuck and what you did to try to get unstuck.

[6]This means your solution will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

required to specify a CFG, as well as a brief explanation about why each string in A_i can be derived in G_i and each string not in A_i cannot be derived in G_i (for $i = 1, 2$).

(c) (*Graded for completeness*) Remember that the definition of set-wise concatenation is: for languages L_1, L_2 over the alphabet Σ, we have the associated set of strings

$$L_1 \circ L_2 = \{w \in \Sigma^* \mid w = uv \text{ for some strings } u \in L_1 \text{ and } v \in L_2\}$$

In class (and in the review quiz) we learned that the class of context-free languages is closed under set-wise concatenation. The proof of this closure claim using CFGs uses the construction: given $G_1 = (V_1, \Sigma, R_1, S_1)$ and $G_2 = (V_2, \Sigma, R_2, S_2)$ with $V_1 \cap V_2 = \emptyset$ and $S_{new} \notin V_1 \cup V_2$, define a new CFG

$$G = (V_1 \cup V_2 \cup \{S_{new}\}, \Sigma, R_1 \cup R_2 \cup \{S_{new} \to S_1 S_2\}, S_{new})$$

Apply this construction to your grammars from part (b) and give a sample derivation of a string in $A_1 \circ A_2$ in this resulting grammar.

(d) (*Graded for correctness*) If we try to extrapolate the construction that we used to prove that the class of regular languages is closed under set-wise concategation, we would get the following construction for PDAs: Given $M_1 = (Q_1, \Sigma, \Gamma_1, \delta_1, q_1, F_1)$ and $M_2 = (Q_2, \Sigma, \Gamma_2, \delta_2, q_2, F_2)$ with $Q_1 \cap Q_2 = \emptyset$, define $Q = Q_1 \cup Q_2$, $\Gamma = \Gamma_1 \cup \Gamma_2$, and

$$M = (Q, \Sigma, \Gamma, \delta, q_1, F_2)$$

with $\delta : Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \to \mathcal{P}(Q \times \Gamma_\varepsilon)$ given by

$$\delta(q, a, b) = \begin{cases} \delta_1(q, a, b) & \text{if } q \in Q_1 \setminus F_1,\ a \in \Sigma_\varepsilon,\ b \in \Gamma_{1\varepsilon} \\ \delta_2(q, a, b) & \text{if } q \in Q_2,\ a \in \Sigma_\varepsilon,\ b \in \Gamma_{2\varepsilon} \\ \delta_1(q, a, b) & \text{if } q \in F_1,\ a \in \Sigma \text{ or } b \in \Gamma_1 \\ \delta_1(q, a, b) \cup \{(q_2, \varepsilon)\} & \text{if } q \in F_1,\ a = \varepsilon,\ b = \varepsilon \\ \emptyset & \text{otherwise} \end{cases}$$

Apply this construction to the machines M_1 and M_2 from part (a), and then use the resulting PDA to prove that *this* construction cannot be used to prove that the class of context-free languages is closed under set-wise concatenation. A complete solution will include (1) the state diagram of the machine M that results from applying this construction to M_1 and M_2, (2) an example of a string that is accepted by this PDA M but that is **not** in the language $A_1 \circ A_2$ with a description of the computation that witnesses that this string is accepted by M and an explanation of why this string is not in $A_1 \circ A_2$ by referring back to the definitions of A_1, A_2, and set-wise concatenation.

2. **Regular languages are context-free** (10 points):

Informally, we think of regular languages as potentially simpler than context-free languages. In this question, you'll make this precise by showing that every regular language is context-free, in two ways.

(a) (*Graded for correctness*) When we first introduced PDAs we saw that any NFA can be transformed to a PDA by not using the stack of the PDA at all. Make this precise by completing the following construction: Given a NFA $N = (Q, \Sigma, \delta_N, q_0, F)$ we define a PDA M with $L(M) = L(N)$ by choosing ... A complete solution will have precise, correct definitions for each of the defining parameters of M: the set of states, the input alphabet, the stack alphabet, the transition function, the start state, and the set of accept states. Be careful to use notation that matches the types of the objects involved.

(b) (*Graded for correctness*) In the book on page 107, the top paragraph describes a procedure for converting DFA to CFGs:

> You can convert any DFA into an equivalent CFG as follows. Make a variable R_i for each state q_i of the DFA. Add the rule $R_i \to aR_j$ to the CFG if $\delta(q_i, a) = q_j$ is a transition in the DFA. Add the rule $R_i \to \varepsilon$ if q_i is an accept state of the DFA. Make R_0 the start variable ofthe grammar, where q_0 is the start state of the machine. Verify on your own that the resulting CFG generates the same language that the DFA recognizes.

Use this construction to get a context-free grammar generating the language

$$\{w \in \{0, 1\}^* \mid w \text{ does not have 11 as a substring}\}$$

by (1) designing a DFA that recognizes this language and then (2) applying the construction from the book to convert the DFA to an equivalent CFG. A complete submission will include the state diagram of the DFA, a brief justification of why it recognizes the language, and then the complete and precise definition of the CFG that results from applying the construction from the book to this DFA. *Ungraded bonus: take a sample string in the language and see how the computation of the DFA on this string translates to a derivation in your grammar.*

3. **Classifying languages** (12 points): On page 4 of the week 4 notes, we have the following list of languages over the alphabet $\{a, b\}$

$$\{a^n b^n \mid 0 \le n \le 5\} \quad \{b^n a^n \mid n \ge 2\} \quad \{a^m b^n \mid 0 \le m \le n\}$$
$$\{a^m b^n \mid m \ge n + 3, n \ge 0\} \quad \{b^m a^n \mid m \ge 1, n \ge 3\}$$
$$\{w \in \{a, b\}^* \mid w = w^{\mathcal{R}}\} \quad \{ww^{\mathcal{R}} \mid w \in \{a, b\}^*\}$$

(a) (*Graded for completeness*) Pick one of the regular languages and design a regular expression that describes it. Briefly justify your regular expression by connecting the subexpressions of it to the intended language and referencing relevant definitions.

(b) (*Graded for completeness*) Pick another one of the regular languages and design a DFA that recognizes it. Draw the state diagram of your DFA. Briefly justify your design by explaining the role each state plays in the machine, as well as a brief justification about how the strings accepted and rejected by the machine connect to the specified language.

(c) (*Graded for completeness*) Pick one of the nonregular languages and design a PDA that recognizes it. Draw the state diagram of your PDA. Briefly justify your design by explaining the role each state plays in the machine, as well as a brief justification about how the strings accepted and rejected by the machine connect to the specified language.

(d) (*Graded for completeness*) Pick one of the nonregular languages and write a CFG that generates it. Briefly justify your design by by demonstrating how derivations in the grammar relate to the intended language.

4. **Turing machines** (10 points): Consider the Turing machine T over the input alphabet $\Sigma = \{0, 1\}$ with the state diagram below (the tape alphabet is $\Gamma = \{0, 1, X, \square\}$). Convention: we do not include the node for the reject state $qrej$ and any missing transitions in the state diagram have value $(qrej, \square, R)$

(a) (*Graded for correctness*) Specify an example string w_1 of length 4 over Σ that is **accepted** by this Turing machine, or explain why there is no such example. A complete solution will include either (1) a precise and clear description of your example string and a precise and clear description of the accepting computation of the Turing machine on this string or (2) a sufficiently general and correct argument why there is no such example, referring back to the relevant definitions.

To describe a computation of a Turing machine, include the contents of the tape, the state of the machine, and the location of the read/write head at each step in the computation.

Hint: In class we've drawn pictures to represent the configuration of the machine at each step in a computation. You may do so or you may choose to describe these configurations in words.

(b) (*Graded for correctness*) Specify an example string w_2 of length 3 over Σ that is **rejected** by this Turing machine or explain why there is no such example. A complete solution will include either (1) a precise and clear description of your example string and a precise and clear description of the rejecting computation of the Turing machine on this string or (2) a sufficiently general and correct argument why there is no such example, referring back to the relevant definitions.

(c) (*Graded for correctness*) Specify an example string w_3 of length 2 over Σ on which the computation of this Turing machine **loops** or explain why there is no such example. A complete solution will include either (1) a precise and clear description of your example string and a precise and clear description of the looping (non-halting) computation of the

Turing machine on this string or (2) a sufficiently general and correct argument why there is no such example, referring back to the relevant definitions.

(d) (*Graded for completeness*) Write an implementation level description of the Turing machine T.

HW4CSE105W24: Homework assignment 4 Due: February 29th at 5pm (no penalty late submission until 8am next morning), via Gradescope

In this assignment, You will practice analyzing, designing, and working with Turing machines. You will use general constructions and specific machines to explore the classes of recognizable and decidable languages. You will explore various ways to encode machines as strings so that computational problems can be recognized.

Resources: To review the topics for this assignment, see the class material from Weeks 5-7. We will post frequently asked questions and our answers to them in a pinned Piazza post.

Reading and extra practice problems: Sipser Sections 3.1, 3.3, 4.1 Chapter 3 exercises 3.1, 3.2, 3.5, 3.8. Chapter 4 exercises 4.1, 4.2, 4.3, 4.4, 4.5.

For all HW assignments: Weekly homework may be done individually or in groups of up to 3 students. You may switch HW partners for different HW assignments. Please ensure your name(s) and PID(s) are clearly visible on the first page of your homework submission and then upload the PDF to Gradescope. If working in a group, submit only one submission per group: one partner uploads the submission through their Gradescope account and then adds the other group member(s) to the Gradescope submission by selecting their name(s) in the "Add Group Members" dialog box. You will need to re-add your group member(s) every time you resubmit a new version of your assignment. Each homework question will be graded either for correctness (including clear and precise explanations and justifications of all answers) or fair effort completeness. For "graded for correctness" questions: collaboration is allowed only with CSE 105 students in your group; if your group has questions about a problem, you may ask in drop-in help hours or post a private post (visible only to the Instructors) on Piazza. For "graded for completeness" questions: collaboration is allowed with any CSE 105 students this quarter; if your group has questions about a problem, you may ask in drop-in help hours or post a public post on Piazza.

All submitted homework for this class must be typed. You can use a word processing editor if you like (Microsoft Word, Open Office, Notepad, Vim, Google Docs, etc.) but you might find it useful to take this opportunity to learn LaTeX. LaTeX is a markup language used widely in computer science and mathematics. The homework assignments are typed using LaTeX and you can use the source files as templates for typesetting your solutions. To generate state diagrams of machines, we recommend using Flap.js or JFLAP. Photographs of clearly hand-drawn diagrams may also be used. We recommend that you submit early drafts to Gradescope so that in case of any technical difficulties, at least some of your work is present. You may update your submission as many times as you'd like up to the deadline.

Integrity reminders

- Problems should be solved together, not divided up between the partners. The homework is designed to give you practice with the main concepts and techniques of the course, while getting to know and learn from your classmates.

- You may not collaborate on homework questions graded for correctness with anyone other than your group members. You may ask questions about the homework in office hours (of the instructor, TAs, and/or tutors) and on Piazza (as private notes viewable only to the Instructors). You *cannot* use any online resources about the course content other than the class material from this quarter – this is primarily to ensure that we all use consistent notation and definitions (aligned with the textbook) and also to protect the learning experience you will have when the 'aha' moments of solving the problem authentically happen.

- Do not share written solutions or partial solutions for homework with other students in the class who are not in your group. Doing so would dilute their learning experience and detract from their success in the class.

You will submit this assignment via Gradescope (https://www.gradescope.com) in the assignment called "hw4CSE105W24".

Assigned questions

1. **Classifying languages** (10 points): Our first example of a more complicated Turing machine was of a Turing machine that recognized the language $\{w\#w \mid w \in \{0,1\}^*\}$, which we know is not context-free. The language
$$\{0^n 1^n 2^n \mid n \geq 0\}$$
is also not context-free.

(a) (*Graded for correctness*) [7] Give an implementation-level description of a Turing machine that recognizes this language.

(b) (*Graded for completeness*) [8] Draw a state diagram of the Turing machine you gave in part (a) and trace the computation of this Turing machine on the input 012. You may use all our usual conventions for state diagrams of Turing machines (we do not include the node for the reject state $qrej$ and any missing transitions in the state diagram have value $(qrej, \square, R)$; $b \to R$ label means $b \to b, R$).

2. **Deciders, Recognizers, Decidability, and Recognizability** (15 points): For this question, consider the alphabet $\Sigma = \{0,1\}$.

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[7]This means your solution will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

[8]This means you will get full credit so long as your submission demonstrates honest effort to answer the question. You will not be penalized for incorrect answers. To demonstrate your honest effort in answering the question, we expect you to include your attempt to answer *each* part of the question. If you get stuck with your attempt, you can still demonstrate your effort by explaining where you got stuck and what you did to try to get unstuck.

(a) (*Graded for correctness*) Give an example of a finite, nonempty language over Σ and two different Turing machines that recognize it: one that is a decider and one that is not. A complete solution will include a precise definition for your example language, along with **both** a state diagram and an implementation-level description of each Turing machines, along with a brief explanation of why each of them recognizes the language and why one is a decider and there other is not.

(b) (*Graded for correctness*) True or false: There is a Turing machine that is not a decider that recognizes the empty set. A complete solution will include a witness Turing machine (given by state diagram or implementation-level description or high-level description) and a justification for why it's not a decider and why it does not accept any strings, or a complete and correct justification for why there is no such Turing machine.

(c) (*Graded for correctness*) True or false: There is a Turing machine that is not a decider that recognizes the set of all string Σ^*. A complete solution will include a witness Turing machine (given by state diagram or implementation-level description or high-level description) and a justification for why it's not a decider and why it accept each string over $\{0, 1\}$, or a complete and correct justification for why there is no such Turing machine.

3. **Closure** (15 points): Suppose M is a Turing machine over the alphabet $\{0, 1\}$. Let $s_1, s_2, \ldots$ be a list of all strings in $\{0, 1\}^*$ in string (shortlex) order. We define a new Turing machine by giving its high-level description as follows:

$M_{new} =$ "On input w :
   1. For $n = 1, 2, \ldots$
   2.   For $j = 1, 2, \ldots n$
   3.   For $k = 1, 2, \ldots, n$
   4.   Run the computation of M on $s_j w s_k$
   5.   If it accepts, accept.
   6.   If it rejects, go to the next iteration of the loop"

Recall the definitions we have: For languages L_1, L_2 over the alphabet $\Sigma = \{0, 1\}$, we have the associated sets of strings

$$SUBSTRING(L_1) = \{w \in \Sigma^* \mid \text{there exist } a, b \in \Sigma^* \text{ such that } awb \in L_1\}$$

and

$$L_1 \circ L_2 = \{w \in \Sigma^* \mid w = uv \text{ for some strings } u \in L_1 \text{ and } v \in L_2\}$$

We say that self-set-wise concatenation of the set L_1 is $L_1 \circ L_1$.

Note: there was a bug in the version of this assignment that was first released.

(a) (*Graded for completeness*) Prove that this Turing machine construction **cannot** be used to prove that the class of decidable languages over $\{0, 1\}$ is closed under **either** of the

above operations ($SUBSTRING$ or self-set-wise concatenation). A complete answer will give a counterexample or general description why the construction doesn't work for both operations.

(b) (*Graded for correctness*) Prove that this Turing machine construction cannot be used to prove that the class of recognizable languages over $\{0,1\}$ is closed under the $SUBSTRING$ set operation. In particular, give a counterexample of a specific language L_1 and Turing machine M_1 recognizing it where M_{new} does not recognize $SUBSTRING(L_1)$.

(c) (*Graded for completeness*) Define a new construction by slightly modifying this one that can be used to prove that the class of recognizable languages over $\{0,1\}$ is closed under $SUBSTRING$. Justify that your construction works. The proof of correctness for the closure claim can be structured like: "Let L_1 be a recognizable language over $\{0,1\}$ and assume we are given a Turing machine M_1 so that $L(M_1) = L_1$. Consider the new Turing machine M_{new} defined above. We will show that $L(M_{new}) = SUBSTRING(L_1)$... *complete the proof by proving subset inclusion in two directions, by tracing the relevant Turing machine computations*"

4. **Computational problems** (10 points): Recall the definitions of some example computational problems from class

Acceptance problem		
...for DFA	A_{DFA}	$\{\langle B, w\rangle \mid B$ is a DFA that accepts input string $w\}$
...for NFA	A_{NFA}	$\{\langle B, w\rangle \mid B$ is a NFA that accepts input string $w\}$
...for regular expressions	A_{REX}	$\{\langle R, w\rangle \mid R$ is a regular expression that generates input string $w\}$
...for CFG	A_{CFG}	$\{\langle G, w\rangle \mid G$ is a context-free grammar that generates input string $w\}$
...for PDA	A_{PDA}	$\{\langle B, w\rangle \mid B$ is a PDA that accepts input string $w\}$

Language emptiness testing		
...for DFA	E_{DFA}	$\{\langle A\rangle \mid A$ is a DFA and $L(A) = \emptyset\}$
...for NFA	E_{NFA}	$\{\langle A\rangle \mid A$ is a NFA and $L(A) = \emptyset\}$
...for regular expressions	E_{REX}	$\{\langle R\rangle \mid R$ is a regular expression and $L(R) = \emptyset\}$
...for CFG	E_{CFG}	$\{\langle G\rangle \mid G$ is a context-free grammar and $L(G) = \emptyset\}$
...for PDA	E_{PDA}	$\{\langle A\rangle \mid A$ is a PDA and $L(A) = \emptyset\}$

Language equality testing		
...for DFA	EQ_{DFA}	$\{\langle A, B\rangle \mid A$ and B are DFAs and $L(A) = L(B)\}$
...for NFA	EQ_{NFA}	$\{\langle A, B\rangle \mid A$ and B are NFAs and $L(A) = L(B)\}$
...for regular expressions	EQ_{REX}	$\{\langle R, R'\rangle \mid R$ and R' are regular expressions and $L(R) = L(R')\}$
...for CFG	EQ_{CFG}	$\{\langle G, G'\rangle \mid G$ and G' are CFGs and $L(G) = L(G')\}$
...for PDA	EQ_{PDA}	$\{\langle A, B\rangle \mid A$ and B are PDAs and $L(A) = L(B)\}$

(a) (*Graded for completeness*) Pick five of the computational problems above and give examples (preferably different from the ones we talked about in class) of strings that are in each of

the corresponding languages. Remember to use the notation $\langle \cdots \rangle$ to denote the string encoding of relevant objects. *Extension, not for credit:* Explain why it's hard to write a specific string of 0s and 1s and make a claim about membership in one of these sets.

(b) (*Graded for completeness*) Computational problems can also be defined about Turing machines. Consider the two high-level descriptions of Turing machines below. Reverse-engineer them to define the computational problem that is being recognized, where $L(M_{DFA})$ is the language corresponding to this computational problem about DFA and $L(M_{TM})$ is the language corresponding to this computational problem about Turing machines. *Hint*: the computational problem is not acceptance, language emptiness, or language equality (but is related to one of them).

Let $s_1, s_2, \ldots$ be a list of all strings in $\{0,1\}^*$ in string (shortlex) order. Consider the following Turing machines

$M_{DFA} =$ "On input $\langle D \rangle$ where D is a DFA :
    1. for $i = 1, 2, 3, \ldots$
    2.   Run D on s_i
    3.   If it accepts, accept.
    4.   If it rejects, go to the next iteration of the loop"

and

$M_{TM} =$ "On input $\langle T \rangle$ where T is a Turing machine :
    1. for $i = 1, 2, 3, \ldots$
    2.   Run T for i steps on each input $s_1, s_2, \ldots, s_i$ in turn
    3.   If T has accepted any of these, accept.
    4.   Otherwise, go to the next iteration of the loop"

HW5CSE105W24: Homework assignment 5 Due: March 14 at 5pm (no penalty late submission until 8am next morning), via Gradescope

In this assignment, You will practice analyzing, designing, and working with Turing machines. You will use general constructions and specific machines to explore the classes of recognizable and decidable languages. You will explore various ways to encode machines as strings so that computational problems can be recognized.

Resources: To review the topics for this assignment, see the class material from Weeks 7-10. We will post frequently asked questions and our answers to them in a pinned Piazza post.

Reading and extra practice problems: Sipser Sections 4.2, 5.3, 5.1. Chapter 4 problems 4.9, 4.12. Chapter 5 exercises 5.4, 5.5, 5.6, 5.7. Chapter 5 problems 5.22, 5.23, 5.24, 5.28

For all HW assignments: Weekly homework may be done individually or in groups of up to 3 students. You may switch HW partners for different HW assignments. Please ensure your name(s) and PID(s) are clearly visible on the first page of your homework submission and then upload the PDF to Gradescope. If working in a group, submit only one submission per group: one partner uploads the submission through their Gradescope account and then adds the other group member(s) to the Gradescope submission by selecting their name(s) in the "Add Group Members" dialog box. You will need to re-add your group member(s) every time you resubmit a new version of your assignment. Each homework question will be graded either for correctness (including clear and precise explanations and justifications of all answers) or fair effort completeness. For "graded for correctness" questions: collaboration is allowed only with CSE 105 students in your group; if your group has questions about a problem, you may ask in drop-in help hours or post a private post (visible only to the Instructors) on Piazza. For "graded for completeness" questions: collaboration is allowed with any CSE 105 students this quarter; if your group has questions about a problem, you may ask in drop-in help hours or post a public post on Piazza.

All submitted homework for this class must be typed. You can use a word processing editor if you like (Microsoft Word, Open Office, Notepad, Vim, Google Docs, etc.) but you might find it useful to take this opportunity to learn LaTeX. LaTeX is a markup language used widely in computer science and mathematics. The homework assignments are typed using LaTeX and you can use the source files as templates for typesetting your solutions. To generate state diagrams of machines, we recommend using Flap.js or JFLAP. Photographs of clearly hand-drawn diagrams may also be used. We recommend that you submit early drafts to Gradescope so that in case of any technical difficulties, at least some of your work is present. You may update your submission as many times as you'd like up to the deadline.

Integrity reminders

- Problems should be solved together, not divided up between the partners. The homework is designed to give you practice with the main concepts and techniques of the course, while getting to know and learn from your classmates.

- You may not collaborate on homework questions graded for correctness with anyone other than your group members. You may ask questions about the homework in office hours (of the instructor, TAs, and/or tutors) and on Piazza (as private notes viewable only to the Instructors). You *cannot* use any online resources about the course content other than the class material from this quarter – this is primarily to ensure that we all use consistent notation and definitions (aligned with the textbook) and also to protect the learning experience you will have when the 'aha' moments of solving the problem authentically happen.

- Do not share written solutions or partial solutions for homework with other students in the class who are not in your group. Doing so would dilute their learning experience and detract from their success in the class.

You will submit this assignment via Gradescope (https://www.gradescope.com) in the assignment called "hw5CSE105W24".

Assigned questions

1. **Computational problems** (14 points): For each of the following statements, determine if it is true or false. Clearly label your choice by starting your solution with True or False and then provide a justification for your answer.

(a) (*Graded for completeness*) [9] True or False: The language

$$\{\langle N_1, N_2 \rangle \mid N_1, N_2 \text{ are both NFAs over } \{0,1\} \text{ and } L(N_1) \subseteq L(N_2)\}$$

is recognizable.

(b) (*Graded for correctness*) [10] True or False: The language

$$\{\langle D \rangle \mid D \text{ is a DFA over } \{0,1\} \text{ and } L(D) = L(0^*)\}$$

is decidable.

(c) (*Graded for correctness*) True or False: The language

$$\{\langle M \rangle \mid M \text{ is a Turing machine and its start state equals its reject state}\}$$

is decidable.

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[9]This means you will get full credit so long as your submission demonstrates honest effort to answer the question. You will not be penalized for incorrect answers. To demonstrate your honest effort in answering the question, we expect you to include your attempt to answer *each* part of the question. If you get stuck with your attempt, you can still demonstrate your effort by explaining where you got stuck and what you did to try to get unstuck.

[10]This means your solution will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

(d) (*Graded for completeness*) True or False: The language

$$\{\langle M\rangle \mid M \text{ is a Turing machine and } L(M) = \emptyset\}$$

is decidable.

Challenge; not graded: what's the difference between (c) and (d)?

2. **What's wrong with these reductions?** (13 points): Suppose your friends are practicing coming up with mapping reductions $A \leq_m B$ and their witnessing functions $f : \Sigma^* \to \Sigma^*$. For each of the following attempts, determine if it is has error(s) or is correct. Do so by labelling each attempt with all and only the labels below that apply, and justifying this labelling.

- *Error Type 1:* The given function can't witness the claimed mapping reduction because there exists an $x \in A$ such that $f(x) \notin B$.
- *Error Type 2:* The given function can't witness the claimed mapping reduction because there exists an $x \notin A$ such that $f(x) \in B$.
- *Error Type 3:* The given function can't witness the claimed mapping reduction because the specified function is not computable.
- *Correct:* The claimed mapping reduction is true and is witnessed by the given function.

Clearly present your answer by providing a brief (3-4 sentences or so) justification for whether **each** of these labels applies to each example.

(a) (*Graded for completeness*) $A_{\text{TM}} \leq_m HALT_{\text{TM}}$ and

$$f(x) = \begin{cases} \langle \text{start} \to q_{acc} \rangle & \text{if } x = \langle M, w\rangle \text{ for a Turing machine } M \text{ and string } w \\ & \text{and } w \in L(M) \\[2em] \langle \text{start} \to q_0 \ \ \ q_{acc} \rangle & \text{otherwise} \end{cases}$$

with transition $0, 1, \_ \to R$ on q_0.

Hint: There are two errors with this attempt.

(b) (*Graded for correctness*) $\{w \mid w \in \{0,1\}^*\} \leq_m \{ww \mid w \in \{0,1\}^*\}$ and $f(x) = xx$ for each $x \in \{0,1\}^*$.

(c) (*Graded for correctness*) $HALT_{\text{TM}} \leq_m EQ_{\text{TM}}$ with

$$f(x) = \begin{cases} \langle \text{start} \to q_{acc}, M_w \rangle & \text{if } x = \langle M, w\rangle \text{ for a Turing machine } M \text{ and string } w \\[2em] \langle \text{start} \to q_{acc}, \ \text{start} \to q_{rej} \ \ q_{acc} \rangle & \text{otherwise.} \end{cases}$$

Where for each Turing machine M, we define

$$M_w = \text{"On input y}$$

1. Simulate M on w.

2. If it accepts, accept.

3. If it rejects, reject."

3. **Using mapping reductions** (14 points): Consider the following computational problems we've discussed

$$A_{TM} = \{\langle M, w \rangle \mid M \text{ is a Turing machine, } w \text{ is a string and } M \text{ accepts } w\}$$
$$HALT_{TM} = \{\langle M, w \rangle \mid M \text{ is a Turing machine, } w \text{ is a string and } M \text{ halts on } w\}$$
$$E_{TM} = \{\langle M \rangle \mid M \text{ is a Turing machine and } L(M) = \emptyset\}$$
$$EQ_{TM} = \{\langle M_1, M_2 \rangle \mid M_1, M_2 \text{ are both Turing machines and } L(M_1) = L(M_2)\}$$

and the new computational problem

$$One_{TM} = \{\langle M \rangle \mid M \text{ is a Turing machine and } M \text{ accepts exactly one string}\}$$

(a) (*Graded for correctness*) Give an example of a string that is an element of One_{TM} and a string that is not an element One_{TM} and briefly justify your choices.

(b) (*Graded for completeness*) Prove that One_{TM} is not decidable by showing that $A_{TM} \leq_m One_{TM}$.

(c) (*Graded for correctness*) Give a different proof that One_{TM} is not decidable by showing that $HALT_{TM} \leq_m One_{TM}$.

(d) (*Graded for completeness*) Is One_{TM} recognizable? Justify your answer.

4. **Examples of languages** (9 points):

For each part of the question, use precise mathematical notation or English to define your examples and then briefly justify why they work.

(a) (*Graded for correctness*) Two undecidable languages L_1 and L_2 over the same alphabet whose union $L_1 \cup L_2$ is decidable, or write **NONE** if there is no such example (and explain why).

(b) (*Graded for correctness*) A regular language L_3 and an unrecognizable language L_4 over the same alphabet whose intersection $L_3 \cap L_4$ is unrecognizable, or write **NONE** if there is no such example (and explain why).

(c) (*Graded for correctness*) A co-recognizable language L_5 that is NP-complete, or write **NONE** if there is no such example (and explain why). Recall the definition: A language L over an alphabet Σ is called **co-recognizable** if its complement, defined as $\Sigma^* \setminus L = \{x \in \Sigma^* \mid x \notin L\}$, is Turing-recognizable.

ProjectCSE105W24: Project Due February 22 at 5pm (no penalty late submission until 8am next day)

The CSE 105 project is designed for you to go deeper and extend your work on assignments and to explore an application of your choosing. The project is an individual assignment and has two tasks:

Task 1: Implementing the construction that converts NFA to DFA, and

Task 2: Illustrating the theorem that every regular language is decidable

What resources can you use? This project must be completed individually, without any help from other people, including the course staff (other than logistics support if you get stuck with screencast). You can use any of this quarter's CSE 105 offering (notes, readings, class videos, homework feedback). Tools for drawing state diagrams (like Flap.js and JFLAP) can be used to help draw the diagrams in the project too. These resources should be more than enough. If you are struggling to get started and want to look elsewhere online, you must acknowledge this by listing and citing any resources you consult (even if you do not explicitly quote them), including any large-language model style resources. Link directly to them and include the name of the author / video creator, any search strings or prompts you used, and the reason you consulted this reference.

The work you submit for the project needs to be your own. Again, you shouldn't need to look anywhere other than this quarter's material and doing so may result in definitions or notations that conflict with our norms in this class so think carefully before you go down this path.

If you get stuck on any part of the project, we encourage you to focus on communicating what you think the question might mean, including bringing an example from class or homework you think might be relevant, and include any submission any aspect where you're unsure. Clear communication about these theoretical ideas and their applications ia one of the main goals of the project.

Submitting the project You will submit a PDF plus a video file for the first task and a PDF for the second task. All file submissions will be in Gradescope.

Task 1: Implementing a construction Nondeterminism is a useful theoretical concept because it can make designs simpler and more modular. However, our actual devices are deterministic. In this part of the project, you'll choose a language that can be represented using nondeterminism in some interesting way, then illustrate the construction that converts NFA to DFA for this example.

Specifically:

1. Choose an alphabet Σ for this entire first task of the project.

2. Write a program in Java, Python, JavaScript, C++ , or another programming language of your choosing that takes as input a representation of an NFA over this alphabet and outputs a representation of a DFA over this alphabet that recognizes the same language. You get to choose the way NFA and DFA are represented, so long as it is general enough to represent any NFA and any DFA over this alphabet. For simplicity: you can restrict your attention to NFA **without spontaneous moves** (in other words, where the transition function has domain $Q \times \Sigma$ when Q is the set of states of the NFA). Informally, this means that the nondeterminism is coming from there being zero, one, or more arrows coming out of each state for each character in the alphabet and there are no arrows with ε labels.

   If you would like, you may use aids such as co-pilot or ChatGPT to help you write this program. However, you should test the code that is produced and be able to explain what it is doing. As a header in your code file, include a comment block describing any resources that were used to help generate your code.

   Whenever your program is run, it should display a representation of the input NFA and of the output DFA of the run.

3. To demonstrate your program, design a NFA over Σ with three states and with no spontaneous moves where the language of the NFA is neither $\emptyset$ nor Σ^* and draw its state diagram. Your NFA should use nondeterminism in some way. In other words, the state diagram you draw can't already be the state diagram of a DFA. Run your program with the NFA you just designed to output a representation of an equivalent DFA and demonstrate its design and the test case on video.

Checklist for submission

For this task, you will submit a PDF plus a video file.

The PDF should include:

- Clear specification of alphabet and state diagram of chosen three-state NFA.

- Documentation for program converting NFA to DFA: include a description of how NFA and DFA are represented in the program and give instructions for running it.

- Printout of code for program converting NFA to DFA.

- Screen shots of demonstration of running your program on your chosen NFA, including the representation of the output DFA.

- Solution is typed or clearly hand drawn with precise language and notation for all terms.

Presenting your reasoning and demonstrating it via screenshare are important skills that also show us a lot of your learning. Getting practice with this style of presentation is a good thing for you to learn in general and a rich way for us to assess your skills. To demonstrate your work, you will create a 3-5 minute screencast video with the following components:

- Start with your face and your student ID for a few seconds at the beginning, and introduce yourself audibly while on screen. You don't have to be on camera for the rest of the video, though it's fine if you are. We are looking for a brief confirmation that it's you creating the video and doing the work you submitted.

- Present the NFA you will be working with. Your video should include a clear and precise explanation of why the language of this NFA is not empty and also not the set of all strings over Σ.

- Show on the screen and explain the code for your program, including the software design choices you made (e.g. which data structures are you using, etc.) and any resources you used. The video should clearly describe which programming language was chosen for the implementation and gives the reasons why.

- Show on the screen and explain the representation of the NFA that you will input to the program.

- Demonstrate running your code on your example input. The video should include screencasts of running the code live. Explain why the output of your program is what you would expect, by connecting the output of the program to a DFA and discussing which strings are accepted / rejected by this DFA.

- Logistics: video needs to load correctly, be between 3 and 5 minutes, show your face and ID, and you introduce yourself audibly while on screen.

Note: Clarity and brevity are both important aspects of your video. In previous years, we've seen students speed up their videos to get below the 5 minute upper bound. This is ok so long as it doesn't compromise clarity.

Task 2: Illustrating a theorem In this part of the project, you'll choose a pattern in an application you care about, define it precisely, build a DFA that recognizes it, and then build the related Turing machine that proves that the language encoding this pattern is decidable.

First, pick **one** application for your example. Here are some ideas to get you started - but you can choose to go in a different direction.

- Data validation for input in text files (e.g. emails with specific domains, dates in specific formats, PIDs in a class list, etc.)

- Finding ASCII codes for punctuation in a binary file.

- The CDC recommended procedure for hand washing (Refer to the guidelines from the CDC here https://www.cdc.gov/handwashing/index.html in your explanation. You might find the first example in chapter 1 about automatic door controllers helpful when starting your design.)

- See more ideas here: https://theory-cs.github.io/files/practical-applications-of-theory-of-computation.pdf

Then:

1. In a paragraph or so, give the context for your chosen application and why you chose it.

2. Specify the alphabet for your example and write a precise (mathematical and/or English) description of a set of strings over this alphabet that is relevant to this application, and include a sentence or so justifying why this set is important and relevant.

3. Give one example of a string over this alphabet in this set and a string over this (same) alphabet not in this set, and explain why you chose these example strings.

4. Design a DFA that recognizes this language. Clearly draw and label the state diagram of this DFA and briefly justify why your design works by describing the role of each state of the DFA and relating it to a plain English description of the language you picked.

5. Use the construction in the proof that all regular languages are decidable (informally described in Theorem 4.1 in the book) to build a Turing machine that simulates your DFA. Hint: your Turing machine will have exactly two more states than your DFA. Draw the state diagram of your Turing machine.

6. For one of the strings from step 4., draw a representation of the computation of your Turing machine on this string. Remember that to describe the computation of a Turing machine, we need to include the contents of the tape, the state of the machine, and the location of the read/write head at each step in the computation. In class we've drawn pictures to represent the configuration of the machine at each step in a computation. You may do so or you may choose to describe these configurations in words.

Checklist for submission

- Solution typed or clearly hand-written/drawn with precise language and notation for all terms and complete, correct, and clear justification.

- Each of the six steps is complete and included in the PDF, with precise language and notation for all terms and complete, correct, and clear justification.

Your video: You may produce screencasts with any software you choose. One option is to record yourself with Zoom; a tutorial on how to use Zoom to record a screencast (courtesy of Prof. Joe Politz) is here:

`https://drive.google.com/open?id=1KROMAQuTCk4OzwrEFotlYSJJQdcG_GUU`.

The video that was produced from that recording session in Zoom is here:

`https://drive.google.com/open?id=1MxJN6CQcXqIbOekDYMxjh7mTt1TyRVMl`

Please send an email to the instructors (minnes@ucsd.edu) if you have concerns about the video / screencast components of this project or cannot complete projects in this style for some reason.