## Day9

**Definition and Theorem**: For an alphabet  $\Sigma$ , a language L over  $\Sigma$  is called **regular** exactly when L is recognized by some DFA, which happens exactly when L is recognized by some NFA, and happens exactly when L is described by some regular expression

We saw that: The class of regular languages is closed under complementation, union, intersection, set-wise concatenation, and Kleene star.

Extra practice:

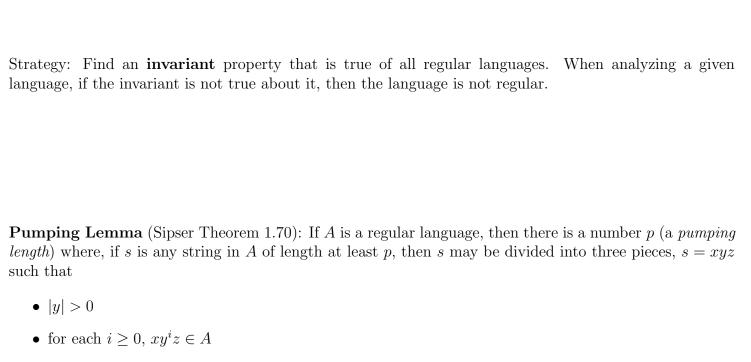
**Disprove**: There is some alphabet  $\Sigma$  for which there is some language recognized by an NFA but not by any DFA.

**Disprove**: There is some alphabet  $\Sigma$  for which there is some finite language not described by any regular expression over  $\Sigma$ .

**Disprove**: If a language is recognized by an NFA then the complement of this language is not recognized by any DFA.

Fix alphabet  $\Sigma$ . Is every language L over  $\Sigma$  regular?

| Set   | Cardinality |
|---|-------------|
| $\{0, 1\}$  |             |
| $\{0,1\}^*$                                       |             |
| $\mathcal{P}(\{0,1\})$                            |             |
| The set of all languages over $\{0,1\}$           |             |
| The set of all regular expressions over $\{0,1\}$ |             |
| The set of all regular languages over $\{0,1\}$   |             |
|   |             |



•  $|xy| \leq p$ .

**Proof idea**: In DFA, the only memory available is in the states. Automata can only "remember" finitely far in the past and finitely much information, because they can have only finitely many states. If a computation path of a DFA visits the same state more than once, the machine can't tell the difference between the first time and future times it visits this state. Thus, if a DFA accepts one long string, then it must accept (infinitely) many similar strings.

**Proof illustration** 

True or False: A pumping length for  $A = \{0, 1\}^*$  is p = 5.

True or False: A pumping length for  $A = \{0, 1\}^*$  is p = 2.

True or False: A pumping length for  $A = \{0, 1\}^*$  is p = 105.

Restating **Pumping Lemma**: If L is a regular language, then it has a pumping length.

Contrapositive: If L has no pumping length, then it is nonregular.

The Pumping Lemma cannot be used to prove that a language is regular.

The Pumping Lemma can be used to prove that a language is not regular.

Extra practice: Exercise 1.49 in the book.

**Proof strategy**: To prove that a language L is **not** regular,

- ullet Consider an arbitrary positive integer p
- ullet Prove that p is not a pumping length for L
- $\bullet$  Conclude that L does not have any pumping length, and therefore it is not regular.

**Negation**: A positive integer p is **not a pumping length** of a language L over  $\Sigma$  iff

$$\exists s \ \big( \ |s| \geq p \land s \in L \land \forall x \forall y \forall z \ \big( \ (s = xyz \land |y| > 0 \land |xy| \leq p \ ) \rightarrow \exists i (i \geq 0 \land xy^iz \not\in L) \big) \ \big)$$

## Day10

**Proof strategy**: To prove that a language L is **not** regular,

- $\bullet$  Consider an arbitrary positive integer p
- Prove that p is not a pumping length for L. A positive integer p is **not a pumping length** of a language L over  $\Sigma$  iff

$$\exists s \ ( \ |s| \ge p \land s \in L \land \forall x \forall y \forall z \ ( \ (s = xyz \land |y| > 0 \land |xy| \le p \ ) \rightarrow \exists i (i \ge 0 \land xy^i z \notin L)) \ )$$

Informally:

• Conclude that L does not have any pumping length, and therefore it is not regular.

**Example**:  $\Sigma = \{0, 1\}, L = \{0^n 1^n \mid n \ge 0\}.$ 

Fix p an arbitrary positive integer. List strings that are in L and have length greater than or equal to p:

Pick s =

Suppose s = xyz with  $|xy| \le p$  and |y| > 0.

Then when i =,  $xy^iz =$ 

**Example**:  $\Sigma = \{0, 1\}$ ,  $L = \{ww^{\mathcal{R}} \mid w \in \{0, 1\}^*\}$ . Remember that the reverse of a string w is denoted  $w^{\mathcal{R}}$  and means to write w in the opposite order, if  $w = w_1 \cdots w_n$  then  $w^{\mathcal{R}} = w_n \cdots w_1$ . Note:  $\varepsilon^{\mathcal{R}} = \varepsilon$ . Fix p an arbitrary positive integer. List strings that are in L and have length greater than or equal to p: Pick s =Suppose s = xyz with  $|xy| \le p$  and |y| > 0.  $, xy^iz =$ Then when i =**Example**:  $\Sigma = \{0, 1\}, L = \{0^j 1^k \mid j \ge k \ge 0\}.$ Fix p an arbitrary positive integer. List strings that are in L and have length greater than or equal to p: Pick s =Suppose s = xyz with  $|xy| \le p$  and |y| > 0.  $, xy^iz =$ Then when i =**Example**:  $\Sigma = \{0, 1\}, L = \{0^n 1^m 0^n \mid m, n \ge 0\}.$ Fix p an arbitrary positive integer. List strings that are in L and have length greater than or equal to p: Pick s =Suppose s = xyz with  $|xy| \le p$  and |y| > 0.

 $, xy^iz =$ 

Then when i =

## Extra practice:

| Language  | $s \in L$ | $s \notin L$ | Is the language regular or nonregular? |
|---|-----------|--------------|--|
| $\{a^nb^n\mid 0\leq n\leq 5\}$                  |           |              |  |
| $\{b^na^n\mid n\geq 2\}$                        |           |              |  |
| $\{a^mb^n\mid 0\leq m\leq n\}$                  |           |              |  |
| $\{a^mb^n\mid m\geq n+3, n\geq 0\}$             |           |              |  |
| $\{b^ma^n\mid m\geq 1, n\geq 3\}$               |           |              |  |
| $\{w \in \{a, b\}^* \mid w = w^{\mathcal{R}}\}$ |           |              |  |
| $\{ww^{\mathcal{R}} \mid w \in \{a, b\}^*\}$    |           |              |  |
|   |           |              |  |