

Regular Expressions

CSE 105 Week 1 Discussion

Deadlines and Logistics

- Make sure you have access to the [class website](#), canvas, piazza and gradescope
- Make sure you can see all our OH in the [class calendar](#)
- Make sure you have access to [PrairieLearn](#) and [PrairieTest](#).
- Read the grading scheme on the class website !
- Schedule your tests asap on [PrairieTest](#) !
- HW1 due next week on 8th (Tuesday) at 5 PM
- Link for the slides :
https://docs.google.com/presentation/d/1rYhfMTnWjxi5GmIOC0Oj13gXquWENJHr5Azs4E_DpOA/edit?usp=sharing

What can you expect from the discussion section

- Recap* of key concepts from lectures
- Lots of informal interaction
- Practice problems from notes, review quizzes, Sipser and HW
- Ask us any questions and get your doubts answered !

PS - All screenshots taken either from Sipser or class notes unless specified otherwise

*See next slide for the extent of the recap

What NOT to expect from the discussion section

- A self contained “lecture” covering CSE 105. The discussion does NOT replace M/W/F lectures
- Answers to “Graded for correctness” HW questions
- Self contained notes to study from

Are the discussion sections podcasted/online/recorded ?

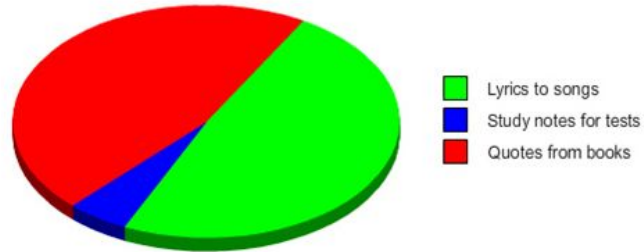


Ask me why

General Information

Do not memorize anything for the sake of completion of HWs, exams etc without first understanding underlying concepts !

What I can memorize



Motivation

1. What are the capabilities and limitations of computers ?
2. Can we answer how “difficult” (or if even possible) a certain computation task is ?
3. Can we mathematically model computational problems ?

Important notation

Alphabet

A non-empty finite set, usually denoted as Σ

Symbol

An element of the alphabet

String over Σ

A finite list of symbols from Σ

Language over Σ

A set of strings over Σ

Σ^*

Set of all possible strings formed from symbols in Σ

Key takeaways

- A language is a **SET** of strings
- Any time you see a language contain anything BUT strings, alarm bells should go off.
- You can have an empty language denoted by \emptyset . This is a language with NO strings

Is this statement correct, given Language $L = \{w \mid w \text{ is a string over } \{0\} \text{ such that } |w| = 1\}$

$L = \emptyset$?

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Is this statement correct, given Language $L = \{w \mid w \text{ is a string over } \{0\} \text{ such that } |w| = 1\}$

$L = \emptyset$?

No : A language can't be just a string, it has to be a set of (0 or more) strings!
 $L = \{0\}$ is the correct notation

Review Quiz Question

Consider the language $\{w \mid w \text{ is a string over } \{0, 1\} \text{ and } |w| \text{ is an integer multiple of } 3\}$. Which of the following are elements of this language? (Select all and only that apply)

- ☐ 000
- ☐ 0
- ☐ The empty set
- ☐ (1, 0, 1)
- ☐ {000}
- ☐ The empty string

Select all possible options that apply. ?

What option(s) can you eliminate right away by virtue of the definition of strings and languages ?

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- ☐ 0
- ☐ The empty string
- ☐ (1, 0, 1)
- ☐ {000}
- ☐ The empty string

Select all possible options that apply. ?

Can eliminate all non string options ! (since only strings can be elements of a language

What option(s) can you eliminate right away by virtue of the definition of strings and languages ?

Regular Expressions

Remember that at the end of the day, Languages are sets. How can we define a set?

1. List out all the elements
2. Use set builder notation and describe membership condition

Or

3. Use recursive definitions (Regular Expressions !)

The language described by a regular expression R is $L(R)$

Recap

Let A and B be languages. We define the regular operations *union*, *concatenation*, and *star* as follows:

- **Union:** $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.
- **Concatenation:** $A \circ B = \{xy \mid x \in A \text{ and } y \in B\}$.
- **Star:** $A^* = \{x_1x_2 \dots x_k \mid k \geq 0 \text{ and each } x_i \in A\}$.

Pop quiz : If $\Sigma = \{0,1\}$ and language $A = \{0, 01, 1\}$ and $B = \{\epsilon, 1\}$ What is

- $A \cup B$
- $A \circ B$
- B^*
- Is $(A^*)^* = A^*$?

Recap

Let A and B be languages. We define the regular operations *union*, *concatenation*, and *star* as follows:

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Pop quiz : If $\Sigma = \{0,1\}$ and language $A = \{0, 01, 1\}$ and $B = \{\epsilon, 1\}$ What is

- $A \cup B : \{\epsilon, 0, 1, 01\}$
- $A \circ B : \{0, 01, 1, 011, 11\}$
- $B^* : \{1, 11, 111 \dots\}$ or $\{1_0 \dots 1_k \mid k \text{ is a non negative integer}\}$
- Is $(A^*)^* = A^*$? : Yes ! Both expressions produce the set of all strings that can be formed by concatenating strings in A with one another as many times as we want

R is a regular expression over the alphabet Σ

1. $R = a$, where $a \in \Sigma$

2. $R = \varepsilon$

3. $R = \emptyset$

4. $R = (R_1 \cup R_2)$, where R_1, R_2 are themselves regular expressions

5. $R = (R_1 \circ R_2)$, where R_1, R_2 are themselves regular expressions

6. (R_1^*) , where R_1 is a regular expression.

Context is super important !

For the following examples assume the alphabet is $\Sigma_1 = \{0, 1\}$:

The language described by the regular expression 0 is $L(0) = \{0\}$

The language described by the regular expression 1 is $L(1) = \{1\}$

The language described by the regular expression ε is $L(\varepsilon) = \{\varepsilon\}$

The language described by the regular expression \emptyset is $L(\emptyset) = \emptyset$

The language described by the regular expression $\Sigma_1 \Sigma_1 \Sigma_1)^*$ is $L((\Sigma_1 \Sigma_1 \Sigma_1)^*) =$

Do both the Σ_1 refer to the same thing ?

Context is super important !

Refers to the alphabet set Σ_1 containing symbols 1 and 0 !

For the following examples assume the alphabet is $\Sigma_1 = \{0, 1\}$:

The language described by the regular expression 0 is $L(0) = \{0\}$

The language described by the regular expression 1 is $L(1) = \{1\}$

The language described by the regular expression ε is $L(\varepsilon) = \{\varepsilon\}$

The language described by the regular expression \emptyset is $L(\emptyset) = \emptyset$

The language described by the regular expression $\Sigma_1 \Sigma_1 \Sigma_1)^*$ is $L((\Sigma_1 \Sigma_1 \Sigma_1)^*) =$

Do both the Σ_1 refer to the same thing ? :
NO !

Refers to any one occurrence of any symbol (0 or 1 in this case) from Σ_1 .

Other conventions

Assuming Σ is the alphabet, we use the following conventions

Σ	regular expression describing language consisting of all strings of length 1 over Σ
$*$ then \circ then \cup	precedence order, unless parentheses are used to change it
$R_1 R_2$	shorthand for $R_1 \circ R_2$ (concatenation symbol is implicit)
R^+	shorthand for $R^* \circ R$
R^k	shorthand for R concatenated with itself k times, where k is a (specific) natural number

Remember

A regular expression should describe a SET of strings over an alphabet since it is descriptive of a Language, which is a SET of strings over an alphabet.

All “operations” and “conventions” you see in a regular expression boil down to some fundamental operation(s) on set(s).

What operations and what sets ?

Remember

A regular expression should describe a SET of strings over an alphabet since it is descriptive of a Language, which is a SET of strings over an alphabet.

All “operations” and “conventions” you see in a regular expression boil down to some fundamental **operation(s)** on **set(s)**.

What **operations** and what **sets** ?

U, * and \circ

As a basis step :

- Element \in alphabet or
- ε
- \emptyset

Inductively:

On any regular expression(s)

When you see... The language described by the regular expression $1^* \circ 1$ is $L(1^* \circ 1) =$

You need to think : sets and operations !

Solve :

When you see... The language described by the regular expression $1^* \circ 1$ is $L(1^* \circ 1) =$

You need to think : sets and operations !

Solve :

$$\begin{aligned} & L(1^* \circ 1) \\ &= L(1^*) \circ L(1) \\ &= L(1)^* \circ L(1) \\ &= \{1\}^* \circ \{1\} \\ &= \{1^k \mid k \geq 0\} \circ \{1\} \\ &= \{1^k 1 \mid k \geq 0\} \text{ (sufficient to leave it at this step)} \\ &= \{1^k \mid k \geq 1\} \end{aligned}$$

From Regex to Language

The language over $\Sigma_1 = \{0, 1\}$ described by the regular expression Σ_1^*1 is $L(\Sigma_1^*1) = \text{BLANK}$

Describe this set in (a) Simple english and (b) Set builder notation

(a) Set of all strings over Σ_1 ending in a 1

(b) $\{x1 \mid x \in \Sigma_1^*\}$

Describe the language generated by these Regex(over $\Sigma = \{0,1\}$) : Basics

1. $(0 \cup 1)$
2. $(0 \cup 1)^*$
3. Σ^*
4. $(0) \cup (1)$
5. $(01) \cup (1)$
6. $(01)^* \cup (1)$
7. $(01 \cup 1)^*$

No, Seriously, try them yourself first !



Describe the language generated by these Regex(over $\Sigma = \{0,1\}$) : Basics

1. $(0 \cup 1) : \{0,1\}$
2. $(0 \cup 1)^* : \{x \mid x \in \Sigma^*\}$
3. $\Sigma^* : \text{Set of all strings that can be created from elements in } \Sigma \text{ (Notice similarity to (2))}$
4. $(0) \cup (1) : \{0, 1\}$
5. $(01) \cup (1) : \{01, 1\}$
6. $(01)^* \cup (1) : \{(01)^k \mid k \geq 0\} \cup \{1\}$
7. $(01 \cup 1)^* : \{x \mid x \in \{01,1\}^*\}$

Describe the language generated by these Regex(over $\Sigma = \{0,1\}$) : Level up

1. $(01)(01)^*(01)$
2. $(01)^+(01)$
3. $1 \cup 11 \cup 111 \cup 1111 \cup 11111^*$
4. $\epsilon^* \Sigma \Sigma \Sigma \epsilon^*$
5. $(\epsilon \cup \Sigma)(\epsilon \cup \Sigma)(\epsilon \cup \Sigma)$
6. $\Sigma^* 1 \Sigma^* 0 \Sigma^* \cup \Sigma^* 0 \Sigma^* 1 \Sigma^*$
7. $(1 \Sigma^* 1) \cup (0 \Sigma^* 0) \cup (\epsilon) \cup (1) \cup (0)$

No, Seriously, try them yourself first !



Describe the language generated by these Regex(over $\Sigma = \{0,1\}$) : Level up

1. $(01)(01)^*(01)$: Set of strings containing repeating units of (01) with at least 2 repeats
2. $(01)^+(01)$: Set of strings containing repeating units of (01) with at least 2 repeats (notice similarity to (1))
3. $1 \cup 11 \cup 111 \cup 1111 \cup 1111^*$: Same language as 11^* or 1^+
4. $\epsilon^* \Sigma \Sigma \Sigma \epsilon^*$: All strings of length 3
5. $(\epsilon \cup \Sigma)(\epsilon \cup \Sigma)(\epsilon \cup \Sigma)$: All strings of length at most 3
6. $\Sigma^* 1 \Sigma^* 0 \Sigma^* \cup \Sigma^* 0 \Sigma^* 1 \Sigma^*$: All strings containing at least one 1 and one 0
7. $(1\Sigma^*1) \cup (0\Sigma^*0) \cup (\epsilon) \cup (1) \cup (0)$: All strings starting and ending with the same symbol

Questions?

Good luck for HW 1 !