CS7GV2: Mathematics of Light and Sound, M.Sc. in Computer Science.

Lecture #4: Light and Sound

Physical Waves!

Described by Wave equation!

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Acoustic waves

solids
liquids
gasses

Sound + vibration

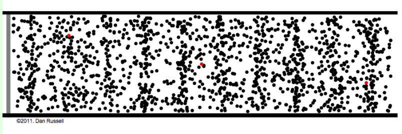
Propagation through matter by oscillation of pressure or of displacement.

No heat or mass is transferred; only energy!

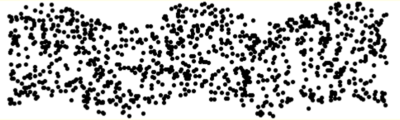
Can be reflected, refracted, diffracted, and/or attenuated.

For medium stiffness K and density ρ, speed $c_s = \sqrt{K/\rho}\,\mathrm{m\,s^{-1}}$.

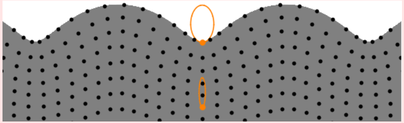
See `https://tinyurl.com/yyv5sajz`



Longitudinal waves have variations around equilibrium pressure due to compression and rarefaction of the medium in the direction of propagation.



Transverse waves have surface deformations perpendicular to the direction of wave propagation.



In solids, superposition of different waves can cause particles to move in elliptical trajectories with depth-dependent direction. In liquids, particles move in anti-clockwise circular trajectories.

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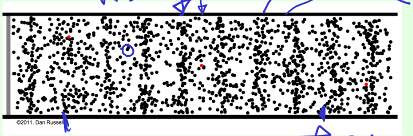
Sound waves!

1 μm   High pressure   Low pressure   A = Pressure   High   low   x



1m   x

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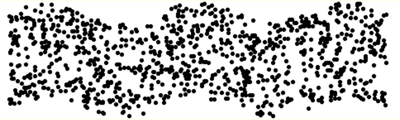

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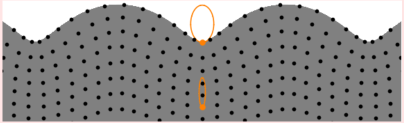

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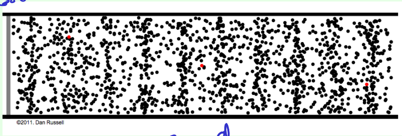
A= pressure



Sound

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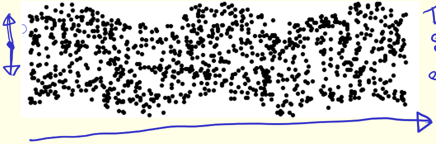
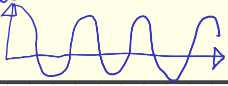


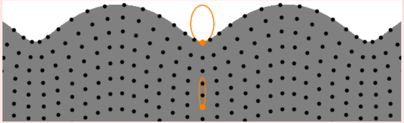
Thin solid e.g. drum

x

Transverse waves have surface deformations perpendicular to the direction of wave propagation.

A= height





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Acoustic waves

Sound & Vibration

NVH

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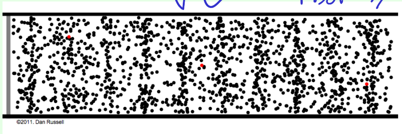
1 DoF // x



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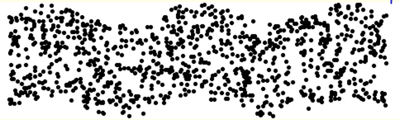
1 DoF // y



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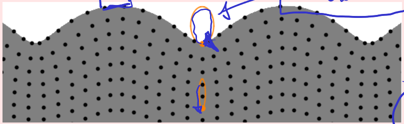
Simulation!

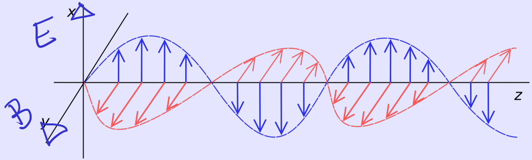


In thick solids, superposition of different waves can cause particles to move in elliptical trajectories with depth-dependent direction. In liquids, particles move in anti-clockwise circular trajectories.

Electromagnetic radiation

Sinusoidal! WAVE EQ!

Light!



Synchronised oscillations of electric and mag. fields propagating at max. speed

$$c \approx 300 \times 10^6\,\mathrm{m\,s^{-1}}.$$ "speed of light"

"Light" is electromagnetic radiation with particular ranges of wavelength λ.

Ultraviolet: 10—390 nm; Visible: 390—760 nm; Infrared: 760—1 000 000 nm.

Frequency $\nu = c/\lambda$, the number of waves that pass a point per second, is sometimes used instead of λ.

For example, $\lambda = 532\,\mathrm{nm}$ is a human-visible "green,"

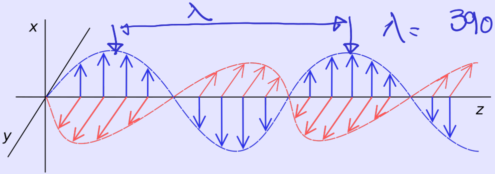
$$\nu \approx \frac{300 \times 10^6\,\mathrm{m\,s^{-1}}}{532 \times 10^{-9}\,\mathrm{m}} = 0.564 \times 10^{15}\,\mathrm{s^{-1}} = 564 \times 10^{12}\,\mathrm{s^{-1}} = 564\,\mathrm{THz}.$$

Light has much higher frequency (shorter wavelength) than the "radio" frequencies used for mobile phones and WiFi (GHz,) FM radio (MHz,) and AM radio (kHz.)

Electromagnetic radiation

Wave equ'.

$\lambda = 390\,\text{nm} \to 760\,\text{nm}$



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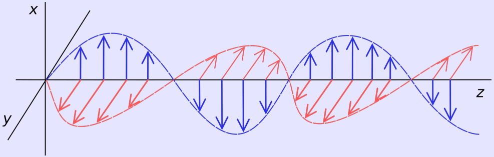

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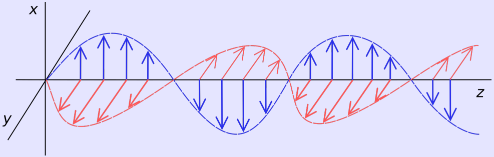

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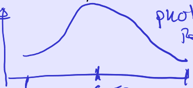

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Hertz

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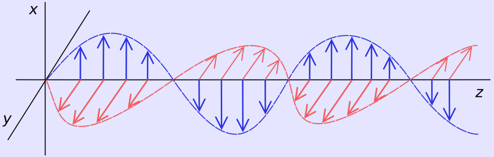
Electromagnetic radiation



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80-110 MHz
50-80 MHz

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2.4GHz
5 GHz

90 MHz
RTÉ 1 !

Vectors

Regular number

For scalars $a, b, c \in \mathbb{R}$, a vector $\boldsymbol{v} \in \mathbb{R}^3$ can be defined as,

definition

$$\boldsymbol{v} := a\,\boldsymbol{i} + b\,\boldsymbol{j} + c\,\boldsymbol{k} = (a, b, c)$$

with standard basis vectors,

$$a\,\boldsymbol{i} = \overset{(a,0,0)}{(1,0,0)} \text{ and } b\,\boldsymbol{j} = \overset{(0,b,0)}{(0,1,0)} \text{ and } \boldsymbol{k} = (0,0,1)$$

in a Euclidean coordinate system. XYZ

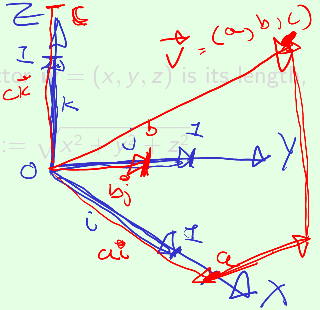
1842

(*Vectors*←*Quaternions*←*Hamilton*←*TCD!*)

Matrix

Magnitude of vector $\boldsymbol{w} = (x, y, z)$ is its length,

$$\|\boldsymbol{w}\| := \sqrt{x^2 + y^2 + z^2}$$



Scalar (or dot) product of two vectors is,

$$\begin{aligned} \boldsymbol{v} \cdot \boldsymbol{w} &:= a\,x + b\,y + c\,z \\ &= \|\boldsymbol{v}\|\|\boldsymbol{w}\| \cos\theta. \end{aligned}$$

Vector (or cross) product of two vectors is,

$$\boldsymbol{v} \times \boldsymbol{w} := (b\,z - c\,y)\boldsymbol{i} + (c\,x - a\,z)\boldsymbol{j} + (a\,y - b\,x)\boldsymbol{k}$$

which is $\perp \boldsymbol{v}$ and $\perp \boldsymbol{w}$.

$$\|\boldsymbol{v} \times \boldsymbol{w}\| = \|\boldsymbol{v}\|\|\boldsymbol{w}\||\sin\theta|$$

Vectors

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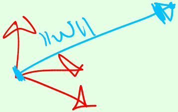
EUCLIDEAN DISTANCE

w

$$|a| = \sqrt{a \times a} = \sqrt{a^2}$$



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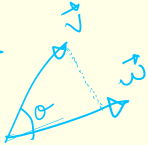
if $\|v\| = \|w\| = 1$

$= \cos\theta$



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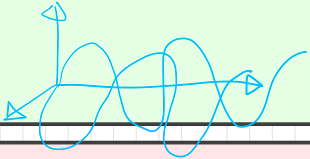

$\in \mathbb{R}^4$

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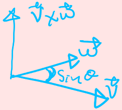

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Vector fields and calculus

$f(x,y,z,t) := x+y+z+t$ or $(x^3-5y^2+4z)xt$

Most functions we encounter are are *scalar-valued*, e.g. $f : \mathbb{R}^4 \to \mathbb{R}$ but they can *vector-valued*, e.g. $f : \mathbb{R}^4 \to \mathbb{R}^3$.

A *scalar field* is an assignment of a scalar to each point in a space; similarly, a *vector field* is an assignment of a vector.

A field can be considered as a function, e.g.

$$\boldsymbol{F} : (x, y, z, t) \to (F_x,\ F_y,\ F_z).$$

coefficients of a vector

Vector calculus is concerned with differentiation and integration of such functions.

$f(x,y,z,t) = (x+y)\vec{i} + (y+z)\vec{j} + (z/t)\vec{k}$

Space and time parameters can be omitted for improved readability but you have to remember this when looking at formulae! E.g.

$$\frac{\partial \mathbf{F}}{\partial t} = (\frac{\partial F_x}{\partial t}, \frac{\partial F_y}{\partial t}, \frac{\partial F_z}{\partial t}).$$

Divergence is,

$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}.$$

A scalar denoting by how much, if at all, the field is like a point source at that position.

Curl is,

$$\nabla \times \boldsymbol{F} = (\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z})\,\boldsymbol{i} + (\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x})\,\boldsymbol{j} + (\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y})\,\boldsymbol{k}.$$

A vector denoting rotation axis and magnitude, for how much the field rotates, if at all, at that position.

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Vector calculus is concerned with differentiation and integration of such functions.

Space and time parameters can be omitted for improved readability but you have to remember this when looking at formulae! E.g.

$$\frac{\partial \boldsymbol{F}}{\partial t} = (\frac{\partial F_x}{\partial t}, \frac{\partial F_y}{\partial t}, \frac{\partial F_z}{\partial t}).$$

Divergence is,

$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}.$$

A scalar denoting by how much, if at all, the field is like a point source at that position.

Curl is,

$$\nabla \times \boldsymbol{F} = (\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z})\,\boldsymbol{i} + (\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x})\,\boldsymbol{j} + (\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y})\,\boldsymbol{k}.$$

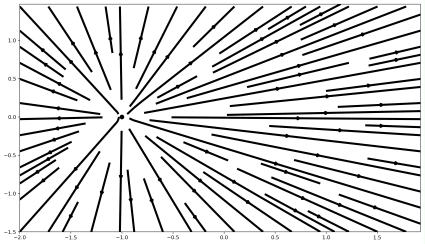
A vector denoting rotation axis and magnitude, for how much the field rotates, if at all, at that position.

Physical vector fields

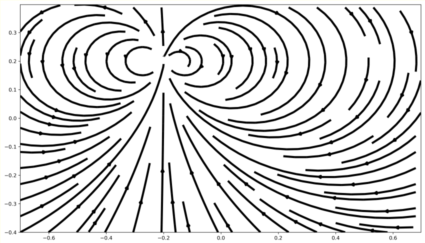
Electricity and magnetism (when considered together) comprise one of the fundamental forces in nature.

An electric field $\mathbf{E}$ exerts force on an electric charge; and when it changes wrt time it creates a magnetic field $\mathbf{B}$.

Consider $\mathbf{E}(\boldsymbol{p}, t)$ as a function denoting force magnitude and direction at position $\boldsymbol{p}$, time t.



Electric monopole field streamlines.



Magnetic dipole field streamlines.

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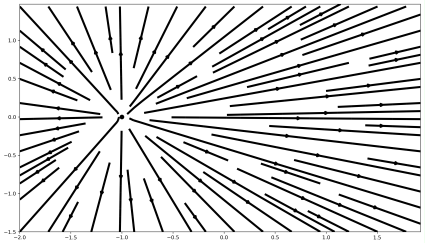

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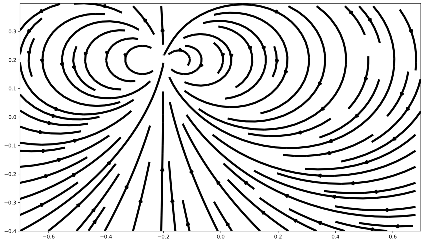

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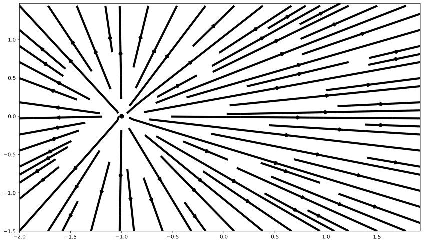

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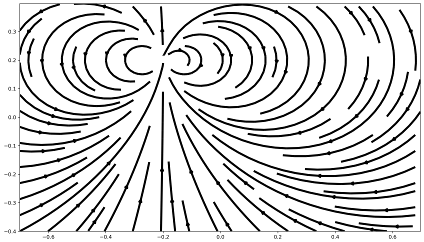

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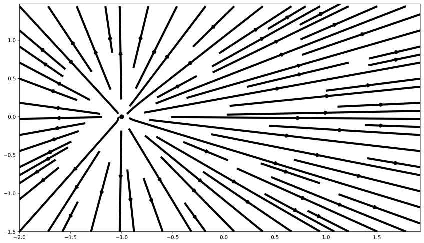

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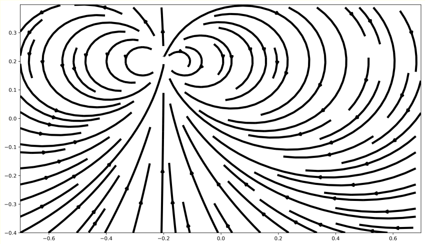

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Some physical laws

Gauss's law for electricity: electric charges generate an electric field.

$$\nabla \cdot \mathbf{E} = \rho/\epsilon$$

where ρ is electric charge and ϵ is electric permittivity.

Gauss's law for magnetism: there are no separate magnetic charges (no monopoles.)

$$\nabla \cdot \mathbf{B} = 0.$$

Faraday's law of induction: A changing magnetic field creates a rotating electric field and vice-versa.

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

Ampère-Maxwell's law: an electric current and a changing electric field create a magnetic field.

$$\nabla \times \mathbf{B} = \mu(\mathbf{J} + \epsilon\frac{\partial \mathbf{E}}{\partial t})$$

where $\mathbf{J}$ is electric current density and μ is magnetic permeability.

Note that $c = 1/\sqrt{\epsilon\mu}$.

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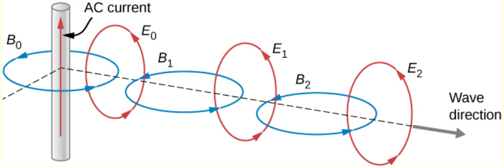
Maxwell's equations

A *system* of equations that describes relationships between electromagnetic radiation field characteristics at a point and time $(\boldsymbol{p}, t)$.

Solving the system at a sequence of points and moments in time allows the *propagation* of radiation to be modelled.

$$\begin{cases} \nabla \cdot \mathbf{E} = \rho/\epsilon \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} = \mu(\mathbf{J} + \epsilon \dfrac{\partial \mathbf{E}}{\partial t}) \end{cases}$$



AC line generating EM radiation

For very good explanations of electromagnetic radiation and Maxwell's equations, see:

https://tinyurl.com/y6dbsrxj (text)

and

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Unknowns!

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System of equations

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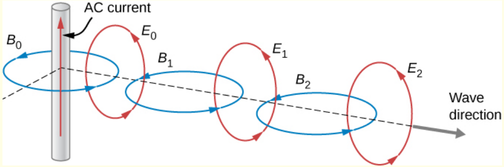
(a, b, c)

(x, y, z)

cos

E, B $\rightarrow$ $\boxed{\text{Sinusoids}}$!



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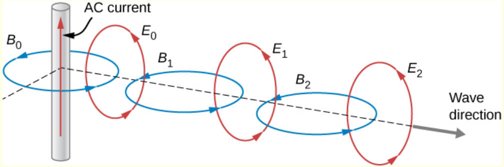

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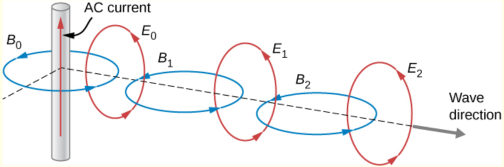

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