CS 3120 Bijections

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August 31 2023

1 $f : \{0,1\}^* \to \mathbb{N}$

$f((x_1, ..., x_n)) := (1, x_1, ..., x_n)_2$
I.e., add a "1" to the beginning of the string, then consider the number in binary.
Here are some examples:

$\mathbf{x} = (x_1, ..., x_n)$	$(1, x_1, ..., x_n)$	$(1, x_1, ..., x_n)_2 = f(\mathbf{x})$
""	1	1
0	10	2
1	11	3
00	100	4
01	101	5
10	110	6
11	111	7
000	1000	8
001	1001	9
010	1010	10

2 $g : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$

$g((a, b)) := \binom{a+b+1}{2} + a$

Here are some examples:

(a, b)	$\binom{a+b+1}{2}$	$\binom{a+b+1}{2} + a$
$(0, 0)$	0	0
$(0, 1)$	1	1
$(1, 0)$	1	2
$(0, 2)$	3	3
$(1, 1)$	3	4
$(2, 0)$	3	5
$(0, 3)$	6	6

If one were to write all the pairs of numbers $\mathbb{N} \times \mathbb{N}$ on a 2-D grid, this would list them going down the top-right to bottom-left diagonal, progressively getting farther from the origin.