

HW #4

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1. A Chemical Bond

(a) With no interference, probability = sum of the individual probabilities:

$$P = |\psi_1|^2 + |\psi_2|^2 = (0.35)^2 + (0.22)^2 = 0.1225 + 0.0484 = \boxed{0.1709}$$

(b) With constructive interference, add the amplitudes before squaring:

$$P = |\psi_1 + \psi_2|^2 = (0.35 + 0.22)^2 = (0.57)^2 = \boxed{0.3249}$$

(c) The wave probability (0.32) is almost double the probability if there was no interference (0.17). This increased electron density between the two positive nuclei creates a covalent bond that holds them together.

Because of the constructive interference, there is a higher probability of finding the electron between the two nuclei, which lowers the overall energy of the system and forms a stable bond.

2. Hydrogen Atom

(a) The longest wavelength has least energy, which is the $E_3 \rightarrow E_2$ transition.

$$E_3 = -1.5 \text{ eV}$$

$$E_2 = -3.4 \text{ eV}$$

$$\Delta E = E_3 - E_2 = -1.5 - (-3.4) = 1.9 \text{ eV}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{1241 \text{ eV} \cdot \text{nm}}{1.9 \text{ eV}} \approx \boxed{653 \text{ nm (a red wavelength)}}$$

(b)

$$E_1 = -13.6 \text{ eV}$$

$$E_5 = -0.54 \text{ eV}$$

$$\Delta E = E_5 - E_1 = -0.54 - (-13.6) = \boxed{13.06 \text{ eV}}$$

(c)

$$\begin{aligned} \frac{P(E_5)}{P(E_1)} &= e^{-\Delta E/k_B T} = \frac{1}{1000} \\ -\frac{13.056}{k_B T} &= \ln\left(\frac{1}{1000}\right) = -6.908 \end{aligned}$$

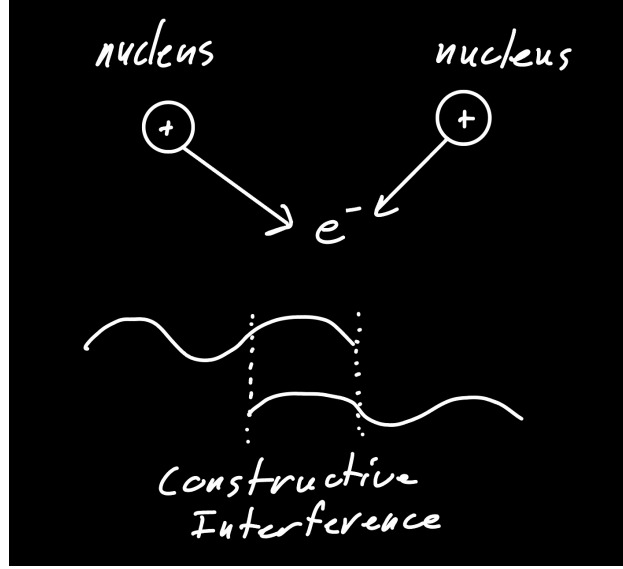


Figure 1: Constructive interference increases electron density between the two nuclei (bonding state).

$$k_B T = \frac{13.056}{6.908} = 1.890 \text{ eV}$$

$$T = \frac{1.890 \text{ eV}}{8.617 \times 10^{-5} \text{ eV/K}} = \boxed{21,900 \text{ K}}$$

3. Chemical Reactions

(a) The diagram has potential $V(x)$ versus position x with wave functions for each state (A, B, and transition)

Each well supports quantized standing-wave states. The $n = 1$ mode is the ground state (lowest energy) in each well; higher n have more nodes and higher energy (as in a box, $E_n = \frac{n^2 h^2}{8mL^2}$). So we draw a localized standing wave for state A and another for state B. To convert $A \rightarrow B$, the system must pass through a delocalized transition-state wave near the barrier top.

(b)

$$\text{rate} = \nu e^{-E_b/(k_B T)}, \quad \text{time} = \frac{1}{\text{rate}}, \quad \nu \approx 10^{13} \text{ s}^{-1}, \quad k_B T = (8.617 \times 10^{-5})(310) = 0.02671 \text{ eV}$$

$$E_b = 0.1 \text{ eV} : \quad \text{rate} = 10^{13} e^{-0.1/0.02671} = 2.4 \times 10^{11} \text{ s}^{-1}, \quad \text{time} = 4 \times 10^{-12} \text{ s}$$

$$E_b = 1 \text{ eV} : \quad \text{rate} = 10^{13} e^{-1/0.02671} = 5.3 \times 10^{-4} \text{ s}^{-1}, \quad \text{time} = 1.9 \times 10^3 \text{ s} \approx 32 \text{ min}$$

$$E_b = 10 \text{ eV} : \quad \text{rate} = 10^{13} e^{-10/0.02671} \approx 10^{-150} \text{ s}^{-1}, \quad \text{time} \approx 10^{150} \text{ s}$$

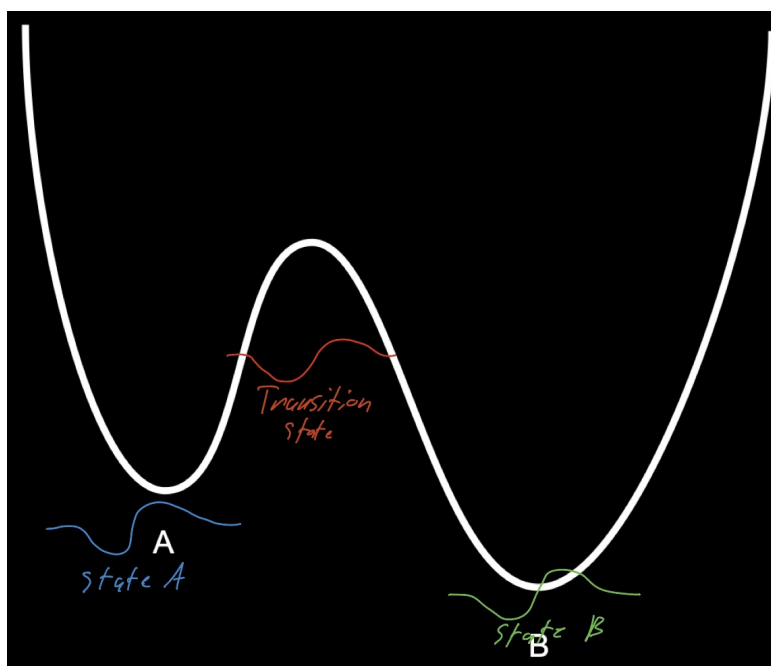


Figure 2: Wave functions for a double-well: state A, state B, and transition state.