

1. Consider the following Turing Machine.

State	Input	Write	Move	Next
q_0	\square	\square	L	q_a
q_0	0	0	R	q_0
q_0	1	1	R	q_1
q_1	\square	\square	L	q_f
q_1	0	0	R	q_1
q_1	1	1	R	q_0

Determine what happens when the Turing Machine is run with the following inputs initially on the tape.

- (a) 0001
- (b) 0111
- (c) 0110
- (d) 0101010001
- (e) 000000000000000111
- (f) 00
- (g)

Solution:

- (a) Fail
- (b) Fail
- (c) Accept
- (d) Accept
- (e) Fail
- (f) Accept
- (g) Accept

2. Give the state table for a Turing Machine that appends a parity bit to a tape with a string of consecutive 0's and 1's.

Solution:

State	Input	Write	Move	Next
q_0	\sqcup	0	L	q_a
q_0	0	0	R	q_0
q_0	1	1	R	q_1
q_1	\sqcup	1	L	q_f
q_1	0	0	R	q_1
q_1	1	1	R	q_0

3. Construct a Turing Machine to compute the sequence $0 \sqcup 1 \sqcup 0 \sqcup 1 \sqcup 0 \sqcup \dots$, that is, 0 blank 1 blank 0 blank, etc [1]. Give the state table for a Turing Machine that appends a parity bit to a tape with a string of consecutive 0's and 1's.

Solution:

State	Input	Write	Move	Next
q_0	\sqcup	0	R	q_1
q_0	0	0	R	q_f
q_0	1	1	R	q_f
q_1	\sqcup	\sqcup	R	q_2
q_1	0	0	R	q_f
q_1	1	1	R	q_f
q_2	\sqcup	1	R	q_3
q_2	0	0	R	q_f
q_2	1	1	R	q_f
q_3	\sqcup	\sqcup	R	q_0
q_3	0	0	R	q_f
q_3	1	1	R	q_f

4. Give the state table for a Turing Machine that multiplies a string of consecutive 0's and 1's by 2. The machine should treat the initial contents of the tape as a natural number written in binary form, with the least significant bit at the end. That is, if the contents of the tape are 01101, then the right-most 1 represents the number 1, the middle 1 represents the number 4 and the left-most 1 represents the number 8. Then the number on the tape is $8 + 4 + 1 = 13$.

Solution:

State	Input	Write	Move	Next
q_0	\sqcup	0	R	q_a
q_0	0	0	R	q_0
q_0	1	1	R	q_0

5. Give the state table for a Turing Machine that multiplies a string of consecutive 0's and 1's by 2. The machine should treat the initial contents of the tape as a natural number written in binary form, with the most significant bit at the end. That is, if the contents of the tape are 01101, then the right-most 1 represents the number 16, the middle 1 represents the number 4 and the left-most 1 represents the number 2. Then the number of the tape is $2 + 4 + 16 = 22$.

Solution:

State	Input	Write	Move	Next
q_0	\sqcup	\sqcup	L	q_1
q_0	0	0	R	q_0
q_0	1	1	R	q_0
q_1	\sqcup	\sqcup	R	q_4
q_1	0	\sqcup	R	q_2
q_1	1	\sqcup	R	q_3
q_2	\sqcup	0	L	q_0
q_2	0	0	R	q_f
q_2	1	1	R	q_f
q_3	\sqcup	1	L	q_0
q_3	0	0	R	q_f
q_3	1	1	R	q_f
q_4	\sqcup	0	R	q_a
q_4	0	0	R	q_f
q_4	1	1	R	q_f

6. Give the state table for a Turing Machine that adds 1 to a string of consecutive 0's and 1's, where the least significant digit is on the right of the input.

Solution:

State	Input	Write	Move	Next
q_0	\sqcup	\sqcup	L	q_1
q_0	0	0	R	q_0
q_0	1	1	R	q_0
q_1	\sqcup	1	L	q_a
q_1	0	1	L	q_a
q_1	1	0	L	q_1

7. Give the state table for a Turing Machine that subtracts 1 to a string of consecutive 0's and 1's, where the least significant digit is on the right of the input.

Solution:

State	Input	Write	Move	Next
q_0	\sqcup	\sqcup	L	q_1
q_0	0	0	R	q_0
q_0	1	1	R	q_0
q_1	\sqcup	\sqcup	L	q_a
q_1	0	1	L	q_1
q_1	1	0	L	q_a

8. List all words of length at most three in Σ^* where Σ is:

- (a) $\{0, 1\}$
- (b) $\{a, b, c\}$
- (c) $\{\}$

Solution:

- (a) $\{\epsilon, 0, 1, 00, 01, 10, 11, 000, 001, 010, 011, 100, 101, 110, 111\}$
- (b) $\{\epsilon, a, b, c, aa, ab, ac, ba, bb, bc, ca, cb, cc, abc, acb, bac, bca, cab, cba\}$
- (c) $\{\epsilon\}$

9. Design a Turing machine to recognise the language $\{0^n 1^n \mid n \geq 0\}$.

Solution:

State	Input	Write	Move	Next
q_0	\sqcup	\sqcup	R	q_a
q_0	0	\sqcup	R	q_1
q_0	1	1	R	q_f
q_1	\sqcup	\sqcup	L	q_2
q_1	0	0	R	q_1
q_1	1	1	R	q_1
q_2	\sqcup	\sqcup	L	q_f
q_2	0	0	R	q_f
q_2	1	\sqcup	L	q_3
q_3	\sqcup	\sqcup	R	q_0
q_3	0	0	L	q_3
q_3	1	1	L	q_3

10. Design a Turing machine to recognise the language $\{ww^R \mid w \in \{0,1\}^*\}$ where w^R is w reversed. For example, when $w = 101011$ then $w^R = 110101$.

Solution:

State	Input	Write	Move	Next
q_0	\sqcup	\sqcup	R	q_a
q_0	0	\sqcup	R	q_1
q_0	1	\sqcup	R	q_3
q_1	\sqcup	\sqcup	L	q_2
q_1	0	0	R	q_1
q_1	1	1	R	q_1
q_2	\sqcup	\sqcup	L	q_f
q_2	0	\sqcup	L	q_5
q_2	1	1	L	q_f
q_3	\sqcup	\sqcup	L	q_4
q_3	0	0	R	q_3
q_3	1	1	R	q_3
q_4	\sqcup	\sqcup	L	q_f
q_4	0	0	L	q_f
q_4	1	\sqcup	L	q_5
q_5	\sqcup	\sqcup	R	q_0
q_5	0	0	L	q_5
q_5	1	1	L	q_5

11. Design a Turing machine to recognise the language $\{a^i b^j c^k \mid i, j, k \in \mathbb{N}_0\}$

Solution:

State	Input	Write	Move	Next
q_0	\sqcup	\sqcup	R	q_a
q_0	a	a	R	q_0
q_0	b	b	R	q_1
q_0	c	c	R	q_2
q_0	\sqcup	\sqcup	R	q_a
q_0	a	a	R	q_f
q_0	b	b	R	q_1
q_0	c	c	R	q_2
q_0	\sqcup	\sqcup	R	q_a
q_0	a	a	R	q_f
q_0	b	b	R	q_f
q_0	c	c	R	q_2

References

- [1] A. M. Turing. On computable numbers, with an application to the entscheidungsproblem. *Proceedings of the London Mathematical Society*, s2-42(1):230–265, 1937.