

1. Explain the terms conjunctive normal form and disjunctive normal form.

Solution: As in notes.

2. Convert the following expressions to Conjunctive Normal Form.

- (a) $a \vee b$
- (b) $a \wedge b$
- (c) $((a \wedge b) \vee (\neg b \wedge c)) \vee \neg d$
- (d) $(a \wedge b) \vee (c \wedge d)$
- (e) $(a \vee b) \wedge (c \vee d)$

Solution:

- (a) $a \vee b$
- (b) $a \wedge b$
- (c) $(a \vee \neg b \vee \neg d) \wedge (b \vee c \vee \neg d)$
- (d) $(a \vee c) \wedge (a \vee d) \wedge (b \vee c) \wedge (b \vee d)$
- (e) $(a \vee b) \wedge (c \vee d)$

3. Convert the following expressions to Disjunctive Normal Form.

- (a) $a \vee b$
- (b) $a \wedge b$
- (c) $((a \wedge b) \vee (\neg b \wedge c)) \vee \neg d$
- (d) $(a \wedge b) \vee (c \wedge d)$
- (e) $(a \vee b) \wedge (c \vee d)$

Solution:

- (a) $a \vee b$
- (b) $a \wedge b$
- (c) $(a \wedge b) \vee (\neg b \wedge c) \vee \neg d$
- (d) $(a \wedge b) \vee (c \wedge d)$
- (e) $(a \wedge c) \vee (a \wedge d) \vee (b \wedge c) \vee (b \wedge d)$

4. Determine if there is a setting of the variables in the following expression that makes the evaluation of the expression true.

(a) $a \vee b$

(b) $a \wedge b$

(c) $((a \wedge b) \vee (\neg b \wedge c)) \vee \neg d$

(d) $(a \wedge b) \vee (c \wedge d)$

(e) $(a \vee b) \wedge (c \vee d)$

Solution:

(a) $(a, b) = (1, 1)$

(b) $(a, b) = (1, 1)$

(c) $(a, b, c, d) = (1, 1, 1, 0)$

(d) $(a, b, c, d) = (1, 1, 1, 0)$

(e) $(a, b, c, d) = (1, 1, 1, 0)$

5. Explain the SAT problem.

Solution: As in notes.

6. Explain the 3-SAT problem.

Solution: As in notes.

7. Explain how to prove that a problem is NP-complete.

Solution: As in notes.

8. Prove that 3-SAT is NP-complete. You may assume that SAT is NP-complete.

Solution: As in notes.