Problem Sheet: Turing machines

1. Consider the following Turing Machine.

State	Input	Write	Move	Next
	В	В		Accept
0	0	0	${ m L}$	0
	1	1	L	1
	В	В		Fail
1	0	0	${ m L}$	1
	1	1	L	0

Determine what happens when the Turing Machine is run with the following inputs initially on the tape.

- (a) 0001
- (b) 0111
- (c) 0110
- (d) 0101010001
- (e) 00000000000000111
- (f) 00
- (g)
- 2. Give the state table for a Turing Machine that appends a parity bit to a tape with a string of consecutive 0's and 1's.
- 3. Construct a Turing Machine to compute the sequence 0_1_0_1_0_1..., that is, 0 blank 1 blank 0 blank, etc [1].
- 4. Give the state table for a Turing Machine that multiplies a string of consecutive 0's and 1's by 2. The machine should treat the initial contents of the tape as a natural number written in binary form.
- 5. Give the state table for a Turing Machine that adds 1 to a string of consecutive 0's and 1's.
- 6. Give the state table for a Turing Machine that subtracts 1 to a string of consecutive 0's and 1's.
- 7. List all words of length at most three in Σ^* where Σ is:
 - (a) $\{0, 1\}$
 - (b) $\{a, b, c\}$
 - (c) {}

- 8. Design Turing Machines to recognise the following languages:
 - (a) $\{0^n 1^n | n \ge 1\}$
 - (b) $\{ww|w \in \{0,1\}^*\}$

References

[1] A. M. Turing. On computable numbers, with an application to the entscheidungsproblem. *Proceedings of the London Mathematical Society*, s2-42(1):230–265, 1937.