Digital Filter Design (EET 3134)

Lab 04: Design of FIR Filters using Least Squares Method

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1 Introduction

FIR filter designs may not be desirable if we want to minimize the energy of the error (between ideal and actual filter) in the passband/stopband. Consequently, if we want to reduce the energy of a signal as much as possible in a certain frequency band, least-squares designs are preferable. We now look at a FIR filter design technique that is based on "experimental" data.

Given an input sequence x(n) and a "desired" filtered output sequence d(n) the task is to design a FIR filter

$$H(z) = \sum_{k=0}^{L-1} h(n) z^{-n} \tag{1}$$

that will minimize the error $e(n) = d(n) - y(n)$ in some sense, where $y(n)$ is the filter output.

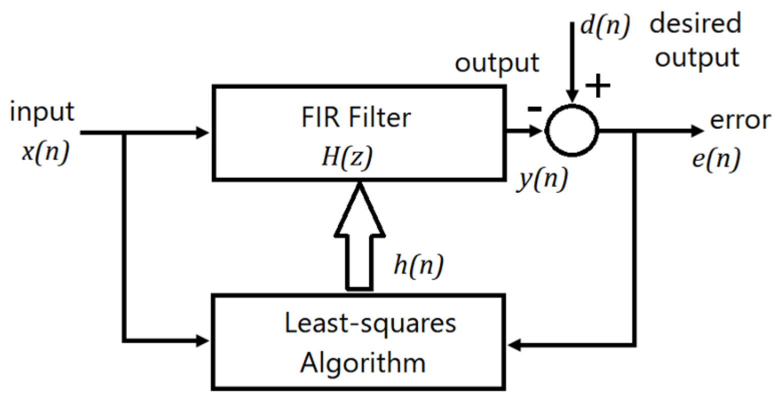


Figure 1: The least-squares filter design technique

In particular, we will look at a filter design method that minimizes the mean-squared-error (MSE), where

$$\begin{aligned} MSE &= \varepsilon[e(n)^2] && (2)\\ &= \varepsilon[(d(n) - y(n))^2] && (3)\\ &= \varepsilon[d(n)^2] + \varepsilon[y(n)^2] - 2\varepsilon[d(n)y(n)] && (4) \end{aligned}$$

In terms of correlation functions

$$MSE = \phi_{dd}(0) + \phi_{yy}(0) - 2\phi_{dy}(0) \tag{5}$$

Solving for solution of filter coefficients h(n) from Equation 6, we get

$$\mathbf{h} = \mathbf{R}^{-1}\mathbf{P} \tag{6}$$

Where, $\mathbf{R}$ is the auto correlation matrix,

$$\mathbf{R} = \begin{bmatrix} \phi_{xx}(0) & \phi_{xx}(1) & \phi_{xx}(2) & \dots & \phi_{xx}(L-1) \\ \phi_{xx}(1) & \phi_{xx}(0) & \phi_{xx}(1) & \dots & \phi_{xx}(L-2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \phi_{xx}(L-1) & \phi_{xx}(L-2) & \phi_{xx}(L-3) & \dots & \phi_{xx}(0) \end{bmatrix} \tag{7}$$

and $\mathbf{P}$ is the cross-correlation vector.

$$\mathbf{P} = \begin{bmatrix} \phi_{xd}(0) & \phi_{xd}(1) & \phi_{xd}(2) & \dots & \phi_{xd}(L-1) \end{bmatrix}^T \tag{8}$$

$\mathbf{h}$ is vector containing the filter coefficients,

$$\mathbf{h} = [h(0), h(1), \dots, h(N-1)]^T \tag{9}$$

Stearns and Hush show that with these coefficients

$$(MSE)_{min} = \phi_{dd}(0) - \mathbf{P}^T\mathbf{h} = \phi_{dd}(0) - \mathbf{P}^T\mathbf{R}^{-1}\mathbf{P} \tag{10}$$

Here

- To evaluate $\mathbf{R}^{-1}$ one needs to compute approximately L^2 number of multiplications. This makes the least square method computationally complex $(O(n^2))$.
- Since $\mathbf{R}$ is a Toeplitz matrix , and efficient algorithms exist for its inversion (e.g., Levinson-Durbin algorithm – see Proakis and Manolakis - 3rd edition, Sec. 12.4.1).
- The development above requires that the processes $x(n)$ and $d(n)$ are stationary.

2 Prelab

Prior to beginning this lab, you must have carefully read over this lab in order to plan how to implement the tasks assigned. Make sure you are aware of all the items you need to include in your lab report.

3 Lab Assignments

3.1 Assignment 01

Write a MATLAB function to perform FIR filter design using least squares method.The function should have the following syntax.

```
function [h,MSE] = LSQFilt(f,d,M)
 %LSQFilt - Demonstration routine for Least-Squares FIR filter design
    %[h,MSE] = LSQFilt(f,d,M)
    % f - rowvector of data samples -length L
    % d - row vector of desired values -length L
    % M - filter order
 % Returns:
    % h - vector of optimal filter coefficients
    % MSE - minimized value of the mean-square-error
```

Solution

function [h,MSE] = LSQFilt(f,d,M)
%LSQFilt -Demonstration routine for Least-Squares FIR filter design %[B,MSE] = LSQFilt(f,d,M)
% f -rowvector of data samples -length N
% d -row vector of desired values -length N
% M -filter order % Returns:
% h -vector of optimal filter coefficients
% MSE -minimized value of the mean-square-error
N = length(f);
xcrf=xcorr(f);
xcrd=xcorr(d,f);
rff=xcrf(N:N+M-1);
R = toeplitz(rff);
P=xcrd(N:N+M-1);
h=inv(R)*P';
phidd=xcorr(d);
MSE=phidd(N)-P*h;

3.2 Assignment 02 : The Linear Predictor

Solve the problem of a one-step linear predictor for an sinusoidal input function

$$s_n = sin\left(\frac{2\pi n}{12}\right), n = 0, 1, 2, ... \tag{11}$$

and show that $\mathbf{h} = [\sqrt{3}, -1]$

Solution

```
clc;clear;close all;
t = 0:199;
s = sin(2*pi*t/12);
d = s;
f = zeros(1,200);
f(2:200) = s(1:199);

[h1,MSE] = LSQFilt(f,d,2)

[h2,MSE] = LSQFilt(f,d,3)
```

```
Output-
h1 =

   1.7321
  -1.0000


MSE =

   0.2500


h2 =

   1.7321
  -1.0000
   0.0000


MSE =

   0.2500
```

3.3 Assignment 03 : System Identification

Suppose we have an "unknown" FIR system and the task is to determine its impulse response. We can construct an experiment, using white noise to excite the system and record the input and output series. The least-squares filter design method is then used to determine the coefficients of a FIR are used as estimates of the plant impulse response.

Use a least-squares filter to estimate the impulse response of an "unknown" FIR system with impulse response as given below.

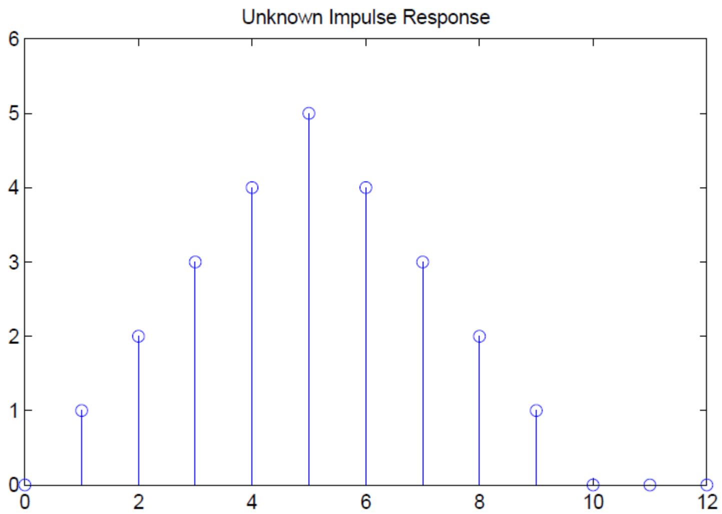
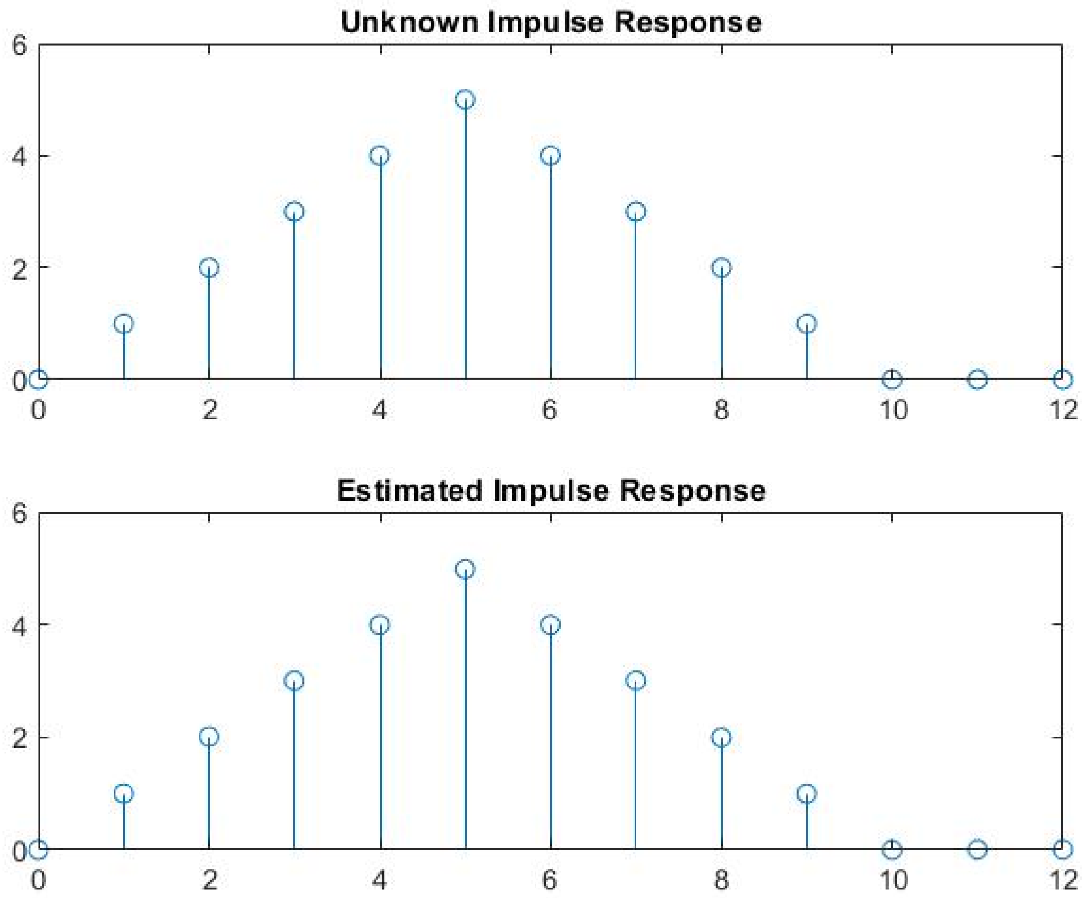


Figure 2: System response for Assignment 03

Solution

```
clc;clear;close all;
h=[0,1,2,3,4,5,4,3,2,1,0,0,0];
f = randn(1,1000);
y = filter(h,1,f);
[h_opt,MSE] = LSQFilt(f,y,15);
subplot(211);
stem(0:length(h)-1,h);
title('Unknown Impulse Response');
subplot(212);
stem(0:length(h)-1,h_opt(1:length(h)));
title('Estimated Impulse Response');
```



Unknown Impulse Response
6
4
2
0
0
2
4
6
8
10
12
Estimated Impulse Response
6
4
2
0
0
2
4
6
8
10
12