New techniques in hex pleating for representational origami design

Brandon Wong

Abstract: We present new developments building off the work of Lang et al. in hex pleating as a method for designing representational origami models, including both uniaxial and non-uniaxial techniques. Compared to box pleating, hex pleating offers greater flexibility of layer distribution, better ability to incorporate structural textures, and a more organic appearance. We will describe how to pack basic uniaxial trees with hex pleating, how packings can be modified to redistribute paper thickness throughout the model, how to incorporate level shifters, and finally discuss hex pleating as a means of creating representational polyhedra.

Hex pleating (HP), simply put, is a system of origami design where the creases are constructed on an isometric grid of equilateral triangles. This method has been well established in the fields of tessellations and modular origami, but has been much less commonly explored in the representational field of origami due to its less intuitive nature. Like box pleating (BP), hex pleating can be uniaxial—that is, applying tree theory to create a 1 dimensional stick figure [Lang 11b], or non-uniaxial, where the model is created without the use of clearly defined flaps and rivers.

Currently, the primary published exposition of representational hex pleating is a 10 page section of Robert Lang's *Origami Design Secrets* [Lang 11b], describing hex pleating in the context of uniaxial polygon packing. Another thorough explanation of hex pleating can be found in Andrey Ermakov's *Origami School of Masters* [Ermakov 12]. This paper expands on both Lang and Ermakov's foundations and presents new developments in hex pleating techniques for representational origami design.

1 Terminology and conventions

Let us first propose some definitions and conventions for the sake of future hex pleating discussion.

1.1 Terminology

• BP, HP, CP: box pleating, hex pleating, and crease pattern, respectively.

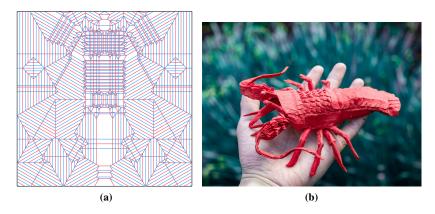


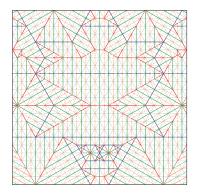
Figure 1: An example model designed and folded by the author that utilizes several techniques described in this paper: paper redistribution, structural texturing, and non-uniaxial transitions. (a) Crease pattern (b) Finished Japanese spiny lobster.

- packing: The arrangement of flaps and rivers in a uniaxial design. Blue lines are used to denote hinge creases, and red lines are used to denote ridge creases.
- axial: an adjective that describes a crease that will lie along the axis of the collapsed base of the CP. Technically, axial creases are only creases that strictly lie along the axis, but in this paper (except the level shifting section) we will use "axial" to refer to both axial creases and axial-parallel creases.
- uniaxial: an adjective that describes the structure of a folded base or the method used to design it: uniaxial HP or uniaxial BP, or non-uniaxial HP or non-uniaxial BP
- on grid: a crease that lies on a grid line. Similarly, off grid refers to a crease that does not lie on any grid line, but may still intersect grid points.

1.2 Grid orientation

First, there is the question of how one should orient a hex grid relative to the paper. In Robert Lang's groundwork of HP fundamentals as described in *Origami Design Secrets*, he orients the grid such that the grid has horizontal lines and the hinge creases are what lie along grid lines. [Lang 11b] In this paper, I propose an alternate convention, where the grid is rotated and scaled such that the grid has vertical lines, and the axial creases of the CP are what lie along grid lines. Let us refer to this alternate convention as an **axial grid.** Figure 2 shows a comparison of both conventions on the same crease pattern.

While Lang's convention is perhaps more intuitive for packing because the hinge creases (the outlines of flaps and rivers) will always lie on grid lines, the main advantage of the axial grid system is that it is much easier to fold. Since axial creases generally make up the large majority of creases in any given uniaxial crease pattern, the folder only needs to fold the grid and will already have most



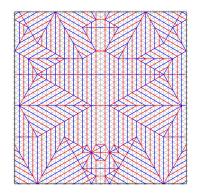


Figure 2: Left: Robert Lang's Brown Widow HP 2 crease pattern [Lang 11a] with Lang's grid convention. Right: the same crease pattern but redrawn using the axial grid convention. On the axial grid, most of the creases of the crease pattern lie on grid lines.

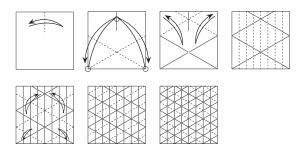
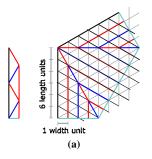


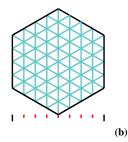
Figure 3: A common folding sequence for precreasing a hex grid from a square. The three circled points in the second step form an equilateral triangle, which is then bisected to construct the first 30 degree creases.

of the creases precreased (much like with box pleating). Additionally, as will be demonstrated in sections 2.2, 2.3, and 2.4, the axial grid allows the designer more flexibility in packing and flap lengths. Regardless, the grid is ultimately just a framework for the crease pattern, and does not affect the CP itself.

1.3 Grid units

Using the axial grid convention, we will designate two types of units when dealing with hex pleating. One **length unit** is equal to the side length of any equilateral triangle on the grid and is used to measure the lengths of flaps and features. One **width unit** is equal to $\frac{\sqrt{3}}{2}$ times one length unit—that is, the altitude of a triangle on the grid. These will be used to measure the widths or elevations of flaps and features. In this paper, the word "unit" by default means a length unit.





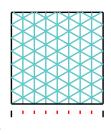


Figure 4: Left: A flap that is 6 length units long and one width unit wide. The length units are the sides lengths of the triangles of the grid, and the width units are the altitudes. Right: An 8x8 grid on a hexagonal paper, and an 8x8 grid on a square paper.

1.4 Grid size

Hex pleating can use both hexagonal and square papers effectively. In either case, let us define the "size" of a given hex grid as the number of sections the vertical pleats of the grid divide the paper into. Figure 4b shows an example of an 8 unit grid in a hexagon and in a square.

In a square paper, the grid lines will never perfectly line up with all of the edges of the paper because the ratio between the height and width of an equilateral triangle is irrational. For the sake of symmetry and easier folding, we will choose the top or bottom edge (which runs along width units) to be the side that doesn't lie on grid points.

1.5 Steep and shallow ridges

Unlike box pleating where all ridge creases are at 45 degree angles with the axial creases, hex pleating features two kinds of ridges: those that meet at 30 degree angles with the axial creases, and those that meet at 60 degree angles with the axial creases. We will refer to the former as **steep ridges** and the latter as **shallow ridges**.

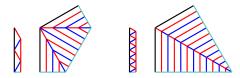


Figure 5: Left: a 6 unit flap made of steep ridges. Right: a 6 unit flap made of shallow ridges, which takes up more space.

2 Uniaxial hex pleating

The high level process for designing a representational model with HP is similar to the process of using other uniaxial design methods. With BP, the design process is

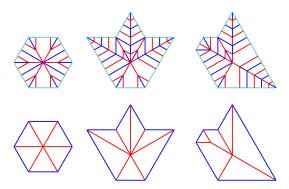


Figure 6: Three examples of ways to make a 3 unit HP center flap. Flaps can use a combination of steep and shallow ridges.

roughly as follows: take a subject and construct an abstracted tree figure, convert the tree into a packing of flaps and rivers, convert the packing to a crease pattern, then finally fold the CP and shape it into the subject [Lang 11b]. In this regard, hex pleating is mostly the same.

2.1 Packing

The first step of designing a model with axial hex pleating is to create a packing of flaps and rivers. Each flap or river can include a combination of steep and shallow ridges. When using the axial grid orientation, creases are as follows:

- Axial creases must lie on grid lines.
- Hinge creases must lie off grid lines.
- Ridge creases can be either on grid lines (shallow ridges) or off grid lines (steep ridges).

Figure 6 demonstrates some creative ways to construct flaps of various shapes. Unlike BP flaps that are almost always rectangular, HP flaps are much more flexible in their shape because there are two types of ridges that can be used.

HP packing is very similar BP or circle packing in the sense that the paper must be completely filled by the flaps and rivers without any empty spaces, and flaps and rivers may not overlap. In other words, the distance between any two points on the tree must be greater than or equal to their distance on the respective crease pattern. [Lang 11b]

2.2 Ridge misalignment

Ridge misalignments are one major difference from BP that arise in the process of packing with HP using the axial grid convention. As an example, let us pack the tree shown on the left in figure 7—a 2 unit and 4 unit flap on the left, a 1 unit river in the middle, and a 3 unit and 1 unit flap on the right. As a first attempt at packing, we might come up with the something like what's shown on the right in figure 7.

The flaps on the left look like they work, and the flaps on the right look like they work, but we are unsure how to fill in the empty space without widening the river.

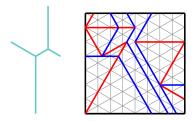


Figure 7: A desired tree figure (left), with a first attempt at packing it (right)

If we try to construct the crease pattern for this incomplete packing (as attempted in figure 8), we run into a problem. Even ignoring jump from the river to the flaps on the right, we still have a misalignment where the 2 unit and 4 unit flap meet on the left. Despite the fact that all of the flaps are centered on grid points and all flaps are integer lengths, we have two points (circled in green) where ridges are meeting at points off the grid. We will refer to this as a **ridge misalignment**. These ridge misalignments are $\frac{2}{3}$ of a width unit away from the nearest grid point, so we could multiply the grid size by 3 to solve this problem. However, this is not the most preferable solution.

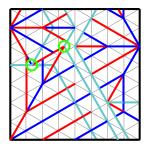


Figure 8: The resulting problematic crease pattern, with ridge misalignments circled in green.

Consider figure 9, which shows a steep ridge marked out for various possible flap lengths. We observe that for every 1 length unit that we make our flap length, we get $\frac{2}{3}$ of a ridge crease—or in other words, one full ridge crease creates $\frac{3}{2}$ units of length for the flap. Therefore, any flap of steep ridges with length that is a multiple of $\frac{3}{2}$ will result in a flap whose ridge ends at a grid point, but any other flap length will result in a ridge misalignment. That is why in figure 8, our 2 unit and 4 unit flaps resulted in ridge misalignments.

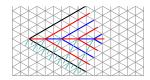


Figure 9: A steep ridge with markings showing where the hinges would be for various flap lengths.

In contrast, with a shallow ridge every 1 length unit that we add to our flap length results in 2 shallow ridges in the flap. Therefore, as long as our flap length is a multiple of $\frac{1}{2}$, our shallow ridge flap will never have a ridge misalignment.

This is a significant difference from box pleating. In box pleating, a flap will add 1 ridge crease for every 1 unit of length, so any integer flap lengths will work, while flaps with non-integer flap lengths would cause box pleating ridge misalignments. In contrast, in hex pleating sometimes integer flap lengths can cause ridge misalignments, while the half unit flaps can be the ones that work. It depends what kind of ridges are used.

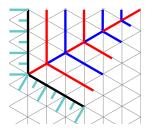


Figure 10: A shallow ridge with markings showing where the hinges would be for various flap lengths.

Let us return to our example packing. One solution would be to only use shallow ridges, and thus be able to assign our flap lengths to any multiple of $\frac{1}{2}$ without incurring ridge misalignments. The problem with this is that shallow ridges take up much more space than steep ridges, so our packing would be much less efficient. Another possible solution would be to make all of our flap lengths to be multiples of $\frac{3}{2}$. This would indeed prevent any ridge misalignments, and is essentially the reason why ridge misalignments do not occur when using Lang's grid convention. However, we may wish to have flaps of other lengths that are not multiples of $\frac{3}{2}$.

We can solve ridge misalignments using the fact that hex pleating flaps can use a combination of steep and shallow ridges. We have just seen that one length unit equates to $\frac{2}{3}$ of a steep ridge, or 2 shallow ridges. Therefore, if the length of our flap is r, and the number of steep and shallow ridge creases along one length of this flap are m and n respectively, then the flap will avoid ridge misalignments if the following equation is satisfied:

$$r = \frac{3m}{2} + \frac{n}{2}$$

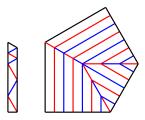
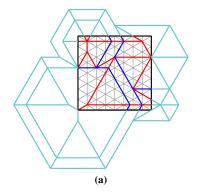


Figure 11: An example of a flap with a mix of steep and shallow ridges, with its x ray on the left. This flap has a length $r = 6\frac{1}{2}$, m = 3 steep ridges and m = 4 shallow ridges, and does not cause ridge misalignments because $6\frac{1}{2} = \frac{3}{2}(3) + \frac{1}{2}(4)$.

By strategically adding in shallow ridges to the 2 unit and 4 unit flaps, we can eliminate all the ridge misalignments. Although there are some ridges that look like they end (intersect a hinge crease) at an off grid point, they are continuing into the colinear ridges of the river and ultimately end at a grid point. Once ridge misalignments are gone, we can finally construct the CP and fold our tree.



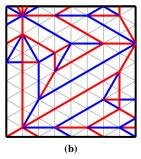


Figure 12: The new packing and creasepattern, fixed by strategically adding shallow ridges to the 2 unit and 4 unit flaps.

Finally, it is worth noting again that Lang's grid convention is not susceptible to ridge misalignments. This is because Lang's grid convention defines 1 unit of length as what the axial grid defines as $\frac{3}{2}$ units (the length of a steep ridge), and shallow ridges always come in groups of 3 [Lang 11b] [Lang 11a], essentially meaning all lengths of the tree would be multiples of $\frac{3}{2}$ if converted to an axial grid. While this is indeed a valid way to avoid ridge misalignments, using the axial grid has the added bonus of being able to create flap lengths that are multiples of smaller increments.

2.3 Ridge sliding

Thus far we have seen how hex pleating can create a stick figure, but this could have been acheived equally well or better through box pleating. We would now like to explore what other advantages HP can offer to continue to structurally improve our design. One of the main advantages of HP is the flexibility of layer distribution of shallow ridges.

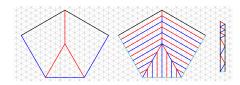


Figure 13: A 9 unit flap that we will use to demonstrate ridge sliding. From left to right: packing, CP, and x-ray.

As a demonstration, let's manipulate the flap in figure 13, which has 6 shallow ridges and 4 steep ridges, and is in total 9 units long. First, shallow ridges can be squashed to create $\frac{1}{2}$ unit long flaps separated by 1 unit long rivers. Let flaps that are formed by ridge squashing be referred to as **ridge bevels.**

Because doing so technically creates several new small flaps and rivers, our packings can quickly get cluttered. We therefore use a shorthand to denote where ridges are being squashed, without having to draw each individual ridge and hinge. Figure 14 shows two packings: the one on the left is the actual packing, that shows the arrangement of flaps and rivers exactly as they would be folded. The packing on the right lumps everything into one abstracted flap by representing the "zigzag" hinges of the ridge bevels into two straight lines. The shorthand notation is essentially replacing the zig-zag hinges of the ridge bevels with two parallel lines whose width is the width of the ridge bevels.

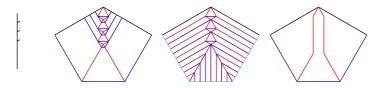


Figure 14: The example flap with squashed shallow ridges (tree, actual packing, crease pattern, shorthand packing)

If one were to fold this physically, one would notice that a given ridge bevel can be sunken to shift it down and make it wider. Figure 15 shows what this would look like from a packing perspective. Doing so is like sliding the ridges up and down the flap, without actually changing the space occupied by the flap itself or the way it interacts with the rest of the packing. Thus, we will refer to this process in general as **ridge sliding.**

This can be useful, for example, if we wanted to incorporate a scale pattern; the horizontal pleats would come from ridge bevels, and the vertical pleats can come from level shifted axial pleats that run perpendicular. This is essentially the method used for scaled section of the losbster shown in figure 1a.

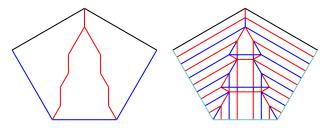


Figure 15: *The ridge bevels become wider as we shift them lower.*

Fully sinking a ridge bevel can shift it down by 1.5 length units (the length of a single steep ridge). However, hex pleated ridge bevels could also be shifted in increments of $\frac{1}{2}$ of a length unit.

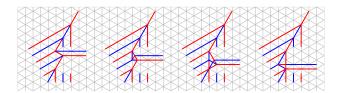


Figure 16: A ridge bevel shifted down in increments of $\frac{1}{2}$ of a length unit.

There are other ways one could slide the ridges around: merging two ridge bevels into one bevel that is twice as large, keeping some shallow ridges unsquashed while others become ridge bevels, swinging the flaps up or down or change the way they intersect the vertical pleats, etc. Essentially, a given HP flap or river can redistribute the paper stored in shallow ridges in useful ways completely independently from the rest of the packing.

2.4 Space allocation

One will sometimes claim that HP is more efficient than BP because hexagonal flaps can approximate a circle better than square flaps. While this is true, in practice the difference is negligible because pythagorean stretches allow both BP and HP to approach the efficiency of circle packing as grid size increases. Nevertheless, although HP flaps may not necessarily take up less space on average compared to BP flaps with pythagorean stretches, they take up space in a way allow the gaps in between to be utilized more fluidly. Free space can be allocated by the designer to form extra details where the model requires it by absorbing the space with shallow ridges and then squashing them to form ridge bevels within the flap or river.

As an explanation by example, consider the packing in figure 17 for a simple human figure. The bare minimum space for each flap is a hexagonal area, leaving 3 pockets of empty space (highlighted in red). There are multiple ways that each pocket could be used in a useful manner. In the top packing and CP, the space is absorbed as shallow ridges in the river. This would be useful, for example, for a

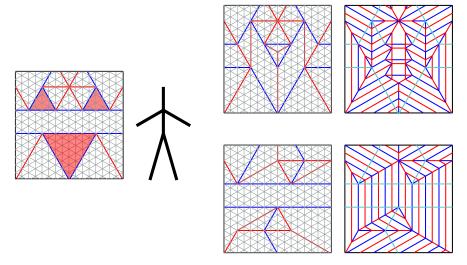


Figure 17: An example of HP's flexibility in space allocation. Left: a packing of a basic human figure, where each flap takes the minimum amount of space and 3 empty areas are left. Top right: a packing and crease pattern that allocates the empty space to form details in the river. Bottom right: a packing and crease pattern that allocates the empty space to form details in the legs, head, and arm flaps.

figure with clothes or other body details. In the bottom packing and CP, the bottom pocket is absorbed by the two legs (albeit asymmetrically), the left small pocket is absorbed by the head flap, and the right small pocket is absorbed by an arm flap. The designer has the freedom to freely allocate the space to different parts of the model without affecting the rest of the packing.

Additionally, this differs significantly from the way BP fills in blank space. Box pleating sometimes is forced to fill space by using 90 degree river meanders or by adding small unused flaps. In both cases, this results in an inconvenient concentration of paper thickness. In contrast, the method of using shallow ridges and ridge bevels to fill space not only makes it easier to utilize the paper for shaping details, but also creates a more even thickness distribution throughout the flap.

2.5 Pythagorean stretches

It also turns out that there exist pythagorean triangles for isometric grids, and pythagorean stretches can be constructed for hex pleating. However, since doing so is very inconvenient for both designing and folding, and only marginally increases the efficiency of a packing, this paper will not explore pythagorean stretches in detail.

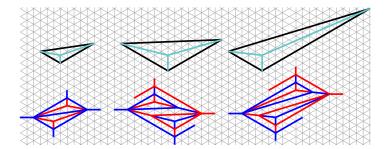


Figure 18: The first 3 pythagorean triples for HP, rabbit eared (top row) with an example of a pythagorean stretch for each one (bottom row). From left to right: the 3-5-7 triangle, the 7-8-13 triangle, and the 5-16-19 triangle.

3 Non-uniaxial techniques

The most basic non-uniaxial structures are level shifters, which can be added to uniaxial HP similarly to how they work in BP. Just like in BP, there are (at least) two kinds of level shifters: ridge level shifters and level shifters over perpendicular pleats. Overall, level shifting in HP is almost identical to BP, but with different angles.

3.1 Ridge level shifters

Starting with ridge level shifters, we find that level shifters can be constructed over both shallow and steep ridges. Similar to how many BP level shifters can be kept on grid by using slopes of 2 and 3, the structure can also usually be kept on grid points. Regardless, these level shifters are just special cases of a generalized ridge transition structure [Wong and Demaine 24].

In practice, ridge level shifters are not any different than level shifters in BP. They are useful because they do not take up any additional paper than what the ridge would have taken up anyways.

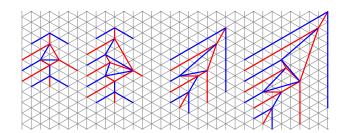


Figure 19: Various ridge level shifters. From left to right: a + 1 and a + 2 shifter over a shallow ridge, a + 1 and a + 2 shifter over a steep ridge.

3.2 Perpendicular pleat level shifters

Moving to perpendicular pleat level shifters, we encounter an interesting advantage of hex pleating. In box pleating, a +1 or +2 level shifter usually requires the perpendicular pleat to be at least one unit wide. But in hex pleating, perpendicular pleat level shifters can be done with pleats that are only half a length unit wide. In other words, this kind of HP level shifter would take on average of $\frac{1}{\sqrt{3}}$ of the space an equivalent BP level shifter (derived from the ratio between length and width units). Furthermore, these half width pleats are extremely convenient because they can be formed within a flap or river by squashing shallow ridges as demonstrated in section 2.3.

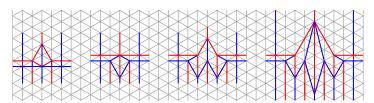


Figure 20: A series of perpendicular pleat shifters. From left to right: a + 1 shifter, another +1 shifter, a + 2 shifter, and a + 3 shifter.

One quirk about perpendicular pleat level shifters in hex pleating is that moving the perpendicular pleat by half a length unit can affect what kind of on-grid level shifters can be constructed around it. As an example, the only difference in the first and second shifter in figure 20 is that the pleat in the first is shifted half a unit down—despite this, the structures necessary to remain on grid points are slightly different. This can be overcome using structures similar to those shown in figure 16 that shift the half width pleat up or down in increments of half a length unit. These level shifters are otherwise very similar to box pleated level shifters. An example of these level shifters applied to a representational model is shown in figure 21a.

3.3 Representational polyhedra

Finally, we introduce what one might call *representational polyhedra*, or in other words, representational models that feature non-flat-foldable but geometrically defined 3d bodies. As a mathematical explanation of why HP is advantageous for this, consider the angular defect of polyhedron vertices. The smallest crease sector angle in HP is 30 degrees, so the angular defect of a polyhedra made from HP is either 60 degrees like an icosahedron or 120 degrees like an octahedron. In contrast, in BP the angular defect can only be 90 degrees like a cube (in fact BP was originally developed as a method for making 3d blocky models like *Mooser's train* [Lang 11b]). Not only do HP vertices have two choices of angular deficiency, but the 60 degree deficient vertices have much more flexibility in distributing the dihedral angles between creases connected to the vertex, analagous to how a vertex of an icosahedron has 5 edges compared to the 3 edges of the vertex of a cube. Overall, this means it's possible to make much more organic looking polyhedra with HP than with BP.

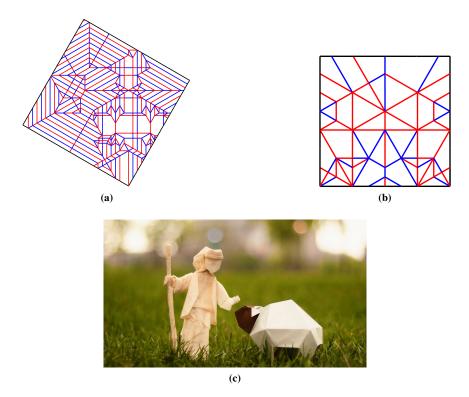


Figure 21: Shepherd HP and Sheep HP, both designed and folded by the author. Top left: the shepherd crease pattern, which features a number of perpendicular level shifters to create the figure's clothes. Top right: the sheep crease pattern, which is another example of a representational polyhedron. Bottom: the folded models together.

One prominent work on designing origami polyhedra is John Montroll's book *Origami Polyhedra Design* [Montroll 11]. In the method described by Montroll, one can start with an unfolded net of a polyhedron, place the net in the square, and then fold away the negative space to form the polyhedron. Several years later, a formalized "Origamizer" algorithm was developed by Erik Demaine and Tomohiro Tachi. This algorithm further generalizes Montroll's work and proves that any polyhedra, including ones with saddle points, can be folded from a single sheet of paper [Demaine and Tachi 17]. However, although the algorithm is highly generalized, the crease patterns it produces are often very complex and impractical to fold.

Non-uniaxial hex pleating offers a more balanced blend of Montroll and Demaine et.al's methods. The polyhedra may not be as precise or complex as what Origamizer can produce, but constraining creases to a grid allows the polyhedra to be more complex and generalizable than Montroll's convex polyhedra but while remaining reasonably practical to fold.

Anecdotally, it generally seems quite possible to design aesthetically pleasing representational models with solid 3d polyhedra bodies using hex pleating techniques, such as the *Stanford Bunny HP* shown in figure 22b. These models can also include locally flat foldable features like an ear or a neck. However, further work is required to fully characterize what 3d shapes are possible with HP, and how exactly to generate such designs. One promising direction is the edge river method developed by Mu-Tsun Tsai [Tsai 21].

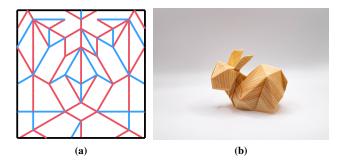


Figure 22: Stanford Bunny HP, designed and folded by the author as an example of a representational polyhedron made with hex pleating. (a) The crease pattern, and (b) the folded model. Compared to Origamizer-generated Stanford Bunny presented in [Demaine and Tachi 17], this model is much less detailed but is much easier to fold.

3.4 Conclusion

In this paper, we have expanded upon the basics of hex pleating. We presented an alternative grid convention that makes it easier to both fold and design models with hex pleating. We covered the basics of designing with HP and then showed a num-

ber of novel techniques that are unique to hex pleating: ridge sliding, more flexible space allocation, and more space-efficient level shifters. Finally, we discuss the possibility of hex pleating applied to creating reasonably foldable representational polyhedra.

Beyond this, there remains much more work to be done to continue exploring what can be done with hex pleating. For example, encorporating HP tessellations and textures into representational models, merging non-uniaxial representational polyhedra structures into detailed uniaxial bases, HP-specific color change techniques, or perhaps others we have yet to dream of. Nevertheless, what is clear is that hex pleating has the potential to shape the next paradigm of representational origami design.

Acknowledgements

The author would like to thank Robert J. Lang for his comments and support early on in these developments, and to Travis Nolan and Alexa D. for their feedback on the manuscript.

References

- [Demaine and Tachi 17] Erik D. Demaine and Tomohiro Tachi. "Origamizer: A Practical Algorithm for Folding Any Polyhedron." *Leibniz International Proceedings in Informatics (LIPIcs)*, pp. 34:1–34:16. doi:10.4230/LIPICS.SOCG.2017.34.
- [Ermakov 12] Andrey Ermakov. Origani School of Masters. CRC Press, 2012.
- [Lang 11a] Robert J. Lang. "Brown Widow HP 2, Opus 586." https://langorigami.com/artwork/brown-widow-hp-2-opus-586/, 2011. Accessed: 2024-03-14.
- [Lang 11b] Robert J. Lang. Origami Design Secrets: Mathematical Methods for an Ancient Art (1st ed.). A K Peters/CRC Press, 2011. doi:10.1201/b10706.
- [Montroll 11] John Montroll. Origami Polyhedra Design (1st ed.). A K Peters/CRC Press, 2011. doi:10.1201/b11163.
- [Tsai 21] Mu-Tsun Tsai. "Geography of Origami: A Brief Introduction to the Edge River Method." https://origami.abstreamace.com/zh/2021/12/12/geography-of-origami-abrief-introduction-to-the-edge-river-method/, 2021. Accessed: 2024-05-10.
- [Wong and Demaine 24] Brandon Wong and Erik D. Demaine. "Algorithmic transitions between parallel pleats." *The 8th International Meeting on Origami in Science, Mathematics and Education.*