This problem sheet is about NP-complete problems [1].

1. Explain what a decision problem is.
2. Explain what the NP complexity class is.
3. Explain the subset sum problem, and the brute-force approach to solving it.
4. Explain the terms conjunctive normal form and disjunctive normal form.
5. Convert the following expressions to Conjunctive Normal Form.
   (a) $a \vee b$
   (b) $a \wedge b$
   (c) $((a \wedge b) \vee (\neg b \wedge c)) \vee \neg d$
   (d) $(a \wedge b) \vee (c \wedge d)$
   (e) $(a \vee b) \wedge (c \vee d)$
6. Convert the following expressions to Disjunctive Normal Form.
   (a) $a \vee b$
   (b) $a \wedge b$
   (c) $((a \wedge b) \vee (\neg b \wedge c)) \vee \neg d$
   (d) $(a \wedge b) \vee (c \wedge d)$
   (e) $(a \vee b) \wedge (c \vee d)$
7. Determine if there is a setting of the variables in the following expression that makes the evaluation of the expression true.
   (a) $a \vee b$
   (b) $a \wedge b$
   (c) $((a \wedge b) \vee (\neg b \wedge c)) \vee \neg d$
   (d) $(a \wedge b) \vee (c \wedge d)$
   (e) $(a \vee b) \wedge (c \vee d)$
8. Explain the SAT problem.
9. Explain the 3-SAT problem.
10. Explain how to prove that a problem is NP-complete.
11. Prove that 3-SAT is NP-complete. You may assume that SAT is NP-complete.

References

[1] Michael Sipser. *Introduction to the Theory of Computation.* International Thomson Publishing, 3rd edition, 1996.