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Multifactor CES general equilibrium: Models and applications

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ABSTRACT

Sector-specific multifactor CES elasticities of substitution and the corresponding productivity growth are jointly measured by regressing the growth of per-factor cost shares against the growth of factor prices. We use linked input-output tables for Japan and the Republic of Korea as the data source for factor price and cost shares in two temporally distant states. We then construct a multisectoral general equilibrium model using the system of estimated CES unit cost functions and evaluate the economy-wide propagation of an exogenous productivity stimulus in terms of welfare. Further, we examine the differences between models based on *a priori* elasticities such as Leontief and Cobb-Douglas.

1. Introduction

In this study, we jointly measure the multifactor CES elasticity of substitution with the productivity growth for multiple industrial sectors by way of two temporally distant cross-sectional data (i.e. linked input–output tables). When we apply the multifactor CES unit cost function, we discover that industry-specific elasticities can be estimated by regressing the growth of per-factor cost shares against the growth of factor prices. We also discover that industry-specific productivity growth can be measured via the intercept of the regression line. Consequently, we make use of the linked input–output tables in order to observe the cost shares and price changes spanning over two periods for multiple industrial sectors.

The two-input constant elasticity of substitution (CES) function was first introduced by Arrow et al. (1961), Uzawa (1962), McFadden (1963) later showed that elasticities were still unique for the case of more than two factor inputs. Empirical analyses concerning the measurement of CES elasticities (e.g. McKibbin and Wilcoxen, 1999; van der Werf, 2008; Koesler and Schymura, 2015) have been based upon time series data, while embedding nest structures into the two-input CES framework conforming to the work by Sato (1967), to handle elasticities between more than two factors of production. The number of factors and thus, of estimable elasticities, can nevertheless be reduced, depending on the availability of time series data. Since we are interested in constructing a multisector general equilibrium model that calls for multifactor production functions, we take the advantage of an alternative approach by exploiting cross-sectional data.

When a multisectoral general equilibrium model is established, assessments can be made of the arbitrary productivity shock resulting from technological innovation in terms of the welfare gained. Previous studies in this regard have assumed constant and uniform unit elasticity (Klein, 1952–1953, Saito and Tokutsu, 1989), or have used empirically estimated elasticities in Translog or multistage (nested) CES functions with a highly aggregated and thus limited number of substitutable factors. Examples include works by Kuroda et al. (1984) and Tokutsu (1994) and many of the works concerning CGE models, such as studies by Böhringer et al. (2015), Go et al. (2016). In contrast, our approach allows us to construct an empirical model of multifactor production with different elasticities of substitution among many (over 350) industrial sectors. Moreover, this approach allows us to prospectively portray the ex post technological structure following any given exogenous productivity shock and to account for welfare in terms of economy-wide input–output performances.

We measure the welfare changes attributed to the exogenous productivity change by SCS (social cost saved), i.e. the difference in the total primary factor inputs required to net produce a fixed amount of final consumption, given the productivity change. We find theoretically that SCS will be positive (primary factor inputs will always be saved) in every sector if the exogenous productivity is improving and, vice versa, under a system with uniform CES elasticity less than unity, which is inclusive of Cobb–Douglas and Leontief systems. Hence, conversely, such a law may not necessarily hold for the case of CES systems with non-uniform elasticities, as verified by the empirical analysis of SCS using the estimated multifactor CES system.

The remainder of this paper is organised as follows. In the next section, we introduce the basics of multifactor CES elasticity and productivity

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growth estimations, and apply the protocol to linked input–output tables for Japan and the Republic of Korea, which have sufficient capacity in terms of the degrees of freedom of the regressions. In Section 3, we replicate the current technological structure as the general equilibrium state of a system of empirically estimated multifactor CES functions; further, we trace how that structure is transformed by exogenous productivity stimuli. Section 4 provides concluding remarks.

2. The model

2.1. Multifactor CES functions

A constant-returns multifactor CES production function of an industrial sector (index j omitted) has the following form:

$$y = zf(\mathbf{x}) = z\left(\sum_{i=0}^{n} \lambda_i^{\frac{1}{\sigma}} x_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

where y denotes the output and x_i denotes the ith factor input. Here the share parameters are assumed to maintain $\lambda_i > 0$ and $\sum_i \lambda_i = 1$, while the elasticity of substitution $\sigma \geq 0$ is subject to estimation. Also, we are interested in measuring the growth of productivity, i.e. $\Delta \ln z$, where Δ represents temporally distant differences.

Displayed below is the unit cost function compatible with the multifactor CES production function:

$$c = z^{-1}h(\mathbf{w}) = \frac{1}{z}\left(\sum_{i=0}^{n} \lambda_i w_i^{\gamma}\right)^{1/\gamma}$$

where c denotes the unit cost of the output and w_i denotes the ith factor price. Here, we use $\gamma = 1 - \sigma$ for convenience. The cost share of the ith input a_i can be determined, in regards to Shephard's lemma, by differentiating the unit cost function:

$$a_i = \frac{\partial c}{\partial w_i}\frac{w_i}{c} = \lambda_i (zc/w_i)^{-\gamma} \tag{1}$$

By taking the log of both sides, we have

$$\ln a_i = \ln \lambda_i - \gamma \ln z + \gamma \ln(w_i/c)$$

As we observe two temporally distant values for cost shares (a_i^0 and a_i^1), factor prices (w_i^0 and w_i^1) and unit costs of outputs as prices ($c^0 = w^0$ and $c^1 = w^1$) reflecting perfect competition, we find two identities regarding the data:

$$\ln a_i^0 = \ln \lambda_i - \gamma \ln z^0 + \gamma \ln(w_i^0/w^0) + \epsilon_i^0$$
$$\ln a_i^1 = \ln \lambda_i - \gamma \ln z^1 + \gamma \ln(w_i^1/w^1) + \epsilon_i^1$$

where we assume that ϵ_i^0 and ϵ_i^1 are identically and normally distributed disturbance terms. Subtraction results in the main regression equation as follows:

$$\Delta \ln a_i = -\gamma \Delta \ln z + \gamma \Delta \ln(w_i/w) + \epsilon_i \tag{2}$$

Here, the disturbance term $\epsilon_i = \epsilon_i^0 - \epsilon_i^1$ is identically and normally distributed, so that one can estimate γ and $\Delta \ln z$ via a simple linear regression Eq. (2). That is, by regressing the growth of per-factor cost shares, i.e. $\Delta \ln a_i$ on the growth of relative prices or $\Delta \ln(w_i/w)$, the slope gives the estimate of γ, while the intercept gives the estimate of $-\gamma \Delta \ln z$. Also, note that λ_i can be calibrated via Eq. (1), as long as we have the estimate for γ.

2.2. The data and estimations

A set of linked input–output tables includes sectoral transactions in both nominal and real terms. Since real value is adjusted for inflation, in order to enable comparison of quantities as if prices had not changed, and since nominal value is not adjusted, we use a price index to convert nominal values into real values. That is, if we standardise the value of a commodity at the reference state as real, its nominal (unadjusted) value at the target state relative to the reference state equals the price index called a deflator. Naturally, the 1995–2000–2005 linked input–output tables for both Japan (MIAC, 2011) and Korea (BOK, 2015) include per-factor deflators (395 factors for Japan and 350 factors for Korea) spanning the fiscal years recorded. These linked input–output tables, however, do not include deflators for primary factor (i.e. labour and capital) and therefore, we use the quality-adjusted price indexes compiled by JIP (2015) for Japan and by KIP (2015) for Korea in order to inflate the primary factor inputs observed as nominal values.

Hence, observations for both the dependent variables (cost shares as input–output coefficients a_{ij}) and independent variables (price ratios w_j/w_i) for estimating Eq. (2) become available with sufficient capacity in terms of degrees of freedom, as we verify that there are $n+1$ inputs; namely, $i = 0, 1, \ldots, n$ and n outputs; namely, $j = 1, \ldots, n$ for an input–output table. In particular, we use the 2000 and 2005 input–output coefficient matrices from the three-period linked input–output tables as the data for the cost share growth (i.e. $\Delta \ln a_{ij}$) and, since we set the reference state at year 2000, the five-year growth of output-relative factor prices becomes simply the log difference between deflators; that is,

$$\Delta \ln w_i/w_j = \ln p_i/p_j$$

where p_i denotes the deflator for commodity i in year 2005 with respect to year 2000.

Fig. 1 displays the estimated CES elasticity (i.e. $\sigma_j = 1 - \gamma_j$) with respect to the statistical significance of γ_j, i.e. the slope of the regression Eq. (2) in terms of the P-value in Japan. Fig. 2 is the version for Korea. Note that CES elasticities are statistically significant (P-value <0.1) for 176 out of 395 sectors for Japan, whereas 166 sectors are significant out of 350 sectors in Korea. The results of the estimations are summarised in the Appendix, Tables A1 and A2, for Japan and Korea, respectively. These tables are confined to sectors whose slopes ($\gamma_j = 1 - \sigma_j$) of the regression (2) are statistically significant and we indicate the level of significance by *** (0.01 level), ** (0.05 level) and * (0.1 level), along with the estimated elasticities. Note that we accept the null hypothesis (i.e. $\gamma_j = 1 - \sigma_j = 0$) for sectors with a statistically insignificant slope and, in that event, the average elasticity, i.e. $\sum_{j=1}^{n} \sigma_j/n$ is 1.32 for Japan and 1.39 for Korea. Alternatively, if we accept all the elasticity estimates, regardless of statistical significance, the average elasticity is 1.46 for Japan and 1.52 for Korea.

These multifactor CES elasticities are comparable to other estimates in the literature. The GTAP (2016) substitution elasticities for intermediate inputs, which are broadly employed in CGE studies (e.g., (Álvarez-Martínez and Polo, 2012; Antimiani et al., 2015)) range from 0.20 to 1.68, while those among internationally traded goods (i.e. Armington elasticities) are generally larger, ranging from 1.15 to 34.40, depending on the industrial sector. Welsch (2006)'s estimate for mean Armington elasticities for France ranges from negative 2.060 to

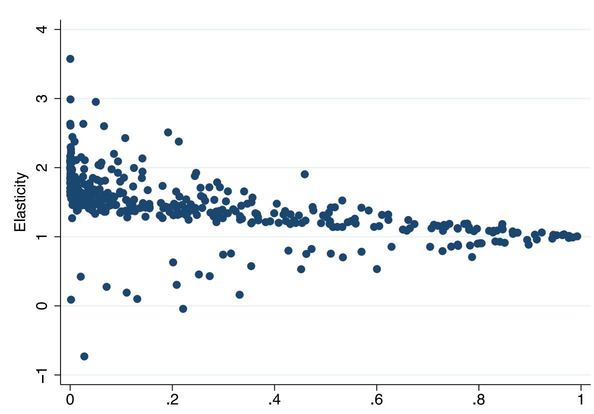

Fig. 1. CES elasticity vs significance (Japan).

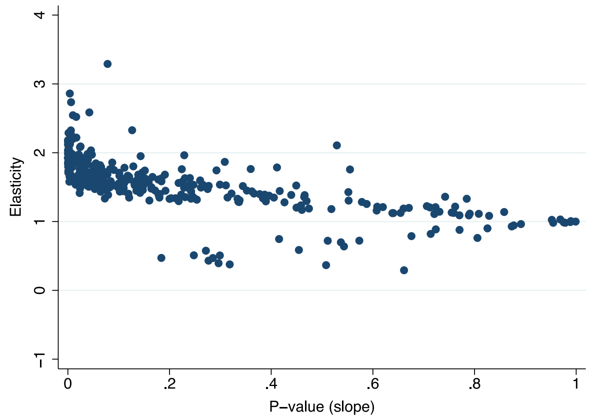

Fig. 2. CES elasticity vs significance (Korea).

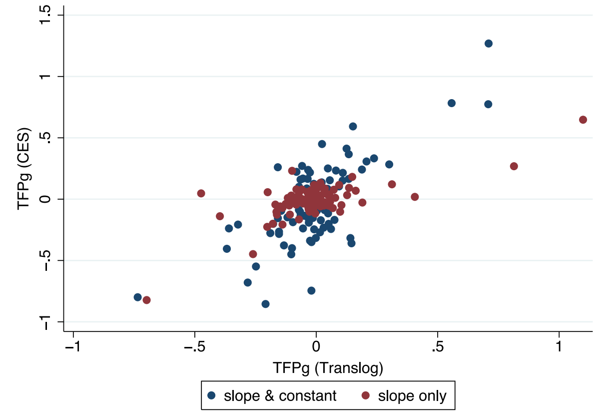

Fig. 3. TFPg of different measurements (Japan).

Fig. 4. TFPg of different measurements (Korea).

positive 2.167 by industry and from 0.178 to 1.024 by time window while oscillating from the early 70's to late 90's. Note that our estimates are also comparable to the Koesler and Schymura (2015)'s KLEM nest-wise CES elasticity estimates for 36 industrial sectors.

In the third column of Tables A1 and A2, we display the productivity growth $\Delta \ln z$, labelled as TFPg (Total Factor Productivity growth), which is the estimated constant of (2) divided by the negative of the corresponding slope. Accordingly, the statistical significances of TFPg are evaluated by way of bootstrapping (with 400 replications) on the basis of regression (2). The statistical significance of the underlying intercept is indicated in parenthesis. Note also that these tables are sorted by the level of the estimated TFPg. We now make some assessments of the estimated TFPg in regards to other possible productivity measurements. Below is the log of the Törnqvist index

$$\mathrm{TFPg(Translog)} = -\ln p + \sum_{i=0}^{n} \left(\frac{a_i^0 + a_i^1}{2}\right) \ln p_i \tag{3}$$

the exactness of which Diewert (1976) showed in measuring the productivity growth of the Translog functions. Thus, we know that Eq. (3) is equal to the productivity growth of the underlying Translog function with or without knowing its parameters. Note that, although it is almost impossible to estimate the parameters of a Translog function with 100 factor inputs, its productivity growth can be measured using the same data (cost shares and price changes) that we use in estimating productivity for a multifactor CES function. Star and Hall (1976) showed that the Törnqvist index is a good approximation of TFPg measurements irrespective of the type of aggregator function and the interval of observations.

In Figs. 3 and 4, we plot the estimated TFPg for a multifactor CES function, tagged as TFPg (CES) for all sectors listed in Tables A1 and A2, against the log of the Törnqvist indexes, tagged as TFPg (Translog). Blue dots indicate sectors whose slope and intercept of regression Eq. (2) are both statistically significant (P-value <0.1) whereas red dots indicate sectors with a slope that is significant but an intercept that is not. In both cases, we observe agreements between the two TFPg measurements; therefore, we evaluate them objectively, as summarised in Table 1. Here, correlation represents Pearson's correlation coefficient, whereas concordance represents Lin (2000) concordance correlation coefficient. Note that, in Figs. 3 and 4, 'Slope Only' significant sectors are represented by red dots and 'Slope and Constant' significant sectors are represented by blue dots; whereas, 'Slope' indicates all significant sectors and thus, the union of red and blue-designated sectors. 'Bootstrapping' indicates sectors with significant TFPg (CES) estimates via bootstrapping. In other words, by way of a multifactor CES function, we obtain TFPg estimates similar to those based on Translog functions that are very general in terms of the elasticities of substitution, set aside their estimability, and yet, a multifactoral elasticity of substitution can be estimated. Note, however, that, in the event that we accept the null for the insignificant slope of regression Eq. (2), we must assume that the function is Cobb–Douglas and that TFPg cannot be measured.

Table 1
Concordances and correlations between Translog and multifactor CES TFPg estimates.

Sectors	Concord.	Correl.	Obs.
Slope (JPN)	0.645	0.669	176
Slope Only (JPN)	0.673	0.707	100
Slope and Constant (JPN)	0.633	0.741	76
Bootstrapping (JPN)	0.794	0.889	21
Slope (KOR)	0.305	0.413	166
Slope Only (KOR)	0.309	0.340	97
Slope and Constant (KOR)	0.370	0.413	69
Bootstrapping (KOR)	0.623	0.707	33

3. Prospective analysis

3.1. Projected prices

In the following section, we construct a multisectoral general equilibrium model that reflects all measured elasticities and observed current cost shares; further, we exogenously impose some productivity change into the model and simulate the multisectoral propagation that can potentially take place. For sake of simplicity, we normalise all current prices at unity. In that event, we know by Eq. (1) that:

$$a_{ij} = \lambda_{ij}, \qquad \sum_{i=0}^{n} a_{ij} = 1, \quad j = 1, 2, \ldots, n$$

Then, the system of multifactor CES unit cost functions in equilibrium under some exogenously given productivity change, i.e.

$\mathbf{z} = (z_1, z_2, \ldots, z_n) \neq \mathbf{1}$, must be in the following state:

$$\begin{aligned}
\pi_1 &= z_1^{-1}(a_{01}\pi_0^{\gamma_1} + a_{11}\pi_1^{\gamma_1} + \cdots a_{n1}\pi_n^{\gamma_1})^{\frac{1}{\gamma_1}}\\
\pi_2 &= z_2^{-1}(a_{02}\pi_0^{\gamma_2} + a_{12}\pi_1^{\gamma_2} + \cdots a_{n2}\pi_n^{\gamma_2})^{\frac{1}{\gamma_2}}\\
&\vdots\\
\pi_n &= z_n^{-1}(a_{0n}\pi_0^{\gamma_n} + a_{1n}\pi_1^{\gamma_n} + \cdots a_{nn}\pi_n^{\gamma_n})^{\frac{1}{\gamma_n}}
\end{aligned} \tag{4}$$

where the projected (ex post) general equilibrium price for factor i is denoted by π_i. Note that the current state, i.e. $\mathbf{z} = \mathbf{1}$, can be reproduced by setting all prices at the current state, i.e. $\boldsymbol{\pi} = \mathbf{1}$ and vice versa.[1]

The projected price, ex post the exogenous productivity change, can be obtained by solving Eq. (4) for $\boldsymbol{\pi}$. By rearranging, we have:

$$\begin{aligned}
z_1^{\gamma_1}\pi_1^{\gamma_1} &= a_{01}\pi_0^{\gamma_1} + a_{11}\pi_1^{\gamma_1} + \cdots a_{n1}\pi_n^{\gamma_1}\\
z_2^{\gamma_2}\pi_2^{\gamma_2} &= a_{02}\pi_0^{\gamma_2} + a_{12}\pi_1^{\gamma_2} + \cdots a_{n2}\pi_n^{\gamma_2}\\
&\vdots\\
z_n^{\gamma_n}\pi_n^{\gamma_n} &= a_{0n}\pi_0^{\gamma_n} + a_{1n}\pi_1^{\gamma_n} + \cdots a_{nn}\pi_n^{\gamma_n}
\end{aligned}$$

Or, by way of row vectors and matrices:

$$\boldsymbol{\pi}^{\gamma}\langle \mathbf{z}^{\gamma}\rangle = \mathbf{a}_0 + \boldsymbol{\pi}^{\gamma}\mathbf{A}$$

where $\boldsymbol{\pi}^{\gamma} = (\pi_1^{\gamma_1}, \ldots, \pi_n^{\gamma_n})$ and $\mathbf{z}^{\gamma} = (z_1^{\gamma_1}, \ldots, z_n^{\gamma_n})$, while we set the price of a primary input as a numéraire, i.e. $\pi_0 = 1$. Angle brackets indicate diagonalisation. Note that $\mathbf{A}$ and $\mathbf{a}_0$ are the current input–output coefficients matrix and value added coefficients vector, respectively. Now, the projected equilibrium price $\boldsymbol{\pi}$ can be obtained in terms of $\mathbf{z}$:

$$\boldsymbol{\pi} = (\mathbf{a}_0[\langle \mathbf{z}^{\gamma}\rangle - \mathbf{A}]^{-1})^{\frac{1}{\gamma}} \tag{5}$$

Besides CES, we may use Eq. (5) to obtain the projected price for the cases of Leontief ($\boldsymbol{\gamma} = \mathbf{1}$) and Cobb–Douglas ($\boldsymbol{\gamma} = \mathbf{0}$). The Leontief case is straightforward:

$$\boldsymbol{\pi} = \mathbf{a}_0[\langle \mathbf{z}\rangle - \mathbf{A}]^{-1} \tag{6}$$

For the Cobb–Douglas case, we first take the log of Eq. (4) and then let $\boldsymbol{\gamma} \to \mathbf{0}$. Below, we work on the unit cost function of any industrial sector j while omitting the subscript:

$$\ln\pi + \ln z = \frac{\ln(a_0 + \sum_{i=1}^{n} a_i\pi_i^{\gamma})}{\gamma} \to \sum_{i=1}^{n} a_i \ln \pi_i$$

Here, we apply l'Hospital's rule when we let $\gamma \to 0$, since, in that event, the numerator and denominator both approach zero. By way of row vectors and matrices, this can be written concisely:

$$\ln\boldsymbol{\pi} = -\ln\mathbf{z} + (\ln\boldsymbol{\pi})\mathbf{A} \tag{7}$$

where the log operators are applied element-wise. The Cobb–Douglas version of the projected price will thus be

$$\begin{aligned}
\boldsymbol{\pi} &= \exp(-(\ln\mathbf{z})[\mathbf{I} - \mathbf{A}]^{-1}) \qquad (8)\\
&= \left(\frac{1}{\prod_{i=1}^{n} z_i^{\ell_{i1}}}, \frac{1}{\prod_{i=1}^{n} z_i^{\ell_{i2}}}, \ldots, \frac{1}{\prod_{i=1}^{n} z_i^{\ell_{in}}}\right)
\end{aligned}$$

where ℓ_{ij} is an element of the Leontief inverse matrix $[\mathbf{I} - \mathbf{A}]^{-1}$.

3.2. Projected structures

Since we set the current price to unity, the final demand in monetary terms will be the same as the physical quantity demanded. Let the current (nominal) final demand be denoted by a column vector $\mathbf{d} = (d_1, \ldots, d_n)^{\top} \geq \mathbf{0}$. Note that the sum of per-product final demand and that of per-sector value added (social cost) equals the GDP. If we have the projected price attributable to some exogenous productivity change, we can evaluate the corresponding welfare change in terms of SCS (social cost saved); that is,

$$\mathrm{SCS} = (\mathbf{1} - \boldsymbol{\pi})\mathbf{d} = \sum_{j=1}^{n} v_j - v'_j \tag{9}$$

Note that v_j and v'_j denote current and projected value added for sector j. The sector-wise distribution of SCS, however, requires further examination of the projected structure of the economy.

According to Eq. (1), the projected cost shares ex post the exogenous productivity change $\mathbf{z}$, which we denote as b_{ij}, can be evaluated by the following identity:

$$b_{ij} = a_{ij}(z_j\pi_j/\pi_i)^{-\gamma_j} \quad i = 0, 1, \ldots, n \tag{10}$$

Hence, under CES, the projected primary factor input (or value added) distribution $\mathbf{v}' = (v'_1, \ldots, v'_n)$ spanning the sectors for a given fixed final demand $\mathbf{d}$ (in physical quantity) can be evaluated as follows:

$$\mathbf{v}' = \mathbf{b}_0[\mathbf{I} - \mathbf{B}]^{-1}\langle\boldsymbol{\pi}\rangle\langle\mathbf{d}\rangle \tag{11}$$

where the entries for $\mathbf{b}_0$ and $\mathbf{B}$ are specified by Eq. (10). Conversely, the current distribution of primary factor inputs (or value added) $\mathbf{v} = (v_1, \ldots, v_n)$ is specified by the current observed cost shares as follows:

$$\mathbf{v} = \mathbf{a}_0[\mathbf{I} - \mathbf{A}]^{-1}\langle\mathbf{d}\rangle \tag{12}$$

Since Eqs. (11) and (12) are row vectors, one can evaluate SCS in terms of a sector-wise distribution.

3.3. Uniform CES elasticity

Here, we examine how SCS will be distributed among sectors, depending on the projected structures pertaining to uniform substitution elasticities, i.e. $\gamma_1 = \gamma_2 = \cdots = \gamma_n = \gamma$. First, by plugging Eq. (10) into Eq. (11) under some uniform elasticity $\sigma = 1 - \gamma$, we have the following exposition for the projected value-added distribution:

$$\begin{aligned}
\mathbf{v}' &= \mathbf{a}_0\langle\boldsymbol{\pi}^{-\gamma}\rangle\langle\mathbf{z}^{-\gamma}\rangle[\mathbf{I} - \langle\boldsymbol{\pi}^{\gamma}\rangle\mathbf{A}\langle\boldsymbol{\pi}^{-\gamma}\rangle\langle\mathbf{z}^{-\gamma}\rangle]^{-1}\langle\boldsymbol{\pi}\rangle\langle\mathbf{d}\rangle\\
&= \mathbf{a}_0[\langle\mathbf{z}^{\gamma}\rangle - \mathbf{A}]^{-1}\langle\boldsymbol{\pi}^{1-\gamma}\rangle\langle\mathbf{d}\rangle
\end{aligned} \tag{13}$$

Hence, we know that, for the Cobb–Douglas and Leontief cases, the projected value added distribution will be

$$\mathbf{v}'(\text{Cobb}--\text{Douglas}) = \mathbf{a}_0[\mathbf{I} - \mathbf{A}]^{-1}\langle\boldsymbol{\pi}\rangle\langle\mathbf{d}\rangle \tag{14}$$

$$\mathbf{v}'(\text{Leontief}) = \mathbf{a}_0[\langle\mathbf{z}\rangle - \mathbf{A}]^{-1}\langle\mathbf{d}\rangle \tag{15}$$

Note that projected equilibrium price Eq. (8) must be applied to Eq. (14) for the Cobb–Douglas case.

Further, we show below that, under uniform substitution elasticities less than unity, the SCS distribution will always be positive (in all sectors) against any exogenous productivity increase, and vice versa. Specifically, we show that

Proposition. Under $0 \leq \gamma \leq 1$, SCS is positive for all sectors such that $\mathbf{v} - \mathbf{v}' \geq \mathbf{0}$, if the exogenous productivity is increasing, i.e. $\mathbf{z} \geq \mathbf{1}$, and SCS is negative in all sectors such that $\mathbf{v} - \mathbf{v}' \leq \mathbf{0}$, if the exogenous productivity is decreasing i.e., $\mathbf{z} \leq \mathbf{1}$.

Proof. Because the input–output coefficient, as well as the productivity, is nonnegative, i.e. $\mathbf{A} \geq \mathbf{0}$ and $\mathbf{z} \geq \mathbf{0}$, we have the following exposition:

$$\begin{aligned}
[\langle\mathbf{z}^{\gamma}\rangle - \mathbf{A}]^{-1} &= \langle\mathbf{z}^{-\gamma}\rangle + \mathbf{A}\langle\mathbf{z}^{-2\gamma}\rangle + \mathbf{A}^2\langle\mathbf{z}^{-3\gamma}\rangle + \cdots\\
[\mathbf{I} - \mathbf{A}]^{-1} &= \mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \cdots
\end{aligned} \tag{16}$$

Thus, by taking $0 \leq \gamma \leq 1$ into account, we know that

$$\begin{aligned}
&[\langle\mathbf{z}^{\gamma}\rangle - \mathbf{A}]^{-1} \leq [\mathbf{I} - \mathbf{A}]^{-1} \quad \text{if } \mathbf{z} \geq \mathbf{1}\\
&[\langle\mathbf{z}^{\gamma}\rangle - \mathbf{A}]^{-1} \geq [\mathbf{I} - \mathbf{A}]^{-1} \quad \text{if } \mathbf{z} \leq \mathbf{1}
\end{aligned} \tag{17}$$

Moreover, as we take for granted that the unit cost mapping Eq. (4) is monotone increasing in price, the projected equilibrium price $\boldsymbol{\pi}$ must be

[1] This may not be so obvious when $\boldsymbol{\gamma} = \mathbf{0}$, until we see Eq. (7).

smaller (larger) than unity when the exogenous productivity $\mathbf{z}$ is increasing (decreasing). Thus, by taking $0 \le \gamma \le 1$ into account, we know that

$$\boldsymbol{\pi} \le \boldsymbol{\pi}^{1-\gamma} \le \mathbf{1} \quad \text{if } \mathbf{z} \ge \mathbf{1} \boldsymbol{\pi} \ge$$
$$\boldsymbol{\pi}^{1-\gamma} \ge \mathbf{1} \quad \text{if } \mathbf{z} \le \mathbf{1} \tag{18}$$

Hence, the structural differences between the reference and projected states can be assessed as follows:

$$[\mathbf{I} - \mathbf{A}]^{-1} \ge [\langle \mathbf{z}^{\gamma} \rangle - \mathbf{A}]^{-1} \langle \boldsymbol{\pi}^{1-\gamma} \rangle \quad \text{if } \mathbf{z} \ge \mathbf{1}$$
$$[\mathbf{I} - \mathbf{A}]^{-1} \le [\langle \mathbf{z}^{\gamma} \rangle - \mathbf{A}]^{-1} \langle \boldsymbol{\pi}^{1-\gamma} \rangle \quad \text{if } \mathbf{z} \le \mathbf{1} \tag{19}$$

Since the SCS distribution $\mathbf{v} - \mathbf{v}'$ is the difference between Eqs. (12) and (13), then Eq. (19) suffices for the proposition.□

Remark. This proposition is inclusive of Cobb–Douglas ($\gamma = 0$) and Leontief ($\gamma = 1$) systems. Uniformity of substitution elasticity γ is required to obtain Eq. (13). For substitution elasticities greater than unity, i.e. $\gamma = 1 - \sigma < 0$, the inequalities for Eq. (17) will be reversed, whereas those for Eq. (18) remain stable so that Eq. (19) may not necessarily hold.

3.4. Simulations

We now apply the framework specified in the previous sections. First, we calibrate the multisectoral models with different elasticities; namely, Leontief, Cobb–Douglas and multifactor CES, as of year 2005. Thus, the cost shares of the current state, i.e. $\mathbf{a}_0$ and $\mathbf{A}$ are as of year 2005. For the multifactor CES system, we make use of the elasticities that are statistically significant, i.e. the sectors displayed in Tables A1 and A2, while we undertake unit elasticity (or the null hypothesis) for the remainder of the sectors.[2]

As for the exogenous productivity change $\mathbf{z}$, we examine the 'productivity doubling' of the 'Ready mixed concrete' (RMC, hereafter) sector, which is the 150th sector for Japan and the 159th sector for Korea. That is,

$$\text{Japan: } z_{j=150} = 2,\ z_{j\neq 150} = 1 \quad (n = 395)$$
$$\text{Korea: } z_{j=159} = 2,\ z_{j\neq 159} = 1 \quad (n = 350) \tag{20}$$

There are a few reasons for choosing this sector. For example, its stimulus is more influential than not throughout the economy. In other words, upstream industrial sectors are preferable because they may influence all downstream sectors, whereas downstream sectors do not have much influence on the upstream sectors. We performed triangulation,[3] in regards to the work of Chenery and Watanabe (1958), on the 2005 input–output coefficient matrices for both Japan and Korea and found that the RMC sector was placed in the upper stream (137th out of 395 sectors for Japan, and 65th out of 350 sectors for Korea) of the supply chain in both economies. Another criterion is whether or not the output of the sector is completely domestic (non imported), as the current study precludes international trade. And most importantly, the sector must be equivalent in both countries. The RMC sector meets all of these criteria.

In Table 2 we summarise the results of calculating SCS via Eq. (9) for the four systems; namely, Leontief, Cobb–Douglas, CES and CES (all estimates), and for two countries; namely, Japan and Korea. The projected equilibrium price $\boldsymbol{\pi}$ for given $\mathbf{z}$, as in Eq. (20), is calculated using Eq. (6) for the Leontief, Eq. (8) for the Cobb–Douglas and Eq. (5) for the CES systems. Along with the SCS, we display the output of the RMC sector of the 2005 input–output table. Notably, the SCS of the Leontief system is very slightly larger than one half the output of the RMC sector, reflecting the productivity doubling of the RMC sector. This is legitimate in regards to Eq. (16), as we consider:

$$[\mathbf{I} - \mathbf{A}]^{-1} - [\langle \mathbf{z} \rangle - \mathbf{A}]^{-1} \approx \mathbf{I} - \langle \mathbf{z} \rangle^{-1} = 1/2$$

[2] For sake of reference, we may also use the estimated elasticities for all sectors, regardless of statistical significance. Such a case will henceforth be indicated as CES (all estimates).

[3] Stages of production leading to final goods are investigated through permutation of sectors. See Kondo (2014) for recent developments.

Table 2
SCS (social cost saved) by productivity doubling of the RMC (ready-mixed concrete) sector. BJPY stands for billion Japanese yens. BKRW stands for billion Korean Republic wons. Values in parentheses are the kurtosis of the corresponding SCS distribution.

	Japan [BJPY]		Korea [BKRW]	
Output	1347		6398	
SCS Leontief	674	(315)	3203	(162)
SCS Cobb–Douglas	926	(52)	4349	(84)
SCS CES	944	(45)	4550	(102)
SCS CES (all estimates)	976	(39)	4643	(75)

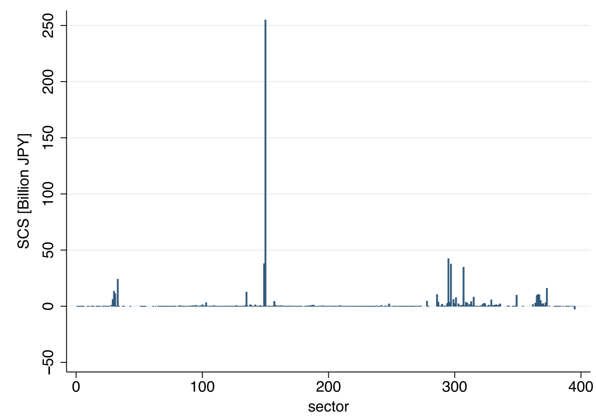

Fig. 5. Sectoral distribution of SCS for productivity doubling of the RMC sector (150th) for the Leontief system (Japan).

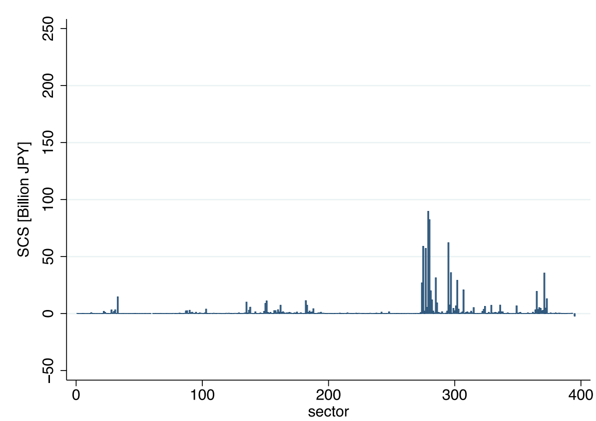

Fig. 6. Sectoral distribution of SCS for productivity doubling of the RMC sector (150th) for the Cobb–Douglas system (Japan).

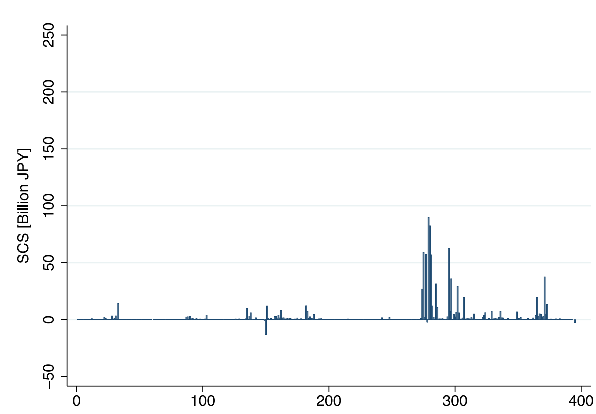

Fig. 7. Sectoral distribution of SCS for productivity doubling of the RMC sector (150th) for the multifactor CES system (Japan).

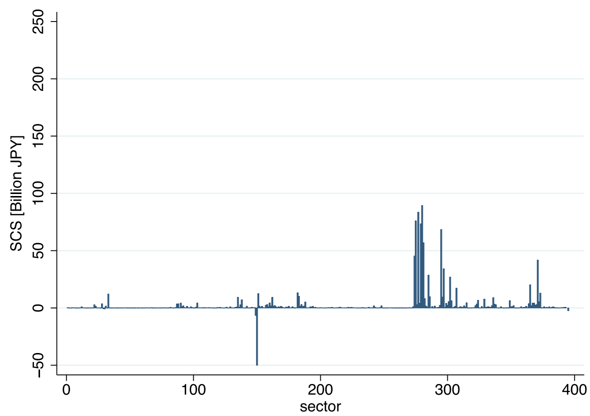

Fig. 8. Sectoral distribution of SCS for productivity doubling of the RMC sector (150th) for the multifactor CES (all estimates) system (Japan).

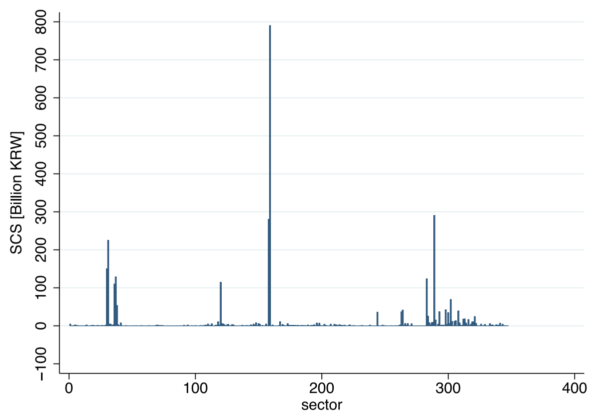

Fig. 9. Sectoral distribution of SCS for productivity doubling of the RMC sector (159th) for the Leontief system (Korea).

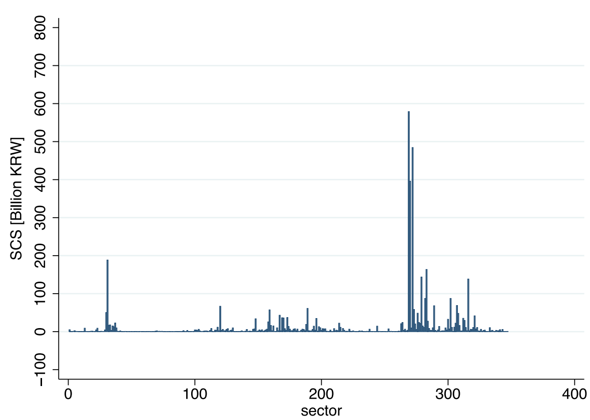

Fig. 10. Sectoral distribution of SCS for productivity doubling of the RMC sector (159th) for the Cobb–Douglas system (Korea).

Conversely, the SCS of the Cobb–Douglas and CES systems is larger than that of the Leontief system, reflecting further propagation across sectors that have greater elasticities.

We now look into the sectoral distribution of the SCS. Figs. 5, 6, 7 and 8 show the projected sector-wise SCS from productivity doubling in the RMC sector under the Leontief, Cobb–Douglas, CES and CES (all estimates) systems, respectively, for Japan. Corresponding figures for Korea are Figs. 9, 10, 11 and 12. As we have anticipated in regards to the previous proposition, the SCS for the Leontief and Cobb–Douglas systems is positively distributed overall.[4] In essence, when there is

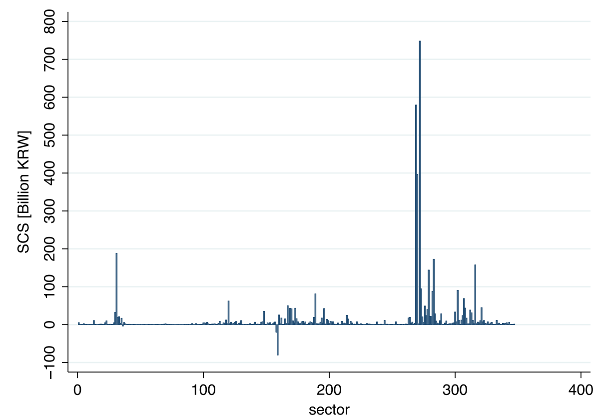

Fig. 11. Sectoral distribution of SCS for productivity doubling of the RMC sector (159th) for the CES system (Korea).

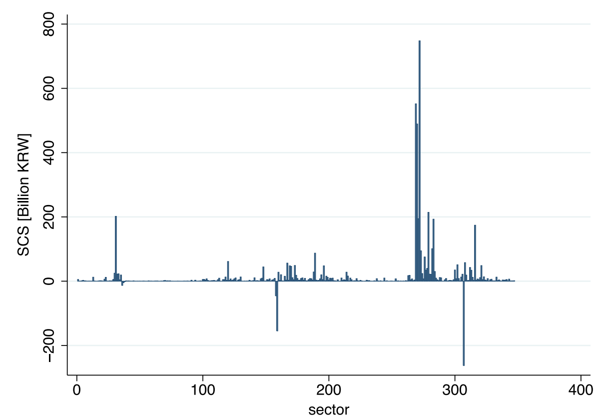

Fig. 12. Sectoral distribution of SCS for productivity doubling of the RMC sector (159th) for the multifactor CES (all estimates) system (Korea).

productivity doubling in one sector, its product price will be cut in half. The inter-sectoral propagation of that price change will nevertheless be different, depending on the elasticity of factor substitution among the interacting sectors. As for the Leontief system, because factor substitution will not exist in any other sector, the price change of RMC to half its former level will have no effect upon RMC's intermediate demand. Thus, in that event, all factor inputs (including the primary factor) for the RMC sector will be reduced by half. This is the main reason why the primary factor for the RMC sector is reduced (as SCS) rather prominently in the Leontief system. Consequently, the intermediate demand of the factors (including the primary factor) will be reduced, respectively, by as much as half the amount that used to go into the RMC sector. Such reduction in intermediate demand and thus, supply, will accumulate due to convergence. In other words, at least half of the primary factor in the RMC sector will be directly reduced and beyond that, the primary factor in any other sector will be indirectly reduced. Figs. 5 and 9 reflect such propagation of productivity doubling in the RMC sector upon primary factor demand under a system of zero elasticity of substitution.

In contrast, as for the Cobb–Douglas system, the intermediate demand for RMC when its price is reduced by half must double, as this is the very definition of the unit elasticity of substitution. Thus, the monetary output and factor inputs (including the primary factor) of the RMC sector will not change. As for an elastic CES system with an elasticity of substitution greater than unity, the factor demand for RMC becomes greater by more than two-fold when the price of RMC is reduced by half. And in that event, the factor inputs of the RMC sector can be increased.[5] In either system, since the system of unit cost functions is strictly concave, the price of all

[4] However, due to the negative entries for **d**, slightly negative values are observed.

[5] This is the main reason why we observe a negative SCS (increased primary factor input) in the RMC sector in Figs. 7 and 11.

factors except that of the primary factor, which will stay constant, will converge in a strictly descending manner. Hence, in equilibrium, the primary factor will be mitigated for the sectors where the primary factor becomes relatively expensive compared with other factor inputs. Notably, Figs. 6 and 7 indicate that the primary factor is reduced (as SCS) rather prominently for the entitled 'Public construction of roads' (279th), 'Public construction of rivers, drainages and others' (280th) and 'Residential construction (non-wooden)' (275th) for Japan. Figs. 10 and 11 indicate that 'Residential building construction' (289th), 'Road construction' (272nd) and 'Non-residential building construction' (270th) are prominent for Korea. These sectors are obviously the ones that extensively utilise RMC for production. In other words, the primary factor in these sectors will be substituted by RMC with reduced prices.

Moreover, we observe from these figures that, not only the magnitude of propagation (in terms of SCS) of the productivity stimulus will be magnified by larger elasticities of substitution, but the distribution of SCS becomes more even. We have measured the 'polarity' of the distribution of SCS over the sectors via kurtosis, as displayed in parentheses in Table 2. The primary factor will be mitigated primarily in the RMC sector, where the productivity is enhanced for the Leontief system, whereas such mitigation of the primary factor will be spread among the sectors for the Cobb–Douglas and CES systems. Put differently, the welfare gain of enhanced productivity in one industry is attained mainly as the curtailment of factor inputs of that particular industry, while keeping the output level consistent, for the Leontief system; whereas, for the Cobb–Douglas and CES systems, the reduced price is appreciated by other industries whose primary factors are reduced by substitution.

4. Concluding remarks

To date, input–output analysis has been a one-of-a-kind framework that considers industry-wide propagation when assessing the costs and benefits of new goods and innovations. Input–output analysis, nonetheless, has based its theory upon the non-substitution theorem, which allows the researcher to study under a fixed technological structure, while restricting the subject of analysis to transformations within final demand. Substitution of technology will, nevertheless, take place in any industry when a new technology is initially introduced into any component (industry) of the economy. Larger influences are typically foreseeable for intermediate industries, as they have much larger and broader feedbacks on economy-wide systems of production.

In order to consider all technology substitution possibilities, we have proposed a methodology to measure the sector specific substitution elasticity for the CES production function, rather than using uniform a priori substitution elasticities (such as zeros and ones) when modelling the economy-wide multisector multifactor production system. A dual analytical method (i.e. unit cost functions) was used to evaluate influences upon general equilibrium technological substitutions and eventually upon social costs and benefits, initiated by the introduction of innovation, which we treat as gains in productivity. We have found that the more elastic production functions (Cobb–Douglas and CES) have more significant and wider propagation effects, whereas inelastic production functions (Leontief) have effects that are relatively less and more polarised. Applications and extensions of this framework could be immense, including internationalisation, dynamicalisation, and quality considerations all remaining for future investigations.

Acknowledgements

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Appendix

Table A1
CES Elasticities and Productivity Growth (Japan 2000–2005).

sector	Elasticity	TFPg	Obs.
Liquid crystal element	2.296***	1.269*** (***)	116
Turbines	1.689***	0.783*** (***)	119
Video recording and playback equipment	2.007***	0.773*** (***)	136
Personal Computers	1.455*	0.647	126
Coal products	1.979**	0.593(***)	91
Frozen fish and shellfish	2.074*	0.449(***)	80
Electronic computing equipment (accessory equipment)	1.871***	0.412** (***)	132
Cyclic intermediates	1.784***	0.367(***)	105
Fowls and broilers	2.199*	0.332(***)	57
Steel ships	1.451***	0.307**(***)	157
Photographic sensitive materials	1.581**	0.283(**)	106
Other business services	2.098***	0.270(***)	122
Electronic computing equipment (except personal computers)	1.668***	0.268	126
Financial service	0.275*	0.260(***)	101
Social welfare (profit-making)	1.268***	0.251(***)	143
Private non-profit institutions serving households, n.e.c. *	1.391*	0.242(***)	105
Repair of ships	1.378**	0.239(***)	142
Inorganic pigment	1.581**	0.233(***)	104
Other iron or steel products	1.345*	0.231	81
Public administration (central) **	1.603***	0.223(***)	219
Boilers	1.646**	0.217(***)	120
Aliphatic intermediates	1.461*	0.214(**)	109
Household electric appliances (except air-conditioners)	1.333**	0.182	153
Medical service (medical corporations, etc.)	1.622**	0.168(**)	156

(continued on next page)

Table A1 *(continued)*

sector	Elasticity	TFPg	Obs.
Synthetic dyes	1.868***	0.165(***)	97
Dishes, sushi and lunch boxes	1.761**	0.165(***)	116
Applied electronic equipment	1.455**	0.160(*)	133
Railway transport (freight)	1.918***	0.154(***)	101
Noodles	1.669**	0.151(*)	108
Motor vehicle parts and accessories	1.701***	0.137* (**)	152
Dextrose, syrup and isomerized sugar	1.405**	0.133(**)	78
Medicaments	1.976*	0.132	135
Electric bulbs	1.570**	0.125(*)	103
Other electrical devices and parts	2.059***	0.121	125
Other general industrial machinery and equipment	1.386*	0.116	140
Other industrial organic chemicals	1.687*	0.115	118
Metal containers, fabricated plate and sheet metal	1.780***	0.104** (**)	134
Metallic furniture and fixture	1.775**	0.103	124
Nursing care (In-facility)	1.585***	0.101(**)	159
Semiconductor making equipment	1.453**	0.099	142
Marine culture	1.717**	0.092	92
Other metal products	1.774***	0.087(*)	145
Bearings	1.627***	0.086	114
Pumps and compressors	2.111***	0.085(**)	129
Wheat, barley and the like	2.952*	0.081	60
Confectionery	1.807***	0.080	121
Other educational and training institutions (profit-making)	1.748**	0.079	74
Sporting and athletic goods	1.578**	0.077	135
Cosmetics, toilet preparations and dentifrices	1.576*	0.074	105
Tires and inner tubes	1.517*	0.072	102
Miscellaneous manufacturing products	1.622***	0.071	180
Gas and oil appliances and heating and cooking apparatus	1.568***	0.069	133
Agricultural public construction	2.039*	0.062	144
Health and hygiene (profit-making)	1.509**	0.059	94
Plumber's supplies, powder metallurgy products and tools	1.596***	0.057	128
Internal combustion engines for vessels	1.808**	0.057	115
Other rubber products	1.740***	0.052	125
Electric wires and cables	1.566***	0.051	121
Other final chemical products	1.782***	0.048	150
Activities not elsewhere classified	3.575***	0.047	179
Paint and varnishes	1.703***	0.047	125
Oil and fat industrial chemicals	1.555*	0.047	91
Compressed gas and liquefied gas	1.593*	0.041	81
Metal products for construction	1.497**	0.040	136
Other pulp, paper and processed paper products	1.517**	0.035	125
Metal molds	1.894***	0.035	127
Health and hygiene (public) **	1.496***	0.033	91
Machinery for agricultural use	1.576**	0.030	142
Publication	1.470*	0.029	105
Other special machinery for industrial use	1.646**	0.026	146
Other industrial inorganic chemicals	1.643**	0.026	116
Abrasive	1.363*	0.025	126
Other services relating to communication	2.444***	0.019	65
Advertising services	1.964***	0.018	103
Electron tubes	1.825***	0.018	116
Retort foods	1.543*	0.012	92
Chemical fertilizer	1.608*	0.012	113
Internal combustion engines for motor vehicles and parts	1.803***	0.010	131
Other structural clay products	1.485**	0.010	107
Newspaper	1.529**	0.007	99
Wooden furniture and fixtures	2.086***	0.004	145
Coated steel	1.981***	0.004	100
Miscellaneous ceramic, stone and clay products	1.455***	0.004	147
Cement	1.577**	0.000	103
Glass fiber and glass fiber products, n.e.c.	1.774***	−0.002	106
Conveyors	1.408**	−0.005	138
Fisheries	1.648***	−0.011	92
Other general machines and parts	1.644***	−0.013	143
Sewage disposal **	1.734***	−0.013	86
Other photographic and optical instruments	0.423**	−0.014	127

(continued on next page)

Table A1 *(continued)*

sector	Elasticity	TFPg	Obs.
Bread	1.664**	−0.015	111
Office supplies	2.608***	−0.015	29
Wiring devices and supplies	1.784***	−0.019	128
Electrical equipment for internal combustion engines	1.483**	−0.021	130
Medical service (non-profit foundations, etc.)	1.812***	−0.021	154
Clay refractories	1.656***	−0.022	109
Cast and forged materials (iron)	2.091***	−0.026	133
Engines	1.859***	−0.026	129
Pulp	2.634**	−0.028	104
Non-ferrous metal castings and forgings	1.615**	−0.034	123
Other wooden products	1.716***	−0.035	160
Railway transport (passengers)	2.086***	−0.040	112
Sugar	1.492**	−0.044	83
News syndicates and private detective agencies	1.434*	−0.045	74
Other electronic components	1.746***	−0.049	152
Electricity	1.476*	−0.052	98
Medical instruments	0.090***	−0.052	151
Repair of motor vehicles	1.442*	−0.052	114
Repair of rolling stock	1.712***	−0.052	117
Other glass products	2.006***	−0.060	107
Bolts, nuts, rivets and springs	1.763***	−0.060	132
Rolled and drawn aluminum	1.824*	−0.063	86
Synthetic fibers	1.636*	−0.065	99
Woven fabric apparel	1.577*	−0.065	101
Whiskey and brandy	2.601*	−0.071	88
Social welfare (private, non-profit) *	1.460***	−0.072	143
Knitted apparel	2.031*	−0.084	107
Accommodations	1.825***	−0.084(**)	161
Medical service (public)	1.808***	−0.087(**)	153
Other transport equipment	1.973***	−0.089	140
Pottery, china and earthenware	2.073***	−0.089(*)	119
Fiber yarns	1.851**	−0.094	94
Plastic footwear	1.965***	−0.095** (**)	108
Nursing care (In-home)	1.552***	−0.095(**)	153
Transformers and reactors	1.600**	−0.102	124
Cast iron pipes and tubes	1.805**	−0.102	90
Cleaning	1.655**	−0.103(*)	88
Aircrafts	1.684**	−0.103	121
Food processing machinery and equipment	1.562**	−0.116(*)	124
Industrial robots	1.520**	−0.117	124
Beauty shops	1.459*	−0.126	91
Plywood	1.713**	−0.126*	86
Passenger motor cars	1.703**	−0.135(*)	123
Audio and video records, other information recording media	1.488*	−0.135(*)	95
Motor vehicle bodies	1.592*	−0.139	125
Barber shops	1.657***	−0.148(***)	86
Repair of machine	1.622**	−0.153(*)	145
Plasticizers	2.262***	−0.153* (***)	84
Other personal services	1.925*	−0.155(**)	113
Rolled and drawn copper and copper alloys	1.829**	−0.166	83
Textile machinery	2.218***	−0.169*(***)	138
Rotating electrical equipment	1.457**	−0.172(**)	127
Chemical machinery	1.528**	−0.176(**)	132
Public baths	1.544*	−0.188(**)	94
Metal processing machinery	1.654***	−0.192** (***)	128
Petrochemical basic products	1.798*	−0.200	89
Image information production and distribution industry	1.678**	−0.201(**)	119
Social welfare (public) **	1.479**	−0.201(***)	142
Hot rolled steel	2.138***	−0.207	97
Crops for feed and forage	2.988***	−0.207*** (***)	58
Crude steel (electric furnaces)	1.870**	−0.226	96
Machinery for service industry	1.378**	−0.233(**)	129
Social education (public) **	1.812*	−0.238(***)	93
Consigned freight forwarding	−0.732**	−0.239(*)	93
Wired communication equipment	2.164***	−0.243* (***)	150
Other electrical devices and parts	1.388**	−0.246(***)	142
Iron and steel shearing and slitting	2.379***	−0.265(*)	83
Other wearing apparel and clothing accessories	1.800*	−0.270(***)	109
Coal mining, crude petroleum and natural	1.850***	−0.277*** (***)	89

(continued on next page)

Table A1 *(continued)*

sector	Elasticity	TFPg	Obs.
gas			
Rolling stock	1.808***	−0.284*** (***)	138
Research and development (intra-enterprise)	1.461**	−0.317* (***)	126
Batteries	1.640**	−0.317(***)	129
Watches and clocks	1.471***	−0.339(***)	121
Wooden chips	1.626*	−0.350(***)	64
Optical fiber cables	1.634**	−0.360(***)	115
Crude steel (converters)	2.635***	−0.377** (***)	99
Electric measuring instruments	1.362*	−0.399(***)	128
Storage facility service	1.602**	−0.404(***)	105
Copper	2.110**	−0.448	77
Private non-profit institutions serving enterprises	1.586*	−0.450(***)	91
Other non-ferrous metal products	2.152**	−0.549*** (**)	88
Pig iron	1.600**	−0.680* (*)	169
Research institutes for natural science (pubic) **	2.090*	−0.745* (***)	90
Metallic ores	1.634***	−0.799*** (***)	82
Ferro alloys	1.652*	−0.823	85
Research institutes for natural sciences (profit-making)	2.108**	−0.855(***)	93

Note: The statistical significances in parentheses are based on the intercept of regression (2).

Table A2
CES Elasticities and Productivity Growth (Korea 2000–2005).

sector	Elasticity	TFPg	Obs.
Photographic and optical instruments	2.116***	0.688*** (***)	165
Computer and peripheral equipment	1.660**	0.619(*)	166
Watches and clocks	1.615**	0.618(***)	147
Electric resistors and storage batteries	2.033***	0.582*** (***)	156
Research institutes(private, non-profit, commercial)	1.498*	0.566(***)	152
Electric household audio equipment	2.141***	0.564* (***)	151
Misc. amusement and recreation services	1.817***	0.514*** (***)	153
Supporting land transport activities	1.555**	0.507(***)	126
Wood furniture	1.495*	0.447(***)	165
Education (commercial)	1.682**	0.417(***)	127
Other audio and visual equipment	1.614*	0.402(*)	164
Bicycles and parts and misc. transportation equipment	1.860***	0.400*** (***)	132
Household laundry equipment	1.480**	0.399(***)	145
Electron tubes	1.709***	0.393*** (**)	159
Semiconductor devices	1.542**	0.371	162
Road freight transport	1.961**	0.370(***)	131
Printed circuit boards	1.550**	0.357* (*)	160
Section steel	1.520**	0.340(*)	121
Supporting air transport activities	2.164***	0.339** (***)	108
Business and professional organizations	2.735***	0.335(***)	95
Passenger automobiles	1.674***	0.334** (***)	155
Office machines and devices	1.536*	0.332(**)	154
Industrial glass products	2.121***	0.292*** (**)	169
Central bank and banking institutions, Non-bank depository institutions	1.864**	0.287(***)	120
Water supply	1.675**	0.285(**)	124
Road passenger transport	1.983***	0.285(**)	131
Clay products for construction	1.800**	0.285(**)	140
Lime, gypsum, and plaster products	1.813*	0.282(***)	134
Food processing machinery	1.592**	0.278** (***)	143
Boiler, Heating apparatus and cooking appliances	1.610*	0.274(**)	164
Pulp	1.526*	0.273	112
Medical instruments and supplies	1.793***	0.271(**)	167
Regulators and Measuring and analytical instruments	1.603**	0.266(**)	167
Coastal and inland water transport	1.552**	0.265(***)	134
Leather	1.831**	0.260* (**)	129
Cosmetics and dentifrices	1.974**	0.255* (**)	165
Non-life insurance	1.586*	0.250(*)	107

(continued on next page)

Table A2 *(continued)*

sector	Elasticity	TFPg	Obs.
Misc. chemical products	1.589**	0.245(**)	172
Sports organizations and sports facility operation	1.635***	0.241(**)	144
Social work activities(other)	1.757**	0.229(**)	137
Trucks and Motor vehicles with special equipment	1.845***	0.229*** (***)	154
Other membership organizations	1.855**	0.225(**)	114
Wooden containers and Other wooden products	2.034**	0.224(**)	124
Bakery and confectionery products	1.819*	0.213(**)	174
Household refrigerators	1.795***	0.213** (***)	152
Asbestos and mineral wool products	1.754**	0.212(**)	145
Air-conditioning equipment and industrial refrigeration equipment	1.524**	0.209	163
Buses and vans	1.736***	0.208(***)	152
Medicaments	1.998***	0.207*** (**)	175
Textile machinery	1.468*	0.199	165
Silk and hempen fabrics	1.982**	0.196*	110
Printing ink	2.049***	0.190** (***)	127
Motors and generators	1.731***	0.187** (**)	161
Misc. non-metallic minerals	2.262***	0.185	108
Sanitary services(public)	1.701**	0.185	130
Concrete blocks, bricks, and other concrete products	1.891***	0.182** (***)	144
Lubricants	1.736*	0.180	131
Pottery	1.560*	0.177	155
Railroad vehicles and parts	1.537**	0.174	157
Metal molds and industrial patterns	1.662**	0.169	152
Luggage and handbags	2.172***	0.161** (***)	118
Pens, pencils, and other artists' materials	1.794***	0.160** (*)	145
Motion picture, Theatrical producers, bands, and entertainers	1.619***	0.158(*)	151
Dairy products	1.971**	0.157(*)	144
Publishing	1.473*	0.154	124
Ship repairing and ship parts	1.799***	0.154** (**)	151
Misc. nonmetallic minerals products	1.680*	0.152	140
Household glass products and others	1.940***	0.143** (*)	136
Agricultural implements and machinery	1.620***	0.129	155
Social work activities(public)	2.169***	0.124	121
Reproduction of recorded media	1.987***	0.123* (**)	136
Anthracite	2.325***	0.122	132
Paints, varnishes, and allied products	1.700**	0.118	155
Line telecommunication apparatuses	1.636**	0.118	161
Leather wearing apparels	1.845*	0.116	108
Library, museum and similar recreation related services(public)	1.843***	0.112	133
Paper containers	1.927***	0.107	132
Knitted clothing accessories	2.204**	0.100	116
Synthetic fiber fabrics	1.852**	0.097	128
Motorcycles and parts	1.687**	0.095	148
Accommodation	1.657**	0.094	132
Ginseng products	1.686*	0.089	104
Sheet glass and primary glass products	1.985***	0.088	129
Electric transformers	1.851***	0.087	150
Salted, dried and smoked seafoods	3.290*	0.084	98
Misc. electric equipment and supplies	1.503*	0.082	155
Printing	1.579***	0.081	143
Abrasives	1.710**	0.074	142
Cement	2.086***	0.070	154
Prepared livestock feeds	1.713*	0.069	154
Library, museum and similar recreation related services(other)	1.578*	0.066	135
Knitted fabrics	1.928**	0.064	111
Internal combustion engines and turbines	1.649***	0.063	156
Fiber bleaching and dyeing	1.949**	0.058	119
Cleaning and disinfection services	1.552*	0.058	104
Other paper products	1.597*	0.054	160
Other raw paper and paperboard	1.808***	0.043	150
Petrochemical intermediate products and Other basic organic chemicals	1.876**	0.042	163
Fastening metal products	1.661**	0.038	137
Household articles of plastic material	1.721**	0.032	124
Stationery paper and office paper	1.497*	0.032	125
Recording media and Photographic chemical products	1.853***	0.031	142

(continued on next page)

Table A2 *(continued)*

sector	Elasticity	TFPg	Obs.
Medical and health services (commercial)	2.288***	0.030	160
Ready mixed concrete	2.040***	0.030	132
Supporting water transport activities	1.637**	0.029	125
Other leather products	1.858*	0.028	91
Construction and mining machinery	1.577**	0.025	156
Nitrogen compounds	1.759**	0.025	114
Road construction	1.389*	0.023	179
Metal products for construction	1.828**	0.019	134
Industrial plastic products	1.674**	0.014	167
Land clearing and reclamation, and irrigation project construction	1.539**	0.009	167
Soy sauce ad bean paste	1.750*	0.008	127
Communications line construction	1.585**	0.006	159
Metal furniture	1.565**	0.006	146
Thread and other fiber yarns	1.915***	−0.004	114
Life insurance	1.627*	−0.005	106
Capacitors and rectifiers, Electric transmission and distribution equipment	1.583***	−0.005	167
Musical instruments	1.506**	−0.005	155
Iron foundries and foundry iron pipe and tubes	1.840***	−0.006	152
Misc. petroleum refinery products	1.793*	−0.011	127
Medical and health services(public)	2.180***	−0.011	138
Pumps and compressors	1.601**	−0.018	158
Adhesives, gelatin and sealants	1.882**	−0.021	143
Rubber products	1.763***	−0.022	154
Canned or cured fruits and vegetables	1.761*	−0.034	139
Corrugated paper and solid fiber boxes	1.662**	−0.040	119
Crushed and broken stone abd Other bulk stones	1.787*	−0.044	120
Railroad construction	1.432*	−0.045	170
Medical and health services(Private, non-profit)	1.946***	−0.046	141
Architectural engineering services	1.606**	−0.048	143
Newspapers	1.873***	−0.049	118
Sporting and athletic goods	1.720*	−0.058	159
Treatment and coating of metals and Misc. fabricated metal products	1.722**	−0.060	171
Synthetic fiber yarn	1.903**	−0.067	124
Plywood	1.769*	−0.067	122
Electric lamps and electric lighting fixtures	1.575**	−0.068	160
Synthetic fibers	1.701*	−0.073	128
Research institutes(public)	1.611**	−0.080	182
Services related to real estate	2.091**	−0.080	91
Lumber	2.081**	−0.080	105
Insulated wires and cables	1.777***	−0.089	169
Other nonferrous metal ingots	1.697*	−0.097	121
Personal services	1.977***	−0.110(*)	124
Conveyors and conveying equipment	1.649**	−0.110	165
Electric power plant construction	1.334*	−0.125	171
Starches	2.220**	−0.137(*)	102
Footwear	1.836***	−0.139* (*)	131
Other edible crops	2.586**	−0.139	58
Explosives and fireworks products	1.637**	−0.156	139
Wooden products for construction	1.953***	−0.164** (**)	114
Bolts, nuts, screws, rivets, and washers	1.688**	−0.168*	139
Pig iron	1.922***	−0.171	138
Railroad passenger transport	2.544***	−0.181**(*)	135
Gold and silver ingots	2.860***	−0.186(*)	112
Sand and gravel	2.520**	−0.201	113
Steel ships	1.549**	−0.203	181
Telecommunications	1.623*	−0.213	123
Other personal repair services	1.925***	−0.225*** (***)	147
Education (public)	1.936***	−0.231*** (**)	169
Gasoline and Jet oil	1.698**	−0.234	127
Other ships	1.888***	−0.287*** (***)	166
Forgings	2.125***	−0.289*** (**)	122
Cargo handling	1.861**	−0.373(***)	122
Research and experiment in enterprise	1.415**	−0.502(***)	225
Education (private, non-profit)	1.525*	−0.509(***)	148

Note: The statistical significances in parentheses are based on the intercept of regression (2).

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