CS7GV2: Mathematics of Light and Sound, M.Sc. in Computer Science.

Lecture #7: Intensity

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November 15, 2024

Notes

Intensity

Forces exerted by electric and magnetic fields can move or heat matter and move charges. This means electromagnetic waves have *energy.*

The energy required to accelerate an object over 1 m distance with 1 N force is,

$$1\,\mathrm{J} = 1\,\mathrm{N}{\cdot}1\,\mathrm{m} = 1\,\mathrm{kg}{\cdot}1\,\mathrm{m\,s^{-2}}{\cdot}1\,\mathrm{m} = 1\,\mathrm{kg\,m^2\,s^{-2}}.$$

Power is energy per unit time, $1\,\mathrm{W} = 1\,\mathrm{J\,s^{-1}}$.

Intensity, or energy flux density, is power per unit area, $\mathrm{W\,m^{-2}}$

Intensity of electromagnetic waves is what what our eyes see; and what is measured by the photosensitive elements in cameras.

Electromagnetic wave intensity at a point in space at time t is $\propto$ to the product of field amplitudes,

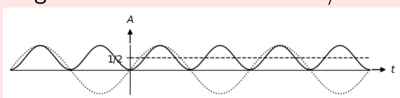
$$1/\mu \|\mathbf{E}(t)\|\,\|\mathbf{B}(t)\|.$$

Average intensity over a wave time period is a more useful quantity,

$$I = \frac{1}{2}\frac{1}{\mu}EB = \frac{E^2}{2\mu c} = \frac{cB^2}{2\mu}$$

where E and B are max. amplitudes.

The product of two in-phase sinusoids of equal amplitude has an average over time of $1/2$ amplitude^2.



For simplicity, scaling factors can be ignored and E and B field amplitudes denoted by A to consider average intensity $I \propto A^2$.

Notes

Intensity PDF

$f_A(a)$ for amplitude A of a random phasor sum was found to follow a Rayleigh distribution. The *derived* PDF for intensity $I = A^2$ is,

$$f_I(i) = f_A(\sqrt{i}) \left| \frac{\mathrm{d}\sqrt{i}}{\mathrm{d}i} \right| = f_A(\sqrt{i}) \frac{1}{2\sqrt{i}}$$

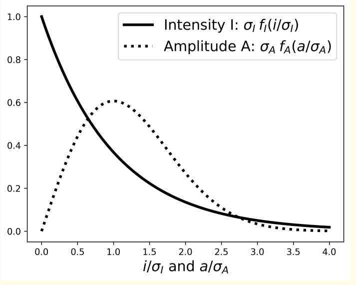
$$\text{since } \frac{\mathrm{d}i^{\frac{1}{2}}}{\mathrm{d}i} = \frac{1}{2} i^{\frac{-1}{2}} = \frac{1}{2} \frac{1}{\sqrt{i}} = \frac{1}{2\sqrt{i}}.$$

$$f_A(a) = \exp\left\{-\frac{a^2}{2\sigma_A^2}\right\} \frac{a}{\sigma_A^2}.$$

$$f_I(i) = \exp\left\{-\frac{i}{2\sigma_A^2}\right\} \frac{\sqrt{i}}{\sigma_A^2} \frac{1}{2\sqrt{i}}$$

$$= \exp\left\{-\frac{i}{2\sigma_A^2}\right\} \frac{1}{2\sigma_A^2}.$$

So intensity PDF follows an *exponential* distribution, i.e. $f_X(x) = \lambda\, \mathrm{e}^{-\lambda x}$.



Mean intensity $\overline{I}$ can be found as $2\sigma_A^2$ so the PDF can be written,

$$f_I(i) = \exp\left\{-\frac{i}{\overline{I}}\right\} \frac{1}{\overline{I}}.$$

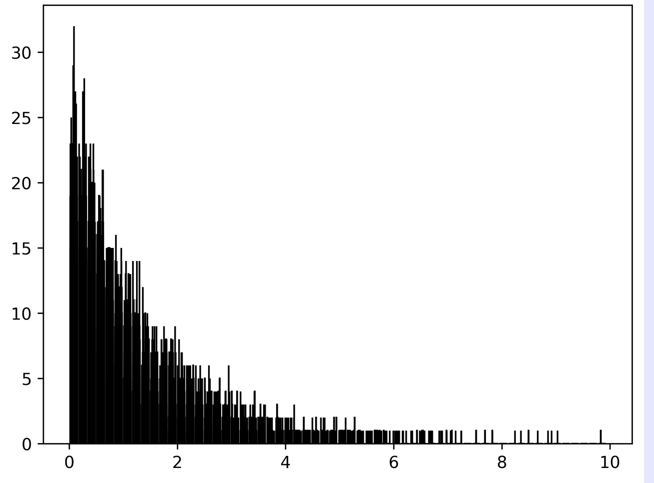
$$\text{Variance } \sigma_I^2 = \overline{I}^{\,2}, \quad \text{Std. dev. } \sigma_I = \overline{I}\,,$$

$$\text{Contrast } C = \sigma_I / \overline{I} = 1.0,$$

$$\text{S/N ratio } = 1/C = \overline{I}/\sigma_I = 1.0.$$

Notes

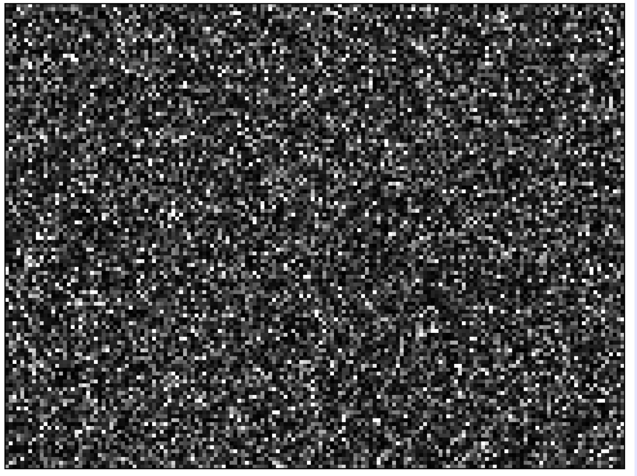
Histogram of simulated intensities



19,200 intensity values chosen randomly in accordance with an exponential PDF with $\overline{I} = 1.0$ and $\sigma_I = 1.0$. These could be many observations over time at a single point in space or many observations over space at a single point in time.

Notes

Image of simulated intensities

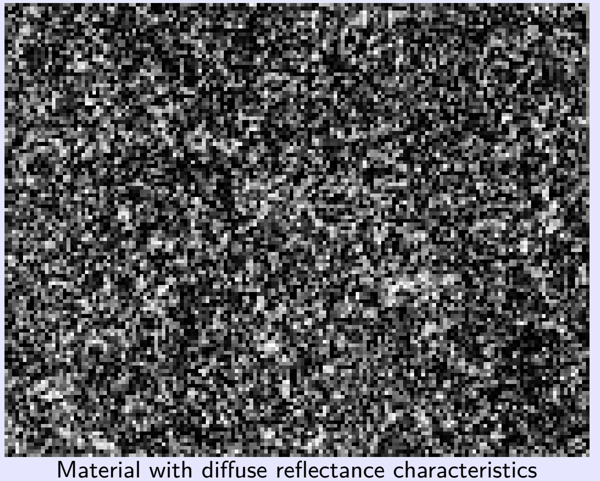


The same[a] intensity values i arranged as a 160×120 pixel image. Contrast $C = \sigma_I/\overline{I} = 1.0$.

[a]These aren't exactly the same as those in the histogram. Values > 4.0 were replaced with 4.0 to avoid too much compression of dynamic range in display.

Notes

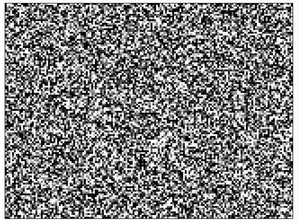
Image of actual intensities



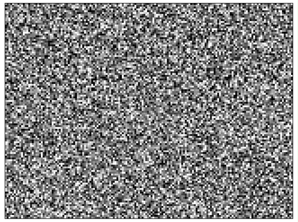
Material with diffuse reflectance characteristics illuminated evenly with *monochromatic* light with *no phase or amplitude change* during the observation time period. $C \approx 0.83$.

Notes

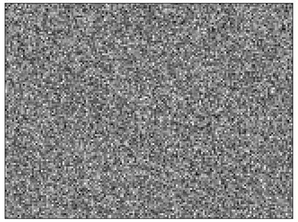
Averages of simulated intensity images



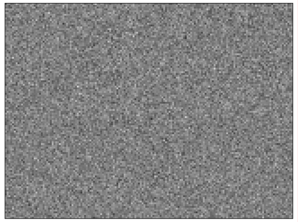
$N = 16$. $C \approx 0.251 = 1/\sqrt{N}$.



$N = 64$. $C \approx 0.125 = 1/\sqrt{N}$.



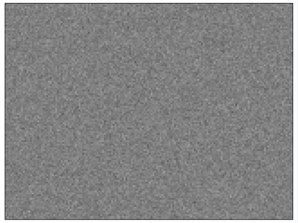
$N = 256$. $C \approx 0.061 = 1/\sqrt{N}$.



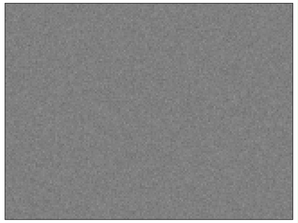
$N = 256$. $C \approx 0.031 = 1/\sqrt{N}$.

Notes

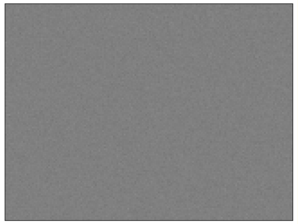
Averages of simulated intensity images



$N = 4,096$. $C \approx 0.015 = 1/\sqrt{N}$.



$N = 16,384$. $C \approx 0.007 = 1/\sqrt{N}$.



$N = 65,536$. $C \approx 0.003 = 1/\sqrt{N}$.

These simulations show, that with appropriate variation of phase and amplitude over a time period, variation of intensity can reduce* from $C = 1$ to $C = 1/\sqrt{N}$ where N is the number of intensities observed.

Notes