1. Consider the following Turing Machine.

| State | Input | Write | Move | Next |
|---|---|---|---|---|
| $q_0$ | $\sqcup$ | $\sqcup$ | L | $q_a$ |
| $q_0$ | 0 | 0 | R | $q_0$ |
| $q_0$ | 1 | 1 | R | $q_1$ |
| $q_1$ | $\sqcup$ | $\sqcup$ | L | $q_f$ |
| $q_1$ | 0 | 0 | R | $q_1$ |
| $q_1$ | 1 | 1 | R | $q_0$ |

Determine what happens when the Turing Machine is run with the following inputs initially on the tape.

(a) 0001

(b) 0111

(c) 0110

(d) 0101010001

(e) 00000000000000111

(f) 00

(g)

**Solution:**

(a) Fail

(b) Fail

(c) Accept

(d) Accept

(e) Fail

(f) Accept

(g) Accept

2. Give the state table for a Turing Machine that appends a parity bit to a tape with a string of consecutive 0's and 1's.

**Solution:**

| State | Input | Write | Move | Next |
|---|---|---|---|---|
| $q_0$ | $\sqcup$ | 0 | L | $q_a$ |
| $q_0$ | 0 | 0 | R | $q_0$ |
| $q_0$ | 1 | 1 | R | $q_1$ |
| $q_1$ | $\sqcup$ | 1 | L | $q_f$ |
| $q_1$ | 0 | 0 | R | $q_1$ |
| $q_1$ | 1 | 1 | R | $q_0$ |

3. Construct a Turing Machine to compute the sequence $0 \sqcup 1 \sqcup 0 \sqcup 1 \sqcup 0 \sqcup \ldots$, that is, 0 blank 1 blank 0 blank, etc [1].

4. Give the state table for a Turing Machine that multiplies a string of consecutive 0's and 1's by 2. The machine should treat the initial contents of the tape as a natural number written in binary form, with the least significant bit at the end. That is, if the contents of the tape are 01101, then the right-most 1 represents the number 1, the middle 1 represents the number 4 and the left-most 1 represents the number 8. Then the number on the tape is $8 + 4 + 1 = 13$.

5. Give the state table for a Turing Machine that multiplies a string of consecutive 0's and 1's by 2. The machine should treat the initial contents of the tape as a natural number written in binary form, with the most significant bit at the end. That is, if the contents of the tape are 01101, then the right-most 1 represents the number 16, the middle 1 represents the number 4 and the left-most 1 represents the number 2. Then the number of the tape is $2 + 4 + 16 = 22$.

6. Give the state table for a Turing Machine that adds 1 to a string of consecutive 0's and 1's.

7. Give the state table for a Turing Machine that subtracts 1 to a string of consecutive 0's and 1's.

8. List all words of length at most three in $\Sigma^*$ where $\Sigma$ is:
   (a) $\{0, 1\}$
   (b) $\{a, b, c\}$
   (c) $\{\}$

9. Design a Turing machine to recognise the language $\{0^n 1^n | n \geq 1\}$.

10. Design a Turing machine to recognise the language $\{ww | w \in \{0, 1\}^*\}$

11. Design a Turing machine to recognise the language $\{a^i b^j c^k | i, j, k \in \mathbb{N}_0\}$

## References

[1] A. M. Turing. On computable numbers, with an application to the entscheidungsproblem. *Proceedings of the London Mathematical Society*, s2-42(1):230–265, 1937.