HW1CSE105F24: Homework assignment 1

CSE105F24

Due: October 8th at 5pm, via Gradescope

In this assignment,

You will practice reading and applying the definitions of alphabets, strings, languages, Kleene star, and regular expressions. You will use regular expressions and relate them to languages and finite automata. You will use precise notation to formally define the state diagram of finite automata, and you will use clear English to describe computations of finite automata informally.

Resources: To review the topics for this assignment, see the class material from Weeks 0 and 1. We will post frequently asked questions and our answers to them in a pinned Piazza post.

Reading and extra practice problems: Sipser Section 0, 1.3, 1.1. Chapter 1 exercises 1.1, 1.2, 1.3, 1.18, 1.23.

For all HW assignments: Weekly homework may be done individually or in groups of up to 3 students. You may switch HW partners for different HW assignments. Please ensure your name(s) and PID(s) are clearly visible on the first page of your homework submission and then upload the PDF to Gradescope. If working in a group, submit only one submission per group: one partner uploads the submission through their Gradescope account and then adds the other group member(s) to the Gradescope submission by selecting their name(s) in the "Add Group Members" dialog box. You will need to re-add your group member(s) every time you resubmit a new version of your assignment. Each homework question will be graded either for correctness (including clear and precise explanations and justifications of all answers) or fair effort completeness. For "graded for correctness" questions: collaboration is allowed only with CSE 105 students in your group; if your group has questions about a problem, you may ask in drop-in help hours or post a private post (visible only to the Instructors) on Piazza. For "graded for completeness" questions: collaboration is allowed with any CSE 105 students this quarter; if your group has questions about a problem, you may ask in drop-in help hours or post a public post on Piazza.

All submitted homework for this class must be typed. You can use a word processing editor if you like (Microsoft Word, Open Office, Notepad, Vim, Google Docs, etc.) but you might find it useful to take this opportunity to learn LaTeX. LaTeX is a markup language used widely in computer science and mathematics. The homework assignments are typed using LaTeX and you can use the source files as templates for typesetting your solutions. To generate state diagrams of machines, we recommend using Flap.js or JFLAP. Photographs of clearly hand-drawn diagrams may also be used. We recommend that you submit early drafts to Gradescope so that in case of any technical difficulties, at least some of your work is present. You may update your submission as many times as you'd like up to the deadline.

Integrity reminders

- Problems should be solved together, not divided up between the partners. The homework is designed to give you practice with the main concepts and techniques of the course, while getting to know and learn from your classmates.
- You may not collaborate on homework questions graded for correctness with anyone other than your group members. You may ask questions about the homework in office hours (of the instructor, TAs, and/or tutors) and on Piazza (as private notes viewable only to the Instructors). You *cannot* use any online resources about the course content other than the class material from this quarter – this is primarily to ensure that we all use consistent notation and definitions (aligned with the textbook) and also to protect the learning experience you will have when the 'aha' moments of solving the problem authentically happen.
- Do not share written solutions or partial solutions for homework with other students in the class who are not in your group. Doing so would dilute their learning experience and detract from their success in the class.

You will submit this assignment via Gradescope (https://www.gradescope.com) in the assignment called "hw1CSE105F24".

Assigned questions

1. **Finding examples and edge cases** (12 points):

With $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $\Gamma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}$

(a) (*Graded for completeness*) [1] Give an example of a string over Σ that is meaningful to you in some way and whose length is between 5 and 20, and explain why this string is meaningful to you.

(b) (*Graded for completeness*) Calculate the number of distinct strings of length 3 over Σ and then explain your calculation.

[1] This means you will get full credit so long as your submission demonstrates honest effort to answer the question. You will not be penalized for incorrect answers. To demonstrate your honest effort in answering the question, we expect you to include your attempt to answer *each* part of the question. If you get stuck with your attempt, you can still demonstrate your effort by explaining where you got stuck and what you did to try to get unstuck.

(c) (*Graded for completeness*) With the ordering $0 < 1 < 2 < 3 < 4 < 5 < 6 < 7 < 8 < 9 < A < B < C < D < E < F$, list the first 50 strings over Γ in string order. Explain how you constructed this list. *Note: you can write a program to generate this list if you'd like, and you may use any external tools to help you write this program. If you do use a program to generate the list, include it (and documentation for how it works) as part of your submission.*

(d) (*Graded for correctness*) [2] Give an example of a finite set that is a language over Σ and over Γ, or explain why there is no such set. A complete and correct answer will use clear and precise notation (consistent with the textbook and class notes) and will include a description of why the given example is a language over Σ and over Γ and is finite, or an explanation why there is no such example.

(e) (*Graded for correctness*) Give an example of an infinite set that is a language over Σ and not over Γ, or explain why there is no such set. A complete and correct answer will use clear and precise notation (consistent with the textbook and class notes) and will include a description of why the given example is a language over Σ and not over Γ and is infinite, or an explanation why there is no such example.

2. **Regular expressions** (10 points):

(a) (*Graded for completeness*) Give three regular expressions that all describe the set of all strings over $\{a, b\}$ that have odd length. Ungraded bonus challenge: Make the expressions as different as possible!

(b) (*Graded for completeness*) A friend tells you that each regular expression that has a Kleene star (*) describes an infinite language. Are they right? Either help them justify their claim or give a counterexample to disprove it and explain your counterexample.

3. **Functions over languages** (15 points):

For each language L over the alphabet $\Sigma_1 = \{0, 1\}$, we have the associated sets of strings

$$SUBSTRING(L) = \{w \in \Sigma_1^* \mid \text{there exist } x, y \in \Sigma_1^* \text{ such that } xwy \in L\}$$

and

$$EXTEND(L) = \{w \in \Sigma_1^* \mid w = uv \text{ for some strings } u \in L \text{ and } v \in \Sigma_1^*\}$$

(a) (*Graded for completeness*) Specify an example language A over Σ_1 such that $SUBSTRING(A) = EXTEND(A)$, or explain why there is no such example. A complete solution will include either (1) a precise and clear description of your example language A and a precise and clear description of the result of computing $SUBSTRING(A)$, $EXTEND(A)$ (using the given definitions) to justify this description and to justify the set equality, or (2) a sufficiently

[2]This means your solution will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

general and correct argument why there is no such example, referring back to the relevant definitions.

(b) (*Graded for correctness*) Specify an example language B over Σ_1 such that

$$SUBSTRING(B) = \{\varepsilon\}$$

and

$$EXTEND(B) = \Sigma_1^*$$

or explain why there is no such example. A complete solution will include either (1) a precise and clear description of your example language B and a precise and clear description of the result of computing $SUBSTRING(B)$, $EXTEND(B)$ (using the given definitions) to justify this description and to justify the set equality with $\{\varepsilon\}$ and Σ_1^* (respectively), or (2) a sufficiently general and correct argument why there is no such example, referring back to the relevant definitions.

(c) (*Graded for correctness*) Specify an example **infinite** language C over Σ_1 such that

$$SUBSTRING(C) \neq \Sigma_1^*$$

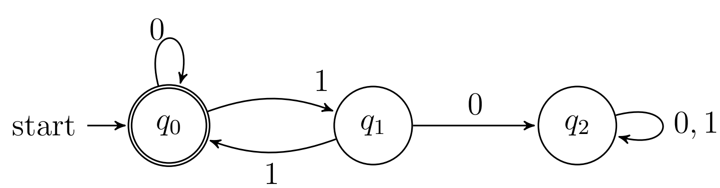
and

$$EXTEND(C) \neq \Sigma_1^*$$

, or explain why there is no such example. A complete solution will include either (1) a precise and clear description of your example language C and a precise and clear description of the result of computing $SUBSTRING(B)$, $EXTEND(B)$ (using the given definitions) to justify this description and to justify the set nonequality claims, or (2) a sufficiently general and correct argument why there is no such example, referring back to the relevant definitions.

4. **Finite automata** (13 points):

Consider the finite automaton $(Q, \Sigma, \delta, q_0, F)$ whose state diagram is depicted below



where $Q = \{q_0, q_1, q_2\}$, $\Sigma = \{0, 1\}$, and $F = \{q_0\}$, and $\delta : Q \times \Sigma \to Q$ is specified by the look-up table

	0	1
q_0	q_0	q_1
q_1	q_2	q_0
q_2	q_2	q_2

(a) (*Graded for completeness*) A friend tries to summarize the transition function with the formula

$$\delta(q_i, x) = \begin{cases} q_0 & \text{when } i = 0 \text{ and } x = 0 \\ q_2 & \text{when } x < i \\ q_j & \text{when } j = (i+1) \mod 2 \text{ and } x = 1 \end{cases}$$

for $x \in \{0,1\}$ and $i \in \{0,1,2\}$. Are they right? Either help them justify their claim or give a counterexample to disprove it and then fix their formula.

(b) (*Graded for correctness*) Give a regular expression R so that $L(R)$ is the language recognized by this finite automaton. Justify your answer by referring to the definition of the semantics of regular expressions and computations of finite automata. Include an explanation for why each string in $L(R)$ is accepted by the finite automaton *and* for why each string not in $L(R)$ is rejected by the finite automaton.

(c) (*Graded for correctness*) Keeping the same set of states $Q = \{q_0, q_1, q_2\}$, alphabet $\Sigma = \{0, 1\}$, same start state q_0, and same transition function δ, choose a new set of accepting states F_{new} so that the new finite automaton that results accepts at least one string that the original one rejected **and** rejects at least one string that the original one accepted, or explain why there is no such choice of F_{new}. A complete solution will include either (1) a precise and clear description of your choice of F_{new} and a precise and clear the two example strings using relevant definitions to justify them, or (2) a sufficiently general and correct argument why there is no such example, referring back to the relevant definitions.

