MAXIMIZATIONSPROBLEM:

$$Y = \max D(c_1, c_2 \dots c_N)$$

→ "constant-returns aggregator"*

→ REAL OUTPUT Y as a result of a max. process based on final consumption

$$\text{s.t.} \quad \sum_{i=1}^{N} p_i c_i = \sum_{f=1}^{F} w_f \bar{l}_f + \sum_{i=1}^{N} \bar{\pi}_i$$

EXPENDITURE APPROACH ← NOMINAL OUTPUT = NOMINAL OUTPUT → INCOME APPROACH

taking into account the following three definitions/conditions:

(1) $y_i = A_i F_i(l_{1i} \dots l_{Fi}, x_{i1} \dots x_{iN})$ → PRODUCTION FUNCTION, LATER MADE MORE CONCRETE BY CES-APPROACH

NOTATION: $X_{USE, SOURCE}$

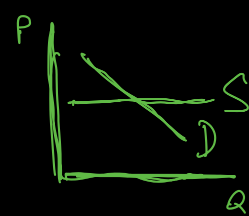
(2) $\bar{\pi}_i = p_i y_i - \sum_{f=1}^{F} w_f l_{if} - \sum_{j=1}^{N} p_j x_{ij}$

(3) $y_i = \sum_{j=1}^{N} x_{ji} + c_i$ AND $\bar{l}_f = \sum_{i=1}^{N} l_{if}$

* SOMETHING LIKE $c_1^{\alpha} \cdot c_2^{\beta} \cdot c_3^{\gamma}$ with $\alpha + \beta + \gamma = 1$

"We derive results in the vicinity of the steady-state, which we normalize to be $(A_1 \dots A_N) = (1, \dots, 1)$"

"We assume that the prod. function F_i of each good has constant returns to scale, which implies that equilibrium profits are 0."



"Efficient economy" → social planning problem → "applying the Envelope-Theorem delivers the result"

↳ identical result to profit/utility max.?!

$$\frac{d \log Y}{d \log A_i} = \lambda_i \quad \text{OK}$$

$$\frac{TS_i}{BIP \neq FS}$$

$$\frac{FS_i}{FS} = \Theta \quad \neq$$

MAX. → GE ↔ $1 - \Theta$

MAPPING SEM?

GÜTERPREISE

Ω

1st ORDER

B&F → 2000

1978

HELL

LOG-LINEAR.

HULTEN-TH.

PROOF THEOREM #1 (HULTEN, APPENDIX) → AGAIN: VERWEIS AUF 1st WELFARE THEOREM

$$Y(A_1 \dots A_n) = \max D(c_1, \dots, c_n) + \sum_i \mu_i \left(A_i F_i((l_{ij})_j, (x_{ij})_j) - \sum_j x_{ij} - c_i\right) + \sum_i \lambda_i \left(\bar{l}_i - \sum_j l_{ji}\right)$$

FROM (1) & (3)

← ONLY FROM (3)

ENVELOPE THEOREM?

SIMPLE TO SEE, BUT WHY? (ENVELOPE!)

$$\frac{dY}{dA_i} = \mu_i F_i((l_{ij})_j, (x_{ij})_j) = \mu_i y_i \qquad \frac{dY}{dA_i} = \mu_i y_i$$

↳ Would also make more sense with $\bar{L}_F$ & L_{jF} in my view...

↳ Eigentlich muss hier doch x_{ji} stehen, weil wir wollen doch eher die Gesamtmenge an Inputs die Sektor i produzieren muss?

↳ $\bar{L}_i$ is „the endowment of each labor type" ←

WENN $\mu_i = p_i$ dann stimmt ja im Grunde der, dass

$$\boxed{\frac{dY}{dA_i} = \text{TOTAL SALES}_i}$$

$$\frac{d \log Y}{d \log A_i} = \lambda_i \qquad \frac{d\log Y}{d\log A_i} = \lambda_i$$

→ Argues that only thing left to show is that $\mu_i = p_i$

↳ WIE KOMMT MAN VON HIER AUF HULTEN?

$$\frac{dY}{dA_i} = Y \frac{d\log Y}{d\log A_i}$$

Out of FOC of planner's problem ✓

$$\mu_i \cdot \frac{\partial F_i}{\partial x_{ij}} = \mu_j$$

$$\frac{\partial D}{\partial c_j} = \mu_j \Rightarrow \text{HENCE } \mu = p \quad \forall i$$

$$\boxed{\frac{\partial Y}{\partial x_{ij}} = \mu_i \cdot \frac{\partial F_i}{\partial x_{ij}} - \mu_j = 0}$$

Competitive Equilibrium ✓

$$p_i \frac{\partial F_i}{\partial x_{ij}} = p_j$$

$$p_c \frac{\partial D}{\partial c_j} = p_j$$

↳ NUMERAIRE!

$$Y \uparrow \qquad \sum c_i$$

$$\frac{dY}{dA_i} = TS_i$$

$$Y \frac{d\log Y}{d\log A_i} = TS_i$$

OFFEN: WAS GEHT DA GENAU AB?

$$\frac{d \log Y}{d\log A_i} = \frac{TS_i}{Y} =$$

OFFEN: ?

$Y = \sum p_i c_i$

↓

REAL VS. NOMINAL

$$\frac{TS_i}{\sum(p_i c_i)} = \lambda_i$$

(ALT): [illegible]

WEITERE ELEMENTE AUS PAPER

$$\frac{\sum INT + \sum FS}{\sum FS}$$

DEFINITION #1: PSEUDO-ELASTICITY OF SUBSTITUTION

DEFINITION #2: INPUT-OUTPUT-MULTIPLIER:

$$\xi = \sum_{i=1}^{N} \frac{d \log Y}{d \log (A_i)} = \sum_{i=1}^{N} \lambda_i$$

CHANGES IN ξ ARE INTERPRETED AS A GE-ELASTICITY BETW. FACTORS & GOODS.

$\xi \downarrow$ GDP, $\xi \uparrow$ SALES

$$\Rightarrow \lambda_i = \frac{FS_i}{\sum FS} \approx \frac{INT_i + FS_i}{\sum FS_i}$$

THEOREM #2: 2^{nd}-ORDER MACRO-IMPACT OF MICRO SHOCKS

$\rho \to$ GE-ELAST. OF SUBSTITUTION

ELASTICITY OF INPUT-OUTPUT MULTIPL.

$$\frac{d^2 \log Y}{d \log A_i^2} = \frac{d \lambda_i}{d \log A_i} = \underbrace{\frac{\lambda_i}{\xi} \sum_{1 \le j \le N, j \neq i} \lambda_j \left(1 - \frac{1}{\rho_{ji}}\right)}_{\text{„CHANGE IN SHARE OF SALES IN OTHER SECTORS j"}} + \underbrace{\lambda_i \frac{d \log \xi}{d \log A_i}}_{\text{„CHANGE IN SALES-TO-GDP RATIO"}}$$

INTUITION: 2^{nd} ORDER CHANGE IS IMPACT OF ΔA_i ON λ_i?

↳ UNSURE WHETHER THIS REALLY HELPS??

→ FROM THIS THE FULL IMPACT CAN BE DERIVED VIA A TAYLOR APPROXIMATION (p. 1163-1164)

↳ MORISHIMA ELASTICITY IS PUZZLING

↳ SEE p. 1161 AT THE BOTTOM!

$$\frac{1}{\rho_{ji}} = \frac{d \log (MRS_{ji})}{d \log A_i} - \frac{d \log \left(\frac{f_j}{f_i}\right)}{d \log A_i} = \frac{d \log \left(\frac{\partial u / \partial A_j}{\partial u / \partial A_i}\right)}{d \log A_i}$$

DEFINITION #3: LEONTIEF

$$\Omega_{ij} = \frac{P_j X_{ij}}{P_i Y_i} \quad (= \Omega^* \text{ UNTEN})$$

USE → SOURCE

i's reliance on j as supplier

$$\Psi_{ij} = (I - \Omega_{ij})^{-1}$$

→ PAPER SPEAKS ABOUT $(N+1+F)$ SECTORS

HOUSEHOLDS, FACTORS

↳ NOT SURE HOW THIS MAPS UNTO THE CODE

DEFINITION #4: INPUT-OUTPUT COVARIANCE-OPERATOR

→ DOESN'T SEEM TOO CRUCIAL ALTHOUGH THEY SAY IT IS :)

→ MAYBE DIG DEEPER?

$$\frac{d \log Y}{d \log A_i} = \lambda_i + \frac{d \log \lambda_i}{d \log A_i}$$

$$e^{\frac{\Omega}{(I - (1-\alpha)\Omega_{int})}} \qquad \log A \quad \frac{\lambda}{P} \qquad = (P, Q)$$

MAXIMIERUNGSPROBLEM IM CODE

```
parfor k = 1:trials
            A = exp(mvnrnd(-1/2*diag(Sigma),diag(diag(Sigma))))';
            A = exp(mvnrnd(-1/2*diag(Sigma_4year),diag(diag(Sigma_4year))))';
            init = [exp(-inv(eye(N)-diag(1-alpha)*Omega)*log(A));(beta'*inv(eye(N)-↙
diag(1-alpha)*Omega))'./exp(-inv(eye(N)-diag(1-alpha)*Omega)*log(A))];  % judicious↙
choice of starting values

            % pick solver type
            %[Soln,~,exitfl] = knitromatlab(@(X) trivial(X),init,[],[],[],[],[],[], @↙
(X)Simulation_Derivs(X,  A, beta, Omega, alpha, epsilon, theta, sigma,L),[],↙
[],'Knitro_options.opt');
            [Soln,~,exitfl] = fmincon(@(X) trivial(X),init,[],[],[],[],[],[], @(X)↙
Simulation_Derivs(X,  A, beta, Omega, alpha, epsilon, theta, sigma,L), optionsf);

            if exitfl == 1 || exitfl == 2% solver no error (fmincon)
            %if exitfl == 0% solver no error (knitro)
                GDP(k) = (Soln(1:N).*(A.^((epsilon-1)/epsilon)).*(alpha.^(1/epsilon)).↙
*(Soln(N+1:2*N).^(1/epsilon)).*(1./L).^(1/epsilon))'*L;
                lambda_simul(:,k) = Soln(1:N).*Soln(1+N:end)/GDP(k);
            end
            parfor_progress;

end
```

INITIALISIERUNG ← (1)

(2) → AUFRUF SOLVER

(3) ↓ BERECHNUNG GDP

(A)

S?

ES GIBT ZWEI RELEVANTE CODE-DOKUMENTE: EIN FILE MIT ALLEN BERECHNUNGEN, DAS DEN OPTIMIZER AN EINER STELLE AUFRUFT (AUSZUG A) UND EIN FILE MIT DER OPTIMIERUNG (B)

→ MEIN VERSTÄNDNIS WÄRE, DASS DER OPTIMIZER EIN „AUKTIONATOR" IST, ALSO PASSENDE PREIS-/MENGEN KOMBINATIONEN FÜR ALLE SEKTOREN SUCHT.

(2) IST EINFACH DER AUF DES SOLVERS

(1) IST IN MEINEM VERSTÄNDNIS EIN VEKTOR MIT 2N ELEMENTEN: ZUERST DIE PREISE FÜR ALLE SEKTOREN i, DANN DIE MENGEN MIT $i = (1 \dots N)$

→ 2 ELEMENTE: $\left(P, \frac{P\cdot Q}{P} = Q\right) = (P, Q)$

IM CODE STECKEN ALSO FORMULIERUNGEN FÜR P & P·Q:

GILT WOHL AUCH HIER....

$$P_i = e^{-(I-(1-\alpha_i)\cdot\Omega)^{-1}\cdot\log(A_i)} = e^{-\underline{(I-\Omega^*)^{-1}}\cdot\log(A_i)}$$

→ TECH IN SECTOR i ($A_i = 1$ IN STEADY STATE $\forall i$)

↳ LEONTIEF-INVERSE!

$$\vec{PQ} = \vec{\beta}\cdot(I-(1-\vec{\alpha})\cdot\Omega)^{-1} = \underline{\vec{\beta}}\cdot\underline{(I-\Omega^*)^{-1}} = \underline{\vec{\lambda}}$$

DAS MUSS MAN WOHL ALS VEKTOR ANSCHREIBEN WEIL SONST DIE MULTIPLIKATION NICHT KLAPPT?

SHARE OF i IN ALL FINAL SALES

MULTIPLIER FOR INTERMEDIATE GOODS OF i USED IN FULL ECONOMY

$\frac{p_i y_i}{\sum_{i=1} p_i c_i}$

↳ „DOMAR WEIGHTS"

↓

TOTAL PRODUCTION IN MONEY TERMS RELATIVE TO GDP (= FINAL SALES)

$$\sum_{i=1}^{N} \lambda_i = \frac{\sum_{i=1}^{N} p_i y_i}{\sum_{i=1}^{N} p_i c_i} = \frac{\text{TOTAL SALES}}{\text{FINAL SALES}}$$

⇒ DER AUSDRUCK FÜR P & Q MACHT FÜR MICH SINN, P NICHT.

(3) HABE ICH (NOCH) NICHT WIRKLICH DURCHBLICKT:

$$GDP = \vec{p}\cdot\vec{A}^{\frac{\epsilon-1}{\epsilon}}\cdot\vec{\alpha}^{\frac{1}{\epsilon}}\cdot\vec{y}^{\frac{1}{\epsilon}}\cdot\left(\frac{1}{L}\right)^{\frac{1}{\epsilon}}\cdot L$$

→ ELASTICITY FACTOR FOR INTERMEDIATE VS. FACTORS IN PRODUCTION

INTERMEDIATE VS. FACTORS (~ 0.5) — PRODUCTION (NAME 0) — KONSUM (NAME 1) — TECHNOLOGY SHOCKS — VALUE ADDED = TOTAL OUTPUT; INCOME / OUTPUT; — INCOME SHARE — LABOR — SHARE OF TOTAL FINAL SALES ($\Sigma = 1$) — INPUTKOEFF.-MATRIX (WIEVIEL € INPUT PRO € OUTPUT?) — DER VON Θ & ϵ INVERS zu PAPER — SUBSTITUTIONS-ELASTIZITÄTEN — EINLESEN INITIAL VECTORS

```
function [outineq, Out, outineq2, OutDeriv]=Simulation_Derivs(X,A, beta, Omega, alpha,↙
epsilon, theta, sigma,L) % no reallocation of labor
N = length(alpha);
p = X(1:N);
y = X(N+1:2*N);
q = (Omega*(p.^(1-theta))).^(1/(1-theta));
w = p.*(A.^((epsilon-1)/epsilon)).*(alpha.^(1/epsilon)).*(y.^(1/epsilon)).*(1./L).^↙
(1/epsilon);
C = w'*L;
```

(1) MENGEN, LÖHNE, KONSUM

$$y^{\frac{1}{\epsilon}} = q^{\frac{1}{\epsilon}} \cdot \alpha$$

A aus LEONTIEF-INVERSE (I-A)

```
Out(1:N) = p - (diag(A)^(epsilon-1)*(alpha.*w.^(1-epsilon)+(1-alpha).*q.^(1-↙
epsilon))).^(1/(1-epsilon));
Out(N+1:2*N) = y' -y'*diag(p)^epsilon*diag(A)^(epsilon-1)*diag(q)^(theta-epsilon)*diag↙
(1-alpha)*Omega*diag(p)^(-theta) - beta'*diag(p)^(-sigma)*C;
Out = Out';
outineq = [];
outineq2 = [];
```

SPUR!

(2) ANPASSUNG VON PREISEN & MENGEN: NÄCHSTER SCHRITT (??)

```
DQDP = bsxfun(@times, (q.^theta), (p.^(-theta))').*Omega; %
DWDP = diag((A.^((epsilon-1)/epsilon)).*(alpha.^(1/epsilon)).*(y.^(1/epsilon)).*(1.↙
/L).^(1/epsilon)); %checked
DWDY = (1/epsilon)*diag(p.*(A.^((epsilon-1)/epsilon)).*(alpha.^(1/epsilon)).*(y.^↙
(1/epsilon-1)).*(L).^(-1/epsilon)); %checked
DCDP = DWDP'*L; %checked
DCDY = DWDY'*L;%checked
DOut1DP = eye(N) - diag(diag(A)^(-1)*((alpha.*(w.^(1-epsilon))+(1-alpha).*(q.^(1-↙
epsilon)))).^(epsilon/(1-epsilon)))*...
    (diag(alpha)*diag(w.^(-epsilon))*DWDP+diag(1-alpha)*diag(q.^(-epsilon))*DQDP);

DOut1DY =  -diag(diag(A.^(-1))*((alpha.*(w.^(1-epsilon))+(1-alpha).*(q.^(1-↙
epsilon)))).^(epsilon/(1-epsilon)))*...
    (diag(alpha)*diag(w.^(-epsilon))*DWDY);

DOut2DP = -(epsilon * diag(p.^(-theta))*Omega'*diag((p.^(epsilon-1)).*(y).*(q.^(theta-↙
epsilon)).*(1-alpha).*(A.^(epsilon-1)))...
    +(theta-epsilon)*diag(p.^(-theta))*Omega'*diag((p.^(epsilon)).*(y).*(q.^(theta-↙
epsilon-1)).*(1-alpha).*(A.^(epsilon-1)))*DQDP ...
    -sigma*diag(beta.*p.^(-sigma-1))*C+ bsxfun(@times, beta.*(p.^(-sigma)), DCDP')...
    - theta* diag(p.^(-theta-1)).*diag(Omega'*diag((p.^(epsilon)).*(q.^(theta-↙
epsilon)).*(1-alpha).*(A.^(epsilon-1)))*y));

DOut2DY = eye(N) - (diag(p)^epsilon*diag(A)^(epsilon-1)*diag(q)^(theta-epsilon)*diag↙
(1-alpha)*Omega*diag(p)^(-theta))' - bsxfun(@times, beta.*(p.^(-sigma)), DCDY');
OutDeriv = [DOut1DP DOut1DY; DOut2DP DOut2DY]';

end
```

(3) ABLEITUNGEN (??)

K σ H F Θ S H A I F ϵ

(B)

DIESER TEIL FEHLT NOCH...

(1)
$$\vec{q} = \left(\Omega \cdot \vec{p}^{\,1-\Theta}\right)^{\frac{1}{1-\Theta}}$$
SUBST. in PRODUCTION

$$\vec{w} = \vec{p}\,\vec{A}^{\frac{\epsilon-1}{\epsilon}}\,\alpha^{\frac{1}{\epsilon}}\,y^{\frac{1}{\epsilon}}\,\frac{1}{L}^{\frac{1}{\epsilon}} = \frac{GDP}{L}$$

↳ SIEHT AUS WIE GDP IN (*) (3)

$$\vec{c} = w \cdot L$$

(2) DEFINE:
$$MA = \begin{pmatrix} A_1 & 0 & 0 & 0 \\ 0 & A_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & A_n \end{pmatrix}$$

$$MP = \begin{pmatrix} p_1 & 0 & 0 & 0 \\ 0 & p_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & p_n \end{pmatrix} \qquad MQ = \ldots$$

$\Rightarrow$

$$\vec{p}\,' = \vec{p} - \left(MA^{\epsilon-1} \cdot \vec{\alpha} \cdot \vec{w}^{(1-\epsilon)} (1-\vec{\alpha}) \cdot \vec{q}^{\,1-\epsilon}\right)^{\frac{1}{1-\epsilon}}$$

$$\vec{y}\,' = \vec{y} - \underline{\vec{y}\, MP^{\epsilon}\, MA^{\epsilon-1}\, MQ^{\Theta-\epsilon}} \cdot \Omega^{*} \cdot \underline{MP}^{-\Theta} - \underline{\beta\, MP^{-\sigma}\, C}$$

↳ IMPACT FROM CONSUMPTION

IMPACT FROM PRODUCTION

(3)
$$DQDP = \frac{\partial Q}{\partial P} = \vec{q}^{\,\Theta} \cdot \vec{p}^{\,-\Theta} \circ \Omega$$

??

$$DWDP = \frac{\partial w}{\partial p} = MA^{\frac{\epsilon-1}{\epsilon}} \cdot \vec{\alpha}^{\frac{1}{\epsilon}} \cdot \vec{y}^{\frac{1}{\epsilon}} \cdot \frac{1}{L}^{\frac{1}{\epsilon}}$$

$$DWDY = \frac{1}{\epsilon}\,\vec{p}\,A^{\frac{\epsilon-1}{\epsilon}}\,\alpha^{\frac{1}{\epsilon}}\,y^{\frac{1}{\epsilon}-1} \cdot L^{\left(-\frac{1}{\epsilon}\right)}$$

IRGENDWIE SCHON SCHRÄG AUFGESCHRIEBEN ...

$$\left.\begin{aligned} DCDP &= DWDP' \cdot L \\ DCDY &= DWDY \cdot L \end{aligned}\right\}$$
HIER WIRD EINFACH DER IMPACT AUF DIE LÖHNE HOCHSKALIERT ?

↳ ABER ES SIEHT WIRKLICH AUS WIE ABLEITUNGEN AUS (B) (1)

$$DOut1DP = I - Diag\left(MA^{-1} \cdot \left(\alpha \cdot w^{1-\epsilon} + (1-\alpha)\, q^{1-\epsilon}\right)^{\frac{\epsilon}{1-\epsilon}} \ldots\right.$$

$$Diag(\alpha) \cdot Diag(w^{-\epsilon}) \cdot DWDP + Diag(1-\alpha)\, Diag(q^{-\epsilon}) \cdot DQDP$$

SIEHT ZUMINDEST AM ANFANG AUS WIE EINE ABLEITUNG VON p'... → WAS DANN WEITER ABGEHT IST MIR SCHON WENIGER KLAR

VERMUTLICH NUTZLOS – NICHT ANSEHEN!

(1) $y_i = A_i F_i(l_{1i} \dots l_{Fi}, x_{i1} \dots x_{iN})$

(2) $\pi_i = p_i y_i - \sum_{f=1}^{F} w_f l_{if} - \sum_{j=1}^{N} p_j x_{ij}$

$\pi_i = p_i A_i F_i(\dots) - \sum_{f=1}^{F} w_f l_{if} - \sum p_j x_{ij}$

$\frac{\partial \pi_i}{\partial x_i}$

BEDARF AN VORLESUNGEN → $-(I-A)^{-1}_i$?

$$\boxed{\frac{\partial \pi}{\partial y_i} = p_i - \sum_{f=1}^{F} w_f \frac{\partial l_{if}}{\partial y_i} - \sum_{j=1}^{N} p_j \frac{\partial x_{ij}}{\partial y_i} = 0} \Leftrightarrow$$

$$\frac{d \log p_i}{d \log A_i} = \frac{\log p_i - \cancel{\log \bar{p}}}{\log A_i - \cancel{\log \bar{A}}} = \text{EXPRESSION}$$

$p = (I-A)^{-}$

$\log p_i =$ EXPRESS $\cdot \log(A_i)$

$p_i = e^{-(I-(1-\alpha_i)\cdot\Omega)^{-1} \cdot \log(A_i)}$

$p_i = \sum^{F} w_f \frac{\partial l_{if}}{\partial y_i} + \sum_{j=1}^{N} p_j \frac{\partial x_{ij}}{\partial y_i}$ → ⊕ ZAHLEN ABER IN $(I-A)^{-1}$ STEHEN

⊖ ZAHLEN

↓

OQ: WAS IST MIT DIAGONALE a_{ii} ?

(WAS NEUES ENTSTEHT?)

$p_i = -(I-A)^{-1}_i$

$p_i = e^{\text{EXPRESS} \cdot \log A_i}$

$p_i \frac{d \log p_i}{d \log A_i} = \frac{d p_i}{d A_i}$

$\frac{d \log p_i}{d \log A_i} = \frac{\log p_i}{\log A_i} =$

⇒ PROFIT MAX. MIT ∄ NB??

AUS WELFARE MAX PROBLEM?

SAUBER AUFSCHREIBEN!

⇒ ODER: p = AVERAGE COSTS?

$p = \frac{\text{COST}}{Y_i} = \frac{\text{COST}}{A F_i}$? TOTAL TRANSFER

$\frac{dp}{dA} = p \cdot \frac{d \log p_i}{d \log A_i} =$

$\frac{\partial p}{\partial A} = -\text{COST} \cdot \frac{1}{F_i}$

↳ oben gleich y_i

$p = \frac{d \log p}{d \log A} = p \cdot \frac{\log p}{\log A} = \frac{-\text{COST}}{Y_i} = \frac{-\text{COST}}{p_i Y_i} = \frac{\log p}{\log A}$

$\pi_i = p_i \cdot (A_i F_i) - \sum w_f l_{if} - \sum p_j x_{ij}$

$\frac{\partial \pi_i}{\partial x_i} = p_i \frac{\partial (A_i F_i)}{\partial x_i} - \sum w_f \frac{\partial l_{if}}{\partial x_i} - \sum p_j \frac{\partial x_{ij}}{\partial x_i} = 0$

$= p_i A_i \frac{\partial F_i}{\partial x_i} - ((I-\Omega)^{-1}) = 0$

$p_i - (I-\Omega)^{-1})_i$

$\log(y) = \lambda_i + \log(A_i)$

$\log(\hat{p}) + \log(\hat{c}) = \lambda_i \cdot \log(A_i)$

$\log(\hat{p}) = \lambda_i \cdot \log(A_i) - \log(\hat{c})$

$$(1)\quad Y_i = A_i F_i \left(l_{1i} \dots l_{Fi},\ x_{i1} \dots x_{iN} \right)$$

$$(2)\quad \Pi_i = p_i Y_i - \sum_{f=1}^{F} w_f l_{if} - \sum_{j=1}^{N} p_j x_{ij}$$

$$\Pi_i = p_i A_i F_i - \sum_{f=1}^{F} w_f l_{if} - \sum_{j=1}^{N} p_j \left(A_j F_j - c_j - \sum_{k \neq i} x_{kj} \right)$$

$$\Pi_i = p_i Y_i - \sum_{f=1}^{F} w_f l_{if} - \sum_{j=1}^{N} p_j x_{ij}$$

$$\vec{\Pi} = \begin{pmatrix} Y_1 & & & \\ & Y_2 & & \\ & & \ddots & \\ & & & Y_N \end{pmatrix} \cdot \vec{p} - M_L \times \vec{w}_F - X \cdot \vec{p}$$

$$\Pi + M_L \vec{w}_F = \left(\begin{pmatrix} Y & & \\ & \ddots & \\ & & Y \end{pmatrix} - X \right) \cdot \vec{p}$$

$$\vec{p} = \left(\Pi + M_L \cdot \vec{w}_F \right) \cdot \left(\begin{pmatrix} Y & & \\ & \ddots & \\ & & Y \end{pmatrix} - X \right)^{-1}$$

$$\vec{p} = \vec{v} \cdot (I - A)^{-1} \ ?$$

$$p_i = f\left[I - \Omega^{*} \right]^{-1}_{i}$$

$$\frac{\partial \Pi_i}{\partial Y_i} = p_i - \sum_{f=1}^{F} w_f \frac{\partial l_{if}}{\partial y} - \sum_{j=1}^{N} p_j \frac{\partial A_j F_j}{\partial y_i} = 0$$

$$\rightarrow p_i = -\left(I - \Omega^{A} \right)^{-1}_{i}$$

$$\left(p_i \frac{d \log p_i}{d \log A_i} = \frac{\partial p_i}{\partial A_i} \right)$$

$$\frac{d \log p_i}{d \log A_i} = \frac{\log p_i}{\log A_i} = \frac{\log p_i}{\log A_i} =$$

$$\frac{d \log Y}{d \log A_i} = \lambda_i = \frac{\sum_j p_i x_{ji} + p_i c_i}{\sum p_i c_i}$$

$$\frac{\partial p_i}{}$$

TEST? 2x2 FALL!

$$Y_1 = A_1 F_1 \left(L_1, x_{11}, x_{12} \right) \qquad Y_2 = A_2 F_2 \left(L_2, x_{21}, x_{22} \right)$$

$$Y_1 = x_{11} + x_{21} + C_1 \qquad Y_2 = x_{12} + x_{22} + C_2$$

$$\Pi_1 = p_1 Y_1 - w L_1 - p_1 \cdot x_{11} - p_2 \cdot x_{12} \qquad \Pi_2 = p_2 Y_2 - w \cdot L_2 - p_1 \cdot x_{21} - p_2 x_{22}$$

WIE SEHEN DIESE TERME AUS?

$$\frac{\partial \Pi_1}{\partial y_1} = p_1 - w \frac{\partial L_1}{\partial y_1} - p_1 \frac{\partial x_{11}}{\partial y_1} - p_2 \frac{\partial x_{21}}{\partial y_1}$$

$$Y_1 = A_1 \left(L_1^{1/3} \cdot x_{11}^{1/3} \cdot x_{12}^{1/3} \right)$$

$$x_{11} = \left(\frac{Y_1}{A L_1^{1/3} \cdot x_{12}^{1/3}} \right)^3 = \frac{Y_1^3}{A^3 L_1 x_{12}}$$

$$Y = \max D(c_1, c_2 \dots c_N)$$

$$(0) \quad \text{s.t.} \sum_{i=1}^{N} p_i c_i = \sum_{f=1}^{F} w_f \bar{L}_f + \sum \pi_i$$

$$(1) \quad Y_i = A_i F_i (L_{1i} \dots L_{Fi}, x_{i1} \dots x_{iN})$$

$$(2) \quad \pi_i = p_i Y_i - \sum_{f=1}^{F} w_f L_{if} - \sum_{j=1}^{N} p_j x_{ij}$$

$$(3) \quad Y_i = \sum_{j=1}^{N} x_{ji} + c_i \quad \text{AND} \quad \bar{L}_f = \sum_{i=1}^{N} L_{if}$$

SO STEHT DAS PROBLEM IM PAPER

VEREINFACHUNG

$N=2$

$F=1$

ANSATZ # 1: SETZE ALLES EINFACH IN NEBENBEDINGUNG EIN

$$\max.\ W = c_1^{1/3} \cdot c_2^{2/3}$$ → STATT DEM ABSTRAKTEN „D" HIER EINE KONKRETE FUNKTION D

$$\text{s.t.} \quad p_1 c_1 + p_2 c_2 = \sum_i wL_i + \sum \pi_i = wL_1 + wL_2 + \pi_1 + \pi_2$$

$$L = c_1^{1/3} \cdot c_2^{2/3} - \lambda \left(p_1 c_1 + p_2 c_2 - \sum_i wL_i - \sum_i \pi_i\right) \quad (2)$$

$$L = c_1^{1/3} \cdot c_2^{2/3} - \lambda \left(p_1 c_1 + p_2 c_2 - \cancel{wL_1} - \cancel{wL_2} - p_1 Y_1 - p_2 Y_2 + \cancel{wL_1} + \cancel{wL_2} + p_1 x_{11} + p_1 x_{21} + p_2 x_{12} + p_2 x_{22}\right)$$

HIER SIEHT MAN (3)

$$L = c_1^{1/3} \cdot c_2^{2/3} - \lambda \left(p_1 (c_1 - Y_1 + x_{11} + x_{21}) + p_2 (c_2 - Y_2 + x_{12} + x_{22})\right)$$

↓ JETZT MIT (1).

WAS IST MIT 2. NB? PREISE?

$$L = c_1^{1/3} \cdot c_2^{2/3} - \lambda \Big(p_1 \big(c_1 - A_1 F_1 (L_1, x_{11}, x_{12}) + x_{11} + x_{21}\big) + p_2 \big(c_2 - A_2 F_2 (L_2, x_{21}, x_{22}) + x_{12} + x_{22}\big)\Big)$$

NACHDEM ICH SO ALLES UNTERBRINGE AUSSER DIE ZWEITE DEFINITION BEI (3) WÜRDE ICH DAFÜR NOCH EINE NB HINZUFÜGEN?

$$\rightarrow L = c_1^{1/3} \cdot c_2^{2/3} - \lambda_1 \Big(p_1 \big(c_1 - A_1 F_1(\dots) + x_{11} + x_{12}\big) + p_2 \big(c_2 - A_2 F_2(\dots) + x_{21} + x_{22}\big)\Big) + \lambda_2 \left(\bar{L} - L_1 - L_2\right)$$

⇒ DAS KANN MAN NUN NACH c_i, x_{ij}, L_i ABLEITEN → 8 FIRST ORDER CONDITIONS, DIE VERMUTL. ÄHNLICH AUSSEHEN WIE IM PAPER ANGEDEUTET

⇒ AUCH DAS MAX.-PROBLEM SIEHT ÄHNLICH AUS WIE IM PAPER (WOBEI DA KEINE PREISE IN DER GLEICHUNG AUFZUTAUCHEN WAS ABER WURSCHT IST DA $p_i > 0 \ \forall \ i$.

ANSATZ #2: STARTE MIT VERGLEICH VON (0) UND (2) OBEN & DANN SCHAU MAL WEITER

TAKING THE SUM OF ALL DEFS IMPLIED BY (2) GIVES: $\sum_i^N \Pi_i = \sum_i^N p_i y_i - \sum_i^N \omega L_i - \sum_i^N \sum_j^N p_j x_{ij} \Leftrightarrow$

PLUGGING THIS $\leftarrow$ $\sum_i \Pi_i = \sum_i p_i c_i + \sum_i p_i \sum_j x_{ji} - \sum_i \omega L - \sum_i \sum_j p_j x_{ij}$ * $(0 = \dots)$

INTO (0) → $\sum p_i c_i - \omega \bar{L} = \sum_i \Pi_i$

GIVES US: $\sum \cancel{p_i c_i} - \cancel{\omega \bar{L}} = \sum_i \cancel{p_i c_i} + \sum_i \cancel{p_i} \sum_j^N x_{ji} - \cancel{\sum \omega L_i} - \sum_i \sum_j \cancel{p_j x_{ij}}$

WIR SEHEN: (0) & (2) SIND ÄQUIVALENT – DA WIR ANNEHMEN DASS $\Pi_i = 0 \ \forall \ i$ WIRD DAS AUTOMATISCH HALTEN – DAS FOLGT AUS MEINER SICHT AUS DER GLEICHUNG MIT DEM GELBEN STERN. VON DAHER BRAUCHEN WIR NUR (1) & (3) BERÜCKSICHTIGEN. FÜR $N = 2$ ERGIBT SICH:

$$W = c_1^{1/3} \cdot c_2^{2/3} \quad \text{s.t.} \quad \begin{aligned} &(1)\ y_1 - c_1 - x_{11} - x_{21} = 0 \longrightarrow \Leftrightarrow \\ &(2)\ y_2 - c_2 - x_{21} - x_{22} = 0 \\ &(3)\ \bar{L} - L_1 - L_2 = 0 \end{aligned} \quad \Bigg| \begin{aligned} &A_1 F_1(\dots) - c_1 - \\ &x_{11} - x_{21} \end{aligned}$$

$$\begin{aligned} L = c_1^{1/3} \cdot c_2^{2/3} &+ \lambda_1 \left(A_1 F_1 (x_{11}, x_{12}, L_1) - c_1 - x_{11} - x_{21} \right) \\ &+ \lambda_2 \left(A_2 F_2 (x_{21}, x_{22}, L_2) - c_2 - x_{12} - x_{22} \right) \\ &+ \lambda_3 \left(\bar{L} - L_1 - L_2 \right) \end{aligned}$$

⇒ DAS SIEHT AUS MEINER SICHT NUN AUS WIE OBEN / IM PAPER, (WOBEI IM PAPER WOHL WIRKLICH EIN INDEX FALSCH IST) & SOLLTE ÄQUIVALENT ZU ANSATZ 1 SEIN. –

$$\frac{\partial L}{C_1} = \frac{1}{3} \cdot \left(\frac{C_2}{C_1}\right)^{\frac{2}{3}} - \lambda_1 = 0$$

$$\frac{\partial L}{x_{11}} = \lambda_1 \left(A_1 \frac{\partial F_1}{\partial x_{11}}\right) - \lambda_1 = 0$$

$$\frac{\partial L}{x_{12}} = \lambda_1 \left(A_1 \frac{\partial F_1}{\partial x_{12}}\right) - \lambda_2 = 0$$

$$\frac{\partial L}{\partial x_{21}} = \lambda_2 \left(A_2 \frac{\partial F_2}{\partial x_{21}}\right) - \lambda_1 = 0$$

$$\lambda_1 \left(\left(A_1 \frac{\partial F_1}{\partial x_{11}}\right) - \cancel{Q}_1 1\right) = 0$$

$$-\left(1 - A_1 \frac{\partial F_F}{\partial x_{11}}\right) = 0$$

$$\lambda_1 \left(A_1 \frac{\partial F_1}{\partial x_{12}}\right) = \lambda_2$$

$$\lambda_1 = \lambda_2 \, (A_2 f.)$$

$$1$$

$$\left(A_1 \frac{\partial F_1}{\partial x_{12}}\right) = \left(A_2 \frac{\partial F_2}{\partial x_{21}}\right)$$

$$p_1 = \left(A_2 \frac{\partial F_2}{\partial x_{21}}\right) \cdot \cancel{A_2}\, p_2$$

$$\frac{p_1}{p_2} = \left(A_2 \frac{\partial F_2}{\partial x_{21}}\right) = 1 \quad \frac{d p_1}{d A}$$

$$\frac{\partial p_1}{\partial A}$$

PROFITMAXIMIERUNG IM 2-SEKTORENFALL?

$Y_1 = A_1 F_1 (L_1, X_{11}, X_{12})$ NOCHMAL WER? $Y_2 = A_2 F_2 (L_2, X_{21}, X_{22})$

$Y_1 = X_{11} + X_{21} + C_1$ $\qquad$ $Y_2 = X_{12} + X_{22} + C_2$

$\Pi_1 = p_1 Y_1 - w L_1 - p_1 \cdot X_{11} - p_2 \cdot X_{12}$ $\qquad$ $\Pi_2 = p_2 Y_2 - w \cdot L_2 - p_1 \cdot X_{21} - p_2 X_{22}$

$$\frac{\partial \Pi_1}{\partial y_1} = p_1 - w \frac{\partial L_1}{\partial y_1} - p_1 \frac{\partial X_{11}}{\partial y_1} - p_2 \frac{\partial X_{12}}{\partial y_1}$$

WIE SEHEN DIESE TERME AUS? (1)

MAL Π NACH A ABLEITEN? (2)

$$Y_1 = A_1 \left(L_1^{1/3} \cdot X_{11}^{1/3} \cdot X_{12}^{1/3}\right)$$

?? $\qquad$ $\frac{\partial X_{12}}{\partial Y_1} = Y_2$

$$X_{11} = \left(\frac{Y_1}{A_1 L_1^{1/3} X_{12}^{1/3}}\right)^3 = \frac{Y_1^3}{A_1^3 L_1 X_{12}}$$

GEHT NICHT

$$\frac{\partial X_{11}}{\partial Y_1} = \frac{(\cdot 3)\, Y_1^2}{A_1^3 L_1 X_{12}}$$

$$\frac{\partial \Pi_1}{\partial y_1} = p_1 - w \frac{3 y_1^2}{A^3 X_{11} X_{12}} - p_1 \frac{3 y_1^2}{A^3 X_{12} L_1} - p_2 \frac{3 y_1^2}{A^3 \cdot X_{11} \cdot L_1} = 0$$

$$\frac{3 \cdot A_1^2 \cdot L_1^{2/3} \cdot X_{11}^{2/3} \cdot X_{12}^{2/3}}{A_1^3 \cdot X_{11} \cdot X_{12}}$$

$\downarrow$

$$\frac{3 \cdot L_1^{2/3}}{A \cdot X_{11}^{1/3} \cdot X_{12}^{1/3}}$$

$$p_1 \left(1 - \frac{3 y_1^2}{A^3 X_{12} L_1}\right) = w \frac{3 y_1^2}{A^3 X_{11} X_{12}} \Bigg| - p_2 \frac{3 y_1^2}{A^3 X_{11} \cdot L_1}$$

$$p_1 = \frac{w \dfrac{3 y_1^2}{A^3 X_{12} L_1} - p_2 \dfrac{3 y_1^2}{A^3 \cdot X_{11} \cdot L_1}}{\left(1 - \dfrac{3 y_1^2}{A^3 X_{12} L_1}\right)}$$

$$p_1 = w \cdot \frac{3 \cdot L_1^{2/3}}{A_1 X_{11}^{1/3} \cdot X_{12}^{1/3}} - p_2 \frac{3 \cdot X_{12}^{2/3}}{A_2 L_1^{1/3} \cdot X_{11}^{1/3}}$$

$$Y_1 = A_1 F_1 (L_1, x_{11}, x_{12}) \qquad Y_2 = A_2 F_2 (L_2, x_{21}, x_{22})$$

$$Y_1 = x_{11} + x_{21} + C_1 \qquad Y_2 = x_{12} + x_{22} + C_2$$

$$\Pi_1 = p_1 Y_1 - w L_1 - p_1 \cdot x_{11} - p_2 \cdot x_{12} \qquad \Pi_2 = p_2 Y_2 - w \cdot L_2 - p_1 \cdot x_{21} - p_2 x_{22}$$

$$\frac{\partial \Pi_1}{\partial y_1} = p_1 - w \frac{\partial L_1}{\partial y_1} - p_1 \frac{\partial x_{11}}{\partial y_1} - p_2 \frac{\partial x_{12}}{\partial y_1}$$

$$x_{11} = Y_1 - x_{21} - C_1$$

$$x_{11} = A_1 F_1 - x_{21} - C_1$$

$$(I - A)^{-1}$$

$$p_1 \left(1 - \frac{\partial x_{11}}{\partial y_1}\right) - w \frac{\partial L_1}{\partial y_1} - p_2 \frac{\partial x_{12}}{\partial y_1} = 0$$

$$p_1 = \frac{w \frac{\partial L_1}{\partial y_1} + p_2 \frac{\partial x_{12}}{\partial y_1}}{\left(1 - \frac{\partial x_{11}}{\partial y_1}\right)}$$

was fehlt?

$$\frac{\partial \Pi_2}{\partial y_2} = p_2 - w \frac{\partial L_2}{\partial y_2} - p_1 \frac{\partial x_{21}}{\partial y_2} - p_2 \frac{\partial x_{22}}{\partial y_2}$$

$$p_2 \left(1 - \frac{\partial x_{22}}{\partial y_2}\right) - w \frac{\partial L_2}{\partial y_2} - p_1 \frac{\partial x_{21}}{\partial y_2}$$

$$\max \Pi_1 = p_1 y_1 - w L_1 - p_1 x_{11} - p_2 x_{12}$$

$$\text{st.} \quad Y_1 = A_1 F_1 \geq x_{11} + x_{21} + C_1$$

$$p_2 = \frac{\partial Y_1}{\partial x_{12}} p_1$$

$$\frac{\partial \Pi_1}{\partial A_1} ?$$

$$(\ln f(x))' = \frac{f'(x)}{f(x)}$$

$$p_i \quad \frac{d \log p}{d \log A} = \frac{d p}{d A}$$

$$\Pi_i \quad \frac{d \log \Pi_i}{d \log y_i} = \frac{d \Pi}{d y_i}$$

$$\frac{(I - A)^{-1} \cancel{?} - w \frac{\partial L}{\partial y_i}}{p_1 Y_1 - wL - p_1 x_{11} - p_2 x_{12}}$$

PROFIT FUNCTION : $\pi_i = p_i y_i - \sum_{f=1}^{F} w_f l_{if} - \sum_{j=1}^{n} p_j x_{ij} = 0$ → OPTIMUM!

$$\Leftrightarrow \quad p_i = \frac{\sum w_f l_{if} + \sum p_j x_{ij}}{y_i} = \frac{\sum w_f l_{if} + \sum p_j x_{ij}}{A_i F_i}$$

THIS IS AVERAGE COSTS, NO?

FUCK UP!

$$\Rightarrow \quad \frac{\partial p_i}{\partial A_i} = - \frac{\left(\sum w_f l_{if} + \sum p_j x_{ij}\right)}{\cancel{y_i}} = AVC$$

→ $F_i = y_i$ MACHEN SIE AUCH IM PAPER / PROOF DORT

↓

WARUM?

$$\Leftrightarrow \quad p_i \cdot \frac{d \log p_i}{d \log A_i} = AVC \quad \Rightarrow$$

$$p_i \, \frac{\log p_i - \underbrace{\cancel{\log \bar{p}_i}}_{=0}}{\log A_i - \underbrace{\cancel{\log \bar{A}_i}}_{=0}} \Rightarrow \log p_i = - \frac{\left(\sum w_f l_{if} + \sum p_j x_{ij}\right)}{p_i y_i} \cdot \log A_i$$

$$\Rightarrow \log p_i = -(I - \Omega)_i^{-1} \cdot \log A_i$$

$$p_i = e^{-(I-\Omega)_i^{-1} \cdot \log A_i}$$

↳ MIR IST NICHT GANZ KLAR, WO DER ~~DER~~ 1er ~~HER~~ KOMMT FÜR DAS $(I - \Omega)$ ....

$$p_i y_i = \sum w l + \sum_{j \neq i} p_j x_{ij} + p_i x_{ii}$$

$$p_i y_i - p_i x_{ii} = \sum w L + \sum p_j x_{ij}$$

$$p_i (y_i - x_{ii}) = \sum w L + \sum p_j x_{ij}$$

$$p_i = \quad \ldots \quad \log p_i = - \frac{\sum w_f l_{if} + \sum p_j x_{ij}}{p_i y_i - p_i x_{ii}} \cdot \log A_i$$

PROFIT FUNCTION : $\pi_i = p_i y_i - \sum_{f=1}^{F} w_f l_{if} - \sum_{j=1}^{n} p_j x_{ij}$

$$\frac{\partial \pi_i}{\partial y_i} = p_i - \sum w_f \frac{\partial l_{if}}{\partial y_i} \cdot A_i - \sum p_j \frac{\partial x_{ij}}{\partial y_i} \cdot A_j = 0$$

$l_f = ?$

$$\frac{\partial p_i}{\partial A_i} = \sum w_f \frac{\partial l_{if}}{\partial y_i} + \sum p_j \frac{\partial x_{ij}}{\partial y_i}$$

$$p_i \frac{d \log p_i}{d \log A_i} \Rightarrow \frac{\sum w_f \frac{\partial l_{if}}{\partial y_i} + \sum p_j \frac{\partial x_{ij}}{\partial y_i}}{p_i} = \frac{\log p_i}{\log A_i}$$

$\cancel{y_i = A_i F_i(l, x)}$

$\cancel{l = \frac{y_i}{A_i} f(x)}$

$$\pi_i = p_i y_i - p_i x_{ii} - \sum w_f l_{if} - \sum_{j \neq i}^{n} p_j x_{ij}$$

$$\frac{\partial \pi_i}{\partial y_i} = p_i - p_i \frac{\partial x_{ii}}{\partial y_i} - \text{—}\sim\text{—}$$

$$p_i \left(1 - \frac{\partial x_{ii}}{\partial y_i}\right) - \text{—}|\text{—} = 0$$

$$p_i = \frac{\frac{\sum w_f l_{if}}{\partial y} \cdot A + \frac{\sum p_j x_{ij}}{\partial y} \cdot A}{\left(1 - \frac{\partial x_{ii}}{\partial y_i}\right) \cdot \cancel{A} \cdot A} + \frac{\left(1 - \frac{\partial x_{ii}}{\partial y}\right) \cdot A}{\left(1 - \frac{\partial x_{ii}}{\partial y}\right) \cdot A}$$

$$\Pi_i = p_i y_i - \sum_{f=1}^{F} w_f l_{if} - \sum_{j=1}^{n} p_j x_{ij}$$ → RICARDO? ODER NUR AND. SEKT.??

$$\Leftrightarrow \quad \Pi_i = p_i c_i + \sum_{j=1}^{n} p_i x_{ji} - \sum w l - \sum_{j=1}^{n} p_j x_{ij}$$

$$\Leftrightarrow \quad \Pi_i = p_i c_i + \sum_{j \neq i} p_i x_{ji} - \sum w L - \sum_{j \neq i} p_j x_{ij}$$

DEMAND FROM OTHER SECTORS ⇒ DOES NOT CHANGE

$$\frac{\partial \Pi_i}{\partial y_i} = p_i \frac{\partial c_i}{\partial y_i} \cdot A_i - \sum w \frac{\partial L}{\partial y_i} \cdot A_i - \sum_{j \neq i} p_j \frac{\partial x_{ij}}{\partial y_i} \cdot A_i = 0$$

$$\frac{\partial \Pi_i}{\partial y_i} = p_i - p_i \frac{\partial x_{ii}}{\partial y_i} \cdot A_i$$

$$\frac{\partial \Pi_i}{\partial y_i} = p_i \frac{\partial c_i}{\partial y_i} - \sum w \frac{\partial L}{\partial y_i} - \sum_{j \neq i} p_j \frac{\partial x_{ij}}{\partial y_i} = 0$$

$$\frac{\partial \Pi_i}{\partial y_i} = p_i - p_i \frac{\partial x_{ii}}{\partial y_i} - \sum w \frac{\partial L}{\partial y_i} - \sum p_j \frac{\partial x_{ij}}{\partial y_i} = 0$$

$$p_i \left(1 - \frac{\partial x_{ii}}{\partial y_i}\right)$$

VERH. D. ABS. Δ

VERH. D. REL. Δ

$$\frac{\partial y}{\partial x} = \frac{y}{(x=1)} \qquad \frac{d \log y}{d \log x}$$

$$\pi_i = p_i y_i - \sum_{f=1}^{F} w_f l_{if} - \sum_{j=1}^{n} p_j x_{ij}$$

$$\frac{\partial \pi_i}{\partial y_i} = p_i - \sum w_f \frac{\partial l_{if}}{\partial y_i} - \sum p_j \frac{\partial x_{ij}}{\partial y_i} = 0$$

$$\Leftrightarrow \frac{\partial \pi_i}{\partial y_i} = p_i - \sum w_f \frac{1}{A_i \frac{\partial F}{\partial L}} - \sum p_j \frac{1}{A_i \frac{\partial F}{\partial x_{ij}}} = 0$$

$$\Leftrightarrow p_i = \sum w_f \frac{1}{A_i \frac{\partial F}{\partial L}} + \sum p_j \frac{1}{A_i \frac{\partial F}{\partial x_{ij}}}$$

$$\Leftrightarrow \frac{\partial p_i}{\partial A_i} = -\left(\sum w_f \frac{1}{A^2 \frac{\partial F}{\partial L}} + \sum p_j \frac{1}{A_i^2 \frac{\partial F}{\partial x_{ij}}}\right)$$

$y = A \cdot F(L, \vec{x})$

$$\frac{\partial y}{\partial x_{ij}} = A \cdot \frac{\partial F}{\partial x_{ij}} \Leftrightarrow$$

$$\frac{\partial x_{ij}}{\partial y_i} = \frac{1}{A \cdot \frac{\partial F}{\partial x_{ij}}}$$

$\Updownarrow$

GENERAL:

$$dy = \frac{\partial y}{\partial x} \cdot dx \Leftrightarrow$$

$$1 = \frac{\partial y}{\partial x} \cdot \frac{dx}{dy} \quad \Big| : \frac{\partial y}{\partial x}$$

$$\frac{dx}{dy} = \frac{1}{\frac{\partial y}{\partial x}}$$

LABOR SCHRAUBEN

$i \to$ AUTOS

$$\frac{\partial p_i}{\partial A_i} = p_i \frac{d \log p_i}{d \log A_i} = p_i \frac{\log p_i - \log \bar{p}_i}{\log A_i - \log \bar{A}_i}$$

$$\Leftrightarrow p_i \frac{\log p_i - \log \bar{p}_i}{\log A_i - \log \bar{A}_i} = -\left(\sum w_f \frac{1}{A^2 \frac{\partial F}{\partial L}} + \sum p_j \frac{1}{A_i^2 \frac{\partial F}{\partial x_{ij}}}\right)$$

EVALUATE IN VICINITY OF STEADY STATE WITH $\bar{p} = \bar{A} = 1$

$5 \cdot 1 + 1 \cdot 5 = 55$ ← $10 \leftarrow a_L = 5 \leftarrow \frac{1}{0{,}2}$ $\quad 5 \cdot a_S = 1 \leftarrow \frac{1}{1}$

$$\Leftrightarrow p_i \frac{\log p_i - \cancel{\log \bar{p}_i}}{\log A_i - \cancel{\log \bar{A}_i}} = -\left(\sum w_f \frac{1}{\cancel{A^2} \frac{\partial F}{\partial L}} + \sum p_j \frac{1}{\cancel{A_i^2} \frac{\partial F}{\partial x_{ij}}}\right)$$

z.B. $\frac{\partial F}{\partial L} = 0.5 \downarrow a_L = 2$

$a_L^\epsilon = 0{,}5 \quad \frac{50}{100} \quad \frac{5}{100} \quad a_S^\epsilon = 0{,}05$ ↳ TECHN. KOEFFIZIENTEN

MONETARISIERTE KOEFFIZIENTEN

$$\frac{\log p_i}{\log A_i} = -\left(\sum \frac{w_f}{p_i} \frac{1}{\frac{\partial F}{\partial L}} + \sum \frac{p_j}{p_i} \frac{1}{\frac{\partial F}{\partial x_{ij}}}\right)$$

$$\log p_i = -\left(\sum \frac{w_f}{p_i} \frac{1}{\frac{\partial F}{\partial L}} + \sum \frac{p_j}{p_i} \frac{1}{\frac{\partial F}{\partial x_{ij}}}\right) \cdot \log(A)$$

- $\frac{Y}{\bar{Y}}$?
- 2 SEKTORENFALL NUMERISCH
- CES - F. PAPER CODE
- MATH - REPLICATION

$\Downarrow$? WO KOMMEN DIE EINSER HER? BRAUCHEN WIR DIE EINSER ÜBERHAUPT?

$$p_i = e^{\left[-(I-\Omega)^{-1}\right]_i \cdot \log(A_i)}$$

$$\frac{\partial F_i}{\partial x_{ij}} > 1 \;??\; \text{(EIGENE PRODUKTION + MARGIN (IMPACT?))}$$

$$\vec{p} = e^{\left[-(I-\Omega)^{-1}\right] \cdot \log(\vec{A})}$$

$$\log\left(\frac{Y}{\bar Y}\right) \approx$$

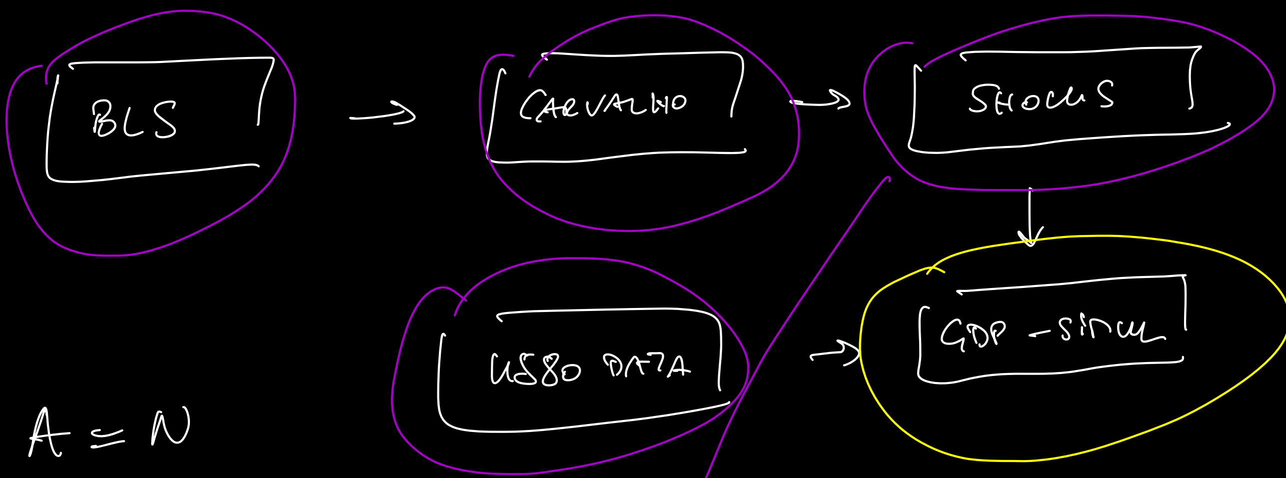
$$p_i = w \cdot \frac{1}{A_i \cdot \frac{\partial F}{\partial L}} + p_i \frac{1}{A_i \frac{\partial F}{\partial x_{ii}}} + p_j \frac{1}{A_i \frac{\partial F}{\partial x_{ij}}}$$

$$p_i \left(1 - \frac{1}{A_i \frac{\partial F}{\partial x_{ii}}}\right) = w \cdot \frac{1}{A_i \frac{\partial F}{\partial L}} + p_j \frac{1}{A_i \frac{\partial F}{\partial x_{ij}}},$$

$$\log Y - \log \bar Y \approx d \log Y$$

$$\frac{Y}{\bar Y} \to \cancel{d}.\ d \log Y$$

$$\hookrightarrow$$



$A \approx N$

$\hookrightarrow A = Y = f(K, L) \cdot A$

ELAST	PAPER	CODE
KONS.	Ø	Ø
PROJ.	∈	~~Ø~~
FACTUS. INTERLED.	~~Ø~~	∈

$GDP = W \cdot L$

$GDP = p \cdot C$

$p \cdot C$ $\quad w \cdot L$

W $\quad \cdot L$

DEMAND = OUTPUT — INCOME

→ WARUM KANN MAN DAS SO AUFSCHREIBEN? IST DAS P∘Q?

$$GDP = \vec{p} \cdot A^{\frac{\epsilon-1}{\epsilon}} \cdot \alpha^{\frac{1}{\epsilon}} \cdot y^{\frac{1}{\epsilon}} \cdot \left(\frac{1}{L}\right)^{\frac{1}{\epsilon}} \cdot L$$

→ ELASTICITY FACTOR FOR INTERMEDIATE VS. FACTORS IN PRODUCTION

$$\vec{p} \qquad \frac{\epsilon-1}{\epsilon}-1 \Leftrightarrow \frac{\epsilon-1}{\epsilon}-\frac{\epsilon}{\epsilon} \Leftrightarrow \left(-\frac{1}{\epsilon}\right) \qquad \frac{C}{L}\circ L$$

$$\frac{\partial GDP}{\partial A} = \vec{p} \cdot \frac{\epsilon-1}{\epsilon} A^{\left(-\frac{1}{\epsilon}\right)} \circ \alpha^{\frac{1}{\epsilon}} \cdot \vec{y}^{\frac{1}{\epsilon}} \circ \left(\frac{1}{L}\right)^{\frac{1}{\epsilon}} =$$

$$\vec{p} \cdot \frac{\epsilon-1}{\epsilon}\left(\frac{\alpha \cdot \vec{y} \cdot \frac{1}{L}}{A}\right)^{\left(+\frac{1}{\epsilon}\right)}$$

INCOME SHARE IN TOTAL PRODUCTION

ENTSPRICHT ENDNACHFR- AUS DEM JEWEILIGEN SEKTOR!

$$\vec{W} = \vec{p}\, \vec{A}^{\frac{\epsilon-1}{\epsilon}}\, \alpha^{\frac{1}{\epsilon}}\, y^{\frac{1}{\epsilon}}\, \frac{1}{L}^{\frac{1}{\epsilon}} = \frac{GDP}{L}$$

↳ SIEHT AUS WIE GDP IN (A) ③

$C = w \cdot L$ → wie GDP!
$\Leftrightarrow \vec{C}$

WIKIPEDIA?
& CES ← NUTZEN / VERBRAUCHE / PRODUKTION

$$W = \frac{GDP}{L}$$

Solow: $Q = F\cdot\left[a\cdot K^{\rho} + (1-a)\cdot L^{(1-\rho)}\right]^{\frac{v}{\rho}}$

gel. to scale

$w \circ L = GDP$

$$\frac{\epsilon-1}{\epsilon}\, z = \beta \cdot \left[\alpha_1 x_1^{-\rho}, \alpha_2 x_2^{-\rho} \dots \alpha_n x_n^{-\rho}\right]^{-\frac{n}{\rho}}$$

meist set $\beta = \rho = 1$

works for $0<\rho<1$ & $\rho > (-1)$

$$Y_i = A_i\left(w_{iL}\cdot L_i^{\frac{\epsilon-1}{\epsilon}} + (1-w_{iL})(X)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \quad \Big|\ ^{\frac{\epsilon-1}{\epsilon}} \Rightarrow y^{\frac{1}{\epsilon}}$$

$$Y_i^{\frac{\epsilon-1}{\epsilon}} = A_i^{\frac{\epsilon-1}{\epsilon}}\left(w_{iL}\cdot L_i^{\frac{\epsilon-1}{\epsilon}} + (1-w_{iL})(X)^{\frac{\epsilon-1}{\epsilon}}\right) \qquad A_i(\dots)^{\frac{1}{\epsilon-1}}$$

↳ $\rho = \frac{\sigma-1}{\sigma}$ ↗ →1 → linear; → →0 → Cobb; ↘ $-\infty$ → Leontief

$$\frac{\partial Y_i}{\partial L_i} = WAGE = A_i\,\frac{\epsilon}{\epsilon-1}\cdot(\dots)^{\frac{1}{\epsilon-1}}\cdot\left(w_{iL}\,\frac{\epsilon-1}{\epsilon}\,L_i^{-\frac{1}{\epsilon}}\right)\, y^{\frac{1}{\epsilon}} \qquad y^{\frac{1}{\epsilon}}\cdot\alpha\cdot\frac{1}{L}^{\frac{1}{\epsilon}}$$

$$\cancel{\frac{\epsilon}{\epsilon-1}}\cdot A_i\cdot(\ )^{\frac{1}{\epsilon-1}}\,\cancel{\frac{\epsilon-1}{\epsilon}}\,\alpha\cdot\frac{1}{L}^{\frac{1}{\epsilon}}$$

③ $DQDP = \frac{\partial Q}{\partial P} = \vec{q}^{\,\Theta} \cdot \vec{p}^{\,-\Theta} \circ \Omega$

??

$$DWDP = \frac{\partial w}{\partial P} = MA^{\frac{\epsilon-1}{\epsilon}} \cdot \vec{\alpha}^{\frac{1}{\epsilon}} \cdot \vec{y}^{\frac{1}{\epsilon}} \cdot \frac{1}{L}^{\frac{1}{\epsilon}}$$

$$DWDY = \frac{1}{\epsilon}\,\vec{p}\,A^{\frac{\epsilon-1}{\epsilon}}\,\alpha^{\frac{1}{\epsilon}}\,y^{\frac{1}{\epsilon}-1}\cdot L^{\left(-\frac{1}{\epsilon}\right)}$$

IRGENDWIE SCHON SAUBER AUFGESCHRIEBEN ...

$DCDP = DWDP' \circ L$
$DCDY = DWDY \cdot L$

HIER WIRD EINFACH DER IMPACT AUF DIE LÖHNE HOCHSKALIERT ?

↳ ABER ES SIEHT WIRKLICH AUS WIE ABLEITUNGEN AUS (B) ①

$$DOut1DP = I - Diag\left(MA^{-1}\cdot\left(\alpha\cdot w^{1-\epsilon} + (1-\alpha)\,q^{1-\epsilon}\right)^{\frac{\epsilon}{1-\epsilon}} \dots\right.$$

$$Diag(\alpha)\cdot Diag(w^{-\epsilon})\cdot DWDP + Diag(1-\alpha)\,Diag(q^{-\epsilon})\cdot DQDP$$

↓ SIEHT ZUMINDEST AM ANFANG AUS WIE EINE ABLEITUNG VON p'... → WAS DANN WEITER ABGEHT IST MIR SCHON WENIGER KLAR

$$Y_i = A_i \left(w_{iL} \cdot L^{\frac{\epsilon-1}{\epsilon}} + (1-w_{iL}) X^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}$$

$$A = A^{\left(\frac{\epsilon-1}{\epsilon}\right)\left(\frac{\epsilon}{\epsilon-1}\right)} \leftarrow$$

LAGRANGE? (1. Bd.)

$$p_i Y_i = \left(A_i^{\frac{\epsilon-1}{\epsilon}} \cdot w_{iL} \cdot L^{\frac{\epsilon-1}{\epsilon}} + A_i^{\frac{\epsilon-1}{\epsilon}} (1-w_{iL}) X^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}$$

CES → GDP, W, P

$$\frac{\partial Y_i}{L} = \text{WAGE} = \frac{\cancel{\epsilon}}{\cancel{\epsilon-1}} \left(\ldots \right)^{\frac{1}{\epsilon-1}} \cdot \left(A_i^{\frac{\epsilon-1}{\epsilon}} \cdot \frac{\cancel{\epsilon-1}}{\cancel{\epsilon}} \cdot L^{-\frac{1}{\epsilon}} \cdot w_{iL} \right)$$

KÖNNTE MAN WOHL HINWEGLASSEN

$$Y^{\frac{1}{\epsilon}} \cdot A^{\frac{\epsilon-1}{\epsilon}} \cdot L^{-\frac{1}{\epsilon}} \cdot \underbrace{w_{iL}}_{=\alpha}$$

(?) → EXP. FEHLT!

$$GDP = \tilde{p} \cdot \tilde{A}^{\frac{\epsilon-1}{\epsilon}} \cdot \alpha^{\frac{1}{\epsilon}} \cdot \tilde{Y}^{\frac{1}{\epsilon}} \cdot \left(\frac{1}{L}\right)^{\frac{1}{\epsilon}} \cdot L$$

?

θL

WAGE

$p?$

$w_{iL}^{\frac{1}{\epsilon}} \cdot L$

$Y = C + X$

$GDP = p \cdot C = pY - pX$

p_i

α

$$\boxed{w_{iL} = w_{iL}^{\frac{1}{\epsilon}} \cdot p_i}$$

$[w]^{\frac{1}{\epsilon}}$

$$w_{iL}^{1-\frac{1}{\epsilon}} = w_{iL}^{\frac{\epsilon-1}{\epsilon}} = p_i$$

$$\cancel{w_{iL}^{\frac{\epsilon-1}{\epsilon}} \cdot w_{iL}}$$

$$w_{iL} \cdot w_{iL}^{-\frac{1}{\epsilon}} = p$$

$$w_{iL}^{\frac{\epsilon}{\epsilon}} \cdot w_{iL}^{-\frac{1}{\epsilon}} = w_{iL}^{\frac{\epsilon-1}{\epsilon}}$$

$\frac{1}{1-\epsilon}$

$$\left(A^{\epsilon-1} \cdot \alpha \cdot w^{1-\epsilon} + (1-\alpha) \cdot \varphi^{1-\epsilon} \right)$$

$$\frac{\epsilon - \epsilon}{1}$$

$$\boxed{\frac{\epsilon}{1} - \epsilon}$$

$$\frac{\epsilon}{1-\epsilon}$$

IDEE: ANDEREN PREIS AUF CES ANPASSEN

$$p_i = \sum w_f \frac{1}{A_i \frac{\partial F}{\partial L}} + \sum p_j \frac{1}{A_i \frac{\partial F}{\partial x_{ij}}}$$

PC

$$GDP = \vec{p} \cdot \vec{A}^{\frac{\epsilon-1}{\epsilon}} \cdot \alpha^{\frac{1}{\epsilon}} \cdot \vec{y}^{\frac{1}{\epsilon}} \cdot \left(\frac{1}{L}\right)^{\frac{1}{\epsilon}} \cdot L$$

$$p_i = w \cdot \frac{1}{A^{\frac{\epsilon-1}{\epsilon}} y^{\frac{1}{\epsilon}} L^{-\frac{1}{\epsilon}} \cdot w_{iL}} + p_j \frac{1}{A^{\frac{\epsilon-1}{\epsilon}} y^{\frac{1}{\epsilon}} x^{-\frac{1}{\epsilon}} \cdot (1-w_{iL})}$$

Sum ?

$$p_i = w \cdot A^{\frac{\epsilon-1}{\epsilon}} \cdot w_{iL} \left(\frac{L}{y}\right)^{\frac{1}{\epsilon}} + p_j A^{\frac{\epsilon-1}{\epsilon}} (1-w_{iL}) \cdot \left(\frac{x}{y}\right)^{\frac{1}{\epsilon}}$$

$$p_i \cdot c_i = (\ldots) \, c_i$$

$$GDP = (\ldots) \, (y_i - x_i) \quad ?$$

$$Y_i = A_i \left(w_{iL} \cdot L^{\frac{\epsilon-1}{\epsilon}} + (1-w_{iL}) X^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$$

$$A = A^{\left(\frac{\epsilon-1}{\epsilon}\right)\left(\frac{\epsilon}{\epsilon-1}\right)}$$

$$y_i = \left(A_i^{\frac{\epsilon-1}{\epsilon}} \cdot w_{iL} \cdot L^{\frac{\epsilon-1}{\epsilon}} + A_i^{\frac{\epsilon-1}{\epsilon}} (1-w_{iL}) x^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$$

$$\frac{\partial y_i}{L} = WAGE = \frac{\epsilon}{\epsilon-1} (\ldots)^{\frac{1}{\epsilon-1}} \cdot \left(A_i^{\frac{\epsilon-1}{\epsilon}} \cdot \frac{\epsilon-1}{\epsilon} \cdot L^{-\frac{1}{\epsilon}} \cdot w_{iL}\right)$$

$$y^{\frac{1}{\epsilon}} \cdot A^{\frac{\epsilon-1}{\epsilon}} \cdot L^{-\frac{1}{\epsilon}} \cdot w_{iL}$$

? → EXP. FEHL

$$GDP = w \cdot \frac{\cancel{A^{\frac{\epsilon-1}{\epsilon}}} \cancel{w_i L^{\frac{\epsilon-1}{\epsilon}}} + (1-w_{iL}) X^{\frac{\epsilon-1}{\epsilon}}}{\cancel{A^{\frac{\epsilon-1}{\epsilon}}} \cancel{y^{\frac{1}{\epsilon}}} \cancel{L^{-\frac{1}{\epsilon}}} \cancel{w_{iL}}}$$

DOCH EHER NUR KOSTEN FUNKTION?

$$\downarrow$$

$$\frac{\cancel{A} L (1-w_{iL}) X^{\frac{\epsilon-1}{\epsilon}}}{Y^{-\frac{1}{\epsilon}}}$$

↑? GAMMA ?

$$\boxed{p \cdot c = GDP = w \cdot L}$$

→ Then the main question is what is w?

$$A^{\frac{\epsilon-1}{\epsilon}} \cdot (-1)$$

$$A^{\frac{1-\epsilon}{\epsilon}} = \frac{1}{\epsilon} - \frac{\epsilon}{\epsilon} = \frac{1}{\epsilon} - 1$$

$$Y_i - \sum_j x_{ji} = c_i$$

$$A \cdot \left(\alpha \cdot L^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha) \sum_j x_{ji}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} - \sum x_{ji}^{\frac{\epsilon-1}{\epsilon} \frac{\epsilon}{\epsilon-1}}$$

OUCH!

$$A \cdot \left(\alpha \cdot L^{\frac{\epsilon-1}{\epsilon}} + \cancel{\sum x_{ji}^{\frac{\epsilon-1}{\epsilon}}} + \alpha x_{ji}^{\frac{\epsilon-1}{\epsilon}} - \cancel{\sum x_{ji}^{\frac{\epsilon-1}{\epsilon}}}\right)^{\frac{\epsilon}{\epsilon-1}} = c_i$$

GEHT NICHT

c_i

$Y_i - x_i \approx L$ Rullier aufgehen?

PROD — INT/FAKTOREN — KONSUM

0 ⊕ — ε — σ 1

RIGIDE — FLEXIBEL ?

OBEN STATT UNTEN SONDERN WIE p · c

hier womöglich Y vor

(SEARCH FOR (OTHER) OCCURRENCES OF GDP INCOME?)

$$\sigma = \frac{1}{1+\rho} \Leftrightarrow$$

$$\sigma + \sigma\rho = 1 \Leftrightarrow$$

http://www.egwald.ca/economics/cescostfunctions.php

STANDARD CES DUALITY EXAMPLE

I. CES (Constant Elasticity of Substitution) Production Function.

A production function is a technological relationship between the levels of output that can be produced using specific combinations of factor inputs.

The three factor **CES** production function is:

q = A * [alpha * (L^-rho) + beta * (K^-rho) + gamma *(M^-rho)]^(-nu/rho) = f(L,K,M).

where L = labour, K = capital, M = materials and supplies, and q = product. The parameter nu is a measure of the economies of scale, while the parameter rho yields the elasticity of substitution:

sigma = 1/(1 + rho).

The coefficients of the production function, A, alpha, beta, gamma, nu, and rho are positive, real numbers. The production function's inputs, L, K, and M, are non-negative real numbers.

The efficiency parameter, A, changes output proportionally for changes in factor inputs, while the distribution parameters, alpha, beta, and gamma, determine the relative shares of the factors in the total cost of producing levels of outputs.

II. CES (Constant Elasticity of Substitution) Cost Function.

A cost function is a relationship between the total cost of producing levels of output, q, at specific values of factor input prices, wL, wK, and wM, using the cost minimizing combinations of factor inputs.

The three factor **CES** cost function is:

C(q;wL,wK,wM) = h(q) * c(wL,wK,wM)

where the **returns to scale function** is:

h(q) = (q/A)^1/nu

a continuous, increasing function of q (q >= 1), with h(0) = 0 and h(1)=1,

and the **unit cost function** is:

c(wL,wK,wM) = [alpha^(1/(1+rho)) * wL^(rho/(1+rho)) + beta^(1/(1+rho)) * wK^(rho/(1+rho)) + gamma^(1/(1+rho)) * wM^(rho/(1+rho))]^((1+rho)/rho)

III. CES Production Function Partial Derivatives.

f(L,K,M)^(1 + rho/nu) = A^(1 + rho/nu) * [alpha * (L^-rho) + beta * (K^-rho) + gamma *(M^-rho)]^((-nu/rho)*(1 + rho/nu)), and
f_L(L,K,M) = nu * alpha * L ^-(1 + rho) * A * [alpha * (L^-rho) + beta * (K^-rho) + gamma *(M^-rho)]^((-nu/rho)*(1 + rho/nu)), →
f_L(L,K,M) = nu * alpha * A^(-rho/nu) * L ^-(1 + rho) * f(L,K,M)^(1 + rho/nu); so similarly
f_K(L,K,M) = nu * beta * A^(-rho/nu) * K ^-(1 + rho) * f(L,K,M)^(1 + rho/nu);
f_M(L,K,M) = nu * gamma * A^(-rho/nu) * M ^-(1 + rho) * f(L,K,M)^(1 + rho/nu).

$\frac{\partial c}{\partial w} = L$

IV. CES Cost Function Partial Derivatives.

∂C(q;wL,wK,wM) / ∂wL = h(q) * alpha^(1/(1+rho)) * (rho/(1+rho)) * wL^(-1/(1+rho)) * ((1+rho)/rho) * [alpha^(1/(1+rho)) * wL^(rho/(1+rho)) + beta^(1/(1+rho)) * wK^(rho/(1+rho)) + gamma^(1/(1+rho)) * wM^(rho/(1+rho))]^(1/rho), →
∂C(q;wL,wK,wM) / ∂wL = h(q) * alpha^(1/(1+rho)) * wL^(-1/(1+rho)) * c(wL, wK, wM)^(1/(1+rho)); so similarly = MENGE L?
∂C(q;wL,wK,wM) / ∂wK = h(q) * beta^(1/(1+rho)) * wK^(-1/(1+rho)) * c(wL, wK, wM)^(1/(1+rho));
∂C(q;wL,wK,wM) / ∂wM = h(q) * gamma^(1/(1+rho)) * wM^(-1/(1+rho)) * c(wL, wK, wM)^(1/(1+rho)).

L, ORDENTLICH ABSCHÄTZEN!

V. Duality: Production to Cost Function.

Find the values of L, K, M, and μ that minimize the Lagrangian:

G(q;L,K,M,μ) = wL * L + wK * K + wM* M + μ * [q - f(L,K,M)]

a. G_L = wL - μ * f_L = 0
b. G_K = wK - μ * f_K = 0
c. G_M = wM - μ * f_M = 0
d. G_μ = q - f(L,K,M) = 0

SHARE OF L IN PROD
L ↔ SHARE/FACTOR →
SHARE/FACTOR
SCALE
SUBSTI $\rho = \frac{1-\sigma}{\sigma}$

$$q = A\cdot\left(\alpha\cdot L^{-\rho} + \beta\cdot K^{-\rho} + \gamma\cdot M^{-\rho}\right)^{-\frac{\eta}{\rho}} = f(L,K,M)$$

$$y_i = A_i\cdot\left(\alpha\cdot L^{-\rho} + (1-\alpha)M^{-\rho}\right)^{-\frac{\eta}{\rho}}$$ → ADAPTED!

UNIT COST

$$C = h(q)\cdot c(w_L, w_K, w_M)$$

$$h(q) = \left(\frac{q}{A}\right)^{\frac{1}{\eta}}$$

$$c = \left(\alpha^{\frac{1}{1+\rho}}\cdot w_L^{\frac{\rho}{1+\rho}} + \beta^{\frac{1}{1+\rho}}\cdot w_K^{\frac{\rho}{1+\rho}} + \gamma^{\frac{1}{1+\rho}}\cdot w_M^{\frac{\rho}{1+\rho}}\right)^{\frac{1+\rho}{\rho}}$$

$$\left(\frac{q}{A}\right)^{\frac{1}{\eta}}$$

EVENTUELL AUCH WIE HIER NACH PREISEN ABLEITEN?

$$TC_i = \left(\frac{y_i}{A}\right)^{\frac{1}{\eta}}\cdot\left(\alpha^{\frac{1}{1+\rho}}\cdot w^{\frac{\rho}{1+\rho}} + (1-\alpha)^{\frac{1}{1+\rho}}\cdot p_j^{\frac{\rho}{1+\rho}}\right)^{\frac{1+\rho}{\rho}}$$

Im Grunde kann man nur $\frac{\partial TC}{\partial y_i}$ sinnvoll aufschreiben → Grenzkosten der Produktion → könnte $\frac{\partial TC}{\partial c_i}$ entsprechen (Aufteilung in 1. Term?) → Gleichsetzung mit Preis → multiplizieren mit c_i? How?

TODOS
(0) ~~COMPARE STARTING EQUATION MORE CAREFULLY~~?
↳ (0A) ~~BETTER ALIGN COST FUNCTION~~
(1) ~~DERIVATIVE COST FUNCTION → INTERPRETATION~~?
(2) COMBINE WITH WHAT WE HAVE? → GDP, WAGES, PROFITFUNCTION
(3) DUALITY: DERIVATION OF COST FUNCTION

CONST. R.T.S. → $\eta = 1$ ok! $\rho = \frac{1-\epsilon}{\epsilon}$ → $\frac{1}{\rho}$ ok!

$$Y_i = A_i\left(\omega_{iL}\cdot l_i^{\frac{\epsilon-1}{\epsilon}} + (1-\omega_{iL})(X_i)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$$ ok!

HIER NOCHMAL DIE PROFITFUNKTION FÜR DEN AKTUELLEN FALL

$$\Pi_i = p_i y_i - \sum_{f=1}^{F} w_f l_{if} - \sum_{j=1}^{n} p_j x_{ij}$$

AUF CES-FALL UMSCHREIBEN MIT TC_i

$$\Pi_i = p_i y_i - \left(\frac{y_i}{A}\right)\cdot\left(\alpha^{\epsilon}\cdot w^{1-\epsilon} + (1+\alpha)^{\epsilon} p_j^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} = 0$$

$$\rho = \frac{1-\epsilon}{\epsilon} \Leftrightarrow \frac{1}{1+\rho} = \epsilon$$

$$\frac{\rho}{1+\rho} = \rho\cdot\frac{1}{1+\rho} = \frac{1-\epsilon}{\epsilon}\cdot\epsilon = (1-\epsilon)$$

$$\frac{1+\rho}{\rho} = \frac{1}{1-\epsilon}$$

$$\frac{\partial \Pi_i}{\partial y_i} = p_i - \frac{1}{A}\cdot\left(\alpha^{\epsilon}\cdot w^{1-\epsilon} + (1+\alpha)^{\epsilon} p_i^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} = 0$$

EVENTUELL DAS HIER EINFACH MAL c_i RECHNEN?

$$p_i = \frac{1}{A}\cdot\left(\alpha^{\epsilon}\cdot w^{1-\epsilon} + (1+\alpha)^{\epsilon} p_i^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}$$

NACH TEER ABLEITEN?

HIER EINFACH WAGE EINSETZEN?

↳ LOG-WAY SOMEWHAT SIMILAR TO PRICE-EQUATION

↳ WAS DANN TUN?

TC NACH PREIS UND ALS L IN W×L?

SIMULATION DERIVS!

WIESO?

$$\frac{\partial TC}{\partial w} = \left(\frac{y_i}{A}\right) \cdot \frac{\cancel{1+\rho}}{\rho} \cdot \left(\ldots\right)^{\frac{1}{\rho}} \cdot \frac{1}{c^{1+\rho}} \frac{\partial p_i}{\partial A} = -\frac{1}{A^2}$$ • WIE GEHABT

LOG(A) KÖNNEN WIR EIGENTL. NICHT BRAUCHEN

$$\frac{\rho}{\cancel{1+\rho}} \cdot w^{-\frac{1}{1+\rho}} \cdot \alpha^{\frac{1}{1+\rho}} = \quad \frac{1}{\rho} = ? \quad \frac{1+\rho}{\rho}$$

$$\left(\frac{y_i}{A}\right) \cdot C^{\epsilon} \cdot w^{-\epsilon} \cdot \alpha^{1-\epsilon} = \frac{1}{1+\rho} \; ?$$

$$\alpha^{\epsilon} \alpha^{1-\epsilon}$$

$$\alpha$$

$$\left(\frac{y_i}{A}\right) \cdot C^{\epsilon} \cdot \left(\frac{\alpha}{w}\right)^{C}$$

UNIT COST?? $\rightarrow = \rho$?

EXPONENT TABLE FOR α

	B&F	B&R & J
PF	$\frac{1}{\epsilon}$	NO
CF	NO	ϵ

$$TC_i = \left(\frac{y_i}{A}\right)^{\frac{1}{\eta}} \cdot \left(\alpha^{\frac{1}{1+\rho}} \cdot w^{\frac{\rho}{1+\rho}} + (1-\alpha)^{\frac{1}{1+\rho}} \cdot p_j^{\frac{\rho}{1+\rho}}\right)^{\frac{1+\rho}{\rho}}$$

$$\frac{\partial TC_i}{\partial w} = \left(\frac{y_i}{A}\right)^{\frac{1}{\eta}} \cdot \frac{\cancel{1+\rho}}{\rho} \left(\ldots\right)^{\frac{1}{\rho}} \cdot \frac{\rho}{\cancel{1+\rho}} \cdot \left(\frac{\alpha}{w}\right)^{\frac{1}{1+\rho}} \cdot$$

Stückzahl • UNIT COST^{-1} • $\left(\frac{\alpha}{w}\right)^{\frac{1}{1+\rho}}$?

$$\alpha^{\epsilon} \cdot w^{-\epsilon}$$

$$\frac{1+\rho}{\rho} - \frac{\rho}{\rho}$$

$$\frac{\rho}{1+\rho} - \frac{1+\rho}{1+\rho} = -\frac{1}{1+\rho}$$

OTHER VERSION OF CES.

$$Q = A\left[\delta K^{-\rho} + (1+\delta) L^{-\rho}\right]^{-\vartheta/\rho}$$

$$\frac{\partial Q}{\partial L} = A \cdot (1+\delta) \cdot -\rho \, L^{-\rho-1} \cdot$$

$$\left(-\frac{\vartheta}{\rho}\right)\left(\ldots\ldots\right)^{-\frac{\vartheta}{\rho}-1} \quad -\frac{\vartheta-\rho}{\rho}$$

$$\frac{\partial Q}{\partial L}^{A} = L_1 \qquad \left(\frac{Q}{L}\right)^{1+\rho}$$

$$\frac{\epsilon}{\epsilon-1} \qquad -\frac{2}{\rho}$$

$$Q = (\ldots)^{-\frac{2}{\rho}}$$

$$\frac{1}{\epsilon-1} \qquad \frac{2-\rho}{\rho}$$

$$Q^{\frac{\rho+1}{1+\rho}} \qquad -\frac{2-\rho}{\rho}+1$$

$$Y^{\frac{1}{\epsilon}} \qquad Y^{1+\rho}$$

$$-\frac{2-\rho+\rho}{\rho}$$

$$-\frac{2+\rho}{\rho}\cdot(1+\rho)$$

$$-\frac{2+\rho}{\rho} - \frac{\rho(\cancel{2\rho}-\rho)}{\rho}$$

$$-\frac{2}{\rho}+1-\frac{2}{\rho}-1 \checkmark$$