1. Consider the following Turing Machine.

State	Input	Write	Move	Next
q_0	$\sqcup$	$\sqcup$	L	q_a
q_0	0	0	R	q_0
q_0	1	1	R	q_1
q_1	$\sqcup$	$\sqcup$	L	q_f
q_1	0	0	R	q_1
q_1	1	1	R	q_0

Determine what happens when the Turing Machine is run with the following inputs initially on the tape.

(a) 0001

(b) 0111

(c) 0110

(d) 0101010001

(e) 0000000000000111

(f) 00

(g)

Solution:

(a) Fail

(b) Fail

(c) Accept

(d) Accept

(e) Fail

(f) Accept

(g) Accept

2. Give the state table for a Turing Machine that appends a parity bit to a tape with a string of consecutive 0's and 1's.

Solution:

State	Input	Write	Move	Next
q_0	$\sqcup$	0	L	q_a
q_0	0	0	R	q_0
q_0	1	1	R	q_1
q_1	$\sqcup$	1	L	q_f
q_1	0	0	R	q_1
q_1	1	1	R	q_0

3. Construct a Turing Machine to compute the sequence $0 \sqcup 1 \sqcup 0 \sqcup 1 \sqcup 0 \sqcup \ldots$, that is, 0 blank 1 blank 0 blank, etc [1]. Give the state table for a Turing Machine that appends a parity bit to a tape with a string of consecutive 0's and 1's.

Solution:

State	Input	Write	Move	Next
q_0	$\sqcup$	0	R	q_1
q_0	0	0	R	q_f
q_0	1	1	R	q_f
q_1	$\sqcup$	$\sqcup$	R	q_2
q_1	0	0	R	q_f
q_1	1	1	R	q_f
q_2	$\sqcup$	1	R	q_3
q_2	0	0	R	q_f
q_2	1	1	R	q_f
q_3	$\sqcup$	$\sqcup$	R	q_0
q_3	0	0	R	q_f
q_3	1	1	R	q_f

4. Give the state table for a Turing Machine that multiplies a string of consecutive 0's and 1's by 2. The machine should treat the initial contents of the tape as a natural number written in binary form, with the least significant bit at the end. That is, if the contents of the tape are 01101, then the right-most 1 represents the number 1, the middle 1 represents the number 4 and the left-most 1 represents the number 8. Then the number on the tape is $8 + 4 + 1 = 13$.

Solution:

State	Input	Write	Move	Next
q_0	$\sqcup$	0	R	q_a
q_0	0	0	R	q_0
q_0	1	1	R	q_0

5. Give the state table for a Turing Machine that multiplies a string of consecutive 0's and 1's by 2. The machine should treat the initial contents of the tape as a natural number written in binary form, with the most significant bit at the end. That is, if the contents of the tape are 01101, then the right-most 1 represents the number 16, the middle 1 represents the number 4 and the left-most 1 represents the number 2. Then the number of the tape is $2 + 4 + 16 = 22$.

Solution:

State	Input	Write	Move	Next
q_0	$\sqcup$	$\sqcup$	L	q_1
q_0	0	0	R	q_0
q_0	1	1	R	q_0
q_1	$\sqcup$	$\sqcup$	R	q_4
q_1	0	$\sqcup$	R	q_2
q_1	1	$\sqcup$	R	q_3
q_2	$\sqcup$	0	L	q_0
q_2	0	0	R	q_f
q_2	1	1	R	q_f
q_3	$\sqcup$	1	L	q_0
q_3	0	0	R	q_f
q_3	1	1	R	q_f
q_4	$\sqcup$	0	R	q_a
q_4	0	0	R	q_f
q_4	1	1	R	q_f

6. Give the state table for a Turing Machine that adds 1 to a string of consecutive 0's and 1's, where the least significant digit is on the right of the input.

Solution:

State	Input	Write	Move	Next
q_0	$\sqcup$	$\sqcup$	L	q_1
q_0	0	0	R	q_0
q_0	1	1	R	q_0
q_1	$\sqcup$	1	L	q_a
q_1	0	1	L	q_a
q_1	1	0	L	q_1

7. Give the state table for a Turing Machine that subtracts 1 to a string of consecutive 0's and 1's, where the least significant digit is on the right of the input.

Solution:

State	Input	Write	Move	Next
q_0	$\sqcup$	$\sqcup$	L	q_1
q_0	0	0	R	q_0
q_0	1	1	R	q_0
q_1	$\sqcup$	$\sqcup$	L	q_a
q_1	0	1	L	q_1
q_1	1	0	L	q_a

8. List all words of length at most three in Σ^* where Σ is:
   (a) $\{0, 1\}$
   (b) $\{a, b, c\}$
   (c) $\{\}$

Solution:

(a) $\{\epsilon, 0, 1, 00, 01, 10, 11, 000, 001, 010, 011, 100, 101, 110, 111\}$

(b) $\{\epsilon, a, b, c, aa, ab, ac, ba, bb, bc, ca, cb, cc, abc, acb, bac, bca, cab, cba\}$

(c) $\{\epsilon\}$

9. Design a Turing machine to recognise the language $\{0^n1^n | n \geq 0\}$.

Solution:

State	Input	Write	Move	Next
q_0	$\sqcup$	$\sqcup$	R	q_a
q_0	0	$\sqcup$	R	q_1
q_0	1	1	R	q_f
q_1	$\sqcup$	$\sqcup$	L	q_2
q_1	0	0	R	q_1
q_1	1	1	R	q_1
q_2	$\sqcup$	$\sqcup$	L	q_f
q_2	0	0	R	q_f
q_2	1	$\sqcup$	L	q_3
q_3	$\sqcup$	$\sqcup$	R	q_0
q_3	0	0	L	q_3
q_3	1	1	L	q_3

10. Design a Turing machine to recognise the language $\{ww^R \mid w \in \{0,1\}^*\}$ where w^R is w reversed. For example, when $w = 101011$ then $w^R = 110101$.

Solution:

State	Input	Write	Move	Next
q_0	$\sqcup$	$\sqcup$	R	q_a
q_0	0	$\sqcup$	R	q_1
q_0	1	$\sqcup$	R	q_3
q_1	$\sqcup$	$\sqcup$	L	q_2
q_1	0	0	R	q_1
q_1	1	1	R	q_1
q_2	$\sqcup$	$\sqcup$	L	q_f
q_2	0	$\sqcup$	L	q_5
q_2	1	1	L	q_f
q_3	$\sqcup$	$\sqcup$	L	q_4
q_3	0	0	R	q_3
q_3	1	1	R	q_3
q_4	$\sqcup$	$\sqcup$	L	q_f
q_4	0	0	L	q_f
q_4	1	$\sqcup$	L	q_5
q_5	$\sqcup$	$\sqcup$	R	q_0
q_5	0	0	L	q_5
q_5	1	1	L	q_5

11. Design a Turing machine to recognise the language $\{a^i b^j c^k | i, j, k \in \mathbb{N}_0\}$

References

[1] A. M. Turing. On computable numbers, with an application to the entscheidungsproblem. *Proceedings of the London Mathematical Society*, s2-42(1):230–265, 1937.