Digital Filter Design (EET 3134)

Lab 06: Design of IIR Filters using Billinear Transformation Method

(Submission by: 11th July 2018)

Branch: ECE			
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1 Bilinear transformation

To avoid the limitations of impulse-invariance transformation caused by the aliasing effect, we need a one-to-one mapping from the s-plane to the s-plane. The bilinear transformation is an invertible nonlinear mapping between the s-plane and the z-plane defined by

$$s = f(z) = \frac{2}{T}\frac{1 - z^{-1}}{1 + z^{-1}} \tag{1}$$

The parameter T, which has no effect on the design process, may be given any value that simplifies the derivations. We emphasize that, in contrast to the impulse-invariance case, T does not have any useful interpretation as a sampling interval because the bilinear transformation does not involve any sampling operation.

The mapping defined by (1) satisfies the three desirable conditions (a)–(c) discussed in the previous assignment; this makes the bilinear transformation the most popular technique for mapping continuous time filters to discrete-time filters.

Another name for this transformation is the linear fractional transformation because when cleared of fractions, we obtain

$$\frac{T}{2}sz + \frac{T}{2}s - z + 1 = 0 \tag{2}$$

which is linear in each variable if the other is fixed, or bilinear in s and z.

Using $s = \sigma + j\Omega$ in (1), we obtain

$$z = \frac{\left(1 + \frac{\sigma T}{2} + j\frac{\Omega T}{2}\right)}{\left(1 - \frac{\sigma T}{2} - j\frac{\Omega T}{2}\right)} \tag{3}$$

Solving for ω as a function of Ω, we obtain

$$\omega = 2\tan^{-1}\left(\frac{\Omega T}{2}\right) \tag{4}$$

$$\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right) \tag{5}$$

This shows that Ω is nonlinearly related to (or warped into) ω but that there is no aliasing. The complex plane mapping under (1) is shown in Fig.1.

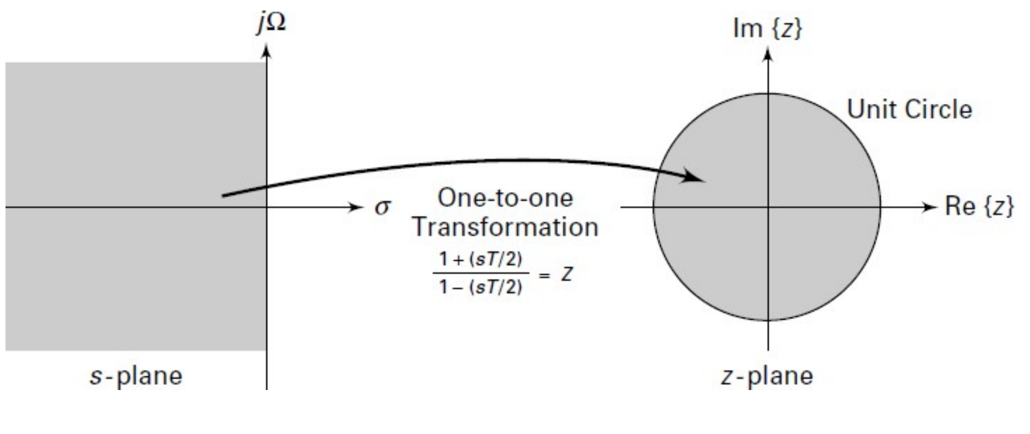


Figure 1: Complex-plane mapping in bilinear transformation

2 Prelab

Prior to beginning this lab, you must have carefully read over this lab in order to plan how to implement the tasks assigned. Make sure you are aware of all the items you need to include in your lab report.

3 Lab Assignments

3.1 Assignment 01

Design a lowpass digital filter using a Butterworth prototype to satisfy

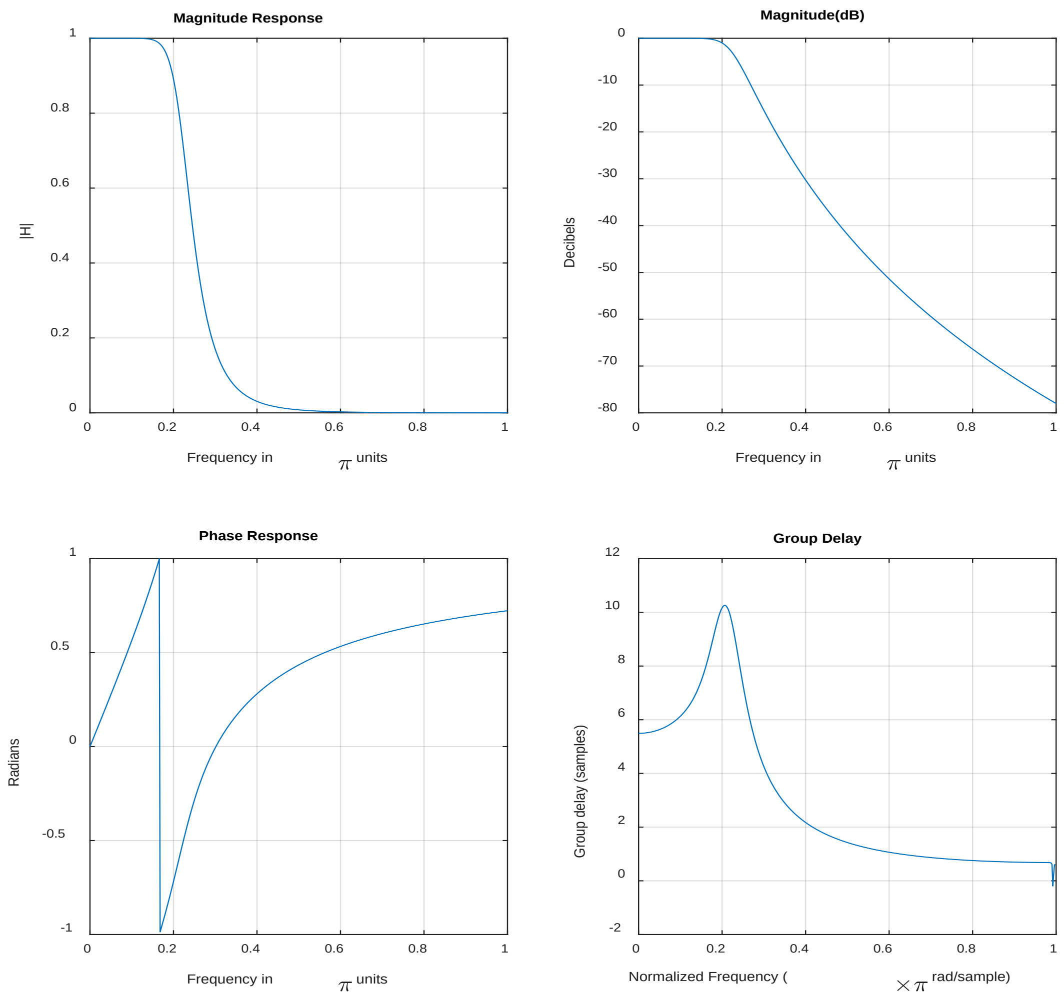
$$\omega_p = 0.2\pi,\ \omega_s = 0.3\pi,\ A_p = 1dB, \text{ and } A_s = 15dB$$

Solution

CODE

```
clc;clear;close all;
% Digital Filter Specifications:
Wp = 0.2*pi; % digital Passband freq in rad
Ws = 0.3*pi; % digital Stopband freq in rad
Rp = 1; % Passband ripple in dB
As = 15; % Stopband attenuation in dB
% Analog Prototype Specifications: Inverse mapping for frequencies
T = 1; Fs = 1/T; % Set T=1
OmegaP = (2/T)*tan(Wp/2); % Prewarp Prototype Passband freq
OmegaS = (2/T)*tan(Ws/2); % Prewarp Prototype Stopband freq
% Analog Butterworth Prototype Filter Calculation:
N = ceil((log10((10^(Rp/10)-1)/(10^(As/10)-1)))/(2*log10(Wp/Ws)));
OmegaC = Wp/((10^(Rp/10)-1)^(1/(2*N)));
[z,p,k] = buttap(N);
p = p*OmegaC;
k = k*OmegaC^N;
B = real(poly(z));
b0 = k; cs = k*B; ds = real(poly(p));
%Butterworth Filter Order = 6
% Bilinear transformation:
[b,a] = bilinear(cs,ds,Fs);
%plots
% Calculation of Frequency Response:
w = [0:1:500]*pi/500; H = freqs(cs,ds,w);
mag = abs(H); db = 20*log10((mag+eps)/max(mag)); pha = angle(H);
figure;
subplot(222);plot(w/pi,db);grid on;title('Magnitude(dB)');xlabel('Frequency in \pi
units');ylabel('Decibels');
subplot(221);plot(w/pi,mag);grid on;title('Magnitude Response');xlabel('Frequency in
\pi units');ylabel('|H|');
subplot(223);plot(w/pi,-pha/pi);grid on;title('Phase Response');xlabel('Frequency in \pi
units');ylabel('Radians');
subplot(224);grpdelay(b,a);grid on;title('Group Delay')
```

PLOT



Magnitude Response
|H|
Frequency in π units
Magnitude(dB)
Decibels
Frequency in π units
Phase Response
Radians
Frequency in π units
Group Delay
Group delay (samples)
Normalized Frequency (×π rad/sample)

3.2 Assignment 02

Design a lowpass digital filter using a Chebyshev - I prototype to satisfy

$$\omega_p = 0.2\pi,\ \omega_s = 0.3\pi,\ A_p = 1dB, \text{ and } A_s = 15dB$$

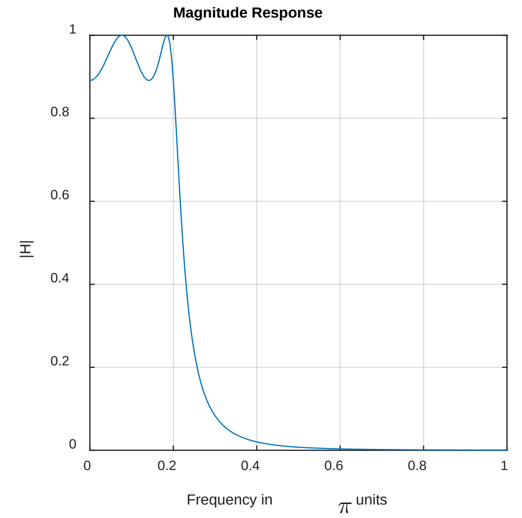
Solution

CODE

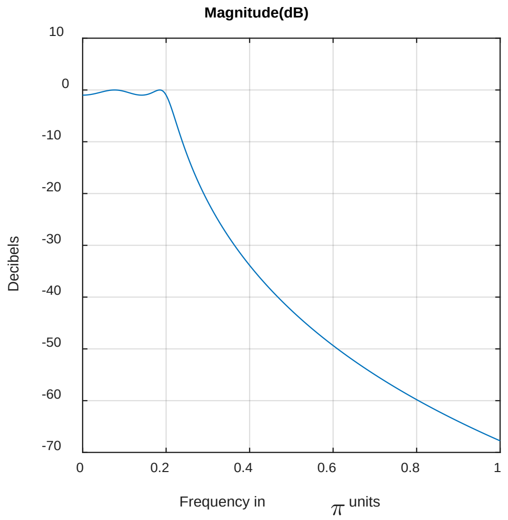
```
clc;clear;close all;
% Digital Filter Specifications:
Wp = 0.2*pi; % digital Passband freq in rad
Ws = 0.3*pi; % digital Stopband freq in rad
Rp = 1; % Passband ripple in dB
As = 15; % Stopband attenuation in dB
% Analog Prototype Specifications: Inverse mapping for frequencies
T = 1; Fs = 1/T; % Set T=1
OmegaP = (2/T)*tan(Wp/2); % Prewarp Prototype Passband freq
OmegaS = (2/T)*tan(Ws/2); % Prewarp Prototype Stopband freq
% Analog Chebyshev-1 Prototype Filter Calculation:
ep = sqrt(10^(Rp/10)-1); A = 10^(As/20);
OmegaC = Wp; OmegaR = Ws/Wp; g = sqrt(A*A-1)/ep;
N = ceil(log10(g+sqrt(g*g-1))/log10(OmegaR+sqrt(OmegaR*OmegaR-1)));
[z,p,k] = cheb1ap(N,Rp); ds = real(poly(p)); aNn = ds(N+1);
p = p*OmegaC; ds = real(poly(p)); aNu = ds(N+1);
k = k*aNu/aNn;
b0 = k; B = real(poly(z)); cs = k*B;
% Chebyshev-1 Filter Order = 4
% Bilinear transformation:
[b,a] = bilinear(cs,ds,Fs);

%plota
w = [0:1:500]*pi/500; H = freqs(cs,ds,w);
mag = abs(H); db = 20*log10((mag+eps)/max(mag)); pha = angle(H);
figure;set(gcf, 'Position', [0, 0, 700, 700])
subplot(222);plot(w/pi,db);grid on;title('Magnitude(dB)');xlabel('Frequency in \pi
units');ylabel('Decibels');
subplot(221);plot(w/pi,mag);grid on;title('Magnitude Response');xlabel('Frequency in
\pi units');ylabel('|H|');
subplot(223);plot(w/pi,-pha/pi);grid on;title('Phase Response');xlabel('Frequency in \pi
units');ylabel('Radians');
subplot(224);grpdelay(b,a);grid on;title('Group Delay')
```

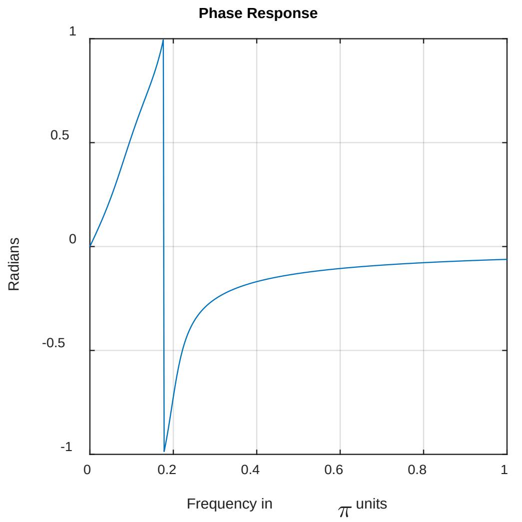
PLOT



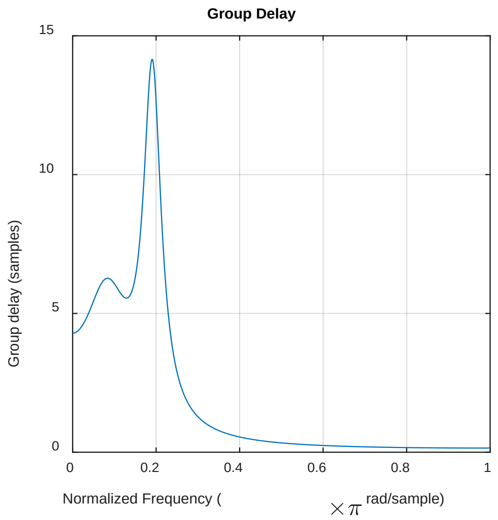
Magnitude Response
1
0.8
0.6
0.4
0.2
0
|H|
0
0.2
0.4
0.6
0.8
1
Frequency in π units



Magnitude(dB)
10
0
-10
-20
-30
-40
-50
-60
-70
Decibels
0
0.2
0.4
0.6
0.8
1
Frequency in π units



Phase Response
1
0.5
0
-0.5
-1
Radians
0
0.2
0.4
0.6
0.8
1
Frequency in π units



Group Delay
15
10
5
0
Group delay (samples)
0
0.2
0.4
0.6
0.8
1
Normalized Frequency (×π rad/sample)

3.3 Assignment 03

Design a lowpass digital filter using a Chebyshev - II prototype to satisfy

$$\omega_p = 0.2\pi,\ \omega_s = 0.3\pi,\ A_p = 1dB, \text{ and } A_s = 15dB$$

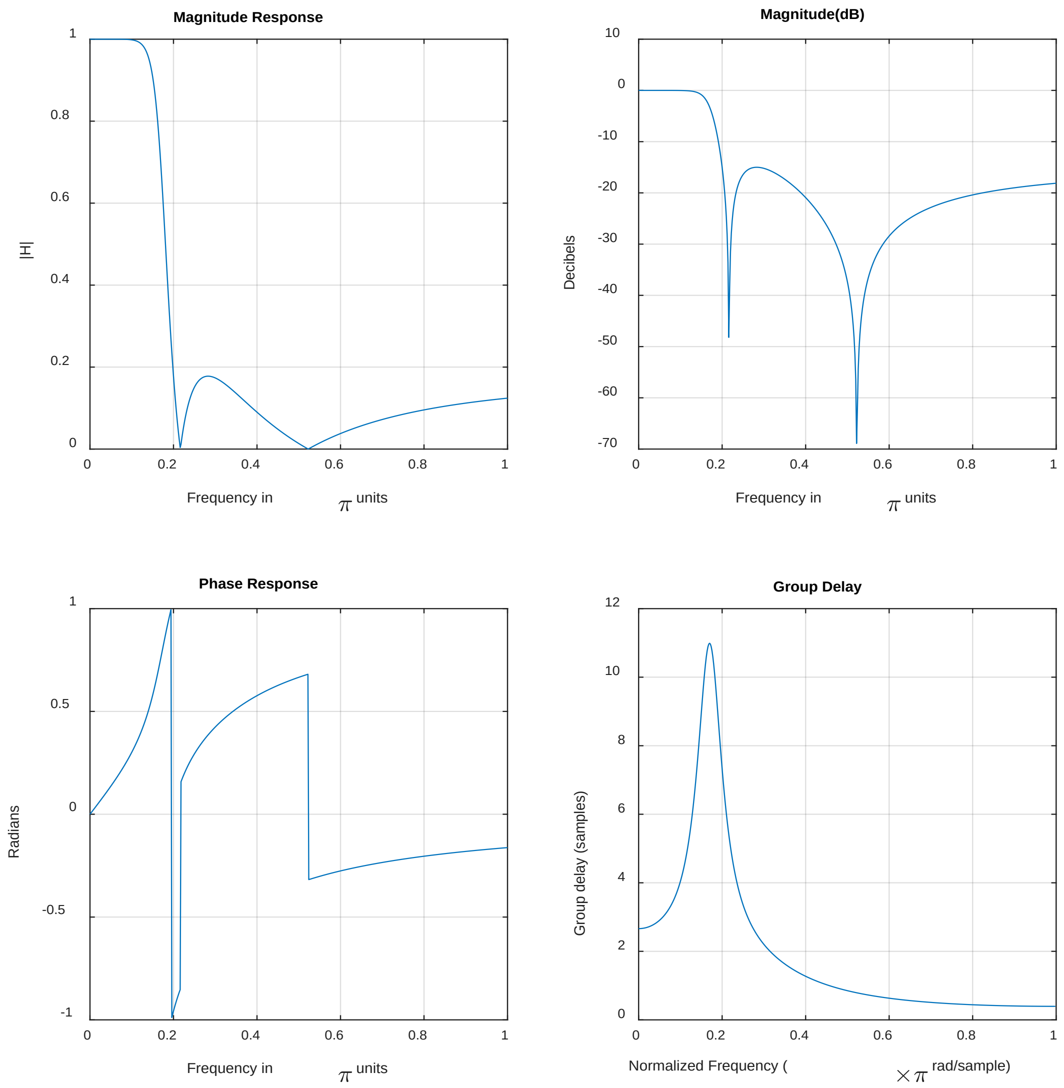
Solution

CODE

```
clc;clear;close all;
% Digital Filter Specifications:
Wp = 0.2*pi; % digital Passband freq in rad
Ws = 0.3*pi; % digital Stopband freq in rad
Rp = 1; % Passband ripple in dB
As = 15; % Stopband attenuation in dB
% Analog Prototype Specifications: Inverse mapping for frequencies
T = 1; Fs = 1/T; % Set T=1
OmegaP = (2/T)*tan(Wp/2); % Prewarp Prototype Passband freq
OmegaS = (2/T)*tan(Ws/2); % Prewarp Prototype Stopband freq
% Analog Chebyshev-2 Prototype Filter Calculation:
ep = sqrt(10^(Rp/10)-1); A = 10^(As/20);
OmegaC = Wp; OmegaR = Ws/Wp; g = sqrt(A*A-1)/ep;
N = ceil(log10(g+sqrt(g*g-1))/log10(OmegaR+sqrt(OmegaR*OmegaR-1)));
[z,p,k] = cheb2ap(N,As);
ds = real(poly(p)); aNn = ds(N+1);
p = p*OmegaC; ds = real(poly(p)); aNu = ds(N+1);
cs = real(poly(z)); M = length(cs); bNn = cs(M);
z = z*OmegaC; cs = real(poly(z)); bNu = cs(M);
k = k*(aNu*bNn)/(aNn*bNu);
b0 = k; cs = k*cs;
% Chebyshev-2 Filter Order = 4
% Bilinear transformation:
[b,a] = bilinear(cs,ds,Fs);

%plota
w = [0:1:500]*pi/500; H = freqs(cs,ds,w);
mag = abs(H); db = 20*log10((mag+eps)/max(mag)); pha = angle(H);
figure;set(gcf, 'Position', [0, 0, 700, 700])
subplot(222);plot(w/pi,db);grid on;title('Magnitude(dB)');xlabel('Frequency in \pi units');ylabel('Decibels');
subplot(221);plot(w/pi,mag);grid on;title('Magnitude Response');xlabel('Frequency in \pi units');ylabel('|H|');
subplot(223);plot(w/pi,-pha/pi);grid on;title('Phase Response');xlabel('Frequency in \pi units');ylabel('Radians');
subplot(224);grpdelay(b,a);grid on;title('Group Delay')
```

PLOT



Magnitude Response
|H|
Frequency in π units
Magnitude(dB)
Decibels
Frequency in π units
Phase Response
Radians
Frequency in π units
Group Delay
Group delay (samples)
Normalized Frequency (×π rad/sample)

3.4 Assignment 04

Design a lowpass digital filter using a elliptic prototype to satisfy

$$\omega_p = 0.2\pi,\ \omega_s = 0.3\pi,\ A_p = 1dB, \text{ and } A_s = 15dB$$

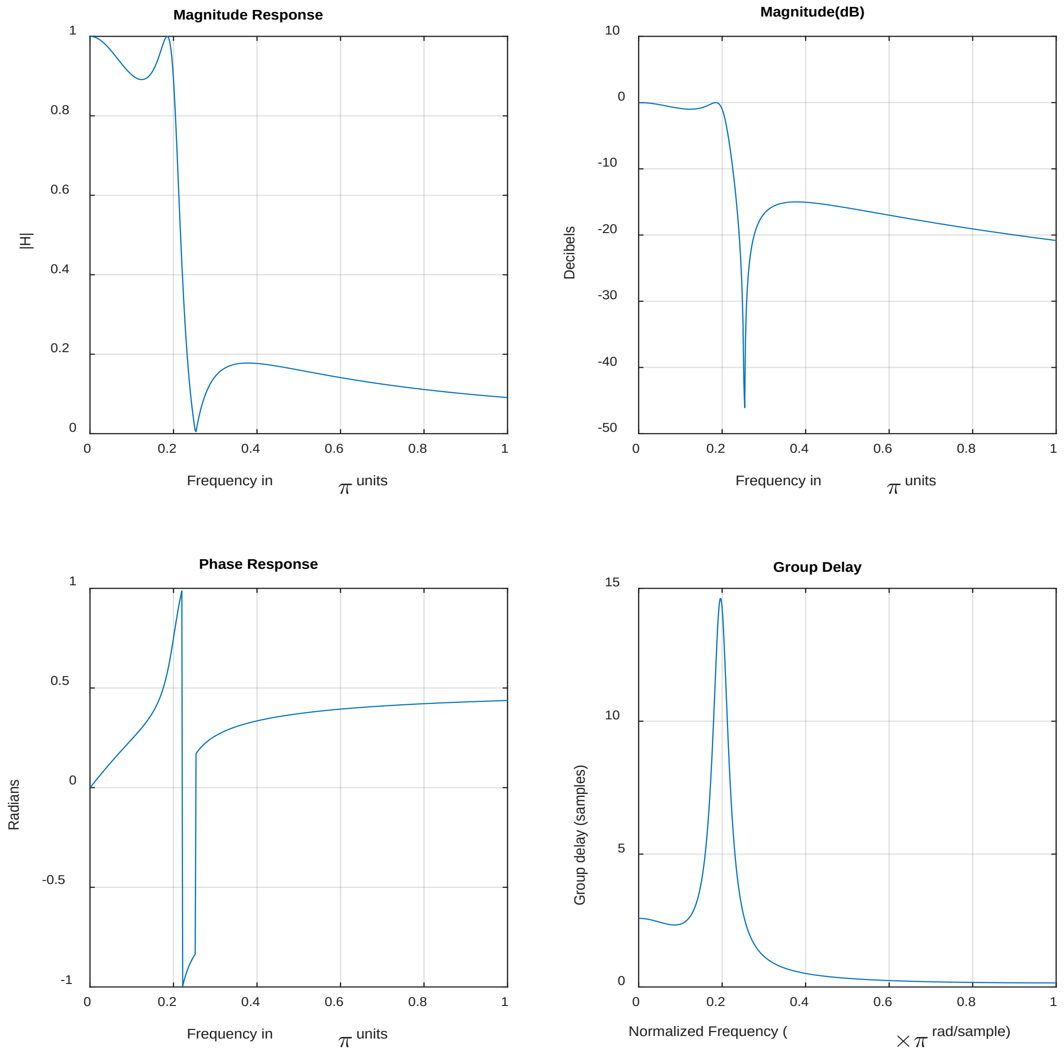
Solution

CODE

```
clc;clear;close all;
% Digital Filter Specifications:
Wp = 0.2*pi; % digital Passband freq in rad
Ws = 0.3*pi; % digital Stopband freq in rad
Rp = 1; % Passband ripple in dB
As = 15; % Stopband attenuation in dB
% Analog Prototype Specifications: Inverse mapping for frequencies
T = 1; Fs = 1/T; % Set T=1
OmegaP = (2/T)*tan(Wp/2); % Prewarp Prototype Passband freq
OmegaS = (2/T)*tan(Ws/2); % Prewarp Prototype Stopband freq
% Analog Elliptic Prototype Filter Calculation:
ep = sqrt(10^(Rp/10)-1); A = 10^(As/20);
OmegaC = Wp; k = Wp/Ws; k1 = ep/sqrt(A*A-1);
capk = ellipke([k.^2 1-k.^2]); % Version 4.0 code
capk1 = ellipke([(k1 .^2) 1-(k1 .^2)]); % Version 4.0 code
N = ceil(capk(1)*capk1(2)/(capk(2)*capk1(1)));
[z,p,k] = ellipap(N,Rp,As);
ds = real(poly(p)); aNn = ds(N+1);
p = p*OmegaC; ds = real(poly(p)); aNu = ds(N+1);
cs = real(poly(z)); M = length(cs); bNn = cs(M);
z = z*OmegaC; cs = real(poly(z)); bNu = cs(M);
k = k*(aNu*bNn)/(aNn*bNu);
b0 = k; cs = k*cs;
%Elliptic Filter Order = 3
% Bilinear transformation:
[b,a] = bilinear(cs,ds,Fs);

%plota
w = [0:1:500]*pi/500; H = freqs(cs,ds,w);
mag = abs(H); db = 20*log10((mag+eps)/max(mag)); pha = angle(H);
figure;set(gcf, 'Position', [0, 0, 700, 700])
subplot(222);plot(w/pi,db);grid on;title('Magnitude(dB)');xlabel('Frequency in \pi units');ylabel('Decibels');
subplot(221);plot(w/pi,mag);grid on;title('Magnitude Response');xlabel('Frequency in \pi units');ylabel('|H|');
subplot(223);plot(w/pi,-pha/pi);grid on;title('Phase Response');xlabel('Frequency in \pi units');ylabel('Radians');
subplot(224);grpdelay(b,a);grid on;title('Group Delay')
```

PLOT



Magnitude Response
|H|
Frequency in π units
Magnitude(dB)
Decibels
Frequency in π units
Phase Response
Radians
Frequency in π units
Group Delay
Group delay (samples)
Normalized Frequency (×π rad/sample)

3.5 Assignment 05

Compare the performances of all the filters designed in this assignment set. Write suitable discussion and draw a proper conclusion from the same.

Solution