

Animation (Hierarchies)

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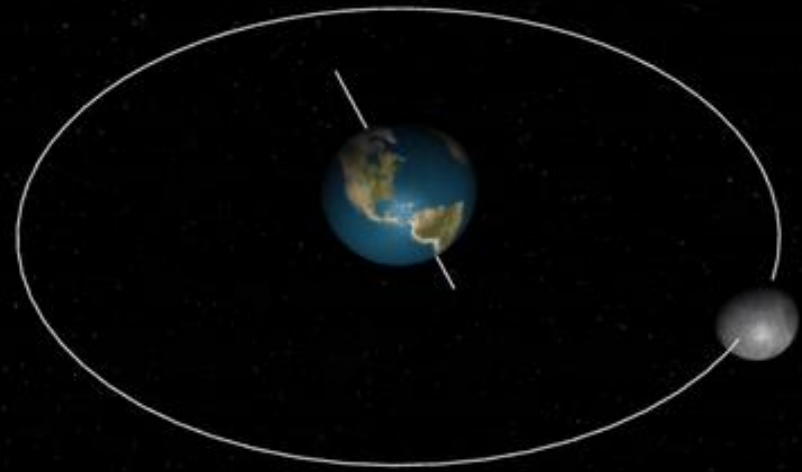
Course www: **Blackboard**

Credits: Some notes taken from Prof. Jeff Chastine

Relative Motion



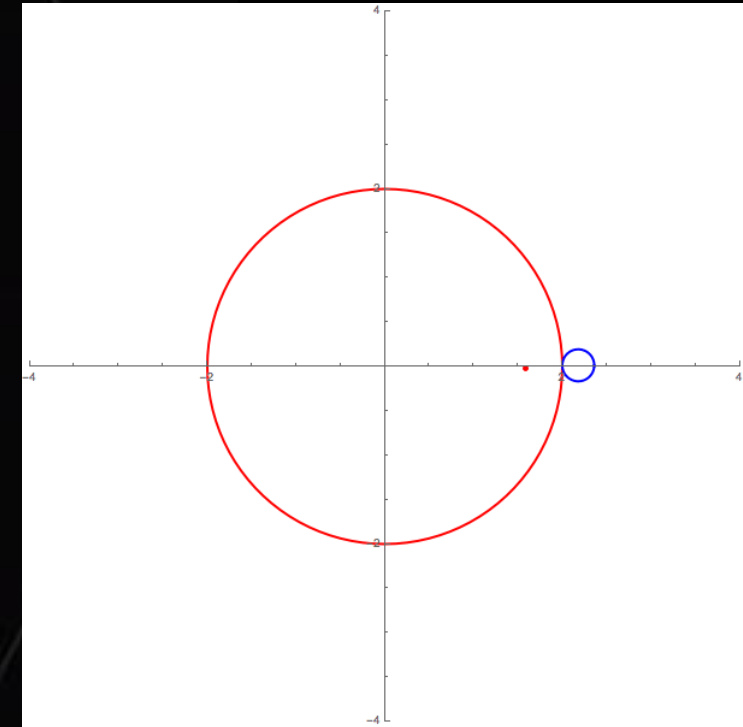
Relative Motion



Relative Motion



Relative Motion

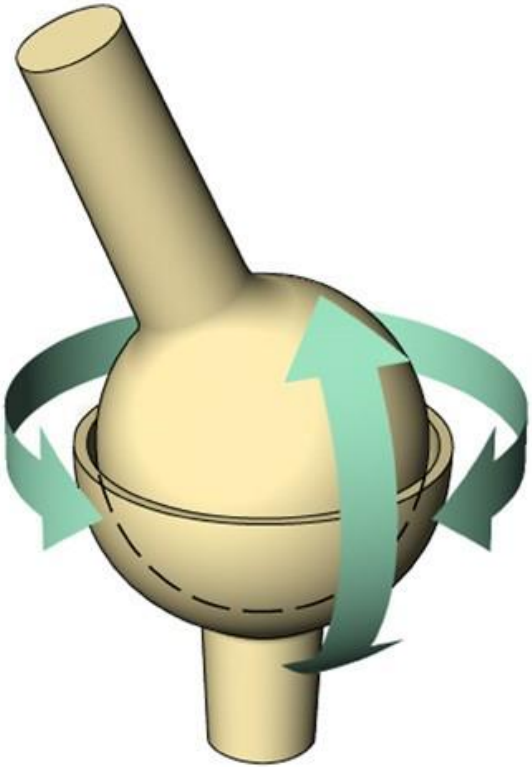


Relative Motion

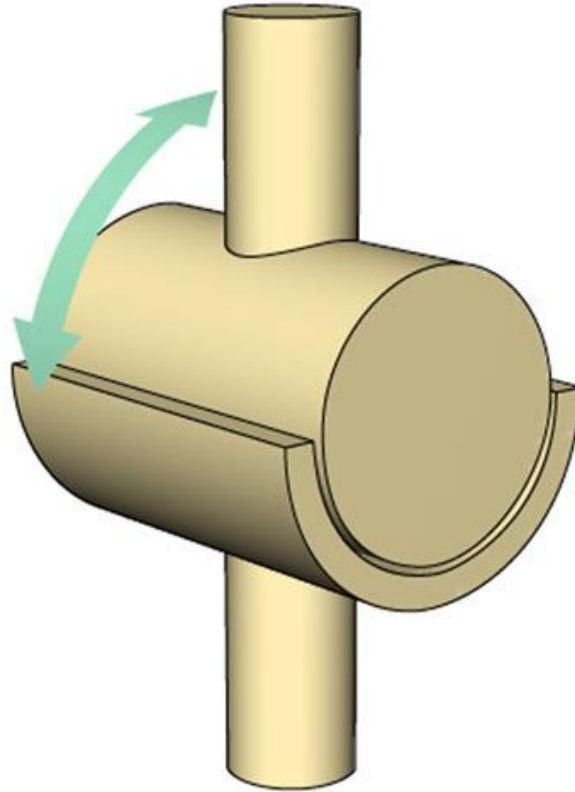
- describe a motion as relative to another one.
 - why?
 - simpler to understand and easy to model with math.
 - useful to animate specific elements (like a human character).
- when more than one is called a *motion hierarchy*.
- typically: components of a hierarchy represent objects that are physically connected or *linked*.
- we can induce constraints with hierarchies:
 - reduced motion's freedom.
- two approaches for animating figures defined by hierarchies: forward & inverse kinematics (future lectures).

Degrees of Freedom

rotational joints example:



3 degrees of freedom
pitch, roll and yaw



1 degrees of freedom

rotational and translational
example:

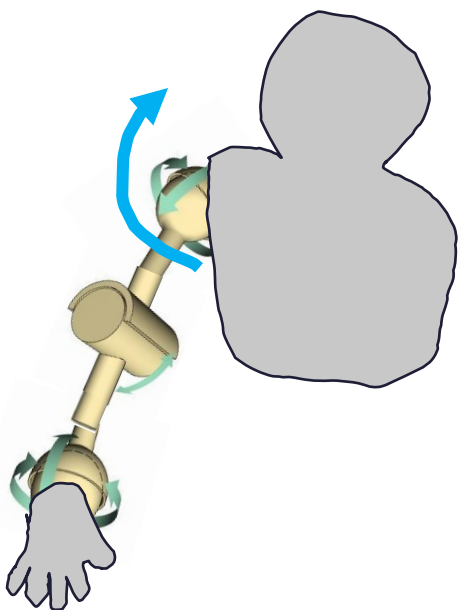


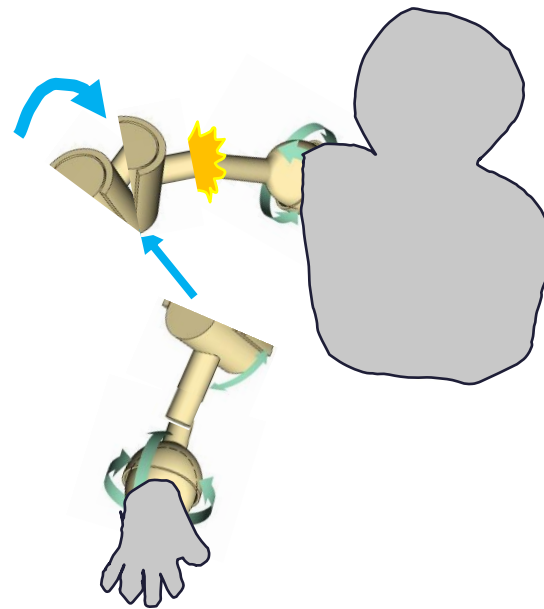
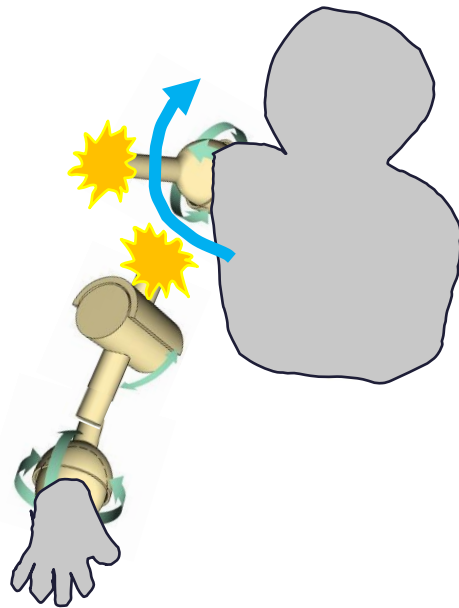
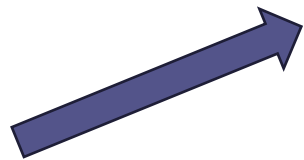
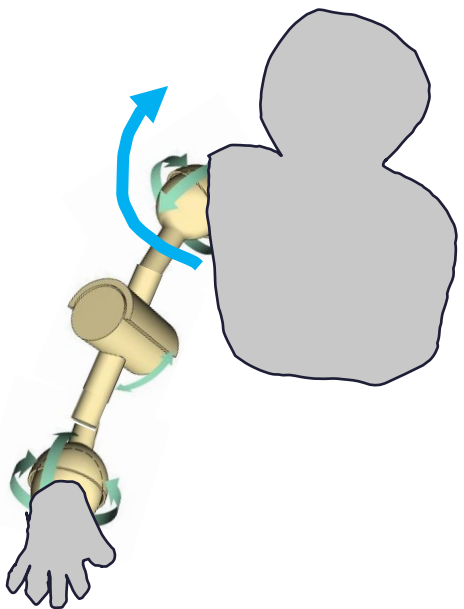
6 degrees of freedom
(no constraints)

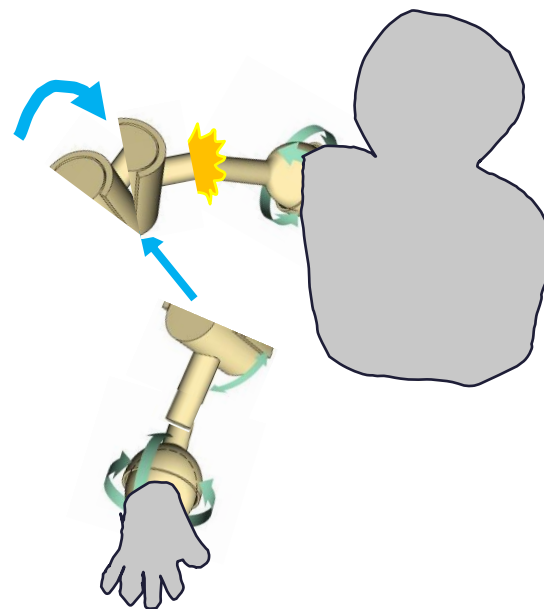
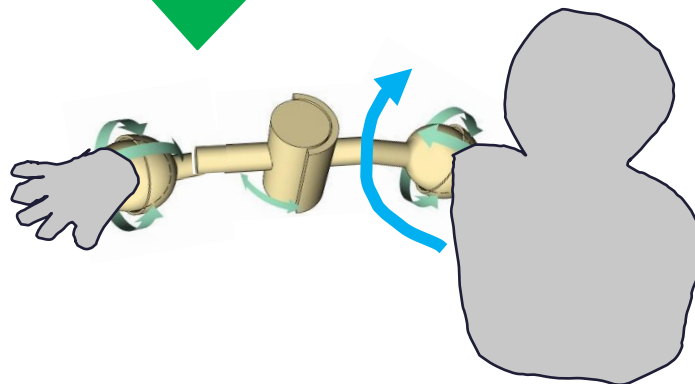
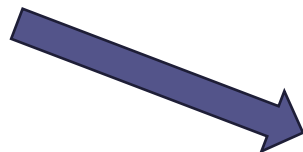
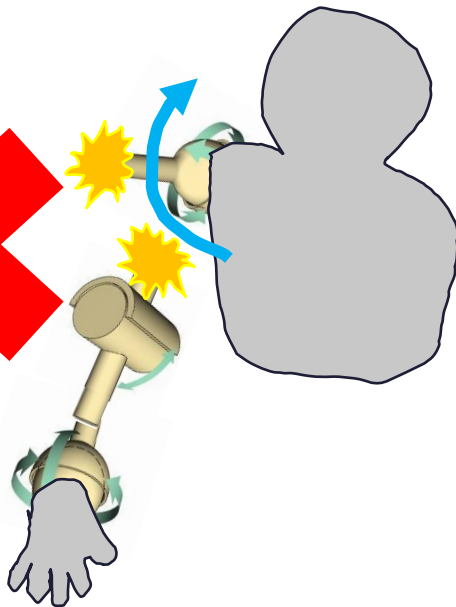
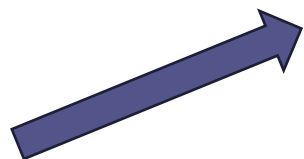
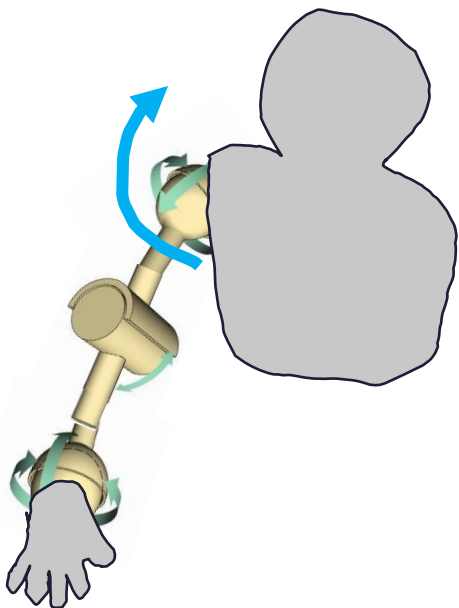
Model Transformations

- thinking in “local frame view” is usually simpler.
- and mathematically easy to extend to a *hierarchical model*.
→ like for *jointed assemblies*, such as articulated figures (animals, robots etc.)
- each sub-component has its own *local frame*.
- changes made to the *parent frame* are propagated down to the *child frames* (thus all models in a branch are globally controlled by the parent).
- This simplifies the specification of *animation*.

note: frame = coordinate system



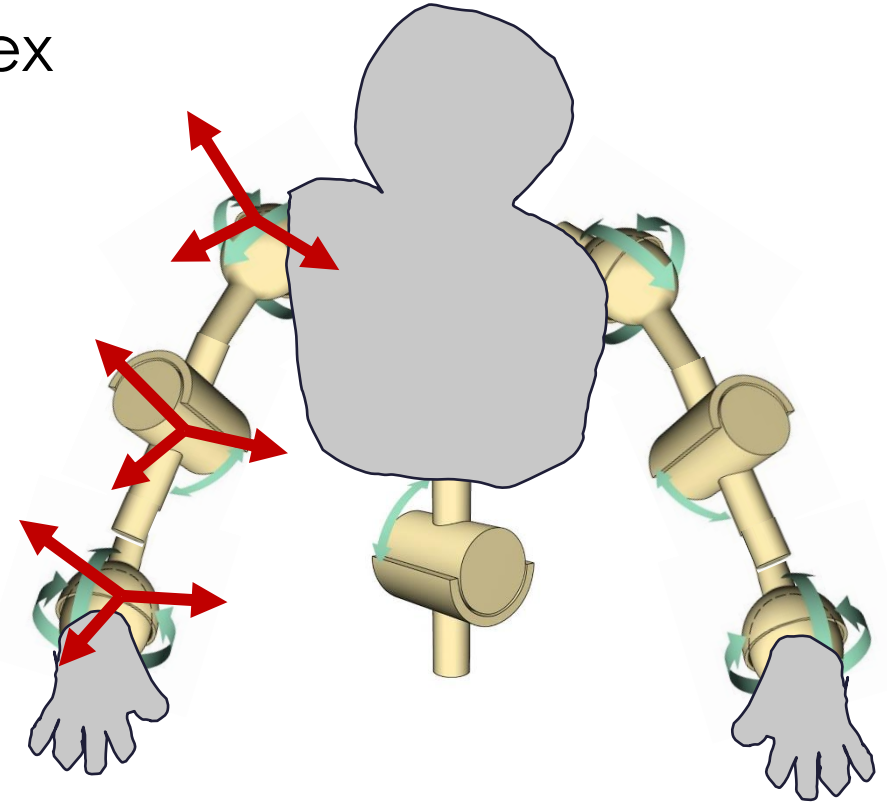
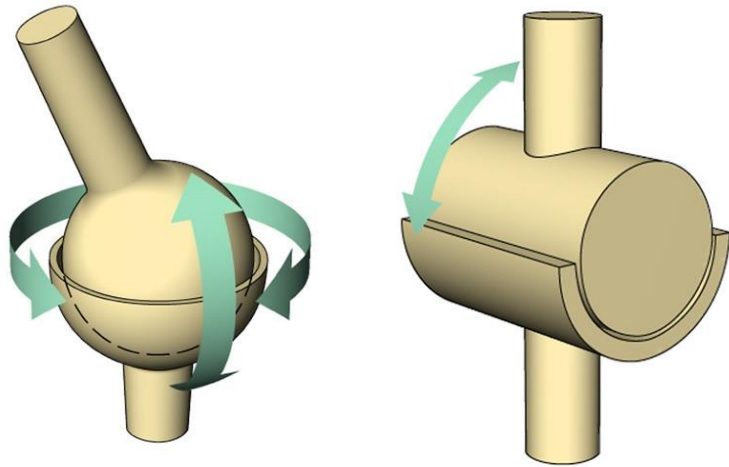




Hierarchical Transformations

This model are typically, used to model complex articulated assembly that already presents an implicit *hierarchy* (like the *human body*).

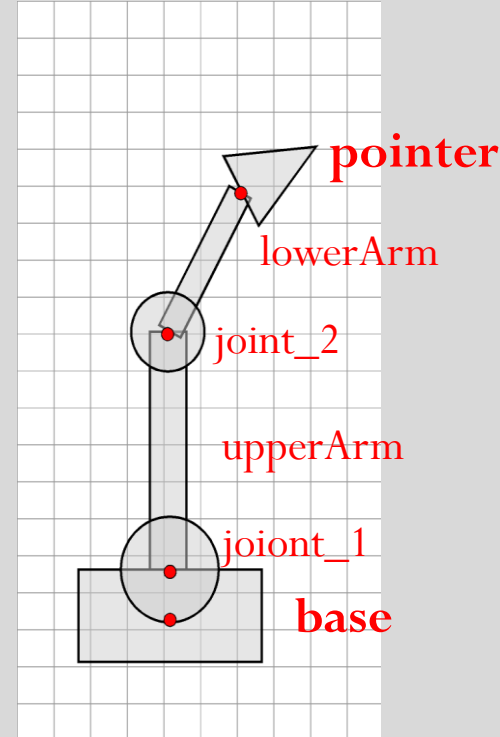
→ to do so we associate *local frames* with each *sub-objects* in the assembly.



Hierarchical Transformations

how we construct a hierarchy?

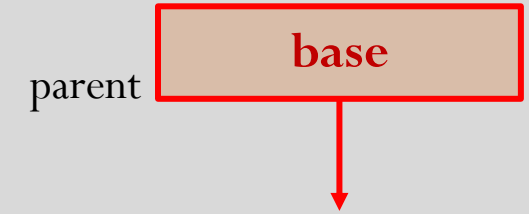
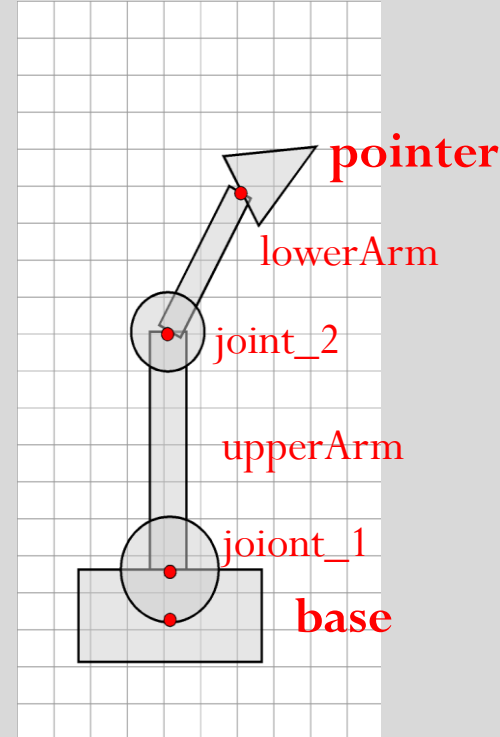
- we *relate parent-child frames* via a transformation (matrix).
 - this relation is described by a *tree*:
 - where each node has its own *local co-ordinate system*.



Hierarchical Transformations

how we construct a hierarchy?

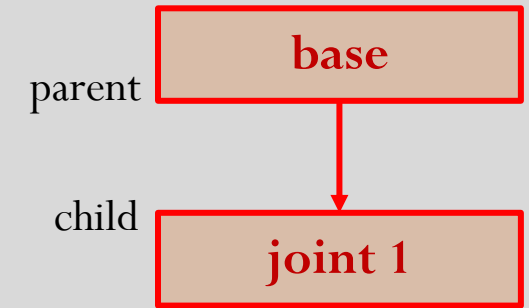
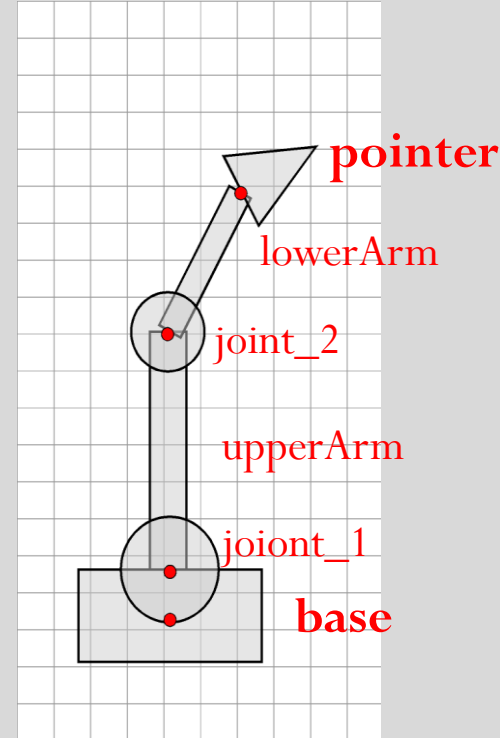
- we *relate parent-child frames* via a transformation (matrix).
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 - where each node has its own *local co-ordinate system*.



Hierarchical Transformations

how we construct a hierarchy?

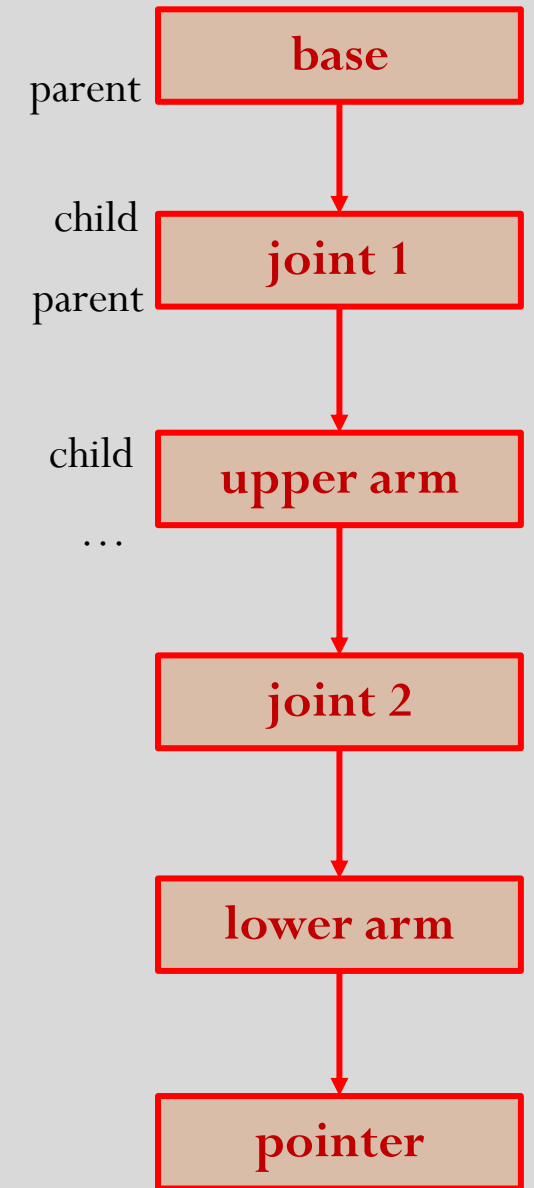
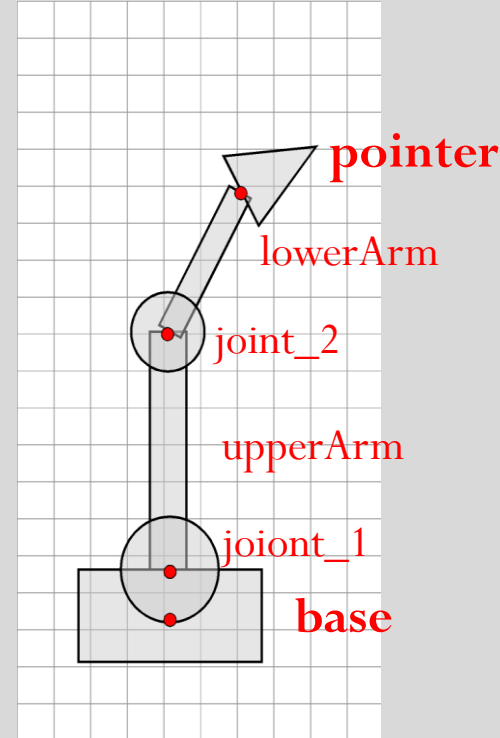
- we *relate parent-child frames* via a transformation (matrix).
 - this relation is described by a *tree*:
 - where each node has its own *local co-ordinate system*.



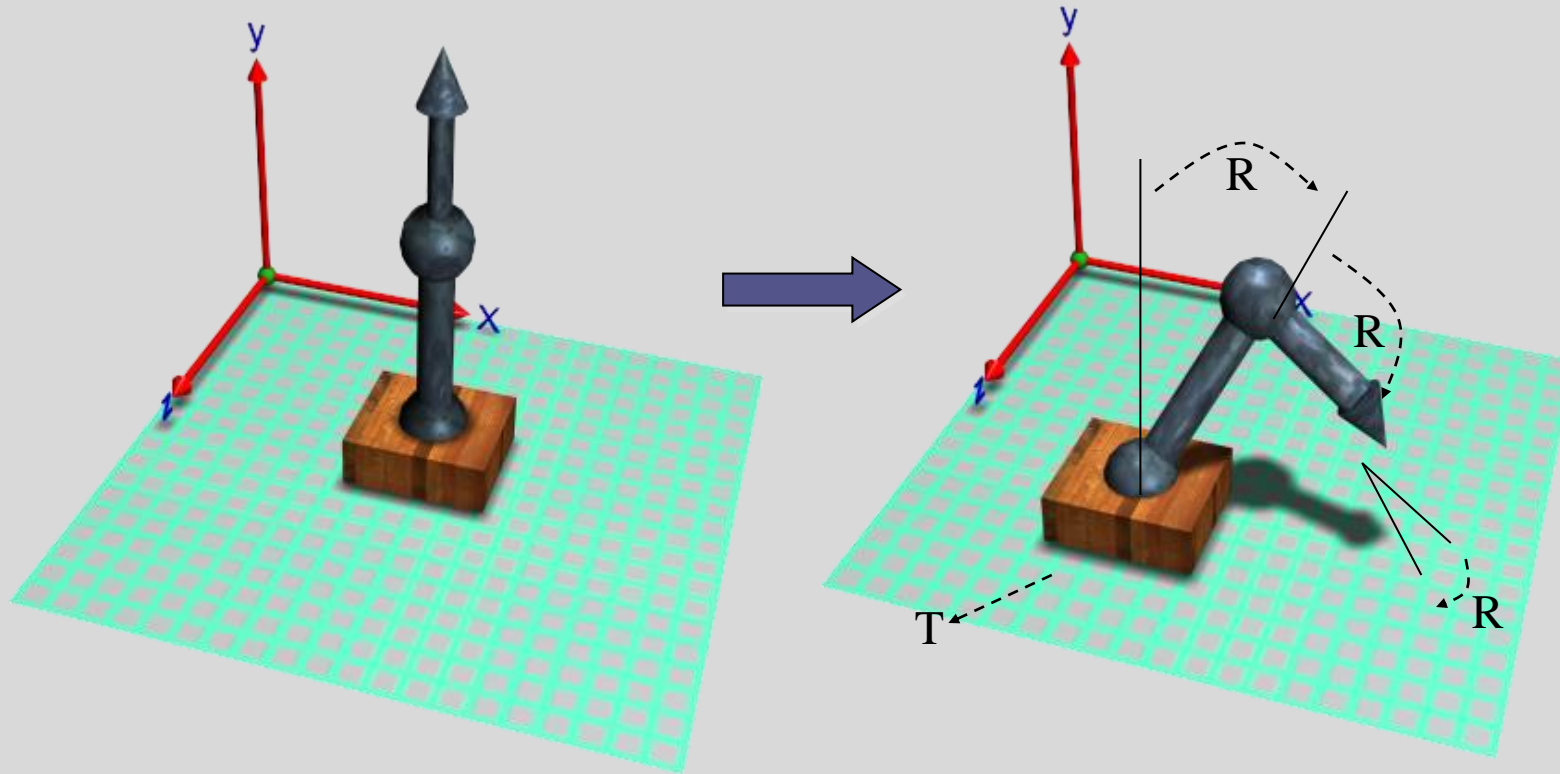
Hierarchical Transformations

how we construct a hierarchy?

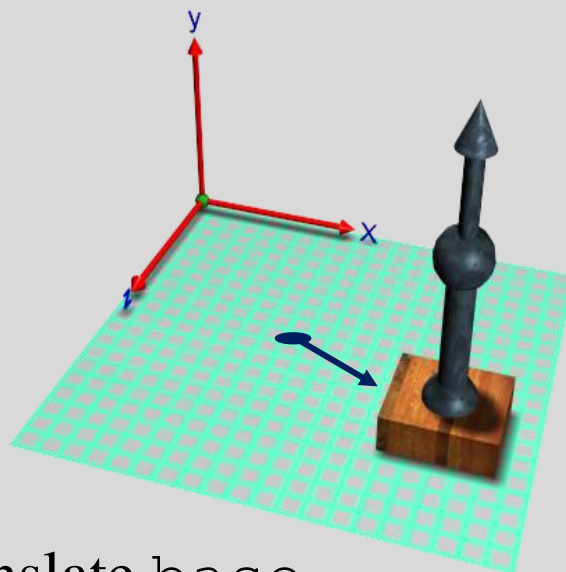
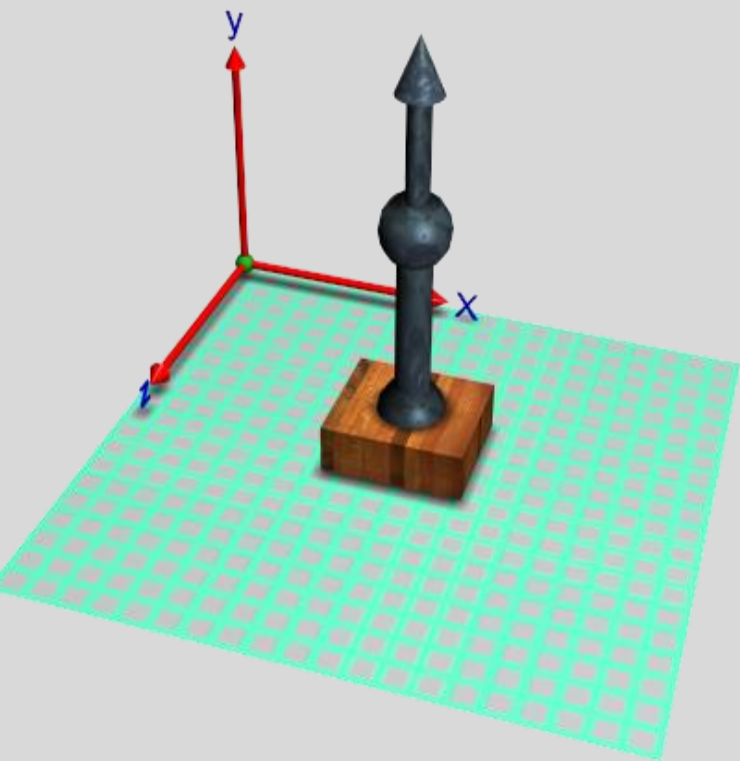
- we relate parent-child frames via a transformation (matrix).
 - this relation is described by a *tree*:
 - where each node has its own *local co-ordinate system*.



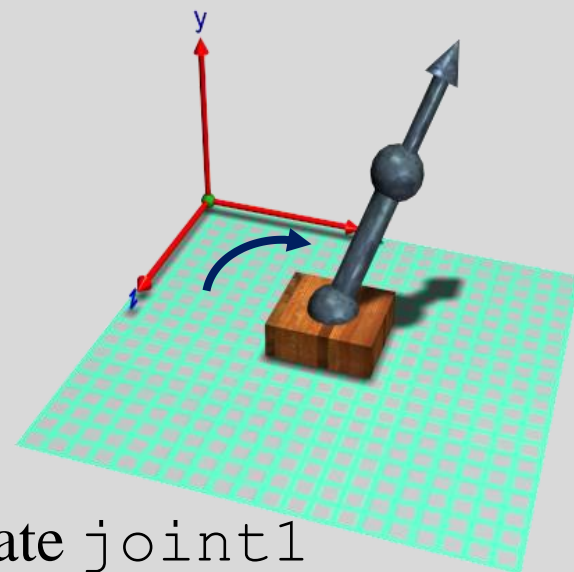
Hierarchical Transformations



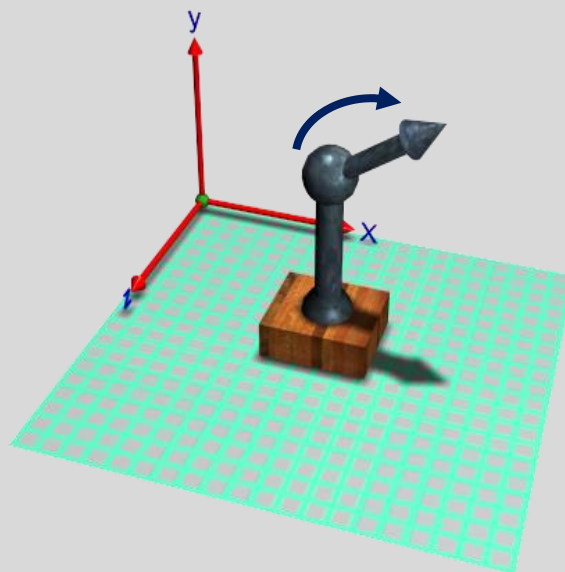
Hierarchical transformation allow independent control over sub-parts of an assembly



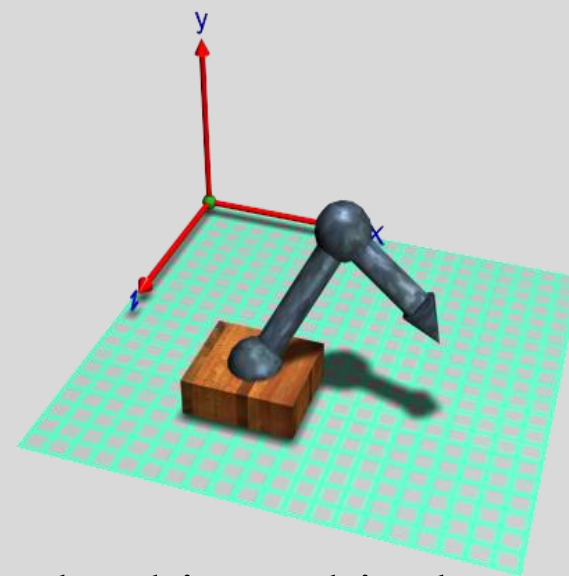
translate base



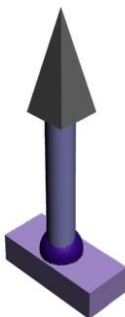
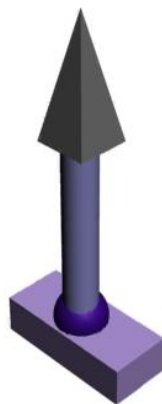
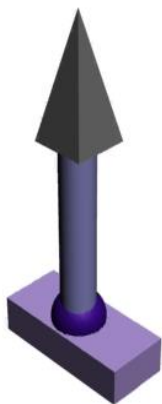
rotate joint1



rotate joint2



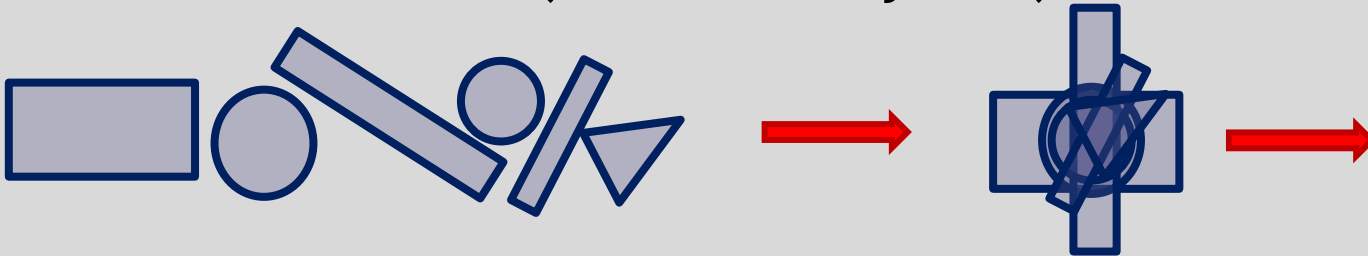
complex hierarchical
transformation



OpenGL[®] Implementation

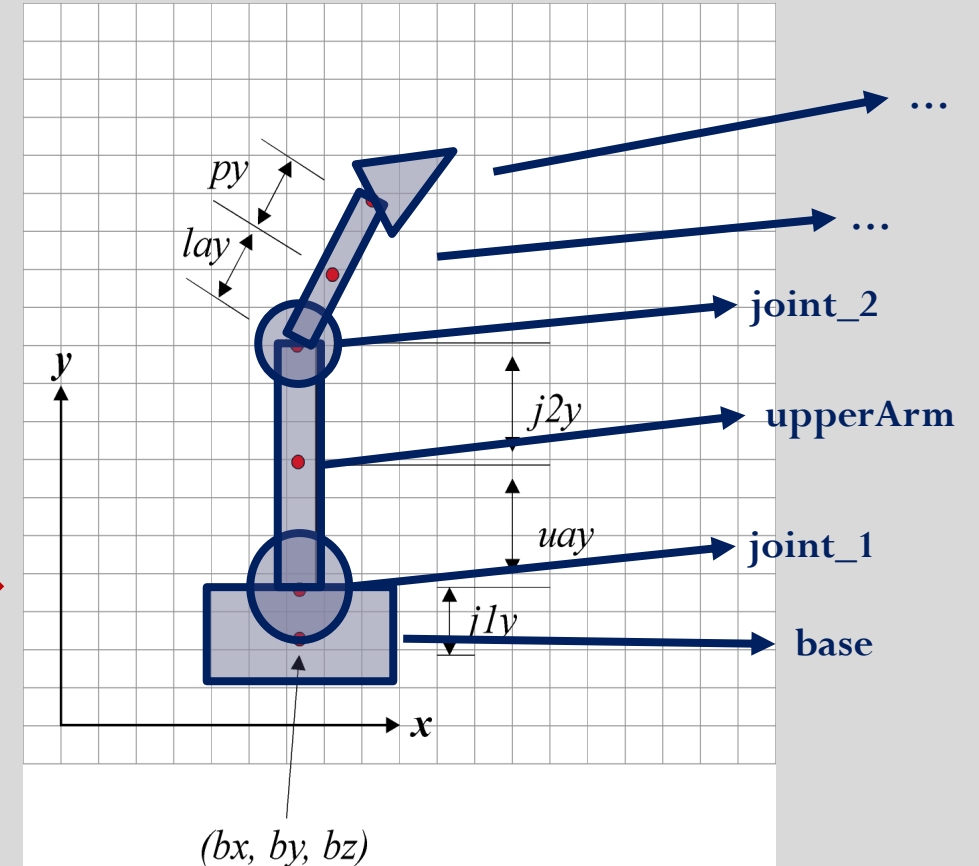
(**input**) what we define:

- a hierarchy (composed of asset of elements *base*, *joint_1*, *upperArm*, *joint_2*, ...).
- location and orientation of each element of the hierarchy in their **local** frame (reference system).



(**output**) what we obtain:

- render all the element of the final structure.
→ this means extract all the **global** coordinates (to send to the shaders).



OpenGL[®] Implementation

base (parent)

```
local1 = identity_mat4 ();  
local1 = rotate(base_orientation) * local1;  
local1 = translate(bx, by, bz) * local1;  
global1 = local1;
```

```
updateUniformVariables(model matrix = global1);  
drawBase();
```

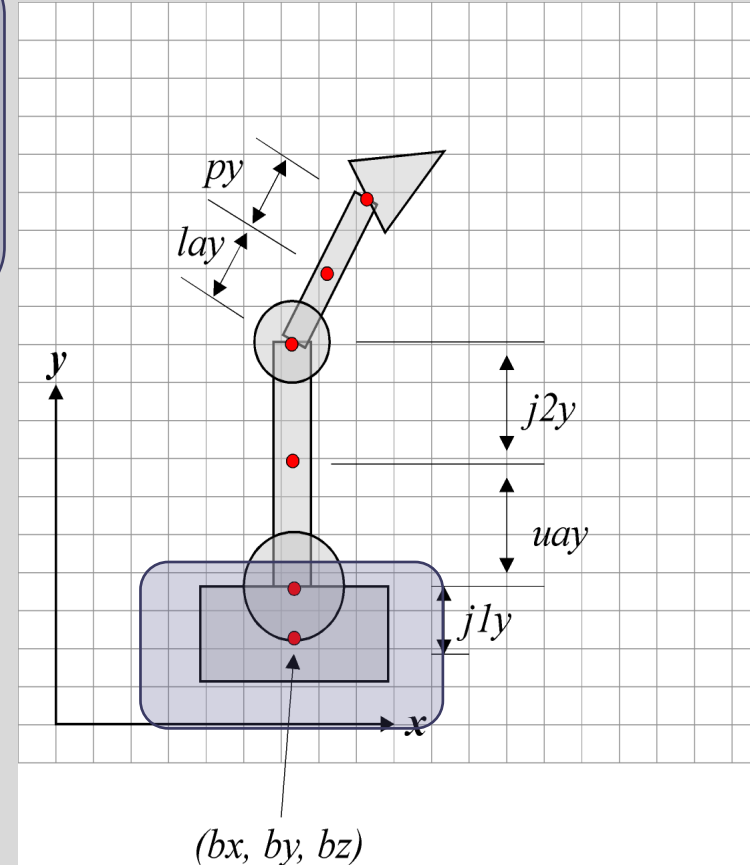
```
local2 = identity_mat4 ();  
local2 = rotate(joint1_orientation) * local2;  
local2 = translate(0, j1y, 0) * local2;  
global2 = local1*local2;
```

```
updateUniformVariables(model matrix = global2);  
drawJoint1();
```

```
local3 = identity_mat4 ();  
local3 = rotate(upperArm_orientation) * local3;  
local3 = translate(0, uay, 0) * local3;  
global3 = local1*local2*local3;
```

```
updateUniformVariables(model matrix = global3);  
drawUpperArm();
```

etc.



OpenGL® Implementation

base (parent)

define position
and orientation
in the world as
transformation
matrix

local1

```
local1 = identity_mat4 ();  
local1 = rotate(base_orientation) * local1;  
local1 = translate(bx, by, bz) * local1;  
global1 = local1;
```

```
updateUniformVariables(model matrix = global1);  
drawBase();
```

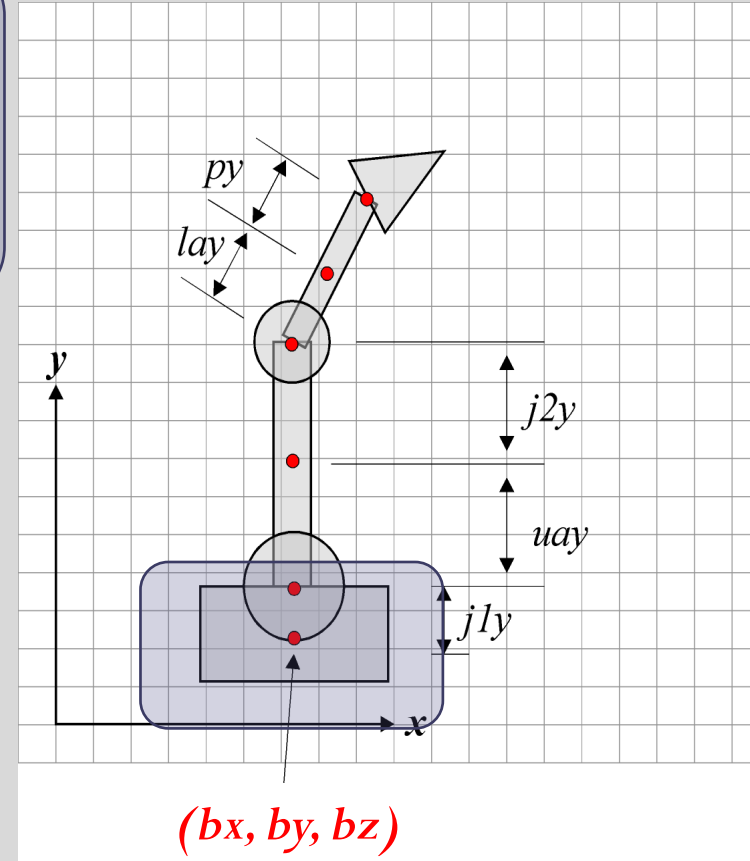
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```

```
updateUniformVariables(model matrix = global2);  
drawJoint1();
```

```
local3 = identity_mat4 ();  
local3 = rotate(upperArm_orientation) * local3;  
local3 = translate(0, uay, 0) * local3;  
global3 = local1*local2*local3;
```

```
updateUniformVariables(model matrix = global3);  
drawUpperArm();
```

etc.



OpenGL® Implementation

base (parent)

in this case
global = local
(no parent)

→ so, we can
draw directly

local1

```
local1 = identity_mat4 ();  
local1 = rotate(base_orientation) * local1;  
local1 = translate(bx, by, bz) * local1;  
global1 = local1;  
  
updateUniformVariables(model matrix = global1);  
drawBase();
```

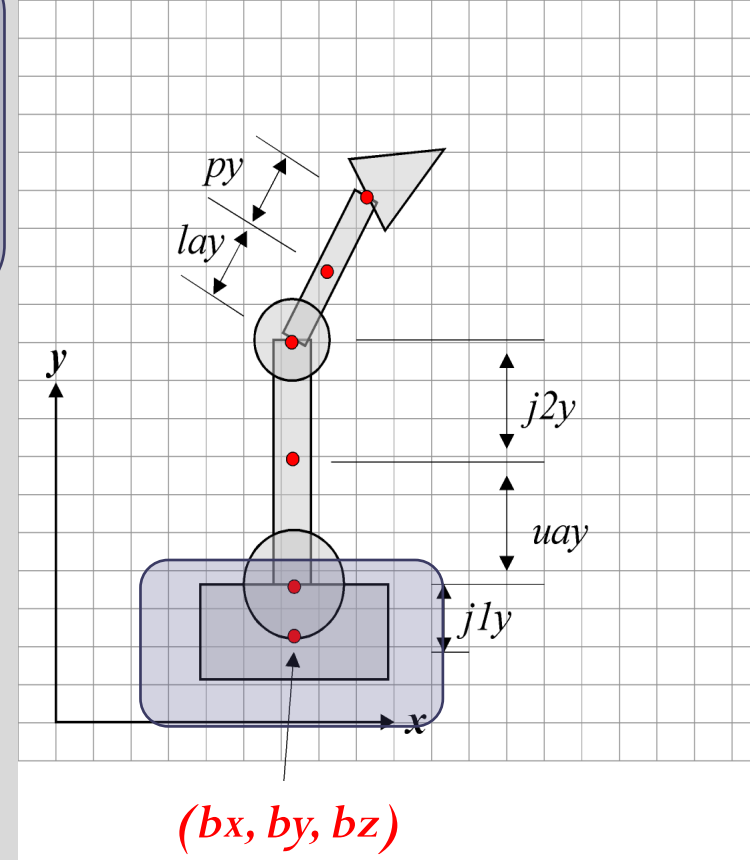
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local2 = translate(0, j1y, 0) * local2;  
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```

```
updateUniformVariables(model matrix = global2);  
drawJoint1();
```

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local3 = identity_mat4 ();  
local3 = rotate(upperArm_orientation) * local3;  
local3 = translate(0, uay, 0) * local3;  
global3 = local1*local2*local3;
```

```
updateUniformVariables(model matrix = global3);  
drawUpperArm();
```

etc.



OpenGL[®] Implementation

joint 1
(child of base)

```
local1 = identity_mat4 ();  
local1 = rotate(base_orientation) * local1;  
local1 = translate(bx, by, bz) * local1;  
global1 = local1;
```

```
updateUniformVariables(model matrix = global1);  
drawBase();
```

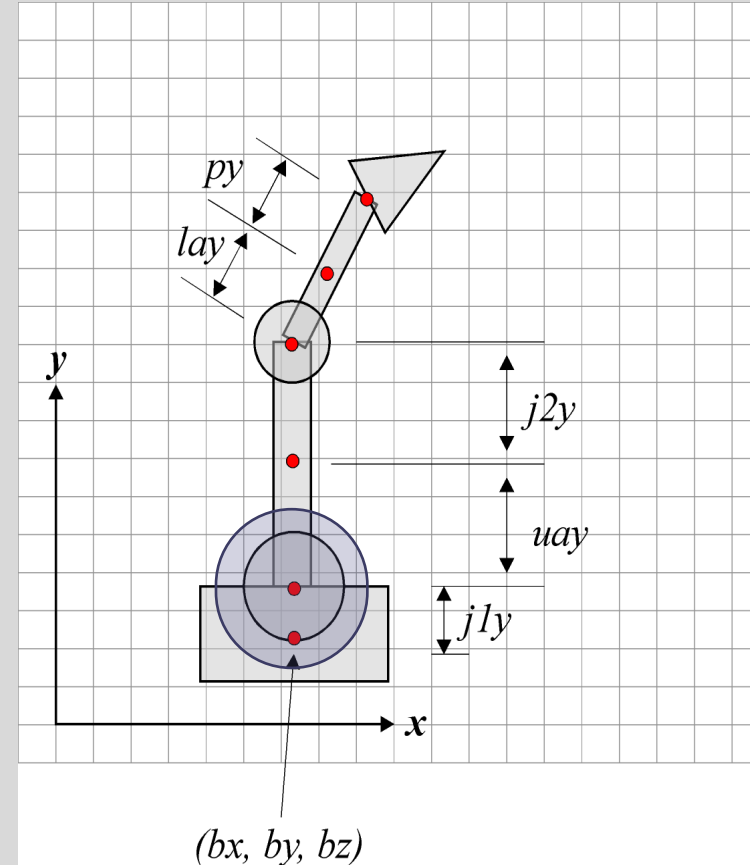
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local2 = rotate(joint1_orientation) * local2;  
local2 = translate(0, j1y, 0) * local2;  
global2 = local1*local2;
```

```
updateUniformVariables(model matrix = global2);  
drawJoint1();
```

```
local3 = identity_mat4 ();  
local3 = rotate(upperArm_orientation) * local3;  
local3 = translate(0, uay, 0) * local3;  
global3 = local1*local2*local3;
```

```
updateUniformVariables(model matrix = global3);  
drawUpperArm();
```

etc.



OpenGL® Implementation

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local1 = identity_mat4 ();  
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updateUniformVariables(model matrix = global1);  
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global2 = local1*local2;
```

```
updateUniformVariables(model matrix = global2);  
drawJoint1();
```

```
local3 = identity_mat4 ();  
local3 = rotate(upperArm_orientation) * local3;  
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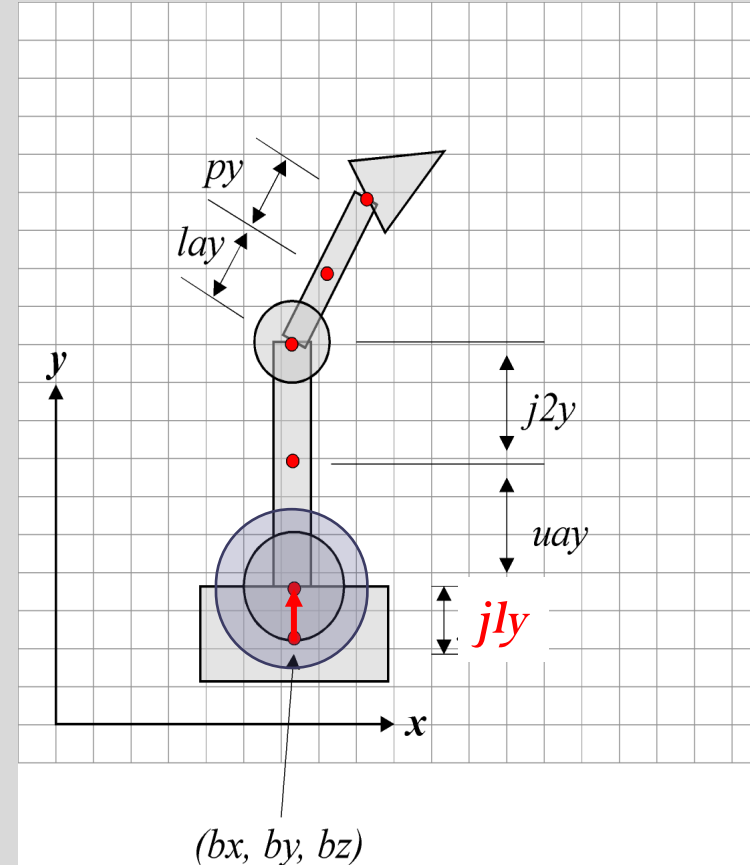
```
updateUniformVariables(model matrix = global3);  
drawUpperArm();
```

etc.

joint 1
(child of base)

define position
and orientation
in the local
frame.

local2



OpenGL® Implementation

```
local1 = identity_mat4 ();  
local1 = rotate(base_orientation) * local1;  
local1 = translate(bx, by, bz) * local1;  
global1 = local1;
```

```
updateUniformVariables(model matrix = global1);  
drawBase();
```

```
local2 = identity_mat4 ();  
local2 = rotate(joint1_orientation) * local2;  
local2 = translate(0, j1y, 0) * local2;  
global2 = local1*local2;
```

```
updateUniformVariables(model matrix = global2);  
drawJoint1();
```

```
local3 = identity_mat4 ();  
local3 = rotate(upperArm_orientation) * local3;  
local3 = translate(0, uay, 0) * local3;  
global3 = local1*local2*local3;
```

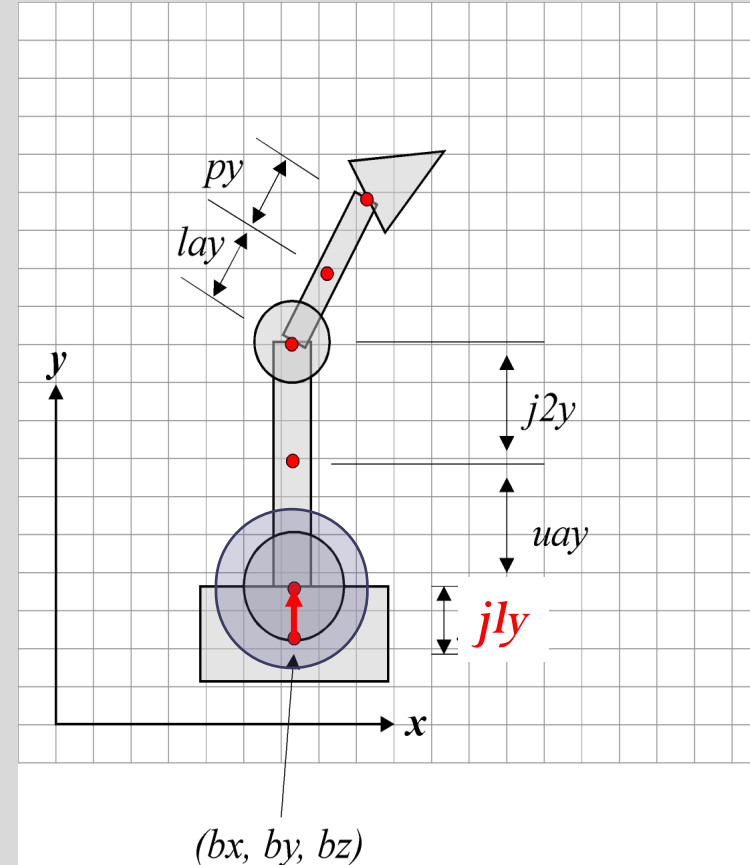
```
updateUniformVariables(model matrix = global3);  
drawUpperArm();
```

etc.

joint 1
(child of base)

we use parent
to transform
from local
→ to global

$\begin{bmatrix} \text{local1} \end{bmatrix} \times \begin{bmatrix} \text{local2} \end{bmatrix}$



OpenGL® Implementation

```
local1 = identity_mat4 ();  
local1 = rotate(base_orientation) * local1;  
local1 = translate(bx, by, bz) * local1;  
global1 = local1;
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```
updateUniformVariables(model matrix = global1);  
drawBase();
```

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local2 = identity_mat4 ();  
local2 = rotate(joint1_orientation) * local2;  
local2 = translate(0, j1y, 0) * local2;  
global2 = local1*local2;
```

```
updateUniformVariables(model matrix = global2);  
drawJoint1();
```

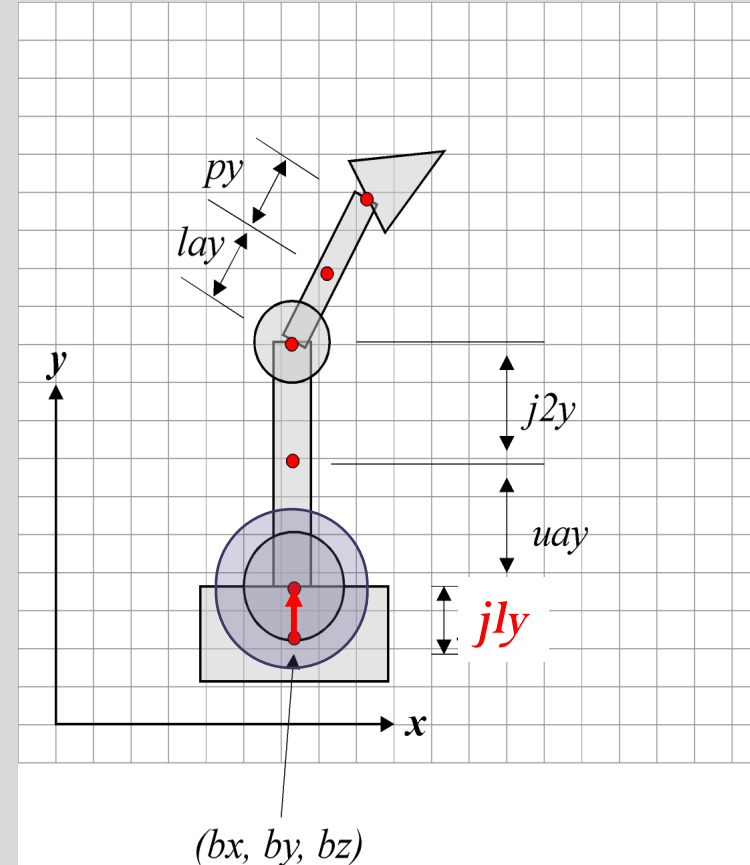
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```

```
updateUniformVariables(model matrix = global3);  
drawUpperArm();
```

etc.

joint 1
(child of base)

we draw the
global
transform



OpenGL[®] Implementation

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local1 = identity_mat4 ();  
local1 = rotate(base_orientation) * local1;  
local1 = translate(bx, by, bz) * local1;  
global1 = local1;
```

```
updateUniformVariables(model matrix = global1);  
drawBase();
```

```
local2 = identity_mat4 ();  
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local2 = translate(0, j1y, 0) * local2;  
global2 = local1*local2;
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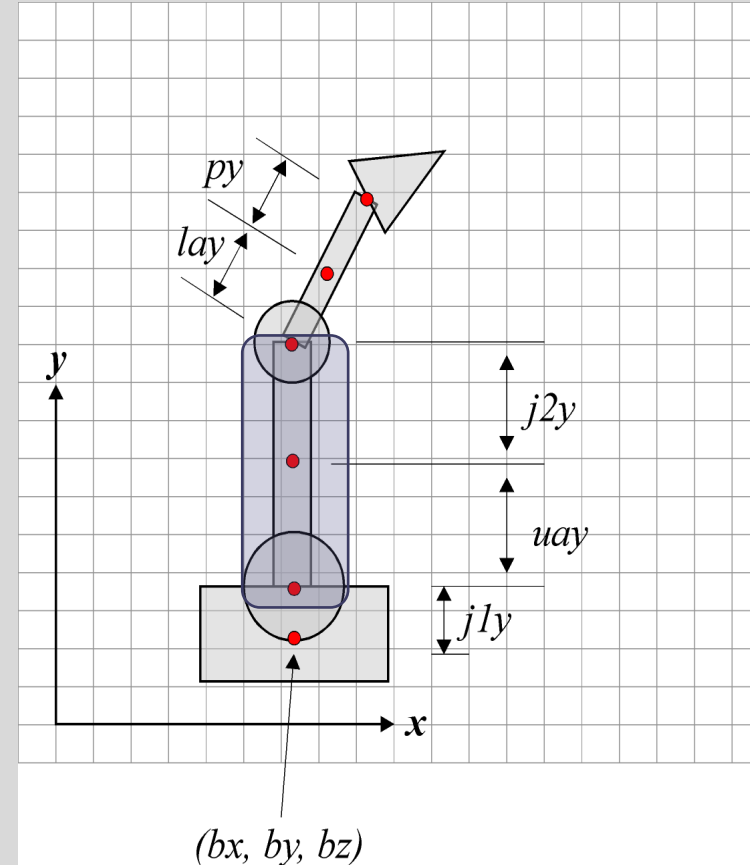
```
updateUniformVariables(model matrix = global2);  
drawJoint1();
```

```
local3 = identity_mat4 ();  
local3 = rotate(upperArm_orientation) * local3;  
local3 = translate(0, uay, 0) * local3;  
global3 = local1*local2*local3;
```

```
updateUniformVariables(model matrix = global3);  
drawUpperArm();
```

etc.

upperArm
(child of joint1)



OpenGL® Implementation

```
local1 = identity_mat4 ();  
local1 = rotate(base_orientation) * local1;  
local1 = translate(bx, by, bz) * local1;  
global1 = local1;
```

```
updateUniformVariables(model matrix = global1);  
drawBase();
```

```
local2 = identity_mat4 ();  
local2 = rotate(joint1_orientation) * local2;  
local2 = translate(0, j1y, 0) * local2;  
global2 = local1*local2;
```

```
updateUniformVariables(model matrix = global2);  
drawJoint1();
```

```
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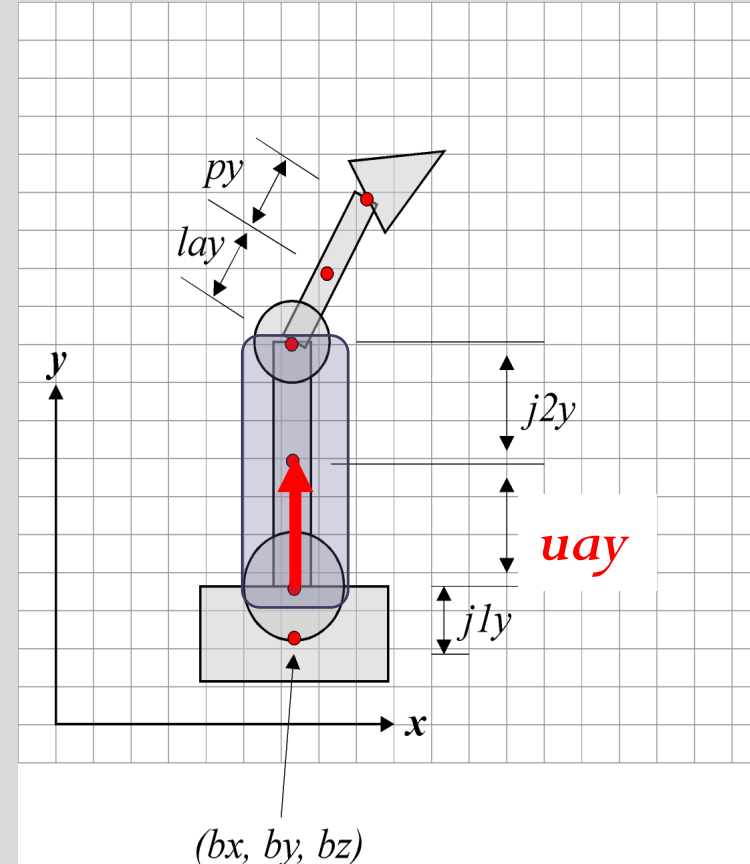
```
updateUniformVariables(model matrix = global3);  
drawUpperArm();
```

etc.

upperArm
(child of joint1)

define position
and orientation
in the local
frame.

local3



OpenGL® Implementation

```
local1 = identity_mat4 ();  
local1 = rotate(base_orientation) * local1;  
local1 = translate(bx, by, bz) * local1;  
global1 = local1;
```

```
updateUniformVariables(model matrix = global1);  
drawBase();
```

```
local2 = identity_mat4 ();  
local2 = rotate(joint1_orientation) * local2;  
local2 = translate(0, j1y, 0) * local2;  
global2 = local1*local2;
```

```
updateUniformVariables(model matrix = global2);  
drawJoint1();
```

```
local3 = identity_mat4 ();  
local3 = rotate(upperArm_orientation) * local3;  
local3 = translate(0, uay, 0) * local3;  
global3 = local1*local2*local3;
```

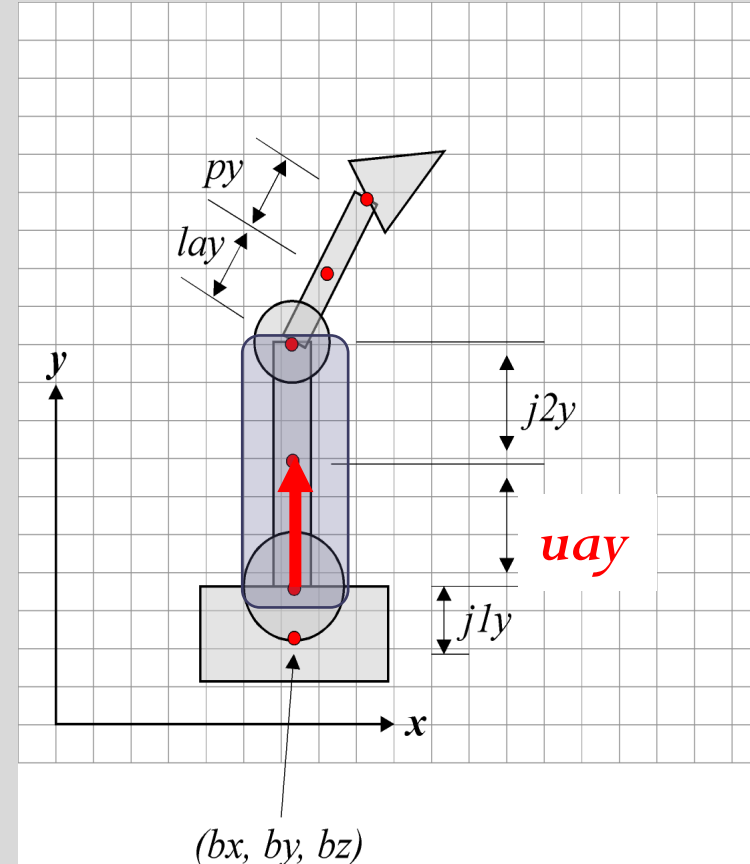
```
updateUniformVariables(model matrix = global3);  
drawUpperArm();
```

etc.

upperArm
(child of joint1)

we use parents
to transform
from local
→ to global

$$\begin{bmatrix} \text{local1} \end{bmatrix} \times \begin{bmatrix} \text{local2} \end{bmatrix} \times \begin{bmatrix} \text{local3} \end{bmatrix}$$



NOTE: this is equivalent to $\text{global3} = \text{global2} * \text{local3}$;

OpenGL® Implementation

```
local1 = identity_mat4 ();  
local1 = rotate(base_orientation) * local1;  
local1 = translate(bx, by, bz) * local1;  
global1 = local1;
```

```
updateUniformVariables(model matrix = global1);  
drawBase();
```

```
local2 = identity_mat4 ();  
local2 = rotate(joint1_orientation) * local2;  
local2 = translate(0, j1y, 0) * local2;  
global2 = local1*local2;
```

```
updateUniformVariables(model matrix = global2);  
drawJoint1();
```

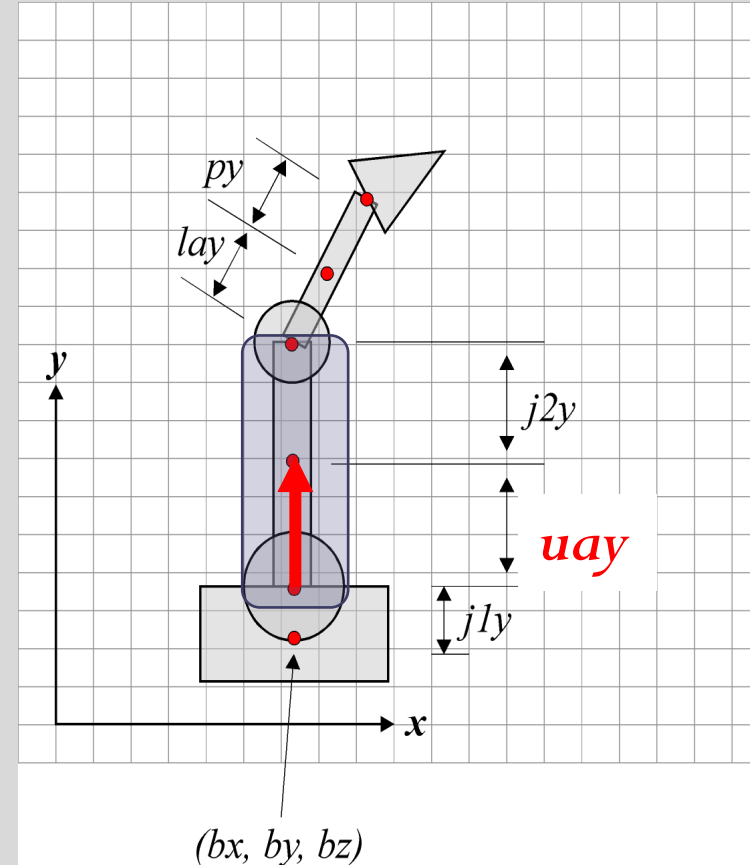
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local3 = translate(0, uay, 0) * local3;  
global3 = local1*local2*local3;
```

```
updateUniformVariables(model matrix = global3);  
drawUpperArm();
```

etc.

upperArm
(child of joint1)

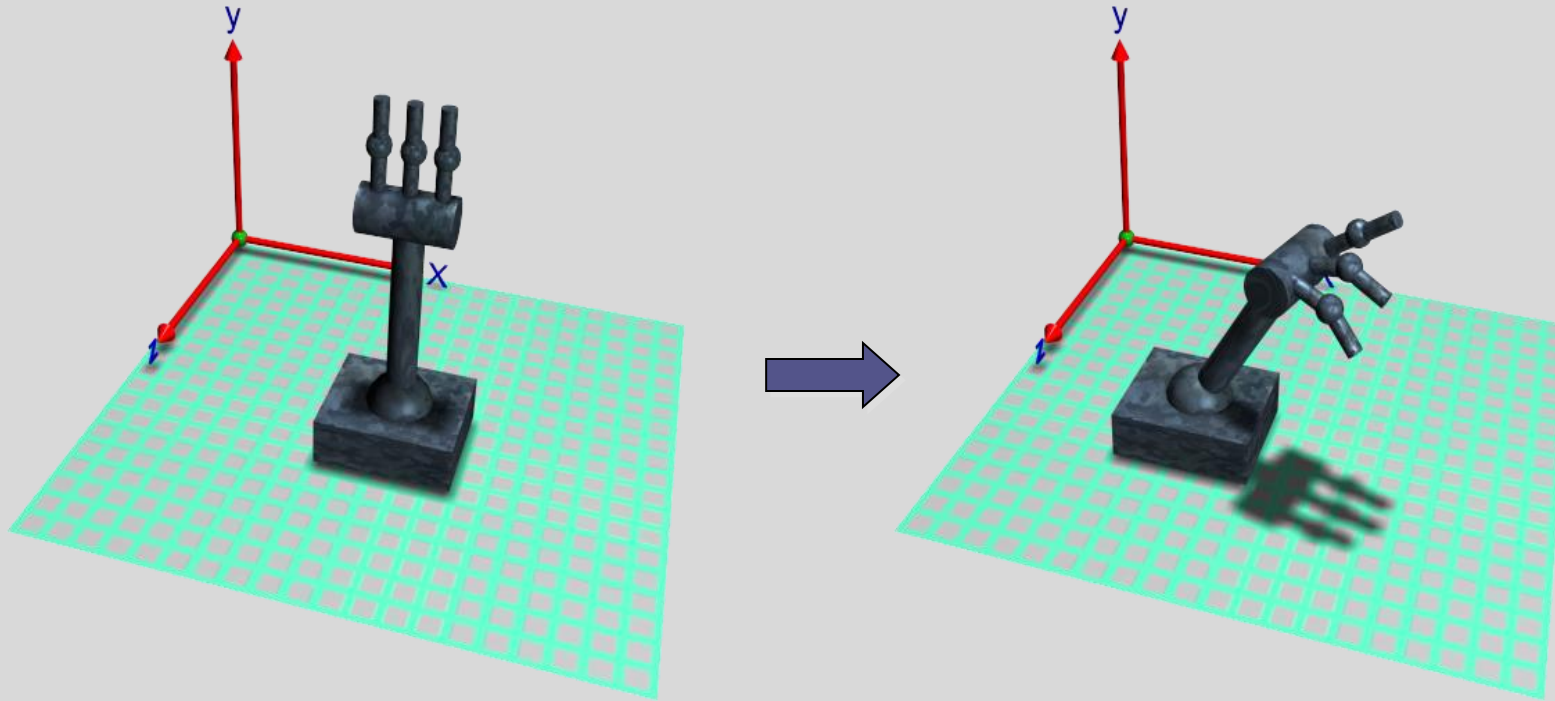
we draw the
global
transform



Hierarchical Transformations

- previous example had simple *one-to-one* parent-child linkages.
- in general there may be many *child frames* derived from a single parent frame.
- we need to remember the parent frame and return to it when creating new children.
 - solution: to keep track of global transformation as we go.

Hierarchical Transformations

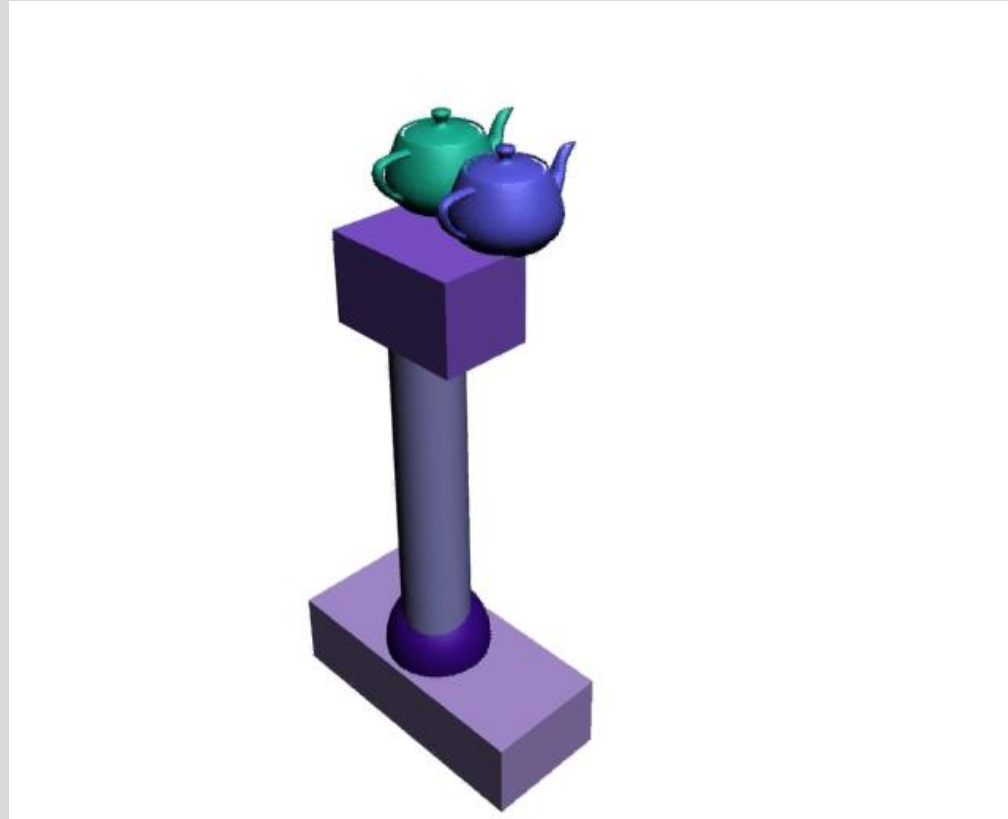


each finger is a child of the wrist (parent)

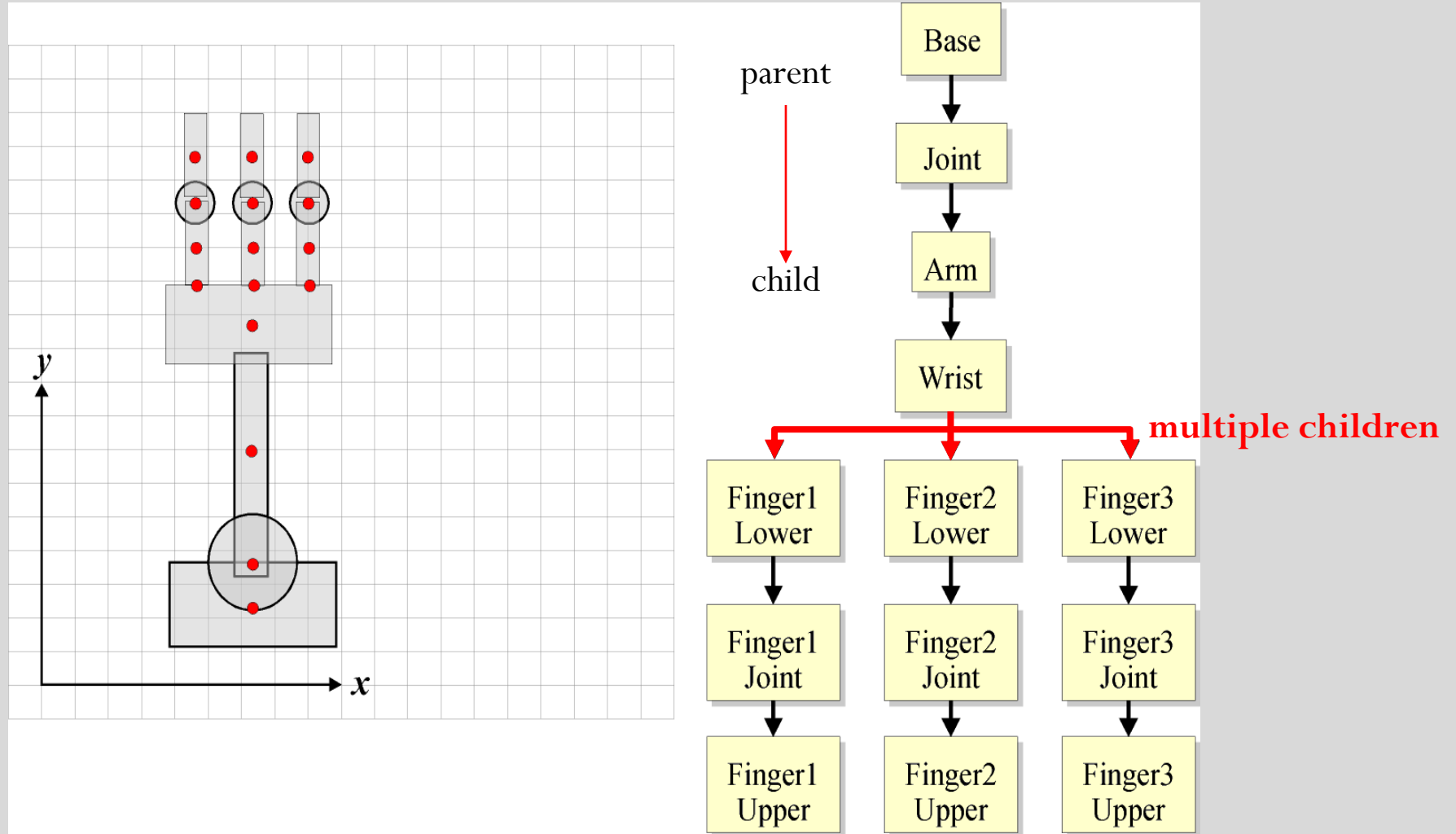
⇒ independent control over the orientation

of each fingers (the movement of the index should not affect the thumb)

Hierarchical Transformations



Hierarchical Transformations



OpenGL[®] Implementation



(**input**) what we define:

- a hierarchy.
- location and orientation of each element of the hierarchy in their **local** frame (reference system).

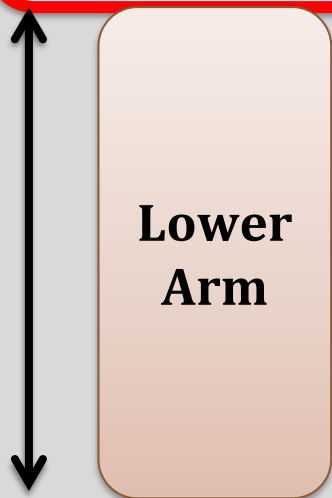
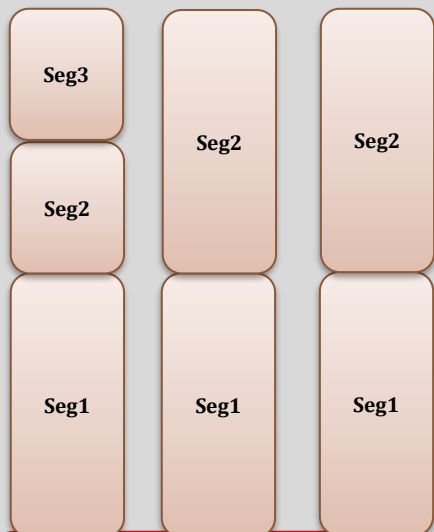
(**output**) what we obtain:

- render all the element of the final structure.
→ extract all the **global** coordinates.

like the
previous
example

(**new problem**) how to manage multiple children?

Finger 1 Finger 2 Finger 3



wrist - do local transformations

```
global_wrist = global_lowerarm*localwrist;
```

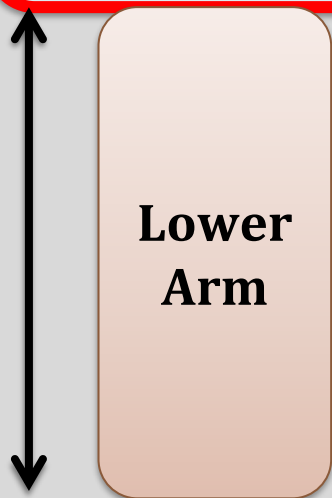
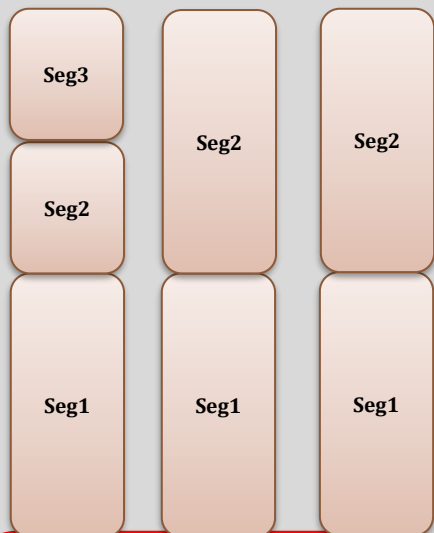
```
updateUniformVariables(model matrix = global_wrist);
```

```
drawWrist();
```



like the
previous
example

Finger 1 Finger 2 Finger 3



wrist - do local transformations

```
global_wrist = global_lowerarm*localwrist;
```

```
updateUniformVariables(model matrix = global_wrist);
```

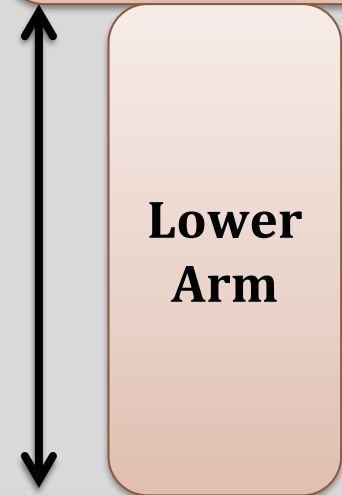
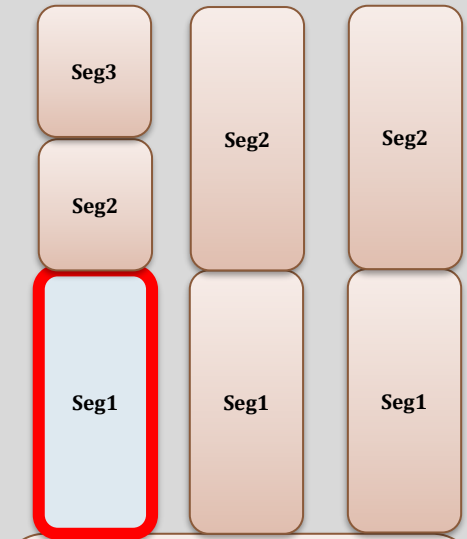
```
drawWrist();
```

we draw the
global
transform

$$\begin{bmatrix} \text{global} \\ \text{lowerarm} \end{bmatrix} \times \begin{bmatrix} \text{local} \\ \text{wrist} \end{bmatrix}$$

like the
previous
example

Finger 1 Finger 2 Finger 3



wrist - do local transformations

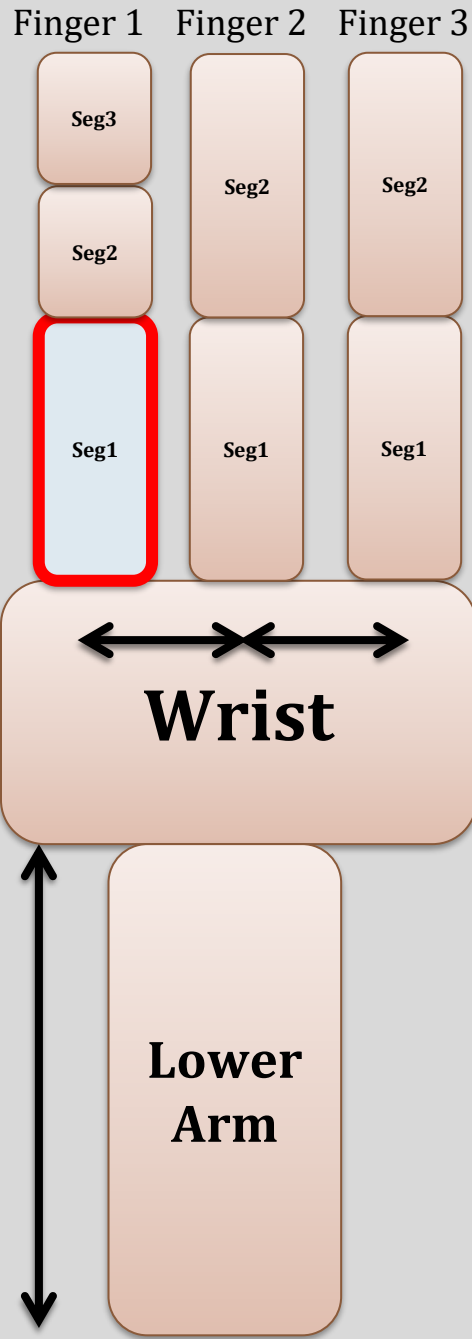
```
global_wrist = global_lowerarm*localwrist;  
updateUniformVariables(model matrix = global_wrist);  
drawWrist();
```

finger1_segment1 - do local transformations

```
global_finger1_1 = global_wrist*localfinger1_segment1;  
updateUniformVariables(model matrix = global_finger1_1);  
drawFinger1Segment1();
```

$$\begin{bmatrix} \text{global} \\ \text{wrist} \end{bmatrix} \times \begin{bmatrix} \text{local} \\ \text{finger1_1} \end{bmatrix}$$

like the
previous
example



wrist - do local transformations

```
global_wrist = global_lowerarm*localwrist;
updateUniformVariables(model matrix = global_wrist);
drawWrist();
```

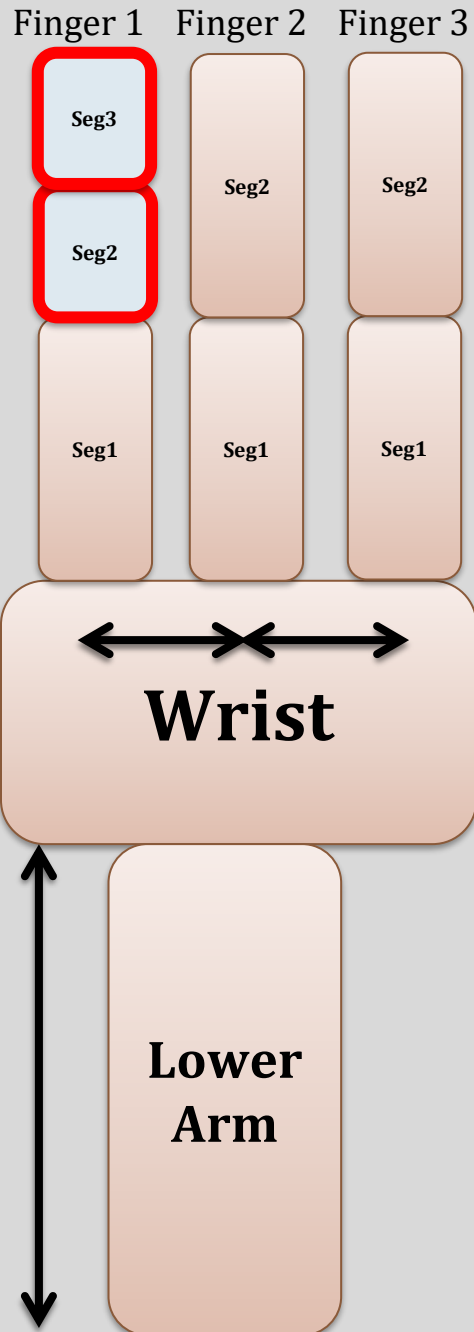
like the
previous
example

finger1_segment1 - do local transformations

```
global_finger1_1 = global_wrist*localfinger1_segment1;
updateUniformVariables(model matrix = global_finger1_1);
drawFinger1Segment1();
```

$$\begin{bmatrix} \text{global} \\ \text{wrist} \end{bmatrix} \times \begin{bmatrix} \text{local} \\ \text{finger1_1} \end{bmatrix}$$

again
we draw the
global
transform



wrist - do local transformations

```
global_wrist = global_lowerarm*localwrist;
updateUniformVariables(model matrix = global_wrist);
drawWrist();
```

like the
previous
example

finger1_segment1 - do local transformations

```
global_finger1_1 = global_wrist*localfinger1 segment1;
updateUniformVariables(model matrix = global_finger1_1);
drawFinger1Segment1();
```

finger1_segment2 - do local transformations

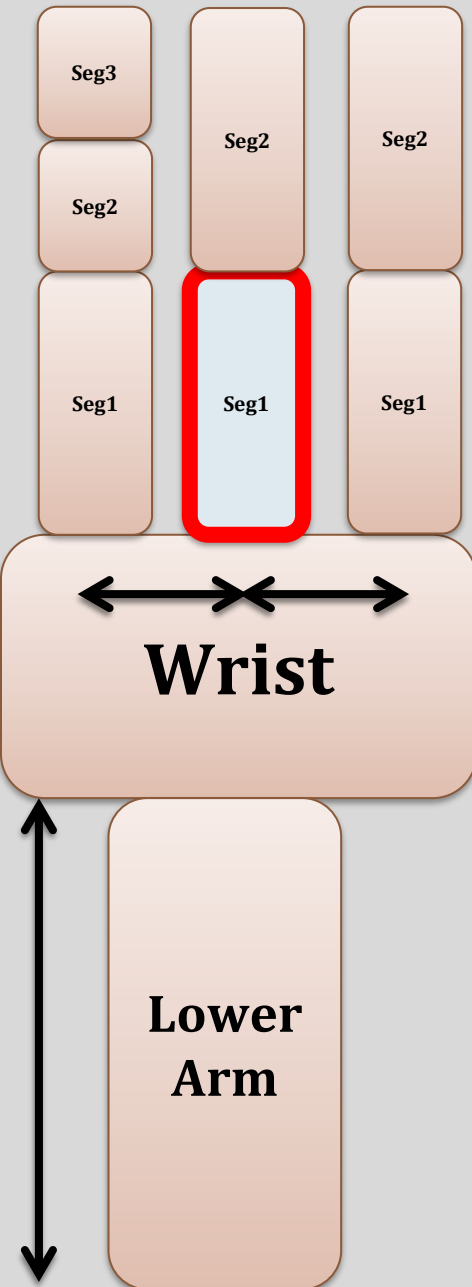
```
global_finger1_2 = global_finger1_1*localfinger1 segment2;
updateUniformVariables(model matrix = global_finger1_2);
drawFinger1Segment2();
```

finger1_segment3 - do local transformations

```
global_finger1_3 = global_finger1_2*localfinger1 segment3;
updateUniformVariables(model matrix = global_finger1_3);
drawFinger1Segment3();
etc.
```

again we extract the global
transform from the parent and
we draw

Finger 1 Finger 2 Finger 3



wrist - do local transformations

```

global_wrist = global_lowerarm*localwrist;
updateUniformVariables(model matrix = global_wrist);
drawWrist();

```

finger1_segment1 - do local transformations

```

global_finger1_1 = global_wrist*localfinger1 segment1;
updateUniformVariables(model matrix = global_finger1_1);
drawFinger1Segment1();

```

finger1_segment2 - do local transformations

```

global_finger1_2 = global_finger1_1 *localfinger1 segment2;
updateUniformVariables(model matrix = global_finger1_2);
drawFinger1Segment2();

```

finger1_segment3 - do local transformations

```

global_finger1_3 = global_finger1_2 *localfinger1 segment3;
updateUniformVariables(model matrix = global_finger1_3);
drawFinger1Segment3();
etc.

```

finger2_segment1 - do local transformations

```

global_finger2_1 = global_wrist*localfinger2 segment1;
updateUniformVariables(model matrix = global_finger2_1);
drawFinger2Segment1();

```

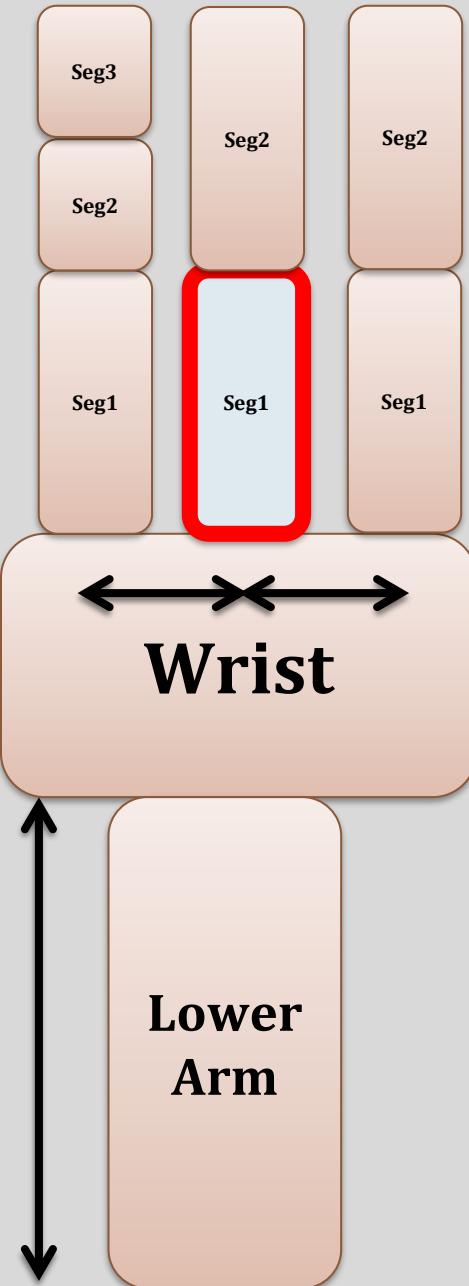
finger2_segment2 - do local transformations

```

global_finger2_2 = global_finger2_1 *localfinger2 segment2;
updateUniformVariables(model matrix = global_finger2_2);
drawFinger2Segment2();

```

Finger 1 Finger 2 Finger 3



wrist - do local transformations

global_wrist = global_lowerarm*localwrist;

updateUniformVariables(model matrix = global_wrist);

drawWrist();

finger1_segment1 - do local transformations

global_finger1_1 = **global_wrist***localfinger1 segment1;

updateUniformVariables(model matrix = global_finger1_1);

drawFinger1Segment1();

finger1_segment2 - do local transformations

global_finger1_2 = **global_finger1_1***localfinger1 segment2;

updateUniformVariables(model matrix = global_finger1_2);

drawFinger1Segment2();

finger1_segment3 - do local transformations

global_finger1_3 = **global_finger1_2***localfinger1 segment3;

updateUniformVariables(model matrix = global_finger1_3);

drawFinger1Segment3();

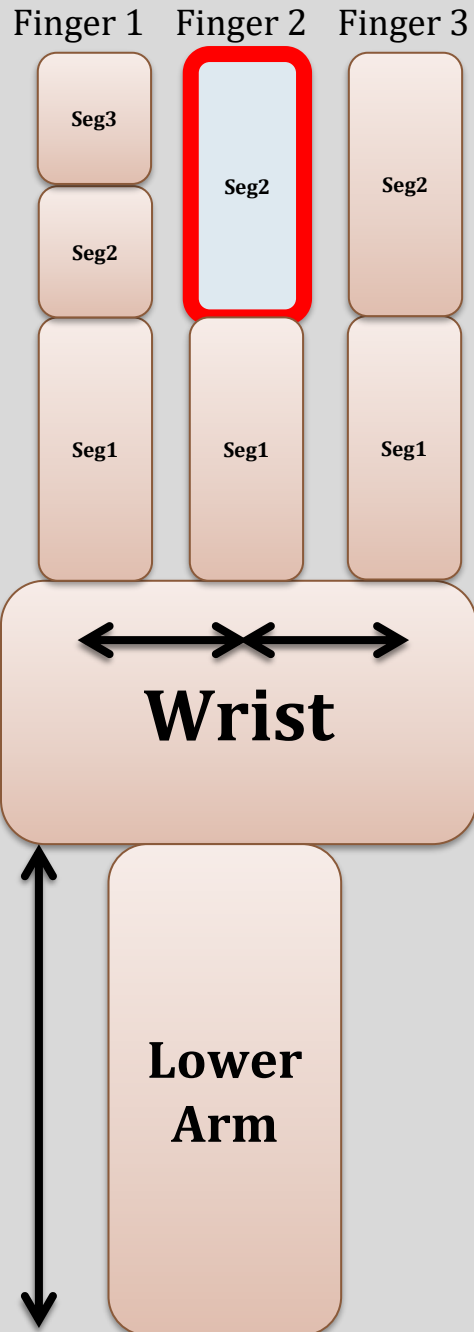
etc.

finger2_segment1 - do local transformations

global_finger2_1 = **global_wrist***localfinger2 segment1;

updateUniformVariables(model matrix = global_finger2_1);

drawFinger2Segment1();



wrist - do local transformations

```
global_wrist = global_lowerarm*localwrist;
updateUniformVariables(model matrix = global_wrist);
drawWrist();
```

finger1_segment1 - do local transformations

```
global_finger1_1 = global_wrist*localfinger1 segment1;
updateUniformVariables(model matrix = global_finger1_1);
drawFinger1Segment1();
```

finger1_segment2 - do local transformations

```
global_finger1_2 = global_finger1_1 *localfinger1 segment2;
updateUniformVariables(model matrix = global_finger1_2);
drawFinger1Segment2();
```

finger1_segment3 - do local transformations

```
global_finger1_3 = global_finger1_2 *localfinger1 segment3;
updateUniformVariables(model matrix = global_finger1_3);
drawFinger1Segment3();
etc.
```

finger2_segment1 - do local transformations

```
global_finger2_1 = global_wrist*localfinger2 segment1;
updateUniformVariables(model matrix = global_finger2_1);
drawFinger2Segment1();
```

finger2_segment2 - do local transformations

```
global_finger2_2 = global_finger2_1 *localfinger2 segment2;
updateUniformVariables(model matrix = global_finger2_2);
drawFinger2Segment2();
```

like the
previous
example



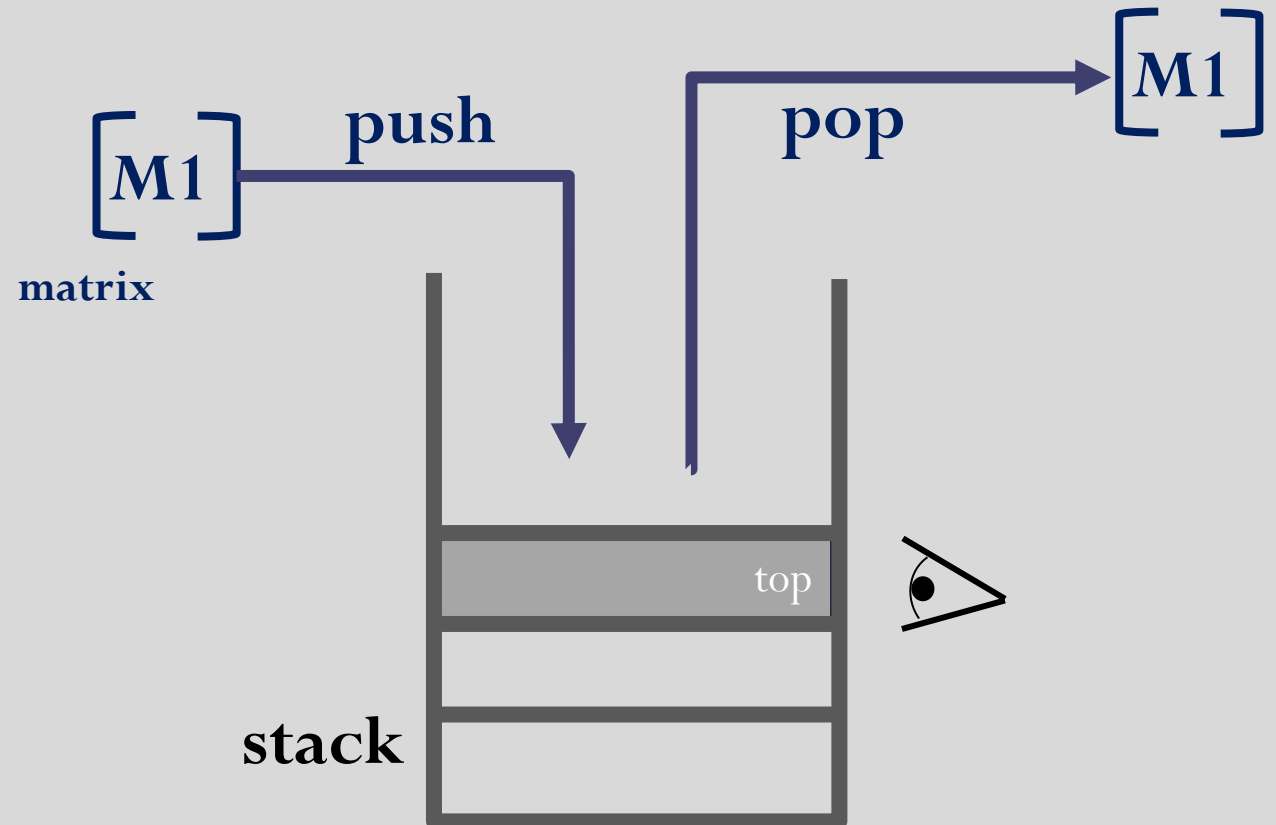
Variation example:

- instead of keep track of each global matrix in the hierarchy → use a memory stack.
- why using a stack?
 - memory efficient.
 - avoid bugs in complex system.



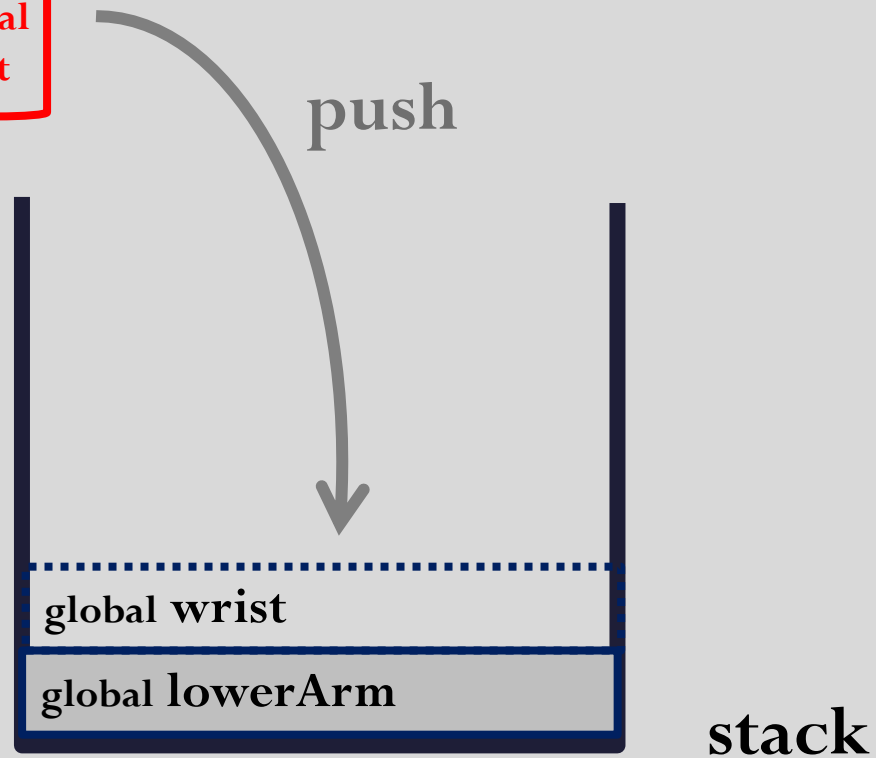
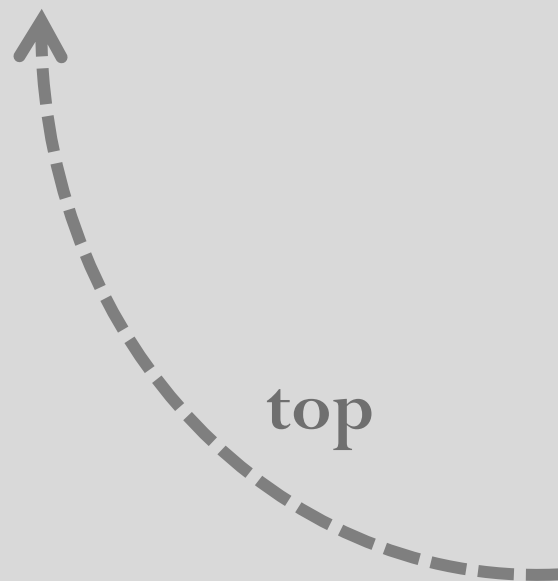
Variation example:

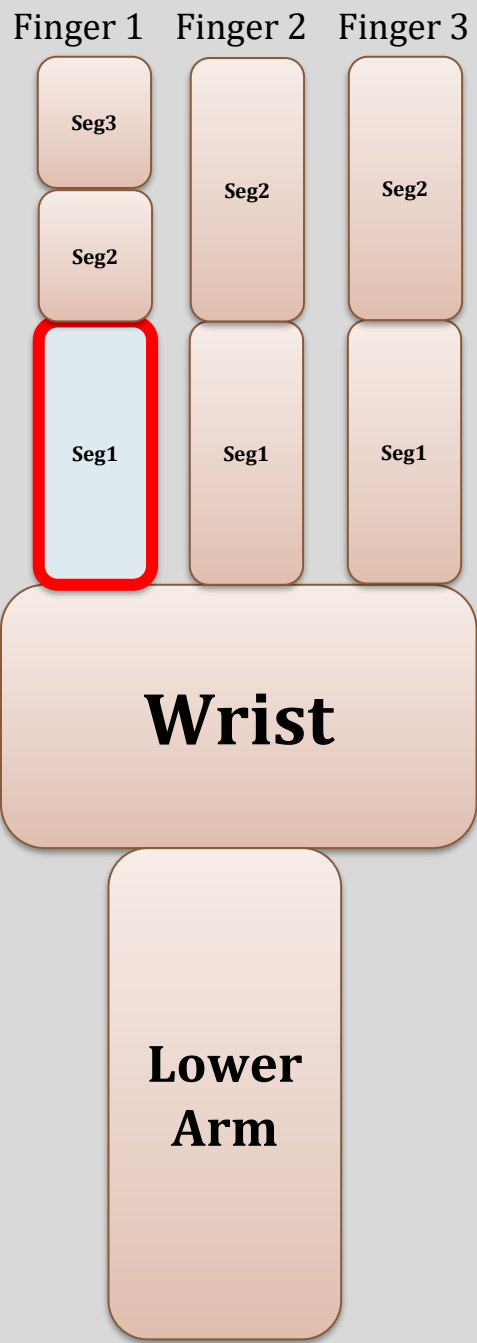
- instead of keep track of each global matrix in the hierarchy → use a memory stack.
- why using a stack?
 - memory efficient.
 - avoid bugs in complex system.



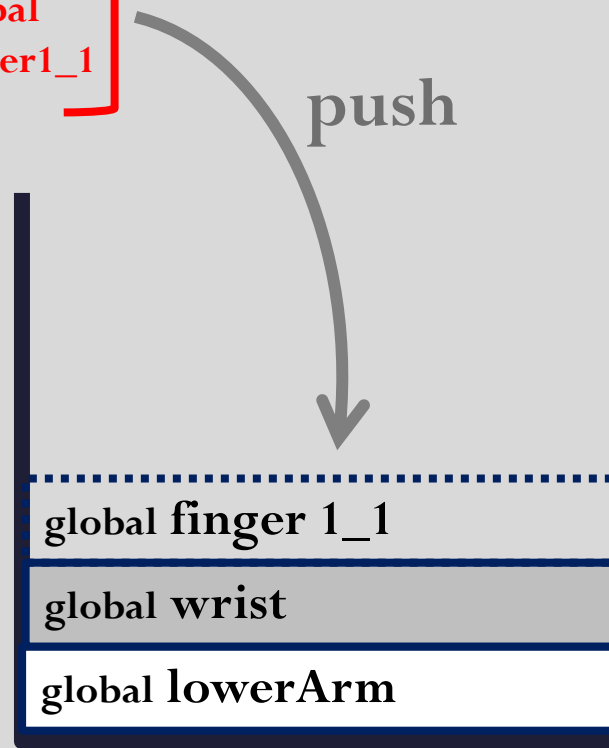
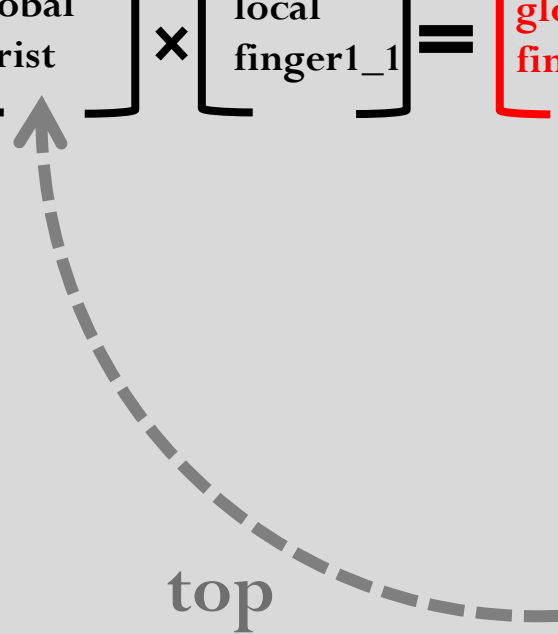


$$\begin{bmatrix} \text{global} \\ \text{lowerarm} \end{bmatrix} \times \begin{bmatrix} \text{local} \\ \text{wrist} \end{bmatrix} = \begin{bmatrix} \text{global} \\ \text{wrist} \end{bmatrix}$$

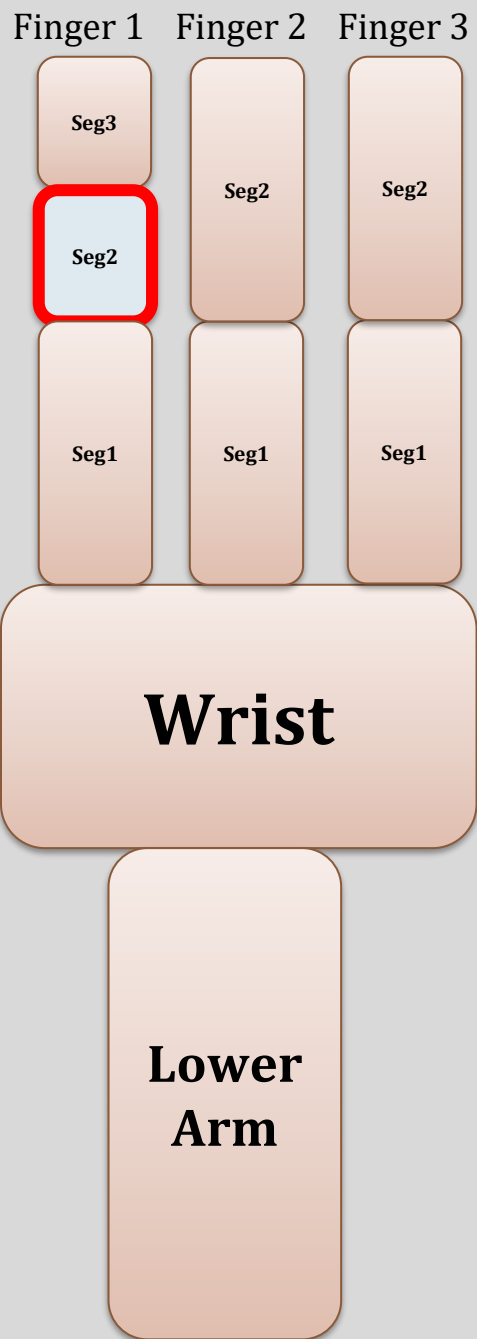




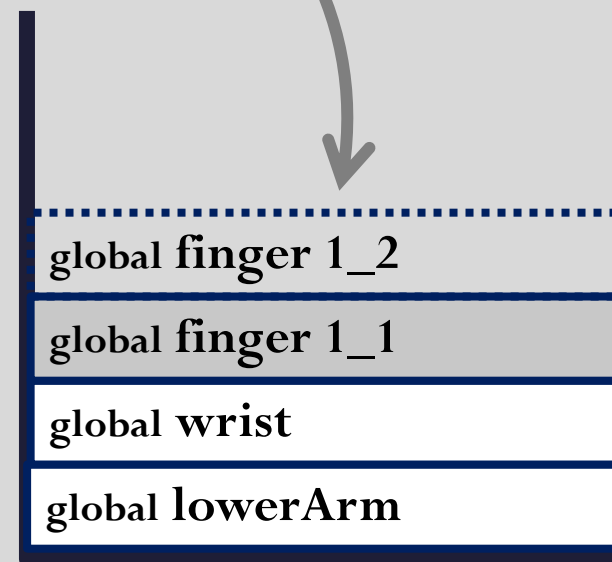
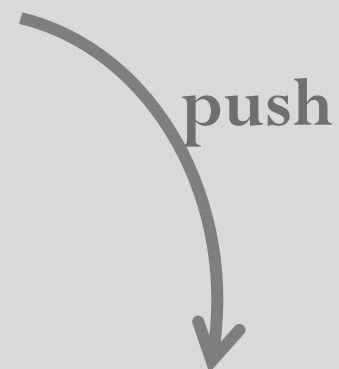
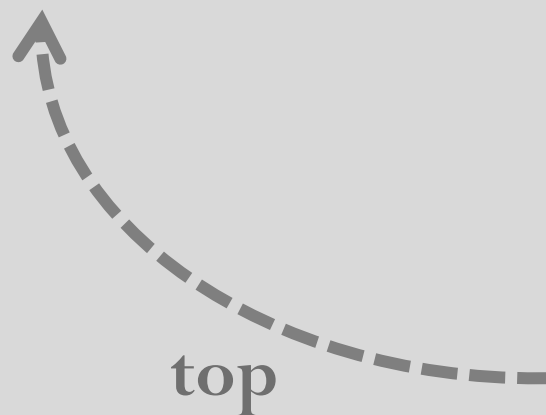
$$\begin{bmatrix} \text{global} \\ \text{wrist} \end{bmatrix} \times \begin{bmatrix} \text{local} \\ \text{finger1_1} \end{bmatrix} = \begin{bmatrix} \text{global} \\ \text{finger1_1} \end{bmatrix}$$



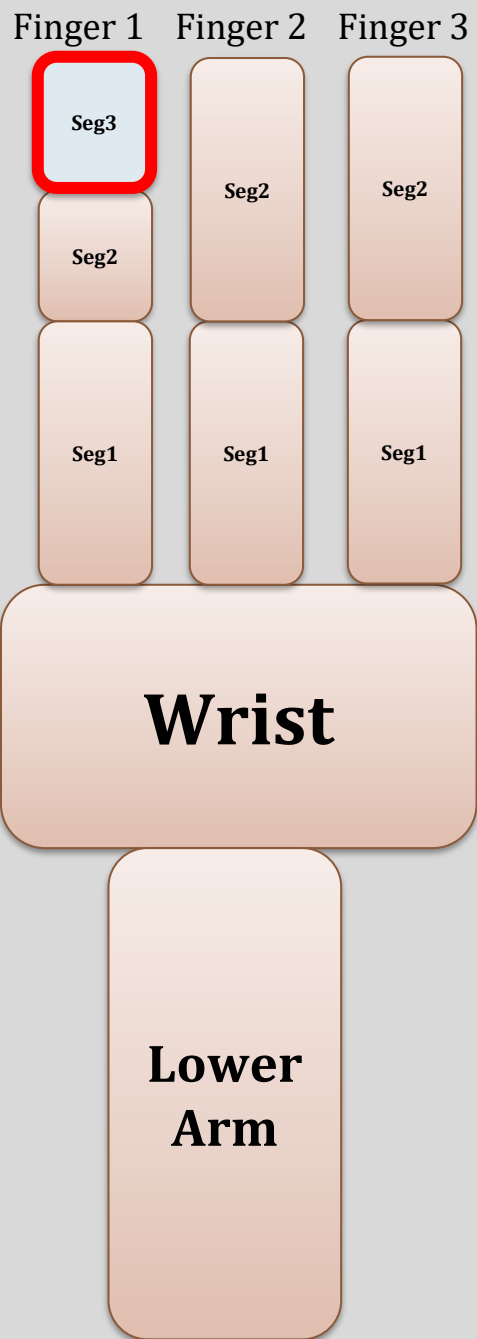
stack



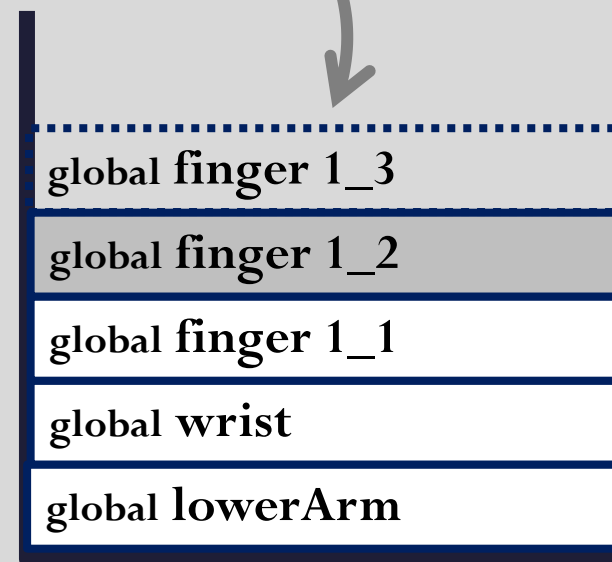
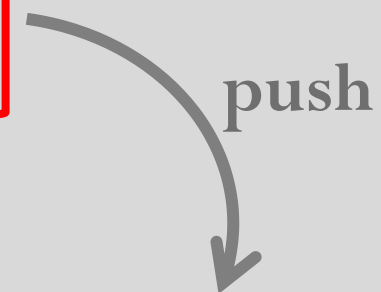
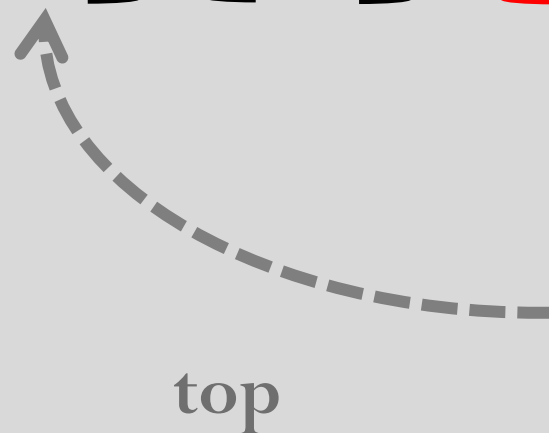
$$\begin{bmatrix} \text{global} \\ \text{finger1_1} \end{bmatrix} \times \begin{bmatrix} \text{local} \\ \text{finger1_2} \end{bmatrix} = \begin{bmatrix} \text{global} \\ \text{finger1_2} \end{bmatrix}$$



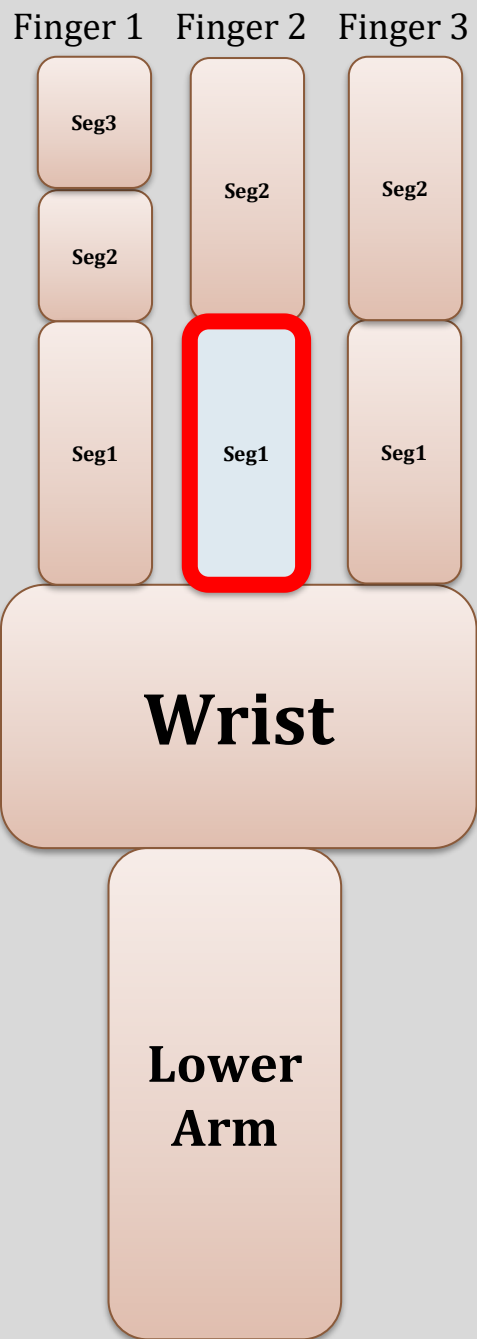
stack



$$\begin{bmatrix} \text{global} \\ \text{finger1_2} \end{bmatrix} \times \begin{bmatrix} \text{local} \\ \text{finger1_3} \end{bmatrix} = \begin{bmatrix} \text{global} \\ \text{finger1_3} \end{bmatrix}$$



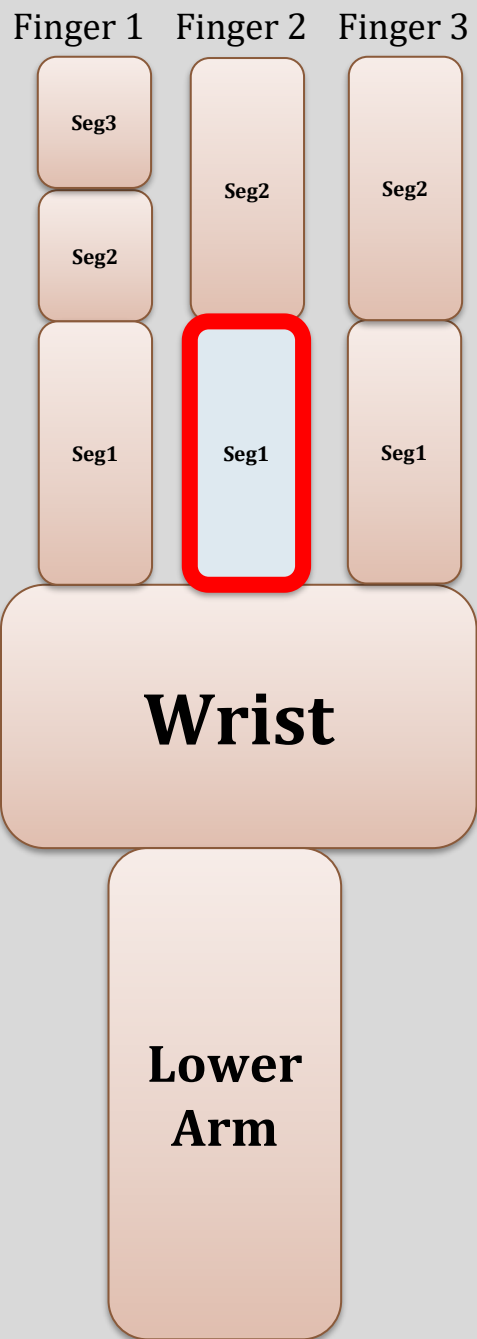
stack



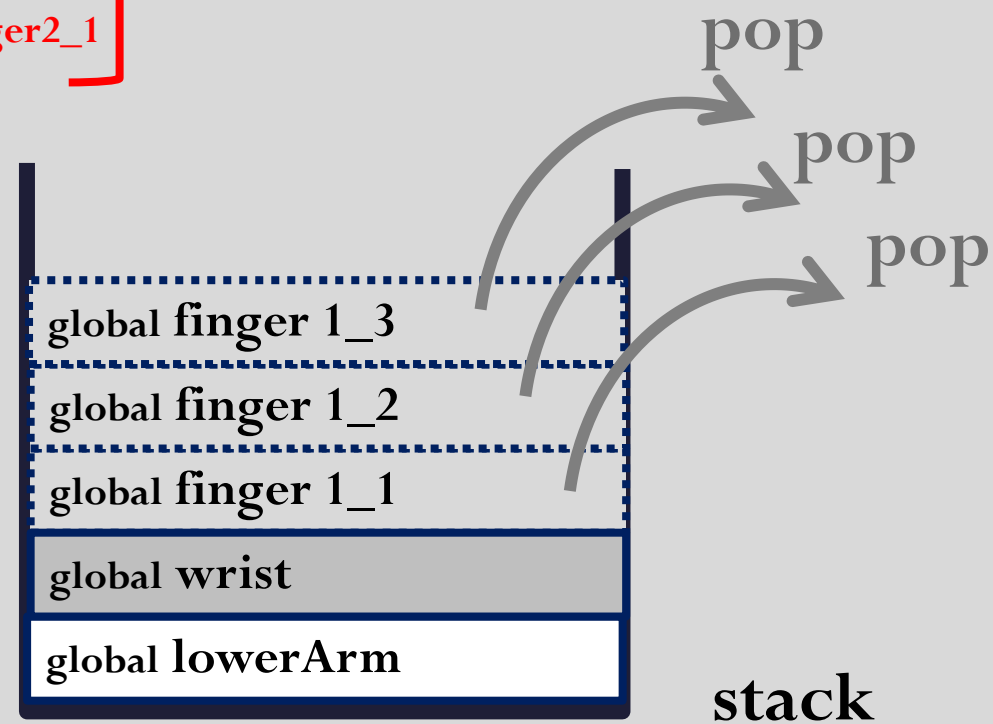
$$\begin{bmatrix} \dots \end{bmatrix} \times \begin{bmatrix} \text{local} \\ \text{finger2_1} \end{bmatrix} = \begin{bmatrix} \text{global} \\ \text{finger2_1} \end{bmatrix}$$

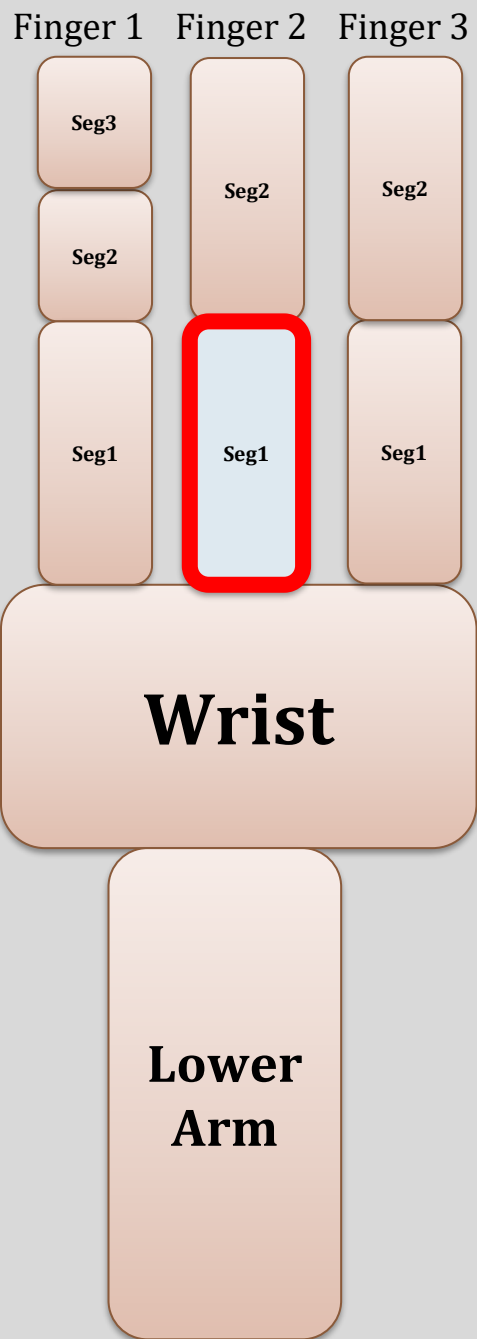


stack

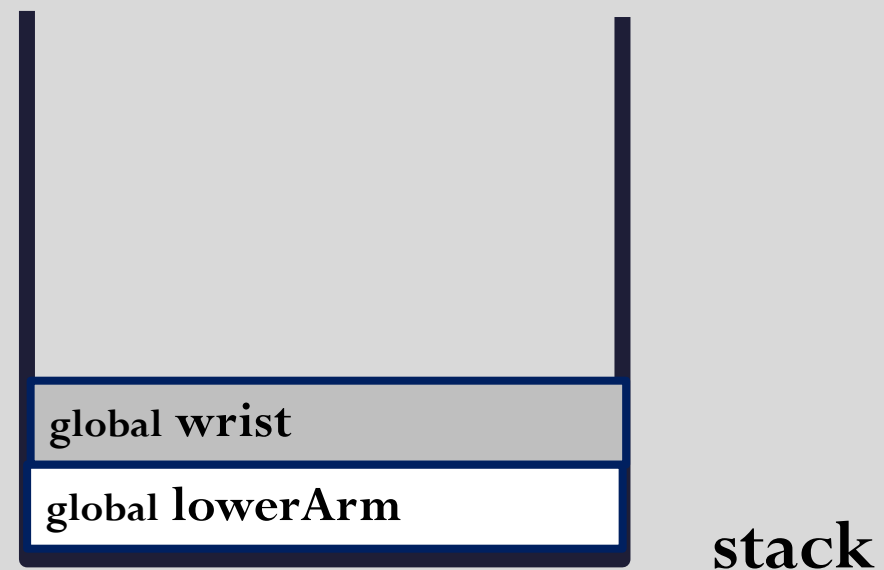


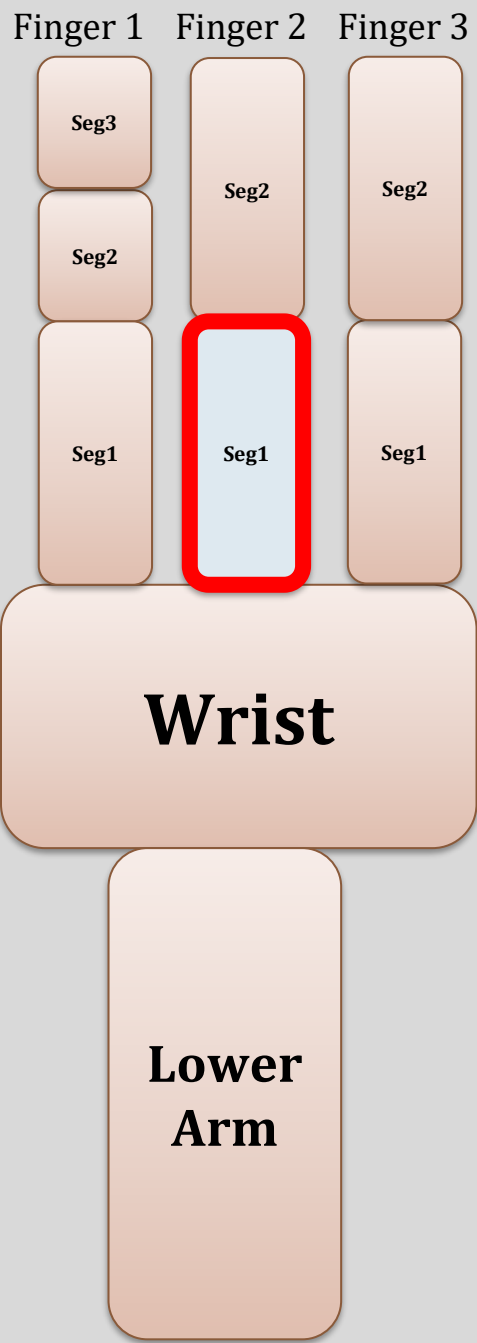
$$\left[\dots \right] \times \left[\begin{array}{c} \text{local} \\ \text{finger2_1} \end{array} \right] = \left[\begin{array}{c} \text{global} \\ \text{finger2_1} \end{array} \right]$$



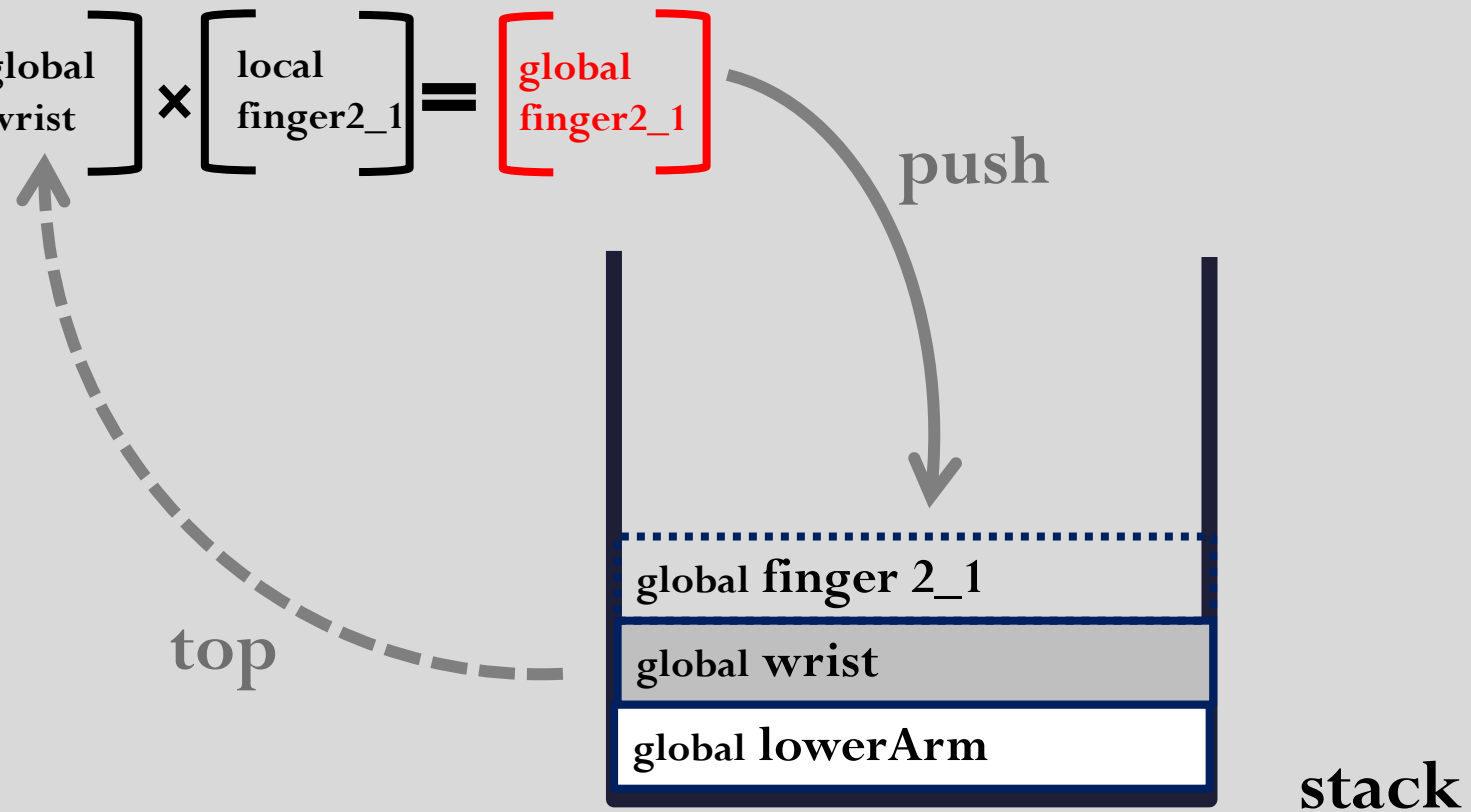


$$\begin{bmatrix} \dots \end{bmatrix} \times \begin{bmatrix} \text{local} \\ \text{finger2_1} \end{bmatrix} = \begin{bmatrix} \text{global} \\ \text{finger2_1} \end{bmatrix}$$





$$\begin{bmatrix} \text{global} \\ \text{wrist} \end{bmatrix} \times \begin{bmatrix} \text{local} \\ \text{finger2_1} \end{bmatrix} = \begin{bmatrix} \text{global} \\ \text{finger2_1} \end{bmatrix}$$



Summary

- Viewing
- Transformations
- Transformations in OpenGL
- Hierarchies
- Next - Animation!