JOHANNES KEPLER
UNIVERSITY LINZ

On energy based vector-hysteresis models

Bachelor-Thesis presentation
Franz Scharnreitner
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JOHANNES KEPLER
UNIVERSITY LINZ
Altenbergerstraße 69
4040 Linz, Austria
jku.at

Code overview

- Theoretical foundations
- Models
- Equivalence
- Code overview

Theoretical foundations

Subdifferential [3]

- Let $f : \mathbb{R}^n \to \mathbb{R}$ be a continuous convex function
- Subdifferential: $\partial f(x) = \{p \in \mathbb{R}^n : f(y) \geq f(x) + \langle p, y - x\rangle \ \forall y \in \operatorname{dom} f\}$
- If $f \in C^1(\mathbb{R}^n, \mathbb{R})$, then $\partial f(x) = \{\nabla f(x)\}$
- Optimality condition: x is a minimum of f iff

$$0 \in \partial f(x) \tag{1}$$

  holds.
- Examples:
  - Subdifferential of $f(x) : x \mapsto \|x\|$: $\partial f(0) = \{x : \|x\| \leq 1\}$
  - Let $K = \{u \in \mathbb{R}^3 : \|u\| \leq \chi\}$ Subdifferential of $I_K(x)$:

$$\partial I_K(x) = \begin{cases} \{0\} & \|x\| < \chi \\ \{p : p = \lambda x \ \forall \lambda > 0\} & \|x\| = \chi \end{cases}$$

Lagrange function [3]

- Given the convex problem $\min_x f(x)$ s.t. $g(x) \leq 0$ (CP). The Lagrange function is then defined as

$$\mathcal{L}_\lambda(x) = f(x) + \lambda g(x) \tag{2}$$

For x to be solution to (CP) it is necessary and sufficient, that the following conditions are fulfilled:

$$\lambda \geq 0, g(x) \leq 0, \lambda g(x) = 0 \tag{3}$$

$$0 \in \partial f(x) + \lambda \partial g(x) = (\nabla \mathcal{L}_\lambda(x)) \tag{4}$$

Legendre-Fenchel Transform [1]

Let f be a convex function. The Legendre-Fenchel transform, or convex conjugate is defined as

$$f^*(x) = \sup_y\{\langle x, y\rangle - f(y)\} \tag{5}$$

- f^* is convex
- If f is differentiable and strictly convex, the equality $f^{**}(x) = f(x)$ holds.
- $f'(x) = \arg\sup_y \langle x, y\rangle - f^*(y)$

Models

Unrestrained Model [2]

Can be derived from the material law $E(b, m) = \frac{\mu_0}{2}\|h\| + U(m)$, where E is the internal energy of a ferromagnetic material and μ_0 is the magnetic permeability of vacuum, b the magnetic flux density, m the magnetization and $h = \frac{1}{\mu_0}b - m$ the magnetic field strength,

Unrestrained model

Find

$$\bar{m} = \arg\min_{m} U(m) - \langle h, m \rangle + \chi\|m - m_p\| \tag{6}$$

Restrained Model [2]

Restrained Model

Find

$$h_r = \arg\min_{u \in K(h)} S(u) - \langle m_p, u \rangle \tag{7}$$

where $K(h) = \{u \in \mathbb{R}^3 : \|u - h\| \leq \chi\}$

S is chosen so that

$$\frac{\partial S}{\partial u}(h_r) = \bar{m} \Leftrightarrow \frac{\partial U}{\partial m}(\bar{m}) = h_r \tag{8}$$

which can be accomplished by choosing S as the Legendre-Fenchel conjugate of U [2]

Equivalence

Equivalence

Theorem

The two proposed models are equivalent in the sense that if $\bar{m}$ minimizes the unrestrained model, then $h_r = \frac{\partial U}{\partial m}(\bar{m})$ minimizes the restrained problem and inversely if h_r minimizes the restrained problem, then $\bar{m} = \frac{\partial S}{\partial u}(h_r)$ minimizes the unrestrained problem.

Proof

- Restrained problem:

$$h_r = \arg\min_{u\in K(h)} S(u) - \langle m_p, u\rangle$$

- Unrestrained problem:

$$\bar{m} = \arg\min_{m} U(m) - \langle h, m\rangle + \chi\|m - m_p\|$$

- Recall:
  - Subdifferential of $f(x) : x \mapsto \|x\|$: $\partial f(0) = \{x : \|x\| \leq 1\}$
  - Lagrange optimality condition
  - $\frac{\partial S}{\partial u}(h_r) = \bar{m} \Leftrightarrow \frac{\partial U}{\partial m}(\bar{m})$

Code overview

Code overview

- Two methods have been implemented in Julia so far.
  - The restrained problem is first solved using a global newton method, the solution is checked, if it is in $K(h)$ and if not it must be on $\partial K(h)$ and newton method is run again, with a parametrization in sphere coordinates
  - For the restrained problem the proximal gradient method is implemented
- We can verify the equivalence for the test functions

$$U(m) = m^T A m + b^T m - c \tag{9}$$

$$S(u) = \frac{1}{2}(m - b)^T A^{-1}(m - b) + c \tag{10}$$

References I

[1] Amir Beck. 2017. First-Order Methods in Optimization. Society for Industrial and Applied Mathematics, Philadelphia, PA. DOI: 10.1137/1.9781611974997. https://epubs.siam.org/doi/abs/10.1137/1.9781611974997.

[2] Manfred Kaltenbacher, Klaus Roppert, Lukas Domenig, and Herbert Egger. 2022. Comparison of Energy Based Hysteresis Models. In (January 2022), pp. 1–4. DOI: 10.1109/COMPUMAG55718.2022.9827509.

[3] R Tyrrell Rockafellar. 1997. Convex analysis. Volume 11. Princeton university press.