Lab 9

1. Show that for any simple graph G (not necessarily connected) having n vertices and m edges, if $m \geq n$, then G contains a cycle.
    - Suppose G has m >= n and G is acyclic. Then G must be a tree and m = n-1 (so m < n) contradicting the fact that m >= n.
      Therefore, m >= n implies G contains a cycle.

2. Suppose G = (V, E) is a connected simple graph. Suppose S = (VS, ES) and T = (VT, ET) are subtrees of G with no vertices in common (in other words, VS and VT are disjoint). Show that for any edge (x,y) in E for which x is in VS and y is in VT, the subgraph obtained by forming the union of S, T and the edge (x,y) (namely, U = (VS U VT, ES U ET U {(x,y)})) is also a tree.
    - We have G is a connected graph and VS & VT are disjoint. So there is only edge (x, y) that connect VS & VT. If exist more than one edge between VS & VT, it will create a cycle, thus making VS & VT no longer subtrees.
      And because S & T are subtrees, they do not contain cycle. Therefore, union between S & T through (x, y) do not create a cycle. Since the union has no cycle, it is a tree.

5. Prove that if T is a tree with at least two vertices, T has at least two vertices having degree 1. Hint. Let v be any vertex in T and think of T as a rooted tree with vertex v. Create the usual levels for the tree. Then use properties of such a tree to solve the problem.
    - Let T as a rooted.
      We have every rooted tree with one or more vertices has at least one vertex of degree 1.
      And every vertex other than the root has a unique parent.
      So T has at least two vertices having degree 1.