

This problem sheet is about computational complexity [1].

1. Classify the following as polynomial, exponential, or logarithmic expressions.
 - (a) $3n + 1$
 - (b) $n^2 + 2n + 1$
 - (c) $\log_b(a)$
 - (d) 10^n
 - (e) $2^n + n^2$
 - (f) $n \log_n$
 - (g) n^n
2. Explain what decision problems are, and how they relate to Turing machines.
3. Explain what the P computational complexity class is, and give an example of a problem in P.
4. Explain the terms conjunctive normal form and disjunctive normal form.
5. Convert the following expressions to Conjunctive Normal Form.
 - (a) $a \vee b$
 - (b) $a \wedge b$
 - (c) $((a \wedge b) \vee (\neg b \wedge c)) \vee \neg d$
 - (d) $(a \wedge b) \vee (c \wedge d)$
 - (e) $(a \vee b) \wedge (c \vee d)$
6. Convert the following expressions to Disjunctive Normal Form.
 - (a) $a \vee b$
 - (b) $a \wedge b$
 - (c) $((a \wedge b) \vee (\neg b \wedge c)) \vee \neg d$
 - (d) $(a \wedge b) \vee (c \wedge d)$
 - (e) $(a \vee b) \wedge (c \vee d)$
7. Determine if there is a setting of the variables in the following expression that makes the evaluation of the expression true.
 - (a) $a \vee b$
 - (b) $a \wedge b$
 - (c) $((a \wedge b) \vee (\neg b \wedge c)) \vee \neg d$
 - (d) $(a \wedge b) \vee (c \wedge d)$
 - (e) $(a \vee b) \wedge (c \vee d)$
8. Explain how to prove that a problem is NP-complete.
9. Prove that 3-SAT is NP-complete. You may assume that SAT is NP-complete.

References

- [1] Michael Sipser. *Introduction to the Theory of Computation*. International Thomson Publishing, 3rd edition, 1996.