CS7GV2: Mathematics of Light and Sound, M.Sc. in Computer Science.

Lecture #1: Waves

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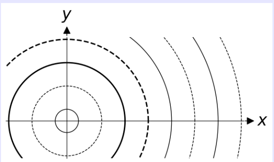
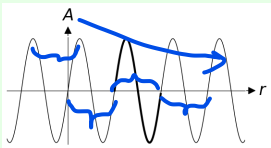
Notes

Physical waves



Periodic disturbance from its equilibrium of matter or field. Propagates radially from a point* source.

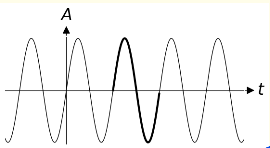
simplify



Amplitude A along a radial line through space.
In 2-D prop., $A \propto r^{-1}$ and in 3-D, $A \propto r^{-2}$.

radius/distance

$\propto$ [illegible]



Amplitude at a distance r wrt time t.

fixed

Prop. speed in $\mathrm{m\,s^{-1}}$ is c
Wavelength in m is λ
Wave period in s is $T = \lambda / c$
Wave frequency in Hz is $\nu = 1/T$
Angular freq. in $\mathrm{rad\,s^{-1}}$ is $\omega = 2\pi/T$
Wave number in $\mathrm{rad\,m^{-1}}$ is $k = 2\pi/\lambda$

Notes

Wave equation

How can waves be described so their behaviour can be analysed mathematically?

A constraint equation can be derived from Newton's and Hooke's physical laws:

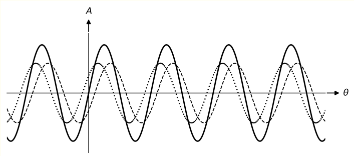
$$\frac{\partial^2 A(r,t)}{\partial r^2} = \frac{1}{c^2}\frac{\partial^2 A(r,t)}{\partial t^2}.$$

An analytic solution* can be expressed as the sum of two "wave shaped" functions of transformed parameters, e.g.

$$A(r,t) = \cos(k\,r - \omega\,t) + \cos(k\,r + \omega\,t).$$

Here the parameters are angles (in radians,)

$$k\,r = r\,2\pi/\lambda \qquad \omega\,t = t\,2\pi/T = t\,c\,2\pi/\lambda.$$



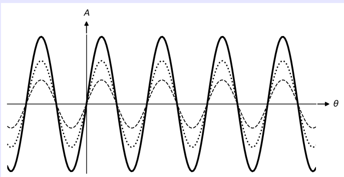
Sum of two sinusoids travelling in opposite directions over time t.

λ

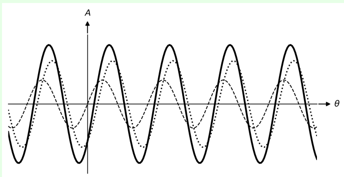
$$\frac{\partial}{\partial t}\left[\frac{\partial}{\partial t}[\cos(r - c\,t) + \cos(r + c\,t)]\right] =$$
$$\frac{\partial}{\partial t}[c\sin(r - c\,t) - c\sin(r + c\,t)] =$$
$$-c^2\cos(r - c\,t) - c^2\cos(r + c\,t).$$
$$\frac{\partial}{\partial r}\left[\frac{\partial}{\partial r}[\cos(r - c\,t) + \cos(r + c\,t)]\right] =$$
$$-\cos(r - c\,t) - \cos(r + c\,t).$$

Notes

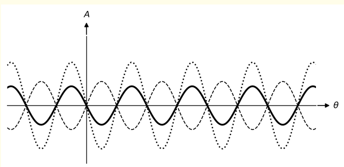
Wave summation, $A_1(\theta) + A_2(\theta)$



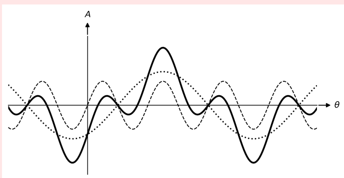
$\lambda_1 = \lambda_2$ and $\phi_1 = \phi_2$.



$\lambda_1 = \lambda_2$ and $\phi_1 > \phi_2$.



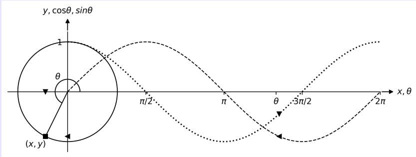
$\lambda_1 = \lambda_2$ and $\phi_1 = \phi_2 + \pi$.



$\lambda_1 > \lambda_2$ and $\phi_1 = \phi_2$.

Notes

Sinusoids



For any point (x, y) on unit circle, $\cos\theta := x$ and $\sin\theta := y$.

Sinusoids with same λ but arbitrary ϕ and A sum to a sinusoid with same λ.

This is how physical waves behave.

Sinusoids are *only* periodic functions with this property.

Geometric contruction is impractical and mathematical expression is complicated:

$$\cos\theta = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\,\theta^{2n}$$

so calculators with programmed buttons or printed tables are used instead.

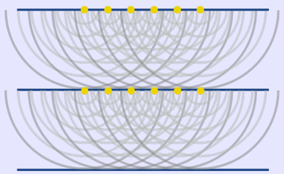
Angle θ corresponding to point (x, y) on the unit circle is $\theta = \arccos x = \arcsin y$.

Notation $\cos^{-1} x$ and $\sin^{-1} y$ is not recommended because of its ambiguity wrt the reciprocal.

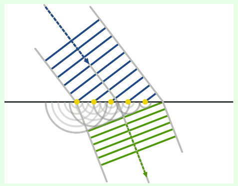
To enhance readability, function parameter need not be enclosed in parentheses.

Notes

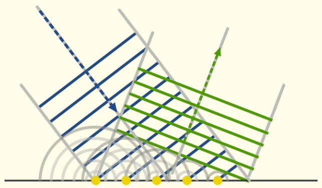
Wavefront propagation



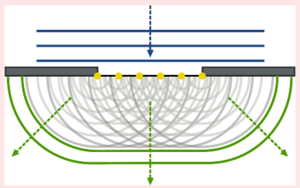
Huygens-Fresnel Construction.



Refraction.



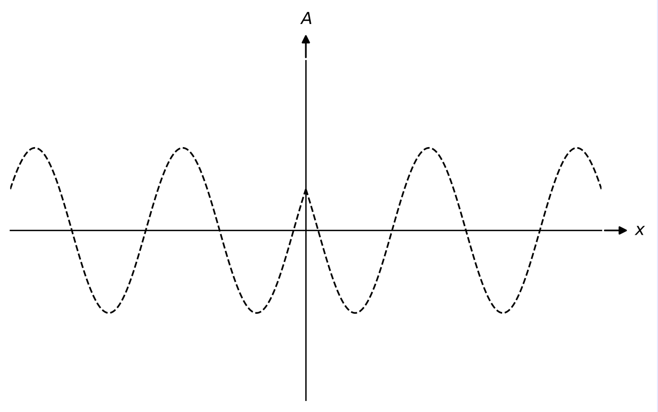
Reflection.



Diffraction.

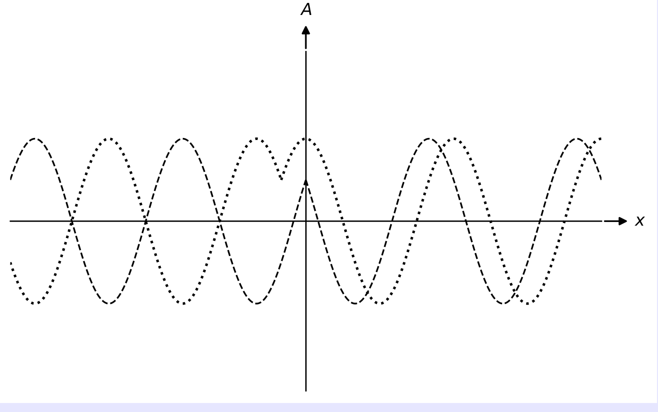
Notes

Propagation from a point where amplitude is increasing



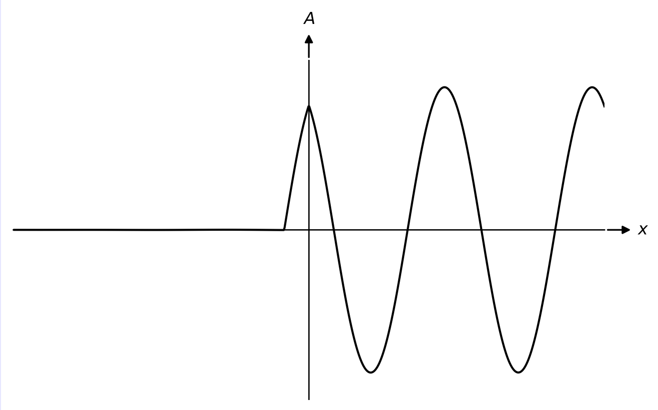
Notes

. . . and from another point where amplitude is decreasing



Notes

Radial propagation . . . from a *dipole*



Huygens-Miller construction.

Notes

Assignment # 1: Wave simulation

- Install the Anaconda distribution of SciPy on your own computer or else make Google Colab or Anaconda Cloud accounts.
- Use the given analytic solution of the Wave Equation to simulate the movement of a radial wave over a distance in space and a period of time.
- Present it in a Jupyter Notebook.
- Make it into a self-contained project repository in your personal account on gitlab.scss.tcd.ie.
- Add `fshevlin@tcd.ie` as a member with "reporter" privileges.

Notes

Greek letters* often used as symbols in mathematics

α	alpha	θ	theta	o	omicron	τ	tau
β	beta	ϑ	caligr. theta	π	pi	υ	upsilon
γ	gamma	ι	iota	ϖ	caligr. pi	ϕ	phi
δ	delta	κ	kappa	ρ	rho	φ	caligr. phi
ϵ	epsilon	λ	lambda	ϱ	caligr. rho	χ	chi
ε	caligr. epsilon	μ	mu	σ	sigma	ψ	psi
ζ	zeta	ν	nu	ς	caligr. sigma	ω	omega
η	eta	ξ	xi				

Γ	big gamma	Λ	big lambda	Σ	big sigma	Ψ	big psi
Δ	big delta	Ξ	big xi	Υ	big upsilon	Ω	big omega
Θ	big theta	Π	big pi	Φ	big phi		

*With their anglophone pronunciations.

Notes

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