EXPERIMENT – 3

<u>AIM</u>: Verification of system characteristics.

<u>OBJECTIVES:</u>

a. Linearity.
b. Time invariance.
c. Stability

<u>SOFTWARE REQUIRED:</u> MATLAB

<u>Pre-Lab</u>

Linearity

A system is said to be linear if it obeys homogeneity and superposition principle.

(a) Homogeneity:

If $x_1(n) \rightarrow y_1(n)$ & If $x_2(n) \rightarrow y_2(n)$ then

$$\boxed{\alpha x_1(n) + \beta x_2(n) \Rightarrow \alpha y_1(n) + \beta y_2(n)}$$

*i.e., zero input leads to zero output.

Similarly

If $x_1(t) \rightarrow y_1(t)$ & If $x_2(t) \rightarrow y_2(t)$ then

$$\boxed{\alpha x_1(t) + \beta x_2(t) \Rightarrow \alpha y_1(t) + \beta y_2(t)}$$

(b) Superposition Principle: if response H is linear then

$H\{a_1 x_1(n) + a_2 x_2(n)\} = a_1 H\{x_1(n)\} + a_2 H\{x_2(n)\}$ Discrete domain

$H\{a_1 x_1(t) + a_2 x_2(t)\} = a_1 H\{x_1(t)\} + a_2 H\{x_2(t)\}$ Continuous domain

If these relations are not obeyed, then the system is Non linear.

Time Invariance:

A system is time- in variant, if its input-output characteristics do not change with time.

$H\{x(n)\} = y(n)$ then $H\{x(n-m)\} = y(n-m) = y(n-m)$

In continuous domain

$x_1(t) \rightarrow y_1(t)$ then $x_1(t-t_0) \rightarrow y_1(t-t_0)$

Causality:

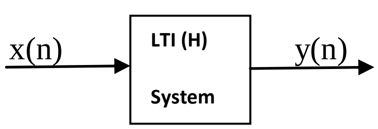
A system where the output is dependent upon the past and present system inputs but not upon the future inputs is a causal system. Either a system will be causal or non causal or anti causal in nature.

An LTI system represented by its impulse response *h(t)* is causal if $h(t) = 0 \; for \; t < 0$ the output of a causal LTI system with a causal input x(t) is $y(t) = \int_0^t x(\tau)h(t-\tau)d\tau$

Stability:

A system is stable, if every bounded input produces a bounded output.

Discrete Domain:-



$y(n) = h(n) * x(n) = \sum_{k=-\infty}^{n} h(k)x(n-k)$

$\Rightarrow |y(n)| = |\sum_{k-\infty}^{\infty} h(k)x(m-k)|$

$\Rightarrow |y(n)| = \sum_{k-\infty}^{\infty} |h(k)||x(n-k)|$ According to the limiting theorem

Now assume bounded input.

i.e $|x(n)| \leq B_x < \infty$ then

$|x(n-k)| \leq B_x$

$$\boxed{|y(n)| \leq B_x \sum_{k=-\infty}^{n} |h(k)|}$$

From this relation the output $|y(n)|$ will be bounded only when

$$\boxed{\sum_{k=-\infty}^{n} |h(k)| < \infty}$$

i.e impulse response is absolutely summable

let

$$|x(t-\tau)| \leq B_x < \infty$$

<u>Similarly in continuous domain:-</u>

$$|y(t)| = \left|\int_{-\infty}^{\infty} h(\tau)x(t-\tau) \cdot d\tau\right|$$

$$\leq \int_{-\infty}^{\infty} |h(\tau)||x(t-\tau)| \cdot d\tau$$

$$\leq B_x \int_{-\infty}^{\infty} |h(\tau)|.$$

$\Rightarrow$ $|y(t)|$ will be bounded only when

$$\boxed{\int_{-\infty}^{\infty} |h(t)| < \infty}$$

Impulse response is absolute integrable.

<u>In-Lab</u> (Write MATLAB programs to do the following)

1. Consider the system $y(t) = x(t)cos(\pi t)$, where $x(t)$ and $y(t)$ are inputs and outputs respectively.
   i. Determine whether the system is time variant or invariant by considering two inputs $x_1(t)$ and $x_2(t)$ where $x_1(t) = u(t)$ $x_2(t) = u(t-1)$. Plot $x_1(t)$, $y_1(t)$, $x_2(t)$ and $y_2(t)$. Check $y_1(t-1)$ is same as $y_2(t)$ or not. If same, then system is TIV, otherwise not.
   ii. Determine whether the system is linear or not by considering same $x_1(t)$ and $x_2(t)$.
   iii. Test whether the system is stable or not. Plot x(t) and y(t) for t is between 0 to 10 and for the decreasing/increasing/constant trend of y(t) and predict the stability. Test for $x(t) = u(t)$ and $x(t) = r(t)$. Compare and discuss.

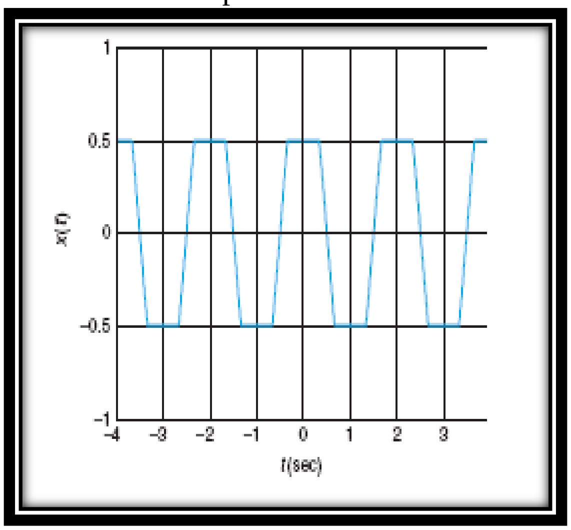
2. Zener Diode-MATLAB

A zener diode circuit is such that the output corresponding to an input $v_s(t)=\cos(\pi * t)$ is a clipped sinusoid

$$x(t)=\begin{cases}0.5 \;\; for \;\; v_s(t)>0.5 \\ -0.5 \; for \, v_s(t)<-0.5 \\ v_s(t) \;\; otherwise\end{cases}$$

as in the following figure for a few periods. Use MATLAB to generate the input and the output signals and plot them in the same plot for $0 \le t \le 4$ at time intervals of 0.001.

(a) Is this system linear? Compare the output obtained from $v_s(t)$ with that obtained from $0.3\, v_s(t)$
(b) Is the system time invariant? Explain.



PROGRAM:-

RESULTS: -

Post-Lab

Let $y(t) = x(t).f(t)$, $f(t) = u(t) - u(t-2)$ and x(t)=u(t), test whether the system is linear , causal, time invariant and stable. Plot, calculate and predict.