This problem sheet is about computational complexity [1].

- 1. Determine the number of comparisons made by Bubble sort on the following inputs.
 - (a) [3, 2, 1]
 - (b) [4, 3, 2, 1]
 - (c) [5, 4, 3, 2, 1]
 - (d) [6, 5, 4, 3, 2, 1]
 - (e) $[20, 19, 18, \dots, 3, 2, 1]$
 - (f) [3,4,5,2,1]
 - (g) [4, 5, 1, 2, 3]

Solution:

- (a) 2+1=3
- (b) 3+2+1=6
- (c) 4+3+2+1=10
- (d) 5+4+3+2+1=15
- (e) 6+4+3+2+1=21
- (f) $(20 \times 19) \div 2 = 190$
- (g) 4+3+2+1=10
- (h) 5+4+3=12, using the fact that if no swaps are made in one pass then the list is sorted.
- $2. \ \,$ Classify the following as polynomial, exponential, or logarithmic expressions.
 - (a) 3n+1
 - (b) $n^2 + 2n + 1$
 - (c) $log_b(a)$
 - (d) 10^n
 - (e) $2^n + n^2$
 - (f) $nlog_n$
 - (g) n^n

Solution:

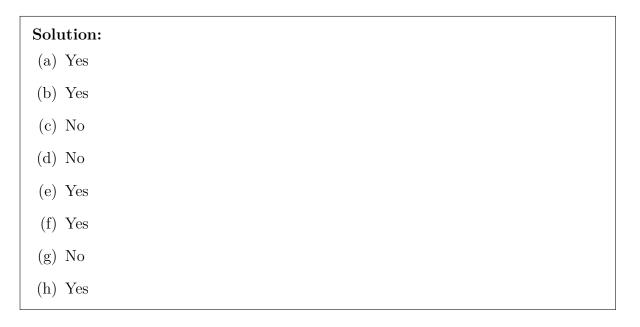
- (a) Polynomial (linear).
- (b) Polynomial.
- (c) Logarithmic.
- (d) Exponential.
- (e) Ambiguous.
- (f) Ambiguous.
- (g) Ambiguous.
- (h) Exponential.
- 3. Explain what the P computational complexity class is, and give an example of a problem known to be in P.
- 4. Explain what PRIMES is.

Solution:

$$PRIMES = \{2, 3, 5, 7, 11, \ldots\}$$

PRIMES is the set of all prime numbers, and is a subset of the natural numbers.

- 5. Describe two different algorithms the check if a number is a prime. The algorithms should accept a single positive integer as input, and output true if the number is prime and false otherwise.
- 6. Determine which of the following are in PRIMES (without Google).
 - (a) 2
 - (b) 3
 - (c) 4
 - (d) 10
 - (e) 11
 - (f) 13,109
 - (g) 100,827
 - (h) 102,203



7. Explain what a decision problem is, and how decision problems relate to Turing machines.

Solution: As in notes.

- 8. Explain the decision problem related to PRIMES.
- 9. Explain the concept of complexity in terms of Turing machines.
- 10. Explain why if we can solve decision problem A in polynomial time, and we can convert decision problem B to problem A in polynomial time, then we can solve problem B in polynomial time too.

References

[1] Michael Sipser. *Introduction to the Theory of Computation*. International Thomson Publishing, 3rd edition, 1996.