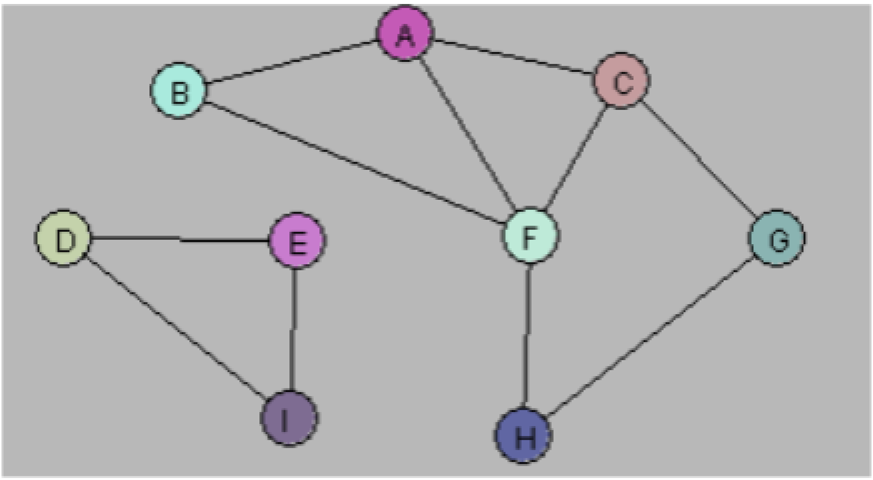
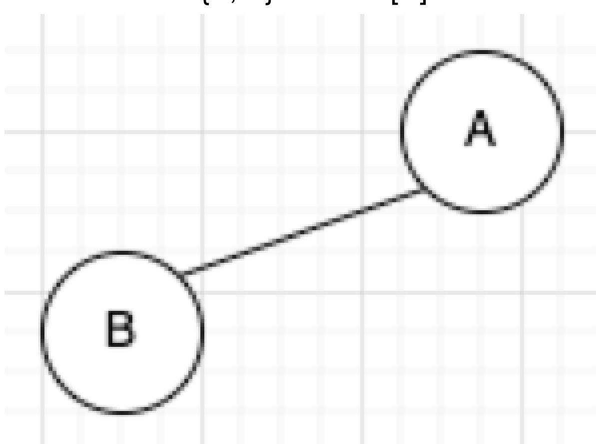
# Lab 8

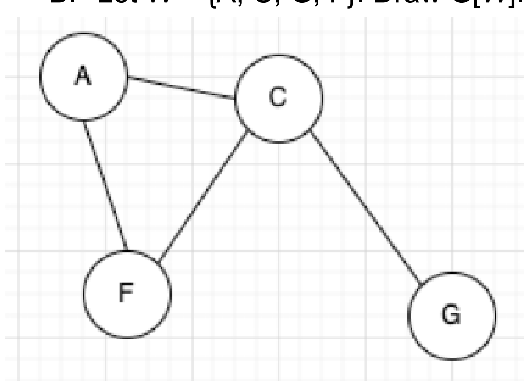
1. Induced Graphs. Answer questions about the graph G = (V,E) displayed below.



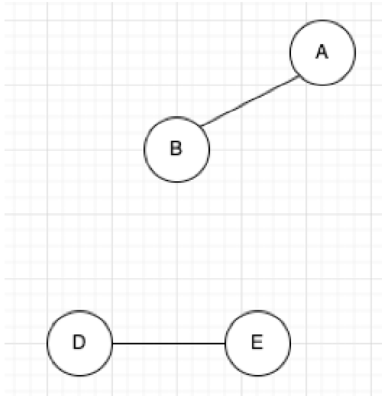

   A. Let U = {A, B}. Draw G[U].



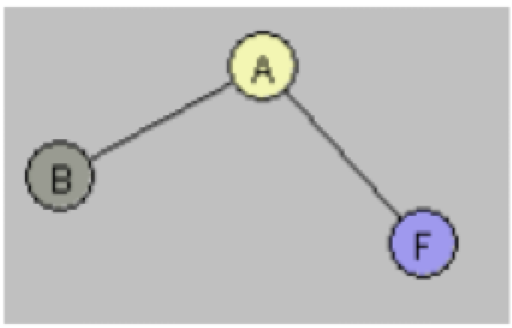

   B. Let W = {A, C, G, F}. Draw G[W].



C. Let Y = {A, B, D, E}. Draw G[Y]




D. Consider the following subgraph H of G:




Is there a subset X of the vertex set V so that H = G[X]? Explain.
-> yes. X = {A, B, F} $\subseteq$ V & H = G[X]

E. Find a way to partition the vertex set V into two subsets V1, V2 so that each of the
induced graphs G[V1] and G[V2] is connected and G = G[V1] U G[V2].

- V1 = {A, B, C, F, G, H}
- V2 = {D, E, I}

G[V1] is connected.
G[V2] is connected.
G = $G[V1] \cup G[V2]$

3. Graph Exercises.

A. Suppose G = (V, E) is a connected simple graph. Suppose V1, V2, . . ., Vk are disjoint subsets of V and that V1 U V2 U . . . U Vk = V. Show that there is an edge (x,y) in E such that for some i≠j, x is in Vi and y is in Vj.
   - Assume that there is no such edge in E. This means that for every edge (x, y) in E, either x and y belong to the same subset Vi or they both belong to different subsets, say Vi and Vj, where i = j.

Since G is a connected graph, there exists a path between any two vertices in V. Let's consider two arbitrary vertices u and v in V, where u is in Vi and v is in Vj, with i ≠ j.

Since there is no edge (x, y) in E such that x is in Vi and y is in Vj, there must be a path P between u and v that consists only of vertices from Vi. However, since v is not in Vi, the path P cannot reach v, which contradicts the assumption that G is connected.

Therefore, our assumption that there is no edge (x, y) in E such that for some i different from j, x is in Vi and y is in Vj must be false. Hence, there must exist at least one edge (x, y) in E such that x is in Vi and y is in Vj, with i ≠ j.

B. In class it was shown that a graph G = (V, E) is connected whenever the following is true,

(*) m > C(n-1, 2)

where n is the number of vertices and m is the number of edges. Is the following true or false?

Every connected graph satisfies the inequality (*).

Prove your answer.

If G is not connected then m <= C(n - 1, 2) = (n - 1)(n - 2) / 2

It can be shown that the graph having n vertices with the maximum possible number edge is: G = $k_{n-1} + k_1$

=> m > C(n - 1, 2)

C. Suppose G is a graph with two vertices. What is the minimum number of edges it must have in order to be a connected graph? Suppose instead G has three vertices; what is the minimum number of edges it must have in order to be connected? Fill in the blank with a reasonable conjecture:

If G has n vertices, G must have at least ____$\frac{(n-1)(n-2)}{2} + 1$____ edges in order to be connected.