<u>EXPERIMENT-5</u>

AIM: Analysis of Signals & Systems with Laplace transforms.

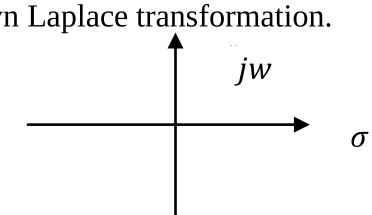
OBJECTIVE:

- Determination of the Laplace transform and ROC of a system/signal.
- Determination of the location of poles and zeros of a system
- Determination of the system stability.
- Determination of inverse Laplace transforms.
- Finding out the response of an arbitrary system to various inputs.

Pre Lab:

Time domain analysis is insufficient to give all the information about the signal. Hence we look for frequency domain analysis achieved by Fourier transform (FT). But problem is FT is achieved on jw-axis only. There are some signals for which Fourier transform does not exist, because they are not absolute integrable. Those signals are expressed as sum of complex exponentials (e^{st}) where s = σ+jw. So they are not restricted to imaginary axis. For these signals we introduce a convergence factor $e^{-\sigma t}$, so that the resulting frequency domain signal $x(t)e^{-jwt}$ $conver$ges i.e FT exist for these signals for appropriate value of 'σ'. and the signal form became $x'(t)$ = $x(t)e^{-\sigma t}e^{-jwt}$. This transformation is known Laplace transformation.

$$x'(t) = x(t)e^{-(\sigma+jw)t} = x(t)e^{-st}$$



So any point in s-plane can be interpret in the form of $\sigma + jw$.

→ This Laplace transform can be applied to analysis of many unstable systems and plays a great role in the investigation of the stability or instability of system. It is of two types

a. Bilateral or two sided.
b. Unilateral or one sided.

<u>Bilateral Laplace transformation</u>:

$$\boxed{L[x(t)] = x(s) = \int_{-\infty}^{\infty} x(t)e^{-st} \cdot dt}$$

Unilateral Laplace transformation :

$$L[x(t)] = x(s) = \int_{0^-}^{\infty} x(t)e^{-st} \cdot dt$$

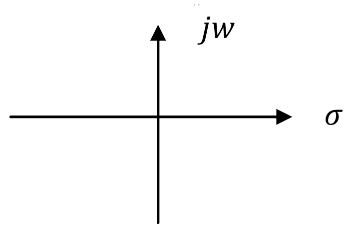
$x(s)\ exist.\ for\ t \geq 0\ \&\ x(s) = 0\ for\ t < 0$

So, it is applicable for casual system.

Inverse Laplace transformation

$$x(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} x(s)e^{st} \cdot dt$$

- It is convenient to represent the complex frequency S=$(\sigma + jw)$ in a complex plane called S-Plane.



- Jw-axis divided the plane in two equal halves. Left half $(\sigma = -ve)$ is known as left half of S-plane & right half $(\sigma = +ve)$ is known as Right half of S-plane.
- The transfer function of a LTI System in Laplace domain is represented as
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$$H(s) = \frac{bms^3 + b_{m-1}s^{m-1} + ----+b_1s + b_0}{a_ns^n + a_{n-1}s^{n-1} + ----+a_1s + a_0} = \frac{N(s)\ (Numerator)}{D(s)\ Denominator}$$

On factorization both Numerator & Denominator

$$H(s) = \frac{N(s)}{D(s)} = K\frac{(S-Z_1)(S-Z_2)----(S-Z_{m-1})(S-Z_m)}{(S-P_1)(S-P_2)----(S-P_{n-1})(s-p_n)}$$

Where K = $\frac{b_m}{a_n}$ (constant Gain)

- Now it is observed that $'Z_i'$ are roots of equation N(s)=0 & P_i are the roots of the equation D(s)=0.
  Also,

$$\boxed{\lim_{s \to Z_i} H(s) = 0}$$

Hence all Z_i are the Zeros of the LTI system.

$$\boxed{\lim_{s \to Z_i} H(s) = 0}$$

Hence all P_i are the poles of the LTI system.

- So poles & zeros are either Real or appear in complex conjugate pair.
- The location of Zeros in S-plane is denoted with the symbol 'o' & Poles with the symbol 'X'.
- If all the poles lie in the left half of S-Plane then the given LTI system is a stable system.

In Lab:

1. Find the Laplace transform of $y(t) = \sin(2\pi t)u(t) - \sin\big(2\pi(t-1)\big)\, u(t-1)$ and its region of convergence.
2. Determine the location of poles and zeros of a system $H(s) = \frac{s^2+2s+1}{s(s+1)(s^2+10s+50)}$. Also check it's stability?
3. Find out the impulse response of the system given in Q.2.
4. The impulse response of the system is $h(t) = u(t) - u(t-1)$. Find the response of the system to following inputs.
   (a) $x(t) = u(t) - u(t-1)$.
   (b) $x(t) = e^{-4t}u(t)$.

   Plot all the inputs and responses. Use Laplace transform.

PROGRAM:-

RESULTS: -

Post Lab:

Let $Y(s) = \frac{X(s)}{s^2+2s+1} + \frac{s+1}{s^2+2s+1}$ be the Laplace transform of the solution of a second order differential equation representing a system with input $x(t)$ and some initial conditions. Using MATLAB

(a) Find the zero-state response for $x(t) = u(t)$.
(b) Find the zero-input response.
(c) Find the complete response when $x(t) = u(t)$ **.**
(d) Find the transient and steady-state response when $x(t) = u(t)$.