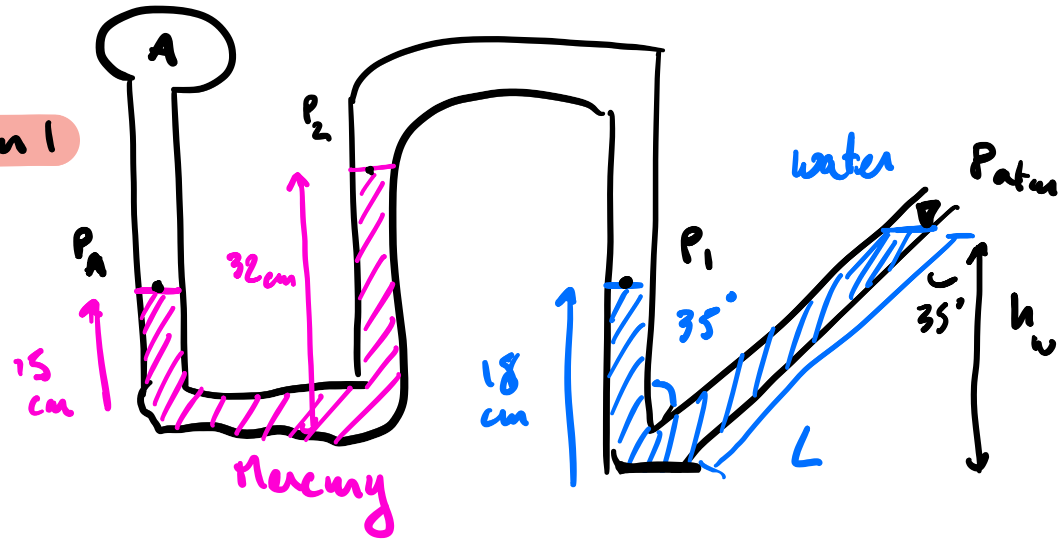
Pset 5

Problem 1



$\rho_w = 1000 \ kg/m^3$

$\rho_m = 13560 \ kg/m^3$

$h_w = L\cos(35°)$

$P_1 = P_{atm} + \rho_w (h_w - 18\times 10^{-2}) g$

$P_2 = P_1$?

$P_A = P_1 + \rho_m (0.32 - 0.15) g$

$$\begin{cases} P_A = P_{atm} + \rho_w (L\cos(35°) - 0.18) g + \rho_m (0.32-0.15) g \\ L\cos(35°) = \dfrac{P_A - \rho_m (0.32-0.15) g - P_{atm}}{\rho_w g} + 0.18 \end{cases}$$

a) $L = 120$ cm

$\Rightarrow P_A = 136.49$ kPa

b) $P_A = 135$ kPa

$\Rightarrow L = 176.11$ cm

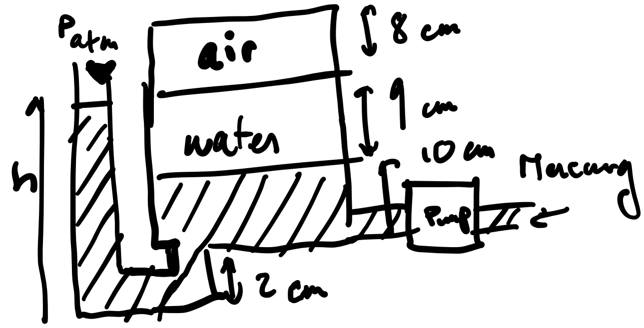
Problem 2

$P_{air} = 110\ kPa$

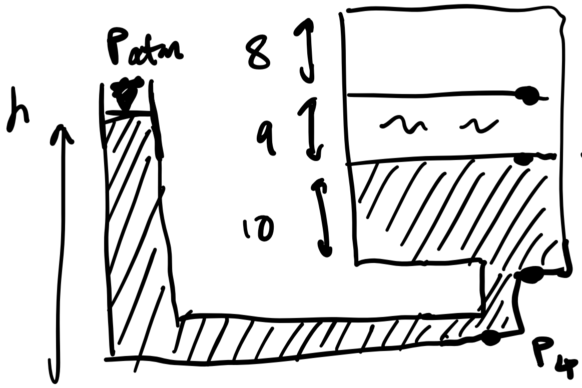
$\rho_w = 999\ kg/m^3$

$\rho_m = 13550\ kg/m^3$

$T = 20°C = 293\ K$



a) find h.



$P_1 = P_{air} = 110\ kPa$

$P_2 = P_{air} + 9\,cm \times \rho_w g$

$P_4 = P_2 + 0.12 \rho_m g.$

$$P_{atm} = P_4 - \rho_m g h$$

$$\Rightarrow h = \frac{P_4 - P_{atm}}{\rho_m g} = \frac{P_{air} + 0.09 \rho_w g + 0.12 \rho_m g - P_{atm}}{\rho_m g}$$

$$= 19.43\ cm$$

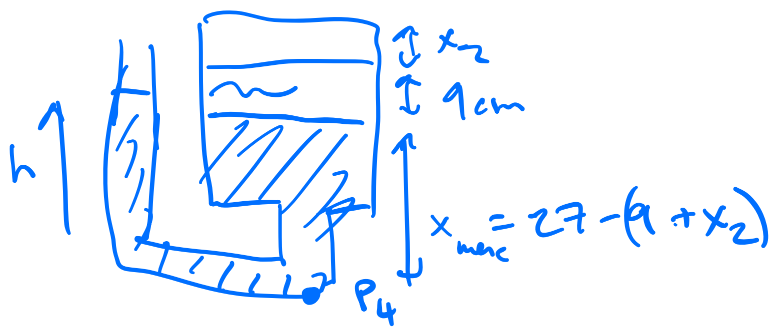
b) Assuming the pressure in the air was originally P_{atm}, $\Delta P = P_{air} - P_{atm} = 9\,kPa$.

c)

$$\left.\begin{aligned} P_2 V_2 &= mRT_2 \\ P_{air} V_{air} &= mRT_{air} \end{aligned}\right\} \quad P_2 V_2 = P_{air} V_{air}$$

$$\Rightarrow P_2 = P_{air} \frac{8cm}{x_2}$$

$$\Rightarrow x_2 = \frac{110 \times 10^3 \times 8 \times 10^{-2}}{150 \times 10^3}$$

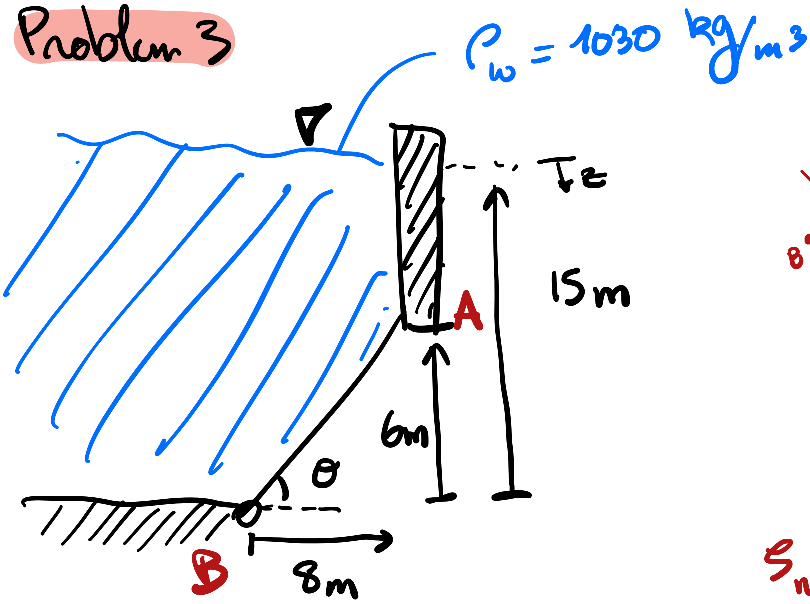


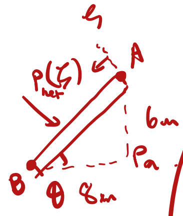
$$P_4 = P_2 + 9\,cm\,\rho_w g + x_{merc} \rho_m g$$

$$\Rightarrow P_{atm} = P_4 - \rho_m g h$$

$$\Rightarrow h = 51.66\ cm$$

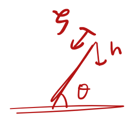
Problem 3





$z = \xi \sin\theta + 9$

$$\left(\begin{aligned} &\tan\theta = \frac{6}{8} \\ &\Rightarrow \sin\theta = 0.6 \\ &z = 0.6\xi + 9 \end{aligned} \right)$$



$$\xi_{max} = \sqrt{6^2 + 8^2} = 10$$

$$P(z) = \rho_w g z + P_{atm}$$

$$P(\xi) = \rho_w g(\sin\theta\,\xi + 9) + P_{atm}$$

b)

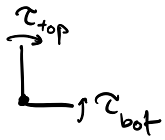
$$\Sigma F_{wall} = \int_0^{\xi_{max}} P(\xi)\,w\,d\xi - \int_0^{\xi_{max}} P_{atm}\,w\,d\xi$$

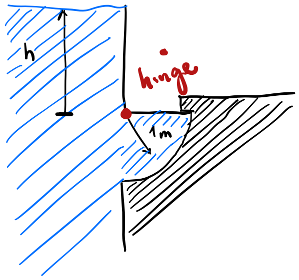
$$= \int_0^{\xi_{max}} \left[\rho_w g(\sin\theta\,\xi + 9) + \cancel{P_{atm}}\right] w\,d\xi - \cancel{\int_0^{\xi_{max}} P_{atm}\,w\,d\xi}$$

$$= \left[\rho_w g \sin\theta \frac{\xi^2}{2} + 9\xi\right] w \Big|_0^{\xi_{max}}$$

$$= \Big(\rho_w g \underbrace{\sin\theta}_{0.6} \frac{\overbrace{\xi_{max}^2}^{10^2}}{2} + 9\overbrace{\xi_{max}}^{10}\Big) w = 1.5161\ \mathrm{MN}$$

Problem 4

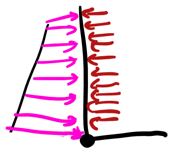




moment balance.
gate opens when
$\tau_{top} > \tau_{bot}$.

At tipping: water height $= h_c$

τ_{top}



$\rightarrow$ net pressure

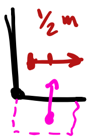
use equivalent load.

$F_{top} = \underbrace{\rho_w g \frac{h_c}{2}}_{} \cdot \underbrace{hw}_{\text{area}}$

$\tau_{top} = F_{top} \times h/3$

τ_{bot}

$\rightarrow$ equivalent net force



$F_{bot} = \rho_w g h_c \cdot (1 \times w)$

$\tau_{bot} = F_{bot} \, 1/2$

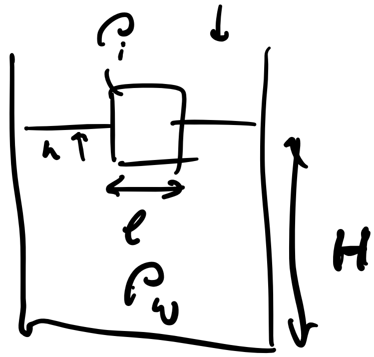
$$\boxed{\text{At tipping} \quad \tau_{top} = \tau_{bot}}$$

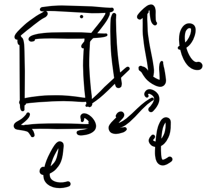
$$\left.\begin{aligned} \tau_{top} &= \rho_w g \frac{h_c^2}{2} w \frac{h_c}{3} \\ \tau_{bot} &= \rho_w g h_c (1m \times w) \frac{1}{2} \end{aligned}\right\} \Rightarrow \cancel{\rho_w} \cancel{g} \frac{h_c^{\cancel{3}\,2}}{6} \cancel{w} = \frac{\cancel{\rho_w g h_c w}}{2} \Rightarrow \boxed{h_c = \sqrt{\frac{6}{2}} = \sqrt{3}\ m}$$

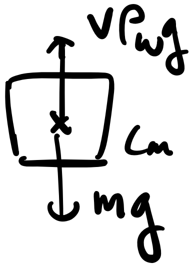
Problem 5

find ΔH vs θ.

$\rightarrow V_{submerge}$ vs V_{melted}.







$\Sigma F_y = 0$

$\Rightarrow V\rho_w g = mg$.

$m = \rho_i \ell^3$

$\Rightarrow V_{submerge} = \dfrac{\rho_i \ell^3}{\rho_w}$

$$\begin{cases} m_w = V_w \rho_w \\ m_w = m_{ice} \end{cases}$$

$\hookrightarrow V_{melted} = \dfrac{m_w}{\rho_w} = \dfrac{\rho_i \ell^3}{\rho_w}$

$= V_{submerged}$

$\rightarrow V_{sub} = V_{melted}$

$\Rightarrow \Delta H = 0.$

the water level stays constant.