This problem sheet is about computational complexity [1].

1. Determine the number of comparisons made by Bubble sort on the following inputs.
   (a) $[3, 2, 1]$
   (b) $[4, 3, 2, 1]$
   (c) $[5, 4, 3, 2, 1]$
   (d) $[6, 5, 4, 3, 2, 1]$
   (e) $[20, 19, 18, \ldots, 3, 2, 1]$
   (f) $[3, 4, 5, 2, 1]$
   (g) $[4, 5, 1, 2, 3]$
2. Classify the following as polynomial, exponential, or logarithmic expressions.
   (a) $3n + 1$
   (b) $n^2 + 2n + 1$
   (c) $log_b(a)$
   (d) 10^n
   (e) $2^n + n^2$
   (f) $nlog_n$
   (g) n^n
3. Explain what the P computational complexity class is, and give an example of a problem known to be in P.
4. Explain what PRIMES is.
5. Describe two different algorithms the check if a number is a prime. The algorithms should accept a single positive integer as input, and output true if the number is prime and false otherwise.
6. Determine which of the following are in PRIMES (without Google).
   (a) 2
   (b) 3
   (c) 4
   (d) 10
   (e) 11
   (f) 13,109
   (g) 100,827
   (h) 102,203

7. Explain what a decision problem is, and how decision problems relate to Turing machines.

8. Explain the decision problem related to PRIMES.

9. Explain the concept of complexity in terms of Turing machines.

10. Explain why if we can solve decision problem A in polynomial time, and we can convert decision problem B to problem A in polynomial time, then we can solve problem B in polynomial time too.

References

[1] Michael Sipser. *Introduction to the Theory of Computation.* International Thomson Publishing, 3rd edition, 1996.