Digital Filter Design (EET 3134)

Lab 03: Design of FIR Filters using Frequency Sampling Method

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1 Introduction

The windowing method requires an analytical expression for the desired frequency response $H_d(e^{j\omega})$, and the impulse response $h_d[n]$ is obtained from the inverse Fourier transform. However, in certain applications, the desired filter is specified by samples of its frequency response function without necessarily knowing an analytical expression for $H_d(e^{j\omega})$. In this lab session, we discuss filter design techniques based upon sampling the desired frequency response function.

2 Basic design approach

Suppose that we are given samples of a desired frequency response at L equally spaced points on the unit circle

$$H_d(k) = H_d(e^{j2\pi k/L}), k = 0, 1, 2, ..., L-1 \tag{1}$$

The desired impulse response $h_d(n)$, which is not available, may have finite or infinite duration. The inverse DFT of $H_d(k)$ is related to $h_d(n)$ by the aliasing relation

$$\tilde{h}(n) = \frac{1}{L}\sum_{k=0}^{L-1} H_d(k)W_N^{-kn} = \sum_{m=-\infty}^{\infty} h_d(n-mL) \tag{2}$$

which is a periodic sequence with fundamental period L. We can design an FIR filter by multiplying $\tilde{h}(n)$ with a window of length L, that is

$$h(n) = \tilde{h}(n)w(n) \tag{3}$$

Since $\tilde{h}(n)$ is periodic, the frequency response of the designed filter is obtained as following.

$$H(e^{j\omega})\frac{1}{L}\sum_{k=0}^{L-1} H_d(k)W(e^{j(\omega-2\pi k/L)}) \tag{4}$$

where $W(e^{j\omega})$ is the Fourier transform of the window.

Thus, the frequency response of the designed filter is obtained by interpolating between the samples $H_d(k)$ using $W(e^{j\omega})$ as an interpolation function. The interpolation process acts similarly to the frequency domain convolution in the windowing method. The windowing method creates an FIR filter by truncating $h_d(n)$; the frequency sampling method creates an FIR filter by aliasing or folding $h_d(n)$.

If $w(n)$ is an L-point rectangular window then $h(n)$ is the primary period of $\tilde{h}(n)$. In this case the approximation error is zero at the sampling frequencies, and finite between them because $W(e^{j\omega})$ is a Dirichlet (periodic sinc) function. For nonrectangular windows, the approximation error is not exactly zero at the sampling frequencies but is very close to zero, however, they have advantages in reducing passband and stopband ripples.

In practice, to minimize the time-domain aliasing distortion, we start with a large value for L in (1), and then we choose a much smaller value for the length of the window in (3)

3 Linear-phase FIR filter design

In general, the approach showed above results in FIR filters with arbitrary phase. To design FIR filters with linear phase we should incorporate the appropriate constraints into the design equations. To assure that the sequence $h_(n)$ is real and satisfies the linear phase constraints

$$h(n) = \pm h(L-1-n) \tag{5}$$

we should form the DFT coefficients $H_d(k)$ from the samples of the desired response very carefully. This is done using the following formulas.

$$\begin{aligned}
H_d(k) &= A_d(k)e^{\psi_d(k)} && (6)\\
A_d(k) &= \begin{cases} A_d(e^{j0})0 & k=0 \\ A_d(e^{j2\pi(L-k)/L}) & k=1,2,...,L \end{cases} && (7)\\
\psi_d(k) &= \begin{cases} \pm\frac{\pi}{2}q - \frac{L-1}{2}\frac{2\pi}{L}k & k=1,2,...,Q \\ \mp\frac{\pi}{2}q + \frac{L-1}{2}\frac{2\pi}{L}(L-k) & k=Q+1,Q+2,...,L-1 \end{cases} && (8)
\end{aligned}$$

where $Q = \frac{L-1}{2}$. The parameter $q = 0$ for type I–II FIR filters, and $q = 1$ for type III–IV FIR filters.

The impulse response $h(n)$ is obtained with a rectangular window.

$H_d(e^{j\omega})$ is zero at the sampling frequencies and the approximation error is larger near the sharp transition and is smaller away from it. This is because of the Gibbs phenomenon (due to a rectangular window) created by the sharp transition between passband and stopband. The maximum approximation error thus depends on the shape of the ideal frequency response; it is smaller for a smoother transition and vice versa.

4 Nonrectangular window design approach

As stated above in the basic design approach, we also can obtain the FIR filter impulse response $h(n)$ by multiplying the periodic $\tilde{h}(n)$ by a nonrectangular window $w(n)$. Although any window function discussed in Lab Assignment 01 can be used, the most popular are Hamming and Kaiser windows.

5 FIR filter design using Frequency Sampling Method

(a) Choose the order of the filter M by placing at least two samples in the transition band.

(b) For a window design approach obtain samples of the desired frequency response $H_d(k)$ using (6),(7) and (8).

(c) Compute the $(M+1)$-point IDFT of $H_d(k)$ to obtain $h(n)$. For a window design approach multiply $h(n)$ by the appropriate window function.

(d) Compute log-magnitude response $H_d(e^{j\omega})$ and verify the design over passband and stopband.

(e) If the specifications are not met, increase M and go back to step (a).

6 Prelab

Prior to beginning this lab, you must have carefully read over this lab in order to plan how to implement the tasks assigned. Make sure you are aware of all the items you need to include in your lab report.

7 Lab Assignments

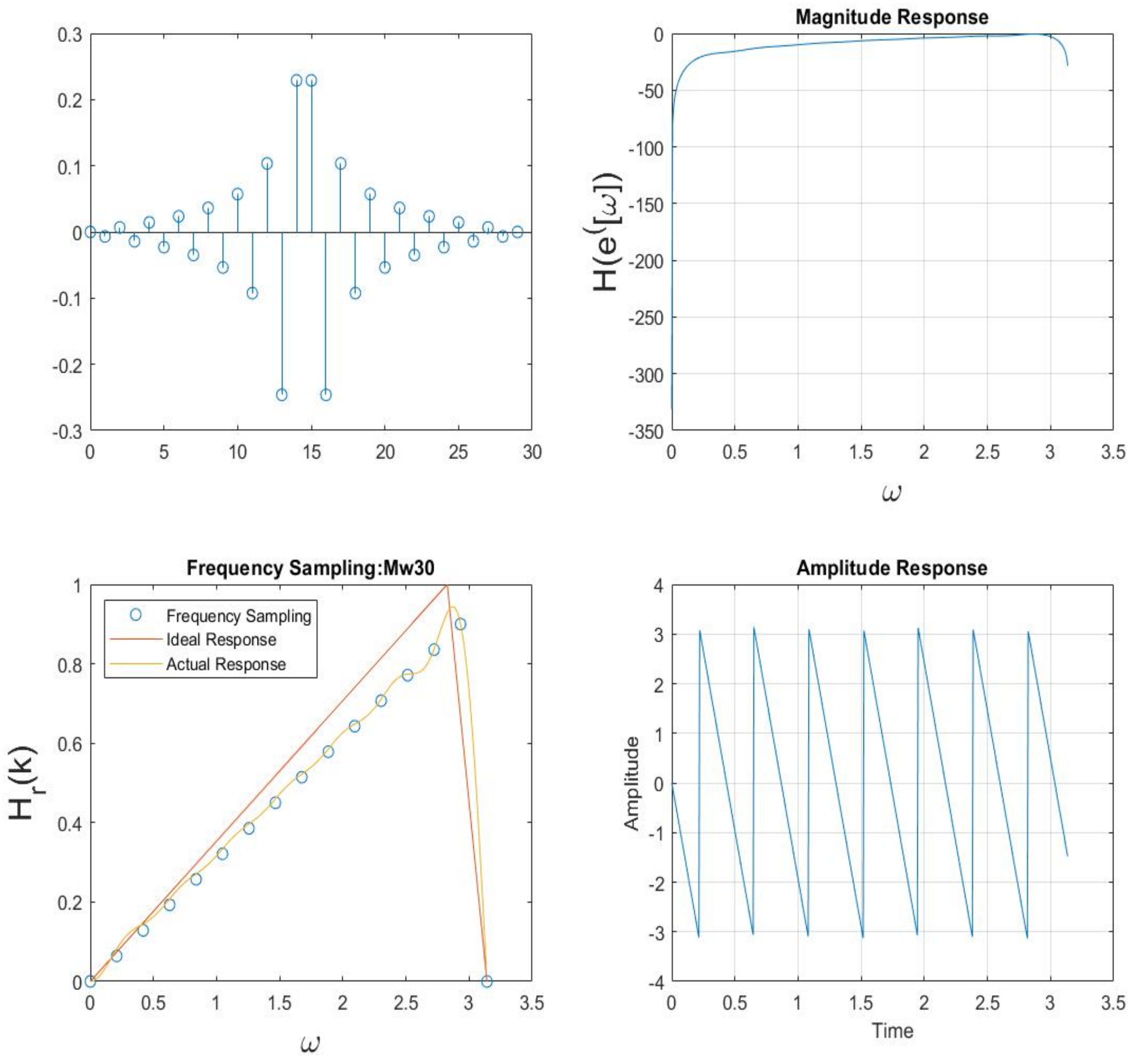
7.1 Assignment 01

An ideal differentiator has a frequency response given by $D(\omega) = j\omega$. Using the frequency sampling method, design a differentiator of order 30 that attenuates frequencies above 0.9π.

Solution

```
clc; close all; clear all;

%specification
N=30;
alpha=(N-1)/2;
l=0:N-1;
wl=(2*pi/N)*l;

%Design Parameters
Hrs=[linspace(0,0.9,15),zeros(1,1),linspace(0.9,0.1,14)];
Hdr=[0,1,0];
wdl=[0,0.9,1]*pi;
k1=0:floor((N-1)/2);
k2=floor((N-1)/2)+1:N-1;
angH= [-alpha*(2*pi)/N*k1, alpha*(2*pi)/N*(N-k2)];
H=Hrs.*exp(1j*angH);
h=real(ifft(H,N));

%PLOT
[MAG,W]=freqz(h,1);
subplot(221)
stem(0:(N-1),h)

subplot(223)
plot(wl(1:(floor(N/2)+1)),Hrs(1:(floor(N/2)+1)),'o', wdl,Hdr);
hold on
plot(W,abs(MAG))
title(['Frequency Sampling:Mw' ,num2str(N)]);
xlabel('\omega','fontsize',18);
ylabel('H_r(k)','fontsize',18);
legend('Frequency Sampling','Ideal Response','Actual Response');
```

```
subplot(222)
plot(W,20*log10(abs(MAG)));grid on
title('Magnitude Response')
xlabel('\omega','fontsize',18);
ylabel('H(e^([\omega])','fontsize',18);

subplot(224)
plot(W,angle(MAG));grid on
title('Amplitude Response')
xlabel('Time');
ylabel('Amplitude');
```



7.2 Assignment 02

Consider the lowpass filter with following specifications.

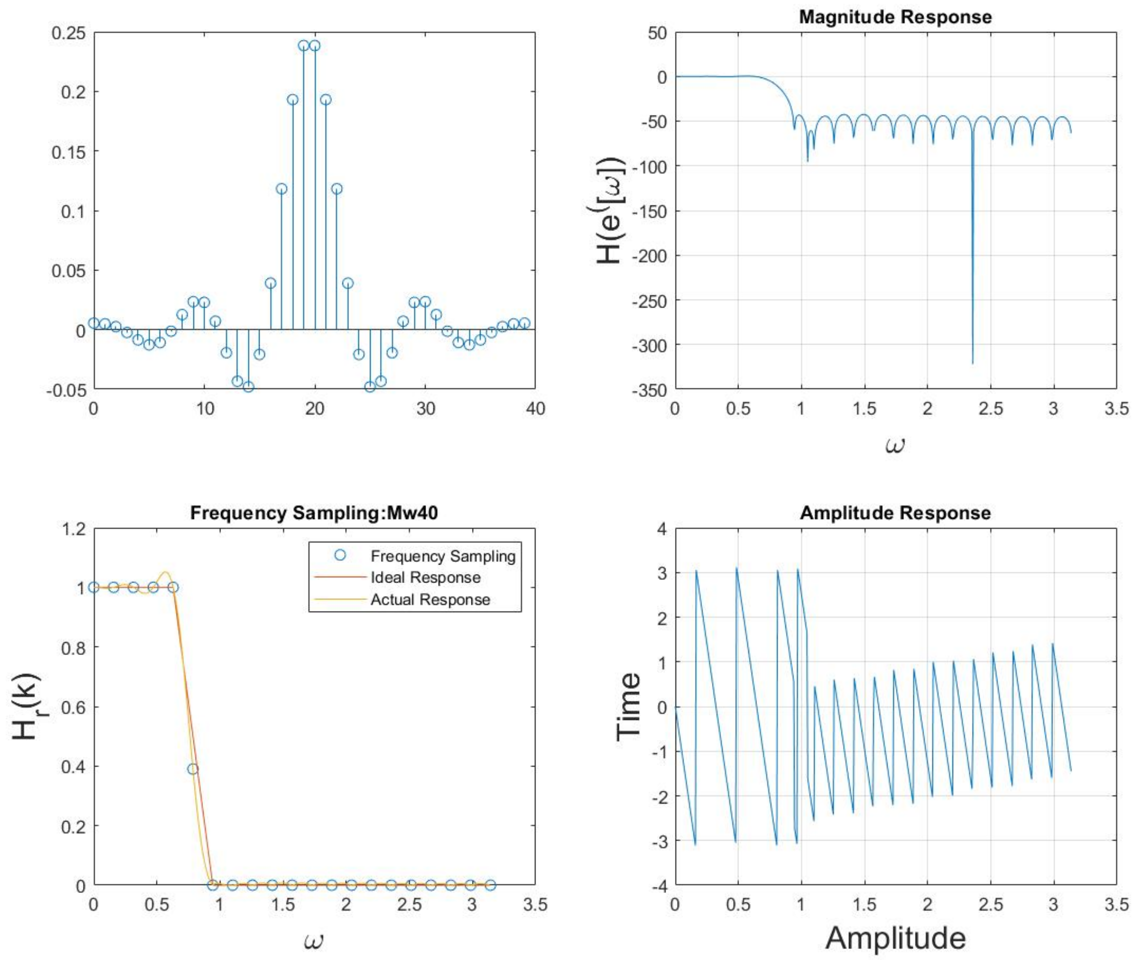
$$\omega_p = 0.2\pi, \omega_s = 0.3\pi, Ap = 0.25dB, As = 50dB$$

Design an FIR filter using the frequency sampling approach. Use a single transition band sample. Check whether the specifications are observed.

Solution

```
clc; clear all; close all;

%specification
wp=0.2*pi
ws=0.3*pi
Ap=-20*log10(0.25)
As=-20*log10(50)
TB=ws-wp;
delta_w=TB/2
N = 40; alpha = (N-1)/2;
L=0:N-1
Wl=(2*pi/N)*L
index=sum(((Wl-wp)>0)==0)

%Design Parameters
% T1 = 0.39

Hrs = [ones(1,5),0.39,zeros(1,29),0.39,ones(1,4)];
Hdr = [1,1,0,0];
Wdl = [0,0.2,0.3,1]*pi;
k1 = 0:floor((N-1)/2); k2 = floor((N-1)/2)+1:N-1;
angH = [-alpha*(2*pi)/N*k1, alpha*(2*pi)/N*(N-k2)];
H = Hrs.*exp(j*angH); h = real(ifft(H,N));

%PLOTS
[MAG,W]=freqz(h,1);
subplot(221)
stem(0:(N-1),h)
subplot(223)
plot(Wl(1:(floor(N/2)+1)),Hrs(1:(floor(N/2)+1)),'o',
Wdl,Hdr);
hold on
plot(W,abs(MAG))
title(['Frequency Sampling:Mw' ,num2str(N)]);
```

```
xlabel('\omega','fontsize',18);
ylabel('H_r(k)','fontsize',18);
legend('Frequency Sampling','Ideal Response','Actual Response');

subplot(222)
plot(W,20*log10(abs(MAG)));grid on
title('Magnitude Response')
xlabel('\omega','fontsize',18);
ylabel('H(e^([\omega])','fontsize',18);

subplot(224)
plot(W,angle(MAG));grid on
title('Amplitude Response')
xlabel('Amplitude','fontsize',18);
ylabel('Time','fontsize',18);
```



7.3 Assignment 03

Consider the lowpass filter with following specifications.

$$\omega_p = 0.2\pi, \omega_s = 0.3\pi, Ap = 0.25dB, As = 50dB$$

Design an FIR filter using the frequency sampling approach. Use two transition band samples. Check whether the specifications are observed.

Solution

```
clc;clear;close all;

%specification
wp=0.2*pi
ws=0.3*pi
Ap=-20*log10(0.25)
As=-20*log10(50)
TB=ws-wp;
delta_w=TB/2
M = 60;
L=0:M-1
wl = (2*pi/M)*L;
alpha = (M-1)/2; l = 0:M-1;
index=sum(((wl-wp)>0)==0)

Hrs = [ones(1,7),0.5925,0.1099,zeros(1,43),0.1099,0.5925,ones(1,6)];
Hdr = [1,1,0,0]; wdl = [0,0.2,0.3,1]*pi;
k1 = 0:floor((M-1)/2); k2 = floor((M-1)/2)+1:M-1;
angH = [-alpha*(2*pi)/M*k1, alpha*(2*pi)/M*(M-k2)];
H = Hrs.*exp(j*angH);
h = real(ifft(H,M));
[MAG,W]=freqz(h,1);

subplot(221)
stem(0:(M-1),h)

subplot(223)
plot(wl(1:(floor(M/2)+1)),Hrs(1:(floor(M/2)+1)),'o', wdl,Hdr);
hold on
plot(W,abs(MAG))
title(['Frequency Sampling:Mw' ,num2str(M)]);
xlabel('\omega','fontsize',18);
ylabel('H_r(k)','fontsize',18);
```

```
egend('Frequency Sampling','Ideal Response','Actual Response');

subplot(222)
plot(W,20*log10(abs(MAG)));grid on
title('Magnitude Response')
xlabel('\omega','fontsize',18);
ylabel('H(e^([\omega])','fontsize',18);

subplot(224)
plot(W,angle(MAG));grid on
title('Amplitude Response')
xlabel('Amplitude','fontsize',18);
ylabel('Time','fontsize,'18');
```