2.005 Pset 3 - Andy Dequin

Problem 1

$g = 9.80 \, m/s^2$

1)

$\Delta T_r = 0$

$$\left(\begin{aligned} W_e &= P \times t \\ &= I V_R t \\ &= \frac{V_R^2}{R} t \end{aligned} \right)$$

2nd law $\quad \Delta S = \int_{t_1}^{t_2} \frac{\delta Q}{T_r} + \cancel{S_{gen}}^{\,0}$ uniform temp

1st law $\quad \Delta E = Q - W_e = 0$ steady operations

$$\Rightarrow W_e = Q$$

$$\Rightarrow \boxed{\Delta S = \frac{W_e}{T_r}}$$

2)

$$S_2 - S_1 = m c_v \ln \frac{P_2}{P_1} + m c_p \ln \frac{V_2}{V_1}$$

$\left.\begin{aligned} P_1V_1 &= mRT_1 \\ P_2V_2 &= mRT_2 \end{aligned}\right\} \rightarrow \frac{P_2}{P_1} = \frac{T_2 V_1}{T_1 V_2}$

TV FORM

$$\begin{aligned} S_2 - S_1 &= m c_v \ln\left(\frac{T_2 V_1}{T_1 V_2}\right) + m c_p \ln\left(\frac{V_2}{V_1}\right) \\ &= m c_v \ln\left(\frac{T_2}{T_1}\right) - m c_v \ln\left(\frac{V_2}{V_1}\right) + m c_p \ln\left(\frac{V_2}{V_1}\right) \\ &= m c_v \ln \frac{T_2}{T_1} + (c_p - c_v) m \ln \frac{V_2}{V_1} \end{aligned}$$

$c_p - c_v = R$

$$S_2 - S_1 = m c_v \ln \frac{T_2}{T_1} + m R \ln \frac{V_2}{V_1}$$

TP FORM

$\left.\begin{aligned} P_1V_1 &= mRT_1 \\ P_2V_2 &= mRT_2 \end{aligned}\right\} \rightarrow \frac{P_2}{P_1} = \frac{T_2 V_1}{T_1 V_2}$

$$\begin{aligned} S_2 - S_1 &= m c_v \ln \frac{P_2}{P_1} + m c_p \ln\left(\frac{T_2 P_1}{T_1 P_2}\right) \\ &= m c_v \ln \frac{P_2}{P_1} + m c_p \left(\ln \frac{T_2}{T_1} + \ln \frac{P_1}{P_2}\right) \\ &= m c_p \ln \frac{T_2}{T_1} + m (c_v - c_p) \ln \frac{P_2}{P_1} \end{aligned}$$

$c_p - c_v = R$

$$S_2 - S_1 = m c_p \ln \frac{T_2}{T_1} - m R \ln \frac{P_2}{P_1}$$

Sign of Terms

PV FORM:

$+ m c_v \ln \frac{P_2}{P_1}$: $\Delta P > 0$ $(\Delta V = 0)$ $\rightarrow \Delta S > 0$

$+ m c_p \ln \frac{V_2}{V_1}$: $\Delta V > 0$ $(\Delta P = 0)$ $\rightarrow \Delta S > 0$.

TV FORM:

$+ m c_v \ln \frac{T_2}{T_1}$: $\Delta T > 0$ $(\Delta V = 0)$ $\rightarrow \Delta S > 0$

$+ m R \ln \frac{V_2}{V_1}$: $\Delta V > 0$ $(\Delta T = 0)$ $\rightarrow \Delta S > 0$

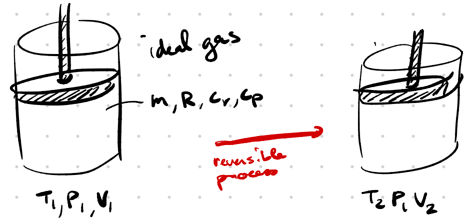
TP FORM

$+ m c_p \ln \frac{T_2}{T_1}$: $\Delta T > 0$ $(\Delta P = 0)$ $\rightarrow \Delta S > 0$

$- m c_p \ln \frac{V_2}{V_1}$: $\Delta P < 0$ $(\Delta T = 0)$ $\rightarrow \Delta S > 0$

3)

find $Q\Big|_{\Delta P=0}$ $(\Delta T, m, R)$



$$W = P_1\Delta V = P_1(V_2 - V_1) = \underbrace{mR(T_2 - T_1)}$$

$$\Delta E = mc_p\Delta T. \quad (\text{constant pressure})$$

$$\begin{cases} P_1V_1 = mRT_1 \\ P_1V_2 = mRT_2 \end{cases}$$

$$\Delta E = Q - W$$

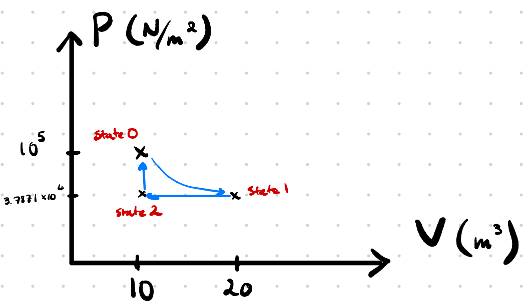
$$\Rightarrow Q = mR\Delta T + mc_p\Delta T$$

$$= \boxed{m(R + c_p)\Delta T}.$$

Problem 2

1)
$$E = E_R + 2.495(PV - P_RV_R)$$

$$S = S_R + 1003\ln\frac{V}{V_R} + 716\ln\frac{P}{P_R}$$



$0 \to 1$

Find P_1, P_2

$$\Delta S_{0\to1} = 0 = S(P_1, V_1) - S(P_0, V_0)$$

$$\Rightarrow 0 = 1003\ln\frac{V_1}{V_0} + 716\ln\frac{P_1}{P_0}$$

$$\Rightarrow 1003\ln\frac{V_1}{V_0} = -716\ln\frac{P_1}{P_0}$$

$$\Rightarrow \frac{V_1}{V_0}^{-\frac{1003}{716}} = \frac{P_1}{P_0}$$

$$\Rightarrow P_1 = P_0\left(\frac{V_1}{V_0}\right)^{-\frac{1003}{716}}$$

$$= \boxed{3.7871 \times 10^4 \ \text{N/m}^2}$$

Find ΔE (matlab)

$$\Delta E_{in\to out} = E(P_0V_0) - E(P_iV_i)$$

$$= 2.495(P_0V_0 - P_iV_i)$$

$$\Delta E_{0\to1} = -6.05 \times 10^5 \text{ J}$$

$$\Delta E_{1\to2} = -9.45 \times 10^5 \text{ J}$$

$$\Delta E_{2\to0} = 1.55 \times 10^6 \text{ J}$$

$$\Delta E_{0\to1\to2\to0} = 0 \quad (\text{cycle})$$

find Q

$$Q_{0\to1} = 0 \quad (\text{adiabatic})$$

$$Q_{1\to2} = \Delta E + W$$

$$= -5.66 \times 10^5 \text{ J}$$

$$Q_{2\to0} = \Delta E + \cancel{W}^{0}$$

$$Q_{0\to0} = 9.839 \times 10^5 \text{ J}$$

$$= W_{0\to0}.$$

find W

$$W_{0\to1} = \cancel{Q}^{0} - \Delta E_{0\to1}$$

$$W_{1\to2} = P(V_1 - V_0) \quad (\text{isobaric})$$

$$= 3.79 \times 10^5 \text{ J}$$

$$W_{2\to0} = \Delta E_{2\to0}$$

$$W_{0\to0} = W_{0\to1} + W_{1\to2} + W_{2\to0} = 9.839 \times 10^5 \text{ J}$$

find ΔS (matlab)

$$\Delta S_{in \to out} = 1003 \ln \frac{V_o}{V_i} + 716 \ln \frac{P_o}{P_i}$$

$$\Delta S_{0 \to 1} = 0$$

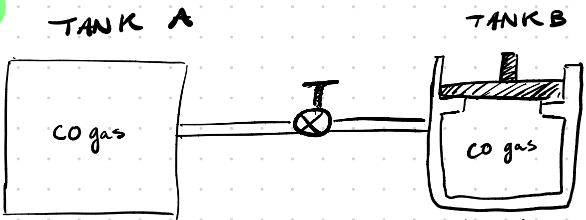
$$\Delta S_{1 \to 2} = -695.2 \text{ J/K}$$

$$\Delta S_{2 \to 0} = 695.2 \text{ J/K}$$

$$\Delta S_{0 \to 0} = 0 \text{ (state parameter)}$$

Process	ΔE (kJ)	Q (kJ)	W (kJ)	ΔS ($\frac{J}{K}$)	S_{transf} ($\frac{J}{K}$)	S_{gen} (J/K)
$0 \to 1$	-6.05×10^2	0	6.05×10^2	0	0	0
$1 \to 2$	-9.45×10^2	-5.66×10^2	3.79×10^2	-695.2 J/K	-695.2 J/K	0
$2 \to 0$	1.55×10^3	1.55×10^3	0	695.2 J/K	695.2 J/K	0
$0 \to 1 \to 2 \to 0$	0 (cycle)	9.84×10^2	9.84×10^2	0	0	0
					($S_{trans} = \Delta S - S_{gen}$)	(all reversible)

Problem 3



$R_u = 8.314 \frac{J}{mol\ K}$

$M_{CO} = 0.28 \frac{kg}{mol}$

State 1 (Tank A)

$m_{A1} = 2$ kg

$T_{A1} = 77°C$

$P_{A_1} = 7 \times 10^4$ Pa

State 1 (Tank B)

$m_{B1} = 8$ kg,

$T_{B1} = 27°C$

$P_{B1} = 1.2 \times 10^5$ Pa

a) IDEAL GAS $PV = nR_uT \Rightarrow V = \frac{nR_uT}{P}$ $\quad n = \frac{m}{M_{CO}}$

$$V_{A,1} = \frac{\frac{m_{A1}}{M_{CO}} R_u T_{A,1}}{P_{A,1}} = \boxed{2.9693 \text{ m}^3}$$

$$V_{B,1} = \boxed{5.9386 \text{ m}^3}$$

b) find P at equilibrium ($P_A = P_B$)

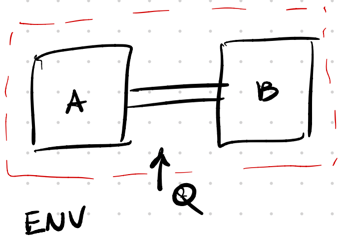
$$P V_{Total} = \frac{m_{Total}}{M_{CO}} R_u T \Rightarrow P = 105000 \text{ Pa}.$$

c) At equilibrium, the mass is proportional to the volume

$$m_{b2} = \frac{V_b}{V_{total}} \times m_{total} = \boxed{6.67\ kg}.$$

d)

$$1 \longrightarrow 2$$



TANK A

$$\Delta U_A = m_{A,2} c_v T_2 - m_{A,1} c_v T_{A1} = 260.75\ kJ$$

TANK B

$$\Delta U_B = m_{B,2} c_v T_2 - m_{B,1} c_v T_1 = -223.5\ kJ$$

$$\Delta U_{1\to 2} = \Delta U_A + \Delta U_B = \boxed{37.25\ kJ} = Q - \cancel{W}^{0}$$

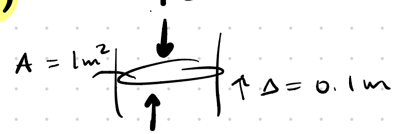
env → A+B.

e) the piston requires 2 bars or 200 000 Pa before rising

$$P V_B = n_{B_2} R_u T$$ where $P = 2$ bar and T is required temperature $\Rightarrow T = 600\ °K$

<u>find Q</u> : $Q = m_{B2} c_v \Delta T = \boxed{1415.5\ kJ}$

f) $F = PA = 10^5 N$



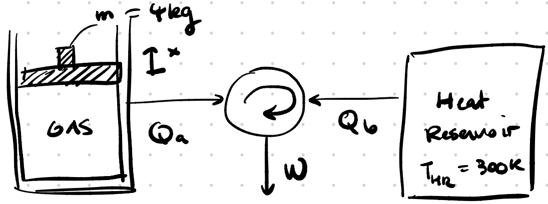
$$W_{3\to4} = F \Delta x_{3\to4} = \boxed{10^4\ J}$$ or 10 kJ

Problem 5

$\downarrow g = 9.80\ m/s^2$

$P_1 = 4 \times 10^5\ Pa$

$T_1 = 400K$



State 1

$C_v = 716 \frac{J}{kgK}$, $R = 287 \frac{J}{kgK}$,

State 2

$T_2 = 200K$

a)

$\Delta U = m c_v \Delta T$

$= -572800\ J$

$\Delta S = \int \frac{\delta Q}{T_1} + \cancel{S_{gen}}^{0 \text{ reversible}}$

$= m c_v \ln\left(\frac{T_2}{T_1}\right)$

$= -1985.2\ \frac{J}{K}$

$Q_{1\to 2} = T_{HR} \times \Delta S = -595.6\ kJ$

(from $S_{trans} = \frac{Q_{HR}}{T_{HR}}$)

$W_{1\to 2} = Q - \Delta U = -22.75\ kJ$

$S_{trans} = \Delta S - \cancel{S_{gen}}^{0 \text{ (reversible)}} = -1985.2\ \frac{J}{K}$

b) $Q_a = -Q_{1\to 2} = 595.6\ kJ$

$\left(\Delta S = 0 = \frac{Q_a}{T_1} + \frac{Q_b}{T_{HR}} + \cancel{S_{gen}}^{0}\right)$

$Q_b = Q_a \frac{T_{HR}}{T_1} = -446.7\ kJ$

$W = Q_a + Q_b = -1042.2\ kJ$

c) $Q_{1-2} = -Q_b = 446.7\ kJ$

$\Delta U_{1\to 2} \sim 0$ for heat resevoir

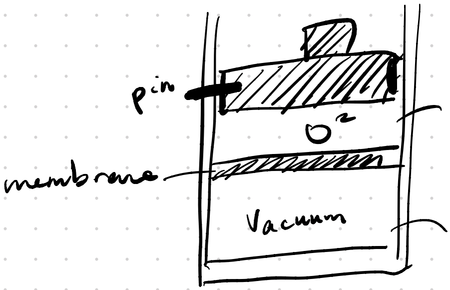
$S_{transfer} = \frac{Q_b}{T_{HR}} - \cancel{S_{gen}}^{0}$

$= 1.489\ kJ/K$

$\Delta S = S_{transf} = 1.489\ kJ/K$

Problem 4

STATE 1



$V_1 = 1m^3$; $P_1 = 8\times10^5 Pa$

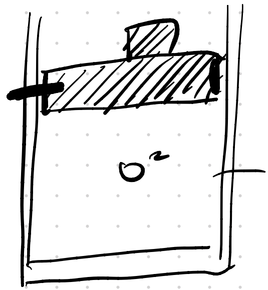
$T_1 = 400K$; $R = 260 \, J/kgK$

$V_v = 2m^3$

a) $P_1V_1 = mRT_1$

$$\Rightarrow m = \frac{P_1V_1}{RT_1} = \underline{7.6923 kg}$$

STATE 2



$V_2 = 3m^3$

$$\Rightarrow \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad \text{(eq 1)}$$

$$\Delta U = Q + W = 0$$

$$\Rightarrow 0 = mc_v \Delta T - P_f \Delta V$$

$$\Rightarrow mc_v(T_2 - T_1) = P_{final}(V_2 - V_1) \quad \text{(eq 2)}$$

find P_2, T_2; solve eq 1 and eq 2 : $P_2 = 362\ 230$ Pa

$T_2 = 543.4 °K$

b) find V_3, T_3 we know that $P_3 = 1.5\times10^5$ Pa

$$P_3V_3 = mRT_3$$

$$mc_v(T_3 - T_2) = P_3(V_3 - V_2)$$

SOLVE : $T_3 = 453°K$, $V_3 = 6.04 \, m^3$

c) $\Delta E_{1-2} = \cancel{Q_{1-2}}^{0} - \cancel{W_{1-2}}^{0} = 0.$

$\Delta E_{2\to 3} = \cancel{Q_{2-3}}^{0} - W_{2-3} = P_3(V_3 - V_2) = 456.25\ kJ$

$\Delta E_{2\to 3} = m c_v (T_3 - T_2) = -456.25\ kJ$ ✓

d)

OXYGEN

$\Delta E_{2\to 3} = \cancel{Q}^{0} - W$
$= m c_v (T_3 - T_2)$

ATM

$\Delta E = \cancel{Q}^{0} - \cancel{W}^{0}$

PISTON-MASS

$\Delta E_{2\to 3} = \cancel{Q}^{0} - W$ = opposite of oxygen.

e) $W = P(V_3 - V_2) = 304\ kJ.$

f) FOR $1\to 2$

$Q = 0 \to S_{trans} = 0.$ (adiabatic)

$\Delta S = m c_v \ln \frac{T_2}{T_1} + mR\ln \frac{V_2}{V_1}$

$= 1556\ J/K = \underline{S_{gen} > 0}$

IRREVERSIBLE

FOR $2\to 3$

$Q = 0 \to S_{trans} = 0$

$\Delta S = m c_v \ln\left(\frac{T_3}{T_2}\right) + mR\ln\left(\frac{V_3}{V_2}\right)$

$= 572\ J/K$

$= \underline{S_{gen} > 0}$