Unit : 4

Propositional and Predicate Logic.

* Logical Equivalences.

1. Identity law:

$$p \wedge T \equiv p$$
$$p \vee F \equiv p$$

2. Domination laws:

$$p \vee T \equiv T$$
$$p \wedge F \equiv F$$

3. Idempotent laws:

$$p \vee p \equiv p$$
$$p \wedge p \equiv p$$

4. Double Negation Law:

$$\sim(\sim p) \equiv p$$

5. Commutative laws:

$$p \vee q \equiv q \vee p$$
$$p \wedge q \equiv q \wedge p$$

6. Associative laws:

$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$
$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$

7. Distributive laws:

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$
$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

8. De Morgan's laws:

$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$
$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

9. Absorption laws:

$$p \vee (p \wedge q) \equiv p$$
$$p \wedge (p \vee q) \equiv p$$

10. Negation laws:

$$p \vee \sim p \equiv T$$
$$p \wedge \sim p \equiv F$$

* Logical Equivalences involving conditional statement:

→ $p \to q \equiv \sim p \vee q$

- $p \to q \equiv \sim q \to \sim p$
- $p \vee q \equiv \sim p \to q$
- $p \wedge q \equiv \sim(p \to \sim q)$

- $\sim(p \to q) \equiv p \wedge \sim q$
- $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
- $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

- $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
- $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

* Logical Equivalences involving Biconditional.

- $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
- $p \leftrightarrow q \equiv \sim p \leftrightarrow \sim q$.
- $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$
- $\sim (p \leftrightarrow q) \equiv p \leftrightarrow \sim q$.

* Normal Forms:

In logic, with help of truth table we can compare if two elements are equivalent.
But when more statements or propositions are involved, then this method is not practical.
One method is to transform S_1 and S_2 to some standard form S_1' and S_2'. Such that a simple comparison of S_1' and S_2' should establish $S_1' \equiv S_2'$
The standard forms are called normal forms or canonical forms.

(1) Conjuctive Normal Form (cnf)

It is a conjunction ($\wedge$) of fundamental disjunction. ($\vee$)

Example: $p \vee q$, $p \vee \sim q$, $\sim p \vee \sim q$, $\sim p$, $\sim q$, p, etc. are fundamental disjunction.
Hence, conjunction of fundamental disjunctions

are joining fundamental disjunction by '$\wedge$'

Example: 1. $(p \vee q) \wedge (q \vee r) \wedge (\sim p \vee \sim r)$
2. $p \wedge (\sim q \vee \sim r)$
3. $(p \vee q \vee \sim r) \wedge (p \vee q) \wedge (\sim p \vee q \vee \sim r)$

(2) Disjunctive Normal form (dnf)

It is a disjunction ($\vee$) of fundamental conjunction. ($\wedge$)

Examples: $p \wedge q$, $\sim p \wedge q$, $\sim p \wedge \sim q$, $p \wedge \sim p$, $q \wedge \sim q$, $p \wedge \sim q$ etc are fundamental conjunctions

Hence, disjunction of fundamental conjunctions are joining fundamental conjunction by '$\vee$'.

Examples: 1. $(p \wedge q) \vee p \vee (q \wedge \sim p)$
2. $(p \wedge q \wedge r) \vee (p \wedge q' \wedge r) \vee (p' \wedge q \wedge r)$
3. $(p' \wedge q \wedge r) \vee (p \wedge q)$
4. $(p \wedge q) \vee r$
5. $(p \wedge q) \vee (r \wedge q) \vee (\sim p \wedge r)$

Remark: $q \wedge \sim q$, $p \wedge \sim p$ are always false.
Hence, if a fundamental conjunction contains at least one pair of (p and $\sim p$) or ($q \wedge \sim q$) etc it will be false.

Ex:1 Obtain the dnf of the form:
Q.B. 152
$p \wedge (p \to q)$

Sol^n:
$p \wedge (p \to q) \equiv p \wedge (\sim p \vee q)$ (cnf) (as $p \to q \equiv \sim p \vee q$)
$\equiv (p \wedge \sim p) \vee (p \wedge q)$
$\equiv F \vee (p \wedge q)$
$\equiv (p \wedge q)$ (dnf)

Ex:2 Obtain conjunctive normal form of

Q.B(i) 158
$(\sim p \to r) \wedge (p \to q)$
$\equiv (\sim(\sim p) \vee r) \wedge (\sim p \vee q)$ (as $p \to q \equiv \sim p \vee q$)
$\equiv (p \vee r) \wedge (\sim p \vee q)$. (cnf). (as $\sim(\sim p) \equiv p$)

Q.B(ii) 159
$(p \wedge q) \vee (\sim p \wedge q \wedge r)$
$\equiv (p \vee (\sim p \wedge q \wedge r)) \wedge (q \vee (\sim p \wedge q \wedge r))$
(use distributive law).
$\equiv ((p \vee \sim p) \wedge (p \vee q) \wedge (p \vee r)) \wedge ((q \vee \sim p) \wedge (q \vee q) \wedge (q \vee r))$
$\equiv (T \wedge (p \vee q) \wedge (p \vee r)) \wedge ((q \vee \sim p) \wedge q \wedge (q \vee r))$
$\equiv (p \vee q) \wedge (p \vee r) \wedge (q \vee \sim p) \wedge q \wedge (q \vee r)$ (cnf).

Ex:3 Find conjunctive normal form and disjunctive
Q.B normal form for the following without using
160 truth table.
$(p \to q) \wedge (q \to p)$

Sol^n:
$(p \to q) \wedge (q \to p)$
$\equiv (\sim p \vee q) \wedge (\sim q \vee p)$ (cnf)
$\equiv ((\sim p) \wedge (\sim q \vee p)) \vee ((q) \wedge (\sim q \vee p))$ (Use D.L)

$\equiv (\sim p \wedge \sim q) \vee (\sim p \wedge p) \vee (q \wedge \sim q) \vee (q \wedge p)$

$\equiv (\sim p \wedge \sim q) \vee F \vee F \vee (q \wedge p)$

$\equiv (\sim p \wedge \sim q) \vee (q \wedge p)$ (dnf)

Ex:4 Obtain disjunction normal form of

Q.B 161 (i) $(p \to q) \wedge (\sim p \wedge q)$

$\equiv (\sim p \vee q) \wedge (\sim p \wedge q)$ (as $p \to q \equiv \sim p \vee q$)

$\equiv (\sim p \wedge (\sim p \wedge q)) \vee (q \wedge (\sim p \wedge q))$ (Use Distributive law)

$\equiv ((\sim p \wedge \sim p) \wedge q) \vee (q \wedge \sim p \wedge q)$

(By associative and commutative law)

$\equiv (\sim p \wedge q) \vee (q \wedge \sim p)$ (dnf) (By idempotent law)

Q.B 131 (ii) $(p \wedge (p \to q)) \to q$.

$\equiv (p \wedge (\sim p \vee q)) \to q$ (as $p \to q \equiv \sim p \vee q$)

$\equiv \sim (p \wedge (\sim p \vee q)) \vee q$

$\equiv \sim p \vee (\sim (\sim p \vee q)) \vee q$

$\equiv \sim p \vee (p \wedge \sim q) \vee q$

Ex: Obtain the conjunctive normal form of each of the following:

QB 162

(i) $p \wedge (p \to q)$

$\equiv p \wedge (\sim p \vee q)$ (cnf)

(ii) $\sim(p\vee q) \leftrightarrow (p\wedge q)$

$\equiv [\sim(\sim(p\vee q)\vee(p\wedge q))] \wedge [\sim(p\wedge q)\vee\sim(p\vee q)]$

$[p\leftrightarrow q \equiv (\sim p\vee q)\wedge(\sim q\vee p)]$

$\equiv [(p\vee q)\vee(p\wedge q)] \wedge [(\sim p\vee\sim q)\vee(\sim p\wedge\sim q)]$

$\equiv [(p\vee q\vee p)\wedge(p\vee q\vee q)] \wedge [(\sim p\vee\sim q\vee\sim p)\wedge(\sim p\vee\sim q\vee\sim p)]$

$\equiv (p\vee q)\wedge(p\vee q)\wedge(\sim p\vee\sim q)\wedge(\sim p\vee\sim q)$

$\equiv (p\vee q)\wedge(\sim p\vee\sim q)$ (cnf) $\left(\because p\wedge p\equiv p,\ p\vee p\equiv p\right)$

(iii) $\sim(p\vee q)\leftrightarrow(p\wedge q)$

$\equiv [\sim(p\vee q)\wedge(p\wedge q)] \vee [(p\vee q)\wedge\sim(p\wedge q)]$

$[p\leftrightarrow q \equiv (p\wedge q)\vee(\sim p\wedge\sim q)]$

$\equiv (\sim p\wedge\sim q\wedge p\wedge q)\vee[(p\vee q)\wedge(\sim p\vee\sim q)]$

[By De' Moogan's law]

$\equiv (\sim p\wedge\sim q\wedge p\wedge q)\vee((p\vee q)\wedge\sim p)\vee((p\vee q)\wedge\sim q)$

[By Distributive law]

$\equiv (\sim p\wedge\sim q\wedge p\wedge q)\vee(p\wedge\sim p)\vee(q\wedge\sim p)\vee(p\wedge\sim q)\vee(q\wedge\sim q)$

(dnf)

Que: Find cnf and dnf of
QB 167

$$(\sim p \to q) \wedge (p \to q)$$

Ans: $(\sim p \to q) \wedge (p \to q)$

$\equiv (\sim(\sim p) \vee q) \wedge (\sim p \vee q)$

$\equiv (p \vee q) \wedge (\sim p \vee q)$ (cnf)

$\equiv [p \wedge (\sim p \vee q)] \vee [q \wedge (\sim p \vee q)]$

$\equiv (p \wedge \sim p) \vee (p \wedge q) \vee (q \wedge \sim p) \vee (q \wedge q)$

$\equiv (p \wedge q) \vee (q \wedge \sim p) \vee q$ (dnf)

Que: Show that $\sim(p \to q)$ and $p \wedge \sim q$ are logically equivalent.

Ans:

$\sim(p \to q)$

$\equiv \sim(\sim p \vee q)$

$\equiv \sim(\sim p) \wedge (\sim q)$ (By De Morgan's law)

$\equiv p \wedge \sim q$ (By Double Negation law)

Que: Show that $\sim(p \vee (\sim p \wedge q))$ and $\sim p \wedge \sim q$ are logically equivalent.

Ans:

$\sim(p \vee (\sim p \wedge q))$

$\equiv \sim p \wedge \sim(\sim p \wedge q)$

$\equiv \sim p \wedge [\sim(\sim p) \vee \sim q]$

$\equiv \sim p \wedge [p \vee \sim q]$ [By Distributive law]

$\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$ $(\sim p \wedge p \equiv F)$

$\equiv F \vee (\sim p \wedge \sim q)$

$\equiv (\sim p \wedge \sim q) \vee F$

$\equiv \sim p \wedge \sim q$

Que! Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

Ans!

$(p \wedge q) \rightarrow (p \vee q)$

$\equiv \sim(p \wedge q) \vee (p \vee q)$

$\equiv (\sim p \vee \sim q) \vee (p \vee q)$

$\equiv (\sim p \vee p) \vee (\sim q \vee q)$ (By associative and commutative law)

$\equiv T \vee T$

$\equiv T$

Que! Prove $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

Ans!

$(p \rightarrow q) \wedge (p \rightarrow r)$

$\equiv (\sim p \vee q) \wedge (\sim p \vee r)$

$\equiv (\sim p) \vee (q \wedge r)$

$\equiv p \rightarrow (q \wedge r)$

Que: Use the law of logic to show that
QB 132 $[(p \to q) \wedge \sim q] \to \sim p$ is a tautology.

Ans:

$[(p \to q) \wedge \sim q] \to \sim p$

$\equiv [(\sim p \vee q) \wedge \sim q] \to \sim p$

$\equiv [(\sim p \wedge \sim q) \vee (q \wedge \sim q)] \to \sim p$

$\equiv [(\sim p \wedge \sim q) \vee F] \to \sim p$

$\equiv (\sim p \wedge \sim q) \to \sim p$

$\equiv \sim(\sim p \wedge \sim q) \vee \sim p$

$\equiv \sim(\sim p) \vee \sim(\sim q) \vee \sim p$

$\equiv p \vee q \vee \sim p$

$\equiv q \vee p \vee \sim p$

$\equiv q \vee T$

$\equiv T$ (Tautology)

Que: Prove using logical equivalence.

$p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

Ans:

$p \leftrightarrow q$

$\equiv (p \to q) \wedge (q \to p)$ $[p \to q \equiv \sim p \vee q]$

$[p \wedge \sim p \equiv F]$

$\equiv (\sim p \vee q) \wedge (\sim q \vee p)$ $[p \vee F \equiv p]$

$\equiv [(\sim p \vee q) \wedge \sim q] \vee [(\sim p \vee q) \wedge p]$

$\equiv [(\sim p \wedge \sim q) \vee (q \wedge \sim q)] \vee [(\sim p \vee q) \wedge p]$

$\equiv [(\sim p \wedge \sim q) \vee F] \vee [(\sim p \vee q) \wedge p]$

$\equiv (\sim p \wedge \sim q) \vee [(\sim p \vee q) \wedge p]$

$\equiv (\sim p \wedge \sim q) \vee [F \vee (q \wedge p)]$

$\equiv (\sim p \wedge \sim q) \vee (q \wedge p)$

$\equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

Que: Show that $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology.

Ans: $[(p \to q) \wedge (q \to r)] \to (p \to r)$

$\equiv \sim [(p \to q) \wedge (q \to r)] \vee (p \to r)$

$\equiv \sim (p \to q) \vee \sim (q \to r) \vee (p \to r)$

$\equiv \sim (\sim p \vee q) \vee \sim (\sim q \vee r) \vee (\sim p \vee r)$

$\equiv (p \wedge \sim q) \vee (q \wedge \sim r) \vee (\sim p \vee r)$

$\equiv (p \wedge \sim q) \vee \{(q \wedge \sim r) \vee r\} \vee \sim p$

$\equiv \{(p \wedge \sim q) \vee \sim p\} \vee \{(q \vee r) \wedge (\sim r \vee r)\}$

$\{(p \vee \sim p) \wedge (\sim q \vee \sim p)\} \vee \{(q \vee r) \wedge T\}$

$\equiv \{T \wedge (\sim q \vee \sim p)\} \vee \{(q \vee r)\}$

$\equiv (\sim q \vee \sim p) \vee (q \vee r)$

$\equiv (\sim q \vee q) \vee \sim p \vee r$

$\equiv T \vee \sim p \vee r$

$\equiv T$

Que: Show that $p \to q$ and $\sim q \to \sim p$ are logically equivalent.

Ans: $p \to q \equiv \sim p \vee q$

$\equiv q \vee \sim p$

$\equiv \sim(\sim q) \vee \sim p$

$\equiv \sim q \to \sim p$

Que: Simplify $(q \wedge p) \vee \sim(q \to p)$

Ans: $(q \wedge p) \vee \sim(q \to p)$

$\equiv (q \wedge p) \vee \sim(\sim q \vee p)$ (Implication law $p \to q \equiv \sim p \vee q$)

$\equiv (q \wedge p) \vee (\sim(\sim q) \wedge \sim p)$ (De Morgan law)

$\equiv (q \wedge p) \vee (q \wedge \sim p)$ (Double Negation law)

$\equiv q \wedge (p \vee \sim p)$ (By Distributive law)

$\equiv q \wedge T$

$\equiv q.$

Que! Show that $\sim(p \vee \sim(p \wedge q))$ is a contradiction

Ans: $\sim(p \vee \sim(p \wedge q))$

$\equiv \sim p \wedge \sim(\sim(p \wedge q))$

$\equiv \sim p \wedge (p \wedge q)$

$\equiv (\sim p \wedge p) \wedge q$

$\equiv F \wedge q$

$\equiv F$

Que! Show that $(\sim r \wedge (r \vee s)) \rightarrow s$ is a tautology.

Ans: $(\sim r \wedge (r \vee s)) \rightarrow s$

$\equiv \sim(\sim r \wedge (r \vee s)) \vee s$

$\equiv (\sim(\sim r) \vee \sim(r \vee s)) \vee s$

$\equiv (r \vee (\sim r \wedge \sim s)) \vee s$

$\equiv ((r \vee \sim r) \wedge (r \vee \sim s)) \vee s$

$\equiv (T \wedge (r \vee \sim s)) \vee s$

$\equiv (r \vee \sim s) \vee s$

$\equiv r \vee (\sim s \vee s)$

$\equiv r \vee T$

$\equiv T$

Que: Show that $q \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$ is a tautology.

$\equiv q \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$

$\equiv q \vee ((p \wedge \sim p) \wedge \sim q)$

$\equiv q \vee (T \wedge \sim q)$

$\equiv q \vee \sim q$

$\equiv T$

Que: Obtain DNF of $p \vee (\sim p \rightarrow (q \vee (q \rightarrow \sim r)))$

Q. B-166

Ans: $p \vee (\sim p \rightarrow (q \vee (q \rightarrow \sim r)))$

$\equiv p \vee (\sim p \rightarrow (q \vee (\sim q \vee \sim r)))$

$\equiv p \vee (\sim (\sim p) \vee (q \vee (\sim q \vee \sim r)))$

$\equiv p \vee (p \vee (q \vee (\sim q \vee \sim r)))$

$\equiv p \vee p \vee q \vee \sim q \vee \sim r$

$\equiv \quad p \vee q \vee \sim q \vee \sim r$ (dnf)

$\equiv \quad p \vee (p \vee (T \vee \sim r))$

$\equiv \quad p \vee (p \vee T)$

$\equiv \quad T$

~~If you are ask to~~ show it is tautology then you need to do this steps.

Que: Obtain cnf of following.

$q \vee (p \wedge r) \wedge \sim [(p \vee r) \wedge q]$

Ans:

$[q \vee (p \wedge r)] \wedge \sim [(p \vee r) \wedge q]$

$\equiv [(q \vee p) \wedge (q \vee r)] \wedge [\sim (p \vee r) \vee \sim q]$

$\equiv [(q \vee p) \wedge (q \vee r)] \wedge [(\sim p \wedge \sim r) \vee \sim q]$

$\equiv (q \vee p) \wedge (q \vee r) \wedge [(\sim p \vee \sim q) \wedge (\sim r \vee \sim q)]$

$\equiv (q \vee p) \wedge (q \vee r) \wedge (\sim p \vee \sim q) \wedge (\sim r \vee \sim q)$

Que: Obtain conjunctive normal form of $(p \to q) \wedge (q \vee (p \wedge r))$

Q.B-168

Ans: $(p \to q) \wedge (q \vee (p \wedge r))$

$\equiv (\sim p \vee q) \wedge (q \vee (p \wedge r))$

$\equiv (\sim p \vee q) \wedge (q \vee p) \wedge (q \vee r)$ (cnf)

Que: Obtain DNF of $\sim(p \vee q) \leftrightarrow (p \wedge q)$

Ans:

$\sim(p \vee q) \leftrightarrow (p \wedge q)$ $[p \leftrightarrow q \Leftrightarrow (p \wedge q) \vee (\sim p \wedge \sim q)]$

$\Leftrightarrow (\sim(p \vee q) \wedge (p \wedge q)) \vee ((p \vee q) \wedge \sim(p \wedge q))$

$\cancel{\Leftrightarrow ((\sim p \wedge \sim q) \wedge (p \wedge q)) \vee ((p \vee q) \wedge (\sim p \vee \sim q))}$

$\equiv (\sim p \wedge \sim q \wedge p \wedge q) \vee [(p \vee q) \wedge \sim p] \vee [(p \vee q) \wedge \sim q]$

$\equiv (\sim p \wedge \sim q \wedge p \wedge q) \vee (p \wedge \sim p) \vee (q \wedge \sim p) \vee (p \wedge \sim q) \vee (q \wedge \sim q)$

Que: Obtain dnf of : $p \wedge \sim(q \wedge r)$

Ans: $p \wedge \sim(q \wedge r)$

$\equiv p \wedge (\sim q \vee \sim r)$

$\equiv (p \wedge \sim q) \vee (p \wedge \sim r)$

Que: obtain dnf of
$(p \rightarrow q) \wedge (q \rightarrow r)$

Ans:
$(p \rightarrow q) \wedge (q \rightarrow r)$

$\equiv (\sim p \vee q) \wedge (\sim q \vee r)$

$\equiv [(\sim p \vee q) \wedge \sim q] \vee [(\sim p \vee q) \wedge r]$

$\equiv (\sim p \wedge \sim q) \vee (q \wedge \sim q) \vee (\sim p \wedge r) \vee (q \wedge r)$

Que: Obtain dnf of
$p \rightarrow ((p \rightarrow q) \wedge \sim(\sim q \vee \sim p))$

Ans:
$\equiv p \rightarrow ((\sim p \vee q) \wedge (q \wedge p))$

$\equiv p \rightarrow ((\sim p \vee q) \wedge (p \wedge q))$

$\equiv p \rightarrow ((\sim p \wedge (p \wedge q)) \vee (q \wedge (p \wedge q)))$

$\equiv p \rightarrow [(F \wedge q) \vee (q \wedge p)]$

$\equiv p \rightarrow (F \vee (q \wedge p))$

$\equiv$ $p \rightarrow (p \wedge q)$

$\equiv$ $\sim p \vee (p \wedge q)$

$\equiv$ $(\sim p \wedge \sim p) \vee (p \wedge q)$

Que: Obtain dnf of $\sim (p \rightarrow (q \wedge r))$

QB-165

Ans: $\sim (p \rightarrow (q \wedge r))$

$\equiv$ $\sim (\sim p \vee (q \wedge r))$

$\equiv$ $\sim (\sim p) \wedge \sim (q \wedge r)$

$\equiv$ $p \wedge \sim (q \wedge r)$

$\equiv$ $p \wedge (\sim q \vee \sim r)$

$\equiv$ $(p \wedge \sim q) \vee (p \wedge \sim r)$

Que: Obtain cnf of $\sim (p \leftrightarrow q)$

Ans: $\sim (p \leftrightarrow q)$

$\equiv$ $\sim [(p \rightarrow q) \wedge (q \rightarrow p)]$ $(\because p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p))$

$\equiv$ $\sim [(\sim p \vee q) \wedge (\sim q \vee p)]$

$\equiv$ $\sim (\sim p \vee q) \vee \sim (\sim q \vee p)$

$\equiv [\sim(\sim p) \wedge \sim q] \vee [\sim(\sim q) \wedge \sim p]$

$\equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

$\equiv [(p \wedge \sim q) \vee q] \wedge [(p \wedge \sim q) \vee \sim p]$

$\equiv (p \vee q) \wedge (\sim q \vee q) \wedge (p \vee \sim p) \wedge (\sim q \vee \sim p)$

$\equiv (p \vee q) \wedge T \wedge T \wedge (\sim q \vee \sim p)$

$\equiv (p \vee q) \wedge (\sim q \vee \sim p)$

Que: Show that $q \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$ is a tautology.

Ans:

$q \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$

$\equiv q \vee [(p \vee \sim p) \wedge \sim q]$

$\equiv q \vee (T \wedge \sim q)$

$\equiv q \vee \sim q$

$\equiv T$

Que: Obtain cnf of following.

$q \vee (p \wedge r) \wedge \sim[(p \vee r) \wedge q]$

Ans:

$q \vee (p \wedge r) \wedge \sim[(p \vee r) \wedge q]$.

$\equiv (q \vee p) \wedge (q \vee r) \wedge (\sim p \vee \sim q) \wedge (\sim r \vee \sim q)$

Scanned by CamScanner

Que: Obtain cnf of $\sim(p \to (q \wedge r))$

Ans:

$\sim(p \to (q \wedge r))$

$\equiv \sim(\sim p \vee (q \wedge r))$

$\equiv \sim(\sim p) \wedge \sim(q \wedge r)$

$\equiv p \wedge (\sim q \vee \sim r)$

$\equiv (p \vee p) \wedge (\sim q \vee \sim r)$

Que: Obtain cnf of (find dnf)

Q.B 163

$(p \to (q \wedge r)) \wedge (\sim p \to (\sim q \wedge \sim r))$

$(p \to (q \wedge r)) \wedge (\sim p \to (\sim q \wedge \sim r))$

$\equiv (\sim p \vee (q \wedge r)) \wedge (p \vee (\sim q \wedge \sim r))$

$\equiv (\sim p \vee q) \wedge (\sim p \vee r) \wedge (p \vee \sim q) \wedge (p \vee \sim r)$.

Que: Obtain cnf of

$\sim((p \vee \sim q) \wedge \sim r)$

$\sim((p \vee \sim q) \wedge \sim r)$

$\equiv \sim(p \vee \sim q) \vee r$

$\equiv (\sim p \wedge q) \vee r$

$\equiv (\sim p \vee r) \wedge (q \vee r)$

Que: QB 163 find cnf and dnf of
$(p \to (q \wedge r)) \wedge (\sim p \to (\sim p \wedge \sim r))$ by truth table method.

Ans:

p	q	r	$\sim p$	$\sim r$	$q \wedge r$	(A) $p \to q \wedge r$	$\sim p \wedge \sim r$	(B) $\sim p \to (\sim p \wedge \sim r)$	$(A) \wedge (B)$
T	T	T	F	F	T	T	F	T	T
T	T	F	F	T	F	F	F	T	F
T	F	T	F	F	F	F	F	T	F
T	F	F	F	T	F	F	F	T	F
F	T	T	T	F	T	T	F	F	F
F	T	F	T	T	F	T	T	T	T
F	F	T	T	F	F	T	F	F	$\boxed{F}$
F	F	F	T	T	F	T	T	T	T

Consider only (T) from last column and choose corresponding values (T) from p, q, r as well as consider (F).

cnf : $(p' \vee q' \vee r) \wedge (p' \vee q \vee r') \wedge (p' \vee q \vee r) \wedge$
(F) $(p \vee q' \vee r') \wedge (p \vee q \vee r')$.

dnf: $(p \wedge q \wedge r) \vee (\sim p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge \sim r)$.
(T)

Que: QB 164 Find cnf and dnf of

$(p \leftrightarrow (q \vee r)) \to \sim p$ using truth table.

Ans:

p	q	r	$q \vee r$	$p \leftrightarrow (q \vee r)$	$\sim p$	$(p \leftrightarrow (q \vee r)) \to \sim p$
T	T	T	T	T	F	F
T	T	F	T	T	F	F
T	F	T	T	T	F	F
T	F	F	F	F	F	T
F	T	T	T	F	T	T
F	T	F	T	F	T	T
F	F	T	T	F	T	T
F	F	F	F	T	T	T

cnf: $(p' \vee q' \vee r') \wedge (p' \vee q' \vee r) \wedge (p' \vee q \vee r')$.
(F)

dnf: $(p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge q \wedge r) \vee (\sim p \wedge q \wedge \sim r)$
(T) $\vee\ (\sim p \wedge \sim q \wedge r) \vee (\sim p \wedge \sim q \wedge \sim r)$

Que: Using truth table method, find dnf and cnf
$(\sim p \to r) \wedge (p \leftrightarrow q)$

Ans:

p	q	r	$\sim p$	$\sim p \to r$	$p \leftrightarrow q$	$(\sim p \to r) \wedge (p \leftrightarrow q)$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	T	F	F
T	F	F	F	T	F	F
F	T	T	T	T	F	F
F	T	F	T	F	F	F
F	F	T	T	T	T	T
F	F	F	T	F	T	F

cnf: $(p' \vee q \vee r') \wedge (p' \vee q \vee r) \wedge (p \vee q' \vee r') \wedge$
(F) $(p \vee q' \vee r) \wedge (p \vee q \vee r)$

dnf: $(p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge r)$.
(T)

$\Rightarrow$ $(\sim p \to r) \wedge (p \leftrightarrow q)$

$\equiv$ $(\sim p \to r) \wedge (p \to q) \wedge (q \to p)$

$\equiv$ $(\sim(\sim p) \vee r) \wedge (\sim p \vee q) \wedge (\sim q \vee p)$

$\equiv$ $(p \vee r) \wedge (\sim p \vee q) \wedge (\sim q \vee p)$ (cnf)

Que: Obtain cnf of $\sim(p \leftrightarrow q)$

Ans: $\sim(p \leftrightarrow q)$

$\equiv$ $\sim[(p \to q) \wedge (q \to p)]$

$\equiv$ $\sim[(\sim p \vee q) \wedge (\sim q \vee p)]$

$\equiv$ $[\sim(\sim p \vee q) \vee \sim(\sim q \vee p)]$

$\equiv$ $(p \wedge \sim q) \vee (q \wedge \sim p)$

$\equiv$ $[p \vee (q \wedge \sim p)] \wedge [\sim q \vee (q \wedge \sim p)]$

$\equiv$ $[(p \vee q) \wedge (p \vee \sim p)] \wedge [(\sim q \vee q) \wedge (\sim q \vee \sim p)]$

$\equiv$ $[(p \vee q) \wedge T] \wedge [T \wedge (\sim q \wedge \sim p)]$ $\equiv (p \vee q) \wedge (\sim q \vee \sim p)$

* **ARGUMENT:** the given set of proposition is called premises (or hypothesis) and the proposition derived from this set is called conclusion.

* **Definition: argument:**

An argument is a process which yield a conclusion from a given set of propositions, called premises.

premises

$$P_1, P_2, P_3, ---, P_n \vdash q \leftarrow \text{conclusion}$$

promotions.

NOTE: Argument $P_1, P_2, P_3, ---, P_n \vdash q$ is said to be valid if q is true whenever all the premises are true.

* **Valid argument:** An argument $P_1, P_2, P_3, ---, P_n \vdash q$ is called valid if q is true whenever all its premises $P_1, P_2, ---, P_n$ are true.

* **Fallacy argument:** An argument which is not valid is said to be a fallacy or an invalid argument.

* **Representation of an argument:**

An argument $P_1, P_2, ---, P_n \vdash q$ is written as

$$\left.\begin{array}{l} P_1 \\ P_2 \\ \vdots \\ P_n \end{array}\right\} \text{(Premises)}$$
$$\overline{\quad q \quad} \text{(conclusion).}$$

Ex: Show that the following argument is valid.

$p \vee q$
$\sim p$
$\overline{\quad q \quad}$

Ans: truth table.

p	q	$\sim p$	$p \vee q$	$(p \vee q) \wedge \sim p$	$[(p \vee q) \wedge \sim p] \to q$
T	T	F	T	F	T
T	F	F	T	F	T
F	$\boxed{T}$	$\boxed{T}$	$\boxed{T}$	T	$\boxed{T}$
F	F	T	F	F	T

Since all enteries in last column are 'T' only.
therefore, $[(p \vee q) \wedge \sim p] \to q$ is tautology.
Hence, the given argument is valid.

Ex: Show that the argument $p, p \to q, q \to r \vdash r$ is valid.

Ans: By using truth table.

Construct truth table for the statement.

$[p \wedge (p \to q) \wedge (q \to r)] \to r$

p	q	r	$p \to q$	$q \to r$	$p \wedge (p \to q)$	$p \wedge (p \to q) \wedge (q \to r)$	$p \to r$
T	T	T	T	T	T	T	T
T	T	F	T	F	T	F	T
T	F	T	F	T	F	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	F	F	T
F	T	F	T	F	F	F	T
F	F	T	T	T	F	F	T
F	F	F	T	T	F	F	T

Since, last column contains only T's,
Hence, the given argument is valid.

<u>Ex:</u> Show that the argument:
$p, p \to q \vdash q$ is valid.

<u>Ans:</u> Construct truth table for the statement
$p \wedge (p \to q) \to q$.

p	q	$p \to q$	$p \wedge (p \to q)$	$[(p \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Since, last column contains only T's,
hence the given argument is valid.
(In both entries are true then conclusion must be true.)

Ex: check the validity of $p \to q, \sim p \vdash \sim q$

Ans: Construct truth table for the statement

$[(p \to q) \wedge \sim p] \to \sim q$.

p	q	$\sim p$	$p \to q$	$(p \to q) \wedge \sim p$	$\sim q$	$[(p \to q) \wedge \sim p] \to \sim q$
T	T	F	T	F	F	T
T	F	F	F	F	T	T
F	T	$\boxed{T}$	$\boxed{T}$	T	$\boxed{F}$	$\boxed{F}$
F	F	$\boxed{T}$	$\boxed{T}$	T	$\boxed{T}$	$\boxed{T}$

Not tautology.

∴ It is not valid.

Ex: check the validity $p \to \sim q, r \to q, r \vdash \sim p$.

Ans:

$[(p \to \sim q) \wedge (r \to q) \wedge r] \to \sim p$.

p	q	r	$\sim q$	$p \to \sim q$	$r \to q$	$(p \to \sim q) \wedge (r \to q)$ =f	(A) $f \wedge r$	(B) $\sim p$	$A \to B$
T	T	T	F	F	T	F	F	F	T
T	T	F	F	F	T	F	F	F	T
T	F	T	T	T	F	F	F	F	T
T	F	F	T	T	T	T	F	F	T
F	T	$\boxed{T}$	F	$\boxed{T}$	$\boxed{T}$	T	T	$\boxed{T}$	T
F	T	F	F	T	T	T	F	T	T
F	F	T	T	T	F	F	F	T	T
F	F	F	T	T	T	T	F	T	T

∴ It is valid.

Que: Test the validity of the following argument:

QB 172

If a man is a bachelor, he is worried. (premise)
If a man is worried, he dies young. (premise)

∴ Bachelors dies young. (Conclusion)

Ans:

Let p: A man (he) is a bachelor.
q: He is worried.
r: He dies young.

Symbolic form:

$$p \to q,\ q \to r \vdash p \to r.$$

p	q	r	$p \to q$ $=A$	$q \to r$ $=B$	$p \to r$ $=C$	$(p\to q)\wedge(q\to r)$ $A \wedge B = \textcircled{c}$	$[(p\to q)\wedge(q\to r)]\to p\to r$ $[(A\wedge B)\to C]$
T	T	T	$\boxed{T}$	$\boxed{T}$	$\boxed{T}$	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	$\boxed{T}$	$\boxed{T}$	$\boxed{T}$	T	T
F	T	F	T	F	T	F	T
F	F	T	$\boxed{T}$	$\boxed{T}$	$\boxed{T}$	T	T
F	F	F	$\boxed{T}$	$\boxed{T}$	$\boxed{T}$	T	T

Hence, argument is valid.

Ex: Test the validity of the following argument. "If it rains then it will be cold. If it is cold then I shall stay at home. Since it rains therefore I shall stay at home."

Ans: let p : It rains.
q : It will be cold.
r : I shall stay at home.

the given ~~formula~~ argument in symbolic form can be

$p \to q$ (a premises)
$q \to r$ (a premises)
p (a premises)
$\overline{\qquad}$
r (conclusion)

Construct table:

p	q	r	$p \to q$	$q \to r$	$(p \to q) \land (q \to r)$ = A	$A \land p$ = B	$B \to r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	F	T
F	T	F	T	F	F	F	T
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

Hence, the given argument is valid.

Que: Test the validity of the arguments:
QB 174
If two sides of a triangle are equal, then the opposite angles are equal. ~~two~~
two sides of a triangle are not equal.

∴ The opposite angles are not equal.

Ans:

Let p: Two sides of a triangle are equal.
q: The opposite angles of a triangle are equal.

Symbolic form: $p \to q$ (a premises)
$\sim p$ (a premises)
$\sim q$ (conclusion)

We can shall construct the truth table for the statement.

$$[(p \to q) \wedge \sim p] \to \sim q$$

p	q	$\sim p$	$\sim q$	$p \to q$	$(p \to q) \wedge \sim p$	$[(p \to q) \wedge \sim p] \to \sim q$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	$\boxed{T}$	$\boxed{F}$	$\boxed{T}$	T	F
F	F	$\boxed{T}$	$\boxed{T}$	$\boxed{T}$	T	T

Last column is not tautology.

Hence, the given argument is not valid.

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Que: State whether the argument given below is valid. It is valid, identify the tautology.

Q13
180

If I drive to work then I will arrive tired.
I drive to work.
∴ I will arrive tired.

Ans:

Let p : I drive to work.
q : I will arrive tired.

Symbolic form: $p \rightarrow q$ (a premises)
p (a premises)
q (conclusion)

the argument is
$p \rightarrow q, p \vdash q$

p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge p$	$[(p \rightarrow q) \wedge p] \rightarrow q$
$\boxed{T}$	$\boxed{T}$	$\boxed{T}$	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Hence, the given argument is valid.

Ex: Test the validity of the argument:

If 8 is even then 2 does not divide 9.
Either 7 is not prime or 2 divides 9.
But 7 is prime, therefore 8 is odd.

Ans:

let p : 8 is even
q : 2 divides 9.
r : 7 is prime.

Symbolic form: $p \to \sim q$ (a premises)
$\sim r \vee q$ (a premises)
r (a premises)
$\sim p$ (conclusion)

p	q	r	$\sim p$	$\sim q$	$\sim r$	$p \to \sim q$	$\sim r \vee q$	f	$f \to \sim p$
T	T	T	F	F	F	F	T	F	T
T	T	F	F	F	T	F	T	F	T
T	F	T	F	T	F	T	F	F	T
T	F	F	F	T	T	T	T	F	T
F	T	$\boxed{T}$	$\boxed{T}$	F	F	$\boxed{T}$	$\boxed{T}$	T	T
F	T	F	T	F	T	T	T	F	T
F	F	T	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T	F	T

Since last column contains only T's.
Hence, the given argument is valid.

Que: State whether the argument given below is valid or not valid. If it is valid identify tautology.

I will become famous or I will be writer
I will not be a writer.
∴ I will become famous.

Ans: Let p: I will become famous.
q: I will be writer.

Symbolic form:

$p \vee q$ (a premises)
$\sim q$ (a premises)
p (conclusion)

The given argument:
$[(p \vee q) \wedge \sim q] \rightarrow p$.

Construct the truth table.

p	q	$\sim q$	$p \vee q$	$(p \vee q) \wedge \sim q$	$[(p \vee q) \wedge \sim q] \rightarrow p$
T	T	T	F	F	T
$\boxed{T}$	F	$\boxed{T}$	$\boxed{T}$	T	T
F	T	T	F	F	T
F	F	F	T	F	T

Since last column contains only T's.

Hence, the given argument is valid.

Que: Test the validity of the argument:

QB 173

S_1: If 7 is less than 4 then 7 is not prime number.
S_2: 7 is not less than 4.

S : 7 is prime number.

Ans:

Here, p : 7 is less than 4.
q : 7 is prime number.

Symbolic form : $p \to \sim q, \sim p \vdash q$.

p	q	~p	~q	p → ~q	(p → ~q) ∧ ~p
T	T	F	F	F	F
T	F	F	T	T	F
F	$\boxed{T}$	$\boxed{T}$	F	$\boxed{T}$	T
F	$\boxed{F}$	$\boxed{T}$	T	$\boxed{T}$	T

Hence, the given argument is not valid.

Que: Determine the validity of argument given:

QB 175

If I study then I will not fail in DM,
If I do not play cricket, then I will study.
But I fail in DM.

Therefore, I must have play cricket.

Ans: let p : I study
q : I will fail in DM.

r : I play cricket.

Symbolic form : $p \to \sim q, \sim r \to p, q \vdash r$

Construct the truth table

p	q	r	$\sim q$	$\sim r$	$p \to \sim q$	$\sim r \to p$
T	T	T	F	F	F	T
T	T	F	F	T	F	T
T	F	T	T	F	T	T
T	F	F	T	T	T	T
F	$\boxed{T}$	$\boxed{T}$	F	F	$\boxed{T}$	$\boxed{T}$
F	T	F	F	T	T	F
F	F	T	T	F	T	T
F	F	F	T	T	T	F

Here, the premises $p \to \sim q$, $\sim r \to p$, and q at 5th entry is 'T'. So, to their corresponding conclusion r at 5th entry is also 'T'.

Hence, the argument is valid.

Que: Determine the validity of argument given:

S_1: If you work hard, then you pass the course.
S_2: You did not pass the course.

S : You did not work hard.

Ans:

Let p : If you work hard.
q : You pass the course.

Symbolic form:

$p \to q$ (a premises)
$\sim q$ (a premises)

$\sim p$ (conclusion)

p	q	~p	~q	p→q	(p→q)∧~q	[(p→q)∧~q]→~p
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	$\boxed{T}$	$\boxed{T}$	$\boxed{T}$	T	T

Hence, the argument is valid.

Que: Determine the validity of argument given:

QB 171

S_1: If I like mathematics I will study

S_2: Either I will study or I will fail.

S: If I fail then I do not like mathematics.

Ans:

Let p: I like mathematics.
q: I will study.
r: I will fail.

Symbolic form: $S_1: p \to q$
$S_2: q \vee r$
$S: r \to \sim p$

Construct the truth table.

p	q	r	~p	p→q	q∨r	r→~p
T	T	T	F	$\boxed{T}$	$\boxed{T}$	$\boxed{F}$
T	T	F	F	$\boxed{T}$	$\boxed{T}$	$\boxed{T}$
T	F	T	F	F	T	F
T	F	F	F	F	F	T
F	T	T	T	$\boxed{T}$	$\boxed{T}$	$\boxed{T}$
F	T	F	T	$\boxed{T}$	$\boxed{T}$	$\boxed{T}$
F	F	T	T	$\boxed{T}$	$\boxed{T}$	$\boxed{T}$
F	F	F	T	$\boxed{T}$	$\boxed{T}$	$\boxed{T}$

Here the premises $p \to q$ and $q \vee r$ at 1st entry all are 'T' but to their corresponding conclusion $r \to \sim p$ at 1st entry 'F'.

Hence, the given argument is not valid.

Que: Consider the following argument and determine whether it is valid or not.

Either I will get good marks or I will not graduate.
If I did not graduate I will go to U.S.A.
I get good marks.
Thus, I would not go to U.S.A

Ans:

Let p: I will get good marks.
q: I will graduate.
r: I will go to U.S.A

Symbolic form:

p	q	r	$\sim q$	$\sim r$	$p \vee \sim q$	$\sim q \to r$
$\boxed{T}$	T	T	F	$\boxed{F}$	$\boxed{T}$	$\boxed{T}$
$\boxed{T}$	T	F	F	$\boxed{T}$	$\boxed{T}$	$\boxed{T}$
$\boxed{T}$	F	T	T	$\boxed{F}$	$\boxed{T}$	$\boxed{T}$
T	F	F	T	T	T	F
F	T	T	F	F	F	T
F	T	F	F	T	F	T
F	F	T	T	F	T	T
F	F	F	T	T	T	F

Hence, the argument is not valid.